

Rogerson, William P.

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On the Optimality of Menus of Linear Contracts

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Northwestern University

2001 Sheridan Road 580 Leverone Hall Evanston, IL 60208-2014 USA

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“On the Optimality of Menus
of Linear Contracts”

William P. Rogerson
Northwestern University

www.kellogg.nwu.edu/research/math

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Discussion Paper No. 714R

**ON THE OPTIMALITY OF MENUS
OF LINEAR CONTRACTS**

by

William P. Rogerson^{*}

January 1987

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Abstract

A number of papers have recently shown that it may be fully optimal in some cases for a principal to offer an agent a menu of contracts, all of which are linear in output. This paper shows that the result (when true) has a very simple and intuitive geometric basis. The result follows from well-known analysis of the standard self-selection problem together with this simple geometric argument. However it is shown that the result only holds for special classes of functional forms and is not generally true.

1. Introduction

The principal-agent literature is concerned with the situation where a principal hires an agent to perform some job, but where the principal cannot directly observe the agent's effort and the outcome is only probabilistically determined by the agent's effort. If the agent is risk neutral and the principal knows the agent's preferences, this informational imperfection poses no problem. The principal offers the agent a contract paying the full value of output on the margin and charges the agent a fee large enough so the agent earns his reservation utility. The principal-agent literature has been primarily concerned with analyzing the nature of the optimal contract when the first of the above two assumptions is violated -- i.e. -- when the agent is risk-averse and the principal knows the agent's preferences.¹ Although this literature has yielded the important insight that an optimal contract will often involve a tradeoff between insuring the agent and providing the agent with incentives to exert effort, a key shortcoming of the literature has been that the optimal contracts are in general extremely complicated.² Thus the theory does not explain the rather simple contracts one often observes in the real world.

Recently Laffont and Tirole [1986], McAfee and McMillan [1986] and Melumad and Reichelstein [1986] have considered the opposite special case to that considered by the standard literature.³ Namely the agent is assumed to be risk neutral but the principal is assumed to only have a probabilistic notion of the agent's preferences. This special case seems at least as reasonable as that considered by the standard literature. In many situations, it is plausible that aspects of the agent's ability are better known to the agent than the principal. Furthermore, risk neutrality may not be an unreasonable assumption in many contracting situations where both parties are firms.

Laffont and Tirole show that in a model with very special functional forms that the optimal contract has a remarkably simple structure. Namely, the principal offers the agent a menu of wage functions where each wage function is linear in output. The agent is allowed to choose which of the linear wage functions he wishes to work under. This contract can be interpreted as one where the agent is required to predict an output. The agent is then paid a fixed fee plus a bonus or penalty which is linear in under-runs or over-runs.

McAfee and McMillan analyze a generalized version of Laffont and Tirole, replacing the assumption of special function forms by the familiar assumptions used in the self-selection literature. They derive a sufficient condition for the result which includes Laffont and Tirole's functional forms as a special case, thus showing that it may be true in more general cases than those considered by Laffont and Tirole. However the condition is difficult to interpret and one is left with no clear economic understanding of why the result is true or what it depends on.

The models of Laffont and Tirole and McAfee and McMillan can be viewed as generalizations of the "standard self-selection problem" first considered by Musa and Rosen [1978] and subsequently analyzed by many others.⁴ In the standard self-selection problem the agent's ability is private information but there is no productive uncertainty -- i.e. -- the agent deterministically chooses the output level. The models of Laffont and Tirole and McAfee and McMillan generalize this by introducing productive uncertainty -- i.e. -- the agent's effort determines the mean of output deterministically but actual output is only stochastically determined by output.

Notice that any generalized self-selection problem can be used to generate a standard self-selection problem by assuming that actual output

always equals the mean of output.⁵ Laffont and Tirole sketch the following idea at the end of their paper. Consider any generalized self-selection problem and the standard self-selection problem generated by it. If the optimal wage contract for the standard problem is convex then a simple geometric argument implies that a menu of linear contracts is optimal in the generalized problem. Melumad and Riechelstein [1986] formalize this argument.⁶

The major point of this paper is to show that McAfee and McMillan's sufficient condition implies that the above convexity property holds. Therefore McAfee and McMillan's result is shown to follow largely from existing well-understood analysis of the standard self-selection problem together with a simple geometric argument. In particular almost no new technical analysis is required. Given the complexity of the analysis in McAfee and McMillan this is a large advantage. For example, McAfee and McMillan simply assume that the first-order approach is valid in their analysis. This paper shows that the validity of the first-order approach in this problem is equivalent to the validity of the first-order approach in the standard self-selection problem which is well understood. The same point can be made for a range of other delicate technical issues which seem to be inherent in this type of problem.

This paper also makes three other points. First it clarifies the generality of the result that menus of linear contracts are optimal. McAfee and McMillan, for example, conclude that "in a broad set of circumstances, the predicted contract is linear in observed output."⁷ By revealing the geometric basis for the result and analyzing a series of examples, this paper shows the result cannot be expected to be generally true. Basically no good economic reason exists to expect a convex wage contract in a standard self-selection

model. In fact, the special functional forms considered by Laffont and Tirole appear to be the only simple class where the result is always true. Therefore the optimality of menus of linear contracts is best viewed as a very special property holding only in a limited class of examples.

Second, a limitation in Melumad and Riechelstein's [1986] proof that the convexity property implies the optimality of a menu of linear contracts is addressed. Because they considered a very general model with almost no assumptions, they were forced to restrict themselves to cases where all types of the agent are hired. In the more structured environment of this paper it can be shown that the argument still holds even when the (realistic) possibility that lower ability types may not be hired is allowed.

Third, Laffont and Tirole consider a slightly more general type of relationship than McAfee and McMillan or Melumad and Riechelstein in that they allow for the possibility that the principal must make some subsidiary decision which he would like to coordinate with the agent's effort choice.⁸ It is shown that the same logic and formal analysis also applies to this more general relationship with a suitable transformation of variables.

Section 2 describes and analyzes the standard self-selection problem. Section 3 describes the generalized self-selection problem and shows how the results of Section 2 can be applied to analyzing it. Concluding remarks follow in Section 4.

2. The Standard Self-Selection Model

The model of this paper is a generalization of the standard self-selection model. Furthermore, the major point of this paper is to show how the results of the standard model can still be used to analyze the generalized model where there is productive uncertainty. Therefore the clearest way of

proceeding is to first describe the standard model and the nature of its solution. Since this model is fairly well-known it will not be described in detail. Many of the analyses of this problem interpret it as a trading problem where "output" is "the probability of trade"⁹ or as a problem where a monopoly produces various product qualities and "output" is "product quality."¹⁰ However, the underlying mathematical structure is essentially the same in all cases. See the references in footnote 4 for more complete discussions of this problem. Sappington [1983] provides a good discussion of the interpretation of this mathematical structure as a principal-agent model.¹¹

A. The Model

The agent is of some type $z \in [0,1]$. Although the agent knows his type the principal does not. The principal simply knows that the agent's type has been drawn according to the distribution $G(z)$ with density $g(z)$. The distribution is assumed to be smooth and to obey the familiar monotone non-increasing inverse hazard rate condition. The inverse hazard rate is defined to be

$$(2.1) \quad H(z) = \frac{1-G(z)}{g(z)} .$$

Formally it is assumed that:

(A.1) G is twice continuously differentiable on $[0,1]$ and g is strictly positive on $[0,1]$.

(A.2) H is non-increasing on $[0,1]$.

The agent chooses how many units of output to produce. Let $c(y,z)$ denote the cost to an agent of type z of producing y units of output. An agent receiving w dollars and choosing to produce y will therefore have utility of¹²

$$(2.2) \quad w - c(y, z).$$

The following assumptions are made about c .

(A.3) c is three times continuously differentiable over $[0, \infty) \times [0, 1]$.

(A.4) $c(0, z) = 0$ for every z .

(A.5) $c_y > 0$, $c_{yy} \geq 0$ for $y > 0$

(A.6) $c_z < 0$ for $y > 0$

(A.7) $c_{yz} < 0$ for $y > 0$

(A.8) $c_{yyz} \leq 0$ for $y > 0$

(A.9) $c_{yzz} \geq 0$ for $y > 0$.

Assumption (A.3) simply requires that c be smooth and assumption (A.4) merely is a normalization. According to (A.5) the cost of producing output is strictly increasing and convex. Assumptions (A.6) and (A.7) describe the essential fashions in which types differ. According to (A.6) higher types

have lower costs of production. According to (A.7) they also have lower marginal costs of production. Finally (A.8) and (A.9) are technical assumptions required for the formal analysis.

The principal values output according to the utility function $u(y)$. If the principal receives y units of output and pays the agent w dollars the principal's utility is given by

$$(2.3) \quad u(y) - w.$$

Assume that u is smooth, strictly increasing and concave. Also, normalize u so that $u(0) = 0$.

$$(A.10) \quad u \text{ is twice continuously differentiable over } [0, \infty);$$

$$(A.11) \quad u(0) = 0$$

$$(A.12) \quad u' > 0, \quad u'' \leq 0.$$

Finally, two technical assumptions need to guarantee the existence and differentiability of a solution must be made. These are given by (A.13) and (A.14). Assumption (A.13) simply states that for high enough values of output, marginal costs eventually exceed marginal benefits.

$$(A.13) \quad \lim_{y \rightarrow \infty} u'(y) - c_y(y, z) < 0 \text{ for every } z \in [0, 1].$$

(A.14) Either c is strictly convex in y or u is strictly concave in y .

Stronger assumptions than are absolutely necessary have been made in order that all the assumptions be fairly simple statements directly about primitives of the model. In particular, the role of (A.1), (A.2), (A.8), (A.9), (A.13) and (A.14) is solely to guarantee the existence of a solution $y(z)$ to (2.7) which is strictly increasing and differentiable. (Expression (2.7) will be defined later on in this section.) Therefore the above six assumptions can be replaced by the much weaker assumption that a strictly increasing differentiable solution exists to (2.7).

B. The Optimal Contract

A contract between the principal and agent will be defined to be a wage function which specifies the wage the agent will be paid contingent upon the output.

Definition: A contract in the standard self selection model is a function ϕ mapping the non-negative reals into the non-negative reals. If the agent produces y units of output, the principal pays the agent a wage of $\phi(y)$ dollars.

When offered the contract $\phi(y)$, an agent can choose to accept the contract and produce any output y (in which case he receives $\phi(y)$) or he can not accept the contract (in which case zero is produced and the agent receives zero). Each type of agent chooses an optimal course of action. Thus any contract results

in a pair of outcome functions $\{w(z), y(z)\}$ which describe the output and income of each type of agent. (If an agent of type z declines to accept the contract then $w(z) = y(z) = 0$.)

Not every pair of functions are the outcome functions for some contract. The revelation principle¹³ provides an elegant characterization of the outcome functions which can be achieved by some contract. Consider the following situation. Choose outcome functions $\{w(z), y(z)\}$. Suppose the agent is asked to announce his type, \hat{z} . The agent can announce any $\hat{z} \in [0,1]$ and need not tell the truth. If he announces \hat{z} he receives $w(\hat{z})$ and must produce $y(\hat{z})$. The agent can also decline to make an announcement in which case he receives zero and produces zero. Then, according to the revelation principle, $\{w(z), y(z)\}$ is an outcome function for some contract if and only if every type of agent finds it optimal to truthfully report his type in the above game.

Formally, then, $\{w(z), y(z)\}$ are outcome functions for some contract if and only if they satisfy the following two properties.

Definition: $\{w(z), y(z)\}$ are said to satisfy incentive compatibility (IC) if

$$(2.4) \quad w(z) - c(y(z), z) \geq w(\hat{z}) - c(y(\hat{z}), z) \text{ for every } z, \hat{z} \in [0,1].$$

Definition: $\{w(z), y(z)\}$ satisfy voluntary participation (VP) if

$$(2.5) \quad w(z) - c(y(z), z) \geq 0 \text{ for every } z \in [0,1].$$

Given outcome functions which satisfy IC and VP, it is straightforward to calculate a contract which implements them. Suppose $\{w(z), y(z)\}$ satisfy IC and VP. Then define $\phi(y)$ as follows. Choose any $\hat{y} > 0$. If there exists a \hat{z} such that $\hat{y} = y(\hat{z})$, then let $\phi(\hat{y}) = w(\hat{z})$.¹⁴ If there does not exist a \hat{z} such that $\hat{y} = y(\hat{z})$ then let $\phi(\hat{y}) = 0$.

An optimal contract is one which maximizes the principal's expected utility given the principal's prior over types of agents and given that each type of agent chooses an optimal course of action. By employing the revelation principal, it is easiest to directly calculate the optimal outcome functions and then derive an optimal contract from them. The optimal outcome functions solve the following program.

Program M

$$(2.6) \quad \text{Maximize} \quad \int [u(y(z)) - w(z)] g(z) dz$$
$$w(z) \leq y(z)$$

Subject to: IC

VP

Thus ϕ is an optimal contract if and only if the outcome functions it induces satisfy Program M.

An important property of any contract (including an optimal one) which will be useful for future analysis is that higher types of agents choose (weakly) higher levels of output -- i.e. -- the outcome function $y(z)$ is weakly increasing. The intuition for this is straightforward. According to (A.7) higher types have a lower marginal cost of production. Thus any increment in output which is profitable for a low type is also at least as

profitable for a higher type. This is stated below as Proposition 1. The proof is standard so is omitted.

Proposition 1:

Suppose $\{w(z), y(z)\}$ satisfy IC and VP. Then $y(z)$ is non-decreasing.

Under Assumptions (A.1) - (A.14) it is possible to calculate the optimal outcome functions. The technique involves showing that the global incentive constraints can be replaced by local incentive constraints. The resulting program can be solved using control theory. Only the result will be reported here.¹⁵

Proposition 2:

Unique optimal outcome functions exist and have the following form. There exists a $z_0 \in [0,1]$ such that $w(z) = y(z) = 0$ for $z < z_0$. For $z \geq z_0$, the outcome functions are differentiable and are determined by

$$(2.7) \quad u'(y(z)) = c_y(y(z), z) - H(z)c_{yz}(y(z), z)$$

$$(2.8) \quad w(z) = c(y(z), z) - \int_{z_0}^z c_z(\hat{y}(z), \hat{z})dz$$

The optimal value of z_0 can also be characterized but this is not necessary for the analysis of this paper so it is omitted. In the optimal outcome functions, z_0 may be zero or greater than zero, depending upon the parameters of the situation. The nature of the solution described in Proposition 2 is as follows. Agents of types less than z_0 decline to participate. Agents of types greater than or equal to z_0 participate. Their

choice of y is determined by (2.7). Then the wage they are paid is determined by (2.8).

Given the optimal outcome functions described in Proposition 2 it is straightforward to calculate an optimal contract which implements them. First, note that if $z \geq z_0$ then $y(z)$ is strictly increasing. To see this, totally differentiate (2.7) to yield

$$(2.9) \quad y'(z) = \frac{-(1-H'(z))c_{yz}(y(z), y) + H(z)c_{yzz}(y(z), z)}{-u''(y(z)) + c_{yy}(y(z), z) - H(z)c_{yyz}(y(z), z)} \cdot$$

By assumptions (A.1) - (A.14) this expression is positive. Therefore y is invertible for $z \geq z_0$. Let $\gamma(y)$ denote the inverse.

Definition:

Suppose that $\{y(z), w(z)\}$ are the optimal outcome functions as defined by (2.7) and (2.8) for some $z_0 \in [0,1]$. Then let $\gamma(y)$ denote the inverse of $y(z)$ defined over $[y(z_0), y(1)]$. Note, for future reference, that the derivative of γ is given by

$$(2.10) \quad \gamma'(y) = \frac{-u''(y) + c_{yy}(y, \gamma(y)) - H(z)c_{yyz}(y, \gamma(y))}{-(1-H'(\gamma(y)))c_{yz}(y, \gamma(y)) + H(\gamma(y))c_{yzz}(y, \gamma(y))} \cdot$$

For values of y in $[y(z_0), y(1)]$, $\phi(y)$ must be defined by

$$(2.11) \quad \phi(y) = w(\gamma(y)).$$

There is some choice in defining $\phi(y)$ for other values of y , however; $\phi(y)$ must simply be chosen low enough so that no type of agent would want to choose a y not in the interval $[y(z_0), y(1)]$. (Except possible to choose

$y = 0$ if $\phi(y) = 0$. This is equivalent to not participating). Obviously defining $\phi(y)$ to be zero for all values of y outside of $[y(z_0), y(1)]$ would suffice. However $\phi(y)$ can be chosen in other ways. Another optimal contract where ϕ is always positive will be defined in the proof of Proposition 4 in the next section.

Proposition 3 summarizes the above discussion.

Proposition 3:

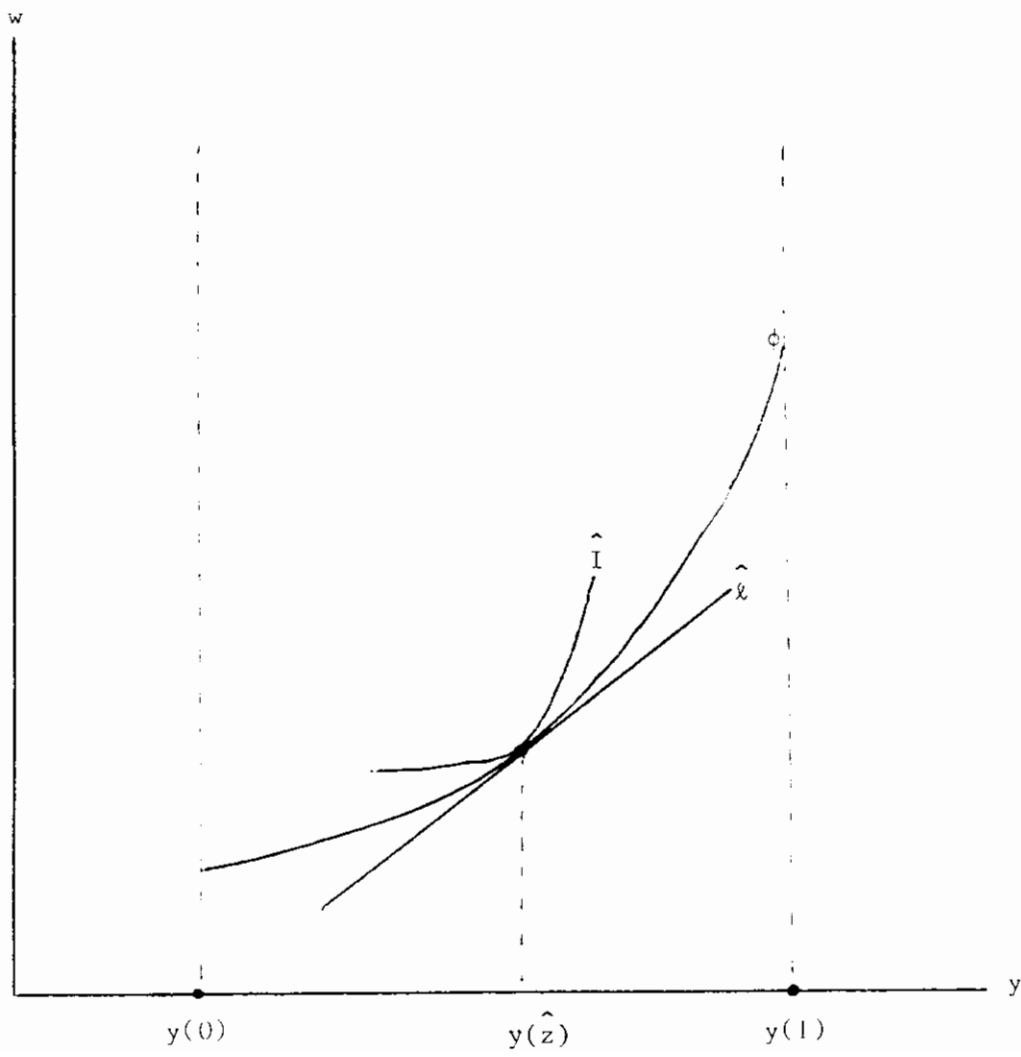
Suppose that $\{y(z), w(z)\}$ are the optimal outcome functions as defined by (2.7) and (2.8) for some $z_0 \in [0,1]$. Then if ϕ is an optimal contract it must satisfy (2.11) for $y \in [y(z_0), y(1)]$. One possible optimal contract is constructed by letting ϕ equal zero for every y outside of $[y(z_0), y(1)]$.

C. Menus of Linear Wage Functions

When there is no productive uncertainty there is no necessity for the principal to offer the agent a menu of wage functions. However the principal could do so if he wished. Consideration of this possibility will yield the key insight of this paper which carries over to the case of productive uncertainty.

For the purposes of developing intuition for this result assume for the moment that $z_0 = 0$ -- i.e. -- all types of agent choose to participate under the optimal contract. Then the optimal contract can be geometrically described as in Figure 1. Agents prefer points in (y,w) space which are up (i.e. -- more income) and to the left (i.e. -- less output.) Furthermore, by assumption the indifference curves of every type of agent are convex.

Figure 1



Therefore the indifference curves of an agent of type \hat{z} reach a tangency with $\varphi(y)$ at $y(\hat{z})$. This is illustrated in Figure 1. The indifference curve \hat{I} is the highest indifference curve that an agent of type \hat{z} can reach.

First suppose that $\varphi(y)$ is convex over the interval $[y(0), y(1)]$ as drawn in Figure 1. Then it is clear that the principal could offer the agent a menu of linear functions where the menu consists of all the tangent lines to $\varphi(y)$ over the interval $[y(0), y(1)]$. An agent of type \hat{z} would choose the line $\hat{\lambda}$ which is tangent to φ at $y(\hat{z})$, and would then choose to produce $y(\hat{z})$.

However, suppose that $\varphi(y)$ is concave over $[y(0), y(1)]$. Then it is equally clear that the principal cannot offer a menu of linear functions in this case. In fact, if $\varphi(y)$ is not convex everywhere over $[y(0), y(1)]$, then a menu of linear functions cannot be offered.

Therefore for the case of $z_0 = 0$ it is geometrically clear that the ability to implement the optimal outcome functions through offering a menu of linear functions is equivalent to the convexity of $w(\gamma(y))$ over $[y(z_0), y(1)]$. Proposition 4 formally shows that this intuition extends to the case where $z_0 > 0$ as well.¹⁶ Since the proof requires development of extra notation, it is relegated to an Appendix.

Proposition 4:

Suppose that $\{y(z), w(z)\}$ are the optimal outcome functions as defined by (2.7) and (2.8) for some $z_0 \in [0,1]$. Then they can be implemented by offering a menu of linear wage functions if and only if $w(\gamma(y))$ is convex over $[y(z_0), y(1)]$. If $w(\gamma(y))$ is convex, the menu of linear functions which

implements the optimal outcome functions consists of all the tangent lines to $w(\gamma(y))$ over the interval $[y(z_0), y(1)]$.

proof:

See Appendix.

QED.

Based on Proposition 4 an interesting question is therefore whether $w(\gamma(y))$ is convex or not and what this depends on. This is answered by Proposition 5.

Proposition 5:

$$(2.12) \quad \frac{d^2}{dy^2} w(\gamma(y)) = c_{yy}(y, \gamma(y)) + c_{yz}(y, \gamma(y))\gamma'(y)$$

where $\gamma'(y)$ is defined by (2.10).

proof:

Since an agent of type $\gamma(y)$ is at a local maximum of utility at y , $w(\gamma(y))$ satisfies

$$(2.13) \quad \frac{d}{dy} w(\gamma(y)) - c_y(y, \gamma(y)) = 0.$$

Differentiate (2.13) with respect to y to yield (2.12).

QED.

It is not possible to sign (2.12) simply by signing its various components. By (A.5) the first term is positive. It was argued above that $\gamma'(y)$ is positive (i.e. -- higher types produces higher output). This and (A.7) imply that the second term is negative. In general it seems that $w(\gamma(y))$ will be neither concave nor convex over its entire domain. In particular, since it is in general not convex, it is not in general possible to implement it by a menu of linear wage functions.

Three special examples will now be considered to illustrate this. The first is a special case where $w(\gamma(y))$ is convex. The second two are special cases where, depending on the parameters, $w(\gamma(y))$ is either globally concave or neither globally convex nor globally concave.

The first special case is where $c(y,z)$ is of the form

$$(2.14) \quad c(y,z) = \alpha(y-z)$$

for some real-valued function α such that

$$(2.15) \quad \alpha' > 0; \alpha'' > 0; \alpha''' \geq 0.$$

and

$$(2.16) \quad \lim_{x \rightarrow \infty} \alpha'(x) > 1.$$

Also assume that

$$(2.17) \quad u(y) = y.$$

It is straightforward to verify that (A.3) - (A.14) are satisfied. Corollary (5.1) describes the result.

Corollary 5.1:

For the case defined by (2.14) - (2.17), $w(\gamma(y))$ is convex.

proof:

Substitute (2.14) into (2.12) to yield

$$(2.18) \quad \frac{d^2}{dy^2} w(\gamma(y)) = \alpha''(y-\gamma(y)) [1-\gamma'(y)] .$$

Therefore $w(\gamma(y))$ is convex if and only if $\gamma'(y) \leq 1$. Substitute (2.14) into (2.10) to yield

$$(2.19) \quad \gamma'(y) = \frac{-u''(y) + \alpha''(y-\gamma(y)) + H(z) \alpha'''(y-\gamma(y))}{+ (1-H'(\gamma(y))) \alpha''(y-\gamma(y)) + H(z) \alpha'''(y-\gamma(y))}$$

By (A.2), H' is non-positive. Therefore if $u'' = 0$ (as it does by (2.17)), then $\gamma' \leq 1$.

QED.

The second case is where costs are given by

$$(2.20) \quad c(y,z) = (1-z) \beta(y)$$

where β is a function satisfying

$$(2.21) \quad \beta' > 0, \beta'' > 0, \beta''' \geq 0$$

and

$$(2.22) \quad \lim_{y \rightarrow \infty} \beta'(y) = \infty.$$

Let $u(y)$ be any strictly increasing weakly concave function. Once again it is straightforward to verify that (A.3) - (A.14) are satisfied. Corollary (5.2) describes the results.

Corollary 5.2:

Suppose that c is defined by (2.20) - (2.22).

(i) A sufficient condition for $w(\gamma(y))$ to be concave is that

$$(2.23) \quad \frac{-H'(z)}{H(z)} \leq \frac{1}{1-z}$$

for every $z \in [0,1]$.

(ii) Suppose that $u''(y) = 0$. Then the above condition is necessary and sufficient -- i.e. --

$$(2.24) \quad \frac{d^2}{dy^2} w(\gamma(y)) \gtrless 0 \text{ if and only if } \frac{-H'(\gamma(y))}{H(\gamma(y))} \gtrless \frac{1}{1-\gamma(y)}$$

proof:

Substitute (2.20) and (2.10) into (2.12) and reorganize to yield

$$(2.25) \quad \frac{d^2}{dy^2} w(\gamma(y)) = \frac{H(z)(1-z)}{1-H'(z)} \left[\frac{-H'(z)}{H(z)} - 1 \right] \beta''(y) + \frac{u''(y)}{1-H'(z)}$$

where $z = \gamma(y)$. By (A.2), $H'(z) \leq 0$. The corollary now follows immediately from (2.25).

QED.

Expression (2.23) is not generally always true or always false for a given distribution. Rather, it will be true for some values of z and not true for others. However one special case where (2.23) is always true is that where $G(z)$ is the uniform distribution. Corollary 5.3 states this result.

Corollary 5.3:

Suppose that c is defined by (2.20) - (2.22) and that

$$(2.26) \quad G(z) = 1 - z.$$

Then (2.23) is true for every $z \in [0,1]$. Therefore $w(\gamma(y))$ is concave.

proof:

Straightforward calculation shows that (2.23) holds with equality.

QED.

The third case is a slight variant on (2.20) - (2.22). Assume that costs are given by

$$(2.27) \quad c(y,z) = (1-z)y$$

and $u(y)$ is a strictly increasing, strictly concave function with

$$(2.28) \quad \lim_{y \rightarrow \infty} u'(y) = \infty.$$

Once again it is straightforward to verify that (A.3) - (A.14) are satisfied. In this case the agent has linear indifference curves in income-output space and (in a graph such as Figure 1 where output is on the horizontal axis and income is on the vertical axis) higher types of agents have flatter indifference curves. This is the case that has been most carefully analyzed in the literature. It is straightforward to see that any contract under which different types choose different outputs must be concave. This is because higher types must choose higher outputs (by Proposition 1) and higher types have flatter indifference curves. Since each agent's indifference curves are tangent to the contract at the agent's optimal choice, the contract must become flatter as output grows. Corollary (5.4) formally verifies this reasoning.

Corollary 5.4:

Suppose that c and u satisfy (2.27) and (2.28). Then $w(\gamma(y))$ is concave.

proof:

Expression (2.25) still determines $\frac{d^2}{dy^2} w(\gamma(y))$, only now $\beta''(y) = 0$. This yields the result.

QED.

In conclusion, one special case where $w(\gamma(y))$ is convex can be identified. This is where (2.14) - (2.17) are satisfied. I have not been able to identify any other simple class of cases where $w(\gamma(y))$ is convex. Furthermore for two equally plausible classes of cases $w(\gamma(y))$ is globally

concave. More generally it seems that $w(\gamma(y))$ will often be neither globally convex nor globally concave. Therefore it seems that other than for the special class of cases described by (2.14) - (2.17), the outcome functions will not generally be implementable by a linear menu of wage functions.

3. The Generalized Self-Selection Model

A. The Model

The standard self-selection model has no productive uncertainty in the sense that the agent deterministically chooses output. The generalized self-selection model allows for a particular form of productive uncertainty. Namely, the cost structure is assumed to be the same as in the previous section only now y is interpreted as the mean of the distribution of output.

Specifically, let x denote output. The agent deterministically chooses a level of production denoted by \hat{y} . However \hat{y} determines output probabilistically. Let $\hat{F}(x/\hat{y})$ and $\hat{f}(x/\hat{y})$ denote the distribution and density of output given \hat{y} . Let $\mu(\hat{y})$ denote the mean of output given \hat{y} and assume that μ is strictly increasing in \hat{y} . Finally let $\hat{c}(\hat{y}, z)$ denote the cost for a type z agent of choosing \hat{y} .

The above formulation is easily translated into one where the agent can be viewed as directly choosing y , the mean of output, instead of \hat{y} . This turns out to be notationally more convenient. Let ξ denote the inverse of μ . Then let $F(x/y)$ and $f(x/y)$ denote the distribution and density of x given y . They are defined by

$$(3.1) \quad F(x/y) = \hat{F}(x/\xi(y))$$
$$f(x/y) = \hat{f}(x/\xi(y)).$$

Finally let $c(y,z)$ denote the cost to an agent of type z of choosing a mean output of y . This is defined by

$$(3.2) \quad c(y,z) = \hat{c}(\xi(y),z)$$

Let $v(x)$ denote the principal's utility over output. Given $v(x)$, the principal's expected utility given the mean of the distribution is y can be defined. Let $u(y)$ denote this.

$$(3.3) \quad u(y) = \int v(x) f(x/y)dy$$

Assume that $c(y,z)$, $u(y)$, and $G(z)$ satisfy (A.1) - (A.14) as described in the previous section.

Thus the structure of the generalized model is very similar to that of the standard model. If the mean of the distribution, y , were observable there would be no essential difference.¹⁷ The principal could offer a wage contract $\varphi(y)$ depending on the mean of the distribution just as in the standard case. However, the principal cannot observe the mean. He can only observe x , the output which results. Therefore a contract is now defined as follows.

Definition: A contract in the generalized self-selection model is a set of real valued functions defined over output $\{\varphi_i(x)\}_{i \in I}$ where I is an index set.

This is interpreted as follows. An agent facing the contract $\{\varphi_i(x)\}_{i \in I}$ can elect to operate under one of the wage functions $\varphi_i(x)$, or to not

participate. In the former event the agent is paid according to the schedule $\psi_i(x)$. In the latter event both parties receive zero.¹⁸

B. The Optimal Contract

Some notation will be useful to describe the approach of this section. Let e be an ordered triple (c,u,F,G) describing the environment of the principal and agent. Let ψ denote a contract as described in Section 3.A above and let Ψ denote the set of all possible contracts. Then let $U(\psi,e)$ denote the principal's expected utility under contract ψ and environment e given that the agent acts to maximize his own expected utility.¹⁹ Define $U^N(e)$ to be the maximum expected utility the principal can attain under any contract given e .

$$(3.4) \quad U^N(e) = \sup_{\psi \in \Psi} U(\psi,e)$$

If a contract ψ attains the supremum it is an optimal contract.

Definition: ψ is an optimal contact for the environment e if

$$(3.5) \quad U^N(e) = U(\psi,e)$$

The superscript "N" is chosen to denote the fact that $U^N(e)$ is the maximum expected utility the principal can attain when y is not observable. It will also be useful to consider the artificial case where y is assumed to be observable by the principal. Define U^0 to be the maximum expected utility the principal can attain when y is observable. This is calculated as in Section 2. It is clear that the principal's welfare can be no larger when y is non-observable.

$$(3.6) \quad U^N(e) < U^0(e)$$

This is because the principal simply has one less piece of information to use when constructing an optimal contract. In fact, in general, one would expect there to be a welfare loss associated with the non-observability of y -- i.e. -- the inequality in (3.6) would be strict.

This motivates the following definition.

Definition: The environment e will be said to exhibit the property of irrelevance of mean observability (IMO) if there exists a contract φ such that

$$(3.7) \quad U(\varphi, e) = U^0(e) .$$

If a contract exists satisfying (3.7) it is clearly optimal. Furthermore there is no welfare loss to the principal associated with the non-observability of y -- i.e. -- the principal could not increase his expected utility even if y was observable and could be contracted upon.

If the principal is risk-neutral in output so that $v(x) = x$ the property of IMO can be given an even stronger interpretation. In this case, assuming that y is observable is equivalent to assuming that the distribution of x given y is degenerate (i.e. -- $F(x, y)$ simply exhibits a mass point on y) and there is no productive uncertainty. Thus if a contract exists such that (3.7) is satisfied, productive uncertainty is irrelevant in that the optimal contract and the expected utility of both the principal and agent remain unchanged no matter what the nature or amount of productive uncertainty.

Clearly IMO is a very strong property and one would not generally expect it to hold. Surprisingly enough, a class of environments can be identified where it does hold. These are the class of environments where, when y is observable, the optimal contract is convex. Proposition 6 contains the result. The optimal contract, for this case, is shown to be a menu of functions linear in output.

Proposition 6:

Fix an environment, e . Suppose that, when y is observable, $\{y(z), w(z)\}$ are the optimal outcome functions defined by (2.7) and (2.8) for some $z_0 \in [0,1]$. Suppose that $w(\gamma(y))$ is convex over $[z_0,1]$ so that the optimal outcome can be implemented by a menu of functions linear in y . Let

$$(3.8) \quad M = \{x_z(y)\}_{z \in [z_0,1]}$$

denote this menu of linear functions where x_z denotes the function chosen by type z .

Define the contract ψ to be the same menu of linear functions as in M only defined over x .

$$(3.9) \quad \psi = \{x_z(x)\}_{z \in [z_0,1]}$$

Then

- (i) Each type's choice of wage function and y is the same under M and ψ .

(ii) The principal's expected utility is the same under M and ψ .

(iii) Therefore e satisfies IMO and ψ is an optimal contract.

proof:

Every type of agent is risk neutral in income. Therefore since the expected value of x is y , an agent of type z views $\lambda_z(y)$ and $\lambda_z(x)$ as the same for every $\hat{z} \in [z_0, 1]$. Therefore each type of agent makes the same choice under M and ψ and receives the same expected payment.

QED.

Thus Proposition 6 describes the following test for determining whether a menu of linear contracts in x is optimal for a given generalized self-selection problem. First create a standard self-selection problem by assuming the mean of output is observable. Then calculate the optimal wage contract as a function of mean output for this standard self-selection problem. If this contract is convex then a menu of linear contracts is optimal.

Note that when y is observable \hat{y} is also observable and a contract could equally well be described as a function from \hat{y} to wages. It is interesting to apply the results of Proposition 6 to this situation. Suppose that $\hat{\phi}(\hat{y})$ is the optimal wage contract expressed as a function of \hat{y} . Then the equivalent contract expressed as a function of y is given by

$$(3.10) \quad \phi(y) = \hat{\phi}(\hat{\zeta}(y)).$$

Thus a menu of linear contracts is optimal if $\hat{\phi}(\xi(y))$ is a convex function of y . Differentiation of (3.10) yields

$$(3.11) \quad \phi''(y) = \hat{\phi}''(\xi(y)) \xi'(y)^2 + \hat{\phi}'(\xi(y)) \xi''(y).$$

In particular notice that if ξ is the convex (or equivalently if μ is concave) that the convexity of $\hat{\phi}$ implies the convexity of ϕ . This is summarized below in the following corollary.

Corollary 6.1

Fix an environment e . Suppose that when y and \hat{y} are observable $\hat{\phi}(\hat{y})$ is an optimal wage contract defined over \hat{y} . Suppose that $\mu(\hat{y})$ is concave. Then if $\hat{\phi}(\hat{y})$ is convex, the optimal outcome when y and \hat{y} are not observable can be implemented by a menu of linear functions in x .

proof

As above.

QED.

C. Previous Work

In this section the results of this paper will be related to those of Laffont and Tirole [1986], McAfee and McMillan [1986] and Melumad and Riechelstein [1986]. First consider Laffont and Tirole [1986]. Laffont and Tirole confine their analysis to the case defined by (2.14) - (2.17) where the optimal contract when y is observable is globally concave. Thus they correctly conclude in their model that the optimal contract always consists of a menu of linear contracts. In Section IV-G and in footnotes 15 and 16 of

their paper they suggest the geometric intuition formally modeled in this paper and speculate that the optimality of menus of linear contracts will continue to hold true under very general conditions. This paper shows that their result is much more special.

Laffont and Tirole's model appears to be somewhat different from that of this paper in three respects. However none of these differences significantly affects the analysis. First, they explicitly include the agent's effort choice as a variable. As explained in footnote 12, this approach is equivalent to that of this paper. Second, they maximize a weighted sum of the principal's and agent's expected utility instead of simply maximizing the principal's expected utility. It is straightforward to show that exactly the same arguments apply in either case. The approach of this paper was chosen so that the correspondence of the generalized and standard self-selection models would be clearest. Most analyses of the standard self-selection problem employ the approach of simply maximizing the principal's expected utility.

Finally, Laffont and Tirole allow for the possibility that the principal also controls some extra decision variable, d . They assume that the decision must be made immediately after the agent's menu selection and before the observation of x . In the notation of this paper this amounts to the following generalization of the model described in Section 3.A. The functions c , F , and G remain unchanged. However now the principal's utility is written as

$$(3.12) \quad v(x,d)$$

where d is a decision which must be made from some set D . Then let $u(y)$ denote the principal's maximum expected utility given the mean y . This is defined by

$$(3.13) \quad u(y) = \max_{d \in D} \int v(x,d) f(x/y) dx .$$

Then if $c, F, G,$ and u satisfy (A.1) - (A.14) the analysis of Section 3 applies in unchanged form. Thus this generalization changes nothing fundamental in the analysis. It only changes the definition of $u(y)$.²⁰

Drawing the correspondence between the model of this paper and that of Laffont and Tirole is somewhat confusing because in their interpretation of the model they use the term "cost" for an entirely different purpose than this paper does. To minimize this confusion, the correspondence will be explicitly described. Laffont and Tirole consider a principal who wishes to construct units of some good. For example perhaps the principal is the Navy which desires to construct a new jet. The per unit cost of construction is given by $(I-x)$ where I is the initial cost and x is the "output" of the agent. That is, the agent exerts effort to produce reductions in the unit cost of production. The agent chooses y at a personal cost of $c(y,z)$ which stochastically determines x according to $F(x/y)$. If a fixed number of units must be constructed this is precisely the generalized self-selection model as described in Section 3. Laffont and Tirole assume that (2.14) - (2.17) are satisfied so the optimality of a linear menu of contracts follows from the analysis of Section 3.

However, Laffont and Tirole also consider the possibility that the principal must choose the number of units to purchase. They assume that this decision must be made after the agent's menu selection is observed but before x is observed. Let d denote the number of units and let $S(d)$ denote the value of d units to the principal. In terms of the above notation in (3.12) and (3.13),

$$(3.14) \quad v(x,d) = S(d) - (I-x)d$$

and

$$(3.15) \quad u(y) = \max_{d \geq 0} S(d) - (I-y)d .$$

Then as described above, a menu of linear contracts is optimal if the optimal contract for the standard self-selection problem when y is observable (and the principal's utility is therefore given by (3.15)) is convex. Laffont and Tirole make sufficient assumptions on $S(d)$, $c(y,z)$, and $G(z)$ for this to be true.

Now consider McAfee and McMillan [1986]. Their condition (14) for the optimality of a menu of linear contracts is simply the requirement that the optimal wage contract as a function of y (calculated when y is observable) be convex. Because McAfee and McMillan choose notation which explicitly includes the agent's effort choice as a variable, a small amount of translation of notation is required to see this. A formal demonstration based on the translation described in footnote 12 is straightforward and will not be given. Instead a geometric explanation will be given. Consider Figure 1 where y is on the horizontal axis and w is on the vertical axis and the optimal contract is $\phi(y)$. Each agent chooses a value of y such that his indifference curve is tangent to $\phi(y)$. Furthermore from Proposition 1 higher types of agents choose higher values of y . Therefore $\phi(y)$ is convex if and only if higher types of agents have steeper indifference curves at the value of y they choose. This is precisely what condition (14) in McAfee and McMillan requires. It requires that the derivative with respect to z of some

complex function be positive. This complex function is easily seen to be the slope of type z 's indifference curve at the value of y he chooses.

The contribution of this paper is to reveal the geometric basis for the result and to show that the result follows largely from existing well-understood analysis of the standard self-selection problem together with this simple geometric argument. Given the complexity of the analysis in McAfee and McMillan, this is a large advantage. For example, McAfee and McMillan simply assume that the first-order approach is valid in their analysis. This paper shows that the validity of the first-order approach in this problem is equivalent to the validity of the first-order approach in the standard self-selection problem which is well-understood. Finally, McAfee and McMillan suggest that the optimality of linear menus of contracts will be a fairly typical result. This paper argues in a series of examples that the result does not hold in general.

The model of McAfee and McMillan differs from that of this paper in one other respect. McAfee and McMillan allow for the existence of more than one agent. The same correspondence between the standard and generalized self-selection problem can be established for this case.

Finally consider Melumad and Riechelstein [1986]. They formally show²¹ that the convexity property implies the optimality of a menu of linear contracts in a very general model with almost no structure. Because of the generality of their model, they were forced to restrict themselves to the case where all types of the agent are hired. In the more structured environment of this paper it is shown that the argument still holds when the possibility that not all types may be hired is allowed. In terms of the formal notation of this paper Melumad and Riechelstein assume that in the optimal contract z_0 equals 0. This paper allows z_0 to assume any value.

4. Conclusion

The optimality of a menu of linear wage functions in the generalized self-selection model really depends on the same property holding true in the standard self-selection problem generated by assuming that y is observable. This in turn depends on the convexity of the optimal contract. However the analysis of Section 2 shows that the optimal contract will not generally be globally convex in the standard self-selection model. One plausible class of examples exists where this property holds but equally plausible classes of examples exist where the exact opposite result holds -- i.e. -- the optimal contract is globally concave. More generally, the optimal contract will usually be neither globally concave nor globally convex. Therefore the optimality of menus of linear contracts in the generalized self-selection model is best viewed as a very special property holding only in a limited class of examples. In general, menus of linear contracts will not be optimal and there will be a welfare loss associated with the non-observability of y .

Appendix

Proof of Proposition 3

Suppose that $\{y(z), w(z)\}$ are the optimal outcome functions as defined by (2.7) and (2.8) for some $z_0 \in [0,1]$. Any optimal contract must be defined by (2.11) over $[y(z_0), y(1)]$. As explained in the body of the paper, one possible choice for an optimal contract would be to define ϕ to be zero for all other values of y . The first major step in the proof is to construct a different optimal contract. To do this the following Lemma is useful. Since this is a standard result in this type of model it will not be proven.

Lemma 1:

Suppose that (\hat{w}, \hat{y}) and (\hat{w}, \hat{y}) are two wage-output pairs and that $\hat{y} < \hat{y}$. Consider two types of agent, \hat{z} and \hat{z} , with $\hat{z} < \hat{z}$. Then

(i) If type \hat{z} prefers (\hat{w}, \hat{y}) , so does type \hat{z} -- i.e. --

$$\text{If } \hat{w} - c(\hat{y}, \hat{z}) \geq \hat{w} - c(\hat{y}, \hat{z}), \text{ then } \hat{w} - c(\hat{y}, \hat{z}) \geq \hat{w} - c(\hat{y}, \hat{z}).$$

(ii) If type \hat{z} prefers (\hat{w}, \hat{y}) , so does type \hat{z} -- i.e. --

$$\text{If } \hat{w} - c(\hat{y}, \hat{z}) \geq \hat{w} - c(\hat{y}, \hat{z}), \text{ then } \hat{w} - c(\hat{y}, \hat{z}) \geq \hat{w} - c(\hat{y}, \hat{z}).$$

This result basically says that higher types of agent are more pre-disposed towards producing more output because they have lower marginal costs of doing so.

Lemma 1 allows construction of the desired optimal contract which is described in Lemma 2.

Lemma 2:

The contract ϕ^* , constructed as follows, is an optimal contract.

$$\begin{aligned} & c(y, z_0), & y \leq y(z_0) \\ (a.1) \quad \phi^*(y) = & w(\gamma(y)), & y(z_0) \leq y \leq y(1) \\ & c(y, 1) + u^*, & y \geq y(1) \end{aligned}$$

where

$$(a.2) \quad u^* = w(1) - c(y(1), 1) .$$

proof:

Notice that $\phi^*(0) = 0$. Given this, by the definition of γ , we know that an agent of type z prefers $y(z)$ to choosing $y = 0$ or $y \in [y(z_0), y(1)]$. Therefore it is sufficient to show that an agent of type z also prefers to choose $y(z)$ over any $y > y(1)$ or $y \in (0, y(z_0))$.

First consider $y > y(1)$. The contract ϕ^* is constructed so a type 1 agent is indifferent between choosing any $y \in [y(1), \infty)$. All other types of agents prefer $y(z)$ to $y(1)$. Therefore by Lemma 1, all other types of agents prefer $y(z)$ to any $y \geq y(1)$.

Now consider $y < y(z_0)$. This must be divided into two sub-cases. First suppose that $z \geq z_0$. Then by an argument very similar to the above argument,

an agent of type z prefers $y(z)$ to any $y \leq y(z_0)$. Second, suppose that $z < z_0$. The contract ϕ^* is constructed so that an agent of type z_0 earns zero profits by choosing any $y \leq y(z_0)$. Therefore an agent of type $z < z_0$ must earn non-positive profits, since costs decrease in type. Therefore choosing $y = 0$ and receiving $w = 0$ is at least as preferable as choosing any other $y \leq y(z_0)$.

QED.

The contract $\phi^*(y)$ has two properties which are crucial for the proof. These are described and proven in Lemma 3.

Lemma 3:

- (i) ϕ^* is convex over $[0, y(z_0)]$ and $[y(1), \infty)$
- (ii) ϕ^* is continuously differentiable. (In particular, ϕ^* is continuously differentiable at $y(z_0)$ and $y(1)$.)

proof:

Property (i) is obvious. Now consider property (ii). The contract ϕ^* is obviously differentiable at all points except $y(z_0)$ and $y(1)$. Furthermore ϕ^* is constructed to be continuous at these two points. Therefore it only remains to show that the derivatives of ϕ^* calculated from the left and right are equal. This will be done for $y(z_0)$. The case of $y(1)$ is almost exactly the same.

For any $y \in [y(z_0), y(z_1)]$, the slope of ϕ^* is determined by the fact that the indifference curve of an agent of type $\gamma(y)$ is tangent to ϕ^* at y . That is,

$$(a.3) \quad \frac{d}{dy} w(\gamma(y)) = c_y(y(z), z)$$

for $z \geq z_0$.¹⁸ In particular, if $\phi^{*'}(y(z_0))$ is calculated from the right it therefore equals $c_y(y(z_0), z_0)$ according to (a.3). However this is the same value as to be had from calculating the derivative from the left according to (a.1).

QED.

Proposition 3 can now be proven. First suppose that $w(\gamma(y))$ is convex over $[y(z_0), y(1)]$. Then by Lemma 3, $\phi^*(y)$ is convex over $[0, \infty)$. Therefore it is geometrically clear that the menu of linear tangents to ϕ^* for $y \in [y(z_0), y(1)]$ implements the outcome.

The other direction of proof of Proposition 3 is straightforward and does not require the above construction. Suppose that the outcome can be implemented by a linear menu of contracts. Then consider any $\hat{y} \in [y(z_0), y(1)]$. There exists some \hat{z} such that $y(\hat{z}) = \hat{y}$. Therefore the menu must include a linear function $\hat{\lambda}(y)$ such that $\hat{\lambda}(\hat{y}) = w(\gamma(\hat{y}))$. Furthermore $\hat{\lambda}(y)$ must lie below $w(\gamma(y))$ for every $y \in [y(z_0), y(1)]$, or else an agent of some other type will not choose his output properly. Therefore $w(\gamma(y))$ is convex at \hat{y} .

QED.

Footnotes

- ¹ See, for example, Grossman and Hart [1983], Holmstrom [1979,1984], Mirrlees [1975, 1979], Rogerson [1985] and Shavell [1979].
- ² See Holmstrom and Milgrom [1985] for an exception.
- ³ Similar models have also been analyzed by Baron and Besanko [1985], and Riordin and Sappington [1986a, 1986b]. However these papers do not consider the issue of implementability by menus of linear contracts which is considered by Laffont and Tirole [1986], McAfee and McMillan [1986], Melumad and Riechelstein [1986] and which is the focus of this paper.
- ⁴ A specific version of the "standard self selection problem" will be described in Section 2. Essentially the same mathematical structure appears in self-selection models of the principal-agent relationship, a monopoly using product quality to price discriminate, trading problems, and taxation problems. See Baron and Myerson [1982], Guesnerie and Laffont [1984], Maskin and Riley [1984], Matthews and Moore [1986,1987], Mirrlees [1985], Myerson and Satterthwaite [1983], and Sappington [1983].
- ⁵ The process by which the standard self-selection problem underlying a given generalized self-selection problem is identified is simplified somewhat for the purposes of this introduction.
- ⁶ Melumad and Riechelstein [1986] also consider a range of other issues relating to the value of communication in this type of model which are not of direct concern to this paper. Also see Picard [1986].
- ⁷ McAfee and McMillan [1986], page 3.
- ⁸ See Section 3-C for a more detailed discussion.
- ⁹ For example, Myerson and Satterthwaite [1983].
- ¹⁰ For example, Mussa and Rosen [1978] and Matthews and Moore [1986,1987].
- ¹¹ Sappington employs a slightly different individual rationality constraint than the model of this paper. However, his discussion and interpretation of the model is relevant in all other respects.
- ¹² This formulation suppresses explicit consideration of the agent's effort choice. This turns out to require the least cumbersome notation. It can be formally derived from a formulation which explicitly includes the agent's effort choice as follows. Suppose that an agent of type z exerting effort e produces output according to

$$y = \gamma(e,z) .$$

An agent of type z exerting effort e and who receives w dollars has utility of

$$w - \varphi(e,z) .$$

Suppose that γ is strictly increasing in e . Let

$$e = \varphi(y, z)$$

denote the inverse of γ . Then $c(y, z)$ is defined by

$$c(y, z) = \varphi(\varphi(y, z), z).$$

¹³See Myerson [1981] and Baron and Myerson [1982] for a more extensive discussion of this.

¹⁴There may exist two different values of z , \hat{z} and \hat{z} , such that $\hat{y} = y(\hat{z})$ and $\hat{y} = y(\hat{z})$. However, then by

$$IC, w(\hat{z}) = w(\hat{z}).$$

¹⁵See the references listed in footnote 4 for various treatments of this result. The only technically delicate issue is establishing absolute continuity of certain functions. See Guesnerie and Laffont [1984], Mirrlees [1985] and Matthews and Moore [1987] for careful discussions of this.

¹⁶The extra difficulty lies in showing that agents of type less than z_0 will not find it attractive to choose one of the linear functions and produce some value of $y \in (0, y(z_0))$ instead of declining to participate.

¹⁷Notice that observation of one of \hat{y} and y immediately also reveals the other since they are related by the invertible function $y = \mu(y)$. Therefore observability of either variable will be spoken of as observability of the mean.

¹⁸An interesting issue investigated by Melumad and Riechelstein [1986] and Picard [1986] concerns whether or not offering a menu of contracts is even necessary for the generalized self-selection model. See these papers for further discussion.

¹⁹If a type of agent is indifferent between two or more actions, assume the agent chooses the one the principal prefers. For some values of ψ there will be no optimal action for some types of agents. Define $U(\psi, e)$ to be $-\infty$ in this case.

²⁰It may be that u is not always convex under natural economic assumptions in this generalized model. This can create extra technical difficulties in the analysis of the standard self-selection problem.

²¹See Corollary to Theorem 2, page 26.

²²This can also be directly calculated using (2.7) and (2.8).

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