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**Working Paper**

## Optimal Coordination Mechanisms in Principal-Agent Problems

Discussion Paper, No. 429

**Provided in Cooperation with:**

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*Suggested Citation:* Myerson, Roger B. (1980) : Optimal Coordination Mechanisms in Principal-Agent Problems, Discussion Paper, No. 429, Northwestern University, Kellogg School of Management, Center for Mathematical Studies in Economics and Management Science, Evanston, IL

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Discussion Paper No. 429  
OPTIMAL COORDINATION MECHANISMS IN GENERALIZED  
PRINCIPAL-AGENT PROBLEMS

by

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June, 1980  
revised, August, 1981

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Abstract The general principal-agent problem is formulated, in which agents have both private information and private decisions, unobservable to the principal. It is shown that the principal can restrict himself to incentive-compatible direct coordination mechanisms, in which agents report their information to the principal, who then recommends to them decisions forming a correlated equilibrium. In the finite case, optimal coordination mechanisms can be found by linear programming. Some basic issues relating to systems with many principals are also discussed. Noncooperative equilibria between interacting principals do not necessarily exist, so quasi-equilibria are defined and shown to exist.

\* The author is indebted to Rick Antle, Paul Milgrom, and especially to Bengt Holmström for valuable discussion and suggestions.

1. Introduction

This is a paper about game design. The central question to be considered is how an individual should structure a social situation which he controls, so as to maximize his expected utility. Following the terminology of Ross [1973], we refer to the individual in control as the principal, and we refer to the other individuals as the agents. That is, we think of an individual as a principal if he is in a position to set the rules of communication and he can establish the structure of incentives to which the other individuals must react. In short, the principal is the individual who has the power to design the game which the other individuals (or agents) must play.

The use of the terms "principal/agent" here may require some explanation. Possible alternative terms could have been "leader/follower", or "exploiter/exploited", or "sovereign/subject". I have chosen to use the principal/agent terminology because it suggests a class of important applications in which economists consider it acceptable for one individual (such as the owner of a corporation) to design a social coordination system (the corporation) so as to optimally exploit some other individuals (his managers and employees). However, we will here characterize the individuals' decisions and information at a level of abstraction in which we lose much of the structure used in most of the literature on principal-agent problems. See Mirrlees [1976], Harris and Raviv [1979], and Holmström [1979, 1980] for some of the important results and applications of principal-agent analysis; Holmström [1980] offers an excellent survey. The problems to be considered in this paper should be properly called generalized principal-agent problems, to distinguish them from the more specific principal-agent structures studied in this literature.

Our generalized principal-agent problem may be viewed as a hybrid between a cooperative and a noncooperative game. The agents are assumed to act noncooperatively as utility maximizers who will passively accept any Nash equilibrium of any game which the principal chooses. The possibilities for communication and cooperation are assumed to be entirely controlled by the principal. Thus, when we identify a particular individual as the "principal", we mean that we want to know what would be the best possible cooperative arrangement for the purposes of this individual, if he had all of the bargaining ability.

Our generalized principal-agent problem can also be interpreted as a social choice problem, where the principal is a social planner and his utility function is a social welfare function. Then the principal's mechanism-design problem can be reinterpreted as the problem of finding a coordination mechanism for the agents which maximizes the expected social welfare.

In designing a game or coordination mechanism, a principal faces two constraining factors: the agents may have private information which he cannot directly observe; and the agents may have private decision domains which he cannot directly control. In the insurance literature, these two factors are known as adverse selection and moral hazard. In order to get information from the agents and influence their decisions, the principal must design a coordination system which gives the agents the incentive to do as he intends. That is, his mechanism must be incentive compatible.

In Section 2, we develop the formal structure of our generalized principal-agent problem. In this formulation, we follow the Bayesian approach to the study of games with incomplete information, as suggested by Harsanyi [1967-8]. We use the Bayesian equilibrium as the solution concept for such games. To compute the equilibria of any game can often be extremely

difficult, so it may be remarkable that the class of all outcomes which the principal can achieve as Bayesian equilibria of games is actually quite easy to characterize. We show this result in Section 3. The key insight is that the principal may, without loss of generality, restrict himself to incentive-compatible direct coordination mechanisms.

This basic insight has been independently discovered by many authors for the case where agents have private information; see Gibbard [1973], Rosenthal [1978], Holmström [1977], Dasgupta, Hammond, and Maskin [1979], Myerson [1979], and Harris and Townsend [1981]. In Myerson [1981], this insight was referred to as the revelation principle, since it asserts that there is no loss of generality in assuming that the principal should structure his incentive system so that all agents will be willing to reveal all of their information to him honestly. The key contribution of Proposition 2 in Section 3 is to extend the revelation principle to situations where there are moral hazard factors, in addition to informational or adverse selection factors, in the principal's problem. Proposition 2 can be best viewed as a synthesis of the model of Holmström [1977] (where moral hazard effects were also considered) with Aumann's [1974] concept of correlated equilibrium. General principal-agent problems with both moral hazard and adverse selection effects have been studied, in the context of auditor contracts, by Antle [1980].

In Section 4, we consider situations in which several principals interact in designing coordination mechanisms for their agents. The obvious notion of equilibrium among principals unfortunately does not have the existence property, so we define a modified concept of quasi-equilibrium among principals, which can be shown to always exist.

## 2. General formulation of the principal-agent problem

In the generalized principal-agent problem, there is one principal and there are  $n$  agents, numbered 1 to  $n$ . There are two reasons why the principal may need to rely on these agents: they may have information which he cannot observe, or they may have the power to make some decisions which he cannot directly control. For each agent  $i$ , we let  $T_i$  denote the set of all possible states of agent  $i$ 's private information, and we let  $D_i$  denote the set of all possible private decisions or actions which agent  $i$  can make. Following Harsanyi [1967-8], we refer to  $T_i$  as the set of all possible types for agent  $i$ , where each type  $t_i$  in  $T_i$  represents a complete description of all the private information  $i$  might have about his environment, his abilities, and his preferences. Each private decision-option  $d_i$  in  $D_i$  may represent, for example, a level of effort which agent  $i$  might exert in working for the principal, and which the principal can not observe or control.

We let  $D_0$  denote the principal's decision domain. Each option  $d_0$  in  $D_0$  may represent a description of how the principal might plan to allocate his resources to the productive agents, or how he might plan to reward them as a function of some future observations about output, etc. Any actions by an agent which the principal can observe and control directly should also be considered as part of the principal's decision domain.

We shall use the notation

$$D = D_0 \times D_1 \times \dots \times D_n, \quad T = T_1 \times \dots \times T_n.$$

That is,  $D$  is the set of all combinations of decisions which could be made by the principal and the agents, and  $T$  is the set of all possible combinations of agents' types. We let  $U_0: D \times T \rightarrow \mathbb{R}$  denote the principal's utility function, and we let  $U_i: D \times T \rightarrow \mathbb{R}$  denote the utility function of agent  $i$ . That is, for any

$(d, t) = (d_0, d_1, \dots, d_n, t_1, \dots, t_n)$  in  $D \times T$ ,  $U_i(d, t)$  represents the expected utility payoff for agent  $i$ , measured in some von Neumann-Morgenstern utility scale, if the principal and the agents act according to the vector of decisions  $d$ , and if the agents' information is as represented by the vector of types  $t$ . Any random variable not observed by any agents should be integrated out, using their conditional distributions given  $t$ , to compute this expected utility  $U_i(d, t)$ .

We let  $P$  denote the probability distribution on  $T$ , so that  $P(t)$  is the probability (as would be assessed by the principal, or by any agent ex ante) that  $t = (t_1, \dots, t_n)$  will be the vector of agents' types. For mathematical simplicity, we shall generally assume that  $D_0$  and all  $D_i$  and  $T_i$  sets are finite.

Given these structures  $(D_0, U_0, (D_i, T_i, U_i)_{i=1}^n, P)$ , the principal's problem is to coordinate his decisions and those of his agents, so as to maximize his expected utility. We assume that the principal has complete control over all communication between the agents, that he can request any information which the agents are willing to send, and that he can send messages and recommendations to the agents. However, the principal cannot directly observe an agent's type in  $T_i$  or control the decisions in any  $D_i$ , except  $D_0$ .

This formulation is rather abstract, so it may be worth seeing how it applies to the conventional principal-agent problem. For example, suppose that there is one agent who gets some private information and then chooses an effort level which affects the quantity of output accruing to the principal. Then  $n=1$ , and the agent's private decision  $d_1$  is his effort level. If the principal can observe the quantity of output before paying the agent, then the principal's decision  $d_0$  is itself a function mapping observed outputs to salary levels for the agent, and so  $D_0$  is a set of functions from  $\mathbb{R}$  to  $\mathbb{R}$ .

If the quantity of output is  $f(d_1, \theta)$ , depending on the agent's effort  $d_1$  and on some random variable  $\theta$  which is correlated with the agent's information  $t_1$ , then the agent's utility function in our formulation is

$$U_1(d_0, d_1, t_1) = E[u_1(d_0(f(d_1, \theta)), d_1) | t_1]$$

where  $u_1(\cdot)$  is the agent's utility for money and effort, and the expectation is taken over  $\theta$  given  $t_1$ . Similarly, if  $u_0(\cdot)$  is the principal's utility for money, then

$$U_0(d_0, d_1, t_1) = E[u_0(f(d_1, \theta) - d_0(f(d_1, \theta))) | t_1].$$

To describe a typical coordination mechanism which could be established by the principal, we let  $M_i$  be the set of all possible messages which agent  $i$  might receive from the principal or from the other agents in the mechanism. We let  $R_i$  be the set of all strategies which agent  $i$  could use for sending reports to the principal or the other agents in the mechanism, given his own type. (We are not assuming that this communication system is necessarily a one-stage affair. To model a multi-stage communication system, let  $R_i$  be the set of all possible strategic plans which agent  $i$  might use to determine the reports he sends at each stage as a function of the messages he has received at earlier stages, and let  $M_i$  be the set of all possible sequences of messages which agent  $i$  might receive over the whole process.) Of course, the messages received by one agent must depend on the reports sent by the others, and the principal's decision in  $D_0$  may also depend on these reports. To describe this dependence, let

$$\pi(d_0, m_1, \dots, m_n | r_1, \dots, r_n)$$

be the probability that the principal will choose  $d_0$  in  $D_0$  and that each agent  $i$  will receive message  $m_i$  in  $M_i$ , given that each agent  $i$  is planning to send reports according to the reporting strategy  $r_i$  in  $R_i$ . Then, these sets  $(R_i, M_i)_{i=1}^n$  together with the probability function  $\pi$  completely describe



the coordination mechanism established by the principal.

In the context of this coordination mechanism  $((R_i, M_i)_{i=1}^n, \tau)$  each agent  $i$  controls his choice of reporting strategy in  $R_i$  as a function of his type, and controls his choice of a decision in  $D_i$  as a function of his type and his message received. That is, agent  $i$  selects a pair of functions  $\rho_i: T_i \rightarrow R_i$  and  $\delta_i: M_i \times T_i \rightarrow D_i$ , such that  $\rho_i(t_i)$  would be  $i$ 's reporting strategy if  $i$  were of type  $t_i$ , and  $\delta_i(m_i, t_i)$  would be  $i$ 's final decision in  $D_i$  after he received message  $m_i$  if his type were  $t_i$ . We may refer to any pair of such functions  $(\rho_i, \delta_i)$  as a participation strategy for  $i$ .

Given any  $n$ -tuple of participation strategies  $((\rho_1, \delta_1), \dots, (\rho_n, \delta_n))$  for the  $n$  agents, the expected utility for agent  $i$  would be

$$(1) \quad V_i((\rho_1, \delta_1), \dots, (\rho_n, \delta_n)) \\ = \sum_{t \in T} \sum_{d_0 \in D_0} \sum_{m \in M} P(t) \pi(d_0, m | \rho(t)) U_i(d_0, \delta(m, t), t)$$

where  $M = M_1 \times \dots \times M_n$ ,  $\rho(t) = (\rho_1(t_1), \dots, \rho_n(t_n))$ , and

$$(\delta(m, t)) = (\delta_1(m_1, t_1), \dots, \delta_n(m_n, t_n)).$$

Thus, the principal's coordination mechanism, in effect, defines a game to be played by the agents. Since each agent's participation strategy is chosen independently (and in particular, cannot be dictated by the principal), the principal should expect the agents to use participation strategies which form a noncooperative equilibrium to this game, in which every agent's participation strategy maximizes his expected utility. Formally, the participation strategies  $((\rho_1, \delta_1), \dots, (\rho_n, \delta_n))$  form an equilibrium if and only if, for every agent  $i$  and every alternative participation strategy  $(\tilde{\rho}_i, \tilde{\delta}_i)$  for  $i$

$$(2) \quad V_i((\rho_1, \delta_1), \dots, (\rho_i, \delta_i), \dots, (\rho_n, \delta_n)) \\ \geq V_i((\rho_1, \delta_1), \dots, (\tilde{\rho}_i, \tilde{\delta}_i), \dots, (\rho_n, \delta_n)).$$

The principal's problem is to design a coordination

mechanism  $((R_i, M_i)_{i=1}^n, \pi)$  such that there is an equilibrium of participation strategies  $((\rho_1, \delta_1), \dots, (\rho_n, \delta_n))$  which gives the principal the highest possible expected utility  $V_0((\rho_1, \delta_1), \dots, (\rho_n, \delta_n))$ , where  $V_0$  is defined as in (1) for  $i=0$ .

### 3. Direct coordination mechanisms and incentive compatibility

The principal's problem might seem to be intractable, because we have put no bound on the complexity of the sets  $M_i$  and  $R_i$ . However, we can now show that there is no loss of generality in restricting our attention to communication and control mechanisms with a very simple structure.

Following the terminology of Dasgupta, Hammond, and Maskin [1979], we say that a coordination mechanism is direct iff each  $M_i = D_i$  and each  $R_i = T_i$ . That is, in a direct coordination mechanism, each agent  $i$  is asked to report a type in  $T_i$  to the principal, and in return, the principal will send each agent  $i$  a suggested decision in  $D_i$ . Thus, a direct mechanism is completely characterized by its probability function  $\pi$ , where  $\pi(d_0, d_1, \dots, d_n | t_1, \dots, t_n)$  is the principal's probability of doing  $d_0$  and recommending  $d_i$  to each agent  $i$  if each agent  $i$  reports  $t_i$ .

In a direct mechanism, agent  $i$  is honest and obedient if he uses the participation strategy  $(\rho_i^*, \delta_i^*)$  satisfying, for all  $t_i$  and  $d_i$ ,

$$\rho_i^*(t_i) = t_i \text{ and } \delta_i^*(d_i, t_i) = d_i.$$

We say that a direct mechanism  $\pi$  is incentive compatible (or, more correctly, Bayesian incentive compatible, in the sense of D'Aspremont and Gerard-Varet [1979]) iff the honest-obedient participation strategies  $((\rho_1^*, \delta_1^*), \dots, (\rho_n^*, \delta_n^*))$  form an equilibrium.

More precisely, we may characterize the set of incentive compatible direct mechanisms as the set of all functions  $\pi: D \times T \rightarrow \mathbb{R}$  satisfying the following inequalities:

$$(3) \quad \pi(d|t) \geq 0 \text{ and } \sum_{e \in D} \pi(e|t) = 1, \quad \forall d \in D, \forall t \in T,$$

and

$$(4) \quad \sum_{t \in T} \sum_{d \in D} P(t) \pi(d|t) U_i(d, t) \\ t_i = \tau_i \\ \geq \sum_{t \in T} \sum_{d \in D} P(t) \pi(d|t_{-i}, \hat{\tau}_i) U_i((d_{-i}, \hat{\delta}_i(d_i)), t), \\ t_i = \tau_i$$

$$\forall i \in \{1, \dots, n\}, \forall \tau_i \in T_i, \forall \hat{\tau}_i \in T_i, \forall \hat{\delta}_i: D_i \rightarrow D_i.$$

(We use here the notation

$$(t_{-i}, \tau_i) = (t_1, \dots, \tau_i, \dots, t_n) \text{ and } (d_{-i}, \hat{\delta}_i(d_i)) = (d_0, d_1, \dots, \hat{\delta}_i(d_i), \dots, d_n).$$

The first summation in (4) is over all  $t$  in  $T$  such that  $t_i = \tau_i$ .)

Condition (3) asserts that  $\pi$  is a valid conditional probability function.

Condition (4) asserts that when agent  $i$  is type  $\tau_i$ , he should do at least as well by being honest and obedient as by reporting  $\hat{\tau}_i$  and then using  $\hat{\delta}_i(d_i)$  when told to do  $d_i$ , given that all other agents are planning to be honest and obedient.

When the  $T_i$  and  $D_i$  sets are finite, the set of incentive-compatible direct coordination mechanisms is characterized by finitely many linear inequalities, since (3) and (4) are linear in  $\pi$ , and the number of mappings  $\delta_i: D_i \rightarrow D_i$  is finite. The principal's expected utility from the direct mechanism  $\pi$  is

$$(5) \quad \sum_{t \in T} \sum_{d \in D} P(t) \pi(d|t) U_0(d, t),$$

if all agents are honest and obedient. Formula (5) is linear in  $\pi$ . Thus, we are led to the following important observation.

Proposition 1 When the type sets  $T_i$  and the decision sets  $D_i$  (including  $D_0$ ) are all finite, then the problem of computing the optimal incentive compatible direct coordination mechanism is a linear programming problem: to choose  $\pi: D \times T \rightarrow \mathbb{R}$  so as to maximize (5) subject to (3) and (4).

We may now ask whether other coordination mechanisms could possibly offer the principal higher expected utility than the best incentive-compatible direct mechanism. Perhaps an equilibrium of participation strategies in which some lying or disobedience is accepted might be better, or some coordination mechanism with  $R_i \neq T_i$  or  $M_i \neq D_i$  might be better. In fact, the answer is No.

Proposition 2 Given any equilibrium of participation strategies  $(\rho_i, \delta_i)_{i=1}^n$  in any coordination mechanism  $((R_i, M_i)_{i=1}^n, \pi)$ , there exists an incentive-compatible direct mechanism  $\pi^*$  in which the principal gets the same expected utility (when the agents are honest and obedient) as in the given equilibrium of the given mechanism. Thus, the optimal incentive-compatible direct coordination mechanism is also optimal in the class of all coordination mechanisms.

Proof Given the equilibrium of participation strategies  $(\rho_i, \delta_i)_{i=1}^n$ , let  $\delta_i^{-1}(d, t)$  be the set of all messages to the agents such that each agent  $i$  would respond by choosing decision  $d_i$  if his type were  $t_i$ .

That is,

$$\delta^{-1}(d, t) = \{m \mid \delta_i(m_i, t_i) = d_i, \text{ for all } i\}.$$

Then, define  $\pi^*: D \times T \rightarrow \mathbb{R}$  so that

$$\pi^*(d|t) = \sum_{m \in \delta^{-1}(d, t)} \pi(d_0, m \mid p_1(t_1), \dots, p_n(t_n)).$$

$\pi^*$  is the direct coordination mechanism which simulates the overall effect of the original mechanism with the given participation strategies, as illustrated in Figure 1. That is,  $\pi^*(d|t)$  is the probability that the principal will choose  $d_0$  and each agent will choose  $d_i$  if  $t$  is the vector of types, when each agent  $i$  chooses his reporting strategy according to  $p_i$ , the principal's decision and the messages to the agents are determined from these reports by  $\pi$ , and each agent  $i$  uses  $\delta_i$  to translate his received message into a decision in  $D_i$ .

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INSERT FIGURE 1 HERE.

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It is easy to see that  $\pi^*$  gives the same expected utility to the principal (and to each agent) as the originally given mechanism, since the probability distribution over decision vectors for any type vector is the same. To check that  $\pi^*$  is incentive compatible, suppose (to the contrary) that agent  $i$  could gain when  $\tau_i$  is his type by reporting  $\hat{\tau}_i$  and then by permuting his instructions by  $\hat{\delta}: D_i \rightarrow D_i$ , in violation of (4). Then, consider the participation strategies  $(\tilde{p}_i, \tilde{\delta}_i)$  for the original mechanism, where

$$\begin{aligned} \tilde{p}_i(\tau_i) &= p_i(\hat{\tau}_i), & \tilde{p}_i(t_i) &= p_i(t_i) \text{ if } t_i \neq \tau_i, \\ \tilde{\delta}_i(m_i, \tau_i) &= \hat{\delta}(\delta_i(m_i, \hat{\tau}_i)), & \tilde{\delta}_i(m_i, t_i) &= \delta_i(m_i, t_i) \text{ if } t_i \neq \tau_i. \end{aligned}$$

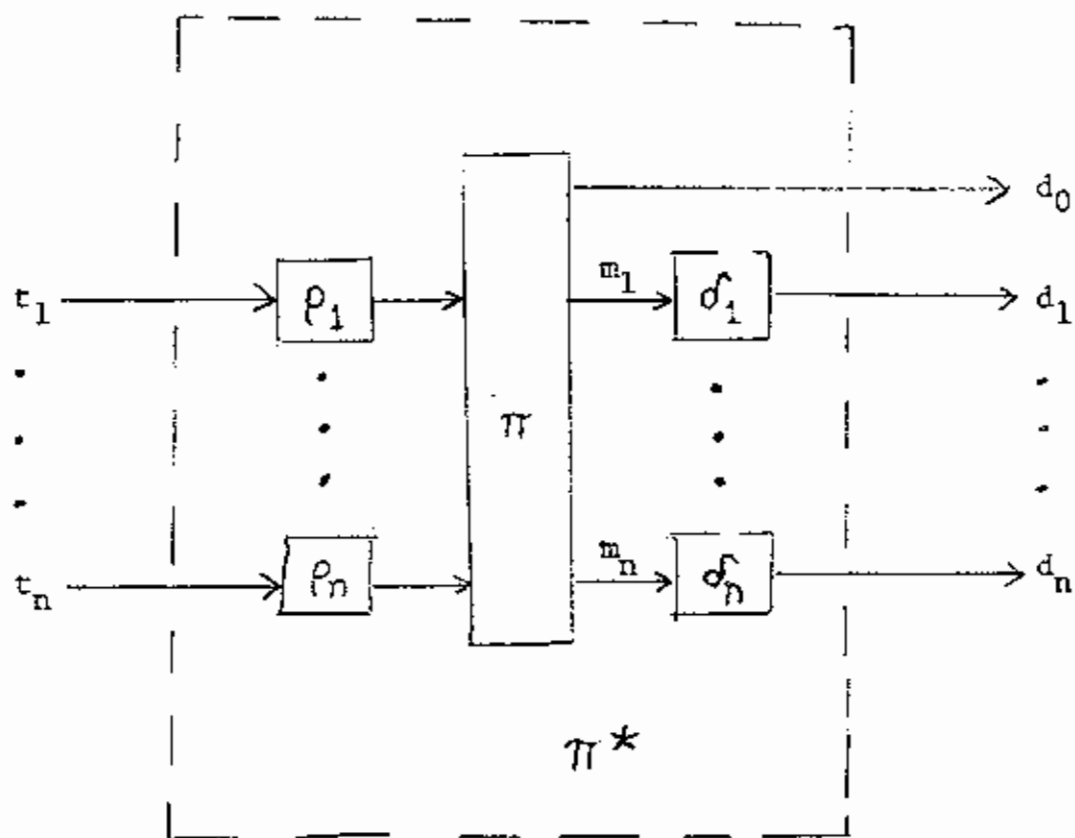


FIGURE 1.

That is,  $(\tilde{\rho}_i, \tilde{\delta}_i)$  differs from  $(\rho_i, \delta_i)$  in that, when  $\tau_i$  is  $i$ 's true type, he sends reports as if  $\hat{\tau}_i$  were his type and then he permutes his planned responses by  $\delta$  from what he would have done if  $\hat{\tau}_i$  were his type. It is straightforward to check that if  $(\hat{\tau}_i, \hat{\delta})$  violated (4) for  $\pi^*$ , then  $(\tilde{\rho}_i, \tilde{\delta}_i)$  would violate (2) for the originally given mechanism, which would contradict the fact that  $(\rho_i, \delta_i)$  is an equilibrium strategy for  $i$ . Thus,  $\pi^*$  must be incentive compatible. Q.E.D.

Consider the special case where  $|D_0| = 1$  and each  $|T_i| = 1$ , so that the principal does not control any decisions directly (he has only one feasible decision) and each agent  $i$  has no private information (he could only be of one type). In this case, the incentive-compatible mechanisms are correlated equilibria, in the sense of Aumann [1974]. That is, the agents are playing a noncooperative game, and the principal, by his control over all communication between agents, can select the correlated equilibrium which maximizes his own expected utility.

It is worth noting that Proposition 2 does not depend in any way on the finiteness of  $T_i$  and  $D_i$ . Even if the  $T_i$ ,  $D_i$ ,  $R_i$ , and  $M_i$  sets were infinite sets, we could still construct an incentive-compatible direct coordination mechanism to simulate any equilibrium of measurable participation strategies. The essential ideas in Figure 1 carry over to the infinite case, except that the simple probability vectors must be replaced by measures. That is  $\pi(\cdot | \rho_1(\tau_1), \dots, \rho_n(\tau_n))$  is now a probability measure on  $D_0 \times M_1 \times \dots \times M_n$ , and  $\pi^*(\cdot | t)$  must be a measure on  $D$  defined so that, for any measurable set  $C \subseteq D$ ,

$$\pi^*(C|t) = \pi(\delta^{-1}(C,t) | p_1(t_1), \dots, p_n(t_n))$$

where

$$\delta^{-1}(C,t) = \{(d_0, m_1, \dots, m_n) | (d_0, \delta_1(m_1, t_1), \dots, \delta_n(m_n, t_n)) \in C\}.$$

We have emphasized the finite case in this section mainly to simplify the logical exposition, and because it is only in the finite case that the principal's problem reduces to a linear programming problem.

#### 4. Equilibria among several principals

A difficulty arises when we try to construct a general theory of principal-agent interaction among several principals, each with his own set of agents. Suppose that there are  $N$  principals, numbered 1 to  $N$ , with  $k$  denoting a typical principal. Each principal  $k$  has  $n_k$  agents, numbered 1 to  $n_k$ , with  $i$  denoting a typical agent. We shall assume that the sets of agents for different principals are disjoint, so that each agent belongs to just one principal. Principal  $k$  together with his agents may be referred to as corporation  $k$ . Extending the notation of Section 2, we let  $T_i^k$  and  $D_i^k$  denote the sets of types and decisions for the  $i^{\text{th}}$  agent belonging to principal  $k$ . We let  $D_0^k$  denote the decision domain directly controlled by principal  $k$ . We use the notation:

$$D^k = D_0^k \times D_1^k \times \dots \times D_{n_k}^k, \quad T^k = T_1^k \times \dots \times T_{n_k}^k,$$

$$D = D^1 \times \dots \times D^N, \quad T = T^1 \times \dots \times T^N.$$

The utility functions of the  $k^{\text{th}}$  principal and his  $i^{\text{th}}$  agent are

$U_0^k: D \times T \rightarrow \mathbb{R}$  and  $U_i^k: D \times T \rightarrow \mathbb{R}$  respectively. Thus, we are allowing that the utility payoff to any principal or any agent may depend on the decisions and



types in all the corporations. For any  $t$  in  $T$ , we let  $P(t)$  denote the probability that  $t$  is the true vector of types for all the agents.

As in Section 3, we may define a direct mechanism for corporation  $k$  to be any function  $\pi^k: D^k \times T^k \rightarrow \mathbb{R}$  where  $\pi^k(d^k|t^k)$  is the probability that the principal will use decision  $d_0^k$  and recommend  $d_1^k$  to his  $i^{\text{th}}$  agent  $i$ , for each  $i$ , if  $t^k$  is the vector of reports from his  $n_k$  agents. (Here,

$$t^k = (t_1^k, \dots, t_{n_k}^k) \text{ and } d^k = (d_0^k, d_1^k, \dots, d_{n_k}^k).$$

Thus,  $\pi^k$  must satisfy

$$(6) \quad \pi^k(d^k|t^k) \geq 0 \quad \text{and} \quad \sum_{e^k \in D^k} \pi^k(e^k|t^k) = 1, \quad \forall d^k \in D^k, \forall t^k \in T^k.$$

Since the payoffs for the agents of one principal may depend on the decisions of other principals and their agents, we cannot define incentive compatibility for any one principal in isolation. Thus, we say that a mechanism  $\pi^k$  for corporation  $k$  is incentive-compatible in the context of mechanisms  $(\pi^1, \dots, \pi^N)$  iff it is an equilibrium for all of  $k$ 's agents to be honest and obedient when principal  $k$  uses mechanism  $\pi^k$ , each other principal  $j$  uses the mechanism  $\pi^j$ , and the agents for every other principal  $j$  ( $j \neq k$ ) are expected to be honest and obedient. Formally,  $\pi^k$  is incentive compatible for corporation  $k$  in the context of  $(\pi^1, \dots, \pi^N)$  iff

$$(7) \quad \sum_{t \in T} \sum_{d \in D} P(t) \pi(d|t) U_i^k(d, t) \\ \sum_{t_i^k = \tau_i^k} \geq \sum_{t \in T} \sum_{d \in D} P(t) \pi^{-k}(d^{-k}|t^{-k}) \pi^k(d^k|t_{-i}^k, \hat{\tau}_i^k) U_i^k((d_{-k, i}, \delta_i^k(d_i^k)), t)$$

$$\forall i \in \{1, \dots, n_k\}, \quad \forall \tau_i^k \in T_i^k, \quad \forall \hat{\tau}_i^k \in T_i^k, \quad \forall \delta_i^k: D_i^k \rightarrow D_i^k,$$

$$\text{where } \pi(d|t) = \prod_{j=1}^N \pi^j(d^j|t^j) \text{ and } \pi^{-k}(d^{-k}|t^{-k}) = \prod_{j \neq k} \pi^j(d^j|t^j).$$

(We use here the notation:  $d = (d^1, \dots, d^N)$ ,  $t = (t^1, \dots, t^N)$ ,  $(t_{-i}^k, \hat{\tau}_i^k)$  is the same as the vector  $t^k$ , except that  $t_i^k$  is replaced by  $\hat{\tau}_i^k$ , and  $(d_{-k,i}^k, \delta_i^k(d_i^k))$  differs from  $d$  in that the component  $d_k^i$  is changed to  $\delta_i^k(d_i^k)$ .) Inequality (7) asserts that  $k$ 's  $i^{\text{th}}$  agent should not be able to increase his expected utility above what he gets with honesty and obedience by reporting  $\hat{\tau}_i^k$ , when  $\tau_i^k$  is his type, and by then choosing decision  $\delta_i^k(d_i^k)$  when told to choose  $d_i^k$ .

When all agents are honest and obedient, principal  $k$ 's expected utility from the mechanisms  $(\pi^1, \dots, \pi^N)$  is

$$(8) \quad v_0^k(\pi^1, \dots, \pi^N) = \sum_{t \in T} \sum_{d \in D} p(t) \left( \prod_{j=1}^N \pi^j(d^j|t^j) \right) u_0^k(d, t).$$

Propositions 1 and 2 can be extended in the obvious way to the case of one principal designing a coordination mechanism in the context of fixed behavior by the other principals and their agents. That is, if  $(\pi^1, \dots, \pi^{k-1}, \pi^{k+1}, \dots, \pi^N)$  characterizes the planned behavior in corporations other than  $k$ , then the optimal mechanism for principal  $k$  should maximize (8) over all  $\pi^k: D^k \times T^k \rightarrow \mathbb{R}$  subject to the constraints (6) and (7). As before, this is a linear programming problem, but now principal  $k$ 's optimal incentive-compatible mechanism depends on the mechanisms chosen by the other principals. Assuming that the principals act noncooperatively in setting up their respective coordination mechanisms, we should then define  $(\pi^1, \dots, \pi^N)$  to be a principals' equilibrium iff, for each principal  $k$ ,  $\pi^k$  maximizes (8) subject to (6) and (7), given the other  $\pi^j$  for

all  $j \neq k$ . Unfortunately, we have the following result.

Proposition 3 Principals' equilibria do not always exist.

Proof It suffices to show one example. So, consider a simple example where there are two principals ( $N=2$ ), each with one agent ( $n_k = 1$ ). Each agent has type set  $T_1^k = \{\alpha, \beta\}$ , and the two agent's types are independent,  $P(\alpha, \alpha) = P(\alpha, \beta) = P(\beta, \alpha) = P(\beta, \beta) = 1/4$ . Each principal has decision domain  $D_0^k = \{A, B, C\}$ , and each agent has no private decision ( $|D_1^k| = 1$ .) For  $k = 1$  or  $2$ , the payoffs to principal  $k$  ( $U_0^k$ ) and his agent ( $U_1^k$ ) depend on the principal's decision and the agent's type according to the following matrix.

$U_0^k, U_1^k$	$t_1^k = \alpha$	$t_1^k = \beta$
$d_0^k = A$	6, 1	0, $z^k$
$d_0^k = B$	0, $z^k$	6, 1
$d_0^k = C$	5, 0	5, 0

Here, the term  $z^k$  is determined by the other principals actions as follows:

for  $k = 1$ ,  $z^1 = \begin{cases} 2 & \text{if principal 2 chooses A or B} \\ 1 & \text{if principal 2 chooses C,} \end{cases}$

for  $k = 2$ ,  $z^2 = \begin{cases} 2 & \text{if principal 1 chooses C} \\ 1 & \text{if principal 1 chooses A or B.} \end{cases}$

If  $z^k = 1$  for sure, then the optimal incentive-compatible mechanism for principal  $k$  is to choose A if the agent says his type is  $\alpha$ , B if  $\beta$ . On the

other hand, if there is any positive probability that  $z^k = 2$ , then the agent will want to steer principal  $k$  to the "wrong" corner (B if  $\alpha$ , A if  $\beta$ ), and the best incentive-compatible mechanism is to choose C for sure. So principal 1 wants to choose C if 2 might choose A or B with positive probability, and principal 2 wants to choose C if 1 might choose C, but principal 1 wants to choose A or B if 2 chooses C for sure, and principal 2 wants to choose A or B if 1 chooses A or B for sure. Thus, there cannot be any principals' equilibrium. Q.E.D.

Essentially, this nonexistence result occurs because the set of incentive-compatible mechanisms for principal  $k$  varies upper-semicontinuously in the other  $\pi^j$ , rather than continuously as is required by the existence theorem of Debreu [1952]. To obtain an existence theorem for the multi-principal problem, we must explore weaker solution concepts than full equilibrium. We shall develop one notion of quasi-equilibrium here. Our quasi-equilibria will differ from equilibria in that each principal's mechanism need not be optimal for him in his class of incentive-compatible mechanisms; instead, we only require that any mechanisms which he might prefer must be "unsafe", in that they would not be incentive-compatible if the other principals made infinitesimal changes in their plans.

In our example, suppose that both principals planned to choose C. This is not a principal's equilibrium because there are better incentive-compatible mechanisms available to principal 1, if principal 2 is absolutely certain to choose C. However, if principal 1 (or his agent) believed that there was even an infinitesimal probability of principal 2 using A or B, then there would be no better incentive-compatible mechanism for principal 1 than to choose C for sure. Thus, we may refer to this pair of mechanisms (both principals choosing

C for sure) as a quasi-equilibrium for the principals, since an infinitesimal perturbation in the mechanisms could render infeasible any mechanism which either principal might prefer to use.

Let  $F^k(\pi^1, \dots, \pi^N)$  be the set of all direct mechanisms  $\bar{\pi}^k$  for corporation k such that  $\bar{\pi}^k$  is incentive compatible in the context of  $(\pi^1, \dots, \bar{\pi}^k, \dots, \pi^N)$ . Let

$$W^k(\pi^1, \dots, \pi^N) = \max_{\bar{\pi}^k \in F^k(\pi^1, \dots, \pi^N)} v_0^k(\pi^1, \dots, \bar{\pi}^k, \dots, \pi^N).$$

That is,  $W^k(\pi^1, \dots, \pi^N)$  is the maximum expected utility which principal k can get from an incentive-compatible mechanism when the other principals use their  $\pi^j$  mechanisms. (Notice that  $F^k$  and  $W^k$  do not actually depend on the argument  $\pi^k$ .)

In general, we say that  $(\bar{\pi}^1, \dots, \bar{\pi}^N)$  is a principals' quasi-equilibrium iff there exists a sequence of direct mechanisms  $\{(\pi_2^1, \dots, \pi_2^N)\}_{\ell=1}^{\infty}$  such that, for every corporation k,

$$(9) \quad \bar{\pi}^k = \lim_{\ell \rightarrow \infty} \pi_2^k,$$

$$(10) \quad v_0^k(\bar{\pi}^1, \dots, \bar{\pi}^N) \geq \lim_{\ell \rightarrow \infty} W^k(\pi_2^1, \dots, \pi_2^N), \quad \text{and}$$

$$(11) \quad \bar{\pi}^k \in F^k(\bar{\pi}^1, \dots, \bar{\pi}^N).$$

Conditions (9) and (10) imply that, for any sequence  $\{\pi_2^k\}_{\ell=1}^{\infty}$  of mechanisms for corporation k,

$$\begin{aligned} \text{if } \hat{\pi}_\ell^k \in F^k(\pi_\ell^1, \dots, \pi_\ell^N) \quad \forall \ell, \\ \text{then } \limsup_{\ell \rightarrow \infty} V_0^k(\pi_\ell^1, \dots, \hat{\pi}_\ell^k, \dots, \pi_\ell^N) &\leq \lim_{\ell \rightarrow \infty} W^k(\pi_\ell^1, \dots, \pi_\ell^N) \\ &\leq V_0^k(\bar{\pi}^1, \dots, \bar{\pi}^N) = \lim_{\ell \rightarrow \infty} V_0^k(\pi_\ell^1, \dots, \pi_\ell^k, \dots, \pi_\ell^N). \end{aligned}$$

So (9) and (10) imply that the sequence of mechanisms  $\{(\pi_\ell^1, \dots, \pi_\ell^N)\}_{\ell=1}^\infty$  is asymptotically optimal for every principal, and (11) implies that the sequence is also asymptotically incentive-compatible for every corporation. The limit of any such sequence is a quasi-equilibrium.

To put it differently, (9) and (10) imply that, for any direct mechanism  $\hat{\pi}^k$ ,

$$\text{if } V_0^k(\bar{\pi}^1, \dots, \hat{\pi}^k, \dots, \bar{\pi}^N) > V_0^k(\bar{\pi}^1, \dots, \bar{\pi}^k, \dots, \bar{\pi}^N) \quad \text{then } \hat{\pi}^k \notin F^k(\pi_\ell^1, \dots, \pi_\ell^N)$$

for all sufficiently large  $\ell$ . That is, we can make arbitrarily small perturbations in our quasi-equilibrium (from  $\bar{\pi}^k$  to  $\pi_\ell^k$ ) in such a way as to render infeasible any mechanism which a principal might have preferred to use.

It should be clear that (letting  $\pi_\ell^k = \bar{\pi}^k$ ) any principals' equilibrium is also a principals' quasi-equilibrium. However, the quasi-equilibria are a larger set, and do always exist.

**Proposition 4** A quasi-equilibrium must exist, for any multi-principal problem with finite type sets and finite decision sets.

**Proof** Given any  $\varepsilon > 0$ , we say that  $\pi^k$  is  $\varepsilon$ -incentive-compatible for corporation  $k$  in the context of  $(\pi^1, \dots, \pi^N)$  iff  $\pi^k$  is a direct mechanism and violates none of the incentive constraints (7) for corporation  $k$  by more than  $\varepsilon$ ; that is,

$$\begin{aligned}
 (12) \quad & \sum_{t \in T} \sum_{d \in D} [P(t) \pi(d|t) U_1^k(d, t)] + \varepsilon \\
 & \sum_{t_1^k = t_1}^k \\
 & \geq \sum_{t \in T} \sum_{d \in D} P(t) \pi^{-k}(d^{-k}|t^{-k}) \pi^k(d^k|t_{-1}^k, \hat{t}_1^k) U_1^k((d_{-k, i}^k, \delta_i^k(d_i^k)), t) \\
 & \sum_{t_1^k = t_1}^k \\
 & \forall i \in \{1, \dots, n_k\}, \quad \forall t_1^k \in T_1^k, \quad \forall \hat{t}_1^k \in T_1^k, \quad \forall \delta_i^k: D_i^k \rightarrow D_i^k.
 \end{aligned}$$

We let  $\bar{F}^k(\pi^1, \dots, \pi^N, \varepsilon)$  be the set of all direct mechanisms  $\tilde{\pi}^k$  such that  $\tilde{\pi}^k$  is  $\varepsilon$ -incentive-compatible for corporation  $k$  in the context of  $(\pi^1, \dots, \tilde{\pi}^k, \dots, \pi^N)$ .

It is straightforward to show that  $\bar{F}^k(\cdot, \varepsilon)$  is an upper-semicontinuous correspondence, and that  $\bar{F}^k(\pi^1, \dots, \pi^N, \varepsilon)$  is always a nonempty compact convex set. The key step is to show that, if  $\varepsilon > 0$ , then  $\bar{F}^k(\cdot, \varepsilon)$  is also lower-semicontinuous. (See Debreu [1959], Chapter 1.) To check lower-semicontinuity, let  $\hat{\pi}^k$  be any direct mechanism for corporation  $k$  such that  $\hat{\pi}^k(d^k|t^k)$  is independent of  $t^k$ . Then, for any  $\pi^k$  in  $\bar{F}^k(\pi^1, \dots, \pi^N, \varepsilon)$  and any  $0 < \lambda < 1$ ,  $(1-\lambda)\pi^k + \lambda\hat{\pi}^k$  will satisfy the constraints in (12) strictly, and so  $(1-\lambda)\pi^k + \lambda\hat{\pi}^k$  will be in  $\bar{F}^k(\tilde{\pi}^1, \dots, \tilde{\pi}^N, \varepsilon)$  for all  $(\tilde{\pi}^1, \dots, \tilde{\pi}^N)$  sufficiently close to  $(\pi^1, \dots, \pi^N)$ . This is sufficient to prove that  $\bar{F}^k(\cdot, \varepsilon)$  is lower-semicontinuous, and thus continuous, for any positive  $\varepsilon$ .

Let  $G^k(\pi^1, \dots, \pi^N, \varepsilon)$  be the set of mechanisms  $\tilde{\pi}^k$  which maximize  $V_0^k(\pi^1, \dots, \tilde{\pi}^k, \dots, \pi^N)$  subject to  $\tilde{\pi}^k \in \bar{F}^k(\pi^1, \dots, \pi^N, \varepsilon)$ . It is straightforward to check that  $G^k(\pi^1, \dots, \pi^N, \varepsilon)$  is a nonempty compact convex subset of  $\bar{F}^k(\pi^1, \dots, \pi^N, \varepsilon)$ ; and  $G^k(\cdot, \varepsilon)$  is upper-semicontinuous, because  $\bar{F}^k(\cdot, \varepsilon)$  is continuous (by the Maximum Theorem of Berge [1959], see Debreu [1959], page 19). By the Kakutani Fixed-Point Theorem, for

any  $\lambda > 0$ , there exists some  $(\pi_\lambda^1, \dots, \pi_\lambda^N)$ , such that  $\pi_\lambda^k \in G^k(\pi_\lambda^1, \dots, \pi_\lambda^N, \frac{1}{\lambda})$  for every  $k$ . These mechanisms satisfy  $\pi_\lambda^k \in \bar{F}^k(\pi_\lambda^1, \dots, \pi_\lambda^N, \frac{1}{\lambda})$ , and  $V_0^k(\pi_\lambda^1, \dots, \pi_\lambda^N) \geq W^k(\pi_\lambda^1, \dots, \pi_\lambda^N)$  because  $\bar{F}^k(\pi_\lambda^1, \dots, \pi_\lambda^N, \frac{1}{\lambda}) \supseteq F^k(\pi_\lambda^1, \dots, \pi_\lambda^N)$ .

The set of all direct mechanisms is compact, so we can select a convergent subsequence of the  $\{(\pi_\lambda^1, \dots, \pi_\lambda^N)\}$ , converging to some  $(\bar{\pi}^1, \dots, \bar{\pi}^N)$  as  $\lambda \rightarrow \infty$ . As  $\lambda \rightarrow \infty$  and  $\varepsilon = \frac{1}{\lambda} \rightarrow 0$ , the  $\varepsilon$ -incentive-compatibility constraints (12) converge to the incentive-compatibility constraints (7). So  $\bar{\pi}^k \in \bar{F}^k(\bar{\pi}^1, \dots, \bar{\pi}^N)$  and  $V_0^k(\bar{\pi}^1, \dots, \bar{\pi}^N) = \lim_{\lambda \rightarrow \infty} V_0^k(\pi_\lambda^1, \dots, \pi_\lambda^N)$

$$\geq \lim_{\lambda \rightarrow \infty} W^k(\pi_\lambda^1, \dots, \pi_\lambda^N).$$

Thus  $(\bar{\pi}^1, \dots, \bar{\pi}^N)$  is a principals' quasi-equilibrium. Q.E.D.



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