

Hauser, John R.; Shugan, Steven M.

**Working Paper**

## Efficient Measurement of Consumer Preference Functions: A General Theory for Intensity of Preference

Discussion Paper, No. 285

**Provided in Cooperation with:**

Kellogg School of Management - Center for Mathematical Studies in Economics and Management Science, Northwestern University

*Suggested Citation:* Hauser, John R.; Shugan, Steven M. (1977) : Efficient Measurement of Consumer Preference Functions: A General Theory for Intensity of Preference, Discussion Paper, No. 285, Northwestern University, Kellogg School of Management, Center for Mathematical Studies in Economics and Management Science, Evanston, IL

This Version is available at:

<https://hdl.handle.net/10419/220645>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

DISCUSSION PAPER NO. 285

EFFICIENT MEASUREMENT OF CONSUMER  
PREFERENCE FUNCTIONS:

A General Theory for Intensity of Preference

by

John R. Hauser <sup>\*/</sup>  
and  
Steven M. Shugan <sup>\*\*/</sup>

July 14, 1977

<sup>\*/</sup>

Assistant Professor of Marketing and Transportation  
Graduate School of Management, Northwestern University

<sup>\*\*/</sup>

Assistant Professor of Marketing, University of Rochester

## ABSTRACT

To design successful new products and services, managers need to understand how consumers form preferences relative to product attributes. This paper develops a general theory of preference measurement making possible more efficient and potentially more accurate measurement of consumer preferences. The theory is based both on the properties of preference scaling functions and on the properties of how consumers react to the measurement task. The theory produces general forms which decompose into (1) utility theory, (2) probabilistic choice theory, (3) interval scaled theory, (4) ratio scaled theory or (5) a hybrid theory, depending on how the consumer reacts to a constant sum paired comparison measurement task.

The theory is implemented and tested with an interactive computer interviewing system. In this system the consumer is asked to make constant sum paired comparisons with respect to two products characterized by attribute levels. The consumer is asked questions to determine his reaction to the measurement task and indicate which sub-theory is appropriate. Further questions provide sufficient data to estimate the consumer's preference function.

The theory is now being applied to the design, location, and evaluation of new telecommunications devices. A numerical example is given and some empirical results are reported.

## 1. INTRODUCTION

To design successful new products and services, managers need to understand how consumers form preferences relative to the attributes of products. To evaluate product or service strategies, managers must have predictions of how consumers will behave if a new product is launched. Accurate predictions of consumer response coupled with models of production costs, tax rates, cash flow, product line considerations, etc. (Shugan and Balachandran [26]) can lead to more successful products and can reduce the risk of failure.

Many researchers have investigated the twin problems of understanding consumer preference and predicting consumer choice. Some techniques estimate consumer preference functions by representing "consumer utility" as a function of the product's attribute levels [8,12,14,19,20,29]. (The higher the "utility" of a product, the more likely a consumer is to choose that product.) Such techniques are useful in the design of products because they indicate the relative effects of changes in the attributes of that product. Other techniques measure interval or ratio scaled preference directly based on actual products prior to test market or national introduction [28,32]. Such techniques are useful in the evaluation of new products because they are based on actual products and on the strong direct measures of preference.

Conjoint analysis (Tversky [33]) is one effective technique to measure preference functions. Although now quite successful in marketing (Green and Davita [6], Green and Wind [8], Wind and Spitz [36]), the application of conjoint analysis can be improved. The consumer task is quite tedious requiring each consumer to rank order 20-40 "products" in terms of preference. (Products can be real or represented by attribute levels.) Furthermore, the

measurement estimates ordinal preference, i.e. a ranking over products, rather than intensity of preference. (Note that although a set of conjoint models over a consumer population estimates how many people choose each product, conjoint analysis does not estimate ratio, interval, or probabilistic preferences which are potentially better indicators of the consumer evaluation process and more accurate predictors of behavior.) Finally, because the measurement task is tedious, it is difficult to ask further questions to check behavioral assumptions underlying the preference measurement.

Tradeoff analysis (Johnson [14]) reduces the consumer task by having consumers rank order products where only two attributes vary at a time. Because this measurement task compares attributes by pairs, tradeoff analysis is restricted in its ability to model interactions among attributes (Farquhar [5]). Preferences are still ordinal and assumptions difficult to check.

Direct utility assessment (Hauser and Urban [11,12]) uses von Neumann-Morgenstern utility theory [35] to develop a more efficient consumer task based on indifference questions. This task, a limiting case of rank order questions, gives more information per question and thus allows both shorter consumer interviews and measurement of more complex utility forms. Preference is still ordinal but the preference function can handle uncertain attributes. Some underlying behavioral assumptions are checked (preferential independence, utility independence, Keeney [16]) but it is infeasible to check all utility assumptions as is done in the lengthy interviews (often 2 days or more) of prescriptive von Neumann-Morgenstern applications (Keeney [17], Keeney and Raiffa [19], Farquhar [5], Bodily [3]). Even with indifference questions and the axiomatic theory, marketing applications still require a 40-50 minute

personal interview to measure a consumer's preference function. Furthermore, the theory cannot yet account for the measurement error inherent in consumer interviews.

Constant sum paired comparison (CSPC) preference measurement directly measures preference. But it is unlike conjoint, tradeoff, and utility analysis which estimate *preference functions*. With CSPC consumers are asked to allocate a fixed sum of "chips" between pairs of actual products or product concepts in proportion to their preferences among those products. Ratio-scaled preference scales are developed from analyses of these responses (Torgenson [32]). Silk and Urban [28] report ease of measurement and excellent predictive capability in over 10 product categories. As high as 80% of the uncertainty is explained. (Pure constant sum measurement, in which chips are allocated simultaneously among all products, is a difficult consumer task often leading to inaccurate results. Pessimier [23].) This consumer task is more efficient than simple paired comparisons because its ratio scale measures intensity of preference.

Unfortunately the axioms of conjoint and utility theories are not directly applicable to CSPC measurement. As a result the more efficient and powerful CSPC consumer task has not yet been used to measure preference functions. If a consistent theory were developed preference functions might be measured more efficiently (fewer questions) and more accurately (intensity of preference and assumption checks).

This paper develops both the theoretical basis and the practical techniques for using efficient measurement, such as CSPC, for measuring consumer preference structure.<sup>+</sup> It begins with a general theory which (1) develops a common structure for all consumer preference measurement, (2) leads to new

<sup>+</sup>The theory is developed for CSPC but can be adapted to the dollar-metric (Pessimier [23]) and other scales which measure intensity of preference.

forms of preference functions, and, most importantly, (3) allows one to test a consumer's responses for preference properties, select the appropriate preference theory, and estimate the appropriate preference function. Based on the general theory this paper then develops a practical procedure, implemented by an interactive computer package, to measure preferences in a market research environment, i.e., quickly and efficiently estimate individual preference functions on a large scale. The paper closes with a numerical example and an empirical case study applying this preference theory to the design of innovative forms of telecommunication for use in small scientific communities.

## 2. THEORY DEVELOPMENT

To understand and predict how consumers will react to new products or services one must have a preference theory that is attribute-based. That is, a preference theory must be able to predict a consumer's preferences by observing his or her perceptions of each product in the choice set relative to a set of attributes (e.g. quality, personalness, convenience, and value for health services). The theory then allows one to measure a real-valued preference function ( $p: \underline{X} \rightarrow R$ ) mapping the attribute perceptions ( $\underline{X}$ ) into a scalar measure of goodness ( $R$ ), such that the consumer is most likely to choose the product with the largest scalar value (e.g., if  $p_{\text{auto}} > p_{\text{bus}}$  then this consumer is more likely to choose auto than bus). Thus the remainder of this paper will assume that any feasible product can be represented by a set of attributes,  $\underline{X} = X_1 \cdot X_2 \cdot \dots \cdot X_k$ . Let  $x_{jk}$  equal the level of attribute  $k$  for product  $j$  and let  $\underline{x}_j = \{x_{j1}, x_{j2}, \dots, x_{jk}\}$ . All analyses and data collection are at the level of the individual consumer. We

suppress the subscript,  $I$ , for notational convenience. Various methods to identify and measure these attributes are factor analysis (Urban [34]), discriminant analysis (Johnson [15], Pessimier [23]), and non-metric scaling (Green and Rao [7]). See Hauser and Koppelman [10] for an empirical comparison of these techniques.

The CSPC measurement, also attribute based, is schematically represented in Figure 1. The consumer is given two product concepts (or in some cases actual products) with known or measured attribute levels,  $\underline{x}_1$  and  $\underline{x}_2$ . He or she is asked to allocate a fixed sum (FS) of "chips" between the products according to his or her preferences for those products.

DIVIDE 100 CHIPS BETWEEN EACH OF THE FOLLOWING PAIRS OF HYPOTHETICAL COMMUNICATION ALTERNATIVES:

<u>PRODUCT A</u>	<u>PRODUCT B</u>
EFFECTIVE	MODERATELY EFFECTIVE
DIFFICULT TO USE	EASY TO USE
. . . ENTER CHIPS FOR PRODUCT A	
<i>24</i>	
. . . ENTER CHIPS FOR PRODUCT B	
<i>76</i>	

Figure 1: An example of constant sum paired comparison measurement. (Respondent's answers are in *italics*.)

Suppose the consumer allocates  $a_1$  chips to  $\underline{x}_1$  and  $a_2 = FS - a_1$  chips to  $\underline{x}_2$ . Since the definition of preference was left unspecified there is



some ambiguity as to what type of preference the pair  $(a_1, a_2)$  represents. Perhaps  $a_1 > a_2$  means only that  $\underline{x}_1$  is preferred to  $\underline{x}_2$ . In this case the CSPC measure is no better than simple pairwise comparisons (Johnson [13]) and one would continue to use conjoint analysis, tradeoff analysis, or direct utility assessment. But suppose that  $a_1 > a_2$  means that the probability of choosing  $\underline{x}_1$  from a choice set is greater than the probability of choosing  $\underline{x}_2$  from that choice set independent of the choice set,  $C$ , where  $\underline{x}_1, \underline{x}_2 \in C$ . Then one could use stochastic theories such as logit analysis (McFadden [22]) or compaction theory (Hauser [9]) to measure  $p(\underline{x}_j)$ . Further properties such as Luce's axiom [20] or Steven's power law [30] could lead to models, such as  $\text{Prob}[\underline{x}_1 \text{ From } \{\underline{x}_1, \underline{x}_2\}] = a_1^\beta / (a_1^\beta + a_2^\beta)$ , which expand to estimate  $\text{Prob}[\underline{x}_1 \text{ from } C]$ . Thus existing techniques may apply to the CSPC measurement depending on how the consumer reacts to the measurement task.

But perhaps more information is contained in the measurement. One possible assumption, based on a cardinal utility theory developed by Shapley [24], is that  $p(\underline{x}_1) - p(\underline{x}_2)$  is proportional to  $a_1 - a_2$ . Another stronger assumption, following Torgenson's ratio-scaled postulates [32], is that  $p(\underline{x}_1)/p(\underline{x}_2) = a_1/a_2$ . By making full use of the measured preferences, either of these assumptions yields preference functions which measure intensity of preference, but each theory requires (a) a test for the appropriateness of the assumptions and (b) a method to estimate the preference function,  $p(\underline{x}_j)$ .

Thus, not only might CSPC provide more efficient data gathering for existing estimation, but it potentially implies more powerful estimation techniques. Clearly, one could choose any of the above assumptions, develop a theory, and estimate preference functions, but today's computer technology

makes possible a more powerful method. Since all assumptions and theories can be shown to lead to the same basic structure, one can use the measurement to first test assumptions and then estimate the appropriate preference model. Further, if the measurement task involves an interactive computer package one can responsively ask questions to test and/or estimate in the most efficient manner. We begin with the general structure and then give assumptions, tests, and estimations for each of the above special cases.

### General Structure

Let  $c: X \rightarrow R$  be a compaction<sup>†</sup> function, let  $*$  be a property operator, and let  $\circ$  be a measurement relation. Suppose that a particular consumer is presented with two potential products,  $i$  and  $j$ , with known attribute levels,  $\underline{x}_i$  and  $\underline{x}_j$ , and suppose he or she responds to the CSPC question with the pair  $(a_{ij}, a_{ji})$  where  $a_{ij} + a_{ji} = FS$ . Then the properties of the compaction function for this consumer are implicitly defined by the equation:

$$c(\underline{x}_i) * c(\underline{x}_j) = a_{ij} \circ a_{ji} \quad (1)$$

If measurement is to be efficient,  $c(\cdot)$  will be measured for a relatively small subset of the possible elements of  $\underline{x}$ . To be useful,  $c(\cdot)$  must apply for all elements of  $\underline{x}$ . This requires consistency in the form of property asymmetry and property transitivity. Asymmetry simply implies that the order in which the consumer is presented  $\underline{x}_i$  and  $\underline{x}_j$  does not affect his evaluation. I.e.

*Axiom 1 (Asymmetry):* If  $c(\underline{x}_i) * c(\underline{x}_j) = a_{ij} \circ a_{ji}$  then  $c(\underline{x}_j) * c(\underline{x}_i) = a_{ji} \circ a_{ij}$ .

<sup>†</sup>We use the more general word, "compaction", rather than "utility" or "preference" to avoid confusion with prescriptive utility theory. See Hauser and Urban [ ].

Asymmetry is a binary relation, property transitivity is a test for consistency among three or more products. In other words, for a specific choice of  $*$  and  $\circ$  property transitivity constrains the relationship among  $a_{ij} \circ a_{ji}$ ,  $a_{jk} \circ a_{kj}$ , and  $a_{ik} \circ a_{ki}$ . This test is specific to each choice of  $*$  and  $\circ$ .

Finally, to simplify measurement  $c(\cdot)$  should be separable into uni-attributed functions. For example,  $c(\underline{x}_i)$  would be simpler to estimate if it were separable into a sum of  $K$  uni-attributed functions,  $c_1(x_{1j})$ ,  $c_2(x_{2j})$ , etc. The separable form depends on  $*$  and  $\circ$ , but in each case there are identifiable independence properties that imply the form of separability. These independence properties are special cases of evaluative independence which is defined as follows.

Suppose that  $X = X_1 \cdot X_2 \cdot \dots \cdot X_K$  can be partitioned into  $Y = X_1 \cdot X_2 \cdot \dots \cdot X_m$  and  $Z = X_{m+1} \cdot X_{m+2} \cdot \dots \cdot X_K$ . Let  $\underline{y}_i, \underline{y}_j \in Y$ , let  $\underline{z}_i, \underline{z}_j \in Z$ , then  $\underline{x}_i = (\underline{y}_i, \underline{z}_i)$  etc. As a special case define  $X_{\overline{k}} = X_1 \cdot X_2 \cdot \dots \cdot X_{k-1} \cdot X_{k+1} \cdot \dots \cdot X_K$  and  $X_{\overline{kj}}$  similarly. Then we can define a general independence property, evaluative independence:

*Definition 1: Let  $Y, Z$  be a partition of  $X$ , then for a given consumer  $Y$  is evaluative independent of  $Z$  (written  $Y$  e.i.  $Z$ ) if for any  $\underline{z}^o \in Z$*

$$c(\underline{y}_i, \underline{z}^o) * c(\underline{y}_j, \underline{z}^o) = a_{ij} \circ a_{ji}$$

then

$$c(\underline{y}_i, \underline{z}) * c(\underline{y}_j, \underline{z}) = a_{ij} \circ a_{ji} \quad \text{for all } \underline{z} \in Z$$

This structure, a generalization of the utility theoretic structure (Keeney and Raiffa [19]), provides a common link between five disparate structures which

potentially explain how consumers react to the CSPC measurement task. Of these five structures utility theory and stochastic theory are now in common use, but the interval, ratio, and hybrid theories are new and until now have not been used to estimate consumer preference functions. The remainder of this section develops each theory within the general structure and indicates how to use each theory to estimate consumer preferences based on CSPC measurement. Specifically, for each theory this section (1) provides a test of property transitivity, (2) provides a specific form and test of evaluative independence, (3) derives a separable function for  $c(\cdot)$ , and (4) indicates how to empirically estimate  $c(\cdot)$ .

### Utility Theory

Suppose that the CSPC question measures only ordinal preference. I.e.,  $a_{ij} > a_{ji}$  means simply that  $\underline{x}_i$  is preferred to  $\underline{x}_j$ . If the functional form of  $c(\cdot)$  is known, then consumer utility measurement (Hauser and Urban [12]) applies directly. If the data structure has certain "nice" properties (Tversky [33]) and sufficient measurements are made, conjoint analysis can be used. To put utility theory in the general structure, define an indicator function,  $\delta(t)$ , such that  $\delta(t) = -1$  if  $t < 0$ ,  $\delta(t) = 0$  if  $t = 0$ , and  $\delta(t) = 1$  if  $t > 0$ . Utility theory is then given by:

$$\delta[c(\underline{x}_i) - c(\underline{x}_j)] = \delta(a_{ij} - a_{ji}) \quad (2)$$

A test for this property<sup>†</sup> is ordinary transitivity. If  $\delta_{ij} = \delta(a_{ij} - a_{ji})$  then this test for property transitivity can be written as:

$$\delta_{ij} + \delta_{jk} - \delta_{ik} = \delta_{ij}\delta_{jk}\delta_{ik} \quad (3)$$

which covers all possible preference and indifference orderings among  $\underline{x}_i$ ,  $\underline{x}_j$ , and  $\underline{x}_k$ . Axiom 1 holds if  $\delta_{ij} = -\delta_{ji}$ .

---

<sup>†</sup>Note that both utility theory and *stronger* properties can hold simultaneously.

Evaluative independence becomes preferential independence (Keeney and Raiffa [19]) and  $Y$  e.i.  $Z$  implies that there exists a value function  $g(\underline{y})$  such that  $c(\underline{x}_j) = v[g(\underline{y}_j), \underline{z}_j]$ . In particular, if each pair of attributes is preferential independent of the other attributes, then there is some *ordinal*  $c(\cdot)$  that is additive. This is stated formally in theorem 1. The proof is contained in Ting [31], Farquhar [5], or Keeney and Raiffa [19].

*Theorem 1: For ordinal utility theory,  $X_k$  e.i.  $X_j$  for all  $k$  implies there is some  $c(\cdot)$  such that:*

$$c(\underline{x}_j) = c_1(x_{1j}) + c_2(x_{2j}) + \dots + c_K(x_{Kj})$$

If the consumer task is extended to product concepts with uncertain attributes, i.e. lotteries, then equation 2 defines von Neumann-Morgenstern [35] utility theory, property transitivity becomes a test of transitivity over lotteries, evaluative independence becomes utility independence (Keeney [18]), and mutual utility independence implies the quasi-additive form (Keeney [18]). This is stated formally in theorem 2.

*Theorem 2: For von Neumann-Morgenstern utility theory,  $X_k$  e.i.  $X_j$  for all  $k$  implies*

$$c(\underline{x}_j) = \sum_k \lambda_k c_k(x_{kj}) + \sum_{m>k} \sum_k \lambda_{mk} c_m(x_{mj}) c_k(x_{kj}) + \\ + \text{third order terms} + \dots + \text{Kth order terms.}$$

where  $\lambda_k$ ,  $\lambda_{mk}$ , etc. are scalar constants.

Ordinal functions can be measured with standard conjoint theory (Green and Wind [ 8]), while von Neumann-Morgenstern functions require direct assessment

(Hauser and Urban [12]). Thus, these standard techniques are a special case of the general structure.

But the theory can be extended by requiring stronger properties than  $\delta(t)$  for  $*$  and  $\circ$ . These stronger properties allow the measurement of intensity of preference and thus provides for more efficient measurement.

### Interval Theory

The first direct extension requires the property that the property operator and measurement relation are not simply indicators but rather interval measures. Fundamental axioms which imply the existence of an interval preference function are derived in Shapley [24]. This theory is summarized by:

$$c(\underline{x}_i) - c(\underline{x}_j) = a_{ij} - a_{ji} \quad (4)$$

A test for this interval property is additive transitivity. I.e., equation 4 is consistent if for all triplets of product concepts,  $\underline{x}_i$ ,  $\underline{x}_j$ , and  $\underline{x}_k$ , equation 5 holds (Test axiom 1 by setting  $\underline{x}_k \equiv \underline{x}_i$ ):

$$(a_{ij} - a_{ji}) + (a_{jk} - a_{kj}) = (a_{ik} - a_{ki}) \quad (5)$$

Evaluative independence (Y e.i. Z) implies that  $c(\underline{y}_i, \underline{z}) - c(\underline{y}_j, \underline{z}) = f(\underline{y}_i, \underline{y}_j)$  for all  $z$ . (Review definition 1.) It is easily shown that this condition implies that  $c(\underline{x}_i) = c(\underline{y}_i) + c(\underline{z}_i)$ . Extending evaluative independence to all  $X_k$  gives the following simple theorem.<sup>+</sup>

*Theorem 3: For the interval theory,  $X_k$  e.i.  $X_k$  for all  $k$  implies*

$$c(\underline{x}_j) = c_1(x_{1j}) + c_2(x_{2j}) + \dots + c_K(x_{Kj})$$

<sup>+</sup>Actually the theorem requires only  $X_k$  e.i.  $X_{k+1} \cdot X_{k+2} \cdot \dots \cdot X_K$  or some equivalent collection of telescoping sets.

If theorem 3 holds, then  $c(\underline{x}_j)$  can be estimated with linear programs based on an absolute error structure (Srinivasen and Shocker [29]) or possibly with ordinary least squares regression (OLS). Simply discretize each attribute and define:

$$c(\underline{x}_j) = \sum_k \sum_m \lambda_{mk} \delta_{mkj} \quad (6)$$

where  $\delta_{mkj} = 1$  if  $x_{kj}$  is at the  $m^{\text{th}}$  level and  $\delta_{mkj} = 0$  otherwise.

The estimation equation is:

$$a_{ij} - a_{ji} = \sum_k \sum_m \lambda_{mk} (\delta_{mkj} - \delta_{mki}) + \text{error} \quad (7)$$

Empirically, mathematical programs perform better because  $\lambda_{mk}$  can be constrained non-negative and when appropriate monotonic in  $m$ . If there are  $L$  levels, a minimum of  $(L-1)K$  questions<sup>+</sup> must be asked to specify  $c(\cdot)$ . More questions are required when an error term is included in equation 7 and estimates,  $\hat{\lambda}_{mk}$ , are obtained.

Thus the stronger interval property, if it applies, implies more efficient measurement and perhaps better insight because an interval scaled compaction function indicates intensity of preference. Tests for interval properties and evaluative independence are easily formulated based on equation 5 and definition 1. See section 4.

### Ratio Theory

The interval theory is useful, but it represents but one way in which consumers might react to CSPC questions. Another theory, initially formulated by Torgerson [32] for objects rather than attribute bundles, is a

---

<sup>+</sup>Base points for  $c_k(\cdot)$  are chosen such that no redundancy exists.

ratio theory.<sup>†</sup> This theory is summarized by:

$$c(\underline{x}_i)/c(\underline{x}_j) = a_{ij}/a_{ji} \quad (8)$$

A test for this ratio property is that of multiplicative transitivity. I.e., equation 8 is consistent if for all triplets of product concepts, equation 9 holds (Test axiom 1 by setting  $\underline{x}_k \equiv \underline{x}_i$ ):

$$(a_{ij}/a_{ji}) \cdot (a_{jk}/a_{kj}) = (a_{ik}/a_{ki}) \quad (9)$$

Evaluative independence (Y e.i. Z) implies that  $c(\underline{y}_i, \underline{z})/c(\underline{y}_i, \underline{z}) = f(\underline{y}_i, \underline{y}_i)$  for all  $\underline{z}$ . This condition implies  $c(\underline{y}_i, \underline{z}) = c(\underline{y}_i) \cdot c(\underline{z})$ . Extending evaluative independence to all  $\underline{x}_k$  gives the following theorem.

*Theorem 4: For the ratio theory,  $\underline{x}_k$  e.i.  $\underline{x}_k$  for all  $k$  implies*

$$c(\underline{x}_j) = c_1(x_{1j}) \cdot c_2(x_{2j}) \cdot \dots \cdot c_K(x_{Kj})$$

If theorem 4 holds, then  $c(\underline{x}_j)$  can be estimated by taking logarithms of equation 8 and formulating a linear program for absolute error or by using OLS. Simply discretize each attribute and define:

$$c(\underline{x}_j) = \prod_k \prod_m (\mu_{mk})^{\delta_{mkj}} \quad (10)$$

where  $\delta_{mkj}$  is defined as before. To estimate  $\mu_{mk}$  use the equation:

$$\log(a_{ij}/a_{ji}) = \sum_k \sum_m (\delta_{mkj} - \delta_{mki}) \log \mu_{mk} + \text{error} \quad (11)$$

Again  $(L-1) \cdot K$  questions are required to specify  $c(\cdot)$ , more if the  $\log \mu_{mk}$ 's are to be estimated.

<sup>†</sup>In Torgerson's measurement, consumers are explicitly requested to use a CSPC scale as a ratio scale.



Like the interval theory, the ratio theory implies more efficient measurement and perhaps better insight into intensity of preference. Ratio scaled preference has been quite successful in predicting consumer behavior (Silk and Urban [28]) and holds great promise for attribute based preference prediction. Tests for ratio properties and evaluative independence are easily formulated based on equation 9 and definition 1. See section 4.

### Hybrid Theories

The interval and ratio theories are two extremes of how consumers might react to CSPC scales. Fortunately, once the general structure has been formulated, many hybrid theories can be posited by carefully defining \* and °. For example:

$$c(\underline{x}_i) - (a_{ij}/a_{ji})^\gamma c(\underline{x}_j) = \beta(a_{ij} - a_{ji}) \quad (12)$$

When  $\gamma \rightarrow 0$ ,  $\beta \rightarrow 1$  the interval theory applies, when  $\gamma \rightarrow 1$ ,  $\beta \rightarrow 0$  the ratio theory applies. This theory is useful to estimate non-linearities that occur at the extreme ends of any constant sum scale. Property transitivity and evaluative independence are complex but non-linear estimation routines such as OPTISEP [1], SUMT [4], and others allow one to estimate certain parameterized forms of  $c(\cdot)$ .

Note that if  $\gamma=0$  then equation 12 is a simple extension of equation 4. Similarly, if  $\beta=0$ , equation 12 is an extension of equation 8. In either case the estimation equation is linear in the new parameter and linear programming or OLS can be used.

## Stochastic Theories

The four preceding theories assume that a deterministic preference function can be estimated subject only to measurement error. Such a preference function is then linked to stochastic choice (Bass [ 2]) by using probability of choice models (Hauser and Urban [11]). An alternative formulation is to assume that stochastic preference can be measured by the CSPC questions.

Let  $P[x_i > x_j]$  be the probability that the consumer will prefer  $x_i$  to  $x_j$  in a paired comparison. Now suppose chips are allocated sequentially to achieve the chip allocation  $(a_{ij}, a_{ji})$ . For each toss, i.e. individual "chip" allocation, there is a probability  $P_{mij}$  that the  $m^{\text{th}}$  "chip" will be allocated to product  $i$  rather than product  $j$ . If one assumes (1) a stationary process (i.e.  $P_{ij} = P_{mij} = P_{nij}$  for all  $n, m$ ), (2)  $P_{ij} = P[x_i > x_j]$ , and since  $P_{ij} + P_{ji} = 1$ , then each individual chip allocation is Bernoulli. The total chip allocation for any fixed sum of "chips" is given by,

$$\text{Prob}[a_{ij}, a_{ji}] = \binom{a_{ij} + a_{ji}}{a_{ij}} [P_{ij}]^{a_{ij}} [P_{ij}]^{a_{ji}} \quad (13)$$

The maximum-likelihood estimator for  $P(x_i > x_j)$  is given by equation 14:

$$P_{ij} = P[x_i > x_j] = a_{ij}/(a_{ij} + a_{ji}) \quad (14)$$

If  $P(x_i > x_j)$  is a function,  $f(\cdot, \cdot)$ , of only the compaction values then equation 14 becomes a special case of the general structure. I.e.,

$$f[c(x_i), c(x_j)] = a_{ij}/(a_{ij} + a_{ji}) \quad (15)$$

A special case of this stochastic theory is the logit model [22] where

$$f[c(x_i), c(x_j)] = \exp[c(x_i)]/(\exp[c(x_i)] + \exp[c(x_j)]) . \quad (16)$$

In this case, property transitivity becomes Luce's choice axiom [20]:

$$(P_{ij}/P_{ji}) \cdot (P_{jk}/P_{kj}) = (P_{ik}/P_{ki}) \tag{17}$$

which reduces to equation 10, the ratio theory test. Furthermore, evaluative independence implies an additive compaction function which implies  $\exp[c(\underline{x}_i)]$  is multiplicative in the attributes. This similarity to ratio theory, although not surprising given one possible interpretation of the logit model as a strict utility model [20], further highlights the ability of the general structure to isolate behavioral assumptions implicit in any estimation based on CSPC questions. In this case,  $c(\cdot)$  is estimated by maximum-likelihood techniques based on equation 13. Note that equation 14 can be extended by including an exponent for  $a_{ij}$  and  $a_{ji}$  (Stevens [30], Pessier [23]) and equation 16 can be extended to probit and other probabilistic models.

Thus, stochastic choice theory, which includes many empirical applications, is a special case of the general structure. But by formulating the stochastic theory as a special case, market researchers can use the more efficient CSPC measurement to improve estimation beyond what is possible with standard (0,1) preference measurement (Johnson [13]).

Summary of General Theory

The general structure,  $c(\underline{x}_i) * c(\underline{x}_j) = a_{ij} \circ a_{ji}$ , provides a common link between the special cases. Without a general structure one would have to make a particular behavioral assumption about how consumers react to CSPC questions. The efficacy of the resulting model would depend on the

choice of a special theory. With the general structure many new options are available; among these are two search/test/estimate procedures.

The first search/test/estimation procedure is individual based. With an interactive computer interviewing system, consumers are asked a number of initial questions which are tested for property transitivity. Based on an heuristic error measure developed in section 4, one theory is selected and further questions are asked to test evaluative independence and to estimate the compaction function. Some questions are reserved for saved data testing. The theoretical advantages of this procedure are more efficient measurement and greater accuracy for prediction. Its disadvantages are that interpersonal comparisons become nearly impossible and managerial diagnostics are difficult to produce. Thus its primary purpose is prediction rather than explanation.

The second search/test/estimation procedure is aggregate. A common set of questions are developed and each consumer is asked the same questions. Based on the heuristic error measure, a common "best representative" theory is selected to model the entire consumer segment. Individual-level parameters for the compaction functions are estimated for prediction and summary statistics (median, interquartile interval) are computed for managerial understanding. This procedure is less efficient, but provides better managerial diagnostics.

The general structure leads to improved understanding of existing theories, development of new theories, and a common structure among theories. It makes possible test/search/estimation procedures for more efficient data collection and model development. Finally, it is sufficiently general to allow further

theories and estimation procedures to be developed. The next sections indicate how to implement the general theory in a market research environment.

### 3. INTERACTIVE COMPUTER INTERVIEWING SYSTEM

Although the aggregate search/test/estimation/ procedure can be implemented with personal interviews, the individual level procedure requires that the questions asked be a function of complex tests on earlier questions. This is difficult without an interactive computer system. Furthermore, data collection via computer terminals leads to consistency of presentation, certain efficiencies in data analysis (coding and key punching are not necessary), improved accuracy (outlying responses are immediately questioned, key punch error avoided), and cost savings (less training is required for interviewers). In fact, in some industrial or scientific community applications, the respondents are so familiar with computer terminals that the questionnaire can be self-administered. Of course, in some applications random, stratified, or choice based [21] samples may be more expensive to obtain for computer interviews. Thus this technique is most appropriate for applications where a computer terminal can be made available to an appropriate sample of the target population.<sup>+</sup> For a more complete discussion of computer interviewing see [13,27].

The gains from computer interviewing can be substantial, but only if the questionnaire can be developed inexpensively for each product category application. To do this, we developed a quasi-compiler and a coding language, PARIS (preference assessment retrieval and information system), to enable managers and analysts to quickly and inexpensively develop a questionnaire specific to their application. See Shugan and Hauser [27].

<sup>+</sup>In some applications responses can be recorded on-line by an interviewer who is in contact with the respondent by telephone.

Compiler: PARIS uses a Fortran-like language that is sufficiently general for most questionnaire applications, yet simple enough that it can be readily used by analysts somewhat familiar with languages such as Fortran, PL-1, or Cobol. With PARIS an analyst can write a questionnaire with instructions, questions, categories for responses, test and branch commands, and macro-instructions to call specific estimation and testing routines. The PARIS compiler produces a quasi-object code for the questionnaire. The resulting compiled questionnaire, which can contain general questions as well as the CSPC questions, is now administered to each consumer by the PARIS system. Answers, plus an abbreviated record of any computations, are stored as consumers' complete responses. A master file retains information regarding the status of all respondents' answers.

Typical Questionnaire: In a study of consumer preferences for telecommunications technology (see next section), a questionnaire has been developed that contains five sections: (1) warmup questions, (2) questions to establish a scenario for usage, (3) consumer rating of attributes of existing products and new product concepts, (4) the CSPC questions, (5) preference ranking and usage intent for the existing products and the concepts, and (6) various personal and demographic questions. Example questions of section 2 are shown in Figure 2. Review Figure 1 for an example of the CSPC questions on the PARIS system. Note that the "questionnaire" handles out of range responses by gently informing the respondent of his mistake and asking for a new response. In this way, response errors will not prevent interviews from being completed. (Of course the respondent may not complete the "questionnaire" in which case partial responses are saved.) Once compiled, the complete questionnaire which contains 96 questions including 16 CSPC questions takes 15-30

WE WOULD LIKE TO KNOW ABOUT YOUR MOST RECENT INTERACTION WITH A COLLEAGUE, OR A VENDOR, ETC., TO DISCUSS A PROBLEM ON WHICH ONE OR MORE OF YOU IS PRESENTLY WORKING. PLEASE CONSIDER INTERACTIONS ONLY WITH THOSE PEOPLE WHO DO NOT WORK IN THE SAME BUILDING AS YOU AND DON'T CONSIDER CALLS JUST TO SET UP APPOINTMENTS.

1. IN ADDITION TO YOURSELF, HOW MANY PEOPLE PARTICIPATED IN THE INTERACTION:  
(PLEASE TYPE IN THE NUMBER OF PEOPLE.)

?3

2. DID YOU USE:

- 1 = TELEPHONE
- 2 = INTEROFFICE MEMO
- 3 = MAIL
- 4 = TELETYPE OR TELECOPIER
- 5 = PERSONAL VISIT (YOU WENT TO HIM [THEM])
- 6 = PERSONAL VISIT (HE [THEY] CAME TO YOU)
- 7 = PERSONAL VISIT (CONFERENCE ROOM, AUDITORIUM, ETC.)
- 8 = OTHER

(PLEASE ANSWER WITH A NUMBER 1 THROUGH 8.)

?10

(PLEASE ANSWER WITH A NUMBER 1 THROUGH 8.)

?8

PLEASE SPECIFY.

?TELEGRAM

Figure 2: An example of interactive computer measurement by PARIS  
(Respondent's answers are in *italics*.)

minutes for an average consumer to complete. The administration cost including on-line hookup is \$1 per respondent on a CDC-6400 (\$510 per cpu hour).

#### 4. NUMERICAL EXAMPLE

Figure 3 is one example of a consumer's response to 16 CSPC questions. This design<sup>+</sup> for 3 attributes at 3 levels contains 10 tests of property transitivity that are used in selecting the appropriate theory. For example, question triplets {1,3,6}, {1,4,5}, and {3,2,4} are three such tests. Further, questions pairs {1,7}, {3,8}, and {6,9} each test evaluative independence for one attribute. Note that FS = 100 chips because empirically the roundoff error inherent in a 10 chip allocation makes it difficult to discriminate between interval and ratio property transitivity (equations 5 and 9). Furthermore, an odd number of chips, such as the usual 11 chips does not allow indifference and is logically inconsistent with equation 5 because  $a_{ij} - a_{ji}$  would always be odd. Consumer reaction to date has been favorable for an allocation of 100 chips.

Transitivity tests: Since equation 3 holds for all 10 tests of property transitivity for utility theory, figure 3 is at least consistent with ordinal data.

To test for stronger properties define  $a_{ik}^I$  as the closest integer such that the interval test, equation 5, holds. Define  $a_{ik}^R$  as the closest integer such that the ratio test, equation 9, holds. These definitions minimize the scale differences inherent when comparing the "errors" in equations 5 and 9. Suppose that  $a_{ik}$  is the respondent's actual answer, then define the following root mean square error tests:

---

<sup>+</sup>This is one possible design. It was chosen as illustrative in testing properties. For a discussion of efficient designs see Cox [4a].



$$T_I^2 = (1/n) \sum_t (a_{ik}^I - a_{ik})_{t}^2 \quad (18)$$

$$T_R^2 = (1/n) \sum_t (a_{ik}^R - a_{ik})_{t}^2 \quad (19)$$

Where the summation is over all possible tests,  $t$ , within the design and  $n$  is the number of such tests. If  $T_I < T_R$  then it is more likely that the consumer is responding via the interval theory than the ratio theory. If  $T_R < T_I$  then he or she is more likely responding via the ratio theory. If both  $T_I$  and  $T_R$  are greater than some cutoff  $T_0$ , reject both theories. (A good rule of thumb is  $T_0 = 2.5$  corresponding to a consumer responding with the nearest "5" cutoff, i.e. 5, 10, 15, ..., 95.)

For the data in table 3,  $T_I = 2.88$  and  $T_R = .54$  which indicates that the "consumer" is most likely responding via a ratio theory.

Estimation: After the transitivity tests, a ratio theory preference function was estimated with a linear program minimizing the absolute error in equation 7. The utility values,  $\lambda_{mk}$ 's, were constrained to be nonnegative and monotonically increasing in  $m$ . The estimated parameters, shown in figure 4a, compare well to the "actual" parameters which produced the data, shown in figure 4b.

### ATTRIBUTE LEVELS

Question	Product i ( $\underline{x}_i$ )			Product j ( $\underline{x}_j$ )			CSPC	
	$x_1$	$x_2$	$x_3$	$x_1$	$x_2$	$x_3$	$a_{ij}$	$a_{ji}$
1	H	H	L	H	L	H	43	52
2	M	M	M	L	H	H	27	73
3	H	L	H	L	H	H	25	75
4	M	M	M	H	L	H	53	47
5	H	H	L	M	M	M	40	60
6	L	H	H	H	H	L	80	70
7	L	H	L	L	L	H	43	57
8	H	L	L	L	H	L	25	75
9	L	L	H	H	L	L	80	20
10	M	M	L	L	H	L	42	53
11	L	M	M	M	M	L	60	40
12	L	M	M	M	M	L	71	29
13	H	L	L	L	M	M	13	87
14	H	L	L	M	M	L	27	73
15	L	L	H	M	L	M	55	45
16	L	H	L	M	L	M	47	53

H, M, L = high, medium, and low levels of each attribute

Figure 3: One "consumer's" response to a 3x3 design.

	$x_1$	$x_2$	$x_3$
H	2.9	8.7	11.6
M	2.0	3.9	4.9
L	1.0	1.0	1.0

a) Estimated preference parameters ( $\hat{\lambda}_{mk}$ 's)

	$x_1$	$x_2$	$x_3$
H	3.0	9.0	12.0
M	2.0	4.0	5.0
L	1.0	1.0	1.0

b) "Actual" preference parameters ( $\lambda_{mk}$ 's)

$$c(\underline{x}_j) = \prod_k \prod_m \lambda_{mk} \delta_{mkj}$$

Figure 4: Preference parameters for the "consumer" responses in figure 3.

## 5. EMPIRICAL EXAMPLE: TELECOMMUNICATIONS

(This application, funded by a grant from the National Science Foundation, is now underway. We report here the managerial problem and the empirical analysis to date. Estimation and field experience will be reported in a revised version in June 1977.)

The managerial problem is to design a mix of telecommunications technology for use in a small scientific community, Los Alamos Scientific Laboratory. The marketing research, which follows the normative methodology described in Hauser and Urban [11], will produce predictions of consumer response and consumer response diagnostics by modeling consumer perceptions, preferences, and choice within each population segment.

Consumer focus groups have been run and analyzed to produce an indication of the choice process, consumer semantics, and a set of 25 attribute scales which attempt to characterize the evaluation process. For the preliminary quantitative analysis, a twenty-page mail-back questionnaire has been carefully designed and pretested (31 returned questionnaires out of 38 mailed out). The actual implementation, now in process, is aiming for 400 returns. Factor analysis and preference regression of the pretest response reveals two perceptual dimensions labeled 'ease of use' and 'efficacy' with relative importances of .18 and .82 in an aggregate linear compensatory model. Ease of use correlates with the ability to find the right person, save time, eliminate paperwork, and get a quick response as well as saving hassle, planning, time, and cost; efficacy correlates with the ability to exchange scientific and technical information, persuade people, convey all forms of information, control the impression you want to make, monitor people, operations, and equipment, yield a high level of human interaction, solve problems, express feelings, and enhance idea development.

Based on these preliminary results, a pretest of evaluation theory is underway with 16 CSPC questions to estimate compaction functions at four levels for each factor. Consumer reaction to the measurement task has been favorable to date. The test will compare property transitivity and predictive ability of the various theories. A further test of predictive ability will compare models estimated by each theory as well as the aggregate linear model in their ability to recover rank order preference among actual product concepts.

## 5. SUMMARY

Consumer preference measurement is important for the design of new products and services. This paper has presented a practical measurement technique for consumer preference functions based on a general theoretic structure.

The measurement technique, based on computer implemented CSPC questions among attribute bundles, provides for more efficient data collection and potentially better prediction. Furthermore, because the measurement includes intensity of preference it can potentially lead to more accurate explanation of the consumer evaluation process.

A general theory is necessary to develop estimation procedures for the measurement technique. This theory, summarized by the equation  $c(\underline{x}_i) * c(\underline{x}_j) = a_{ij} \circ a_{ji}$ , provides a common structure and test to select the appropriate special theory for estimation of a consumer preference function. Its special cases include existing consumer utility theories and stochastic preference theories as well as the interval and ratio theories which measure intensity of preference. Besides providing an efficient method

for assessing individual preference functions this structure provides a useful taxonomy for understanding the implications of CSPC measurement and provides a useful generalization for the development of more powerful theories.

The general theory coupled with the interactive computer implementation of CSPC questions holds great potential for improved preference measurement.

## BIBLIOGRAPHY

1. Abelman, H., "OPTISEP -- A Package of Subroutines for Non-Linear Optimization," Vogelback Computing Center, Northwestern University, Evanston, Ill., Document No. 322, July 1976.
2. Bass, F., "The Theory of Stochastic Preference and Brand Switching," Journal of Marketing Research, Vol. 11, February 1974, pp. 1-20.
3. Bodily, S., "Collective Choice with Multidimensional Consequences," Ph.D. Thesis, MIT Operations Research Center, Cambridge, Mass., February 1976.
4. Cohen, C., "SUMT Version 4: Sequential Unconstrained Minimization Techniques for Non-Linear Programming," Vogelback Computing Center, Northwestern University, Evanston, Ill., Document No. 200 (Rev. A), February 1975.
- 4a. Cox, D.R., Planning of Experiments, (New York: John Wiley & Sons, 1958).
5. Farquhar, P.H., "A Survey of Multiattribute Utility Theory and Applications," Management Science, (forthcoming, 1977).
6. Green, P. and M. Devita, "An Interaction Model of Consumer Utility," Journal of Consumer Research, Vol. 2, September 1975.
7. Green, P. and V. Rao, Applied Multidimensional Scaling, (New York: Holt, Rinehart, and Winston, Inc., 1972).
8. Green, P.E. and Y. Wind, Multiattribute Decisions in Marketing, (Hinsdale, Ill.: The Dryden Press, 1973).
9. Hauser, J.R., "Consumer Preference Axioms: Behavioral Postulates for Describing and Predicting Stochastic Choice," Working Paper, Department of Marketing, Northwestern University, Evanston, Ill., November 1976, (submitted, Management Science, 1977).
10. Hauser, J.R. and F.S. Koppelman, "Effective Marketing Research: An Empirical Comparison of Techniques to Model Consumers' Perceptions and Preferences," Technical Report, Transportation Center, Northwestern University, Evanston, Ill., January 1976, (submitted, Journal of Marketing Research, 1977).
11. Hauser, J.R. and G.L. Urban, "A Normative Methodology for Modeling Consumer Response to Innovation," (forthcoming, Operations Research, 1977).
12. Hauser, J.R. and G.L. Urban, "Direct Assessment of Consumer Utility Functions: von Neumann-Morgenstern Theory Applied to Marketing," Working Paper, MIT Sloan School, Revised January 1977, (forthcoming, Management Science, 1977).
13. Johnson, R.

14. Johnson, R.M., "Tradeoff Analysis of Consumer Values," Journal of Marketing Research, Vol. 11, May 1974, pp. 121-127.
15. Johnson, R.M., Multiple Discriminant Analysis Applications to Marketing Research, Market Facts, Inc., January 1970.
16. Keeney, R.L., "Multiplicative Utility Functions," Operations Research, Vol. 22, No. 1, January 1974, pp. 22-23.
17. Keeney, R.L., "A Decision Analysis with Multiple Objectives: The Mexico City Airport," Bell Journal of Economics and Management Science, Vol. 4, 1973, pp. 101-117.
18. Keeney, R.L., "Utility Functions for Multiattributed Consequences," Management Science 18, 1972, pp. 276-287.
19. Keeney, R.L. and H. Raiffa, Decision Analysis with Multiple Conflicting Objectives, (New York: John Wiley & Sons, 1976).
20. Luce, R.D., Individual Choice Behavior, (New York: John Wiley & Sons, 1959).
21. Manski, C.F. and S.R. Lerman, "The Estimation of Choice Probabilities from Choice Based Samples," (forthcoming, Econometrica, 1977).
22. McFadden, D., "Conditional Logit Analysis of Qualitative Choice Behavior," in Paul Zarembka, ed., Frontiers in Econometrics, (New York: Academic Press, 1970), pp. 105-142.
23. Pessemier, E.A., "Market Structure Analysis of New Product, Market, and Communication Opportunities," Marketing Science Institute, Report No. 76-106, June 1976, Cambridge, Mass.
24. Shapley, L.S., "Cardinal Utility from Intensity Comparisons," RAND Report R-1683-PR, July 1975, Santa Monica, Calif.
25. Shugan, S.M., "Descriptive Stochastic Preference Theory and Dynamic Optimization: Applications Toward Predicting Consumer Choice," Ph.D. Dissertation, Northwestern University, Department of Managerial Economics and Decision Sciences, Evanston, Ill., August 1977.
26. Shugan, S.M. and V. Balachandran, "A Mathematical Programming Model For Optimal Product Line Structuring," Discussion Paper No. 265, Northwestern University, The Center for Mathematical Studies in Economics and Management Science, Evanston, Ill., April 1977, (submitted, Management Science, 1977).
27. Shugan, S.M. and J. Hauser, "PARIS: An Interactive Market Research Information System," Working Paper, May 1977, Northwestern University, Department of Marketing, Evanston, Ill.
28. Silk, A.J. and G.L. Urban, "Pretest Market Evaluation of New Packaged Goods: A Model and Measurement Methodology," Working Paper, Alfred P. Sloan School of Management, MIT, Cambridge, Mass., February 1976, (forthcoming, Journal of Marketing Research, 1977).



29. Srinivasan, V. and A. Shocker, "Linear Programming Techniques For Multi-dimensional Analysis of Preferences," Psychometrika, Vol. 38, September 1973, pp. 337-370.
30. Stevens, S.S., "Ratio Scales of Opinion," Handbook of Measurement and Assessment in Behavioral Sciences, D.K. Whitla, ed., (Reading, Mass.: Addison-Wesley Publishing Co., 1968), pp. 171-199.
31. Ting, H.M., "Aggregation of Attributes for Multiattributed Utility Assessment," Technical Report No. 66, Operations Research Center, MIT, Cambridge, Mass., August 1971.
32. Torgenson, W.S., Methods of Scaling, (New York: John Wiley & Sons, 1958), pp. 105-112.
33. Tversky, A., "A General Theory of Polynomial Conjoint Measurement," Journal of Mathematical Psychology 4, 1967, pp. 1-20.
34. Urban, G.L., "PERCEPTOR: A Model for Product Positioning," Management Science 8, April 1975, pp. 858-871.
35. von Neumann, J. and O. Morgenstern, The Theory of Games and Economic Behavior, 2nd ed., (Princeton, NJ.: Princeton University Press, 1947).
36. Wind, Y. and L.K. Spitz, "Analytical Approach to Marketing Decisions in Health-Care Organizations," Operations Research, Vol. 24, No. 5, Sept.-Oct. 1976.