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Allocative efficiency measurement revisited—
Do we really need input prices?

By Oleg Badunenko, Michael Fritsch and Andreas Stephan*

January 16, 2006

Abstract

The traditional approach to measuring allocative efficiency is based on input prices, which are rarely known at the firm level. This paper proposes a new approach to measure allocative efficiency which is based on the output-oriented distance to the frontier in a profit–technical efficiency space and which does not require information on input prices. To validate the new approach, we perform a Monte-Carlo experiment which provides evidence that the estimates of the new and the traditional approach are highly correlated. Finally, as an illustration, we apply the new approach to a sample of about 900 enterprises from the chemical industry in Germany.

JEL Classification: D61, L23, L25, L65

Keywords: allocative efficiency, data envelopment analysis, frontier analysis, technical efficiency, Monte-Carlo study, chemical industry

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1 Introduction

A significant number of empirical studies have investigated the extent and determinants of technical efficiency within and across industries (see Alvarez and Crespi 2003, Caves and Barton 1990, Gumbau-Albert and Joaquín 2002, Green and Mayes 1991, and Fritsch and Stephan 2004a). Comprehensive literature reviews of the variety of empirical applications are made by Lovell 1993 and Seiford 1996, 1997). Compared to this literature, attempts to quantify the extent and distribution of allocative efficiency are relatively rare (see Greene 1997).\(^1\) This is quite surprising since allocative efficiency has traditionally attracted the attention of economists: what is the optimal combination of inputs so that output is produced at minimal cost? How much could the profits be increased by simply reallocating resources? To what extent does competitive pressure reduce the heterogeneity of allocative inefficiency within industries?\(^2\) A firm is said to have realized allocative efficiency if it is operating with the optimal combination of inputs given prices of inputs. The traditional approach to measuring allocative efficiency requires input prices (see Atkinson and Cornwell 1994, Greene 1997, Kumbhakar 1991, Kumbhakar and Tsionas 2005, and Oum and Zhang 1995) which are hardly available in reality.\(^3\) This explains why empirical studies of allocative efficiency are highly concentrated on certain industries, particularly banking, because information on input price can be obtained for these industries.

This paper introduces a new approach to estimating allocative efficiency, which is solely based on quantities and profits and does not require information on input prices. An indicator for allocative efficiency is derived as the output-oriented distance to a frontier in a profit–technical efficiency space. What is, however, needed is an assessment of input-saving technical efficiency; \textit{i.e.}, how less input could be used to produce given outputs.

The paper proceeds as follows: Section 2 theoretically derives a new method for estimating allocative efficiency and introduces a theoretical framework for activity analysis models. Section 3 presents the results of the Monte-Carlo experiment on comparison of...
allocative efficiency scores calculated using both traditional and new approaches. Section 4 provides a rationale and a simple illustration using the new approach; Section 5 concludes.

2 Measurement of Allocative Efficiency

2.1 Traditional Approach to Allocative Efficiency

A definition of technical and allocative efficiency was made by Farrell 1957. According to this definition, a firm is technically efficient if it uses the minimal possible combination of inputs for producing a certain output (input orientation). Allocative efficiency, or as Farrell called it price efficiency, refers to the ability of a firm to choose the optimal combination of inputs given input prices. If a firm has realized both technical and allocative efficiency, it is then cost efficient (overall efficient).

![Diagram](attachment:figure1.png)

**Figure 1:** Measurement and decomposition of cost efficiency
Figure 1, similarly to Kumbhakar and Lovell 2000, shows firm A producing output $y^A$ represented by the isoquant $L(y^A)$. Dotted lines are the isocosts which show level of expenditures for a certain combination of inputs. The slope of the isocosts is equal to the ratio of input prices, $w(w_1, w_2)$. If the firm is producing output $y^A$ with the factor combination $x^A$ ($a$ in Figure 1), it is operating technically inefficient. Potentially, it could produce the same output contracting both inputs $x_1$ and $x_2$ (available at prices $w$), proportionally (radial approach); the smallest possible contraction is in point $b$, representing $(\theta x^A)$ a factor combination. Having reached this point, the firm is considered to be technically efficient. Formally, technical efficiency is measured by the ratio of the current input level to the lowest attainable input level for producing a given amount of output. In terms of Figure 1, technical inefficiency of unit $x^A$ is given by

$$TE(y^A, x^A) = \theta = \frac{w(\theta x^A)}{wx^A}$$

or geometrically by $ob/oa$. The measure of cost inefficiency (overall efficiency) is given by the ratio of potentially minimal cost to actual cost:

$$CE(y^A, x^A, w^A) = \frac{wx^E}{wx^A}$$

or geometrically by $oc/oa$. Thus, cost inefficiency is the ratio of expenditures at $x^E$ to expenditures at $x^A$ while technical efficiency is the ratio of expenditures at $(\theta x^A)$ to expenditures at $x^A$. The remaining portion of the cost efficiency is given by the ratio of expenditures at $x^E$ to expenditures at $(\theta x^A)$. It is attributable to the misallocation of inputs given input prices and is known as allocative efficiency:

$$AE = \frac{CE}{TE} = \frac{wx^E/wx^A}{w(\theta x^A)/wx^A}$$

or in terms of Figure 1 is given by $oc/ob$. 

3
2.2 A New Approach to Allocative Efficiency

When input prices are available, allocative efficiency in the pure Farrell sense can be calculated using, for example, a non-parametric frontier approach Färe, Grosskopf and Lovell 1994 or a parametric one (Greene 1997, among others). However, if input prices are not available these approaches are not applicable. In contrast to this, the new approach we propose allows measuring allocative efficiency without information on input prices. An estimate of allocative efficiency can be obtained with the new approach that is solely based on information on input and output quantities and on profits.

The first step of this new approach involves the estimation of technical efficiency; whereby, in the second step allocative efficiency is estimated as an output-oriented distance to the frontier in a profit–technical efficiency space.

Proposition 1 Existence of the frontier in profit–technical efficiency space

A profit maximum exists for any level of technical efficiency.

In Figure 2, three firms, A, B, and C using inputs $x_A$, $x_B$ and $x_C$, available at prices $w$, produce output $y_A$, which is measured by the isoquant $L(y_A)$. For the sake of argument, firms A, B, and C are all equally technically efficient (the level of technical efficiency $\theta$, however, is arbitrarily chosen) which is read from expenditure levels at $(\theta x_A)$, $(\theta x_B)$, and at $(\theta x_C)$, respectively. In geometrical terms $ob_A/oa_A = ob_B/oa_B = ob_C/oa_C$. The costs of these three firms are determined by $wx_A$, $wx_B$, and by $wx_C$. The isocost corresponding to expenditures at $x_C$ is the closest possible to the origin $o$ for this level of technical efficiency and, therefore, implies the lowest level of cost. This is because $x_C$ is the combination of inputs lying on the ray from origin and going through the tangent point of the isocost (corresponding to expenditure level of $wx_E$) to the isoquant $L(y_A)$. This implies that for $\theta$-level of technical efficiency costs have a lower bound and using the fact that firms are producing the same output $y_A$, profits have an upper bound. Without loss of generality, for each level $\theta$ of technical efficiency there is a profit maximum, which proves the existence of a frontier in profit–technical efficiency space.
Remark 1 Frontier in profit–technical efficiency space is sloped upwards

In Figure 3, two firms, C and D, use inputs \( x^C \) and \( x^D \) to produce output \( y^A \), which is measured by the isoquant \( L(y^A) \). Both firms are allocatively efficient because they lie on the same ray from the origin that goes through the tangent point \( x^E \); thus, in terms of Proposition 1 we only look at the frontier points. These firms operate, however, at different levels of technical efficiency \( \theta^C \) and \( \theta^D \), respectively. Since the isocost representing the level of expenditure \( wx^C \) is closer to the origin than that of the expenditure level \( wx^D \), costs of firm C are smaller than those of firm D and firm C is more profitable than firm D. Since \( \frac{ob^C}{oa^C} > \frac{ob^D}{oa^D} \), \( \theta^C > \theta^D \), larger technical efficiency is associated with larger profits for points forming the frontier in profit–technical efficiency space. This proves that such frontier is upward sloping.

Proposition 2 The higher the allocative efficiency the higher the profit For any arbitrarily chosen level of technical efficiency, the closer the input combination to the
optimal one (i.e., the larger the allocative efficiency) the larger the profit will be.

Equation 3 suggests that in terms of Figure 2 (all three firms are equally technically efficient) expenditures solely depend on allocative efficiency. Moreover, the smaller the allocative efficiency the larger the expenditure. Keeping in mind that these firms produce the same output $y^A$, we conclude that for $\theta$-level of technical efficiency (again chosen arbitrarily) the larger the allocative efficiency the lower the costs and the larger the profit is; as allocative efficiency reaches its maximum (for firm C), the maximal profit is also achieved. Without loss of generality, this statement is true for any level of technical efficiency.

Proposition 3 Allocative efficiency in profit–technical efficiency space Output-oriented distance to the frontier in profit–technical efficiency space measures allocative efficiency.
In Figure 4, the frontier is the locus of the maximum attainable profits as defined in Proposition 1. The firms A, B, and C have the same technical efficiency level $TE^0$; however, they have different profit levels: $p_1$, $p_2$, and $\bar{p}$, respectively. The potential level of profit which firms can reach is $\bar{p}$. The closer the observation is to the frontier, the larger the profit is. As we recall from Figure 2, the shift from firm A to firm C is only possible when the input-mix is changed; i.e., allocative efficiency is improved. Thus, in Figure 4 the shift from firm A to firm B means an increase in allocative efficiency (distance $AE^A$ is larger than distance $AE^B$), and further increase in allocative efficiency within the same level of technical efficiency is only possible up to firm C’s observation, for which both profit and allocative efficiency are at the maximum. Thus, which is most remarkable, a vertical distance from the observation to the frontier serves as a measure of the allocative efficiency.

To summarize, we have defined a new way of estimating allocative efficiency, specif-
ically, this is the output-oriented distance to the frontier in profit–technical efficiency space.

3 Monte-Carlo simulation

To analyze whether our new approach to measuring allocative efficiency yields valid estimates, we conducted several Monte-Carlo experiments. According to a micro-economic theory, a firm which chooses such a combination of inputs, that their ratio cost shares is equal to the ratio of output elasticities of the respective inputs, will be most profitable. When we speak of optimal combination of inputs, the original notion of allocative efficiency comes into play, and we suggest that the closer the cost share ratio of inputs to the ratio of elasticities the larger a firm’s allocative efficiency will be.

3.1 Empirical implementation of the traditional approach

The traditional approach can be used when input prices are known. Under technology $T$ such that

$$T = \{(x, y) \text{ can produce } y\}$$

we measure input-oriented technical efficiency as the greatest proportion that the inputs can be reduced and still produce the same outputs:

$$F^i(y, x) = \inf \{\lambda: \lambda x \text{ can still produce } y\}$$

We employ the Data Envelopment Analysis (DEA) all the way through the empirical estimation. For $K$ observations, $M$ outputs, and $N$ inputs an estimate of the Farrell Input-Saving Measure of Technical Efficiency can be calculated by solving a linear programming problem for each observation $j$ ($j = 1, \ldots, K$):

$$\hat{\text{TE}}_j = \hat{F}^i_j(y, x|C) = \min \left\{\lambda: \sum_{k=1}^K z_k y_{km} \geq y_j, \sum_{k=1}^K z_k x_{kn} \leq x_j \lambda, z_k \geq 0\right\}$$
for \( m = 1, \ldots, M \) and \( n = 1, \ldots, N \). Note that superscript \( i \) stands for input orientation while \( C \) denotes constant returns-to-scale. Other returns-to-scale are modeled adjusting process operating levels \( z_k \)s (see Färe and Primont 1995 for details).

When input prices and quantities are given we can calculate the total costs and the minimum attainable cost (solve linear programming problem) and then compute an estimate of cost efficiency for each observation \( j \) \( (j = 1, \ldots, K) \) as in equation (2):

\[
\hat{C}_i^j(y,x,w|C) = \min \left\{ \frac{\sum_{n=1}^N w_{jn}x_{jn}}{\sum_{n=1}^N w_{jn}x_{jn}} : \sum_{k=1}^K z'_k y_{km} \geq y_j, \sum_{k=1}^K z'_k x_{kn} \geq x_j, z'_k \geq 0 \right\}
\]

for \( m = 1, \ldots, M \) and \( n = 1, \ldots, N \). We refer to the residual of technical and cost efficiencies as Input Allocative Efficiency, which can be computed for each observation \( j \) \( (j = 1, \ldots, K) \) as:

\[
\hat{AE}_i^j(y,x,w|C) = \frac{\hat{C}_i^j(y,x,w|C)}{\hat{EF}_j^i(y,x,w|C)}
\]

### 3.2 Empirical implementation of the new approach

As mentioned above, the main virtue of the new approach is that we do not necessarily need input prices for measuring allocative efficiency. Technically, we need output-oriented distances to the frontier in the profit–technical efficiencies space. We take advantage of the technical efficiency estimates (denoted by TE) obtained as in equation (6) and profitability measure (denoted by Pr) to calculate (solve linear programming problem) allocative efficiency for each observation \( j \) \( (j = 1, \ldots, K) \) as follows:

\[
\hat{AE}_j^i(y,x|C) = \max \left\{ \theta : \sum_{k=1}^K z''_k Pr_k \geq Pr_j \theta, \sum_{k=1}^K z''_k TE_k \leq TE_j, z''_k \geq 0 \right\}
\]

### 3.3 Design of the Monte-Carlo experiments

In each of the Monte-Carlo trials, we study a production process which uses two inputs to produce one output. Data for the \( i^{\text{th}} \) observation in each Monte-Carlo experiment were
generated using the following algorithm.

(i). We chose output elasticities of two inputs to be 0.2 and 0.8.

(ii). Draw $x_1 \sim (\phi + \lambda \cdot \text{uniform})$; uniform on the interval [0;1].

(iii). Draw $r \sim \text{uniform}$; uniform on the interval [0;8]. This is meant to be an experimental ratio of used inputs.

(iv). Set $x_2 = rx_1$.

(v). Choose $\epsilon$. In doing so, we allow the ratio of inputs in each Monte-Carlo trial to vary on the interval $[\epsilon; 8 - \epsilon]$. Therefore, we obtain enough variation of inefficient combinations of inputs, or in other words, enough variation of allocative inefficiency.

(vi). Draw $u \sim N^+(0, \sigma_u^2)$ and set ‘te\_drawn’ equal $\exp(-u)$.

(vii). Generate output data assuming trans-log production function, which will contain inefficiency component:\footnote{5}

$$y_i = 0.2x_{1i} + 0.8x_{2i} + \gamma_{11}x_{1i}^2 + \gamma_{22}x_{2i}^2 + \frac{1}{2}\gamma_{12}x_{1i}x_{2i} + \text{te\_drawn}_i, \quad i = 1, \ldots, N \quad (10)$$

The chosen parameter values ensure homogeneity of degree one. We run the simulations with $N = 100$ and with $N = 400$.

(viii). Draw price of input $x_1$: $w_1 \sim (\varphi + \psi \cdot \text{uniform})$, uniform on the interval [0;1]. The price of input $x_2$ is calculated as $w_2 = \theta w_1$—we want to keep the ratio of input prices constant to have the isoquants parallel (recall Figure 2).

(ix). Set profit as output (we set output price equal to 1) minus cost and this is divided by output.

(x). DEA traditional allocative efficiency as in equation (8).
(xi). DEA our measures of allocative efficiency using technical efficiency drawn in step (vi) as in equation (9).

(xii). Solve for technical efficiency as in equation (6), and DEA our measure of allocative efficiency using these solved technical efficiency scores.

(xiii). Calculate rank correlation coefficient between allocative efficiency estimates based on traditional and our approaches.

(xiv). Repeat steps (i) through (xiii) L times.

In each of our experiments we set $\phi = 1$, $\lambda = 7$, $\varphi = 1$, $\psi = 0.05$, $\gamma_{11} = 0.01$, $\gamma_{22} = 0.01$, and $\gamma_{12} = -0.02$. In order to look at different variabilities of inappropriately chosen ratios of inputs, we set $\epsilon = 0.5$, $\epsilon = 1$, and $\epsilon = 2$. With $\epsilon = 2$, variability of allocative efficiency is expected to have been reduced considerably—range becomes [2;6]; and vice versa, $\epsilon = 0.5$ ensures very large variability—range increases to [0.5;7.5]. We conduct three sets of experiments setting $\sigma_u^2$ to 0.0025, 0.025, and 0.25; this ensures covering a plausible range of standard deviations of technical efficiency. In each experiment we ran $L=500$ Monte-Carlo trials.$^7$

### 3.4 Results

From Tables 1-6 it is clearly seen that in all three cases the DEA estimates the drawn technical efficiency scores fairly accurately—the rank correlation coefficient (Corr4) is close to one. This is an expected outcome since we do not assume a stochastic term in the production output generation (step (vii) of the experiment). The same argument applies to the rank correlation coefficient between allocative efficiency calculated in step (xi) and that calculated in step (xii) (Corr3). Thus, there is not much difference in using the true or the estimated technical efficiency in the new approach. However, what is of most interest to us are the rank correlation coefficients between allocative efficiency estimates from the traditional and our new approach (Corr1 and Corr2). Corr1 has been computed with the
estimates of allocative efficiency based on “true” technical efficiency while Corr2 has been
computed with the estimates of allocative efficiency based on estimated values of technical
efficiency. As previously mentioned, the rank correlation between these measures is quite
high (Corr3). We argue that it is more appropriate to draw conclusions from Corr2 since
we do not know the “true” technical efficiency in practice.

The first observation worth mentioning is that when variability of sub-optimal ratios
decreases (ε increases): our method is less successful in yielding similar estimates as
the traditional one. Hence, our method deteriorates in terms of exactness when “true”
allocative efficiency is not very heterogeneous.

Furthermore, the results show that our approach is robust with respect to variance
of the drawn technical efficiency, σ²_u. Looking closely at correspondent ratios, one can
notice that for the same θ’s Corr2 is increasing when σ²_u increases, whereas for other
θ’s Corr2 decreases when we increase σ²_u; however, the changes are minor. The same
argument applies to the standard deviation of Corr2. This implies that for different
levels of σ²_u distributions of Corr2 are virtually the same. The skewness of the variable
Corr2 is always negative and is about –0.6 which means that the distribution of Corr2
is skewed to the left and more values are clustered to the right of the mean. Kurtosis is
about 0.6, but it varies more than the skewness; it increases with increase of σ²_u. Kernel
density estimates of Corr2 for the case θ = 0.75 are shown in Figure 5. Note that we
use the Gaussian kernel function and the Sheather and Jones 1991 rule to determine the
“optimal” bandwidth.

The results are better when the sample size is increased to 400 (Tables 4-6). However,
the improvement does not change our main conclusions based on the experiments with
sample size 100. As expected, standard deviations of rank coefficients are almost halved
when the sample size is quadrupled.

Results of one run⁸ (sample size 500) are summarized in Figure 6. Our methodology
almost completely repeats the trend of the traditional approach for ε = 0.5 which is
backed by a high correlation coefficient in Tables 1 and 4; as ε becomes larger Figure 6
suggests that our methodology is less able to predicts allocative efficiency. However, it is most remarkable that our methodology is in line with the traditional approach.

4 Empirical illustration of the new approach

4.1 Data

To illustrate the usefulness of the new approach for measuring allocative efficiency when input prices are not available, we apply it to micro-data from the German Cost Structure Census\(^9\) of manufacturing for the year 2003. Our sample comprises only enterprises from the chemical industry. The measure of output is gross production. This mainly consists of the turnover and the net-change of the stock of the final products.\(^10\)

The Cost Structure Census contains information for a number of input categories.\(^11\) These categories are payroll, employers’ contribution to the social security system, fringe benefits, expenditure for material inputs, self-provided equipment, and goods for resale, for energy, for external wage-work, external maintenance and repair, tax depreciation of fixed assets, subsidies, rents and leases, insurance costs, sales tax, other taxes and public fees, interest on outside capital as well as “other” costs such as license fees, bank charges and postage, or expenses for marketing and transport.

Some of the cost categories which include expenditures for external wage-work and external maintenance and repair contain a relatively high share of reported zero values because many firms do not utilize these types of inputs. Such zeros make the firms incomparable and, thus, might bias the DEA results. In order to reduce the number of reported zero input quantities, we aggregated the inputs into the following categories: (i) material inputs (intermediate material consumption plus commodity inputs), (ii) labor compensation (salaries and wages plus employer’s social insurance contributions), (iii) energy consumption, (iv) user cost of capital (depreciation plus rents and leases), (v) external services (\textit{e.g.}, repair costs and external wage-work), and (vi) “other” inputs related to production (\textit{e.g.}, transportation services, consulting, or marketing).
Profits are computed as one minus the total costs divided by the turnover. Since the DEA requires positive values, we standardize the profit measure to the interval (0,1) by adding the minimum profit and dividing this by the range of profits.

4.2 Results

Figure 7 shows profitability plotted against estimated technical efficiency. Remarkably, a frontier, as could be theoretically expected from Proposition 1, indeed exists. Another observation worth mentioning is that within a certain level of technical efficiency (i) profitability greatly varies suggesting variation in allocative efficiency (as firms A, B, and C in Proposition 3) and (ii) profits are bounded from above. Moreover, the frontier is positively sloped as was stated in the first theoretical part of this paper. Interestingly, Figure 7 suggests that even with 100 percent technical efficiency enterprises can be allocatively inefficient.

We calculated technical efficiency scores as in equation (6). Table 7, which contains descriptive statistics of the estimated technical efficiencies, suggests that an average German chemical manufacturing enterprise is fairly inefficient. The median of technical efficiency implies that half of firms have an efficiency of 68 percent or less. The scores for allocative efficiency are obtained solving the linear programming problem as in equation (9). Descriptive statistics on allocative efficiency are also presented in Table 7. At a first glance, the mean and the variation of allocative efficiency appear to be strikingly similar to that of technical efficiency. However, the distribution of allocative efficiency is more symmetric and has a lower variance compared to the technical efficiency distribution.

Kernel estimated density of technical efficiency is shown in the left panel of Figure 8; we use Gaussian kernel function and the Sheather and Jones 1991 rule to determine the “optimal” bandwidth. Although the number of firms is quite large, we analyze the sensitivity of efficiency scores relative to the sampling variations of the estimated frontier in an additional step. Consequently, we perform the homogeneous bootstrap as described by Simar and Wilson 1998. The geometric mean of the bias-corrected efficiency scores
is 0.6066, which is on average 0.0886 lower than that estimated via the DEA; the mean variance of bias is 0.0036. In comparison to other studies, however, the bias of estimates and its standard error are rather low, thereby indicating a robustness of the technical efficiency scores.

5 Conclusions

Allocative inefficiency, introduced in the seminal work by Farrell 1957, has important implications from the perspective of the firm. How much could firms increase their profits—given a certain output they produce—just by reallocating resources? On the other hand, the existing empirical evidence on the extent and determinants of allocative efficiency within and across industries is rather limited. The main reason is that the traditional approach to assessing allocative efficiency requires input prices. However, input prices are rarely accessible, which per se, precludes the analysis of the allocative efficiency with non-parametric approach.

In this paper, a new method is developed which enables calculating allocative efficiency without knowing input prices. This indicator is derived as the output-oriented distance to the frontier in profit–technical efficiency space. Thus, besides input and output quantities, only the profits of the firms are needed for calculating allocative efficiency. A simple Monte-Carlo experiment was performed to check the validity of the new methodology. We obtain high-rank correlation coefficients between allocative efficiency estimates based on both traditional and new approaches for different parameter constellations. Moreover, the new approach proved to be quite robust with respect to variance of true technical efficiency. Finally, we applied the new approach to a sample of about 900 enterprises in the German chemical industry. The results suggest a large variation of allocative efficiency even for technically efficient enterprises. Thus, the example highlights the usefulness of our method for obtaining allocative efficiency measures when input prices are not available.
Endnotes

*The research on this project has benefited from the comments of participants of the Royal Economic Society Conference [2005], 3rd International Industrial Organization Conference, 10th Spring Meeting of Young Economists, and IX European Workshop on Efficiency and Productivity Analysis.

1For studies in the financial sector, see the review by Berger and Humphrey 1997 and also Topuz, Darrat and Shelor 2005, Färe, Grosskopf and Weber 2004, Isik and Hassan 2002. Some studies have been performed for the agricultural sector (e.g., Coelli, Rahman and Thirtle 2002, Chavas and Aliber 1993, Chavas, Petrie and Roth 2005, and Grazhdaninova and Zvi 2005). Studies for manufacturing sector are relatively rare (e.g., Burki, Khan and Bratsberg 1997, and Kim and Gwangho 2001)

2Moreover, allocative efficiency is also import for the analysis of the production process; e.g., to estimate the bias of (i) the cost function parameters, (ii) returns to scale, (iii) input price elasticities, and (iv) cost-inefficiency Kumbhakar and Wang forthcoming or to validate the aggregation of productivity index Raa 2005).

3This includes retrieving allocative efficiency using shadow prices (see Greene 1997 and Lovell 1993).

4Let us assume that the ratios of input prices are equal for each firm. This assumption is needed to have the isocosts parallel to each other.

5Since the DEA is deterministic, we do not incorporate a stochastic term in the Monte-Carlo trials.

6Using a different experiment, Greene 2005 obtains estimates of technical efficiency with standard deviations from 0.09 to 0.43.

7The simulation is programmed in SAS 9.1.3; computationally, one run with N=100, L=500 takes about 7 hours on a Pentium IV processor running at 3GHz. Thus, we defined relatively few parameter constellations in the performed experiment.

8We repeated this experiment many times and the general picture was always similar; however, due to space constraints it is not possible to present all results here.

9Aggregate figures are published annually in Fachserie 4, Reihe 4.3 of Kostenstrukturerhebung im Verarbeitenden Gewerbe (diverse years). For more details on the Cost Structure Census, see Appendix A1.

10We do not include turnover from activities that are classified as miscellaneous such as license fees, commissions, rents, leasing etc. because this kind of revenue cannot adequately be explained by the means of a production function.

11Though the production theory framework requires real quantities, using expenditures as proxies for inputs in the production function is quite common in the literature (see e.g., Paul, Nehring, Banker and Somwaru 2004, Paul and Nehring 2005).
References


Appendix A1

The Cost Structure Census is gathered and compiled by the German Federal Statistical Office (Statistisches Bundesamt). Enterprises are legally obliged to respond to the Cost Structure Census; hence, missing observations due to non-response are precluded. The survey comprises all large German manufacturing enterprises which have 500 or more employees. Enterprises with 20-499 employees are included as a random sample that is representative for this size category in a particular industry. For more information about cost structure census surveys in Germany, we refer the reader to Görzig, Fritsch, Hennchen and Stephan 2004.
Table 1: Means of Rank Correlations. $\epsilon = 0.5$, $N = 100$.

<table>
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<th>$\theta$</th>
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<th>1</th>
<th>1.25</th>
<th>0.025</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
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<td>0.8712</td>
<td>0.7885</td>
<td>0.7365</td>
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<td></td>
<td>0.0416</td>
<td>0.059</td>
<td>0.0663</td>
<td>0.0407</td>
<td>0.0589</td>
<td>0.0664</td>
<td>0.0469</td>
<td>0.0687</td>
<td>0.0818</td>
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<tr>
<td>Corr3</td>
<td></td>
<td>0.9899</td>
<td>0.988</td>
<td>0.9894</td>
<td>0.9915</td>
<td>0.9901</td>
<td>0.9895</td>
<td>0.9468</td>
<td>0.9419</td>
<td>0.9464</td>
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<tr>
<td></td>
<td></td>
<td>0.0194</td>
<td>0.0212</td>
<td>0.0188</td>
<td>0.0148</td>
<td>0.0159</td>
<td>0.0168</td>
<td>0.0531</td>
<td>0.0492</td>
<td>0.0397</td>
<td></td>
<td></td>
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<tr>
<td>Corr4</td>
<td></td>
<td>0.8928</td>
<td>0.8937</td>
<td>0.8893</td>
<td>0.9524</td>
<td>0.9528</td>
<td>0.956</td>
<td>0.983</td>
<td>0.9816</td>
<td>0.9825</td>
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<tr>
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<td></td>
<td>0.0489</td>
<td>0.0495</td>
<td>0.0423</td>
<td>0.0275</td>
<td>0.0268</td>
<td>0.0254</td>
<td>0.0124</td>
<td>0.0148</td>
<td>0.0141</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Standard errors in italics

$^b$ Corr1 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xi).

$^c$ Corr2 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xii).

$^d$ Corr3 is the rank correlation between allocative efficiency calculated in step (xi) and that calculated in step (xii).

$^e$ Corr4 is the rank correlation between technical efficiency calculated in equation (6) and that drawn in step (vi).

Table 2: Means of Rank Correlations. $\epsilon = 1$, $N = 100$.

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>$\theta$</th>
<th>0.0025</th>
<th>1</th>
<th>1.25</th>
<th>0.025</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>0.25</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr1</td>
<td></td>
<td>0.8569</td>
<td>0.7043</td>
<td>0.6192</td>
<td>0.8519</td>
<td>0.6991</td>
<td>0.6053</td>
<td>0.7851</td>
<td>0.6381</td>
<td>0.5476</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>0.0412</td>
<td>0.0653</td>
<td>0.0744</td>
<td>0.0429</td>
<td>0.0685</td>
<td>0.0779</td>
<td>0.0706</td>
<td>0.0803</td>
<td>0.0838</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr2</td>
<td></td>
<td>0.8611</td>
<td>0.7111</td>
<td>0.6264</td>
<td>0.8598</td>
<td>0.7197</td>
<td>0.6263</td>
<td>0.847</td>
<td>0.7481</td>
<td>0.6709</td>
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<tr>
<td></td>
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<td>0.0641</td>
<td>0.0722</td>
<td>0.0405</td>
<td>0.0654</td>
<td>0.0771</td>
<td>0.048</td>
<td>0.0753</td>
<td>0.0944</td>
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<td></td>
</tr>
<tr>
<td>Corr3</td>
<td></td>
<td>0.9928</td>
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<td>0.0163</td>
<td>0.0152</td>
<td>0.0157</td>
<td>0.0149</td>
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<tr>
<td>Corr4</td>
<td></td>
<td>0.9183</td>
<td>0.9209</td>
<td>0.9196</td>
<td>0.959</td>
<td>0.9633</td>
<td>0.9626</td>
<td>0.9874</td>
<td>0.9878</td>
<td>0.9869</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0341</td>
<td>0.0344</td>
<td>0.0353</td>
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<td>0.0248</td>
<td>0.0254</td>
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<td>0.0111</td>
<td>0.0113</td>
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</tr>
</tbody>
</table>

Notes from Table 1 apply.

Table 3: Means of Rank Correlations. $\epsilon = 2$, $N = 100$.

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>$\theta$</th>
<th>0.0025</th>
<th>1</th>
<th>1.25</th>
<th>0.025</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>0.25</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr1</td>
<td></td>
<td>0.814</td>
<td>0.5782</td>
<td>0.3386</td>
<td>0.8042</td>
<td>0.5561</td>
<td>0.3168</td>
<td>0.6841</td>
<td>0.4515</td>
<td>0.2602</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0553</td>
<td>0.0762</td>
<td>0.0835</td>
<td>0.0438</td>
<td>0.0794</td>
<td>0.0928</td>
<td>0.103</td>
<td>0.1063</td>
<td>0.0984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr2</td>
<td></td>
<td>0.8155</td>
<td>0.5837</td>
<td>0.3448</td>
<td>0.8091</td>
<td>0.575</td>
<td>0.3498</td>
<td>0.7638</td>
<td>0.6048</td>
<td>0.4864</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0437</td>
<td>0.0738</td>
<td>0.0828</td>
<td>0.0425</td>
<td>0.0791</td>
<td>0.0917</td>
<td>0.0609</td>
<td>0.0992</td>
<td>0.1294</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr3</td>
<td></td>
<td>0.9939</td>
<td>0.9948</td>
<td>0.9938</td>
<td>0.9917</td>
<td>0.9904</td>
<td>0.9878</td>
<td>0.9265</td>
<td>0.9117</td>
<td>0.9049</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.0144</td>
<td>0.0124</td>
<td>0.013</td>
<td>0.0152</td>
<td>0.0156</td>
<td>0.0202</td>
<td>0.0765</td>
<td>0.0838</td>
<td>0.0652</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr4</td>
<td></td>
<td>0.9455</td>
<td>0.9449</td>
<td>0.9443</td>
<td>0.9749</td>
<td>0.9743</td>
<td>0.9731</td>
<td>0.991</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.0283</td>
<td>0.03</td>
<td>0.03</td>
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<td>0.0197</td>
<td>0.0206</td>
<td>0.009</td>
<td>0.0089</td>
<td>0.0075</td>
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</tr>
</tbody>
</table>

Notes from Table 1 apply.
Table 4: Means of Rank Correlations. $\epsilon = 0.5$, $N = 500$.

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>0.0025</th>
<th>0.025</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Corr1</td>
<td>0.8812</td>
<td>0.7551</td>
<td>0.7132</td>
</tr>
<tr>
<td></td>
<td>0.0182</td>
<td>0.0288</td>
<td>0.0311</td>
</tr>
<tr>
<td>Corr2</td>
<td>0.8824</td>
<td>0.7567</td>
<td>0.7144</td>
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<tr>
<td></td>
<td>0.0176</td>
<td>0.0287</td>
<td>0.0307</td>
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<td>Corr3</td>
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<tr>
<td></td>
<td>0.0035</td>
<td>0.0031</td>
<td>0.0036</td>
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<td>Corr4</td>
<td>0.9726</td>
<td>0.973</td>
<td>0.9733</td>
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<tr>
<td></td>
<td>0.0096</td>
<td>0.0106</td>
<td>0.0099</td>
</tr>
</tbody>
</table>

Notes from Table 1 apply.

Table 5: Means of Rank Correlations. $\epsilon = 1$, $N = 400$.

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>0.0025</th>
<th>0.025</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Corr1</td>
<td>0.876</td>
<td>0.7169</td>
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<tr>
<td></td>
<td>0.0178</td>
<td>0.0334</td>
<td>0.035</td>
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<tr>
<td>Corr2</td>
<td>0.8766</td>
<td>0.7185</td>
<td>0.6375</td>
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<tr>
<td></td>
<td>0.0176</td>
<td>0.0333</td>
<td>0.0349</td>
</tr>
<tr>
<td>Corr3</td>
<td>0.9992</td>
<td>0.9991</td>
<td>0.9992</td>
</tr>
<tr>
<td></td>
<td>0.0026</td>
<td>0.0028</td>
<td>0.0025</td>
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<tr>
<td>Corr4</td>
<td>0.9814</td>
<td>0.9809</td>
<td>0.9821</td>
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<tr>
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<td>0.0086</td>
<td>0.0086</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

Notes from Table 1 apply.

Table 6: Means of Rank Correlations. $\epsilon = 2$, $N = 400$.

<table>
<thead>
<tr>
<th>$\sigma^2_u$</th>
<th>0.0025</th>
<th>0.025</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Corr1</td>
<td>0.8337</td>
<td>0.5911</td>
<td>0.341</td>
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<tr>
<td></td>
<td>0.0195</td>
<td>0.0361</td>
<td>0.0455</td>
</tr>
<tr>
<td>Corr2</td>
<td>0.8339</td>
<td>0.5924</td>
<td>0.3422</td>
</tr>
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<td>0.0362</td>
<td>0.0455</td>
</tr>
<tr>
<td>Corr3</td>
<td>0.9994</td>
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<td>0.9995</td>
</tr>
<tr>
<td></td>
<td>0.0025</td>
<td>0.0022</td>
<td>0.0017</td>
</tr>
<tr>
<td>Corr4</td>
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</table>

Notes from Table 1 apply.

Table 7: Descriptive statistics of technical and allocative efficiency, $N=905$.

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>mean</th>
<th>st.d.</th>
<th>coef. of var</th>
<th>skewness</th>
<th>min</th>
<th>10th</th>
<th>25th</th>
<th>median</th>
<th>75th</th>
<th>90th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technical</td>
<td>0.689</td>
<td>0.151</td>
<td>0.214</td>
<td>0.440</td>
<td>0.325</td>
<td>0.529</td>
<td>0.591</td>
<td>0.682</td>
<td>0.803</td>
<td>1.000</td>
</tr>
<tr>
<td>Allocative</td>
<td>0.696</td>
<td>0.118</td>
<td>0.170</td>
<td>-0.002</td>
<td>0.310</td>
<td>0.540</td>
<td>0.608</td>
<td>0.697</td>
<td>0.780</td>
<td>0.852</td>
</tr>
</tbody>
</table>
Note: *In each panel the vertical dashed line is the mean value of the corresponding density.*

**Figure 5:** Estimates of Sampling Densities of Corr2 ($\theta = 0.75$, $L = 500$, $\epsilon = 0.5$, $\epsilon = 1$ and $\epsilon = 2$)
Figure 6: Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, $w_2x_2/w_1x_1$ ($\theta = 0.75$, $N = 400$, $\epsilon = 0.5$, $\epsilon = 1$ and $\epsilon = 2$)
Figure 6 continued: Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, $w_2 x_2 / w_1 x_1$ ($\theta = 0.75$, $N = 400$, $\epsilon = 0.5$, $\epsilon = 1$ and $\epsilon = 2$)
Figure 6 continued: Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, \( w_2x_2/w_1x_1 \) (\( \theta = 0.75, N = 400, \epsilon = 0.5, \epsilon = 1 \) and \( \epsilon = 2 \))
Figure 7: Profitability plotted against estimated technical efficiency scores for about 900 German enterprises from the chemical industry
Note: In each panel the vertical dashed line is the mean value of the corresponding density.

Figure 8: Estimates of sampling densities of technical and allocative efficiency scores
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Working Paper Series

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