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**Working Paper**

# The Relation Between Social Security, Saving, and Investment in a Life-Cycle Model

Discussion Paper, No. 492

**Provided in Cooperation with:**
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*Suggested Citation:* Bewley, Truman (1981) : The Relation Between Social Security, Saving, and Investment in a Life-Cycle Model, Discussion Paper, No. 492, Northwestern University, Kellogg School of Management, Center for Mathematical Studies in Economics and Management Science, Evanston, IL

This Version is available at:
https://hdl.handle.net/10419/220852

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# THE RELATION BETWEEN SOCIAL SECURITY, SAVING, AND INVESTMENT IN A LIFE-CYCLE MODEL */

by

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August 1981

Revised October 1981

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*/ The research reported here was done with the support of National Science Foundation Grant No. SOC 79-06672

## 1) Introduction

In this paper, I try to clarify the theoretical effects of social security on the stock and flow of personal saving, on investment, and on the capital stock. This paper was stimulated by reading Feldstein's well-known paper on the subject (1974), which I found perplexing. I have found no other paper written since which has treated the subject to my satisfaction, though Hymans(1980) and Eisner (1980) point to the flaw in Feldstein's argument which caught my attention. Feldstein argues that social security decreases personal saving and that this decrease in turn depresses investment. It is true that social security might well depress personal saving. But it is wrong to assert that a decrease in personal saving implies a decrease in real investment, for investment equals the sum of private and government saving, and government saving can be affected by social security. It is this error that was caught by Hymans and Eisner. Eisner also makes the important observation that when social security is increased, the increase in the government's social security liability can be offset by a decline in its debt.

I have also found that Feldstein uses a conceptually incorrect measure of social security wealth when estimating his consumption function. I discuss this matter as well.

I use a version of Diamond's model from his paper on national debt (1965). This model is an overlapping generations model with a single production sector. There are no bequests, though I discuss bequests briefly in the concluding section. There is a government in the model which can issue debt. The model is of a closed economy, so that there is no external debt. The

government finances the interest on its debt by lump-sum taxes. Social security consists of lump-sum tax and benefit payments. My analysis is comparative static. I compare steady state equilibria before and after the introduction of social security.

From the point of view of a purist, one can make few general statements about the effect of social security on investment and the capital stock. However, if one takes a more relaxed attitude, one can identify circumstances which seem normal and in which some interesting statements are possible. In the normal circumstances, an increase in taxes increases the equilibrium interest rate and reduces the capital stock. Roughly speaking, this is so because the government's budget must balance, so that an increase in taxes implies an increase in interest payments on the government's debt. Such payments would normally increase only because of higher interest rates.

Thus, an increase in social security contributions reduces the capital stock and an increase in benefits increases it. This reasoning leads to the conclusion that the form of social security which gives the largest capital stock is one with no contributions at all. However, this reasoning is misleading. The proper conclusion to be drawn from the model of this paper seems to be that the effect of social security on the capital stock should be ignored since the equilibrium capital stock may be manipulated at will by changing the magnitude of real government debt. (I do not mean to imply that such manipulation is possible in reality, but only that it is possible in the model.)

In determining the effects of social security, the key number to look at is the present value to a young person of the lifetime contributions and benefits of social security. If this number is zero, social security has no effect on the capital stock or any other real variable, as Samuelson (1975), Hymans (1980) and

Eisner (1980) have pointed out. If this number is positive, the capital stock increases under normal circumstances.

I also argue that a correct measure of social security wealth must start from the idea that the key number is the present value of social security to people when they are young. More precisely, if one uses a life-cycle theory of consumption, then one should look at the wealth of an individual when he plans his spending, if one wants to estimate the relation between wealth and consumption. Feldstein adds up the wealth of individuals of all ages, ignoring the fact that once an individual's plans are fixed, then the evolution of his wealth during his lifetime does not determine or change his plans.

The organization of the paper is as follows. In the next section, I describe the model, assuming no growth in population or productivity. In section 3, I discuss the real effects of changes in taxes or stationary equilibria and identify what I call normal circumstances. In section 4, I discuss the effects of social security on the interest rate and capital stock. In section 5, I discuss the effects of social security on consumers' equilibrium stocks of assets and on the government debt. In section 6, I introduce into the model growth in population and productivity. In section 7, I discuss the effects of social security on investment and on the flows of personal and government saving. This discussion makes sense only if there is growth, for without growth all these variables are zero in a stationary state. In section 8, I discuss measures of social security wealth. In section 9, I estimate the drop in the stock and flow of personal saving for 1971 caused by social security. These estimates are made under the absurd assumption that social security had present value zero to young people. I make these estimates only in order to illustrate how the results of the previous sections fit together. In section 10, I discuss Feldstein's paper.

2) The Model

As I have said, the model is essentially that of Diamond, (1965, 1973) in that it combines Samuelson's consumption loan model (Samuelson, 1958) with a one sector growth model. All consumers have identical utility functions and endowments. One consumer is born each period, so that there is no population growth. Each consumer lives $A + 1$ periods, where $A \geq 1$. There are two goods, these being labor and one produced good, which I call food. $L_a$ denotes the labor supply of a consumer in his $a^{th}$ period of life, where $a = 0,1,\ldots,A$. The utility function of a consumer is of the simple form $\sum_{a=0}^{A} (1+\rho)^{-a} u(x_a)$, where $x_a$ is his consumption of food in the $a^{th}$ period of life. The number $\rho$ is his pure rate of time preference. I assume that $u$ is continuously differentiable, strictly increasing and strictly concave.

There is one industry. The production function of the industry is $y = f(k,L)$, where $k$ is the input of food, $L$ is the input of labor, and $y$ is the output of food. I assume that $f$ is continuously differentiable, concave, homogeneous of degree one, non-decreasing, and such that $f(k,L) = 0$ if either $k$ or $L$ is zero. Also, $\lim_{k \to 0} \frac{\partial f(k,1)}{\partial k} = \infty$.

Since my analysis will be of a comparative static nature, I define only equilibria which are stationary.

A stationary allocation is of the form $((x_a),k)$, where $(x_a) = (x_0,\ldots,x_A)$. The allocation is feasible if $\sum_{a=0}^{A} x_a + k = f(k, \sum_{a=0}^{A} L_a)$. Notice that the capital stock does not depreciate.

The only prices are the wage rate, $w$, and an interest rate, $r$. The unit of account is food. The government collects a lump-sum tax of $\tau_a$ each period

from the consumer of age $a$, for $a = 0,1,\ldots,A$. $\tau_a$ may be negative, in which case it represents a subsidy. Consumers can borrow and lend freely. Their only budget constraint is that they not be in debt when they die. There are no bequests, so that the budget constraint of a consumer is

$$\sum_{a=0}^{A} (1+r)^{-a} x_a = \sum_{a=0}^{A} (1+r)^{-a}(wL_a - \tau_a).$$

A stationary equilibrium is of the form $((x_a),k;\ w,r,(\tau_a))$, where this vector must satisfy the following conditions:

(i) $((x_a),k)$ is a feasible allocation,

(ii) $(x_a)$ solves the problem

$$\max \sum_{a=0}^{A} (1+\rho)^{-a} u(y_a)$$

$$\text{subject to} \quad \sum_{a=0}^{A} (1+r)^{-a} y_a = \sum_{a=0}^{A} (1+r)^{-a}(wL_a - \tau_a),$$

(iii) $(k, \sum_{a=0}^{A} L_a)$ solves the problem

$$\max_{K \geq 0, L \geq 0} [f(K,L) - (1+r)K - wL].$$

It is easy to prove that

2.1) for any $r \geq 0$, there exists a stationary equilibrium with interest rate $r$. The taxes $\tau_a$ may be chosen so that $\tau_a = \tau_0$, for all $a$.

It is also true that all such equilibrium are Pareto optimal. These facts, and related matters are discussed extensively in Diamond (1973) and Bewley (1981).

It is not true that a stationary equilibrium exists for any specification of the taxes $(\tau_a)$. They may simply be too large or too small.

The aggregate net asset position of consumers in a stationary equilibrium

$((x_a),k;\ w,r,\ (\tau_a)$ is

$K = \sum_{a=0}^{A} \sum_{s=0}^{a} (1+r)^{a-s}(wL_s - x_s - \tau_s)$. It is not hard to prove that $K$ obeys the following equation

2.2) $$r(K - k) = \sum_{a=0}^{A} \tau_a$$

This equation may be interpreted as saying that the government's budget balances. Since consumers can hold only real assets or government debt, the difference $K - k$ is the government's debt. Thus $r(K - k)$ is the interest payment on the government debt and $\sum_{a=0}^{A} \tau_a$ is the tax revenue.

Some authors have made the equation $K = k$ a part of the definition of equilibrium. This condition amounts to assuming that the government has no debt. It does not seem appropriate to fix $K - k$, for to do so is to deprive the government of one of its policy instruments, that instrument being the degree of its indebtedness. How can one fix this instrument while allowing the government to carry out a major change such as the introduction of social security? Also, it is a fact of life that governments tend to be large debtors, and that their indebtedness changes as a result of deficits and inflation.

The addition of the equation $K = k$ determines the interest rate. In the equilibrium I have defined above, the interest rate is not determined, but is a free variable. Another effect of requiring $K = k$ is to reduce the range of possible social security systems, for the taxes must always satisfy $\sum_{a=0}^{A} \tau_a = 0$. Also, the requirement that $K = k$ makes it much more likely that social security would reduce the capital stock, for social security is likely to reduce $K$.

Kotlikoff (1979a) has required $K = k$ in his analysis of the effect of social security on the capital stock. (See p. 238, beginning of the first full paragraph.) It is for this reason that his theoretical model predicts that social security would have a large negative effect on the capital stock.

Remark It is important to understand that in the model of this paper, equation (2.2) is not an equilibrium condition, but is a consequence of equilibrium. It is a form of Walras' law.

The number $\sum_{a=0}^{A} \tau_a$ may be negative, in which case the government is a net lender to the public. In fact, it is easy to make up examples in which $\Sigma \tau_a$ is negative. ($\Sigma \tau_a$ is negative in the example of the next section.) If a bequest motive were included, then $\Sigma \tau_a$ would be less likely to be negative, for the public would hold more assets.

Equilibrium is possible only if the government is willing to provide the quantity $K - k$ of debt. Sudden changes in this quantity could be brought about by a capital levy or distribution. More gradual changes in government debt could be brought about by the inflation or deflation resulting from changes in the vector of taxes $(\tau_a)$. Of course, inflation or deflation could not change the sign of the government's indebtedness.

Questions of timing turn out to be very important in this paper, especially when I examine measures of social security wealth in section 8. For this reason, I now explain carefully the chronometric conventions I use throughout the paper. I think of time as divided into days and nights. Business activity takes place during the day. Interest is paid or earned on assets held overnight and the interest is

paid during the following day. All accounts are as of the end of a day, so that all stock variables are measured at the end of a day. Thus, $k$ is the stock of capital at the end of a day and $K$ is the aggregate asset position of consumers at the end of a day. All flows occur during a day. Thus, a consumer of age $a$ pays $\tau_a$ in taxes during a day, consumes $x_a$ during a day, and contributes $L_a$ units of labor during a day. Firms use the capital stock held at the end of the previous day and the labor of the current day to produce the output of the current day.

For the moment, I index by time $t$ all variables other than $r$ and $w$. $x_{at}$ is the consumption during period $t + a$ of a consumer born in period $t$, $k_t$ is the capital stock at the end of period $t$, and so on. The feasibility condition for day $t$ is $\sum_{a=0}^{A} x_{at} = f(k_{t-1}, \sum_{a=0}^{A} L_{at})$. If we imagine that firms borrow during day $t-1$ in order to pay for $k_{t-1}$, then their profit during day $t$ is $f(k_{t-1}, \sum_{a=0}^{A} L_{at}) - (1+r)k_{t-1} - w \sum_{a=0}^{A} L_{at}$. (Because of constant returns to scale, the maximum profit is zero.) The number $K_t$ is defined as $K_t = \sum_{a=0}^{A} \sum_{s=0}^{a} (1+r)^{a-s}(wL_{s,t-a+s} - x_{s,t-a+s} - \tau_{s,t-a+s})$. $G_t \equiv K_t - k_t$ is the debt of the government at the end of day $t$. Interest on this is paid during day $t+1$, so that equation (2.2) should be read as $r G_t = \sum_{a=0}^{A} \tau_{a,t+1}$.

3) Comparative Statics in the Model

It is important to realize that taxes affect equilibrium only through the number $\sum_{a=0}^{A} (1+r)^{-a}\tau_a$. That is,

3.1) if $((x_a),k;w,r,(\tau_a))$ is a stationary equilibrium, then so is $((x_a),k;\ w,r,\ (\hat{\tau}_a))$, provided that $\Sigma(1+r)^{-a}\tau_a = \Sigma(1+r)^{-a}\hat{\tau}_a$ .

The taxes affect the equilibrium only through the budget constraint on consumers, and the number appearing in the budget constraint is $\Sigma(1+r)^{-a}\tau_a$.

I now argue that under normal circumstances, an increase in any tax $\tau_a$ will increase the interest rate and decrease the capital stock.

First of all, observe that the interest rate determines all the real variables of a stationary equilibrium $((x_0),k;w,r,(\tau_a))$. The interest rate even determines the sum $\sum_{a=0}^{A} (1+r)^{-a}\tau_a$ . I define the function $T(r)$ by $T(r) = \sum_{a=0}^{A} (1+r)^{-a}\tau_a$ .

Fix an equilibrium $((x^a),k;w,r,(\bar{\tau}_a))$ and fix $a$, where $0 \le a \le A$. Imagine that for each interest rate $R$, the $a^{th}$ tax is adjusted so that equilibrium is possible with interest rate $R$. This makes the $a^{th}$ tax a function of $R$, call it $\tau_a(R)$. This function satisfies the equation $T(R) = \sum_{i=0}^{A} (1+r)^{-i}\bar{\tau} + (1+r)^{-a}(\tau_a(R) - \bar{\tau}_a)$. Let $G(R)$ be the level of government debt in the new equilibrium. Then, $\sum_{i=1} \bar{\tau}_i + (\tau_a(R) - \bar{\tau}_a) = RG(R)$, by equation (2.2). Hence, $\frac{d\tau_a(R)}{dR} = G(R) + \frac{RdG(R)}{dR}$ , assuming that $G$ is differentiable.

In most real economies, the level of domestically held government debt is very large, so that we should think of $G$ as large. If we do, it seems likely that $\frac{d\tau_a(R)}{dR}$ would be positive as well. If $\frac{d\tau_a(R)}{dR} > 0$, then we

may invert the function $\tau_a$, and $\frac{dR(\tau_a)}{dt_a} > 0$. It is for these reasons that I assert that under normal circumstances an increase in taxes increases that interest rate.

There seems to be no predictable relation between government debt and the interest rate. In fact, the behavior of the function $G(R)$ defined above is erratic in that it varies markedly from example to example. In many examples, the sign of $\frac{dG(R)}{dR}$ changes as $R$ varies. (See, for instance, example (3.3) below.) We cannot say that an increase in taxes would lead to an increase in government debt, for $\frac{dG(R(\tau_a))}{d\tau_a} = \frac{dG(R(\tau_a))}{dR} \frac{dR(\tau_a)}{d\tau_a}$, and the sign of $\frac{dG}{dR}$ is indefinite, whereas that of $\frac{dR}{d\tau_a}$ is not.

It is easy to see why the effect of the interest rate on the government's debt is indefinite. The government's debt is the difference between the asset holdings of consumers and the real capital stock. Increasing the interest rate decreases the capital stock, and this effect tends to increase the government's debt. However, the decrease in capital decreases consumer income, and this second effect tends to reduce consumers' assets and hence the government debt. It is well-known that if consumer income is held fixed, then the effect of the interest rate alone on consumer asset holdings is indefinite, for changes in the interest rate have both income and substitution effects.

I now return to the effect of taxes on the interest rate. Suppose that all the taxes are parameterized by a single variable $s$, so that we may write $\tau_a(s)$, for $a = 0,1,\ldots,A$. Let $r(s)$ be the equilibrium interest rate when taxes are $(\tau_a(s))$. I now argue that

3.2) under normal circumstances, the sign of $\frac{dr(s)}{ds}$ is the same as the sign of $\sum_{a=0}^{A} (1+r(s))^{-a} \frac{d\tau_a(s)}{ds}$.

In order to see that this statement makes sense, fix a value of $s$, call it $\bar{s}$, and define $\tau_0(r)$ by the equation $T(r) = \tau_0(r) + \sum_{a=1}^{A} (1+r)^{-a} \tau_a(r(\bar{s}))$. Then $\sum_{a=0}^{A} (1+r(s))^{-a} \tau_a(s) = T(r(s)) = \tau_0(r(s)) +$ $+ \sum_{a=1}^{A} (1+r(s))^{-a} \tau_a(r(\bar{s}))$. If one takes the derivative with respect to $s$ at $s = \bar{s}$ of both of the far left and far right sides of these equations and cancels like terms, one obtains

$$\sum_{a=0}^{A} (1+r(\bar{s}))^{-a} \frac{d\tau_a(\bar{s})}{ds} = \frac{d\tau_0(r(\bar{s}))}{dr} \frac{dr(\bar{s})}{ds}.$$

Statement (3.3) now follows from the fact that $\frac{d\tau_0(r(\bar{s}))}{dr} > 0$ under normal circumstances.

The appeal to "normal circumstances" that has been made is a reference to economic reality and not to properties of the model. In fact, it is easy to make up examples of the model in which increases in taxes decrease the interest rate. The following example is a case in point.

3.3) Example $A = 1$, $u(x) = \log x$, $\rho = 0$, $L_0 = 1$, $L_1 = 0$, and $f(k,L) = 2\sqrt{kL}$.

That is, each consumer lives two periods, his utility function is $\log x_0 + \log x_1$, he has one unit of labor in youth and none in old age, and the production function is $2\sqrt{kL}$.

Suppose that $\tau_0 = \tau_1 = \tau(r)$. It is not hard to calculate that $\tau$ is the following function of the interest rate $r$. $\tau(r) = r(r-1)(r+2)^{-2}(r+1)^{-1}$. Then $\frac{d\tau(r)}{dr} = (-2 + 5r + 4r^2 - r^3)(2+r)^{-3}(1+r)^{-2}$, which is negative for small $r$ and for large $r$ and is positive in between. The changes of sign occur somewhere between $r = 1/4$ and $r = 1/2$ and somewhere between 4 and 5. These interest rates may seem too large, but one must remember that since consumers live only two periods, one period equals a generation.

In the above example, the government debt is $G(r) = 2(r-1)(r+2)^{-1}(r+1)^{-1}$, which is not large. $G(r)$ is even negative for $r < 1$. Government debt is rarely large in examples. The model is to this extent not consistent with reality. This lack of consistency is probably due to the absence of bequests. One can make government debt large by introducing bequests.

The comparative static analysis of this section differs somewhat from that of Diamond (1965). He analyzes the effects of changes in government debt on the interest rate and other variables, whereas I analyze the effects of taxes. Also Diamond determines the sign of these effects by assuming a stability condition. The stability can be described as follows. Imagine an infinite horizon, non-stationary equilibrium with rational expectations and suppose that the single produced good is the numéraire in each period. Suppose that the government debt is constant when measured in terms of the numéraire. This means that tax collections each period equal the interest payments on the debt at the current interest rate. Diamond requires that this equilibrium converge to a steady state given any initial conditions. I do not include such a stability condition, for I have in mind that the government has perfect knowledge of the economy, controls the interest rate, and uses taxes and the interest rate in order to bring the economy to a stationary equilibrium. I discuss a model reflecting this description in (1981).

4) Real Effects of Social Security

In what follows, $((x_a),k;w,r,(\tau_a))$ denotes a stationary equilibrium without social security and $((x_a + \Delta x_0),k + \Delta k;\ w + \Delta w,\ r + \Delta r,\ (\tau_a + \Delta\tau_a))$ denotes an equilibrium with social security. The numbers are $\Delta\tau_a$ are the social security contributions and benefits. Contributions correspond to positive values of $\Delta\tau_a$ and benefits to negative values.

I assume that the $\Delta\tau_a$ are such that for some $\underline{a} < A$, $\Delta\tau_a \geqq 0$, for $a < \underline{a}$ and $\Delta\tau_a < 0$, for $a \geqq \underline{a}$. I also assume that $r > 0$. Because of these assumptions,

4.1) $\sum_{a=0}^{A} \Delta\tau_a < 0$ if $\sum_{a=0}^{A} (1+r)^{-a}\,\Delta\tau_a = 0$ and

$\sum_{a=0}^{A} (1+r)^{-a}\,\Delta\tau_a > 0$ if $\sum_{a=0}^{A} \Delta\tau_a = 0.$

I now discuss the real effects of social security. First of all, we have the following.

4.2) If $\sum_{a=0}^{A} (1+r)^{-a}\,\Delta\tau_a = 0$, then social security has no effect on any of the variables $(x_a)$, k, w and r.

This statement is simply another form of statement (3.1). I call social security fully contributory if it satisfies $\sum_{a=0}^{A} (1+r)^{-a}\,\Delta\tau_a = 0$. Social security is fully contributory if it has present value zero to a young person.

Statement (4.2) appears as Theorem 2 in Samuelson (1975). Eisner (1980) and Hymans (1980) have also observed that social security has no real effects when it is fully contributory

I now analyze the real effects of social security when it is not fully contributory. Statement (3.2) implies the following.

4.3) Under normal circumstances, social security increases the interest rate and reduces the capital stock if $\sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a > 0$. Similarly, under normal circumstances social security reduces the interest rate and increases the capital stock if $\sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a < 0$.

In order to see the link between the above statement and (3.2), let $\tau_a(s) = \tau_a + s\Delta\tau_a$ and let $r(s)$ be as in (3.2). Then $\Delta r = r(1) - r(0)$. As long as $\sum_{a=0}^{A} (1+r(s))^{-a} \Delta\tau_a$ does not change sign as $s$ goes from zero to one, then statement (3.2) implies that $\Delta r$ has the same sign as $\sum_{a=0}^{A} (1+r(0))^{-1} \Delta\tau_a = \sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a$. It seems reasonable to suppose that $\sum_{a=0}^{A} (1+r(s))^{-a} \Delta\tau_a$ would change sign only in exceptional cases.

In order to see the significance of statement (4.3), consider the following two special cases. Call social security <u>pay-as-you-go</u> if $\sum_{a=0}^{A} \Delta\tau_a = 0$ and call it <u>free</u> if $\Delta\tau_a \leqq 0$, for all $a$. If social security is pay-as-you-go, then the government takes in in contributions every period as much as it pays out in benefits. If social security is free, no one ever pays any contributions. By (4.1), $\sum_{a=0}^{A} (1+r)^{-a}\Delta\tau_a > 0$ if social security is pay-as-you-go, so that pay-as-you-go social security increases the interest rate and reduces the capital stock. If social security is free, then $\sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a < 0$, so that the interest rate declines and the capital stock increases. Notice that in these two cases, the sign of $\sum_{a=0}^{A} (1+r(s))^{-a} \Delta\tau_a$ would never change.

The fact that free social security would increase the capital stock might seem strange, for one might imagine that the free benefits would increase consumption from a given income and so reduce saving and investment. However, it must be remembered that by assumption the government's budget is in balance both before and after the introduction of social security.

5) Effects on Financial Stocks

In this section, I examine the effects of social security on the asset holdings of consumers and on government debt. Let $K$ be consumers' aggregate asset position before social security, and let $K + \Delta K$ be this position after social security. Similarly, let $G = K - k$ and $G + \Delta G$ be the government's debt before and after social security, respectively.

The effects of social security on financial stocks may be analyzed by using (2.2), which says that $r(K - k) = rG = \sum_{a=0}^{A} \tau_a$. If social security is fully contributory, it does not affect real variables, so that by applying $\Delta$ to the above equation one obtains

$$5.1) \qquad \Delta K = \Delta G = r^{-1} \sum_{a=0}^{A} \Delta\tau_a \ .$$

Also by (4.1), $\Sigma\Delta\tau_a < 0$, so that consumers' asset position and the government's debt decrease. The number $- r^{-1} \sum_{a=0}^{A} \Delta\tau_a$ is the present value of government's future social security liability. Therefore, equation (5.1) simply says that the liability for social security is offset exactly by a decrease in government debt. Notice that the stock of consumer assets decreases even though there are no real changes in the economy.

Eisner (1980) and Samuelson (1975) have already pointed out that when social security is fully contributory, then the changes in social security liability and government debt cancel. This idea also appears in Diamond (1973, p. 222) and in Bierwag, Groves, and Khang (1969).

Now suppose that social security is not fully contributory. As in section 3, consider the taxes $(\tau_a(s)) = (\tau_a + s\Delta\tau_a)$, for $0 \leq s \leq 1$; and consider $K, k$, $G$, and $r$ as functions of $s$. By taking the derivative with respect to $s$ in the equations $r(s)(K(s) - k(s)) = r(s)G(s) = \sum_{a=0}^{A} \tau_a(s)$, one obtains that

5.2) $$\frac{dK(s)}{ds} = \frac{dk(s)}{ds} - r(s)^{-1}\frac{dr(s)}{ds}(K(s) - k(s)) + r(s)^{-1}\sum_{a=0}^{A}\Delta\tau_a,$$

or

5.3) $$\frac{dG(s)}{ds} = -r(s)^{-1}\frac{dr(s)}{ds}G(s) + r(s)^{-1}\sum_{a=0}^{A}\Delta\tau_a.$$

Now suppose that social security is pay-as-you-go. Then, equations (5.1) and (5.2) become $\frac{dK}{ds} = \frac{dk}{ds} - r^{-1}\frac{dr}{ds}G$ and $G^{-1}\frac{dG}{ds} = -r^{-1}\frac{dr}{ds}$. Recall that in this case, $\frac{dk}{ds} < 0$ and $\frac{dr}{ds} > 0$ under normal circumstances. Also, $G > 0$. These inequalities imply that $\frac{dK}{ds} < 0$ and $\frac{dG}{ds} < 0$. That is, pay-as-you-go social security decreases consumer assets and the government debt.

Finally, suppose that social security is free. Then, $\sum_{a=0}^{A}\Delta\tau_a < 0$ and under normal circumstances $\frac{dr}{ds} < 0$ and $\frac{dk}{ds} > 0$. Nothing can be said about the sign of either $\frac{dK}{ds}$ or $\frac{dG}{ds}$. This might seem strange, for one might imagine that free social security would have to be accompanied by a decrease in government debt. The drop in debt would be needed to decrease the government's interest payments and so balance its budget. However, the drop in the interest rate alone might decrease the government's interest payments enough to offset the social security payments. It is hard to imagine how this could occur if social security were on a large scale. In the United States, social security benefits far exceed the interest on the national debt.

6) Growth

I now introduce growth in population and productivity into the model of section 2. Recall that in the model of section 2, stationary equilibrium corresponds to capital saturation at a given interest rate. I now want a model in which capital saturation implies that all per capita variables grow at a fixed rate $g$. Also,the population should grow at rate $n$. In order to have such a model, it is necessary to make special assumptions about the utility function and the form of technical change.

Assume that the utility function of each consumer is of the form $u(x) = \log x$ or $u(x) = x^r$, where $0 < r < 1$. Assume also that the production function at time $t$ is of the form $f_t(k,L) = f_0(k,(1+g)^t L)$, where $f_0$ satisfies the same assumptions as $f$ did in section 2.

The number of young people in period zero is denoted by $N$, so that the number alive in period $t$ is $(1+n)^t N$.

A stationary equilibrium is still written as $((x_a),k;w,r,(\tau_a))$. However, the interpretation is now different, since the equilibrium is not really stationary. The consumption of a person of age $a$ who is born in period $t$ is $(1+g)^t x_a$. The capital stock at the end of period $t$ is $(1+g)^t(1+n)^t N k$, so that $k$ is capital stock at the end of period zero per young person. The wage in period $t$ is $(1+g)^t w$. The tax paid by a person of age $a$ born in period $t$ is $(1+g)^t \tau_a$.

The allocation $((x_a),k)$ is said to be <u>feasible</u> if $\sum_{a=0}^{A} (1+n)^{-a} x_a + k = f_0((1+g)^{-1}(1+n)^{-1} k, \sum_{a=0}^{A} (1+n)^{-a} L_a)$.

$((x_a),k;w,r,(\tau_a))$ is a <u>stationary equilibrium</u> if

i) $((x_a),k)$ is a feasible allocation,

ii) $(x_a)$ solves the problem

$$\max \sum_{a=0}^{A} (1+\rho)^{-a} u(y_a)$$

$$\text{subject to } \sum_{a=0}^{A} (1+r)^{-a} y_a = \sum_{a=0}^{A} (1+r)^{-a}(wL_a - \tau_a),$$

iii) $(k, \sum_{a=0}^{A} (1+n)^{-a} L_a)$ solves the problem $\max_{K \geq 0, L \geq 0} [f_0(K,L) - (1+r)K - wL]$.

Because of the special assumptions about $u$ and the $f_t$, we have the following. For all $t$,

$((1+g)^t x_a)$ solves the problem

$$\max \sum_{a=0}^{A} (1+\rho)^{-a} u(y_a)$$

$$\text{s.t. } \sum_{a=0}^{A} (1+r)^{-a} y_a = (1+g)^t \sum_{a=0}^{A} (1+r)^{-a}(wL_a - \tau_a), \text{ and}$$

$((1+g)^t(1+n)^t k, \sum_{a=0}^{A} (1+n)^{t-a} L_a)$ solves

$$\max_{K \geq 0, L \geq 0} [f_t(K,L) - (1+r)K - (1+g)^t wL], \text{ and}$$

$$(1+g)^t \sum_{a=0}^{A} (1+n)^{t-a} x_a + (1+g)^t(1+n)^t k$$

$$= f_t((1+g)^{t-1}(1+n)^{t-1}k, \sum_{a=0}^{A} (1+n)^{t-a} L_a).$$

That is, the economy is in equilibrium in every period.

Let $K$ be the aggregate asset position of consumers at the end of period zero per young person, so that $(1+g)^t(1+n)^t NK$ is the aggregate asset position of

all consumers at the end of period t. Let $G = K - k$.

It is not hard to see that equation (2.2) becomes

$$(6.1) \qquad (r - n - g - ng)(1+g)^{-1}(1+n)^{-1}(K-k) = \sum_{a=0}^{A} (1+n)^{-a}\tau_a \ .$$

This equation may be rewritten as $r(1+g)^{-1}(1+n)^{-1}NG =$

$= N \sum_{a=0}^{A} (1+n)^{-a}\tau_a + (n+g+ng)(1+g)^{-1}(1+n)^{-1}NG$ . The term on the left side of this new equation is the interest to be paid in period zero on government debt held at the end of period minus one. The first term on the right is the government's tax revenue in period zero. The second term on the right is the amount of new debt issued by the government in period zero. The equation simply says that the government's flow of funds balances.

It is now easy to see that the results of sections 3 - 5 remain valid, when suitably interpreted. Fully contributory social security is still defined by the equation $\sum_{a=0}^{A} (1+r)^{-a}\Delta\tau_a = 0$. Pay-as-you-go social security is defined by the equation $\sum_{a=0}^{A} (1+n)^{-a}\Delta\tau_a = 0$.

Equation (5.1) becomes

$$(6.2) \qquad \Delta K = \Delta G = (1+g)(1+n)\ (r - n - g - ng)^{-1} \sum_{a=0}^{A} (1+n)^{-a}\Delta\tau_a$$

Equations (5.2) and (5.3) become

$$(6.3) \qquad (r - n - g - ng)\frac{dK}{ds} = (r - n - g - ng)\frac{dk}{ds} - \frac{dr}{ds}(K-k)$$

$$+ (1+g)(1+n) \sum_{a=0}^{A} (1+n)^{-a}\Delta\tau_a \quad \text{and}$$

$$(6.4) \qquad (r - n - g - ng)\frac{dG}{ds} = -\frac{dr}{ds}G + (1+g)(1+n)\sum_{a=0}^{A}(1+n)^{-a}\Delta\tau_a \ .$$

As long as $r > n + g + ng$, the effects of social security on financial stocks are as before. The effects change when $r < n + g + ng$. For instance, suppose that $n < r < n + g + ng$ and that social security is pay-as-you-go. then, $\sum_{a=0}^{A}(1+r)^{-a}\Delta\tau_a > 0$, so that under normal circumstances social security increases the interest rate and reduces the capital stock. We see from equations (6.3) and (6.4) that social security increases the government's debt and has an indeterminate effect on consumers' asset holdings.

7) Effects on the Flow of Personal Saving

I first define personal saving. Let $((x_a),k;w,r,(\tau_a))$ be a stationary equilibrium and let $y$ be aggregate disposable income in period zero divided by the number of young people then alive. That is, $y = r(1+g)^{-1}(1+n)^{-1}K - \sum_{a=0}^{A}(1+n)^{-a}\tau_a + w\sum_{a=0}^{A}(1+n)^{-a}L_a$. Let $S = y - \sum_{a=0}^{A}(1+n)^{-a}x_a$. $S$ is aggregate personal saving in period zero per young person. $S$ satisfies the following equation.

7.1) $$S = (n+g+ng)\,(1+n)^{-1}(1+g)^{-1}K \ .$$

In order to verify that this equation is valid, eliminate $\sum_{a=0}^{A}(1+n)^{-a}\tau_a$ from the equation defining $y$ and from equation (6.1). This operation gives

$$y = (n+g+ng)(1+g)^{-1}(1+n)^{-1}K + (r-n-g-ng)(1+g)^{-1}(1+n)^{-1}k$$

$$+ w\sum_{a=0}^{A}L_a. \quad \text{Clearly,} \quad k + \sum_{a=0}^{A}(1+n)^{-a}x_a$$

$$= f_0\left((1+g)^{-1}(1+n)^{-1}k\,,\ \sum_{a=0}^{A}(1+n)^{-a}L_a\right) = (1+r)(1+n)^{-1}(1+g)^{-1}\,k$$

$$+ w\sum_{a=0}^{A}(1+n)^{-a}L_a, \quad \text{so that}$$

$$(r-n-g-ng)(1+n)^{-1}(1+g)^{-1}\,k + w\sum_{a=0}^{A}(1+n)^{-a}L_a = \sum_{a=0}^{A}(1+n)^{-a}x_a\ .$$

Substitution into the previous equation now gives (7.1).

Equation (7.1) may be interpreted further. Real investment is $I = (n+g+ng)(1+n)^{-1}(1+g)^{-1}k$ in period zero per young person. Therefore, $S - I = (n+g+ng)(1+n)^{-1}(1+g)^{-1}(K-k)$. The right hand side of this equation is simply the expression for the increase in government debt in period zero

per young person. The increase in government debt is the negative of government saving. Therefore, if we let $S_G = -(n+g+ng)(1+n)^{-1}(1+g)^{-1}(K-k)$ be government saving in period zero per young person, we have

$$I = S + S_G .$$

That is, investment equals personal plus government saving. This is as it should be since the business sector earns no profits and so cannot save.

Let $\Delta$ denote, as before, the change resulting from the introduction of social security. Then by equation (7.1),

$$7.2) \qquad \Delta S = (n+g+ng)(1+n)^{-1}(1+g)^{-1}\Delta K .$$

Also,

$$7.3) \qquad \Delta I = \Delta S + \Delta S_G .$$

That is, we cannot use $\Delta S$ to predict $\Delta I$, for they differ by $\Delta S_G$. $\Delta S_G$ is in fact hard to predict, except in the case of fully contributory social security. By applying $\Delta$ to equation (6.1) and rearranging, one obtains

$$7.4) \qquad \Delta S_G = \Delta r\,(1+g)^{-1}(1+n)^{-1}(K-k) - \sum_{a=0}^{A}(1+n)^{-a}\,\Delta\tau_a .$$

Thus in the case of pay-as-you-go social security, $\Delta S_G$ is positive if

government debt is positive and if social security increases the interest rate. Hence under normal circumstances an increase in government saving partially offsets the decline in private saving. In the case of fully contributory social security, the change in government saving completely offsets the change in personal saving.

The three formulas are related. In fact,

$$8.1) \qquad W3 = W2 - (A+1)^{-1} r^{-1} W1 \ .$$

This formula implies that $W3 = W2$ if social security is fully contributory and $W3 = -(A+1)^{-1} r^{-1} W1$ if social security is pay-as-you-go. Note that $W3 > 0$ if social security is either fully contributory or pay-as-you-go.

In order to verify equation (8.1), notice that $-W3 = \sum_{a=0}^{A} \sum_{t=0}^{a} (1+r)^{-t} \Delta\tau_a$. This equation is obtained by reversing the order of summation in the original formula given for $W3$. It now follows that

$$-r(1+r)^{-1} W3 = -W3 - (1+r)^{-1}(-W3)$$

$$= \sum_{a=0}^{A} \left[ \sum_{t=0}^{a} (1+r)^{-t} - \sum_{t=1}^{a+1} (1+r)^{-t} \right] \Delta\tau_a$$

$$= \sum_{a=0}^{A} [1 - (1+r)^{-a-1}] \Delta\tau_a = \sum_{a=0}^{A} \Delta\tau_a \ .$$

$$-(1+r)^{-1} \sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a \ . \quad \text{Hence,} \quad W3 = -(1+r) r^{-1} \sum_{a=0}^{A} \Delta\tau_a$$

$$+ r^{-1} \sum_{a=0}^{A} (1+r)^{-a} \Delta\tau_a = W2 - (A+1)^{-1} r^{-1} W1 \ .$$

If we now return to the model with growth of section 6, then the formulas for $W1$, $W2$, and $W3$ become

$$8.2) \qquad W1 = -N(A+1) \sum_{a=0}^{A} (1+g)^a (1+r)^{-a} \Delta\tau_a \ ,$$

$$W2 = -N(1+r)(r-n-g-ng)^{-1} \sum_{a=0}^{A} (1+n)^{-a} \Delta\tau_a \ , \quad \text{and}$$

$$W3 = -N \sum_{a=0}^{A} (1+n)^{-a} \sum_{s=a}^{A} (1+g)^{s-a} (1+r)^{-(s-a)} \Delta\tau_s \ .$$

8) Measures of Social Security Wealth

I here argue that Feldstein (1974) and others have not used appropriate measures of social security wealth when estimating the aggregate consumption function. Feldstein's formulas for social security wealth measure expected consumer wealth at a moment in time. In the life-cycle model, this wealth is not directly relevant to consumers' spending behavior. What is relevant is expected wealth at the time spending plans are made. The distinction is delicate, but I imagine that it could prove important in econometric work.

Return for the moment to the model of section 2 in which there is no growth and there is one young consumer in each period. I discuss measures of social security wealth which in a stationary equilibrium have the following values.

$$W1 = -(A+1)\sum_{a=0}^{A}(1+r)^{-a}\Delta\tau_a ,$$

$$W2 = -(1+r)r^{-1}\sum_{a=0}^{A}\Delta\tau_a , \quad \text{and}$$

$$W3 = -\sum_{a=0}^{A}\sum_{s=a}^{A}(1+r)^{-(s-a)}\Delta\tau_s .$$

W1 is the sum over all people of the present value of social security to them when they are young. W2 is the present value to the government of its social security liability. W3 is what Feldstein's definition of net social security wealth would be in a stationary equilibrium for the model of section 2. (See Feldstein (1974), pp. 912-913.) The same definition is used by Leimer and Lesnoy (1980). (See pages 26 and 27 of their paper.) I will argue that the appropriate measure of social security wealth is one that in a steady state reduces to W1 and not W3.

Of course, W2 makes sense only if $r - n - g - ng \neq 0$. Equation (8.1) now becomes

$$8.3) \qquad W3 = W2 - (1+g)(1+n)(r-n-g-ng)(A+1)^{-1} W1 \quad .$$

The verification of this equation is completely analogous to that of (8.1).

I now address the question of why W3 is not a good measure of social security wealth for the purpose of estimating a consumption function. In order to explain the error associated with W3, I return again to the model of section 2 in which there is no growth. Suppose that changes in social security are made in such a way that no consumer is ever surprised. That is, changes affect only young people, so that the taxes and benefits that a consumer plans for when he is young are the ones he actually pays and receives. Let $\Delta\tau_{at}$ be the social security tax paid (or minus the benefit received) by a consumer of age a born in period t. The statistic corresponding to W1 has value

$$8.4) \qquad W1_t = - \sum_{a=0}^{A} \sum_{s=0}^{A} (1+r)^{-s} \Delta\tau_{s,t-a}$$

at time t. Similarly, the statistic corresponding to W3 is

$$W3_t = - \sum_{a=0}^{A} \sum_{s=a}^{A} (1+r)^{-(s-a)} \Delta\tau_{s,t-a} \quad .$$

Remark Equations (8.1) and (8.3) do not apply in the non-stationary case. That is, there is no equation of the form

$W3_t = W2_t - b\,r^{-1}W1_t$, where $b > 0$.

A simple example suffices to show that $W3_t$ gives false signals as to consumers' ability to spend. Suppose that people live two periods and that the interest rate is zero. Let $\Delta\tau_{at} = 0$ for $a = 0,1$, if $t < 1$, and $\Delta\tau_{0t} = -\Delta\tau_{1t} = 1$, if $t \geq 1$. Notice that social security is introduced in period one and that it then covers only young people and thereafter covers both young and old. Social security is fully contributory, so that the interest rate and consumption are not affected. In this example, $W1_t = -\Delta\tau_{0t} - \Delta\tau_{1t} - \Delta\tau_{0,t-1} - \Delta\tau_{1,t-1} = 0$, for all $t$, whereas $W3_t = -\Delta\tau_{0t} - \Delta\tau_{1t} - \Delta\tau_{1,t-1} = 0$, if $t \leq 1$ and $W3_t = 1$, if $t > 1$. Thus, $W3_t$ gives a false signal of an increase in ability to spend during period two.

One can correct this false signal by adding consumer holdings of assets to W3. Let $K_t$ be consumers' asset holdings at the end of period $t$ and let $K$ be the level of these holdings before the introduction of social security. Then, $K_t = K - \Delta\tau_{0t} = K$, for $t < 1$ and $K_t = K - 1$, for $t \geq 1$. The correct way to combine $K_t$ and $W3_t$ is to form $K_{t-1} + W3_t$, since $K_{t-1}$ measures assets at the end of period $t-1$ and $W3_t$ measures wealth at the beginning of period $t$. Then, $K_{t-1} + W3_t = K - \Delta\tau_{0,t-1} - \Delta\tau_{1,t-1} - \Delta\tau_{0t} - \Delta\tau_{1t} = K + W1_t = K$, for all $t$, in the above example.

$K_{t-1} + W3_t$ does not give correct signals if $\sum_{a=0}^{A} (1+r)^{-a}\,\Delta\tau_{at} \neq 0$. In order to see the problem, change the contribution in the above example to zero, so that social security is free. Suppose that only the person born in period one is covered by social security. Hence, $\Delta\tau_{11} = -1$ and $\Delta\tau_{at} = 0$ otherwise, Suppose also that the consumer born in period one increases

consumption by 1/2 in each of periods one and two. Finally, suppose that the government consumes goods and that it decreases consumption by exactly the amount needed to offset the increase in private consumption. Then, the equilibrium interest rate is not affected by social security. In the new example, $W1_t = 1$, for $t = 1$ and 2, and $W1_t = 0$ otherwise, and $K_0 + W3_1 = K + 1$, $K_1 + W3_2 = K + 1/2$, and $K_{t-1} + W3_t = K$, otherwise. The decrease by 1/2 from period one to two is explained by the fact that the increase in consumption of the young consumer of period one reduces his assets by 1/2. Thus, $K_{t-1} + W3_t$ gives a false signal of a drop in ability to spend during period one. This drop may seem unimportant, but it must be remembered that in a regression analysis it is the change in variables which determines the parameter estimates.

The problem just described would occur whenever consumers' asset position was included as a variable in a consumption function. It seems that the explanatory variables of a consumption function should include only variables that describe innovations or changes in consumers' ability to spend. Consumers' asset position is simply a residual reflecting past saving.

Remark Feldstein includes as explanatory variables the current value of social security wealth, consumers' assets lagged one period, and the current value of business retained earnings. These variables are not added, but appear with separate coefficients.

It is not easy to define a good index of social security wealth if one allows for the possibility that consumers can be surprised. The consumption of older people should react more to changes in wealth than the consumption of younger people. Therefore, a good index should give

greater weight to the wealth of older people. In order to illustrate what I mean, I assume that an increase in wealth causes a person to increase his consumption by the same quantity in each of the remaining periods of his life. I suppose also that he spends a proportion $c$ of this increase in wealth on himself. ($c$ would be one if there were no bequest motive.) From these assumptions, it follows that if the consumer has an increase in wealth of $\Delta W$ when he is of age $a$, he will spend $c[\sum_{s=0}^{A-a}(1+r)^{-s}]^{-1}\Delta w$ more in that and each succeeding period of his life. I assume that the population of young people in period $t$ is $N_t$. Under these assumptions, an appropriate measure of social security wealth is

$$Wl_t = -\sum_{a=0}^{A} N_{t-a}\Big[\sum_{s=0}^{A}(1+r)^{-s}\,\Delta\tau_{s,t-a,t-a} + \sum_{n=1}^{a} b_n \sum_{s=n}^{A}(1+r)^{-(s-n)}\left(\Delta\tau_{s,t-a,t-a+n} - \Delta\tau_{s,t-a,t-a+n-1}\right)\Big],$$

where $b_n = [\sum_{a=0}^{A}(1+r)^{-a}][\sum_{a=0}^{A-n}(1+r)^{-a}]^{-1}$ and where $\Delta\tau_{atn}$ is the social security tax which a consumer born at time $t$ expects at time $n$ to pay at time $t+a$. Thus, $\Delta\tau_{att} = \Delta\tau_{at}$, where $\Delta\tau_{at}$ is as in equation (8.4). If no consumer is ever surprised, then $\Delta\tau_{atn} = \Delta\tau_{at}$, for all $n$, so that $Wl_t$ is as in (8.4).

I have found no paper which mentions the error described here in Feldstein's specification of the consumption function. However, in cross section studies I have seen of consumer asset accumulation, the social security wealth variable and other wealth variables are treated nearly as I suggest they should be. (See, for instance Kotlikoff (1979b) and Kurz (1981).)

9) Numerical Estimates

I illustrate how the previous results fit together by estimating the reduction in the stock and flow of personal saving due to social security. I do not intend that these estimates be taken seriously.

I assume that social security has been fully contributory, that contributions and benefits have grown exponentially at the rate of population growth, and that population has grown at a constant rate. These are bad assumptions, especially the first one, but they allow me to use Feldstein's measure of social security wealth as an estimate of the present value of the government's social security liability. Because I assume exponential growth, equation (8.3) applies. Because I assume that social security is fully contributory, $W1 = 0$ and $W3 = W2$.

The reduction in the flow of personal saving may be estimated using equations (6.2) and (7.2). These equations imply that the drop in personal saving equals $N(n + g + ng)(r - n - g - ng)^{-1} \sum_{a=0}^{A} (1+n)^{-a} \Delta\tau_a$, where $N$ is the population of young people. By equations (8.2) and (8.3), this expression equals $(n+g+ng)(1+r)^{-1} W3$. Therefore, the reduction in saving due to social security is approximately $(n+g)\,W3$. Feldstein's estimate of net social security wealth in 1971, as corrected by Leimer and Lesnoy (1980) is about 800 billion 1972 dollars. If $n+g = 3\%$, then the reduction in a flow of private saving due to social security was about 24 billion 1972 dollars in 1971. Feldstein's own estimate for the same year was 61 billion 1971 dollars, but he made a conceptual error which has been corrected by Eisner (1980). The corrected estimate is 10 billion 1971 dollars. After correcting an error in Feldstein's computer program, Leimer and Lesnoy (1980)

conclude that social security has had no significant effect on personal saving.

Equation (8.2) and the equation W3 = W2 imply that Feldstein's measure of social security wealth also measures the decrease in national debt due to social security. That is, the national debt would have been about twice what it actually was if there had been no social security. All these calculations have been made under assumptions which imply that social security would have had no real effects at all.

10) Feldstein's Paper

It seems fair to compare Feldstein's treatment of social security (Feldstein, 1974) with the results of this paper, for it is clear that he had in mind an overlapping generations model like that of this paper. I find his discussion very confusing. The thesis of his paper is that social security inhibits personal saving and hence capital accumulation. However, the theoretical analysis in the first section of this paper is, for the most part, an analysis of fully contributory social security, which has no effect on the capital stock. But Feldstein does not seem to have had this case in mind throughout the paper. His empirical work in sections 2 - 5 bears little relation to his earlier theoretical model, as is evidenced by his measure of social security wealth. Near the end of his paper, he asserts that social security was pay-as-you-go.

> "The lower level of GNP reflects the pay-as-you-go nature of our social security system. Because social security contributions are used to pay concurrent benefits, the capital stock is smaller and income is less." (See p. 923.)

Feldstein may well be correct in believing that pay-as-you-go social security depresses the capital stock, as I indicated earlier. However, his calculation of the effect on the capital stock is almost surely incorrect, for he assumes that any reduction in personal saving implies an equal reduction in investment. He seems to overlook completely the fact that the government also contributes to saving (see equation (7.3)). In fact, it would probably be very difficult to measure the change in government saving induced by social security. In order to do so, it would be necessary to estimate the production function (see equation (7.4)). At the end of section 7, I pointed out that if social security is pay-as-you-go, then the decline in private saving would

be partly offset by an increase in government saving.

In any case, if Feldstein believes that social security depresses the capital stock because social security is pay-as-you-go, then he should criticize the way social security is administered. He should not imply that social security itself depresses the capital stock. If he is so interested in the capital stock, perhaps he should advocate free social security. It would increase the capital stock under the same set of "normal circumstances" which would make pay-as-you-go social security depress the capital stock.

## 11) Conclusion

For the purpose of exposition, I have dealt with a very simple model. My conclusions would not change if one generalized the model by including heterogeneous consumers and many commodities. The conclusions would be strengthened if one allowed lifetimes to be random variables. Consumers would then accumulate assets in order to insure themselves against a long old age. (Assets held at death would go to heirs.) Thus, a random lifetime would tend to increase consumers' holdings of government debt and so would make the "normal circumstances" described in section 3 more consistent with the model.

Inclusion of a bequest motive could change the properties of the model greatly. By a bequest motive, I mean a utility to the legator for the bequest. Barro (1974) has suggested that this utility depends on the utility of the heir. If the bequest motive is of this form, then under certain assumptions social security has no effect at all on any real variable, as Barro pointed out (1974, 1978). Barro has also suggested that social security could be offset by diminished support of parents by their children.

It is hard for me to believe that Barro's description of the bequest motive is realistic. If his description were accurate, then the flow of personal saving would be very sensitive to interest rates. But econometricians have had trouble measuring any effect at all of interest rates on saving. (See, for example, Howrey and Hymans (1978).) I have analyzed and criticized Barro's model in another paper (1981).

One cannot deny that the bequest motive is important. Recent empirical work suggests that bequests are responsible for a large portion of private wealth. (See, for example, Kotlikoff and Summers (1980), Kurz (1981) and

White (1978).) However, this empirical work does not tell us what the nature of the bequest motive is. Perhaps a large part of what heirs receive is simply what legatees hold against a possibly longer old age.

ACKNOWLEDGEMENT

I would like to thank Robert Eisner for helpful discussions and criticism and for guidance to the literature.

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