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Discussion Paper #634

THE ECONOMICS OF QUALITY TESTING AND DISCLOSURE*

by

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Abstract

Sellers are often more able than consumers to test product quality. We ask first whether such firms will voluntarily test quality and disclose what they learn. The answer depends in a paradoxical way upon the presence of mandatory disclosure rules: only if disclosure is mandatory will a seller not test and disclose. We then ask if it is even desirable for consumers to be informed about quality. We show that if income effects for the product are negligible, then consumers and firms will agree that a regime in which consumers are uninformed (informed) is preferable to a regime in which they are informed (uninformed) if income and quality are complements (substitutes) in utility. Consumers and firms can disagree -- in either way -- about which regime is better if income effects are not negligible. We conclude by discussing the desirability of mandatory testing laws.

1. Introduction

Much attention has been given to the consequences of sellers having better information than prospective buyers about product quality. In contrast, the consequences of sellers having better access to information have not been studied. A seller has better access to information when he is better able to test his products, which is the case when consumers do not know what tests are available, or they lack the expertise to administer and interpret tests, or they cannot purchase enough units to obtain statistical significance. Testing by private experts, such as Consumers Union, who then publish the results is also often infeasible, both because outside experts do not have the testing expertise of inside experts, and because the free rider problem can undermine markets for information. We ask in this paper whether sellers will voluntarily test quality and inform consumers of what they learn, and whether it is desirable for consumers to be so informed.

When testing is not an issue, both the market and the government can provide informed sellers with incentives to communicate. The most basic incentives are to be truthful, i.e., to not lie or mislead. Market competition may reward a firm that periodically brings out new products with a reputation for honesty; customers who have been misled should take their future business to a more honest vendor. The government provides incentives to be truthful by enacting antifraud laws, such as those regarding false advertising, perjury, and warranties of fitness. For an antifraud law to be effective, there must be a way to establish ex post the veracity of the seller. This is accomplished most directly when a buyer observes, and can verify to a third party, the true quality of a product at some time after purchasing it. Even if buyers cannot observe quality ex post, antifraud laws may be obeyed if experts can be hired to determine why a product failed, or if

In Section 2, we extend these arguments to sellers who can acquire as well as disclose information. We consider the polar case in which sellers and buyers initially have the same information, but sellers can costlessly test the quality of their products. Voluntary testing and disclosure does not necessarily occur in this case. For example, suppose a seller can make it verifiably known before the purchase date that he has not tested quality. The seller can then decide to not test without causing consumers to disbelieve the resulting claim of ignorance. If the seller is driven to disclose when he tests, he will not test if the profits obtained when buyers are ignorant exceed the expected profits obtained when they are informed.

We consider in Section 2 the more realistic case in which consumers cannot know for sure that the seller has not tested quality. Whether the market provides incentives for the seller to test then depends upon whether disclosure rules are effective. The dependence is, we believe, counterintuitive and perverse. The seller will test and fully disclose if disclosure rules are not in effect. If disclosure rules are in effect, the seller will not test (and therefore not disclose) whenever he prefers buyers to be uninformed. This is because the seller can claim ignorance when disclosure rules are effective only if he has not tested, so that a claim of ignorance must then be taken at face value.

Thus, Section 2 concludes with the observation that sellers will not necessarily test and inform buyers of product quality. In Section 3 we ask whether it is even desirable for consumers to be informed. The type of product we consider in this section is not one that creates a significant hazard if its quality is low, for which the answer is obvious. Rather, we consider a product that is put to the same use regardless of its quality -- only its effectiveness or operating cost in that use depends upon its quality.

intuition is straightforward. If quality and income are substitutes, consumers would like to consume relatively more income in states where quality is low. This is what happens when they are informed, since then the price of the product is lower when it has low quality. On the other hand, if quality and income are complements, consumers would prefer to have more income in high quality states than in low quality states, which is the opposite of what happens when they are informed.

This intuition can be overturned when utility is not quasilinear. We present, in Section 3, two examples to show that then producers and consumers can disagree in both of the two possible ways about which regime is better. Thus, all four possibilities can occur for agreement and disagreement between consumers and producers about which regime is better.

We show in Section 4 that the model in Section 2 is a reduced form of that in Section 3. We conclude in Section 5 with a brief discussion of the desirability of mandatory testing laws.

2. Voluntary Testing and Disclosure

In this section we construct a model of the firm to address the question of whether firms will voluntarily test and disclose product quality. We assume that testing is costless and surely determines the true quality in order to make the results as sharp as possible. We also assume that the firm tests discreetly, so that consumers cannot know for sure whether the firm knows the quality at the time they purchase.

The model is in the spirit of the "games of persuasion" studied by Grossman [1981], Milgrom [1981], and Milgrom and Roberts [1984]. The equilibrium is in the spirit of Kreps and Wilson's [1982] sequential

probability on one state, then the corresponding profit levels have the same ordering:

$$(1) \quad \text{For any } \gamma \text{ and } e_s \neq \gamma, \pi(e_s) \begin{cases} < \pi(\gamma) & \text{if } \sum_{t < s} \gamma_t = 0 \\ > \pi(\gamma) & \text{if } \sum_{t > s} \gamma_t = 0 . \end{cases}$$

Note that (1) implies $\pi(e_s) < \pi(e_{s+1})$ and $\pi(\gamma) < \pi(e_s)$ for all $\gamma \neq e_s$.

The firm has two decisions to make, whether to test the product and what to claim about its quality. We view its quality announcement as identifying a subset of quality levels in which the true quality lies, which is the approach taken by Grossman [1981], Milgrom [1981], and Milgrom and Roberts [1984]. The firm will be truthful because of the presence of effective antifraud incentives. Hence, to avoid even a chance of being caught in a lie, the firm must report total ignorance if it does not test, which we view formally as reporting the set $A = Q$. If it does test, to avoid lying the firm must report a superset of the true quality level. The firm can report according to a mixed strategy, which allows the firm to be as vague as possible.

Thus, a behavioral strategy for the firm is a pair (α, β) , where α is the probability of testing and β is a conditional probability function that determines announcements when the firm tests: if the firm tests quality and learns the true state is s , then $\beta(A|s)$ is the probability it reports $A \subseteq Q$. The strategy (α, β) is feasible in the game without disclosure rules if and only if it satisfies

Conditions (4a) and (4b) are merely Bayes' rule for updating when the firm announces either total ignorance, $A = Q$, or a more informative report, $A \subsetneq Q$. Both conditions reflect the antifraud requirement that the firm must report total ignorance when it does not test. Condition (4c) requires that γ be consistent with the restriction (2c) that the firm can only report supersets of the truth. Note that (4c) applies even to subsets A that have zero probability of being reported according to (α, β) .

If the adopted strategies are (α, β, γ) , the firm's expected profit is

$$\Pi(\alpha, \beta, \gamma) = \alpha \sum_{s \in Q} \sum_{A \subseteq Q} \phi_s \beta(A|s) \pi(\gamma(A)) + (1-\alpha) \pi(\gamma(Q)),$$

where the last term follows because the firm must report Q when it does not test. A sequential equilibrium without disclosure rules is a triple (α, β, γ) that satisfies the feasibility conditions (2) and (3), the consumer rationality conditions (4), and the firm rationality condition

$$(5) \quad \Pi(\alpha, \beta, \gamma) \geq \Pi(\hat{\alpha}, \hat{\beta}, \gamma) \quad \text{for all } (\hat{\alpha}, \hat{\beta}) \text{ satisfying (2)}.$$

The surely testing, truthfully disclosing, and skeptically inferring strategies are $\alpha = 1$, $\beta(\{s\}|s) = 1$, and $\gamma(m(A)|A) = 1$, where $m(A)$ is the minimum quality level in the set A . The skeptical inference rule is obviously a rational response to the truthful disclosing strategy, and truthful disclosing is obviously the best the firm can do when consumers are so skeptical. Their skepticism also makes testing optimal for the firm. This is because the firm must report total ignorance if it does not test, thereby causing consumers to believe the quality level is the worst possible even when it is not. Testing, together with truthful reporting, dispels this most

be another state such that $\beta(A|s) > 0$. Then (4b) implies $\gamma(t|A) < 1$. Since $\beta(A|w) = 0$ for $w > t$, (4b) also implies that $\gamma(w|A) = 0$ for all $w > t$. Hence, e_t stochastically dominates $\gamma(A)$. The firm could thus do better by reporting $\{t\}$ instead of A if the state is t , contradicting $\beta(A|t) > 0$.

Now, assume $\beta(A|s) > 0$ for some $s \neq m(A)$. Since s is the only state in which A is reported with positive probability, $\gamma(A) = e_s$ (see 4(b)). Similarly, letting $B \subseteq Q$ be a set such that $\beta(B|m(A)) > 0$, $\gamma(B) = e_{m(A)}$. But e_s stochastically dominates $e_{m(A)}$, so that the firm could do better by announcing A and never announcing B when it learns the state is $m(A)$. This contradiction proves (ii).

(iii) Suppose $\gamma(m(A)|A) < 1$ for some $A \subseteq Q$. As in the previous paragraph, if we let $B \subseteq Q$ be a set such that $\beta(B|m(A)) > 0$, then $\gamma(B) = e_{m(A)}$. But, since $\gamma(m(A)|A) < 1$, (4c) implies that $\gamma(A)$ stochastically dominates $e_{m(A)}$. Hence, the firm should never announce B if the state is $m(A)$, a contradiction. ////

Proposition 1 implies that antifraud incentives alone, without the addition of disclosure rules, will cause consumers to be so skeptical that the firm will be forced to fully test and disclose. Disclosure is full in the sense that consumers are able to "invert" the firm's reporting strategy to determine the true quality level, since no announcement will be made with positive probability in more than one state.

The situation is different when effective disclosure rules are present. We take a disclosure rule, as discussed in the introduction, to be an incentive for the firm to fully reveal its information about quality. The presence of effective disclosure rules simply restricts further the definition

Proposition 2: Any (α, β, γ) satisfying (2') and (3) is a sequential equilibrium with disclosure rules if and only if

$$\alpha = \begin{cases} 0 & \text{if } \pi(\phi) < \sum_{s \in Q} \phi_s \pi(e_s) \\ 1 & \text{if } \pi(\phi) > \sum_{s \in Q} \phi_s \pi(e_s). \end{cases}$$

Proof: Since β specifies full disclosure, expected profit at (α, β, γ) is

$$\Pi(\alpha, \beta, \gamma) = \alpha \sum_{s \in Q} \phi_s \pi(e_s) + (1-\alpha)\pi(\gamma(Q)).$$

Since (4'a) implies that no updating occurs if Q is announced, $\gamma(Q) = \phi$.

Hence,

$$\Pi(\alpha, \beta, \gamma) = \alpha \sum_{s \in Q} \phi_s \pi(e_s) + (1-\alpha)\pi(\phi).$$

The firm's best strategy is thus to always test ($\alpha=1$) if $\sum_{s \in Q} \phi_s \pi(e_s) > \pi(\phi)$, and to never test ($\alpha=0$) if $\sum_{s \in Q} \phi_s \pi(e_s) < \pi(\phi)$. ////

Proposition 2 states that when disclosure rules are effective, the firm will not test (and hence not disclose) exactly when it prefers its customers to be uninformed rather than informed. Since without disclosure rules the firm will test and disclose, any change caused by a disclosure rule will be just the opposite of its intent. In any case, in the context of our model, mandatory disclosure laws are exactly what firms who can choose whether to acquire information should want.

When disclosure rules are effective, a firm is not forced to test in order to dispel skepticism simply because consumers cannot be skeptical: an

3. Welfare Comparisons

We turn now to the welfare consequences of informing consumers about quality, which requires a more detailed description of firm and consumer preferences. We compare a market regime in which consumers are informed to one in which they are uninformed, not specifying now why information is communicated in one regime but not in the other. The most straightforward interpretation is that the environment is one in which the firm does not voluntarily test and disclose. The regime with informed consumers then differs from the regime with uninformed consumers by the presence of a mandatory testing and disclosing law. We discuss later the consistency of this interpretation with the results of Section 2.

A single firm produces a product of uncertain quality and sells it to a consumer in exchange for a good called "money." Both traders are competitive price-takers, the consumer acting to maximize expected utility and the firm to maximize expected profit. These assumptions are made for simplicity; the results extend to many consumers and, provided the quality of their outputs are perfectly correlated, to many firms producing the same product. Most results also hold if the firm is a monopoly, as we shall indicate.

The product with uncertain quality is denoted by x . Good y is money, or rather, a composite representing expenditure on all other goods. The consumer has an endowment Y of money and no endowment of x . Because the benefit she obtains from a consumption bundle (x,y) depends on the quality of x , we assume the consumer has a state-dependent utility function $u^s(x,y)$. Her ex ante preferences over contingent commodity bundles $\{(x_s, y_s)\}_{s=1, S}$ are represented by the expected utility $Eu^s(x_s, y_s) = \int \phi_s u^s(x_s, y_s)$. To capture the notion that quality varies positively with the state, our basic assumption is that the marginal rate of substitution of y for x increases in s :

We assume the consumer receives positive amounts of both goods, so that the inverse demand function $p_U(x)$ near the equilibrium satisfies

$$(9) \quad p_U(x) = \frac{Eu_1^S(x, Y - p_U(x)x)}{Eu_2^S(x, Y - p_U(x)x)} .$$

We assume that for any x near x^U , a unique $p_U(x)$ is defined by (9). The equilibrium is determined by equating price to marginal cost:

$$(10) \quad p_U(x^U) = c'(x^U) \quad \text{and} \quad p^U = p_U(x^U).$$

An informed consumer equilibrium (ICE) is a $(2S + 1)$ -tuple $(x^I, (y_s^I, p_s^I)_{s=1, S})$ satisfying

$$(11) \quad (x^I, y_s^I) \in \underset{x, y \geq 0}{\operatorname{argmax}} u^S(x, y) \quad \text{S.T.} \quad p_s^I x + y \leq Y$$

$$(12) \quad x^I \in \underset{x \geq 0}{\operatorname{argmax}} (E p_s^I) x - c(x).$$

Notice that the firm is assumed in (12) to have perfect foresight regarding state contingent prices. The consumption of x does not depend upon the state because the price adjusts to clear the market in every state — the assumption that $u_1^S > 0$ implies that demand for x would be infinite at a zero price. We again assumed that all equilibrium quantities are positive, that inverse demand curves are well-defined by

$$(13) \quad p_s(x) = \frac{u_1^S(x, Y - p_s(x)x)}{u_2^S(x, Y - p_s(x)x)},$$

Proof: Substituting into (9) and (13) yields, respectively,

$p_U(x) = (Eq_s/Er_s)v'(x)$ and $p_S(x) = (q_s/r_s)v'(x)$. Substituting these expressions into $Eu^S(x^U, Y - p_U(x^U)x^U)$ and $Eu^S(x^I, Y - p_S(x^I)x^I)$ yields, respectively,

$$Eu^S(x^U, y^U) = (v(x^U) - x^U v'(x^U))Eq_s - YEr_s,$$

$$Eu^S(x^I, y^I) = (v(x^I) - x^I v'(x^I))Eq_s - YEr_s.$$

The consumer is therefore indifferent if $x^U = x^I$, which is the case if supply is perfectly inelastic. Since $v'' < 0$ implies that $v(x) - xv'(x)$ increases in x , the consumer prefers the UCE (ICE) if $x^U > x^I$ ($x^U < x^I$). Thus, given that $c' > 0$ (upward sloping supply), the consumer prefers the UCE if the inverse demand function $p_U(x)$ is above the expected inverse demand function $Ep_S(x)$, i.e., if $Eq_s/Er_s > E(q_s/r_s)$. The firm maximizes expected profit and hence also prefers the regime with the higher expected demand curve; the only exception is if marginal cost is constant, in which case expected profit is always zero and the firm is indifferent. Now, note that

$$\frac{Eq_s}{Er_s} = E\left(\frac{q_s}{r_s}\right) + \frac{\text{cov}(r_s, q_s/r_s)}{Er_s}.$$

The definition of increasing quality in (6) implies that q_s/r_s increases in s . The covariance of r_s and q_s/r_s is therefore positive (negative) if r_s increases (decreases) in s . Hence, $Eq_s/Er_s > (<) E(q_s/r_s)$, so that both traders prefer the UCE (ICE), if r_s increases (decreases) in s . ////

ICE. The firm has the opposite preference, preferring the ICE to the UCE. (In fact, the ICE allocation is fully efficient.)

Thus, the consumer may prefer the uninformed consumer regime even if income and quality are substitutes. The following proposition shows that she always prefers the uninformed consumer regime if income and quality are complements and marginal costs are constant.

Proposition 4: If $u_2^s < u_2^{s+1}$ for all s , and if marginal costs are constant, then the consumer prefers the UCE to the ICE and the firm is indifferent. Furthermore, the UCE is efficient if $u_2^s = u_2^{s+1}$ for all s .

Proof: We first show the efficiency of the UCE if u_2^s does not depend on s . Given our curvature and differentiability assumptions, a sufficient condition for an efficient allocation is

$$\frac{Eu_1^s(x, y_s)}{u_2^t(x, y_s)} = c'(x) \quad \text{for all } t = 1, \dots, S.$$

The UCE is therefore efficient, since by (9), (10), and the hypothesis that $u_2^s = u_2^{s+1}$ for all s , for every t it satisfies

$$\frac{Eu_1^s(x^U, y_s^U)}{u_2^t(x^U, y_s^U)} = \frac{Eu_1^s(x^U, Y - p^U x^U)}{Eu_2^s(x^U, Y - p^U x^U)} = c'(x^U).$$

Now assume u_2^s increases in s , and that c' is constant. Let $\bar{y} = Ey_s^I$. Then, since y_s^I decreases in s , there exists a state $t \geq 2$ such that $y_s^I - \bar{y} > 0$ if $s < t$ and $y_s^I - \bar{y} \leq 0$ if $s \geq t$. The sign of $y_s^I - \bar{y}$ is therefore the opposite that of $u_2^s(x^I, \bar{y}) - u_2^t(x^I, \bar{y})$ for every s . Hence,

$$\begin{aligned}
Eu^S(x^U, y^U) - Eu^S(x^I, y^I) &= E(1-q_s) \ln(Y - p^U x^U) - E(1-q_s) \ln(Y - p^I x^I) \\
&= (1-Eq_s) \ln(Y - (Eq_s)Y) - E(1-q_s) \ln(Y - q_s Y) \\
&< 0,
\end{aligned}$$

where the inequality follows from Jensen's Inequality applied to the convex function $f(q) = (1-q)\ln(Y-qY)$.

The example can now be altered so that the firm strictly prefers the UCE. Let $u^S(x, y) = \theta q_s \ln(x) + (1-q_s) \ln(y)$, where $\theta > 1$. By continuity, the consumer prefers the ICE if $\theta \approx 1$. But the firm prefers the UCE because it yields a higher demand curve:

$$\begin{aligned}
P_N(x) &= \left(\frac{Eq_s}{1+(\theta-1)(Eq_s)} \right) \left(\frac{\theta Y}{x} \right) \\
&= E \left(\frac{q_s}{1+(\theta-1)q_s} \right) \left(\frac{\theta Y}{x} \right) - \text{cov} \left(q_s, \frac{1}{1+(\theta-1)q_s} \right) \left(\frac{\theta Y}{x} \right) \\
&> E \left(\frac{q_s}{1+(\theta-1)q_s} \right) \left(\frac{\theta Y}{x} \right) = Ep_s(x).
\end{aligned}$$

We remarked at the beginning of this section that most of its results would hold if the firm were to act as a monopoly. A monopoly sets expected marginal revenue equal to marginal cost. Thus, condition (10) becomes $R_U^c(x^U) = c^c(x^U)$, where $R_U(x) = xp_U(x)$, and condition (14) becomes $R_I^c(x) = c^c(x^I)$, where $R_I(x) = Exp_s(x)$. Now, examples 1 and 2 are worded so as to be completely valid regardless of whether the firm is a competitor or a monopoly. Proposition 3 is also true if the firm is a monopoly, although its proof should be modified. Thus, all four patterns of preference can still

$$(16a) \quad (x^*, \alpha^*, \beta^*) \in \operatorname{argmax} \left(\alpha \sum_{s \in Q} \sum_{A \subseteq Q} \phi_s \beta(A|s) p^*(A) + (1-\alpha) p^*(Q) \right) x - c(x),$$

where the constraint set is given by $x \geq 0$ and (2) or (2'); and the prices clear the market:

$$(16b) \quad p^*(A) = p(x^*, \gamma^*(A)) \quad \text{for all } A \subseteq Q.$$

If the firm is a monopoly, an equilibrium is a four-tuple $(\alpha^M, \beta^M, \gamma^M, x^M)$ such that $(\alpha^M, \beta^M, \gamma^M)$ satisfies the same conditions (2)-(4) (or (2'), (3) and (4')), but (16) is replaced by

$$(17) \quad (x^M, \alpha^M, \beta^M) \in \operatorname{argmax} \left(\alpha \sum_{s \in Q} \sum_{A \subseteq Q} \phi_s \beta(A|s) p(x, \gamma^M(A)) \right. \\ \left. + (1-\alpha) p(x, \gamma^M(Q)) \right) x - c(x).$$

From (16b), a competitive firm's perceived profit when it produces x and consumers have beliefs γ is $\pi^*(\gamma, x) = p(x^*, \gamma)x - c(x)$, where x^* is the equilibrium output. Condition (16a) then implies

$$(18) \quad (\alpha^*, \beta^*) \in \operatorname{argmax} \alpha \sum_{s \in Q} \sum_{A \subseteq Q} \phi_s \beta(A|s) \pi^*(\gamma^*(A), x^*) + (1-\alpha) \pi^*(\gamma^*(Q), x^*).$$

Expression (18) is the same as (5) or (5') with $\pi(\gamma) = \pi^*(\gamma, x^*)$. If we let $\pi^M(\gamma, x) = p(x, \gamma)x - c(x)$, then replacing every "*" in (18) by an "M" yields a condition for the monopoly equilibrium that is also the same as (5)

since \hat{x} is positive and maximizes $h(x, e_s)$. Hence,

$$\begin{aligned} h_1(\hat{x}, \gamma) &= \sum_{t \leq s} \gamma_t (u_1^t(\hat{x}, Y - p\hat{x}) - pu_2^t(\hat{x}, Y - p\hat{x})) \\ &= \sum_{t \leq s} \gamma_t \left[\frac{u_1^t(\hat{x}, Y - p\hat{x})}{u_2^t(\hat{x}, Y - p\hat{x})} - \frac{u_1^s(\hat{x}, Y - p\hat{x})}{u_2^s(\hat{x}, Y - p\hat{x})} \right] u_2^t(\hat{x}, Y - p\hat{x}) \\ &< 0, \end{aligned}$$

since $u_1^t/u_2^t < u_1^s/u_2^s$ for all $t < s$. Because h is quasiconcave in x , this shows that $x(p, \gamma) < \hat{x} = x(p, e_s)$. Thus, the demand curve $x(\cdot, e_s)$ is to the right of the demand curve $x(\cdot, \gamma)$ at every p for which $x(p, e_s) > 0$. Because $p(\cdot, e_s)$ and $p(\cdot, \gamma)$ are the corresponding inverse demand functions, this implies $p(x, e_s) > p(x, \gamma)$ for all $x > 0$. // //

5. Conclusion

We conclude by noting the positive and normative consequences, in our model, of a law which requires firms to test and disclose product quality. Because firms will voluntarily test and disclose if they cannot credibly claim they are ignorant, a law designed to induce firms to test and disclose will be redundant in this case. Hence, a necessary condition for the law to have an effect is that claims of ignorance be credible. We have shown that disclosure rules forcing firms to disclose all that they know will serve to make claims of ignorance credible, as will sufficiently high testing costs and, we believe, sufficient uncertainty on the part of consumers about the nature of testing.

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auditors and lawyers can be hired to study the records of an accused firm. We assume here that for whatever reason, sellers will not lie.

The flow of information from sellers to buyers is enhanced more if sellers are not only honest, but also if they disclose everything relevant that they know. An example of a government-provided incentive to disclose is a mandatory disclosure law, such as is prevalent in securities markets. A less obvious example is the common law practice of awarding greater damages to plaintiffs in product liability cases when it is discovered the seller had secretly known of design flaws. The intent in both cases is to cause a seller to expect to pay a penalty in the event of product failure unless he had fully revealed his knowledge of possible defects. As long as there is positive probability that the information held by a seller can be discovered ex post, the penalties for not disclosing can be made large enough to make full disclosure optimal. For lack of a better term, we shall refer to all government-provided incentives to disclose as disclosure rules.

The market can also provide incentives for exogenously informed sellers to disclose. The argument for this is contained in Grossman [1981], Milgrom [1981], and Milgrom and Roberts [1984]. These authors consider a seller and prospective buyers to be playing a "game of persuasion" in which the strategy of the seller is to reveal information. Although the seller cannot lie because of the presence of effective antifraud laws, he can be vague, up to claiming total ignorance. If the profits of the seller increase in the quality of the product, the seller would like to convince buyers that quality is high. Buyers will therefore assume the worst--any vague claim about product quality will be interpreted as revealing that the true quality is at the lowest level consistent with the claim being truthful. The seller responds to this skepticism by precisely disclosing the true quality.

Some examples might be a software package whose job-specific effectiveness is in doubt, an air conditioning design whose efficiency is in doubt, a financial security whose return is in doubt, or a demonstrably safe drug intended to cure cancer (Laetrile?) whose efficacy is in doubt. For such products it is not clear that welfare is greatest in a regime where consumers are informed before they purchase. Releasing quality information will only cause the price of a product to be positively correlated with its quality.

A well-known result regarding the welfare effects of public information is that of Hirshleifer [1971], which is generalized by Marshall [1974]. These authors compare a regime characterized by a complete set of contingent claims exchange markets to another regime that differs only by the public announcement of information before the markets open. They find that the outcome in the second regime, the one with informed traders, is inefficient because of the uninsurable price risk created by the announcement.

We find instead that in some cases the regime with informed consumers is unanimously preferred by producers and consumers, although in other cases the regime with uninformed consumers is unanimously preferred. Our results differ because we assume, in order to allow demand to depend on quality, that consumers have state-dependent utility. As Hirshleifer and Riley [1979] note, demand for insurance can be negative if utility is state dependent. Public information cannot then be shown undesirable simply because it creates an uninsurable price risk.

Which regime is better depends in part upon whether income and quality are substitutes or complements in utility. We first assume utility is quasilinear, the case in which income effects are zero. We show then that the regime with informed consumers is Pareto superior if quality and income are substitutes, whereas it is Pareto inferior if they are complements. The

equilibrium. However, instead of defining a game between the firm and consumers, we define in this section only a "reduced form game" that is more in the tradition of partial equilibrium models of the firm. The model specifies only (i) how the firm's profit depends on the probability beliefs of consumers about quality, and (ii) how the consumers change those beliefs when the firm makes various claims about quality. Left unspecified are the objectives and actions of consumers, and the decisions of the firm other than its testing and announcement decisions. As is shown in Section 4, the reduced form game defined here is consistent with a model in which the firm is a monopoly price-setter and the consumers are competitive maximizers of expected utility, where the probabilities they use depend upon what the firm claims about quality. It will also be shown to be consistent with a Walrasian model in which the equilibrium price depends upon the beliefs of consumers through their demand functions.

The set of states of nature that determine quality is $Q = \{1, \dots, S\}$. The interpretation will be that states are numbered so that quality increases with the state; we shall sometimes refer to a state as a quality level. The initial beliefs of both the firm and the consumers are specified by a probability vector $\phi = (\phi_1, \dots, \phi_S)$, where each ϕ_s is positive.

Let e_s be the probability vector putting all probability on state s . The firm's profit is given by a function $\pi(\gamma)$ of the beliefs of consumers at the time they purchase, $\gamma = (\gamma_1, \dots, \gamma_S)$. The central assumption is that the firm wants consumers to believe quality is high. We could capture this by assuming $\pi(\gamma)$ increases when γ increases in the sense of stochastic dominance. Instead, we make the weaker assumption that if two probability vectors are ordered by stochastic dominance, and if one of them puts all

$$(2a) \quad 0 \leq \alpha \leq 1;$$

$$(2b) \quad \beta(\cdot|\cdot) \geq 0 \text{ and } \sum_{A \subseteq Q} \beta(A|s) = 1 \text{ for all } s \in Q; \text{ and}$$

$$(2c) \quad \beta(A|s) = 0 \text{ for all } A \subseteq Q \text{ and } s \notin A.$$

Conditions (2a) and (2b) require that α and β be probability functions.

Condition (2c) requires the firm to report a superset of the true state when it tests, which reflects the effectiveness of antifraud laws.

Consumers will update their beliefs about quality when they hear an announcement by the firm. Their rule for updating is given by an inference function $\gamma(A) = (\gamma(1|A), \dots, \gamma(S|A))$, where $\gamma(s|A)$ is the probability put on state s when the firm reports subset A . An inference function is feasible if and only if it satisfies the laws of probability:

$$(3) \quad \gamma(\cdot|\cdot) \geq 0 \text{ and } \sum_{s \in Q} \gamma(s|A) = 1 \text{ for all } A \subseteq Q.$$

For (α, β, γ) to be an equilibrium, we require γ to be a rational response to (α, β) in the sense of Kreps and Wilson [1982]. That is, γ must be consistent with both Bayes' rule and with the fact that the firm can only report supersets of the truth. Equilibrium therefore requires

$$(4a) \quad \gamma(s|Q) = \frac{\phi_s [\alpha \beta(Q|s) + 1 - \alpha]}{\alpha \sum_{t \in Q} \phi_t \beta(Q|t) + 1 - \alpha} \quad \text{if} \quad \alpha \sum_{t \in Q} \phi_t \beta(Q|t) + 1 - \alpha > 0;$$

$$(4b) \quad \gamma(s|A) = \frac{\phi_s \beta(A|s)}{\sum_{t \in Q} \phi_t \beta(A|t)} \quad \text{if } A \neq Q \text{ and } \sum_{t \in Q} \phi_t \beta(A|t) > 0; \text{ and}$$

$$(4c) \quad \gamma(s|A) = 0 \text{ for all } A \subseteq Q \text{ and } s \notin A.$$

pessimistic of beliefs whenever it is unwarranted. Thus, these strategies constitute a sequential equilibrium without disclosure rules. The following proposition characterizes all such equilibria.

Proposition 1: Any (α, β, γ) satisfying (2) and (3) is a sequential equilibrium without disclosure rules if and only if

- (i) $\alpha = 1$;
- (ii) $\beta(A|s) > 0$ implies $s = m(A)$; and
- (iii) $\gamma(m(A)|A) = 1$ for all $A \subseteq Q$.

Proof We show only the necessity of (i)-(iii); the proof of sufficiency is straightforward. So let (α, β, γ) be an equilibrium.

(i) Suppose $\alpha < 1$. Then there is positive probability that Q will be reported and the true state is not S . Hence, $\gamma(S|Q) < 1$ (see (4a)). Therefore (1) implies $\pi(\gamma(Q)) < \pi(e_S)$, so that expected profit conditional on S is less than $\pi(e_S)$:

$$\begin{aligned} \Pi(\alpha, \beta, \gamma|S) &= \alpha \sum_{A \subseteq Q} \pi(\gamma(A))\beta(A|S) + (1-\alpha)\pi(\gamma(Q)) \\ &< \alpha \sum_{A \subseteq Q} \pi(e_S)\beta(A|S) + (1-\alpha)\pi(e_S) = \pi(e_S). \end{aligned}$$

But the firm could obtain $\pi(e_S)$ in state S by testing for sure and announcing $A = \{S\}$ whenever S is the true state. Because it does not have to disclose, the firm need not alter its reporting strategy in other states. This yields a new strategy that is feasible and increases expected profit, a contradiction.

(ii) We first show that for a given A , $\beta(A|s) > 0$, for at most one s . Assuming not, let t be the largest state such that $\beta(A|t) > 0$, and let $s < t$

of a feasible strategy for the firm. We say (α, β) is feasible in the game with disclosure rules if and only if it satisfies

$$(2'a) \quad 0 \leq \alpha \leq 1, \text{ and}$$

$$(2'b) \quad \beta(A|s) = \begin{cases} 0 & \text{if } A \neq \{s\} \\ 1 & \text{if } A = \{s\}. \end{cases}$$

Condition (2'a) is the same as (2a), whereas (2'b) strengthens (2b) and (2c) to require that the firm fully disclose the state when it tests.

An inference function remains feasible provided it satisfies (3), the rules of probability. It is a rational response to any (α, β) satisfying (2') provided it satisfies

$$(4'a) \quad \gamma(s|Q) = \phi_s \text{ for all } s \in Q, \text{ and}$$

$$(4'b) \quad \gamma(s|A) = 0 \text{ for all } s \notin A.$$

Condition (4'a) reflects knowledge of the effectiveness of disclosure rules. Because it is known that the firm must fully disclose when it tests, an announcement of total ignorance must mean the firm did not test, so that consumers should not infer anything from such an announcement. Condition (4'b) is the same as (4c), and it merely reflects the effect of antifraud laws, namely, that the firm must announce a superset of the true state. This minimal rationality requirement is all that is required.

The following proposition characterizes the sequential equilibria with disclosure rules, which are triples (α, β, γ) satisfying (2'), (3), (4'), and

$$(5') \quad \Pi(\alpha, \beta, \gamma) \geq \pi(\hat{\alpha}, \hat{\beta}, \gamma) \text{ for all } (\hat{\alpha}, \hat{\beta}) \text{ satisfying (2')}.$$

announcement of ignorance must be taken at face value when it is known that disclosure rules cause the firm to disclose all that it knows. There are other reasons why announcements of ignorance might be credible, such as (i) a necessity for testing publicly, so that consumers would always know whether quality had been tested; (ii) a nontrivial cost to testing; or (iii) a more complicated information structure, so that consumers would not know how much the firm knew before testing or what it learns from testing. Including these factors, particularly (iii), is beyond the scope of this paper.

We have also not considered the conditions under which disclosure rules would be effective. To do so would involve introducing a third agent with an ex post information structure who would enforce penalties when discontented customers brought legal action against the firm, somewhat along the lines of Palfrey and Romer [1983]. Disclosure rules would then alter the firm's payoffs instead of its strategy set; the latter was justified here only because we assumed that disclosure rules would be either fully effective or fully ineffective. This extension is also beyond the scope of this paper. However, we conjecture that it would yield the following result: if a disclosure rule could be effective, then even without it the firm will test and disclose if and only if it prefers to have its customers informed. The logic for this is that a disclosure rule could be effective only if the third party can determine ex post whether the firm had tested. But then, the firm should be able to sign, and make public, a binding contract to the effect that it will pay a large penalty if it claims ignorance but the third party discovers it had tested quality. This would serve to make its announcements of ignorance credible even in the absence of disclosure rules, giving us another situation in which the firm would not test and disclose necessarily.

$$(6) \quad u_1^s/u_2^s < u_1^{s+1}/u_2^{s+1} \quad \text{for all } s=1, \dots, S-1.$$

This assumption will imply that the consumer's demand for x increases in s if she is informed. We also assume that for every probability vector $\hat{\gamma}$, the function $\sum \hat{\gamma}_s u^s(x, y_s)$ is twice continuously differentiable and strictly quasiconcave in (x, y_1, \dots, y_S) , with each $u_1^s > 0$ and $u_2^s > 0$.

Good x is produced from good y according to a cost function $y = c(x)$, which has continuous derivatives $c' > 0$ and $c'' \geq 0$. The basic technological assumption is that the output decision must be made before anyone knows the true state of nature. This is in keeping with the interpretation that quality information can be collected only by testing a finished product.

The consumer is not informed about quality at the time she purchases in the uninformed consumer regime, whereas she is informed at that time in the informed consumer regime. In the informed consumer regime, the price of x will depend on its quality. The firm is assumed to have perfect foresight regarding the state-dependent prices. In both regimes only the product market opens; in neither regime is there a separate insurance market allowing income to be shifted from one state to another.

Formally, an uninformed consumer equilibrium (UCE) is a triple (x^U, y^U, p^U) such that (x^U, y^U) maximizes both traders' objective functions given the price p^U :

$$(7) \quad (x^U, y^U) \in \operatorname{argmax}_{x, y \geq 0} E u^s(x, y) \quad \text{S.T.} \quad p^U x + y \leq Y;$$

$$(8) \quad x^U \in \operatorname{argmax}_{x \geq 0} p^U x - c(x).$$

and that equilibrium is given by equating the expected price to marginal cost:

$$(14) \quad E p_S(x^I) = c'(x^I) \quad \text{and} \quad p_S^I = p_S(x^I).$$

Condition (6) implies $p_1(x) < \dots < p_S(x)$ (see Section 4). Hence, the equilibrium prices and incomes satisfy $p_1^I < \dots < p_S^I$ and $y_1^I > \dots > y_S^I$.

Our first result is based on the observation that informing the consumer of product quality can act as a partial substitute for missing insurance markets by allowing her to shift income from high quality states to low quality states. In the informed consumer regime, the consumer will pay more, and hence consume less income, in high quality states. It is as though insurance against low quality is bundled with the product. The consumer would purchase such insurance at an actuarially fair rate if her marginal utility of income is low when the quality is high, i.e., if income and quality are substitutes in utility. She would regard such insurance as harmful if income and quality are complements, even if she is risk averse in the sense of each utility function u^S being concave. A conjecture, then, is that the informed consumer regime is preferred if income and quality are substitutes, but the uninformed consumer regime is preferred if they are complements. This conjecture is correct if the consumer has quasilinear utility, as we now show.

Proposition 3: Suppose that for each s , $u^S(x,y) = q_S v(x) + r_S y$, with $v' > 0$ and $v'' < 0$. Then, both the consumer and the firm prefer the UCE to the ICE if income and quality are complements ($r_1 < \dots < r_S$), whereas they both prefer the ICE to the UCE if income and quality are substitutes ($r_1 > \dots > r_S$). (The firm is indifferent if marginal cost is constant; the consumer is indifferent if supply is perfectly inelastic.)

Proposition 3 is striking: for the given class of utility functions, either regime can Pareto dominate the other, depending upon whether the marginal utility of income is higher or lower in high quality states. As an extreme example, consider a new drug that has been proven safe and which can perhaps cure an otherwise incurable fatal disease. The drug should obviously be used even though it is not certain to work. A low quality state entails death and hence, arguably, a lower marginal utility of income. Given its assumptions, Proposition 3 implies that no attempt should be made to acquire and disseminate information about the efficacy of the drug.

Proposition 3 does not hold in all respects for more general utility functions. While one regime may better equalize the marginal utilities of income across states, the other regime may still be preferred by one of the traders because of its more favorable terms of trade. Consider the following.

Example 1: The firm sells an asset that returns q_s dollars per unit in state s . The consumer cares only about income and is risk averse. Thus, $u^s(x, y) = u(q_s x + y)$, and u is strictly concave. Note that the marginal utility of income $u'(q_s x + y)$ decreases in s because of concavity and $q_1 < \dots < q_S$, so that income and quality are substitutes. Nevertheless, contrary to the conclusion of Proposition 3, the consumer prefers the UCE to the ICE. This is because her demand in each state in the informed consumer regime is perfectly elastic at $p_s = q_s$, so that she obtains no surplus, receiving only the utility $u(Y)$ of her endowment in every state. Her expected utility is greater than $u(Y)$ in the uninformed consumer regime because the inverse demand function $p_U(x)$ does slope down. She prefers the UCE to the ICE, despite being perfectly insured against the risk of low quality in the

$$E\{(y_s^I - \bar{y})u_2^s(x^I, \bar{y})\} = E\{(y_s^I - \bar{y})(u_2^s(x^I, \bar{y}) - u_2^t(x^I, \bar{y}))\} < 0.$$

The allocation $(x^I, y_1^I, \dots, y_s^I)$ thus lies below the hyperplane tangent to the indifference curve at $(x^I, \bar{y}, \dots, \bar{y})$. Since $Eu^s(x, y_s)$ is quasiconcave in (y_1, \dots, y_s) , this implies that the consumer prefers $(x^I, \bar{y}, \dots, \bar{y})$ to the ICE allocation. Now, the consumer prefers, at least weakly, the UCE allocation (x^U, y^U, \dots, y^U) to $(x^I, \bar{y}, \dots, \bar{y})$ by a revealed preference argument: constant marginal cost implies $p^U = c' = Ep_s^I$, so that the consumer could have afforded $(x^I, \bar{y}, \dots, \bar{y})$ when she chose (x^U, y^U, \dots, y^U) , as $p^U x^I + \bar{y} = (Ep_s^I)x^I + E(Y - p_s^I x^I) = Y$. Transitivity therefore implies that the consumer prefers the UCE to the ICE. The firm, because marginal cost is constant, receives zero expected profit in both regimes and is hence indifferent. ////

Between them, Propositions 3 and 4 make a good case for the superiority of the uninformed consumer regime when quality and income are complements in utility. Example 1 shows that consumers may prefer not being informed even when quality and income are substitutes. Proposition 3 and Example 1 together illustrate three of the four patterns of agreement and disagreement between the firm and the consumer. The next example shows the fourth possibility, that in which the consumer prefers the ICE and the firm prefers the UCE.

Example 2: Let $u^s(x, y) = q_s \ln(x) + (1 - q_s) \ln(y)$, where $0 < q_1 < \dots < q_s < 1$.

Then calculation yields $p_U(x) = (Eq_s)Y/x$ and $p_s(x) = q_s Y/x$. The firm therefore faces the same ex ante demand curve in both regimes,

$p_U(x) = Ep_s(x)$. Hence, the regimes result in the same outputs, $x^U = x^I$, and

the firm is indifferent between them. The consumer prefers the ICE, since

occur, and the determining factor in the case of quasilinear utility remains whether income and quality are substitutes or complements.

4. Relating the Models

We now fit the models together. Instead of assuming the consumer in Section 3 is either informed or not, assume now that the firm controls how much the consumer knows by its testing and reporting choice. We can then let the firm's strategy be a triple (x, α, β) , where x is an output level and (α, β) is a testing and reporting strategy as in Section 2. The consumer's beliefs conditional on an announcement $A \subseteq Q$ by the firm will be given by an inference function $\gamma(s|A)$. If her beliefs after the announcement are γ , the resulting inverse demand curve is given implicitly by

$$(15) \quad p(x, \gamma) = \frac{\sum_{s \in Q} \gamma_s u_1^s(x, Y - p(x, \gamma)x)}{\sum_{s \in Q} \gamma_s u_2^s(x, Y - p(x, \gamma)x)} .$$

If the firm is competitive, it believes it can influence price only by its reporting strategy, not by its quantity choice. The price is therefore determined by a function $p(A)$. An equilibrium is a five-tuple $(\alpha^*, \beta^*, \gamma^*, p^*, x^*)$ such that (α^*, β^*) satisfies the feasibility condition (2) ((2') if disclosure rules are effective); γ^* satisfies the feasibility condition (3) and the rationality condition (4) ((4') if disclosure rules are effective); the firm maximizes expected profit:

or (5')). We conclude that in either case, the reduced form of the model is exactly as defined in Section 2.

Proposition 1 and 2 regarding voluntary disclosure and testing will hold in both the competitive and the monopoly cases if $\pi^*(\gamma, x^*)$ and $\pi^M(\gamma, x^M)$ are monotonic in γ in the sense of (1). Their monotonicity follows immediately if the price function $p(x, \gamma)$ is monotonic in the same sense, i.e., if

$$(19) \quad \text{For any } x > 0 \text{ and } e_s \neq \gamma, \quad p(x, e_s) \begin{cases} < p(x, \gamma) & \text{if } \sum_{t < s} \gamma_t = 0 \\ > p(x, \gamma) & \text{if } \sum_{t > s} \gamma_t = 0. \end{cases}$$

Note that (19) implies the assertion made in Section 3 that $p_s(x) < p_{s+1}(x)$. Our last proposition is that (19) follows from assumption (6), which was that $u_1^s/u_2^s < u_1^{s+1}/u_2^{s+1}$.

Proposition 5: The inverse demand function $p(x, \gamma)$ satisfies (19).

Proof: Let $s \in Q$ and $\gamma \neq e_s$ satisfy $\gamma_{s+1} = \dots = \gamma_s = 0$. We show that $p(x, e_s) > p(x, \gamma)$ for all $x > 0$. The proof of the other part of (19) is essentially the same.

Let $x(p, \hat{\gamma})$ be the consumer's demand for x when its price is p and her beliefs are $\hat{\gamma}$. Let $p > 0$ be a price such that $x(p, e_s) > 0$. Lastly, let $h(x, \hat{\gamma}) = \sum_{s \in Q} \hat{\gamma}_s u(x, Y - px)$, and $\hat{x} = x(p, e_s)$. Then

$$0 = h_1(\hat{x}, e_s) = u_1^s(\hat{x}, Y - p\hat{x}) - pu_2^s(\hat{x}, Y - p\hat{x}),$$

Even if factors exist to make claims of ignorance credible, firms will still voluntarily test and disclose if they prefer, net of testing costs, to have informed rather than uninformed customers. A second necessary condition for the law to have an effect is, therefore, that firms prefer their customers to be uninformed. But if the demand functions of informed consumers exhibit negligible income effects, i.e., if their utility functions are quasilinear, they will agree with firms about whether they should be informed. Thus, a necessary condition for a mandatory testing and disclosure law to be both effective and desired by consumers is that demand should depend significantly on income, which is the only case in which consumers could prefer to be informed when the firm prefers them to be uninformed.

Any policy conclusions based on these arguments should be taken strictly within the context of the model. In particular, it must be remembered that we have assumed information about quality to have no productive use, such as allowing the firm to make better production decisions or consumers to make better use of the product. While we believe the model has relevance if testing reveals information about the worth of financial securities or the efficacy of a new drug, it is not so relevant if testing reveals information about the safety of a new drug. Another caveat is that by assuming quality is exogenous, we have not addressed the interesting question of whether mandatory testing and disclosure laws can induce firms to provide higher quality.