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DISCUSSION PAPER NO. 278

A SHORT NOTE ON A LINEAR EQUATION
ASSOCIATED WITH CERTAIN ECONOMIC
RESPONSE SYSTEMS⁺

by

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A Short Note on a Linear Equation
Associated with Certain Economic
Response Systems

We consider three problems:

Problem 1: Find the sign of $\frac{dp_i}{d\alpha}$ for the system of the equations [1]
(the equations represent demand and supply in a
service differentiated transport sector):

$$\begin{aligned} p_i(q_i, \alpha) &= R_1(\sum q_k) \\ p_i(q_j) &= R_j(\sum q_k) \quad j \neq i \\ MC_k^h(q_k^h) &= p_k(q_k) \quad k=1, \dots, n \\ & \quad h=1, \dots, H_k \\ q_k &= \sum_h q_k^h \end{aligned}$$

where suitable conditions are given for $\frac{\partial p_k}{\partial q_k}$, R'_k
and $\frac{\partial p_i}{\partial \alpha}$, and there are H_k firms providing mode k
service with n modes.

Problem 2: Does a solution exist (and what is it) for the follow-
ing system of first order conditions [4*]:

$$\varphi_{ih}(-2 + 2\gamma_i + \frac{1}{n}) + \bar{\varphi}_h(1 - 2\gamma_i) = -\Pi_{ih}(1 - \frac{1}{n})$$

where n is the number of agents, γ_i is the i^{th} agents
share of surplus from trade, Π_{ih} is the agents actual
marginal rate of substitution between good h and the
numeraire good while φ_{ih} is the announced marginal rate
substitution ($\bar{\varphi}_h = \frac{1}{n} \sum_i \varphi_{ih}$)?

* A related problem is in [3].

Problem 3: Given the $n-1$ demands $(m_j, j \neq 1)$ for a public good y ($y = \sum m_j$), consider a consumer who desires to minimize his Groves-Ledyard tax [2] C_i :

$$C_i = \alpha_i q y + \frac{\gamma}{2} \left[\frac{n-1}{n} (m_i - \hat{\mu}_i)^2 - \frac{1}{n-2} \sum_{j \neq i} (m_j - \hat{\mu}_i)^2 \right]$$

where q is the price of the good, $\gamma > 0$ is a coefficient set by the planning, α_i is the consumer's share of the cost of the good and

$$\hat{\mu}_i = \frac{1}{n-1} \sum_{j \neq i} m_j. \text{ If on the other hand all the } m_i$$

are to be found, is there a solution?

What these problems all have in common is that their solution depends on a system of linear equations that is considered in the following theorem:

Theorem: The system of equations $(R-BC)x = d$ with $|R| \neq 0$, $C = FEP$, all matrices $n \times n$, E the matrix of ones and x, d are $n \times 1$ vectors, has the solution:

$$x = R^{-1} \left[d + \frac{e' P R^{-1} d}{1 - e' P R^{-1} B F e} B F e \right]$$

(where e is an $n \times 1$ vector of ones) on the condition that $e' P R^{-1} B F e \neq 1$.

Proof:

$$(R-BC)x = d$$

$$\Rightarrow Rx = B(x+d)$$

$$\Rightarrow x = R^{-1}BCx + R^{-1}d$$

$$= R^{-1}BFEPx + R^{-1}d$$

$$= R^{-1}BF e \cdot e'Px + R^{-1}d$$

$$= (e'Px) \cdot (R^{-1}BF e) + R^{-1}d$$

$$\Rightarrow e'Px = e'Px \cdot e'PR^{-1}BF e + e'PR^{-1}d$$

$$\Rightarrow e'Px = \frac{e'PR^{-1}d}{1-e'PR^{-1}BF e}$$

$$\Rightarrow x = R^{-1}d + \frac{e'PR^{-1}d}{1-e'PR^{-1}BF e} \cdot R^{-1}BF e$$

In the first problem the comparative statics conditions can be written as a system of (n-1) equations:

$$(I - MT)dp = d$$

where

$$M = \begin{bmatrix} R'_2 & \dots & R'_2 \\ \vdots & & \vdots \\ \vdots & & \vdots \\ R'_n & \dots & R'_n \end{bmatrix} = \begin{bmatrix} R'_2 & & 0 \\ & R'_3 & \\ 0 & \dots & \\ & & R'_n \end{bmatrix} \begin{bmatrix} 1 & \dots & 1 \\ \vdots & & \vdots \\ \vdots & \cdot & \vdots \\ \vdots & & \vdots \\ \vdots & \dots & \vdots \\ \vdots & & 1 \end{bmatrix}$$

$$= CE$$

$$T = \begin{bmatrix} k_1 r_2 & & 0 \\ & k_1 r_3 & \\ & 0 & \dots \\ & & & k_1 r_n \end{bmatrix} = F$$

and where $k_1 = \frac{\partial p_1 / \partial q_1}{\partial p_1 / \partial q_1 - R_1'}$ and $r_j = \sum_h [1 / (MC_j^h)']$

$$d = \frac{-\partial p_1 / \partial \alpha}{\partial p_1 / \partial q_1 - R_1'} \cdot \begin{bmatrix} R_2' \\ \vdots \\ \vdots \\ R_n' \end{bmatrix}$$

and finally $dp = \begin{bmatrix} dp_2 / d\alpha \\ \vdots \\ \vdots \\ dp_n / d\alpha \end{bmatrix} = x$

In this case (I-MT) has an inverse (i.e. the conditions of the theorem are met).

In the second problem if we let $B = P = I$, and R and C be diagonal matrices with

$$r_{ii} = -2 + 2\gamma_i + \frac{1}{n}$$

and

$$c_{ii} = (1 - 2\gamma_i) / n$$

and if $\varphi_h = (\varphi_{1h}, \dots, \varphi_{nh})'$ is the $n \times 1$ solution vector while $d = (-\pi_{1h} / (1 - \frac{1}{n}), \dots, -\pi_{nh} / (1 - \frac{1}{n}))'$ then we can represent problem two as:

$$(R - CE)\varphi_h = d .$$

Here again we find the conditions of the theorem are met and the announced marginal rates of substitution (φ_h) can be expressed in terms of the actual marginal value of substitution (d).

Finally, in the third problem we find that the condition that $e'PR^{-1}BFe \neq 1$ is not met. The system of equations is

$$(nI - E)m = - \frac{qn}{\gamma} \alpha .$$

Letting $R = nI$, $B = P = F = I$ we see that $e'PR^{-1}BFe = 1$.

REFERENCES

- [1] Daughety, A.F., and Inaba, F.S., "Spatial Price Equilibrium with Multiple Transport Modes." Working Paper 601-76-01, The Transportation Center, Northwestern University, October 1975.
- [2] Groves, T., and Ledyard, J., "Optimal Allocation of Public Goods: A Solution of the 'Free Rider' Problem." Discussion Paper No. 144, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1975. Forthcoming in Econometrica.
- [3] Roberts, J., "Incentives in Planning Procedures for the Provision of Public Goods." Discussion Paper No. 264, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, January 1977.
- [4] Schoumaker, F., "Strategic Behavior in Planning Procedures with Private Goods." Discussion Paper No. 279, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, April 1977.