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## Specific Human Capital and Labor Turnover

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DISCUSSION PAPER NO. 289

SPECIFIC HUMAN CAPITAL AND LABOR TURNOVER

by

Dale T. Mortensen

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## 1. Introduction

Two sources of job-worker attachment are emphasized in the labor turnover literature.<sup>1</sup> First, the accumulation of job specific skills raises the productivity of a given job-worker match relative to alternatives. Second, limited information restricts both the employer's and the worker's ability to locate a substitute for the match. Both sources create human capital in the form of a future stream of quasi-rents that is specific to the job-worker match. The likelihood of a separation decreases with the size of this stream.

A separation can be initiated by either the employer or the worker involved in an existing match; call the first a dismissal and the second a quit. Becker [1964] and Parsons [1972] argue that the division of the rents by the wage payment affects the quit and dismissal probabilities. Specifically, an increase in the wage decreases the former and increases the latter other things equal. As a consequence, they argue, each party to a match has an incentive to share the rents with the other as a means of reducing the expected capital loss attributable to the future possibility that the other will separate.

In this paper a different view is expressed - one based on the following observations.<sup>2</sup> First, the wage determination problem is one of bilateral bargaining in general when specific capital exists.<sup>3</sup> Second, because a separation initiated by one party to a match affects the welfare of the other, the problems of choosing search strategies is a two-person game. In this context the cooperative solution concept, a search strategy pair that maximizes joint welfare in the Pareto sense, is appealing. In the context of the model developed in this paper, the cooperative solution is unique and independent of the nature of the wage bargain. Consequently, marginal changes in the wage have no effect on either the dismissal or the quit rate if the cooperative strategies are pursued.

Moreover, a wage contract exists, a possible outcome of the bargaining process, that motivates each party to pursue his cooperative strategy. Finally, we argue that the existing empirical evidence on labor turnover is consistent with the hypothesis that both parties search for alternatives in a cooperative manner.

The body of the paper is organized as follows. In section 2, a model of job-worker matching is sketched that accounts for both specific skill accumulation and imperfect information as sources of job-worker attachment. Within this framework, turnover is a characteristic of a dynamic process by which job-worker matches are improved. The analytic purpose of the section is to derive the relationship between each parties search strategies and the probability that he will separate.

Section 3 is devoted to a derivation and a characterization of the cooperative solution to the game of search. The problem of calculating the cooperative solution strategies is one of dynamic programming when each party prefers future uncertain income streams that have higher expected present values. These techniques yield the principal theoretical conclusions of the paper. The cooperative search strategies do not depend upon the division of product of the match between employer and worker.

The question of individual incentive compatability is treated in section 4. As a method of studying the question, each party is assumed to pursue his non-cooperative Nash strategy given the wage contract, the agreement concerning the division of the net product. A Nash solution to the game generally does depend on the terms of the contract. For example, if the agreement simply specifies a wage-tenure schedule, then both turnover rates associated with the Nash solution are too large relative to those implied by the cooperative solu-

tion. An externality exists in this case because each party ignores the expected capital loss that his search imposes on the other. However, if the contract is such that each party is required to compensate for this loss, then the Nash and cooperative solutions are equivalent.

Severance pay and non-vested pension plans are institutional arrangements that can be interpreted as terminal compensation schemes. In section 5, other quantitative implications of the cooperative search strategy hypothesis are derived and compared with the existing empirical evidence.

## 2. TURNOVER AND SEARCH STRATEGIES

In a world of heterogeneous workers and jobs, a problem of matching jobs and workers in some best way exists. A centralized competitive market under conditions of perfect information about technology and worker abilities solves the problem by sorting workers among jobs in a manner that maximizes aggregate output, appropriately defined.<sup>4</sup>

When the locations of particular jobs and workers are uncertain, the instantaneous attainment of a solution that is optimal in this sense is not possible. Because it is not in the interest of either the employer or worker to wait until the best alternative is located, matches are formed that are imperfect. The fact that better alternatives generally exist for both the employer and the worker involved in an existing job-worker match motivates search. A separation is the possible consequence. Turnover of this type occurs when at least one of the two parties finds a preferred alternative. From this viewpoint, turnover is a characteristic of a dynamic process by which job-worker matches are improved in a decentralized labor market.<sup>5</sup>

In this section a partial model of such a matching process is outlined. The analysis is partial in the sense that the alternatives available to the employer and the worker matched are taken as given. In each case available alternatives are represented as a given distribution. To capture the notion that locations are uncertain, each party is assumed to search by sampling from his own distribution. The theory of optional search incorporated in the model is an extension of that developed by Burdett [1977] and Mortensen [1977].

Every agent, whether employer or worker, orders alternative income stream available to him at a given date according to their expected present values. In the theoretical analysis we abstract from worker age effects by assuming that both parties have horizons that are indefinitely far in the future. However, in the last section this assumption is relaxed for the purpose of deriving the empirical effects of aging. The expected capitalized future income stream given the continuation of an existing match depends in general on the job type and worker type involved and on the tenure of the match. Dependency on tenure arises because skills are acquired on the job.

Alternative income streams available to either employer or worker are characterized by a known stationary distribution of their expected present values. Each searches by randomly drawing a sequential sample, without recall, from his own distribution. Consequently, elements of the sample to date are independent and identically distributed. Finally, because the skills acquired with tenure are job specific, the distribution of alternatives do not depend on the tenure of the match.

Consider an existing match that has attained tenure  $t$ . At that date each party makes two decisions. First, given that the match is to be continued,

each selects the intensity with which to search for an alternative during the short future interval  $[t, t+h)$ . Let  $\alpha_{1t}$  and  $\alpha_{2t}$  represent measures of the employer's and the worker's search intensities respectively. Second, each party must decide whether or not to accept any alternative located during the interval  $[t, t+h)$ . By definition, an alternative is acceptable if and only if the expected present value of the future income stream that it offers is at least as large as the agent's reservation value. Let  $\beta_{1t}$  and  $\beta_{2t}$  respectively represent the employer's and the worker's reservation values during the interval  $[t, t+h)$ . The pairs  $(\alpha_{1t}, \beta_{1t})$  and  $(\alpha_{2t}, \beta_{2t})$  are referred to as the employer's and worker's respective search strategies during the interval.

Suppose that alternatives for each party arrive according to a Poisson process and that the two processes are independent. Interpret the two search intensities,  $\alpha_{1t}$  and  $\alpha_{2t}$ , as the means of the two arrival processes. Each of these are selected subject to a cost. The idea is that each party "buys" a probability distribution defined on the number of alternatives located per periods and that the mean of the distribution increases with the "price".<sup>6</sup> If the length of the interval  $[t, t+h)$  is sufficiently short, then the following three outcomes of the joint search process during the interval are approximately exhaustive and mutually exclusive given the independent Poisson arrival processes assumption:<sup>7</sup>  $n_t = 1$ , the employer finds a new alternative during the interval;  $n_t = 2$ , the worker locates a new alternative offer; and,  $n_t = 3$ , neither finds alternative. The frequency distribution on these three possibilities is given by the vector  $(h\alpha_{1t}, h\alpha_{2t}, 1-h\alpha_{1t} - h\alpha_{2t})$ .

Let  $z_{1t}$  and  $z_{2t}$  represent the expected present value of the future income stream offered by a random alternative located during the interval  $[t, t+h)$  by the employer and worker respectively.  $F_1(z)$  and  $F_2(z)$  denote their respective



probability distribution functions. Since at most one of the two find an alternative during the interval,  $F_i(z)$  is the conditional probability that  $z_{it} \leq z$  given  $n_t = i$ . For convenience, we extend the definition by letting  $z_{it} = 0$  when  $n_t \neq i$ . Of course,  $z_{it} \geq \beta_{1t}$  implies that the employer's dismisses the job's current occupant by replacing him with the newly discovered alternative. Similarly, if  $z_{2t} \geq \beta_{2t}$ , the worker quits to take the alternative job discovered during the interval.

In the case of search without recall, the worker is unemployed at the end of the interval  $[t, t+h)$  if the employer finds an acceptable alternative and the job is vacant if the worker quits to accept an alternative job. Let  $x_1$  denote the expected capitalized future profit stream that the employer can expect given that the job is currently vacant. Analogously, let  $x_2$  denote the expected capitalized future income stream accruing to the worker given unemployment. Obviously,  $x_1$  and  $x_2$  are lower bounds on the employer's and worker's absolute shares of the capitalized value of the future income generated by an existing job-worker match. In other words,  $y_{1t+h} \geq x_1$  and  $y_{2t+h} \geq x_2$  are both necessary for continuation beyond tenure date  $t+h$  where  $y_{1t+h}$  and  $y_{2t+h}$  represent the employer's and worker's expected capitalized future income streams given continuation. When both conditions are satisfied,

$$hp_{1t} = \Pr\{n_t = 1\} \Pr\{z_{1t} \geq \beta_{1t} | n_t = 1\} = hx_{1t}[1 - F_1(\beta_{1t})] \quad (1)$$

and

$$hp_{2t} = \Pr\{n_t = 2\} \Pr\{z_{2t} \geq \beta_{2t} | n_t = 2\} = hx_{2t}[1 - F_2(\beta_{2t})] \quad (2)$$

respectively equal the probability that the worker will be dismissed and will quit during the interval  $[t, t+h)$ . In the sequel,  $p_{1t}$  and  $p_{2t}$  are referred to as the dismissal rate and the quit rate respectively.

### 3. COOPERATIVE SEARCH STRATEGIES

Both the employer's and the worker's future income streams depend on the future duration of a job-worker match. Since the duration is determined by the search strategies that both pursue, the decision problem can be formulated as a two person game. The cooperative solution to the game is characterized in this section. Three principal results are presented. First, the cooperative search strategies pair is unique if search costs are convex and exists given reasonable boundary conditions. Second, the turnover rates are independent of the way that future rents specific to the match are divided between the employer and worker. Finally, both the quit rate and dismissal rate declines with the specific capital value of the match.

The cooperative solution maximizes the expected capitalized future income streams of both employer and worker if the match continues,  $(y_{1t}, y_{2t})$ , in the vector sense subject to the restriction that neither is better off separated; i.e.,  $(y_{1t}, y_{2t}) \geq (x_1, x_2)$ . Because "utility" is transferable, the cooperative solution necessarily maximizes the sum

$$y_t = y_{1t} + y_{2t} \tag{3}$$

at each tenure date. This problem is easily formulated and solved using dynamic programming techniques.

The following specification is assumed. The value of the product flow net of any training cost during the interval  $[t, t+h)$ ,  $v_t$ , is non-decreasing in tenure. The assumption reflects the accumulation of job specific skills with tenure. The cost of search,  $c_1(\alpha_1)$  in the employer's case and  $c_2(\alpha_2)$  in the worker's, is an increasing strictly convex function of intensity, is zero given no search, and tends to infinity as intensity increases without bound. The interest rate common to both employer and worker is a positive constant denoted

by  $r$ . The support of the distribution of alternatives is bounded from above in both cases, i.e.,  $F_i(\bar{z}_i) = 1$  for some  $\bar{z}_i < \infty$   $i = 1, 2$ . Finally, the expected capitalized values of holding a vacancy and of searching while unemployed are both non-negative; i.e.  $(x_1, x_2) \geq 0$ .

The capital value of continuing the match,  $y_t$ , equals the present value of the net income obtained by both employer and worker during the interval  $[t, t+h)$ ,  $h[v_t - c_1(\alpha_{1t}) - c_2(\alpha_{2t})]$ , plus the present value of the expected sum of their future incomes as of the end of the interval. Since  $y_t$  is maximum only if the second term is as large as possible, the match terminates at the end of the interval if and only if

$$y_{t+h}^* \leq \max[z_{1t}, x_1] + \max[z_{2t}, x_2] \quad (4)$$

where  $y_{t+h}^*$  is the maximum expected present value were the match to continue beyond tenure date  $t+h$  and the right side is the maximum expected present value sum were the match to terminate at tenure date  $t$ . The cooperative reservation value for agent  $i$  is then that value of an alternative  $z_{it}, \beta_{1t}^*$ , that equates the two sides of (4) when  $n_t = i$ .

Given (4), Bellman's principle of dynamic optimality implies that

$$(1+rh)y_t^* = \max_{(\alpha_1, \alpha_2) \geq 0} [h[v_t - c_1(\alpha_1) - c_2(\alpha_2)] + E\{\max[y_{t+h}^*, \max[z_{1t}, x_1] + \max[z_{2t}, x_2]] | (\alpha_1, \alpha_2)\}] \quad (5)$$

where  $E\{\cdot | (\alpha_1, \alpha_2)\}$  denotes the expectation operator taken with respect to the joint distribution of the search outcome  $(z_1, z_2)$  given some search intensity pair. Since an increase in  $\alpha_i$  is an increase in the probability that  $z_i > 0$ , an increase in either augments the end of interval expected present value sum. The cooperative search intensities  $(\alpha_{1t}^*, \alpha_{2t}^*)$  are chosen to balance this return against the additional current cost of search.

At most one party finds an alternative during the interval  $[t, t+h)$ ; i.e.,  $z_{it} = 0$  if  $n_t \neq i$ . The probability that agent  $i$  finds the alternative is  $h\alpha_{it}$ ; i.e.  $\Pr\{n_t = i\} = h\alpha_{it}$ . These facts and (3) imply

$$\begin{aligned} (1 + rh)y_t^* &= \max_{(\alpha_1, \alpha_2) \geq 0} \{ [hv_t - hc_1(\alpha_1) - hc_2(\alpha_2)] \\ &\quad + h\alpha_1 \int \max[y_{t+h}^*, \max[z_1, x_1] + x_2] dF_1(z_1) \\ &\quad + h\alpha_2 \int \max[y_{t+h}^*, x_1 + \max[z_2, x_2]] dF_2(z_2) \\ &\quad + (1 - h\alpha_1 - h\alpha_2) \max[y_{t+h}^*, x_1 + x_2] \} . \end{aligned}$$

Finally, since  $\beta_{it}^*$  equates the two sides of (4) given  $n_t = i$ , the relationship can be rewritten as

$$\begin{aligned} (1 + rh)y_t^* &= hv_t + \max[y_{t+h}^*, x_1 + x_2] \tag{6} \\ &\quad + h \max_{(\alpha_1, \beta_1) \geq 0} [\alpha_1 \int_{\beta_1}^{\infty} (z_1 + x_2 - \max[y_{t+h}^*, x_1 + x_2]) dF_1(z_1) - c_1(\alpha_1)] \\ &\quad + h \max_{(\alpha_2, \beta_2) \geq 0} [\alpha_2 \int_{\beta_2}^{\infty} (z_2 + x_1 - \max[y_{t+h}^*, x_1 + x_2]) dF_2(z_2) - c_2(\alpha_2)] \end{aligned}$$

where  $(\alpha_1^*, \beta_1^*)$  is the solution to the maximization problem defined by the second to last term the right side of (6) and  $(\alpha_2^*, \beta_2^*)$  is the solution to the problem defined by the last term. In other words, the cooperative search strategy for agent  $i$  maximizes the difference between the total return and cost attributable to the search of agent  $i$ .

The previously stated assumptions imply that the cooperative search strategies are unique. Obviously, they are the solutions to the following first order conditions.

$$c_i^*(\alpha_{it}^*) = \int_{\beta_{it}^*}^{\infty} [z_i - \beta_{it}^*] dF_i(z_i), \quad i = 1, 2, \quad (7)$$

$$\beta_{1t}^* + x_2 = \max[y_{t+h}^*, x_1 + x_2] \quad (8.a)$$

$$x_1 + \beta_{2t}^* = \max[y_{t+h}^*, x_1 + x_2]. \quad (8.b)$$

The essential nature of the cooperative solution is reflected in (4) and (8). First, the match terminates at tenure date  $t+h$  with certainty if the total capital value of continuing beyond that date  $y_{t+h}^*$  is less than the capital value of holding the job vacant plus the capital value of being unemployed. Second, if  $y_{t+h}^* \geq x_1 + x_2$ , then an alternative generated by agent  $i$ ,  $z_{it}$ , is acceptable if and only if

$$z_{it} - x_i \geq \beta_{it}^* - x_i = y_{t+h}^* - x_1 - x_2.$$

The rent gained by agent  $i$  were he to accept, the difference  $z_{it} - x_i$ , must be at least as large as the sum of the rents given up by both agents, the difference  $y_{t+h}^* - x_1 - x_2$ . Finally, in each case the cooperative search intensity equates the marginal cost and marginal return attribute to searching more intensely.

Obviously, if the match existed during the interval  $[t, t+h)$  it must have been true that  $y_t^* \geq x_1 + x_2$  since (4) holds for all tenure dates. Note that the last two terms on the right side of (6) are non-negative given  $c_1(0) = c_2(0) = 0$ . Consequently,

$$v_t \geq r(x_1 + x_2) \Rightarrow y_t^* \geq x_1 + x_2. \quad (9)$$

In other words, future rents accrue to a match whenever the value of the product exceeds the opportunity cost of the match, the imputed interest on the capitalized values to both agents of not being matched. Of course, the hypothesis is not necessary for the conclusion of (9) to hold. Specifically, if  $v_t$  increases with tenure as a consequence of specific skill accumulation, then rents may exist

early in the tenure of the match even though the current value of the product is less than the current opportunity cost flow. Such a period is one in which the match is investing in specific human capital.

Obviously, equations (1), (2), (7) and (8) imply that both turnover rates depend on the capital value of the match,  $y^*$ , but not on its division between the two parties. Specifically, an increase in the expected present value of both agents future incomes given continuation of the match reduces the reservation values of both agents by virtue of (8). Because an acceptable offer is less likely given a higher reservation value, the marginal return to search, the right sides of (7), falls as  $y^*$  increases. The search intensities fall in response to such an increase given convex search costs. Finally, because both reservation values increase and both search intensities decrease, the dismissal rate falls by virtue of (1) and the quit rate falls by virtue of (2). Later we investigate the empirical implications of the hypothesis that both employer and worker pursue their cooperative strategies in more detail.

#### 4. INCENTIVES AND THE WAGE CONTRACT

A wage contract is an agreement between the employer and worker involved in a match that allocates the value of their joint product between them. That the nature of the contract will have an effect on the search behavior of both parties is a relatively obvious point. However, the contract itself is endogenously determined by negotiation. In negotiating, it seems reasonable that each party will take account of the effect of proposed contracts on the search behavior of the other. Since each party prefers that the other pursue his cooperative strategy, a plausible solution to the bilateral bargain is a contract that motivates both to search in a manner that maximizes their joint welfare.

In this section we attempt to formalize these ideas as follows. Assume that each party pursues his own Nash non-cooperative search strategy given the wage contract. In this case any coordination of search activities must be motivated by the form of the contract. Call the solution to the bargaining proposed above an equilibrium contract. A wage contract is an equilibrium then, if and only if the Nash strategy of each party is also his cooperative strategy.<sup>8</sup> In this section we consider a particular wage contract that is not an equilibrium in this sense and we construct another that is an equilibrium contract.

A feasible wage contract is a sequence of functions  $\{\omega_t(\cdot)\}$ . The element  $\omega_t(\cdot)$  determines the net transfer from the employer to worker as of the end of interval  $[t, t+h)$ . In general, its domain is the information available to both parties at the end of the interval. Potentially, then, the domain includes the outcome of the joint search process to date and the search strategies of both parties in addition to the worker and job types matched.

Consider first a simple contract that merely specifies the wage payment at each tenure date. In this case,

$$\omega_t(\cdot) = hw_t \tag{10}$$

where  $w_t$  is the wage rate during the interval  $[t, t+h)$ , a prespecified number independent of the search outcome realized during the interval. Let  $y_{1t}$  and  $y_{2t}$  respectively represent the employer's and worker's expected present values of future income if the match were to continue. In the case of agent  $i$ , his Nash strategy is a rule that sets current and future choices of his search intensity and reservation value in a manner that maximizes  $y_{it}$  given (10) and the other agent's strategy. Let  $(\alpha_{it}^\circ, \beta_{1t}^\circ)$  denote this choice for the interval  $[t, t+h)$  and let  $(y_{1t}^\circ, y_{2t}^\circ)$  represent the capital values of continuing the match given that both employer and worker pursue Nash strategies.

The maximal end of interval capitalized value of agent  $i$ 's future income, given the other's strategy, in  $\max[y_{it+h}^\circ, z_{it}, x_i]$  if he locates an alternative during the interval; i.e.,  $n_t = i$ . The alternative found,  $z_{it}$ , is acceptable if and only if it is at least as large as the other two numbers. Consequently, the Nash reservation value,  $\beta_{it}^\circ$ , is  $\max[y_{it+h}^\circ, x_i]$ . Given that it is agent  $i$  who locates an alternative, then agent  $j$  obtains an expected future income stream equal in capitalized value to  $\max[y_{jt+h}^\circ, x_j]$  only if agent  $i$  does not terminate the match. If agent  $i$  does separate, then agent  $j$  obtains  $x_j$  only, because continuation is not an option. Consequently, Bellman's principle and the definition of a Nash non-cooperative strategy imply

$$\begin{aligned} (1+rh)y_{1t}^\circ = & \max_{\alpha_1 \geq 0} [h[v_t - w_t - c_1(\alpha_1)] \\ & + h\alpha_{1t} \int \max[y_{1t+h}^\circ, z_{1t}] dF_1(z_1) \\ & + h\alpha_{2t} [F_2(y_{2t+h}^\circ) y_{1t+h}^\circ + (1 - F_2(y_{2t+h}^\circ)) x_1] \\ & + (1 - h\alpha_{1t} - h\alpha_{2t}) y_{1t+h}^\circ] \end{aligned}$$

and analogously

$$\begin{aligned} (1+rh)y_{2t}^\circ = & \max_{\alpha_2 \geq 0} [h[w_t - c_2(\alpha_2)] \\ & + h\alpha_{1t} [F_1(y_{1t+h}^\circ) y_{2t+h}^\circ + (1 - F_1(y_{1t+h}^\circ)) x_2] \\ & + h\alpha_{2t} \int \max[y_{2t+h}^\circ, z_{2t}] dF_2(z_2) \\ & + (1 - h\alpha_{1t} - h\alpha_{2t}) y_{2t+h}^\circ] \end{aligned}$$

if  $(y_{1t+h}^\circ, y_{2t+h}^\circ) \geq (x_1, x_2)$ . Because the match terminates with certainty if



$y_{it+h}^\circ < x_i$  for either agent, the same conditions hold but with  $x_i$  replacing  $y_{it+h}^\circ$  for both values of  $i$  in this case. Given that the reservation value for agent  $i$  is the smallest  $z_i$  such that  $z_i = \max[y_i^\circ, z_i]$ , the conditions above can be written in the following equivalent but more convenient form:

$$(1+rh)y_{1t}^\circ = h(v_t - w_t) + (1 - p_{2t}^\circ)y_{1t+h}^\circ + hp_{2t}^\circ x_1 \quad (11.a)$$

$$+ h \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 - y_{1t+h}^\circ] dF_1(z_1) - c_1(\alpha_1) \right].$$

$$(1+rh)y_{2t}^\circ = hw_t + (1 - hp_{1t}^\circ)y_{2t+h}^\circ + hp_{1t}^\circ x_2 \quad (11.b)$$

$$+ h \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\infty} [z_2 - y_{2t+h}^\circ] dF_2(z_2) - c_2(\alpha_2) \right].$$

Of course,  $p_{1t}^\circ$  and  $p_{2t}^\circ$  are the two turnover rates associated with the Nash strategies as defined by (1) and (2) respectively.

Obviously, (11) implies

$$c_i'(\alpha_{it}^\circ) = \int_{\beta_{it}^\circ}^{\infty} [z_i - \beta_{it}^\circ] dF_i(z_i), \quad i = 1, 2, \quad (12)$$

and

$$\beta_{it}^\circ = y_{it+h}^\circ, \quad (13)$$

if  $(y_{it+h}^\circ, y_{it+h}^\circ) \geq (x_1, x_2)$ . Further more, since  $x_i$  replaces  $y_{it+h}^\circ$

everywhere if  $y_{it+h}^\circ < x_i$  for either value of  $i$ ,

$$v_t - rx_1 \geq w_t \geq rx_2 \Rightarrow (y_{1t}^\circ, y_{2t}^\circ) \geq (x_1, x_2) \quad (14)$$

in general.

The search intensity is set to equate its marginal cost and return

by virtue of (12). Any alternative that yields an expected future income stream to agent  $i$  that exceeds the capitalized value of that associated with the match is acceptable by virtue of (13). Finally, if the value of the product exceeds the opportunity cost of the match, then a wage rate exists that guarantees rents to both parties by virtue of (14). Presumably the wage agreed to will be one of these. If not, the match terminates.

Given Nash strategies relative to the wage contract (10), a marginal change in the wage rate does affect the turnover rates. Specifically, an increase in any future wage rate increases  $y_{2t+h}^{\circ}$  but reduces  $y_{1t+h}^{\circ}$  since (11.a) and (11.b) hold for all  $t$ . Therefore, (12), (13), (1) and (2) imply that an increase in future wage rates increase the current dismissal rate and decrease the current quit rate. The fact that it is only future wage rates that affect current turnover is not fully appreciated in the literature.

A comparison of (8) and (13) reveal another important difference between the strategies that are Nash relative to (10) and the cooperative strategies.

Since  $y_{1t+h}^{\circ} + y_{2t+h}^{\circ} \leq y_{t+h}^*$  by definition of the cooperative solution,

$$\beta_{1t}^* = y_{t+h}^* - x_2 \geq y_{1t+h}^{\circ} + y_{2t+h}^{\circ} - x_2 \geq y_{1t+h}^{\circ} = \beta_{1t}^{\circ}$$

and

$$\beta_{2t}^* = y_{t+h}^* - x_1 \geq y_{1t+h}^{\circ} + y_{2t+h}^{\circ} - x_1 \geq y_{2t+h}^{\circ} = \beta_{2t}^{\circ}$$

given  $(y_{1t+h}^{\circ}, y_{2t+h}^{\circ}) \geq (x_1, x_2)$ , with strict equality holding if rents accrue to both parties. This fact, (7) and (12) imply that  $\alpha_{it}^* \leq \alpha_{it}^{\circ}$ ,  $i = 1, 2$ .

Consequently, both the dismissal rate and the quit rate given the Nash strategies, what ever the wage, are larger than those associated with the cooperative strategies.

This inefficiency arises because neither party takes account of the loss that the other suffers when the first terminates the match. The employer's is equal to  $y_{1t+h}^{\circ} - x_1$  when the worker quits. The worker loses  $y_{2t+h}^{\circ} - x_2$  when dismissed. Given the Nash reservation values, the match terminates in situations where one party's net gain  $z_{it} - y_{it+h}^{\circ}$  is less than the other's net loss,  $y_{jt+h}^{\circ} - x_j$ .

The externality can be internalized by a wage contract that requires each party to compensate the other as a precondition to initiating a separation. Specifically, the following contract, a wage plus contingent terminal side payment, is an equilibrium in the sense defined above:

$$\omega_t(\cdot) = hw_t + \begin{cases} \max[0, y_{2t+h} - x_1] & \text{if } z_{1t} \geq \beta_{1t} \\ -\max[0, y_{1t+h} - x_2] & \text{if } z_{2t} \geq \beta_{2t} \end{cases} \quad (15)$$

Because of the required compensation, an offer is acceptable to the employer if and only if its expected present value plus the compensation exceeds his share of the matches' capital value. Analogously, a worker quits to accept an alternative if and only if  $z_{2t} + \max[0, y_{1t+h} - x_2] \geq \max[y_{2t+h}, x_2]$ . Consequently, the Nash reservation values relative to (15) are  $\beta_{1t}^*$  and  $\beta_{2t}^*$  if both pursue cooperative strategies in the future. Finally, given those reservation values, the Nash search intensities are also cooperative because the private marginal return to search is the right side of (7) in each case. The fact that  $y_{1t} = y_{2t} = 0$  and  $y_t^* = 0$  at the horizon date T completes the equivalence proof by induction.

Because agent j obtains  $\max[y_{jt+h}^*, x_2]$  given (15) whether agent i finds

an alternative which is acceptable or not, the two agents' absolute shares of the maximal expected present value of continuing the match are determined by

$$(1+rh)y_{1t}^* = h(v_t - w_t) + y_{1t+h}^* + h \max_{(\alpha_1, \beta_1)} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 - \beta_{1t}^*] dF_1(z_1) - c_1(\alpha_1) \right] \quad (16.a)$$

and

$$(1+rh)y_{2t}^* = hv_t + y_{2t+h}^* + h \max_{(\alpha_2, \beta_2)} \left[ \alpha_2 \int_{\beta_2}^{\infty} [z_2 - \beta_{2t}^*] dF_2(z_2) - c_2(\alpha_2) \right] \quad (16.b)$$

if  $(y_{1t+h}^*, y_{2t+h}^*) \geq (x_1, x_2)$ . Therefore, so long as the wage at each tenure date is chosen to guarantee that  $(y_{1t}^*, y_{2t}^*) \geq (x_1, x_2)$  its choice only serves to divide the rents between employer and worker. Any marginal change in the wage has no effect on turnover behavior.

The practice of granting severance pay to a dismissed worker is precisely the type of terminal side payment specified in (15). Other restrictions on the employer's freedom to initiate a termination are consistent with the general framework. Arrangements that require direct compensation of the employer before quitting are not common, however. In part, the lack of such compensation reflects the fact that contracts that give the employer a property right to human capital are not legally enforceable. However, paid vacations and non-vested pension plans, payments made contingent on specified periods of previous employment, serve as means of raising the cost of quitting to the worker. All of these practices are equivalent to paying some portion of the worker's wage into a contingent escrow account. That sum then serves at least to partially compensate the employer if the worker does quit.

## 5. EMPIRICAL TURNOVER BEHAVIOR

To what extent does the hypothesis that the employer and worker pursue cooperative search strategies explain econometric relationships between turnover rates and other variables? In this section we attempt an answer by deriving the implications of the hypothesis and by comparing these implications with existing empirical results. The studies included for this purpose include those reviewed by Parsons [1977]. In addition, two recent studies that analyze cross section data on individuals, Viscusi [1976] and Bartel and Borjces [1976], are of particular interest. Below we consider in sequence a number of explanatory variables that appear to be important in one or more of these studies.

Tenure: That both the quit and the layoff rate are negatively associated with duration on the job is well established in the literature. The accumulation of skills specific to the job is a natural explanation although the "job shopping" hypothesis is also a contender.<sup>9</sup> To obtain a formal derivation of the relationship, start by assuming that the value of the product of a match,  $v(t)$ , increases the  $t$  but is bounded; i.e.,  $v'(t) > 0$  and  $v(t) \rightarrow v(\infty) < \infty$  as  $t \rightarrow \infty$ . The typical "learning curve" satisfies both conditions. Given "diminishing returns", the assumption is also consistent with an investment in training formulation.

In the simplest case workers are ageless: i.e., the potential tenure of every match is infinite. In this case, the maximal capitalized sum of the employer's and worker's future income given contribution of their match beyond tenure date  $t$ ,  $y^*(t)$ , is in continuous time a solution to the following differential equation analogue to equation (6):

$$\frac{dy(t)}{dt} = \dot{y}(t, y) = ry - v(t) \quad (17)$$

$$- \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 + x_2 - y] dF_1(z_1) - c_1(\alpha_1) \right]$$

$$- \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\infty} [z_2 + x_1 - y] dF_2(z_2) - c_2(\alpha_2) \right]$$

in the region  $y \geq x_1 + x_2$ . Equation (17) is obtained by dividing both sides of (6) by  $h$  and then taking limits appropriately as  $h$  tends to zero.

The solutions to (17) are graphically represented as trajectories in Figure 1. The properties of these trajectories are derived as follows. Let  $f(t)$  represent the value of  $y$  satisfying  $y'(t) = 0$  at each  $t$ . In other words,  $f(t)$  is implicitly defined as follows

$$rf(t) = v(t) + \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 + x_2 - f(t)] dF_1(z_1) - c_1(\alpha_1) \right]$$

$$+ \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\infty} [z_2 + x_1 - f(t)] dF_2(z_2) - c_2(\alpha_2) \right] .$$

That  $f(t)$  exists for all  $t$  is easily established. In addition,  $f(t)$  is bounded if  $v(t)$  is bounded; let  $f(\infty)$  denote the bound. By virtue of the envelope theorem and equations (1) and (2), an implicit differentiation yields

$$f'(t) = \frac{v'(t)}{r + p_1(t) + p_2(t)} > 0$$

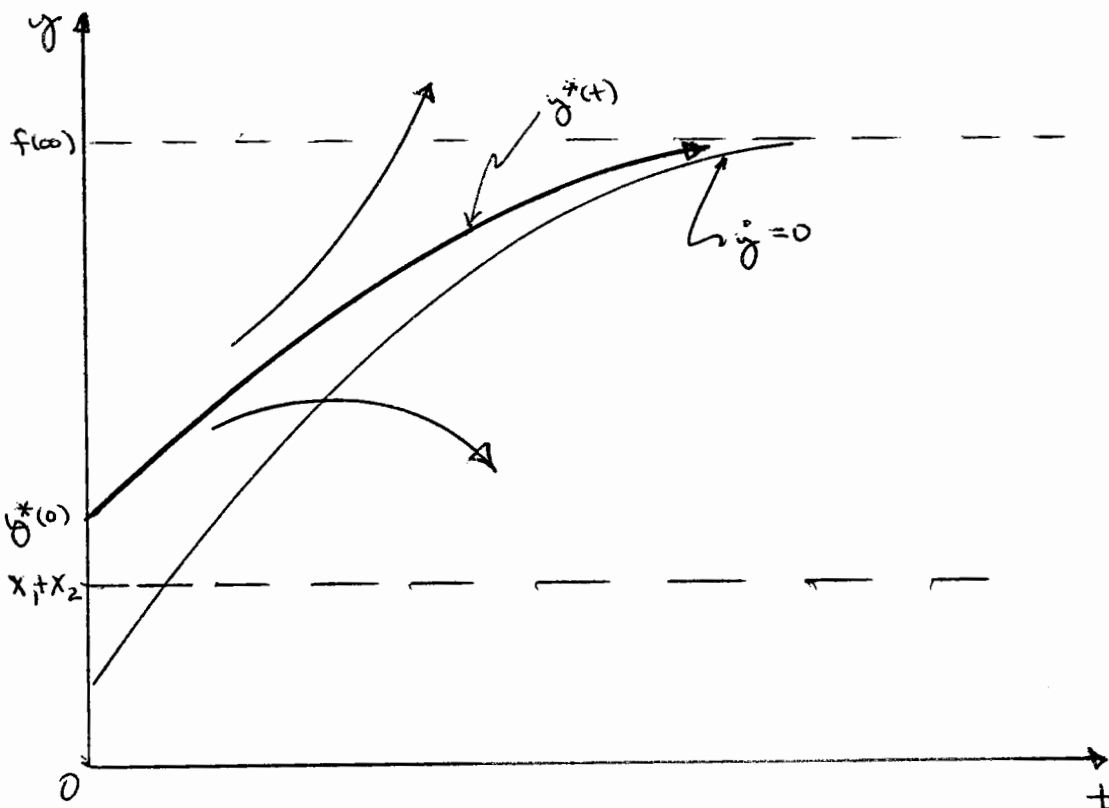
where  $p_i(t)$  is the rate at which agent  $i$  initiates turnover associated with the optimal choices of  $\alpha_i(t)$  and  $\beta_i(t)$ . Finally, (17) and the envelope theorem also imply

$$\frac{\partial \dot{y}(t)}{\partial y} = r + p_1(t) + p_2(t) > 0.$$

Consequently,  $y'(t) \geq 0$  as  $y(t) \geq f(t)$  for all  $t$ .

Figure 1

Specific Capital and Tenure: Infinite Life



As suggested in Figure 1, only a single trajectory limits to  $f(\infty)$  as  $t \rightarrow \infty$  and it lies everywhere in the region  $y'(t) > 0$ . All other solutions either violate the transversality conditions,  $\lim_{t \rightarrow \infty} e^{-rt} y(t) = 0$ , or are dominated by the converging solution in the sense that each yields a lower capital value of the match at all tenure dates. Therefore,  $y^*(t)$  is the converging solution as labeled in Figure 1. Equations (1), (2), (7) and (8) imply that both turnover rates decline with tenure since  $dy^*(t)/dt > 0$ .

Age: The most important effects of aging in a human capital model arises from the fact that the length of future streams of income to retirement shorten as a worker's age increases. Because a worker's age and the same worker's tenure on a specific job are perfectly correlated, we must relax the assumption that the worker's distribution of alternative capital values is independent of tenure in order to incorporate this effect. Specifically, let  $T$  denote the length of the period to the worker's retirement date as of the beginning of his tenure on his current job. Then,

$$s = T - t \tag{18}$$

is the length of his remaining working life at tenure date  $t$ .

Let  $F_2(z_2, s)$  denote the distribution of alternative expected capitalized future income streams available to the worker given a remaining working life of length  $s$ . Because the present value of every stream decreases as the remaining working life shortens and because all tend to zero as the retirement date is approached, this family of distribution functions is such that both the mean and dispersion about the mean increase with  $s$ , at least for small values of  $s$ . In other words, in the case of the mean,

$$E\{z_2|s\} = \int_0^{\infty} z_2 dF_2(z_2, s) ,$$
$$\frac{\partial E\{z_2|s\}}{\partial s} > 0. \tag{19.a}$$

The dispersion of alternatives about the mean increases with  $s$  in the "mean preserving sense" of Rothschild and Stiglitz [1970] if and only if

$$\int_0^x \frac{\partial F_2(z_2, s)}{\partial s} dz_2 \geq 0 \text{ for all } 0 < x < \infty . \tag{19.b}$$



with equality holding at one  $x$  at least. Because of discounting, the magnitude of both effects are virtually zero for large values of  $s$ . The magnitudes are significant, however, when  $s$  is small since

$$E\{z_2|s\} = 0 \quad \text{with certainty when } s = 0.$$

The conditions of (19) form the hypothesis for the results reported in this section.

Obviously, the length of the period to the worker's retirement at the commencement of the match,  $T$ , is the maximum tenure that the match can attain. In other words, the problem of computing the cooperative strategies has a finite horizon equal to  $T$ . As a consequence, the maximal sum of the worker's and employer's future income at tenure date  $t$  depends on the length of the horizon. Let  $y_t^*(T)$  denote this number. Because the match terminates with certainty at tenure date  $T$  and because a retiring worker has no incentive to search for a new job in the final interval, the analogue to condition (6) for the interval  $[T-h, t)$  is

$$(1+rh)y_{T-h}^*(T) = hv_{T-h} + x_1 + x_2 + \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 \beta_1 t^*] dF_1 - c_1(\alpha_1) \right].$$

By taking the limits of both sides as  $h \rightarrow 0$ , we obtain

$$y^*(t, T) = \lim_{t \rightarrow T} y_{T-h}^*(T) = x_1 + x_2 \tag{20}$$

as the right hand end point condition for the optimization problem when viewed in continuous time. Since (6) holds for all  $t < T-h$  provided that  $F_2(z)$  is replaced by  $F_2(z, s)$  to account for the aging of the worker,  $y^*(t, T)$  is a solution to the following partial differential equation analogue.

$$\frac{\partial y}{\partial t} = \dot{y}(t, T, y) = ry - v(t) \quad (21)$$

$$\begin{aligned} & \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\omega} [z + x_2 - y] dF_1(z) - c_1(\alpha_1) \right] \\ & \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\omega} [z + x_1 - y] dF_2(z, T-t) - c_2(\alpha_2) \right] \end{aligned}$$

in the regime  $y \geq x_1 + x_2$ . Of course, the quit rate is now given by

$$p_2 = \alpha_2 [1 - F_2(\beta_2, T-t)] \quad (22)$$

The solutions to (21) for a given value of  $T$  are illustrated in Figure 2.

The locus of points  $\dot{y} = 0$  is the curve  $y = f(t, T)$  where from (21)

$$\begin{aligned} rf(t, T) = v(t) + & \max_{(\alpha_1, \beta_1) \geq 0} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z_1 + x_2 - f(t, T)] dF_1(z_1) - c_1(\alpha_1) \right] \\ & + \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\infty} [z_2 + x_1 - f(t, T)] dF_2(z_2, T-t) - c_2(\alpha_2) \right]. \end{aligned} \quad (23)$$

Let  $(\alpha_2^\circ, \beta_2^\circ)$  denote the solution to the maximization problem defined in the last term on the right side of (23). Since  $\beta_2^\circ(t) = f(t, T) - x_1$ , the marginal return to the worker's search can be written as follows:

$$\begin{aligned} \int_{\beta_2^\circ(t)}^{\infty} [z_2 + x_1 - f(t)] dF_2(\cdot) &= \int_{\beta_2^\circ(t)}^{\infty} [z_2 - \beta_2^\circ(t)] dF_2(\cdot) \\ &= \int_0^{\infty} [z_2 - \beta_2^\circ(t)] dF_2(\cdot) + \int_0^{\beta_2^\circ(t)} [\beta_2^\circ(t) - z_2] dF_2(\cdot) \\ &= E\{z_2, T-t\} - \beta_2^\circ(t) + \int_0^{\beta_2^\circ(t)} F_2(z_2, T-t) dz_2 \\ &= E\{z_2, T-t\} + x_1 - f(t, T) + \int_0^{f(t, T) - x_1} F_2(z_2, T-t) dz_2. \end{aligned}$$

Consequently, an implicit differentiation of (23) yields

$$\frac{\partial f(t, T)}{\partial t} = \frac{1}{r+p_1(t)+p_2(t)} \left[ v'(t) - \alpha_2^{\circ}(t) \left( \frac{\partial E\{z_2, s\}}{\partial s} + \int_0^{\beta_2^{\circ}(t)} \frac{\partial F_2}{\partial s} dz_2 \right) \right]$$

by virtue of the envelope theorem and equations (18) and (22).

The two terms in the square brackets respectively reflect the two tenure related effects. The first  $v'(t)$  is positive as a consequence of skill accumulation. The second is negative from (19) because a worker's age and tenure are perfectly and positively correlated on a specific job. In the case of a worker who commences his current job while young, one expects the first effect to dominate for small values of  $t$  because with discounting the reduction of his future working life given a small increase in tenure has little consequence. However, when  $t$  is large the second effect dominates because the skill accumulation process is over. This "typical" case is represented in Figure 2. Of course, if  $T$  is small, the worker was old when the match commenced, either effect can dominate throughout the worker's potential tenure. Since (21) and the envelope theorem imply

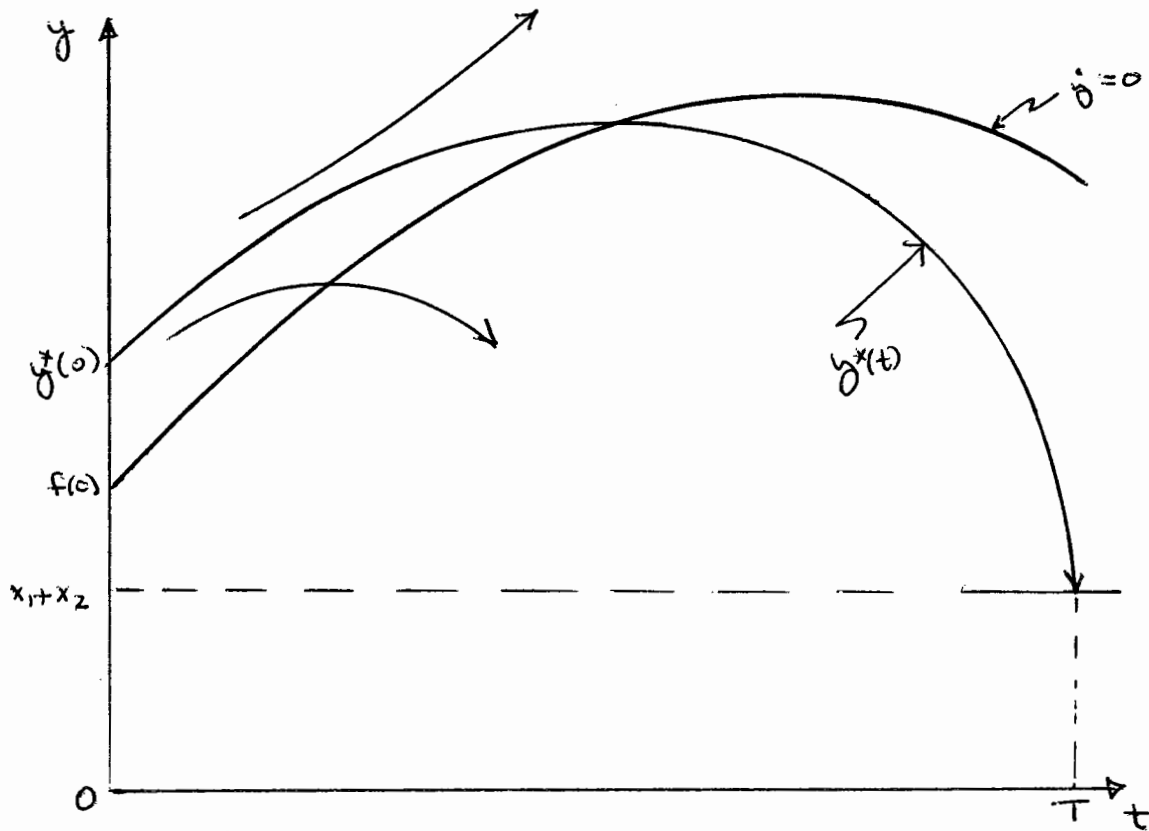
$$\frac{\partial \dot{y}}{\partial y} = r + p_1(t) + p_2(t) > 0, \tag{24}$$

we know that  $\dot{y} \gtrless 0$  as  $y \gtrless f(t)$ . The right hand end point condition, (20), identifies the solution of interest as the trajectory labeled  $y^*(t, T)$ . In the case illustrated, the capital value of the match increases with tenure at the beginning of the match as a consequence of specific training and then declines later as the worker's retirement date is approached. Of course,  $y^*(0, T) \geq x_1 + x_2$  was necessary for the formation of the match and was sufficient as well if the worker were unemployed and the job vacant. As we have

shown  $v(0) > r(x_1, + x_2)$  implies  $y^*(0,T) > x_1 + x_2$ . The same condition also guarantees that  $f(0) > x_1 + x_2$  by virtue of (23). Consequently,  $f(0) > y^*(0,T) > x_1 + x_2$  is a possibility. In this case,  $y^*(t,T)$  declines with tenure throughout the duration of the match.

Figure 2

Specific Capital and Tenure: Finite Worker Life



One lesson to be learned from Figure 2 is that the effect of tenure on turnover is ambiguous in general if one fails to control for age and vice versa. However, by controlling for specific skills with tenure, unambiguous age effects are predicted by the model. All follow from the fact that the total specific capital value of continuing the match decreases with age holding tenure constant. Formally

$$\frac{\partial y^*}{\partial T} > 0, \quad \forall t \leq T .$$

In other words, the capital value of a match involving the same job but a younger worker is larger than that involving an identical older worker with the same tenure.

A proof of (25) follows. Consider two different matches involving the same job type. Suppose that the two workers are of the same type, that the tenure of the two matches are identical, but that one worker is slightly younger than the other. Specifically, at the formation dates,  $T$  is the remaining working life length of the older and  $T + h > T$  is the remaining working life length of the younger. We know that  $y^*(T, T+h) > y^*(T+h, T+h) = y^*(T, T) = x_1 + x_2$  from (20) and the fact that the match involving the younger worker would terminate with certainty at tenure date  $T$  or before otherwise. As a consequence,

$$\frac{\partial y^*(T, T)}{\partial T} = \lim_{h \rightarrow 0} \left[ \frac{y^*(T, T+h) - y^*(T, T)}{h} \right] > 0 ;$$

i.e., (25) holds for  $t = T$ . Suppose contrary to (25), that a  $\hat{t} < T$  exists such that  $\hat{y} = y^*(\hat{t}, T) = y^*(\hat{t}, T+h)$ . The fact that (25) holds at  $t = T$  implies that  $y^*(t, T+h)$  intersects  $y^*(t, T)$  from below for at least one value of  $\hat{t}$  that nearest  $T$ . Formally,

$$\frac{\partial \hat{y}^*(\hat{t}, T, \hat{y})}{\partial T} = \lim_{h \rightarrow 0} \left[ \frac{\hat{y}^*(\hat{t}, T+h, \hat{y}) - \hat{y}^*(\hat{t}, T, \hat{y})}{h} \right] \geq 0 .$$

This result contradicts the following implication of (21) and (19):

$$\frac{\partial \dot{y}(t, T, y)}{\partial T} = -\alpha_2^\circ(t) \left[ \frac{\partial E\{z_2 | s\}}{\partial s} + \int_0^{\beta_2^\circ(t)} \frac{\partial F_2(\cdot)}{\partial s} \right] < 0$$

for all  $t$  and  $y$ .

Because the alternatives available to the employer are independent of the worker's age, (25) immediately implies that an older worker's suffers a

larger dismissal rate. Specifically, the employer finds a preferred alternative more easily and, because the return to search is longer, searches for one more intensely. Of course, if cooperative search strategies are pursued because terminal compensation is required, the difference between the dismissal rate of the older and younger worker can be interpreted as an early but voluntary retirement rate.

The effect of age on the quit rate is ambiguous in general because the capital values of all the worker's alternatives also decline with age. This fact is clear where one inspects the conditions defining the worker's cooperative strategy,

$$\beta_2^* = y^* - x_1 \quad (26.a)$$

and

$$c'_2(\alpha^*) = \int_{\beta_2^*}^{\omega} [z_2 - \beta_2^*] dF_2(z_2, T-t) = E\{z_2 | T-t\} + \int_0^{\beta_2^*} F_2(z_2, T-t) dz_2 - \beta_2^*. \quad (26.b)$$

Although  $\beta_2^*$  increase with T by virtue of (25) and (26.a), the first term on the right side of (26.a) also increases with T given  $\beta_2^*$  from (19.a) and (19.b). Therefore the affect of age on the search intensity is ambiguous. However, because the capital value of all alternative jobs tend to zero as the retirement date is approached but  $\beta_2^* \geq x_2 > 0$  for all  $t \leq T$ , a tenure date  $\hat{t}$  exists at which the right side of (26.b), the marginal return to search, is zero. Because they don't search, very old workers don't quit to take another job.

Although it is generally accepted that younger workers quit more frequently, the age coefficients in many empirical studies is biased upward due to the positive correlation between age and tenure in any sample. Parsons [1972] study which does include "rough" measures of age and tenure reports a statistically significant negative age effects across industries. Viscusi [1976] confirms

these results across individuals with data from both the National Longitudinal Survey and the Michigan Survey of Working Conditions. Interestingly, his regression estimates of the age effect is larger in absolute value when tenure is not included in all cases. Of course, these results are quite consistent with the hypothesis under study. Moreover, Parsons [1972] in the same study finds that "layoff" rates across industries increase with the proportion of workers over 55 years of age, a result which is predicted by the hypothesis.

### Tenure Reconsidered

In all studies known to the author both worker and employer initiated turnover rates decline with tenure. In the case of the National Longitudinal Survey of older men, results reported by Hall [1972] and by Bartel and Borjas [1976] suggest that it may be the single most important explanatory variable in both cases. Viscusi [1976] confirms these results in the case of quit rates even when age is appropriately controlled for although his results suggest that tenure is somewhat less significant in the case of younger men. For these reasons, showing that the infinite life results generalize in the finite life case is crucial.

For this purpose substitute  $T-s$  for  $t$  and define the new variable

$$q(s,T) \equiv y(T-s, T). \tag{27}$$

Since  $s = T-t$  is the remaining length of a workers life at tenure date  $t$ , an increase in  $T$  given  $s$  is an increase in tenure holding age constant. In terms of the new variables (20) is

$$q^*(0,T) = y^*(T,T) = x_1 + x_2.$$

Let  $\dot{q}$  denote the derivative of  $q$  with respect to  $s$  for given  $T$ . Since  $\dot{q} = -\dot{y}$ .

(21) can be rewritten as

$$\dot{q}(s, T, q) = -v(T-s) + \max_{(\alpha_1, \beta_1)} \left[ \alpha_1 \int_{\beta_1}^{\infty} [z + x_2 - q] dF_1(z) - c_1(\alpha_1) \right] \\ + \max_{(\alpha_2, \beta_2) \geq 0} \left[ \alpha_2 \int_{\beta_2}^{\omega} [z + x_2 - q] dF_2(z, s) - c_2(\alpha_2) \right] - rq.$$

The fact that  $q(s, T) = x_1 + x_2$  for all  $T$  and

$$\frac{\partial \dot{q}(s, T, q)}{\partial T} = v'(T-s) > 0$$

together imply the desired result. Namely,

$$\frac{dy^*}{dt} \Big|_s \equiv \dot{y}^*(t, T) + \frac{dy^*(t, T)}{\partial T} \equiv \frac{\partial q^*(s, T)}{\partial T} > 0. \quad (28)$$

Market Conditions: The fact that quit rates move procyclically and layoff rates move countercyclically has long been noted. A search theoretic explanation, common in the literature, emphasizes the effect of changes in the availability of alternatives to the employer and worker over the business cycle. In our model, these effects can be derived by assuming that the cost of search falls at the margin when the availability of alternatives increases. Specifically,

$\frac{\partial c_i'(\alpha_i)}{\partial u_i} < 0$  where  $u_i$  is the unemployment rate in the employer's case,  $i = 1$ , and the vacancy rate is the workers,  $i = 2$ . Then, since

$$c_i(\alpha_i^*) = \int_{\beta_i^*}^{\beta_i^*} [z - \beta_i^*] dF_i(z),$$

$$\frac{\partial \alpha_i^*}{\partial u_i} = - \frac{1}{c''} \frac{\partial c_i'}{\partial u_i} > 0. \quad (29)$$

In other words, the employer and worker respectively search more intensely when the unemployment rate and vacancy rate is higher. Since the unemployment and vacancy rates are negatively correlated over time so are the two turnover rates.



Most authors use the inverse of the unemployment rate in quit rate studies rather than the vacancy rate. Typically the justification is that the existence of unemployment discourages quits. However, Parson [1972] reports that even an imperfect vacancy measure provides a better explanation of the procyclic movement in quit rates. He also finds that the unemployment rate is not dominated by the inverse of the vacancy rate as an explanation of the countercyclic movement in layoff rates. These results support the availability hypothesis.

The observant reader has noticed that the derivations of (29) ignores a second effect of an increase in the availability of alternatives. Since  $c_i(0) = 0$ , an increase in availability of alternatives to agent  $i$  lowers his total cost of search at a given intensity if it lowers marginal cost. One can show, as a consequence, that the capital value of continuing the match increases with availability.<sup>12</sup> This effect tends to reduce the incentive to search by lowering the marginal return. However, the size of this effect is small if the change in the measure of availability is temporary. Consequently, the analysis presented above is the appropriate one when the interest is in the effect of temporary fluctuations in the unemployment rate and vacancy rate about their "natural" levels.

In time series studies, turnover rates are typically industry averages rather than propensities for groups of workers with known characteristics. Consequently, an alternative explanation for the observed behavior over the cycle exists based on the systematic changes in the tenure distribution of the employed work force. Because the quit propensity of an individual worker is inversely related to his tenure, the new workers hired in an unswing raise the average

quit rate of those employed. The fact that a worker's probability of being dismissed is also inversely related to tenure implies that the quit rate will fall in the downswing as the most recently hired workers are let go.<sup>13</sup> In a recent paper, Brechling [1977] reports that the inclusion of a recently hired work force variable has a significantly positive sign in quit equations estimated with industry times series data as this argument would predict. Further empirical analysis based on a pooled cross section-time series data base is needed to distinguish between and/or to determine the relative importance of the "availability" and "aggregation" hypotheses.

Product Demand: In the preceeding analysis, the derived demand for labor is taken as given. Consequently, any employer initiated separation is part of an act of replacing an existing employer with a new one. Although separations of this type continually take place, they are conceptually different from layoffs, the bulk of employer initiated turnover.

For our purpose a layoff is a separation resulting in unemployment attributable to a reduction in the demand for the product of the match. A satisfactory general explanation for layoffs is an unsolved theoretical problem of long standing. Central to the problem is the question, why do workers accept unemployment rather than reduction in wages when product demand falls? We do not attempt an answer here.<sup>14</sup> Instead we show that a worker can be laid off in our model even when willing to accept a reduction in compensation. Moreover, the theory's prediction concerning who is most likely to be let go is born out by the evidence.

Suppose that the value of the product of a job-worker match falls from  $v(t)$  to  $v_0(t)$ ; i.e.,  $v(t) > v_0(t)$  for all  $t$ . As a consequence, the capital value of continuing the match falls from  $y^*(t, T)$  to  $y_0^*(t, T)$ . There are three possibilities.

In the first case,  $y_o^*(t,T)$  still exceeds  $x_1 + x_2$ , the sum of the capital value associated with holding the job vacant and being unemployed, for all  $t < T$ .

In this case a future wage stream exists that will insure the continuation of the match independent of its tenure and the worker's age. Of course, that wage stream may be smaller than the original. Nevertheless, it is in the interest of both the employer and worker to renegotiate. The hypothesis that cooperative strategies are pursued presumes that the appropriate renegotiation will take place. In the second case,  $y_o^*(t,T) < x_1 + x_2$  for all  $t < T$ . Because in this case there exists no wage that can divide the total capital value of the match between worker and employer so that they both prefer to continue, the match terminates.<sup>15</sup>

The third possibility is a mixture of the first two cases -  $y_o^*(t,T) \leq x_1 + x_2$  for some tenure dates but  $y_o^*(t,T) > x_1 + x_2$  for others. Since the capital value of continuing increases with tenure holding age constant, those laid off in any age cohort are those most recently hired. However, older workers are disproportionately represented among those let go at a given tenure because the capital value of continuing declines with age holding tenure constant. The hypothesis, then, is consistent with the standard practice of laying off workers in inverse tenure order and provides a possible explanation for the observation that older workers are often "retired early" in response to reductions in product demand.

Compensation: Perhaps the single unique implications of the cooperative search strategy hypothesis is that marginal changes in the division of the match's specific capital value won't affect turnover. This inference seems to be contradicted by numerous studies that report negative association between wage and quit rates across industries; e.g., Stoikov and Raimon [1968], Burton and Parker [1969], Pencavel [1970], and Parsons [1972]. One problem common

to all of these studies is the inadequacy with which job tenure and/or age are measured. Measures used are at best only rough indices of the tenure and/or age distribution within one industry. Since it is well known that wage rates increase with age and tenure across individuals, it may be that wage differences across industries reflect the differences in the age and tenure distribution more accurately than the actual measures used in these studies.

At a more fundamental level, it may be that inter industry wage differences in wage rates reflect differences in specific human capital. Although the wage rate is only restricted to a range given cooperative strategies, the hypothesis does not rule out the possibility that specific human capital is shared in say more or less fixed proportions. Indeed, since a proposed wage contract must be viewed as "equitable" by both the employer and the worker before an agreement can be reached, we might expect sharing as a means of obtaining agreement on a contract that motivates cooperative search behavior.

In any case, to the extent that wage differences proxy for age, and tenure differences, the estimated magnitude of the wage effect in quit equations will be lower when accurate measures are included. Fortunately, such data exist in the case of individual worker surveys. Viscusi [1976] reports that the wage effects though negative, is weak. The coefficient estimates are particularly non-robust in quit intention regressions for individuals estimated with both the Michigan Survey and the National Longitudinal Survey of young and older men. Other variables included that have robust coefficients of the expected signs include age, tenure, injury rate, union membership and measures of fringe benefits. The results reported by Bartel and Borjas [1976] tell a similar story.

For older men in the NLS, Viscusi finds that coverage by a private pension plan significantly reduces the estimated probability of quitting. In the Michigan Survey the same effect is reported for the presence of fringe benefits among which is coverage by a private pension plan. Bartel and Borjas report the same effect in their quit probability regressions and also report a significant negative coefficient for pension plan coverage in layoff regressions. To the extent that fringe benefits provided contingent on employment can be interpreted as a scheme equivalent to terminal compensation by the worker to his employer, these results support the cooperative strategy hypothesis.

Offsetting this supporting evidence for the hypothesis is the positive association between layoff rates and the wage across industries reported by Parsons [1972]. The same association across individuals is also reported by Bartel and Borjas [1976] for older men in the National Longitudinal Survey. Of course, a layoff probability is not the same concept as our dismissal probability and, indeed, this effect may be attributable to downward wage rigidity.

## 6. Summary

In the paper a model of labor turnover is developed. The existence of job specific skills and imperfect information about the location of alternatives are complementary sources of job-worker attachment in the model. Because the strategy used by the worker to search for alternative jobs affects the employer's future income stream and vice versa, the choice of a strategy by both is a two person game. Moreover, the existence of the two imperfections responsible for job-worker attachment imply that the problem of dividing the rents associated with any given job-worker match by an appropriate choice of a wage contract is one of bilateral bargaining.

That a wage contract exists with the property that it motivates each party involved in a match to search in a manner that maximize the sum of both expected discounted future income streams in the principal theoretical result presented in the paper. Specifically, if each party to a match is required to compensate the other for the capital loss suffered as a consequence of separation, then the non-cooperative Nash search strategies are also cooperative search strategies. Both severance pay and non-vested pension plans are prevalent institutional arrangements that can be interpreted as the means needed to effect terminal compensation.

The fact that employment contracts often do include schemes that allow for compensation suggests that cooperative search strategies should be seriously considered as possible explanations for observed turnover behavior. In the paper we argue that existing evidence is consistent with the hypothesis that cooperative search strategies are pursued. Even the implication that the wage rate does not affect turnover rates is not contradicted by recent evidence on cross individual worker differences in turnover behavior.

FOOTNOTES

1. For a recent survey of the literature see Parsons [1977].
2. Another interesting objection to the Becker-Parsons conclusion is developed in Donaldson and Eaton [1976].
3. In his survey article, Parsons [1977] makes this point and calls for its development.
4. See Rosen [1977] for an excellent recent discussion of comparative advantage and market sorting.
5. The kinship of this description and Becker's [1974] theory of marriage is obvious.
6. A similar specification is used by Wilde [1977].
7. This is a standard "trick" in the theory of stochastic processes. See, for example, Feller [1968, pp. 447-448].
8. This concept is very closely related to what M. Bailey [1976] calls an optimal wage contract.
9. Reynolds [1951] seems to be the originator of the idea that quit-tenure relationships might be explained by a process of learning about the attributes of a job while working at it. Only recently this idea has flowered in a series of papers by Johnson [1977], Viscusi [1977], Jovanovic [1977], and Wilde [1977].
10. See also Diamond and Stiglitz [1974].
11. Note that we are assuming, correctly, that the age of a job's current occupant does not affect the distribution of alternatives available to the employer.
12. If as a consequence the capital values of all alternatives also increase, the possibilities are even more numerous.
13. This point was suggested to me by L. Jacobson in conversation. He uses it to explain some of his results in Jacobson [1975].
14. The recently developed implicit contract approach does provide an explanation and appears to be a generalization of the cooperative strategy hypothesis for an economy in which worker preferences are defined as income and leisure. See particularly the discussion in Bailey [1977] which includes a definition of an optimal implicit contract.
15. However, the probability of termination is lower the larger is the original difference between  $y^*$  and  $x_1 + x_2$ . This seems to be the essence of Oi's argument in his classic contribution [1962].

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