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## Involuntary Unemployment and Implicit Contracts

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Discussion Paper # 459

INVOLUNTARY UNEMPLOYMENT AND IMPLICIT CONTRACTS

by

V. V. Chari

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## 1. INTRODUCTION

The fundamental theorems of welfare economics argue that in the presence of complete markets, a competitive equilibrium is Pareto-optimal. Markets do not exist for the direct exchange of claims on future labor services; moral hazard and the abolition of slavery make it questionable that they ever can. Consequently, the risk inherent in an uncertain stream of labor income cannot be shifted entirely onto the capital markets and the resulting allocation is, in general, not Pareto-optimal. Recent developments in the theory of implicit contracts<sup>1</sup> suggest that at least part of the risk can be transferred to the capital markets through the institution of an employing firm which is assumed to be less risk-averse than its employees. Inevitably, this diversification of risk is accompanied by a pattern of employment quite different from that which might occur in an otherwise identical auction market. In the extreme case when the mediating firm is risk-neutral and the utility functions of employees are separable in consumption and leisure, the firm absorbs all the risk, guaranteeing a constant wage to workers and reallocating employment from less to more favorable states of nature. If workers are endowed with indivisible units of leisure, then the contract may involve the practice of "laying-off" workers in some states. It is also possible in some (though not all) versions of these models that the resulting underemployment equilibria may contain the feature that the laid-off workers would prefer to work at the wage rate paid the rest of the labor force rather than stay out of work. In this sense it has been argued that implicit contract theory explains "involuntary" employment.

Arguments have appeared in a quite unrelated strand of the literature that involuntary unemployment is best understood within the confines of disequilibrium theory.<sup>2</sup> Rejecting the proposition that wages and prices clear

kets,<sup>3</sup> the protagonists argue that "what looks like involuntary employment is involuntary unemployment" (Solow (1980)). Work in this area has in large part been motivated by the observation that prices and wages move "slowly" to clear markets. As a first approximation, prices are exogenously fixed (Malinvaud (1977) or Barro and Grossman (1971)). This is, of course, a serious shortcoming (though there are disequilibrium models (Hahn (1978)) that attempt to incorporate price-setting behavior). This paper attempts to forge a link between disequilibrium theory and the theory of implicit contracts.

I will define involuntary unemployment to arise when the marginal rate of substitution between consumption and labor is less than the marginal rate of transformation between production and the labor input and it is feasible to increase labor supplied. In such a situation there exist obvious gains to trade. The worker gains by supplying additional labor services and the firm profits by accepting them. The difference between this formulation and that traditionally used by implicit contract theorists lies in the fact that in those formulations when the worker desires to supply his labor services at the wage rate his colleagues are receiving, the firm has no incentive to hire him.

A basic partial equilibrium model is presented in section 2. A central assumption is that firms are better informed than workers about the true "state of nature." Since there are no markets in human capital in the model the firm and the workers must negotiate a contract prior to the revelation of the state of nature which attempts to reconcile the potentially conflicting objectives of risk-sharing and information revelation.

The optimal contract is characterized in section 3 and is found to yield a compensation schedule which is convex in the level of employment, i.e., the marginal wage rate increases with the number of hours worked. This is a consequence of the incentive compatibility constraints and is independent of

the worker's preferences. Unlike the pure risk sharing contract, compensation is not independent of the state of nature. Necessary conditions for the existence of involuntary unemployment are studied in this section.

Section 4 compares the allocations obtained above with a contract where there are no asymmetries in information and argues that involuntary unemployment is impossible in such an environment. Section 5 concludes the paper.

## 2. THE MODEL

There are two types of agents in the economy: workers and firms. The firms are in an industry which produces a single homogeneous good at a constant price which is normalized to equal 1. The technology for producing the good is affected by a productivity shock.<sup>4</sup> Firms draw their productivity shocks independently from a known fixed distribution. For simplicity, assume that productivity shocks  $\theta$  are drawn from a set of discrete states  $\theta$ ;

$\Theta = \{\theta \mid \theta = \theta_1, \theta_2, \dots, \theta_N\}$  according to a probability distribution

$Q = \{q \mid q = q_1, q_2, \dots, q_N; \sum_{i=1}^N q_i = 1\}$ . The probability that  $\theta = \theta_i$  is  $q_i$ . Then, without loss of generality, order productivity shocks so that  $\theta_1 < \theta_2 < \dots < \theta_N$ .

Workers know the probability distribution. One interpretation of this scenario is that there are a large number of firms so that  $q_i$  is the fraction of firms that have labor productivity  $\theta_i$ .

The only input into the production process is labor services offered by workers. The technology is constant returns to scale in the labor input and is additive across the number of workers employed by a firm.

$$y_i = \theta_i \sum_{j=1}^J n_j \tag{2.1}$$

where  $y_i$  is the output of firm  $i$

$n_j$  is the labor supplied by worker  $j$

$J$  is the number of workers hired by the firm

$\theta_i$  is the productivity of firm  $i$

The preferences of each worker is represented by an identical utility function

$U(c, n)$ .

where  $c$  is consumption

$n$  is labor supplied

$U(c,n): \mathbb{R}^+ \times [0, \bar{n}] \rightarrow \mathbb{R}$  is bounded, twice differentiable in both arguments and strictly concave.

$$U_c > 0, U_n < 0, U_{cc} < 0, U_{nn} < 0, U_{cc}U_{nn} - (U_{cn})^2 > 0.$$

$\bar{n}$  is the maximum amount of labor that can be supplied.

Prior to the firm's observing its productivity shock, firms and workers enter into a "contract" specifying a compensation-employment schedule. Once workers and firms have signed their private contracts and the state of nature has been realized, workers cannot move to another firm. This assumption (which is uncomfortably close to involuntary servitude) can to some extent be justified by costs of moving from one location to another. Empirical evidence (Feldstein (1975)) suggests that about 70 percent of laid-off workers return to their old jobs. The lack of markets to diversify the risk in labor incomes ensures that it will, in general, be optimal for workers and firms to agree to a contract rather than resort to auction markets to organize production.

The information structure of the model is that each firm observes its productivity shock while other agents in the economy do not. The objective function of firms is assumed to be maximization of expected profits while workers seek to maximize expected utility. At the time of signing the contract each worker has an alternative source of income that yields a utility of  $z$ . Thus, no worker will accept a contract unless he is guaranteed at least  $z$  units of expected utility.



The additive separability of technology ensures that each firm will seek to maximize expected profits from each contract offered to each worker. We may safely therefore consider only contracts between a single firm and a single worker. All other contracts will be identical to this one. The contract specifies the total compensation  $c_i$  and the labor to be supplied  $n_i$  in each state of nature  $\theta_i$ ,  $i = 1, 2, \dots, N$ .

The contract is thus a pair of functions

$$\{c(\theta), n(\theta)\} : \Theta \times \Theta \rightarrow \mathbb{R}^+ \times [0, \bar{n}] \quad (2.2)$$

It will be noted that we have used the true value of  $\theta$  as an argument of the functions. It is straightforward to show that any contract between the worker and the firm can be represented as shown above (Myerson (1979), Harris and Townsend (1977)). Suppose the firm were to lie about the true value of  $\theta$ . At the time of negotiating the contract the worker knows that the firm will lie if it has an incentive to do so. The contract designed between the firm and worker will therefore be such that the firm will not have an incentive to lie. The optimal contract must therefore satisfy incentive compatibility or self-selection constraints<sup>5</sup> which are of the form

$$\theta_i n_i - c_i \geq \theta_i n_j - c_j \quad \text{all } i, j = 1, 2, \dots, N. \quad (2.3)$$

The firm's profits if it reports the true value of  $\theta$  are at least as large as if it lies.

The optimal contract is chosen by solving the following program:<sup>6</sup>

$$\begin{aligned} \bar{n} \geq n_i \geq 0 \quad & \text{Max} \quad \sum_{i=1}^N q_i [\theta_i n_i - c_i] \\ c_i \geq 0 \end{aligned} \quad (2.4)$$

subject to 
$$\sum_{i=1}^N q_i [U(c_i, n_i)] \geq z \quad (2.5)$$

and 
$$\theta_i n_i - c_i \geq \theta_j n_j - c_j \quad \text{all } i, j = 1, 2, \dots, N \quad (2.6)$$

The only novelty in this problem that distinguishes it from the standard risk-sharing problem is the incentive compatibility constraints 2.6.

### 3. CHARACTERIZATION OF THE OPTIMAL CONTRACT

The preliminary steps in characterizing the optimal contract are to note certain results that follow immediately from the incentive compatibility constraints.

PROPOSITION 1:  $c$  and  $n$  are nondecreasing in  $\theta$ . Furthermore, the compensation-employment schedule is convex (see Figure 1) and profits are nondecreasing in  $\theta$ .

PROOF: From Equation (2.6), we have

$$\theta_i n_i - c_i \geq \theta_i n_j - c_j$$

and  $\theta_j n_j - c_j \geq \theta_j n_i - c_i$

or  $\theta_i (n_i - n_j) \geq c_i - c_j \geq \theta_j (n_i - n_j)$

It follows that

$$n_i \geq n_j \text{ and } c_i \geq c_j \quad \text{iff } \theta_i \geq \theta_j.$$

Thus,  $n_1 \leq n_2 \leq \dots \leq n_N$  and  $c_1 \leq c_2 \leq \dots \leq c_N$ . Profits are nondecreasing since

$$\theta_i n_i - c_i \geq \theta_i n_j - c_j \geq \theta_j n_j - c_j \quad \text{if } p_i \geq p_j \quad (3.1)$$

Furthermore,

$$\theta_1 \leq \frac{c_2 - c_1}{n_2 - n_1} \leq \theta_2 \leq \frac{c_3 - c_2}{n_3 - n_2} \dots \leq \theta_N.$$

Convexity in this discrete state case is simply the statement that the line passing through  $(c_{i+2}, n_{i+2})$  and  $(c_i, n_i)$  does not lie below the line passing through  $(c_i, n_i)$  and  $(c_{i+1}, n_{i+1})$  or the line joining  $(c_{i+1}, n_{i+1})$  and  $(c_{i+2}, n_{i+2})$ .

Q.E.D.

It is to be noted that these conclusions are independent of any special assumptions about the preferences of workers and arise purely from the informational asymmetry in the model. In particular, convexity of the contract implies that the marginal wage rate required to induce additional units of employment is increasing as employment increases. This provides one justification for institutions such as overtime pay, incentive payments, bonuses, etc., quite independently of what workers' preferences are. This unusual feature does not occur in the pure risk sharing contracts.<sup>7</sup>

Further investigation of the incentive compatibility constraints yields useful results.<sup>8</sup> First, we need consider only sequential pairwise constraints, i.e.

$$\theta_i n_i - c_i \geq \theta_i n_{i+1} - c_{i+1} \quad i = 1, \dots, N-2$$

and

$$\theta_{i+1} n_{i+1} - c_{i+1} \geq \theta_{i+1} n_{i+2} - c_{i+2}$$

together imply

$$\theta_i n_i - c_i \geq \theta_i n_{i+2} - c_{i+2}$$

The proof is immediate. We have (recalling that  $\theta_1 < \theta_2 < \dots < \theta_N$ )

$$\begin{aligned} c_{i+2} - c_{i+1} &\geq \theta_{i+1}(n_{i+2} - n_{i+1}) \\ &\geq \theta_i(n_{i+2} - n_{i+1}) \end{aligned}$$

Hence,

$$\theta_i n_{i+1} - c_{i+1} \geq \theta_i n_{i+2} - c_{i+2}$$

But

$$\theta_i n_i - c_i \geq \theta_i n_{i+1} - c_{i+1}$$

Hence

$$\theta_i n_i - c_i \geq \theta_i n_{i+2} - c_{i+2} .$$

Similar results hold when comparing allocations for higher values of  $\theta$  with those for lower values of  $\theta$ .

We may therefore rewrite the programming problem as

$$0 \leq n_i \leq \bar{n} \quad \text{Max} \quad \sum_{i=1}^N q_i [p_i n_i - c_i] \tag{3.2}$$

subject to

$$\sum_{i=1}^N q_i [U(c_i, n_i)] \geq z$$

and

$$\theta_i n_i - c_i \geq \theta_{i-1} n_{i-1} - c_{i-1} \quad i = 1, 2, \dots, N-1$$

$$\theta_i n_i - c_i \geq \theta_{i-1} n_{i-1} - c_{i-1} \quad i = 2, \dots, N$$

PROPOSITION 2: The programming problem (3.2) has a unique solution.

PROOF: The constraint set is clearly convex. We may restrict consideration of the optimum to a compact set. Define  $\bar{c}$  by  $U(\bar{c}, 0) = z$ .

Thus,

$$\sum_{i=1}^N q_i [p_i n_i - c_i] \geq -\bar{c}$$

Since

$$0 \leq n_i \leq \bar{n}$$

and

$$0 \leq c_i$$

it is clear that  $c$  is bounded from above. The maximand is a continuous function and thus attains its maximum. Uniqueness follows from the fact that the worker's constraint must be binding (if not, reduce all  $c_i$ , thus increasing profits without violating incentive compatibility) and the fact that the worker's utility function is strictly concave.

Q.E.D.

What makes the model interesting is, of course, the possibility of involuntary unemployment. Within the context of this model, involuntary unemployment (or more precisely, involuntary underemployment) is defined in DEFINITION 1: The worker is said to be involuntarily underemployed in state  $i$  if

$$-\frac{U_n}{U_c} < \theta_i \tag{3.3}$$

and

$$n_i < \bar{n}$$

The requirement is simply that the marginal rate of substitution be less than the marginal rate of transformation. The worker is involuntarily unemployed if (3.3) holds and  $n_i = 0$ .

The gains to trade in such a situation are evident. Both the firm and the worker would like to see a larger value of  $n$  for increased  $c$ .

To see the possibilities of involuntary unemployment, recast the programming problem in Lagrangian form

$$L = \underset{0 \leq n_i \leq \bar{n}}{\text{Max}} \sum_{i=1}^N q_i [\theta_i n_i - c_i + \lambda U(c_i, n_i) - \lambda z]$$

$$0 \leq c_i$$

$$+ \sum_{i=1}^{N-1} \mu_i [\theta_i n_i - c_i - \theta_i n_{i+1} + c_{i+1}]$$

$$+ \sum_{i=2}^N \gamma_i [\theta_i n_i - c_i - \theta_i n_{i-1} + c_{i-1}]$$

This yields

$$- q_i [1 - \lambda U_c(c_i, n_i)] + [\mu_{i-1} - \mu_i] + [\gamma_{i+1} - \gamma_i] \leq 0. \quad (3.4)$$

with strict equality if  $c_i > 0$

$$q_i [\theta_i + \lambda U_n(c_i, n_i)] + [\mu_i \theta_i - \mu_{i-1} \theta_{i-1}] + [\gamma_i \theta_i - \gamma_{i-1} \theta_{i+1}] \quad (3.5)$$

$$\leq 0 \text{ if } n_i < \bar{n}$$

$$\geq 0 \text{ if } n_i = \bar{n}$$

$$i = 1, 2, \dots, N$$

with the convention that  $\mu_0 = 0$  and  $\gamma_{N+1} = 0$ .

To consider the possibility that the worker may be involuntarily underemployed, suppose that for some  $i$ ,  $c_i > 0$  and  $0 < n_i < \bar{n}$ . Then, by multiplying equation (3.4) by  $\theta_i$  and adding equation (3.5), we have

$$q_i \lambda [\theta_i U_c(c_i, n_i) + U_n(c_i, n_i)] + \mu_{i-1} (\theta_i - \theta_{i-1}) + \gamma_{i+1} (\theta_i - \theta_{i+1}) = 0 \quad (3.6)$$



Thus, involuntary underemployment is possible if

$$\theta_{i+1}n_{i+1} - c_{i+1} = \theta_{i+1}n_i - c_i$$

and

$$\theta_{i-1}n_{i-1} - c_{i-1} > \theta_{i-1}n_i - c_i$$

for then  $\gamma_{i+1} \geq 0$  and  $\mu_{i-1} = 0$ .

It is instructive to note that along with the possibility of involuntary underemployment goes the possibility of involuntary overemployment which is defined symmetrically.

Certain facts about the nature of the optimal contract and the occurrence of involuntary unemployment are also obvious from the first order conditions. There can be no involuntary unemployment in the highest possible state, i.e., when  $\theta = \theta_N$ . If  $0 < n_N < \bar{n}$ , then equation (3.6) can be rearranged to read

$$-\frac{U_n(c_N, n_N)}{U_c(c_N, n_N)} = \frac{\mu_{N-1}(\theta_N - \theta_{N-1})}{q_N \lambda U_c(c_N, n_N)} + \theta_N > \theta_N$$

The marginal rate of substitution between leisure and consumption is at least as large as the marginal rate of transformation (recall that the productivity shocks are ranked in increasing order).

The economic rationalization for the existence of ex-ante optimal contracts that involve involuntary unemployment ex-post is straightforward. Against the gains to trade from increased employment in some states must be balanced incentives for the firm to tell the truth in the next higher state of

productivity. In particular, if the next higher state is vastly more productive it may be necessary to "take the lumps" in the form of involuntary unemployment to induce the firm to tell the truth when times are good.

We now establish necessary and sufficient conditions for the existence of involuntary unemployment in this model. It turns out that inferiority of leisure is crucial for the existence of involuntary unemployment. The implications of this proposition are explored in section 4.

Before proving the theorem, we establish a useful lemma:

Define  $X_j(\theta) \equiv \{c, n \mid \theta n - c = \theta n_j - c_j, n > n_j, c > c_j$

$$\left. - \frac{U_n(c, n)}{U_c(c, n)} < \theta \right\}$$

$X_j(\theta)$  is the set of allocations along a line with slope  $\theta$  as we move north west of  $(c_j, n_j)$ . Note that  $X_j(\theta)$  is nonempty if and only if  $-\frac{U_n(c_j, n_j)}{U_c(c_j, n_j)} < \theta$ . This follows from the strict concavity of the utility function.

Lemma: If consumption and leisure are normal goods then

$U_c(c, n) < U_c(c_j, n_j)$  for all  $(c, n) \in X_j(\theta)$  (if  $X_j(\theta)$  is nonempty) for any choice of  $(c_j, n_j)$ .

Proof: Consider the change in the marginal utility of the consumption as we move along the line with slope  $\theta$  from  $(c_j, n_j)$

$$\begin{aligned} \frac{dU_c(c, n)}{dn} \Big|_{\theta n - c = \theta n_j - c_j} \\ = U_{cc} \theta + U_{cn} \end{aligned}$$

By assumption  $-\frac{U_n(c, n)}{U_c(c, n)} < \theta$

Hence, the expression above reduces to

$$\frac{dU_c(c,n)}{dn} < U_{cc} \left(-\frac{U_n}{U_c}\right) + U_{cn} < 0$$

The last inequality follows from the assumption of normality of consumption and leisure.

Hence, the marginal utility of consumption is decreasing along  $X_j(\theta)$ .

Q.E.D

We are thus led to

Theorem: If consumption and leisure are normal goods then there cannot be involuntary unemployment.

Proof: Suppose that for some state  $i$  the worker was involuntarily unemployed. Then a study of equation (3.6) reveals that necessarily

$$\gamma_{i+1} > 0 \text{ or}$$

$$\theta_{i+1} n_{i+1} - c_{i+1} = \theta_{i+1} n_i - c_i$$

Consider the set  $K(j,k)$  defined so that there is involuntary unemployment in the states between  $j$  and  $k$  and there is no involuntary unemployment<sup>9</sup> at  $j-1$  or  $k+1$

$$K(j,k) \equiv \{i | j < i < k, \theta_{i+1} n_{i+1} - c_{i+1} = \theta_{i+1} n_i - c_i,$$

$$\theta_j n_j - c_j > \theta_j n_{j-1} - c_{j-1}$$

$$\theta_j n_j - c_j > \theta_j n_{j+1} - c_{j+1}$$

$$\theta_{k+2} n_{k+2} - c_{k+2} > \theta_{k+2} n_{k+1} - c_{k+1}$$

$$\theta_k n_k - c_k > \theta_k n_{k+1} - c_{k+1} \}$$

Applying equation (3.4) to  $j$  and  $k+1$  we see that (assuming an interior solution)

$$\begin{aligned} 1 - \lambda U_c(c_j, n_j) &= \frac{[\mu_{j-1} - \mu_j]}{q_j} - \frac{[\gamma_{j+1} - \gamma_j]}{q_j} \\ &= 1 - \lambda U_c(c_{k+1}, n_{k+1}) - \frac{[\mu_k - \mu_{k-1}]}{q_{k+1}} - \frac{[\gamma_{k+2} - \gamma_{k+1}]}{q_{k+1}} \end{aligned} \quad (3.7)$$

However, from the definition of  $K(j,k)$  it follows that

$$\mu_j = 0$$

$$\gamma_j = 0$$

$$\mu_k = 0$$

$$\gamma_{k+2} = 0$$

Consequently, it follows from equation (3.7) that

$$U_c(c_j, n_j) < U_c(c_{k+1}, n_{k+1})$$

However, note that  $(c_j, n_j)$  and  $(c_{k+1}, n_{k+1})$  lie on a line with a slope at

least as steep as  $\theta_j$  and  $-\frac{U_n(c_j, n_j)}{U_c(c_j, n_j)} < \theta_j$ . Consequently, from Lemma 1,

$$U_c(c_j, n_j) > U_c(c_{k+1}, n_{k+1})$$

which yields the desired contradiction.

Q.E.D.

This result establishes that leisure must be an inferior good for involuntary unemployment to occur.

It may seem at first sight that the perfect enforceability of ex-ante (but not ex-post) contracts is an unduly stringent restriction. Indeed there seems no cause for either party to call in an enforcing authority (such as the government) when there are evident gains to trade<sup>10</sup> that they can exercise themselves. One rationalization is to suppose that the firm negotiates with different<sup>11</sup> workers in subsequent periods and that deviations from ex-ante optimality will become known to workers in the future. Then, the firm would be unable to hire workers in the future and would have an incentive to stick to the ex-ante optimal contract.<sup>12</sup>

4. A COMPARISON WITH THE PURE RISK SHARING CONTRACT

The model discussed in the preceding sections is not quite so specialized as may seem at first glance. In fact, a model such as that of Azariadis (1975) is mathematically very similar to the model of section 2.

Suppose that a firm, such as the one in section 2, has a production function  $y = f(x)$  where  $x$  is the number of employees who work for the firm. There are  $J$  employees in a labor pool who are available to work for the firm. All workers are identical and have preferences described by

$$\begin{aligned} V(w, \ell) &= U(w) + K \quad \text{if } \ell = 1 \\ &= U(w) \quad \ell = 0 \end{aligned}$$

$\ell$  is leisure. All workers are endowed with one indivisible unit of leisure.  $w$  is wages. Let  $x_i$  be the probability that a given worker will work in state  $i$ . The programming problem that the optimal contract must solve is

$$\begin{aligned} \text{Max} \\ 1 \geq x_i \geq 0 \quad \sum_{i=1}^n q_i [\theta_i f(x_i, J) - x_i w_i^1 - (1 - x_i) w_i^2] \\ w_i \geq 0 \end{aligned}$$

subject to

$$\sum_{i=1}^N q_i [x_i U(w_i^1) + (J - x_i) U(w_i^2) + (1 - x_i) K] \geq z$$

and

$$\begin{aligned} \theta_i f(x_i, J) - x_i w_i^1 - (1 - x_i) w_i^2 \geq \theta_j f(x_j, J) - x_j w_j^1 - (1 - x_j) w_j^2 \\ i, j = 1, 2, \dots \end{aligned}$$

where  $w_i^1$  is the wage paid to a worker

$w_i^2$  is the wage paid to those laid off.

The convexity of the constraint set and strict concavity of the utility function ensure that  $w_i^1 = w_i^2 = w_i$  (to see this, let  $\hat{w}_i = x_i w_i^1 + (1 - x_i) w_i^2$ ). The firm is indifferent between the options. The worker strictly prefers the allocation marked with a caret.

Let  $n_i = f(x_i J)$

and  $g(n_i) = x_i K = \frac{K}{J} f^{-1}(n_i)$

$c_i = w_i$

Then, it is immediate that the problems are formally similar.

Of course, in this formulation of the risk-sharing model, in the absence of informational asymmetries and with severance pay, there can be no involuntary unemployment in the sense implicitly used by contract theorists. No worker wishes to work at the wage rate offered his co-workers. In fact, every worker desires to be laid off. With a different formulation of the utility function of workers (see footnote 13), involuntary unemployment in this sense is possible.

It is trivial to verify that involuntary unemployment, as defined in section 3, is impossible in the absence of informational asymmetries. It might be conjectured that with informational asymmetries involuntary unemployment as defined here involving layoffs instead of reductions in work-hours might be possible. The theorem, however, assures us that the separability induced by the 0-1 leisure decision which implies that leisure is a normal good also precludes the possibility of involuntary unemployment. Thus, for the particular structure studied here, though there are layoffs in

equilibrium there is never unemployment. Indeed, whenever there are gains to trade, there is overemployment: the marginal rate of substitution exceeds the marginal rate of transformation. In the original contract-theoretic formulation of the labor market, the absence of severance pay played a central role in the rationalization of involuntary unemployment. This lack of payment was never explained. The implicit idea was that unemployment benefits and other job opportunities made severance pay a costly luxury for the firm and workers. Risk sharing, however, implies that (with separable utility functions) the pie available for consumption will be evenly distributed, but workers in productive industries will have to work more than those in less productive industries. To put it differently, risk sharing implies a separation between contribution to the social product (through labor supply) and consumption or receipts from the social product. The marginal utility of consumption is kept constant while the marginal disutility from labor supplied varies widely. Informational asymmetries allow the explanation of involuntary underemployment without arbitrary restrictions on severance pay. As The theorem cautions, however, informational asymmetries are not sufficient to explain the existence of involuntary unemployment.

Thus far, we have considered two extremes. One, where all variation occurs in hours of work and involuntary underemployment is possible, and another, where all variations occur as layoffs and unemployment is never involuntary. By relaxing the strong assumption of additive separability in the number of workers employed which was made in section 2 and by introducing diminishing returns to the number of workers employed, it is possible to allow for layoffs as well as changes in the workweek. It may then be possible to observe involuntary unemployment manifest itself as layoffs. In any of these formulations it seems essential that the marginal utility of consumption be an



increasing function of labor supplied for involuntary unemployment to be possible.

## 5. CONCLUSIONS

Hall and Lilien (1979) and Calvo and Phelps (1977) in analyses similar to the one carried out here arrive at quite different conclusions. They contend that equilibrium contracts must involve ex-post profit maximization. It is evident that an incentive-compatible contract that also involves ex-post profit maximization can be dominated on average by one that does not. Unfortunately, this involves possible dissatisfaction on both sides. Employees are laid off who are quite sincere when they respond to the government's unemployment survey that they wish to work. The apparent observation is that markets are not cleared. Yet the paradox disappears when viewed as an ex-ante optimal contract. A casual observer of the economy described in this paper would observe the firm unilaterally deciding upon employment and compensation. He would observe workers sincerely declaring their desire to work. He would observe firms which seemingly have plenty of job-seekers at offered wage rates but apparently rationing by quantity.

Several issues remain unexplored in this analysis. Among them are contract length (see Dye (1979)), the lack of full indexing (see Chari (1980) and Gray (1978)) of contracts and questions such as the nature of the optimal contract when both layoffs and changes in the work week are possible methods of altering labor supply. The last is particularly interesting since what we do seem to observe is that a worker's compensation does not seem to depend upon the labor supplied by others in the firm but is strongly dependent upon the number of hours he works. I conjecture that layoffs are a signal to workers that demand in future periods is anticipated to be low but wages are not cut since that would violate incentive-compatibility. For example, if  $\theta_i$  contained a permanent and a transitory component, the incentive compatible contract that would allow workers to distinguish between the components may

well involve layoffs when there is a drop in the permanent component and work week changes when the transitory component is affected.

FOOTNOTES

<sup>1</sup>The original work is due to Azariadis (1975), Baily (1974) and Gordon (1974). Dreze (1979) puts the implicit contract theory within a broader framework dealing with problems of bearing the risk associated with human capital. Holmström (1979) considers generalizations of Azariadis's model.

<sup>2</sup>Drazen (1980) provides an excellent survey of recent work.

<sup>3</sup>It is not clear whether the proposition that markets clear is testable. Lucas and Sargent (1979) argue forcefully that it is not.

<sup>4</sup>The same analysis would go through if the element of private information was the price of the good produced by the firm. It must be noted, however, that then the price of any firm's product is highly correlated across the industry and we need to impose the additional assumption that workers of different firms cannot communicate with one another.

<sup>5</sup>See Harris and Townsend (1977) or Myerson (1979).

<sup>6</sup>It is obvious from the programming problem that the assumption of constant returns to scale is not a serious restriction. Suppose the production function is concave:  $y = f(n)$ . Then replace the labor term in the utility function by  $f^{-1}(y)$  which is convex and the same analysis holds.

<sup>7</sup>Constant returns to scale play an important, though not crucial, role in this result.

<sup>8</sup>Many of these results are similar to those obtained by Spatt (1980) and indicate that for a broad variety of models with asymmetric information we should expect such results.

<sup>9</sup>It is possible that there is incomplete separation in the sense that the same allocation  $(c_j, n_j)$  may prevail for a number of states. In this case, choose the lowest state in this set and the highest in the set corresponding to  $(c_k, n_k)$ .

<sup>10</sup>This, perhaps, is one reason why overtime pay is often legislatively fixed rather than a subject of bargaining.

<sup>11</sup>This qualification is necessary to avoid the messy problem of multi-period contracts.

<sup>12</sup>These statements are conjectural. Selten's chain-store paradox shows that if the firm is in operation only for a finite number of periods, it will not build a reputation for honesty. The reasoning is simple. In the last period, there will be deviation from ex-ante optimality. Then, in the second last period, the firm does not lose by an ex-post trade. By induction, the firm will always make ex-post trades. See Milgrom and Roberts (1980) for a way out of the paradox.

<sup>13</sup>In Azariadis's formulation, preferences were

$$\begin{aligned} V(c, \ell) &= K && \text{if } \ell = 1 \\ &= U(c) && \text{if } \ell = 0 \end{aligned}$$

The firm could not pay laid-off workers. This seems to me to be an entirely arbitrary restriction.

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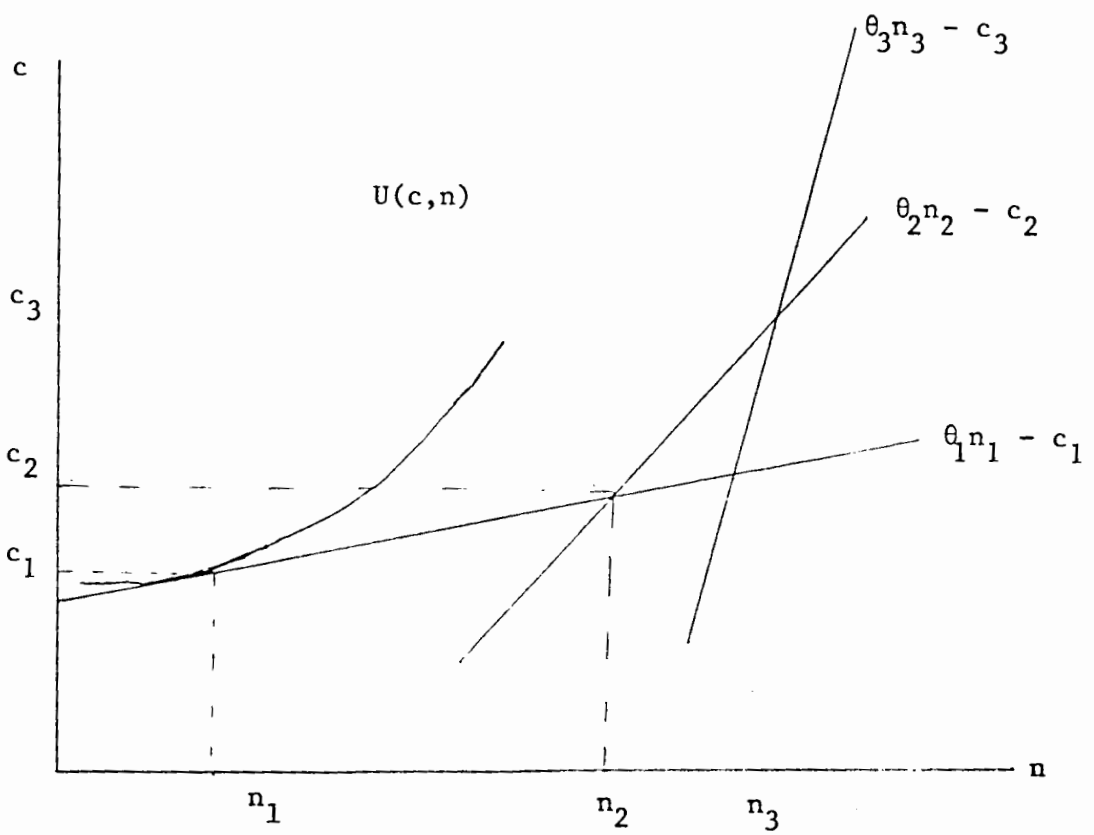


Figure 1