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## The Welfare Cost of Factor Taxation in a Perfect Foresight Model

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DISCUSSION PAPER NO. 643

THE WELFARE COST OF FACTOR TAXATION IN A PERFECT FORESIGHT MODEL\*

by

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## Abstract

This paper determines the marginal welfare cost of various taxes and their impact on resource allocation in a dynamic general equilibrium model where capital accumulation is determined by intertemporal maximizing behavior. First we derive the crucial formulae for the excess burdens and impacts of anticipated and unanticipated, temporary and permanent, tax policies. Second, using a range of estimates for taste and technology parameters suggested by the empirical literature, we compute various examples, finding that excess burdens are large, very sensitive to parameter estimates, and also sensitive to anticipation effects. However, the rankings of the tax policies turn out to be insensitive to these estimates.

## 1. Introduction

A major problem in public finance is the incidence and welfare cost of various taxes. This paper examines these issues in a representative agent perfect foresight model of equilibrium growth, accomplishing two tasks. First, it analytically derives formulae for the exact marginal welfare costs of various tax policy changes, both unanticipated and partially anticipated. Second, it combines these formulae with current econometric estimates of the critical structural parameters to give examples of the magnitude of these excess burdens. The analysis indicates that the marginal cost of factor taxation may be higher than indicated by earlier analyses and indicates that the differential costs among various instruments may be quite large. It also shows that anticipation aspects of tax policies are as important as the values of the relevant demand and supply elasticities in determining the costs of taxation.

The approach of this model differs from previous work in several important fashions. First, we study a dynamic general equilibrium where infinite-life agents maximize an intertemporal utility function. This feature distinguishes it from the static analyses of Browning (1976) and Stuart (1984), the savings-rate function approach of Feldstein (1974a,b) and Bernheim (1981), and from the model of Fullerton-King-Shoven-Whalley (1981) and Ballard-Shoven-Whalley (1985), which puts savings in the utility function. These models either ignore capital accumulation or treat it in an ad hoc fashion. We use the representative infinitely-lived agent model instead of the life-cycle model used in Auerbach-Kotlikoff (1983) and Summers (1981), implicitly adopting the Barro (1975) treatment of bequests. Since evidence concerning bequests often indicate that they are of significant importance (for example, see Kotlikoff-Summers (1981)) this model is an appropriate

benchmark to study, and compare with the overlapping generations analyses. Also, the equilibrium is unique, whereas overlapping generations models often have a continuum of equilibria, making comparative dynamic exercises impossible.

Second, we derive analytical expressions for the transition paths resulting from a change in tax policy, allowing us to compute the exact marginal impacts and efficiency cost of raising revenue for each instrument. This distinguishes our analysis from the numerical approximations of nonmarginal changes by Auerbach-Kotlikoff-Skinner (1983) and the quadratic approximation of Chamley (1981).

Third, we can examine anticipation effects and study temporary tax changes or phased-in tax reforms. These effects do not exist in any model where current savings is unaffected by any net rate other than the current net rate of return, as in Feldstein (1974), Fullerton, et. al., and Ballard, et. al. Since anticipation effects are often used in discussions of tax policy, it is better to examine them instead of ruling them out a priori.

Using a broad range of parameterizations suggested by examination of the current U.S. tax laws and by the econometric literature on aggregate consumption demand, labor supply, and factor substitutability in production, we find a number of interesting and suggestive results. First, for these parameter values, the welfare gain of an immediate and permanent cut in capital taxation is substantial, being at least 25 cents per dollar of lost capital tax revenue when factors are only poor substitutes ranging up to eight dollars per dollar of lost capital tax revenue when higher estimates of factor substitutability are used. These efficiency losses are at least doubled when the impacts on labor tax revenues are taken into account. Overall, a conservative value of the marginal cost of capital income taxation, at a tax

rate of .5, is about one dollar per dollar of revenue gain, exceeding the range given by the roughly comparable analysis of Ballard-Shoven-Whalley. The calculations concerning the marginal cost of labor taxation, at a tax rate of .3, also exceeds earlier estimates, in particular, those of Browning, Stuart, and Ballard-Shoven-Whalley, even when comparably parameterized.

Even more striking are the results for the investment tax credit, a policy parameter ignored in those earlier examinations. Over this range of parameterizations, we find that the efficiency gain of a dollar in extra permanent investment tax credits is at least a dollar, more likely two to three dollars. In fact, an increase in the investment tax credit quite plausibly pays for itself since the induced capital formation may increase factor income tax revenues sufficient to pay for the increased tax credits. In particular, temporary investment tax credits are the most likely to be self-financing since anticipated future investment tax credits depress investment in the short run. Also, this shows that these self-financing results do not rely on intergenerational differences in marginal propensities to consume, as Auerbach and Kotlikoff (1983) claim in their discussion of similar results in an overlapping generations context.

While the level of excess burden for various taxes cannot be determined with precision due to our imprecise knowledge of the important structural parameters, several qualitative conclusions are robust. Over this same broad range of parameterizations, labor taxation has an efficiency cost substantially lower than capital taxation, even when we use the most extreme estimates of labor supply elasticities. In this model, using any proposed estimates for taste and technology parameters, welfare would be improved substantially at the margin by moving away from capital income taxation and towards higher labor taxation and more investment subsidies at current tax

levels. This conclusion is, of course, purely an efficiency result since it ignores redistribution, as do earlier analyses.

One particularly valuable feature of our analysis, and missing in earlier examinations of marginal excess burden, is the ease with which anticipation effects of tax policies may be analyzed. We find that anticipation effects are of substantial importance with these parameterizations. As expected, an unanticipated short-lived temporary tax on capital income is effectively a lump-sum tax. However, we find that the efficiency cost of capital taxation increases rapidly as the tax increase becomes more anticipated and as the duration of the tax increase lengthens. Somewhat less expected is the fact that the opposite is true for the investment tax credit, with anticipated tax credits being far less valuable than unanticipated credits. This difference is due to the fact that any capital income tax change, current or future, reduces current investment, whereas any extra future investment tax credit increase will depress current investment, but a current increase will encourage investment. Therefore, temporary investment tax credits are more efficient than permanent tax credits.

Most surprising is the finding that the efficiency cost of labor taxation is substantially affected by anticipation, with anticipated future labor taxation usually being less distortionary than immediate labor taxation. This is due to the fact that future labor taxation often encourages capital formation initially since the negative income effect increases labor supply and investment, and reduces current consumption, whereas an immediate and permanent increase in labor taxation immediately reduces labor supply, the rate of return to investment, and investment. In general, one finds that the timing of a change in any of these tax policies is as crucial in determining the marginal cost of tax revenue as are the underlying structural parameters

and the specific tax instrument yielding the extra dollar.

Given these results concerning the large difference between the value of changing various instruments, one is initially struck by the apparent irrationality of the current tax structure. However, when we take into account the fact that current policymakers have control only over the current tax structure, we find that the current mix between labor and capital income taxation may not be grossly irrational in this model since the welfare gains of short-term changes in labor and capital taxation are similar. This suggests that the inability of current policymakers to determine future tax policies consistency has a substantial impact on the long-run structure of taxation.

## 2. The Model

We assume that all agents have an intertemporal utility function over consumption paths,  $c(t)$ , and labor paths,  $l(t)$ :

$$U = \int_0^{\infty} e^{-\rho t} u(c, l) dt$$

where  $u$  is a concave in consumption and labor with  $u_1 > 0 > u_2$ . We assume that the net production function,  $F(K, l)$ , is concave in capital and labor and displays constant returns to scale. Agents hold two perfectly substitutable assets, capital stock and government bonds. For our purposes it is sufficient to assume that either the initial stock of bonds is zero or bonds are continuously rolled over. In both cases we can ignore the bond market in examining the real evolution of the economy (see Judd 1983a or Brock and Turnovsky 1982). Taxes are assessed on capital income at the rate of  $\tau_K$  (we assume true economic depreciation) and on labor income at  $\tau_L$ . An investment credit  $\theta$  on gross capital investment and lump sum transfers of  $Tr$  are made



each period. The gross-of-tax returns on labor and capital are  $w$  and  $r$ , respectively.

Since bonds and capital are perfect substitutes, we can determine the intertemporal demand for consumption and leisure by examining a representative agent who holds only capital. This representative agent's problem is

$$\text{Max}_{c,\ell} \int_0^{\infty} e^{-\rho t} u(c,\ell) dt$$

$$\dot{K} = (1 - \tau_K)rK + (1 - \tau_L)w\ell - c + \theta(\delta K + \dot{K}) + Tr$$

where  $\delta$  is the rate of depreciation. If  $\lambda(t)$  is the private marginal value of capital at  $t$ , the Hamiltonian equations describing the agent's choice are

$$(1a) \quad \dot{\lambda} = \lambda \left( \rho - \frac{r(1 - \tau_K) + \delta\theta}{1 - \theta} \right)$$

$$(1b) \quad \dot{K} = [(1 - \tau_K)rK + (1 - \tau_L)w\ell - c + Tr + \delta\theta K]/(1 - \theta)$$

$$(1c) \quad u_1(c,\ell) = \lambda/(1 - \theta)$$

$$(1d) \quad -\frac{u_2(c,\ell)}{u_1(c,\ell)} = w(1 - \tau_L)$$

In equilibrium, both factors must receive their marginal product:

$$(2a) \quad F_K = r = f'(k)$$

$$(2b) \quad F_\ell = w = f - kf'(k)$$

where  $k \equiv K/\ell$  is the capital-labor ratio and  $f(k) \equiv f(K,\ell)/\ell$  is output per unit of labor input. This implies the general equilibrium equations

$$(3a) \quad \dot{\lambda} = \lambda[\rho - (f'(k)(1 - \tau_K) + \delta\theta)/(1-\theta))]$$

$$(3b) \quad \dot{K} = L(\lambda, K, \tau_L, \theta)f(k) - C(\lambda, K, \tau_L, \theta)$$

where  $k=K/L$ , and  $L(\lambda, K, \tau_L, \theta)$  and  $C(\lambda, K, \tau_L, \theta)$  are labor supply and consumption demand for fixed  $K$ ,  $\lambda$ ,  $\tau_L$ , and  $\theta$ , determined by the first-order conditions:

$$(4a) \quad u_1(C, L) = \lambda/(1 - \theta)$$

$$(4b) \quad -\frac{u_2(C, L)}{u_1(C, L)} = F_L(K, L)(1 - \tau_L)$$

In order to analyze changes in taxes, we need to know how  $C$  and  $L$  are affected by changes in  $\lambda$ ,  $K$ ,  $\tau_L$ , and  $\theta$ . We need compute only the first order properties of  $C$  and  $L$ . If we define  $\beta$  to be the intertemporal elasticity of consumption demand,  $\eta$  the compensated elasticity of labor supply,  $v$  the intertemporal elasticity of labor supply, and  $\alpha$  the elasticity of consumption with respect to the contemporaneous wage and use the notation  $\hat{x} = dx/x$ , we may represent the local dependence of consumption demand and labor supply on the net wage and opportunity cost of consumption as

$$(6a) \quad \hat{C} = \alpha(\hat{w} - \frac{d\tau_L}{1 - \tau_L}) + \beta(\hat{\lambda} + \frac{d\theta}{1 - \theta})$$

$$(6b) \quad \hat{L} = \eta(\hat{w} - \frac{d\tau_L}{1 - \tau_L}) + v(\hat{\lambda} + \frac{d\theta}{1 - \theta})$$

From the definition of  $\sigma$ , the elasticity of substitution between capital and labor in net output, and letting  $\theta_K$  and  $\theta_L$  denote the capital and labor shares of income, respectively, we also know that

$$(6c) \quad \hat{K} - \hat{L} = \sigma(\hat{w} - \hat{r})$$

The final local expression, (6d), follows from  $F_L(K, l) = w$ , the demand curve for labor.

$$(6d) \quad \hat{w} = \frac{\theta_K}{\sigma} (\hat{K} - \hat{L})$$

From (6), we can solve for the instantaneous equilibrium responses of  $C$ ,  $L$ ,  $w$ , and  $r$  to  $K$ ,  $\lambda$ ,  $\tau_L$ , and  $\theta$ . Rewriting 6(b,d) in matrix form, we have

$$(7) \quad \begin{pmatrix} 1 & -\eta \\ \frac{\theta_K}{\sigma} & 1 \end{pmatrix} \begin{pmatrix} \hat{L} \\ \hat{w} \end{pmatrix} = \begin{pmatrix} v(\hat{\lambda} + \frac{d\theta}{1-\theta}) - \eta \frac{d\tau_L}{1-\tau_L} \\ \frac{\theta_K}{\sigma} \hat{K} \end{pmatrix}$$

Solving for  $\hat{L}$  and  $\hat{w}$ , from (7) we have

$$(8a) \quad \hat{L} = v^e(\hat{\lambda} + \frac{d\theta}{1-\theta}) - \eta^e(\frac{d\tau_L}{1-\tau_L} - \frac{\theta_K}{\sigma} \hat{K})$$

$$(8b) \quad \hat{w} = -\frac{\theta_K}{\sigma} [v^e(\hat{\lambda} + \frac{d\theta}{1-\theta}) - \eta^e \frac{d\tau_L}{1-\tau_L} - \hat{K}]$$

where

$$(9) \quad v^e \equiv \frac{v}{1 + \eta\theta_K/\sigma}, \quad \eta^e \equiv \frac{\eta}{1 + \eta\theta_K/\sigma}$$

that is,  $\eta^e$  and  $v^e$  represent the net response of labor supply to changes in the price of consumption, and capital stock and wage taxes, respectively, after one takes into account the change in the wage induced by the change in labor supply.  $\hat{r}$  and  $\hat{C}$  are found by substituting (8) into 6(a,c).

Suppose that the economy is in the steady state corresponding to a set of

constant government policy parameters,  $\tau_K$ ,  $\tau_L$ ,  $\theta$ ,  $\bar{G}$ , and  $\overline{Tr}$ . We will model a policy change by examining the response of the economy to the announcement and implementation of a "small" policy change. That new policy is expressed by

$$(11a) \quad \tau_K^*(t) = \tau_K + \varepsilon h_K(t)$$

$$(11b) \quad \tau_L^*(t) = \tau_L + \varepsilon h_L(t)$$

$$(11c) \quad \theta^*(t) = \theta + \varepsilon z(t)$$

$$(11d) \quad G^*(t) = \bar{G} + \varepsilon g(t)$$

$$(11e) \quad Tr^*(t) = \overline{Tr} + \varepsilon Tr(t)$$

where  $\varepsilon$  is understood to be small. If we substitute policies (11) into the equilibrium equations (3), we will have solutions for  $K$  and  $\lambda$  for each  $\varepsilon$ , denoted by  $K(t, \varepsilon)$  and  $\lambda(t, \varepsilon)$ , respectively. We will want to know how these small policy changes will affect the economy. This is best done by examining the impact on the paths of  $\lambda$  and  $K$ . Define the perturbations of  $\lambda$  and  $K$  caused by the policy change as follows:

$$\begin{aligned} \lambda_\varepsilon(t) &= \frac{\partial}{\partial \varepsilon} \lambda(t, 0) & K_\varepsilon(t) &= \frac{\partial}{\partial \varepsilon} K(t, 0) \\ \dot{\lambda}_\varepsilon(t) &= \frac{\partial}{\partial \varepsilon} \frac{\partial}{\partial t} \lambda(t, 0) & \dot{K}_\varepsilon(t) &= \frac{\partial}{\partial \varepsilon} \frac{\partial}{\partial t} K(t, 0) \end{aligned}$$

We similarly define  $l(t, \varepsilon)$ ,  $l_\varepsilon(t, \varepsilon)$ ,  $w(t, \varepsilon)$ , and  $w_\varepsilon(t, \varepsilon)$  to represent labor supply, the labor supply change, wage rate and its rate of change. If we substitute (11) into (3), differentiate the result with respect to  $\varepsilon$  and evaluate at  $\varepsilon = 0$ , we find that  $\lambda_\varepsilon$  and  $K_\varepsilon$  solve

$$(13a) \quad \frac{\dot{\lambda}_\varepsilon}{\lambda} = - \frac{(1 - \tau_K)}{(1 - \theta)} k f'' (\hat{K} - \hat{\lambda}) - \frac{(\rho + \delta)z(t)}{1 - \theta} + \frac{f'}{1 - \theta} h_K(t)$$

$$(13b) \quad \dot{K}_\varepsilon = F(\theta_L \hat{\lambda} + \theta_K \hat{K}) - \beta c(\hat{\lambda} + \frac{z(t)}{1 - \theta}) - g(t) - \alpha c(\hat{w} - \frac{h_L(t)}{1 - \tau_L})$$

where  $\hat{x}$  is understood to mean  $x_\varepsilon(t)/x(0)$ .

Using equations (8) and (9), we can write (13) as a linear differential equation:

$$(14) \quad \begin{pmatrix} \frac{\dot{\lambda}_\varepsilon}{\lambda} \\ \frac{\dot{K}_\varepsilon}{F} \end{pmatrix} = J \begin{pmatrix} \frac{\lambda_\varepsilon}{\lambda} \\ \frac{K_\varepsilon}{F} \end{pmatrix} + \vec{v}(t)$$

where

$$v_1(t) = f' \frac{1 - \tau_K}{1 - \theta} \frac{\theta_L}{\sigma} (\eta^e \frac{h_L(t)}{1 - \tau_L} - v^e \frac{z(t)}{1 - \theta}) - (\rho + \delta) \frac{z(t)}{1 - \theta} + f' \frac{h_K(t)}{1 - \theta}$$

$$v_2(t) = \theta_L (v^e \frac{z(t)}{1 - \theta} - \eta^e \frac{h_L(t)}{1 - \tau_L}) + \alpha \theta_c \{ \frac{h_L(t)}{1 - \tau_L} + \frac{\theta_K}{\sigma} (v^e \frac{z(t)}{1 - \theta} - \eta^e \frac{h_L(t)}{1 - \tau_L}) \}$$

$$- \beta \theta_c \frac{z(t)}{1 - \theta} - \frac{g(t)}{F}$$

and J is the Jacobian matrix of the system (3) evaluated at the initial steady-state values of K and  $\lambda$ , and the initial tax rates:

$$(15) \quad J \equiv \begin{pmatrix} - \frac{1 - \tau_K}{1 - \theta} \frac{\theta_L}{\sigma} v^e f' & \frac{1 - \tau_K}{1 - \theta} \frac{\theta_L}{\sigma \theta_K} (1 - \frac{\theta_K}{\sigma} \eta^e) f' f' \\ - \beta \theta_c + \theta_L v^e + \alpha v^e \frac{\theta_K}{\sigma} \theta_c & f' (1 + \frac{\theta_L}{\sigma} \eta^e - \alpha \frac{\theta_c}{\sigma} (1 + \eta \theta_K / \sigma)^{-1}) \end{pmatrix}$$

and  $\theta_c$  is the share of net output going to private consumption in the steady

state.

Define, for  $s > 0$

$$\Lambda_{\epsilon}(s) \equiv \int_0^{\infty} e^{-st} \lambda_{\epsilon}(t) dt$$

$$X_{\epsilon}(s) \equiv \int_0^{\infty} e^{-st} K_{\epsilon}(t) dt$$

to be the Laplace transforms of  $\lambda_{\epsilon}$  and  $K_{\epsilon}$ , respectively<sup>1</sup>. Taking the Laplace transform of (14) and solving the resulting algebraic equation for each  $s$  yields the solution for  $\Lambda_{\epsilon}(s)$  and  $X_{\epsilon}(s)$ :

$$(16) \quad \begin{pmatrix} \frac{\Lambda_{\epsilon}(s)}{\lambda} \\ \frac{X_{\epsilon}(s)}{F} \end{pmatrix} = (sI - J)^{-1} \left( V(s) + \begin{pmatrix} \lambda_{\epsilon}(0)/\lambda \\ 0 \end{pmatrix} \right)$$

where  $V(s)$  is the Laplace transform of  $v(t)$  evaluated at  $s$ .

We shall assume that leisure is normal, that is,  $v > 0$ . Therefore,  $\det(J) < 0$  and  $J$  has two eigenvalues of opposite signs,  $\zeta < 0 < \mu$ . Since this is a saddle-point stable system and the capital stock is bounded for any  $\epsilon$ , the shadow price of capital moves smoothly in  $\epsilon$  to insure  $X_{\epsilon}(\mu) < \infty$ . (See Judd (forthcoming)). Therefore

$$(17) \quad \frac{\lambda_{\epsilon}(0)}{\lambda} = \frac{J_{11} - \mu}{J_{21}} \left\{ \theta_L \left( v^e \frac{Z(\mu)}{1 - \theta} - \eta^e \frac{H_L(\mu)}{1 - \tau_L} \right) + \alpha \theta_c \left\{ \left( \frac{H_L(\mu)}{1 - \tau_L} \right) (1 - \eta^e \frac{\theta_K}{\sigma}) + v^e \frac{\theta_K Z(\mu)}{\sigma(1 - \theta)} \right\} \right\}$$

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<sup>1</sup>The Laplace transform of function of  $t$ ,  $f(t)$ , is a function of  $s$ ,  $F(s)$ , defined to be the present value of  $f(t)$  discounted at the rate  $s$ , as indicated in the definitions of  $\Lambda_{\epsilon}$  and  $X_{\epsilon}$ . In general,  $F(s)$  is defined only for sufficiently large  $s$ , however any positive  $s$  will be valid in this application since all functions will be bounded. The crucial property used below is that the Laplace transform of  $f'(t)$  is  $sF(s) - f(0)$ .

$$\begin{aligned}
 & -\beta\theta_c \left\{ \frac{Z(\mu)}{1-\theta} - \frac{G(\mu)}{F} + \frac{K_\varepsilon(0)}{F} \right\} \\
 & - f' \frac{1-\tau_K}{1-\theta} \frac{\theta_L}{\sigma} \left( \eta^e \frac{H_L(\mu)}{1-\tau_L} - v^e \frac{Z(\mu)}{1-\theta} \right) \\
 & + (\rho + \delta) \frac{Z(\mu)}{1-\theta} - f' \frac{H_K(\mu)}{1-\theta}
 \end{aligned}$$

where  $H_K$ ,  $H_L$ ,  $Z$ , and  $G$  are the Laplace transforms of  $h_K$ ,  $h_L$ ,  $z$ , and  $g$ , respectively. With (17) giving  $\lambda_\varepsilon(0)$ , (16) expresses the complete solution for  $\Lambda_\varepsilon$  and  $X_\varepsilon$ . This is all the information about transition paths we need.

(17) gives the impact of the policy change on the private shadow value of capital. First, examination of the  $H_K(\mu)$  term shows that any future increase in  $\tau_K$  reduces  $\lambda$  at  $t = 0$ . From (6) and (14), this shows that consumption will drop and labor supply will rise in response, with the net effect reducing investment. Similarly, one sees that a current investment tax credit increase will encourage investment whereas future increases may reduce investment, forcing us to examine reasonable parameter values to determine which direction is most plausible. The impact of future wage taxation is similarly ambiguous. However, these impacts on the shadow value of capital will affect the supply of capital and efficiency in a distorted capital market.

We next determine the impacts on revenue and lifetime utility. Let  $\Psi_\varepsilon(s)$  and  $\Gamma_\varepsilon(s)$  denote the sums, discounted at the rate  $s$ , of the changes in consumption and labor supply, respectively. Brock and Turnovsky (1981) show that if the initial stock of bonds is zero the government budget constraint is

$$(18) \quad 0 = \int_0^\infty \left( (g + \text{Tr} - \tau_K^{\text{KF}} K - \tau_L^{\text{LF}} L + \theta(\delta K + \dot{K})) e^{-\int_0^s \psi(\tau) d\tau} \right) ds$$

where the tax and spending rates are arbitrary functions of time and

$$(19) \quad \phi(\tau) = \rho - \dot{p}/p, \quad p = u_1(C, \lambda)$$

represents the intemporal prices determined in the bond market. When we substitute our policy shocks, (11), into the budget constraint and differentiate the result with respect to  $\varepsilon$ , we find (20), the condition that the budget constraint imposes on our shocks:

$$(20) \quad 0 = Kf'(k)H_K(\rho) + \lambda(f(k) - kf'(k))H_L(\rho) + \tau_L(f(k) - kf'(k))\Gamma_\varepsilon(\rho) \\ - \delta KZ(\rho) - G(\rho) + \tau_K f'(k)X_\varepsilon(\rho) - \theta(\rho + \delta)X_\varepsilon(\rho) \\ + Kf''(k)(\tau_K - \tau_L)\left(\frac{X_\varepsilon(\rho)}{K} - \frac{\Gamma_\varepsilon(\rho)}{L}\right)\frac{K}{L} - TR(\rho)$$

where  $TR(\rho)$  and  $G(\rho)$  are the present values of  $Tr(t)$  and  $g(t)$  discounted at  $\rho$ , respectively. The key point to note is that when we compare the present values of revenue changes and expenditure changes, the appropriate rate of discount is  $\rho$ , the after-tax rate of return.<sup>2</sup> Also note that revenue in these expressions include only capital taxation revenues on privately-issued assets, physical capital and privately-issued bonds backed by such assets. If we were to include tax collections on debt paid on government debt, then the appropriate rate of discount is the marginal product of capital. However, that procedure would be less direct since it would necessitate the solution to the path of government debt issue.

The change in welfare, in terms of the consumption good at  $t = 0$ , is

$$(21) \quad \frac{dU}{u_1} = \Psi_\varepsilon(\rho) + \frac{u_2}{u_1}\Gamma_\varepsilon(\rho)$$

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<sup>2</sup>After writing this paper, the author became aware of Chamley's (1982) attempt to generalize Judd (1981) to handle the case of elastic labor supply. However, he used  $f'$  to discount revenue streams, invalidating all his consumption equivalent excess burden calculations.



$$\begin{aligned}
 &= \int_0^{\infty} e^{-\rho t} [F_K K_{\varepsilon} - \delta K_{\varepsilon} - \dot{K}_{\varepsilon} + F_L L_{\varepsilon} + \frac{u_2}{u_1} L_{\varepsilon}] dt \\
 &= (F_K - \rho) X_{\varepsilon}(\rho) + (F_L - \bar{w}) \Gamma_{\varepsilon}(\rho) + K_{\varepsilon}(0)
 \end{aligned}$$

where  $\bar{w}$  equals the net-of-tax wage. This is the classic form for change in welfare, where we have expressed the change in welfare in terms of the gap between marginal cost and marginal benefits, and the change in quantities.

In our computed examples below we will need a measure of revenue. "dR" will be the present value of the change in revenue from taxation of labor income and income from physical capital or privately-issued debt, not privately-held public debt, as a fraction of output. Since we assume a bond market, the timing of the lump-sum rebates is immaterial. As always in these problems there is a problem concerning the choice of discount rates. This issue is settled by examining the balanced budget condition (20). This condition says that  $TR(\rho)$ , the present value of changes in transfers discounted at  $\rho$ , the after-tax interest rate, must equal the change in tax revenues, discounted at  $\rho$ . Since this implies that the change in taxes can finance a constant increment in transfers of  $\rho TR(\rho)$ ,  $\rho dR$  is the constant change in lump-sum transfers which would bring the budget back into balance after the enactment of a change in some tax parameter.

The change in utility from a tax change is equivalent to consumption being incremented by a constant equal to  $\rho dU/u_1$ . We define the marginal deadweight loss, MDWL, to be the ratio between the change in real income due to the tax change and the constant increment in the transfer financed by the tax change,  $(\rho dU/u_1)/dR$ . When we discuss any of the tax parameters, MDWL is also the welfare change of using a distortionary tax instead of a constant one dollar lump-sum tax. MDWL is negative if revenue and welfare move in opposite directions, the nonperverse case.

We must note that, if the extra revenue were spent on government consumption, MDWL is not the loss in real income from private consumption in excess of the direct resource cost. If the extra revenue is used to increase government consumption and labor is a normal good, then the welfare loss of the direct resource cost will cause labor supply, investment, and the steady-state capital stock to increase, reducing the distortions and increasing revenues. In Judd (1984), a discussion of cost-benefit analysis in this framework, we find that this effect is not trivial.

Our MDWL figure also is not the compensating capital stock change at  $t=0$ . Straightforward calculations show that if the capital stock is shocked by  $K_\epsilon(0)$  at 0, then

$$(22) \quad \text{MUK} \equiv \frac{\rho}{\rho - \lambda} \left[ \frac{\tau_K}{1 - \tau_K} + \tau_L \theta_L \left( \frac{v^e (J_{11} - \mu)}{J_{21} \rho} + \frac{\eta^e}{\sigma(1 - \tau_K)} \right) \right] + 1$$

is the capitalized marginal utility gain per unit of capital. If taxes were absent then  $\text{MUK} = 1$ . Otherwise, lifetime welfare is affected as if consumption were increased by  $\rho \text{MUK}$  units per period per unit increase in  $K$  at  $t = 0$ , holding labor supply and investment fixed. With taxes, the utility gain equals the direct improvement plus the welfare impacts due to marginal factor supply changes which are not neutral in the presence of taxes. The net welfare impact of these changes is ambiguous, the ambiguity being reflected in the expression for  $\text{MUK}$ . The increased capital stock raises output which when consumed yields an efficiency gain of  $F_K - \rho$  per unit of extra output. It also raises wages and labor supply, reflected in the term containing  $\eta^e$ , and also lowers the marginal utility of consumption, affecting labor supply through  $v^e$ , the equilibrium intertemporal elasticity. Since  $\eta > 0$  and  $\tau_L > 0$ , the increase in wages increases labor supply and efficiency, but the fall in  $\lambda$

will reduce labor supply and efficiency if  $v^e > 0$ , this fall reflected by the negative value of  $(J_{11} - \mu)J_{21}^{-1}$ . The net labor efficiency of effect is ambiguous, with both signs realized in our parameterizations.

Given MDWL and MUK, the shock to capital stock at  $t=0$  which would compensate for the utility impact of a tax change is  $-MDWL/MUK$  per dollar of extra revenue.

This completes the formal derivation of the impact of local tax changes around a steady state of our economy. We next examine MDWL for a broad range of parameter values meant to encompass views of the U.S. economy.

### 3. Parameterized Examples

In this section and Tables I through VIII we discuss examples of the magnitude of revenue changes, short-run effects, and marginal excess burdens under various assumptions concerning tax rates and their changes, capital-labor substitutability, labor supply, and consumption demand. In all tables we set capital share at .25 and assume capital consumption allowances to be .12 of net product, numbers suggested by casual examination of national income accounts. These numbers implicitly ignore consumer durables, an appropriate assumption here since their services are not taxed. We assume that consumption equals .8 of net output, a figure suggested by the national income accounts. Except where noted, the results below are insensitive to reasonable changes in capital and depreciation share assumptions, especially when compared the sensitivity to most other parameters.

Opinions differ substantially on the recent level of the effective tax rate on capital income. Feldstein, Dicks-Mireaux, and Poterba (1983) argue for rates in the U.S. ranging from .6 to .85 during the 1970s, whereas Fullerton and King (1984) argue for about .35 during the 1970s and .3 after the revisions of 1981 and 1982. We choose to be neutral in the disagreement

and use these estimates as bounds on the "truth". There is less disagreement concerning  $\tau_L$ , so we shall assume  $\tau_L = .3$  in most tables.

First, we present Tables I, II, and III to point out some basic features when  $u$  is of the form  $c^\beta + \gamma l^\eta$ , where  $\gamma$  is chosen to assure  $\theta_L = .75$ . This is a useful case to examine since  $\beta$  and  $\eta$  can be fixed independent of tax rates and each other, allowing us to hold other parameters fixed in examining changes in individual parameters. Since differentiation would be an excessively tedious fashion to elicit the dependence of MDWL on these structural parameters, Tables I through III are presented as a substitute. In these tables, we set  $\tau_K = .5$  and  $\tau_L = .3$ , and  $\theta = .05$ . The resulting effective tax rate on capital is .42, between the Fullerton and King estimates and the higher estimates of Feldstein, et. al.

Table I examines the case of an immediate, permanent, and unanticipated increase in  $\tau_K$ ,  $\tau_L$ , or  $\theta$ . In the notation used above, this is represented by constant positive  $h_K$ ,  $h_L$ , or  $z$  functions. We allow the elasticity of labor supply with respect to current wage to be 0 or 1.00, again a "reasonable" range for the compensated instantaneous elasticity of labor supply (see Killingsworth 1983). We allow  $\beta$  to be  $-.1$ ,  $-.3$ ,  $-.5$ ,  $-1.0$ , or  $-2.0$ , a range suggested by the macroeconometric literature, in particular, the work of Hansen and Singleton (1982), Weber (1970, 1975) and Hall (1981).

In Table I, we allow  $\sigma$  to be either .7 or 1.0, bracketing most estimates (see Berndt 1976). In thinking about  $\sigma$ , we must keep in mind that our  $\sigma$  is the substitutability in the net output function, which is less than in the gross output function by about .2. The results indicate that the marginal cost of factor taxation is substantial and varies greatly across instruments. The wage tax has an efficiency cost of at most 47 cents per dollar of revenue whereas the capital income tax has an efficiency cost of at least 24 cents.

The important feature to note is that for all parameters, the excess burden of permanent capital taxation far exceeds that of permanent wage taxation. Both pale however when compared with the results for the investment tax credit; in fact, an increase in the investment tax credit may increase revenues.

These results should be compared with the quadratic approximation method used by Chamley (1981). Our marginal efficiency costs substantially exceed those computed by Chamley. For example, in the Cobb-Douglas production and logarithmic utility case, with  $\tau_K = .5$  and  $\tau_L = .3$ , his method found a marginal cost of about fifty cents, whereas our method finds it to be well over two dollars when we take into account the impact on wage taxes and if we ignore the wage tax impact (as Chamley does) it is one dollar. Also, Chamley finds (making the same assumptions about  $\eta$ ,  $v$ , and  $\alpha$ ) that the addition of an elastic labor supply increases the excess burden of capital taxation by a factor of between 1.0 and 1.33, whereas we find a somewhat larger range of possibilities. These differences are not surprising since we study a local approximation around a taxed equilibrium and compute the true marginal efficiency losses, whereas he takes the local approximation around the untaxed equilibrium and uses it as a global approximation. One crucial feature is surely the fact that the rate of convergence increases as  $\tau_K$  increases, indicating that the rates of adjustment used by Chamley were biased downward, causing the welfare losses to be underestimated.

Table II indicates the sensitivity to outlying estimates of  $\sigma$ . Overall we find that the marginal cost of capital taxation rises with  $\sigma$ ,  $|\beta|$  and  $\eta$ . All three factors indicate high elasticities of substitution, naturally leading to higher excess burdens. In fact, for sufficiently high values, for example,  $\sigma=1.3$ ,  $\beta=-2.0$ , and  $\eta=1.0$ , we find that an increase in  $\tau_K$  will lead to a decrease in revenues. The excess burden associated with a marginal increase

in  $\tau_L$  has a similar dependence, but is relatively moderate for all parameter values displayed. In the case of the investment tax credit, we find that an increase in the credit will lead to an increase in revenues if  $\sigma$  is one or greater. In general, we find that substantial welfare gains would be achieved by an increase in  $\theta$  financed by either a capital or labor tax increase.

In all of our examples so far, we have assumed that  $\tau_K = .5$ ,  $\theta = .05$ , and  $\tau_L = .3$ . There is considerable disagreement as to what the true marginal tax rates are. Therefore, we explore the sensitivity of our results in Table I to the marginal tax rates in Table III when  $\sigma = 1$ ,  $\beta = -1$ , and  $\eta = .2$ . We find considerable sensitivity to choice of  $\tau_K$  as  $\tau_K$  varies from .4 to .7, a range encompassing most estimates. For fixed values of all other parameters, the marginal excess burden may vary by factors of two to three over this range of values for  $\tau_K$ . Similarly, the welfare impacts of a change in  $\theta$  would vary significantly. The marginal excess burden of labor taxation is also affected by  $\tau_K$  since labor supply responses to labor taxation will affect investment incentives. This example shows that MDWL can deviate substantially from the proportional-to-tax-rate rule commonly used in approximations. These results are robust over the parameterizations.

In Tables I through III, we examined the case of additively separable utility functions to gain some handle on the interactions among labor supply, consumption, and factor demand elasticities and the incidence of taxation. We next examine more detailed issues using specifications, some nonadditive, suggested by the empirical literature.

In Tables IV through VII, we do two things: we use parameter estimates of particular studies and examine the effects of various anticipated tax policy changes. One advantage of our approach is the ease with which one can determine such effects, which are clearly important since tax changes are

often phased-in and few policy changes are permanent. In each of these tables,  $T_1$  is the date at which a tax parameter is increased and  $T_2$  is the date at which it is returned to its original value. We define one period as that amount of time,  $t_{.01}$ , such that  $e^{-\rho t_{.01}} = .99$ , that is, utility is discounted by 1% during one period. This normalization affects no substantive result. For each  $T_1, T_2$  pair, we report MDWL for such a perturbation. Each table has six columns, the first three corresponding to  $\tau_K = .4$  and the second three assuming  $\tau_K = .65$ . We assume  $\tau_L = .3$ ,  $\theta = .05$ , and  $\theta_c = .8$  throughout Tables IV through VII. Each set of three columns in each table yields MDWL for a marginal change in  $\tau_L$ ,  $\tau_K$ , and  $\theta$ , respectively. Therefore, each row indicates the timing of the policy change announced at  $t = 0$  and the columns indicate the initial level of  $\tau_K$  and the affected tax instrument.

We also use various point estimates from the empirical labor supply literature. Since we are now going to examine the importance of the timing of the policy, space limitations argue that we fix a representative collection of estimates for the taste parameters. Since there are few truly dynamic empirical labor supply studies, the work of Macurdy (1981, 1983) being the most recent, we will also use static and synthetic cohort estimates of uncompensated and total income elasticities which we must convert to our  $\alpha$ ,  $\beta$ ,  $\eta$  and  $v$ . Since these labor supply elasticities are more commonly discussed, we will use them to describe our parameterizations. Define  $e_w$  to be the uncompensated wage elasticity--that is, if a worker's wage is 1% greater in all periods, then labor supply is  $e_w\%$  greater in all periods. Since our typical agent has no life-cycle variability in labor quality, if he faces, as he does in our steady state, a net interest rate  $\rho$  on his assets,  $A$ , he will react to a permanent wage increase just as a single period agent with nonlabor income of  $\rho A$  would. Therefore, if we interpret cross-sectional wage

differences as reflecting permanent wage differences among agents with identical preferences, then the uncompensated wage elasticities reported there equal our  $e_w$ . This interpretation is possibly not exactly true, but since we present a variety of parameterizations for  $e_w$ , this is not a severe limitation of our study. (Also, I know of no way to reconcile estimates which assume no net savings with the interpretation of temporary wage differences). Define  $e_I$  to be the total income elasticity--that is, the product of net wage and the derivative of labor supply with respect to nonlabor income.  $\alpha$  is determined once we make choices of  $\beta$ ,  $e_w$ ,  $e_I$ , and the net wage share of total income. Under each separate table, we list the important parameters,  $\sigma$ ,  $\beta$ ,  $e_w$ , and  $e_I$ .

Table IV assumes the utility function<sup>3</sup> used in Auerbach, Kotlikoff, and Skinner, based on Ghez and Becker. This parameterization allows one to compare the results of our representative formulation to their overlapping generations framework. The first half of Table IV uses  $\sigma = 1.0$  and the second half uses  $\sigma = .4$ .

We also use the explicitly dynamic work of Macurdy (1981, 1983). Table V uses a utility function estimated in Macurdy (1983)<sup>4</sup>, whereas Table VI uses estimates from Macurdy (1981) for  $e_w$ ,  $e_I$ , and a value of  $\beta$  implied jointly by his estimates and the assumption of separable in consumption and leisure. We let  $\sigma = 1.0$  and  $.4$  in V and Table VI assumes  $\sigma = .8$ .

Table VII displays excess burden results when we use parameterizations

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<sup>3</sup>That is, we take the utility function they use and assume no growth in labor quality over time. Our uncompensated elasticity differs from what they called an uncompensated elasticity since, in interpreting Ghez-Becker, they assumed that wage changes were temporary. We use their utility function instead of interpreting Ghez-Becker differently since this gives agents in the two models the same tastes.

<sup>4</sup>We evaluate his estimated utility function at the means of the wage, income, and various demographic factors.



suggested by the static empirical labor analysis of Hausman (1981). Hausman explicitly considers the nonlinearities in the budget constraint induced by taxation and reports several values for  $e_w$  and  $e_I$ . His sample of husbands suggests  $e_w = 0$  and  $e_I = -.7$  whereas  $e_w = .9$  and  $e_I = -.5$  for wives, and  $e_w = .5$  and  $e_I = -.36$  for female heads of households. In each table we allow  $\beta$  to be  $-1$  or  $-.1$ . If we weight Hausman's estimate for  $e_w$  and  $e_I$  for husbands, married women workers, and female heads of households by labor income, we get  $e_w = .2$  and  $e_I = -.6$  for the aggregate elasticities. Table VII assumes these parameters. Since static analysis cannot estimate  $\beta$ , we must be careful that the  $\beta$  we choose is consistent with the concavity of  $u$ . In particular,  $\beta < -.5$  is inconsistent with this labor parameterization. We choose  $\beta = -.6$  since  $\beta$  between  $-1.0$  and  $-.7$  yield results essentially similar other examples whereas  $\beta = -.6$  gives us a case where future labor taxation reduces current investment, a feature which alters some conclusions.

Before discussing the welfare analysis of various tax perturbations, we should examine the crucial positive aspects of these examples. In the interest of conserving space, we will summarize the relevant qualitative features. First, in all these cases, any increase in  $\tau_K$ , immediate or future, temporary or permanent, reduces current investment. Second, immediate and permanent increases in  $\tau_L$  reduces current investment, current labor supply, and the steady-state capital stock, and, except in Table VII, anticipated future labor taxation increases causes both current investment and labor supply to rise. This stimulative impact of future labor taxation occurs when the adverse income effect of future labor taxation dominates and reduces the current consumption of goods and leisure. Third, immediate and permanent  $\theta$  increases cause labor supply and current investment to rise, but future increases cause both to fall. Fourth, those cases with relatively greater

factor substitutability and labor supply elasticities display the greatest steady-state impacts of taxation on factor supply. These positive impacts will help in understanding the welfare results below.

A word of caution concerning these parameterizations is in order. We do not claim that these are the only valid parameterizations available. These are chosen because they represent the range of current estimates of labor supply elasticities. Macurdy (1981) estimates low compensated and uncompensated labor supply elasticities. The utility function used in Auerbach, Kotlikoff, and Skinner (1983) is used to facilitate comparisons between that line of work and this one. Also, it displays a negative  $e_w$ . The Macurdy (1983) estimate is high, but it is only for men. It should be used here since it is currently the most unrestricted and explicitly dynamic estimate. This estimate will also show that our conclusions continue to hold even when we use the highest estimate of labor supply elasticity. This collection covers a range of data sets including the Panel Study of Income Dynamics, Seattle-Denver Income Maintenance Experiment, and the synthetic cohort approach based on Census data. This collection is an interesting and valuable one since it covers the broad range of current estimates and data sources. When other commonly cited estimates for labor supply elasticities, such as those of Lucas and Rapping (1969), Abbot and Ashenfelter (1976, 1979), and others noted in Killingsworth (1983), are used the results for this analysis are not substantially affected. It is important that we do examine a broad collection of taste parameters instead of choosing a single set since the results are quite varied in magnitudes. Despite this diversity, we do find some robust results concerning the ranking of various policies. This shows that while we may have low confidence in the magnitudes of the efficiency costs of various tax policies, we may have some confidence along

some qualitative dimensions.

Using Tables IV through VII, we may discuss the effects of various intertemporal policy shocks announced at  $t=0$ . In Tables IV through VII, when  $0 < T_1 = T_2 - 1$  we are modelling the effects of a one period change in an instrument, that is, it is announced at  $t = 0$  that an increase in a tax parameter will take effect at  $t = T_1$  and last until  $t = T_1 + 1$ . Since our analysis is linear, the impacts on revenue and welfare of a permanent change is just a weighted sum of the impacts of such one period changes since  $T_1$  may range from zero to infinity. This decomposition of any effect into its single period components makes these perturbations the natural ones to first examine in studying the nature and importance of anticipation effects. Several interesting features are robust across our parameterizations. First, the efficiency cost of increasing  $\tau_k$  during  $[T_1, T_1 + 1]$  rises substantially as  $T_1$  increases. In fact, after 200 periods (not displayed) the tax increase often has a perverse impact on revenue.

Second, we find that the cost of labor taxation is affected by anticipation effects. Pulses of future labor tax increases generally have lower efficiency costs as  $T$  increases, with Table VII providing the only exception. The efficiency cost of anticipated wage taxation depends on its impact on current investment. For most of our parameterizations, capital accumulation and labor supply are initially stimulated by a future wage tax increase. This initial increase in factor supply yields an immediate improvement in efficiency which partially counters the increased distortion in the labor market which occurs when the tax is actually imposed. Since this initial benefit can occur only when there is a lag between the announcement and the actual change, the marginal efficiency loss decreases as the lag increases from zero. In Table VII, future labor taxation reduces current

investment, providing the opposite impact on efficiency.

Third, the impacts of pulses of  $\theta$  are even more dramatic. In all of our examples future investment credits reduce current investment. In many of our examples, immediate increases in  $\theta$  increase efficiency since revenues rise enough to pay for the extra credits, causing both utility and revenues to rise. As  $T_1$  rises, the revenue impacts usually cease to be perverse. Therefore, the excess burden of raising revenues by decreasing the investment tax credit between  $T_1$  and  $T_1 + 1$  drops substantially as  $T_1$  increases. However, for distant one period changes, the excess burden of raising revenue from a reduction in  $\theta$  is substantially greater than from increasing labor or capital income taxation.

When  $T_1=0$ , we represent an announcement that a policy instrument is increased immediately, but for only  $T_2$  periods, after which the instrument is assumed to return to its original value. Tables IV through VII display the efficiency costs of such a change for  $T_2 = 1, 4, 8, 20$ , and 5000. We find that when  $T_2$  is small, usually when less than 8 periods, capital taxation has a lower efficiency cost, whereas when  $T_2$  exceeds 20, capital taxation generally becomes inferior at the margin, under our assumptions concerning the tax rates. This is an interesting perturbation to consider for understanding policy formation. Almost any optimal intertemporal policy is dynamically inconsistent in this model due to the presence of the capital stock. In the short-run, the capital stock is fixed and capital taxation has a low efficiency cost, whereas future capital taxation is more distortionary since people will decumulate in response to it. Hence, an optimal program will likely call for a greater reliance on capital taxation in the short run than in the long run. (See Judd 1983b for a proof that in the long run there should be no net capital income taxation in this model even if capital markets

are not perfect, long-run capital supply has a finite elasticity, and the government has a redistributive motive.) However, when the economy gets near the "long run", policymakers will want to go back on the earlier "promise" of low capital taxation and again impose high taxation of capital in the short run. In an environment where policymakers have influence only over current and short-run policies, the  $t = 0$  policymaker can only effect a change in policy between  $t = 0$  and  $t = T$ , where  $T$  is the date when his control ends, even though he may care about the long-run impact on welfare. Hence, he will examine only the consequences of what he can do, that is, a  $T$ -period immediate change in policy. (See Judd 1983b for analysis of Phelps-Pollak equilibrium consistent plans for a similar problem.) In this model, the efficiency cost difference between capital and labor income taxation is small from the point of view of any one policymaker with control over only 8 to 20 periods of the immediate future. Therefore, there is less incentive to any actual policymaker to deviate from current levels of capital and labor taxation than indicated by Table I, which implicitly takes the perspective of a policymaker with control over the entire future. Although further work beyond the scope of this study is needed, this indicates that problems of dynamic consistency may play a quantitatively significant role in pushing our tax structure away from an "optimal" one.

The third type of policy examined in Tables IV through VII is that of a phased-in permanent change. This is represented when we let  $T_2$  to be infinite and let the time of the initial change,  $T_1$ , be positive. Since these are anticipated changes, they move in response to  $T_1$  just as the one period changes do, but are less sensitive to  $T_1$  since they are averages of the corresponding one period changes.

In comparing the first three columns in each table with the second three

columns, we find that the higher capital tax rate associated with the latter columns leads to a greater efficiency cost for all tax instruments. This is expected for the investment tax credit and capital income tax since they are directly associated with the capital stock. However, the nontrivial impact of  $\tau_K$  on the welfare cost of labor taxation is due to the cross-effect of labor supply on investment. Since labor supply falls as labor is taxed more, the return to capital falls and investment is reduced. In a capital market distorted by a positive  $\tau_K$ , this reduction in capital stock will reduce the economy's efficiency, and increasingly so as  $\tau_K$  is greater.

We end the discussion of excess burdens by noting the size of MUK, the marginal social value of an extra unit of capital. For all of our examples, MUK substantially exceeds unity. Since MUK is much greater in those cases where MDWL is also large, we find that the range of excess burdens in terms of initial capital stock equivalents is much smaller. The smallest excess burdens in these terms still belong to Table VI, where the excess burden of an immediate and permanent  $\tau_K$  increase is equal to a 46¢ loss in initial capital stock per dollar of revenue.

One must take care in interpreting these computed excess burdens. We are examining a highly aggregated model. On the production side, we are abstracting from heterogeneous effective tax rates over various types of capital due to the asymmetric treatment of equipment and structures and the lack of true economic depreciation. On the labor side we are also abstracting from heterogeneous treatment due to progressivity. While we can only speculate, it would appear that our procedure of examining average tax rates and average supplies would bias our computations downward since excess burden is a convex function of tax rates.

The representative agent formulation is the major aggregation assumption,

abstracting from heterogeneity in the distribution of factor endowments. However, examination of factor incomes indicate that many of our conclusions continue to be pure efficiency results. While replacing permanent capital taxation with labor taxation will increase the value of capital income, the induced capital accumulation may cause a net increase in the value of labor income in the higher  $\tau_K = .65$  case, generalizing the results of the inelastic labor supply analysis in Judd (1981, 1983c). Furthermore, using labor taxation to finance a permanent investment tax credit will increase labor income in all cases examined in Tables IV through VII. Therefore, if our model describes a disaggregated model, many of our efficiency-improving policies are actually Pareto improvements in the disaggregated model.

We see that we cannot come away from this study with any one number for excess burden. First, the excess burdens turn out to be very sensitive to the taste and technology parameters. Second, the excess burdens turn out to be very sensitive to the timing of the tax. In fact, knowing the timing of the policy which yields the marginal dollar of revenue is as important as knowing tastes and technology.

The last item to discuss is a comparison with other work. The major alternative approach to excess burden calculations is exemplified by Ballard, et. al. In that analysis, the major parameter is the elasticity of savings,  $\epsilon_s$ , which is usually fixed between 0 and 1. While there is no analogous structural parameter in the perfect foresight model, an econometrician proceeding as in Boskin (1978) or Howrey and Hymans (1980) would still find one. In this model, the elasticity of savings,  $\epsilon_s$ , estimated by regressing gross savings against the net-of-tax rate of return as the economy approached its steady-state would be  $-(\zeta + \delta)\sigma / (\theta_L \delta)$ . In this model, further computations show that  $\epsilon_s$  is extremely sensitive to  $\delta$  and  $\zeta$ , but bears no monotonic

relation with excess burdens. In fact, with  $\delta=.12$  the savings elasticities in Tables IV through VII range from .3 to 3, whereas if  $\delta=.25$  of net output, all of our parameterizations would display negative  $\epsilon_s$ , but no excess burden number in Tables IV through VI for labor or capital taxation would be affected by more than five percent. (Of course, the excess burden for the investment tax credit would be substantially affected because of the much greater level of replacement investment.) In contrast, excess burden is substantially affected by the savings elasticity in the Ballard-Shoven-Whalley model<sup>5</sup>. Therefore, if one were truly in a world described by this intertemporal optimization model, estimating  $\epsilon_s$  and plugging it into a savings rate model would not give a reliable approximation. Furthermore, such an approach would miss the nonnegligible anticipation effects of future taxation we found above.

Comparing our results with the static labor market analyses indicates that they are also not good approximations of the excess burden of labor taxation in a truly dynamic context. The relatively moderate central range of excess burdens computed by Browning and Stuart, five to thirty cents, is conservative relative to that computed here. Even when we leave out the extreme case of Macurdy (1983), we have a somewhat higher range. One reason is that the compensated supply elasticities for labor supply are somewhat higher, reflecting more recent labor estimates. Second, labor taxation affects the return on capital and the incentives to invest, resulting in a change in welfare since capital does not receive its marginal product. The

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<sup>5</sup>Direct comparison of our numerical results with Ballard, et. al., is not valid since they examined the cost of a dollar of taxation removed from the economy. However, after making the appropriate adjustments, the biases indicated by comparing our examples with theirs continue to hold, particularly for small labor supply elasticities. The qualitative comparisons of the two approaches are unaffected by this difference. Also note that their effective tax rate of .5 is closest to our  $\tau_K = .65$  case since  $\theta = .05$ .



importance of this effect is seen when we compare the  $\tau_K = .4$  columns with the  $\tau_K = .65$  columns in Tables IV through VII. The excess burden of labor taxation is higher, generally substantially so, under the higher capital income tax. This comes from the cross-effect that higher labor taxation has on the capital supply market because the tax rate on labor and the compensated labor supply elasticity is held fixed in each table. This effect and the anticipation effects that we find for labor taxation are both ignored in the static models of Browning and Stuart.

These various parameterizations give several robust implications. First, the marginal excess burden of permanent capital taxation is high relative to that of permanent wage taxation, but substantially less than that of permanent change in the investment tax credit. This ranking holds in all cases, even when the compensated labor supply elasticity is at the high end of estimates. This happens because an elastic labor supply is associated with a more sensitive steady-state factor supply.

Second, the obvious point that temporary unanticipated capital taxation has little loss is verified, but we further see that only moderate levels of anticipation produce substantial excess burden. In most of our examples, for some  $T$  around 20 or less,  $T$ -period labor and capital tax changes produce the same marginal efficiency losses. Therefore, only very short-run unanticipated capital income taxation is lump-sum in nature and current tax rates may be a reflection of the dynamic inconsistency problem in dynamic taxation.

Finally, we note that for no parameterization considered above would an equal decrease in both capital and labor tax rates would revenues increase. However, such a perverse movement of revenue is more likely in this model than indicated in the analysis of Fullerton (1982). He argues that even if income tax rates were around .8, further tax increases would increase revenue. In

this intertemporal optimizing framework, tax rate increases at that level of taxation reduce revenues unless intertemporal substitution in consumption were very low.

## 6. Caveats and Conclusions

In this paper we have succeeded in exactly determining the short-run impacts and the marginal efficiency cost of factor taxation of intertemporally complex tax changes around the steady state of a simple perfect foresight model. First, we show that such an exercise is tractable. It is also clear that the method used here is easily extendable to cases of multiple types of capital, human capital accumulation, and nonadditive preferences. Second, we find that when we take standard reference values for parameters of tastes and technology the resulting estimates for excess are large, with substantial differences across various instruments. This analysis strongly indicates that the desired direction of permanent tax reform is an increase in the investment tax credit and increase in labor taxation, since this result is robust across a broad range of estimates for the crucial parameters. Examination of impacts on factor incomes indicate that such a policy change could be a Pareto improvement.

There are several aspects of reality ignored by this analysis. First, we ignored adjustment costs which have played an important role in investment theory. Adjustment costs convex in the rate of change, the usual specification, will slow down the adjustment rate and reduce the efficiency cost of capital taxation but probably not affect the cost of labor taxation as much, thereby reducing the efficiency gap between these instruments. In the case of inelastic labor supply, computations show that the range of excess burdens is not affected by adjustment cost parameters suggested by the empirical literature. However, it is not clear that the convex specification

is the best. Rothschild (1971) has argued for concave costs of adjustment whereas Kydland and Prescott (1982) studied a time-to-build specification. These, particularly the latter, would appear to just delay the beginning of adjustment, not slow the ultimate rate. Hence, it is not clear that adjustment costs would substantially alter our conclusions concerning any but the most short-run policies. Given the variety of possible specifications of adjustment and that they have been ignored in other examinations of excess burden, we also ignored them.

Uncertainty is another important aspect missing in our analysis. However, it is not clear what the bias of that omission is since the impact of uncertainty on savings often depends on third-order properties of the utility function.

Given all these aspects of reality ignored in our model, we cannot claim that these results are directly applicable to the U.S. economy. The primary goal was to analyze excess burden in a commonly used model of dynamic general equilibrium, examine the impact of various technical and taste parameters, and compare the results with other efforts. While levels will certainly be affected by these extra elements, this model was examined in the belief that the biases found will continue to hold and that the techniques used here will be useful in developing the more general models.

Table I

$\sigma$	$ \beta $	$\eta$	$\tau_K$		$\tau_L$		$\theta$	
			dR	-MDWL	dR	-MDWL	dR	-MDWL
.7	.1	0	.219	.244	.75	0	.09	.584
		1	.191	.474	.74	.049	.062	1.43
	.3	0	.193	.518	.75	0	.064	1.53
		1	.179	.67	.72	.128	.05	2.35
	.5	0	.175	.755	.750	0	-.046	2.80
		1.0	.163	.967	.70	.22	-.032	4.426
	1.0	0	.160	.988	.75	0	-.032	4.9
		1.0	.142	1.465	.675	.338	-.014	14.10
	2.0	0	.148	1.207	.75	0	-.020	8.75
		1.0	.118	2.27	.700	.469	.009	-27.91*
1.0	.1	0	.208	.285	.75	0	.079	.733
		1	.151	.769	.74	.049	.023	4.96
	.3	0	.166	.722	.75	0	.038	3.12
		1	.136	1.1	.724	.127	.008	18.3
	.5	0	.133	1.25	.75	0	-.006	28.8
		1.0	.116	1.654	.704	.219	.011	-16.4*
	1.0	0	.105	1.978	.75	0	.02	-9.14*
		1.0	.089	2.814	.680	.335	.038	-6.43*
	2.0	0	.081	2.96	.75	0	.045	-5.21*
		1.0	.056	5.67	.653	.46	.070	-4.46*

Assume  $\tau_K = .5$ ,  $\tau_L = .3$ ,  $\theta = 0$ ,  $\theta_K = .25$ ,  $\delta k/f = .12$ ,  $\rho = .04$ , and that all government revenue is lump-sum rebated to the agents.

\* indicates that there was a perverse movement in revenue. The negative "-MDWL" entry indicates the utility gained per dollar of extra revenue.

Table II

$\sigma$	$ \beta $	$\eta$	$\tau_K$		$\tau_L$		$\theta$	
			dR	MDWL	dR	MDWL	dR	MDWL
.4	.5	.2	.223	-.45	.726	-.088	-.094	-1.05
		1.0	.216	-.56	.693	-.223	-.087	-1.35
	2.0	.2	.212	-.65	.719	-.117	-.083	-1.64
		1.0	.185	-1.15	.639	-.475	-.056	-3.73
1.3	.5	.2	.091	-2.24	.734	-.083	.036	.20
		1.0	.073	-3.04	.708	-.217	.053	4.09
	2.0	.5	.015	-21.2	.725	-.112	.110	2.81
		1.0	-.0015	244.	.658	-.458	.126	2.84

Assume  $\tau_K = .5$ ,  $\tau_L = .3$ ,  $\theta = .05$ ,  $\theta_K = .25$ ,  $\delta k/f = .12$ ,  $\theta_c = .8$ .

Table III

$\sigma$	$ \beta $	$\eta$	$\tau_K$	$\tau_L =$	Increase $\tau_K$			Increase $\tau_L$		
					0	0.2	0.4	0	0.2	0.4
1.0	1.0	.2	.1	-.09	-.13	-.18	-.003	-.047	-.119	
			.2	-.21	-.28	-.37	-.007	-.051	-.125	
			.3	-.39	-.51	-.69	-.010	-.056	-.132	
			.4	-.65	-.89	-1.34	-.014	-.061	-.139	
			.5	-1.10	-1.66	-3.25	-.018	-.066	-.147	
			.6	-1.98	-3.92	-78.1	-.022	-.072	-.155	
			.7	-5.41	-99.4	5.0	-.027	-.078	-.164	
			.8	-61.3	5.9	2.8	-.032	-.084	-.173	

Assume  $\tau_K = .5$ ,  $\tau_L = .3$ ,  $\theta = .05$ ,  $\theta_K = .25$ ,  $\delta k/f = .12$ ,  $\theta_c = .8$ . Entries give MDWL if  $\tau_K$  or  $\tau_L$  is raised immediately and permanently.

Table IV

$T_1$	$T_2$	$\tau_K = .4$			$\tau_K = .65$		
		$\tau_L$	$\tau_K$	$\theta$	$\tau_L$	$\tau_K$	$\theta$
0	1	-.49	-.03	3.4	-1.07	-.1	2.5
8	9	-.31	-.54	97.4	-.34	-4.5	5.1
40	41	-.18	-1.30	-1.4	-.18	-27.4	-29.3
0	4	-.44	-.13	4.1	-.82	-.6	2.7
0	8	-.40	-.24	5.2	-.63	-1.2	3.0
0	20	-.32	-.47	17.1	-.43	2.9	4.0
0	40	-.28	-.64	-17.6	-.34	-4.4	5.1
0	5000	-.26	-.75	-7.2	-.31	-5.6	6.0
4	5000	-.23	-.92	-3.6	-.24	-10.0	11.8
8	5000	-.22	-1.04	-2.5	-.21	-15.2	32.4
40	5000	-.18	-1.33	-1.3	-.18	-28.0	-28.4
MUK = 1.47				MUK = 2.11			

$\sigma = 1$ ,  $\beta = -.25$ ,  $e_w = -.13$ ,  $e_I = -.85$ .

0	1	-.36	-.03	-6.8	-.59	-.10	-4.9
4	5	-.30	-.21	-3.6	-.39	-.64	-2.5
8	9	-.27	-.34	-1.5	-.31	-.87	-1.6
40	41	-.23	-.53	-.5	-.50	-.33	-3.8
0	8	-.31	-.17	-3.1	-.44	-.52	-3.0
0	20	-.28	-.29	-1.8	-.36	-.74	-2.1
0	40	-.27	-.36	-1.4	-.33	-.83	-1.8
0	5000	-.26	-.39	-1.2	-.31	-.87	-1.6
4	5000	-.25	-.45	-.9	-.28	-.96	-1.3
8	5000	-.24	-.48	-.7	-.27	-1.00	-1.1
40	5000	-.23	-.53	-.5	-.26	-1.02	-1.0
MUK = 1.68				MUK = 2.40			

$\sigma = .4$ ,  $\beta = -.25$ ,  $e_w = -.13$ ,  $e_I = -.85$

Table V

T <sub>1</sub>	T <sub>2</sub>	$\tau_K = .4$			$\tau_K = .65$		
		$\tau_L$	$\tau_K$	$\theta$	$\tau_L$	$\tau_K$	$\theta$
0	1	-2.03	-.08	2.5	-121.55	-.33	2.2
8	9	-.63	-1.19	11.8	-.61	54.91	4.9
40	41	-.36	-2.02	-.20	-.38	16.20	16.2
0	4	-1.50	-.28	2.9	-5.08	-1.55	2.4
0	8	-1.15	-.52	3.3	-2.31	-3.75	2.6
0	20	-.77	-.94	5.6	-1.11	-14.1	3.2
0	40	-.63	-1.19	11.8	-.84	-44.17	3.8
0	5000	-.57	-1.33	42.7	-.73	-305.68	4.2
8	5000	-.41	-1.81	-3.5	-.41	17.82	11.3
40	5000	-.36	-2.02	-2.0	-.38	16.30	16.2
MUK = 2.05				MUK = 3.09			

$\sigma = 1, \beta = -1.37, e_w = .7, e_I = -.77$

0	1	-.97	-.06	10.2	-2.06	-.23	16.5
8	9	-.56	-.68	-2.7	-.66	-1.82	-2.9
40	41	-.46	-.90	-.9	-.58	-1.97	-1.9
0	4	-.84	-.22	23.5	-1.48	-.73	66.2
0	8	-.74	-.36	-33.5	-1.14	-1.11	-24.3
0	40	-.57	-.66	-3.1	-.77	-1.63	-4.8
0	5000	-.55	-.71	-2.4	-.73	-1.70	-4.0
8	5000	-.48	-.85	-1.1	-.59	-1.95	-2.0
40	5000	-.46	-.90	-.9	-.58	-1.97	-1.9
MUK = 2.32				MUK = 3.53			

$\sigma = .4, \beta = -1.37, e_w = .7, e_I = .77$

Table VI

$T_1$	$T_2$	$\tau_K = .4$			$\tau_K = .65$		
		$\tau_L$	$\tau_K$	$\theta$	$\tau_L$	$\tau_K$	$\theta$
0	1	-.11	-.03	6.9	-.19	-.12	4.0
8	9	-.08	-.43	-6.4	-.10	-2.50	15.4
40	41	-.06	-.98	-1.1	-.06	-6.10	-6.2
0	4	-.10	-.10	10.5	-.17	-.48	4.6
0	8	-.10	-.19	28.6	-.14	-.92	5.5
0	40	-.08	-.51	-4.2	-.09	-2.61	17.6
0	5000	-.07	-.59	-3.0	-.09	-3.06	38.6
8	5000	-.06	-.79	-1.6	-.06	-4.91	-11.0
40	5000	-.06	-1.00	-1.0	-.06	-6.15	-6.1
MUK = 1.27				MUK = 1.61			

$\sigma = .8$ ,  $\beta = -1.2$ ,  $e_w = .08$ ,  $e_I = -.06$

Table VII

$T_1$	$T_2$	$\tau_K = .4$			$\tau_K = .65$		
		$\tau_L$	$\tau_K$	$\theta$	$\tau_L$	$\tau_K$	$\theta$
0	1	-.08	-.03	7.10	.07	-.12	4.14
8	9	-.25	-.55	-68.28	-.31	-3.19	7.20
40	41	-.49	-1.78	-2.40	-.66	-43.02	-84.97
0	4	-.12	-.12	8.78	-.01	-.49	4.48
0	8	-.15	-.24	12.37	-.10	-1.00	4.95
0	40	-.30	-.73	-14.54	-.36	-4.03	8.15
0	5000	-.34	-.90	-8.13	-.42	-5.38	9.84
4	5000	-.38	-1.11	-5.26	-.50	-8.85	15.13
8	5000	-.41	-1.29	-4.01	-.56	-13.40	25.43
40	5000	-.51	-1.92	-2.15	-.67	-51.07	-61.02
MUK = 1.82				MUK = 2.27			

$\sigma = .8$ ,  $\beta = -.6$ ,  $e_w = .2$ ,  $e_I = -.6$



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