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A Dynamic Theory of Racial Income Differences

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Discussion Paper 225

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“A Dynamic Theory of Racial
Income Differences”

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A square frame containing a circle. The text "CMS-EMS" is centered at the top, and "The Center for Mathematical Studies in Economics & Management Sciences" is centered below it.

CMS-EMS
The Center for
Mathematical Studies
in Economics &
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I. Introduction

Public decision makers have long been concerned with the problem of inequality in the distribution of income. Within the last fifteen years in this country considerable attention has been focused on racial income inequality. Though the problem of racial income differences has been with us for some time--the black-white median income ratio being below two-thirds throughout the past century¹--it is only recently that explicit public policies have been formulated to address the situation. The moral and political developments of the late fifties and early sixties created a climate where the legitimacy of a system which denied the full opportunity for achievement to its minority citizens could be broadly questioned. As a result of this questioning a set of federal statutes and judicial rulings have evolved affirming the ideal of equal opportunity.

It is important to understand the relationship between this moral and legal change and the actual living standard of minority Americans. The decade of the 1960's was marked by a significant growth of direct public action of the federal level. This included a massive assault on poverty and racial inequality. While continuing depression in the black community is evident, blacks have experienced significant gains in their relative economic position.² Whether these advances may be attributed to specific government action, however, remains a topic of debate.³ The implication for the relative economic position of minorities of having moved toward equality of opportunity is not entirely clear.

This uncertainty pervades much of the current policy debate over the role of government in mitigating racial inequality. Some have observed that a mere guarantee of equal opportunity does not break the barrier of systemic discrimination as evidenced in segregated housing patterns and the differential quality of public education. According to this view, racial inequality could persist for sometime. It is argued that more affirmative policies are needed if the current black-white earnings differential is to be significantly lowered in our lifetime. Others note that action oriented policies, like the enforcement of goals and timetables for minority employment, are not needed and unduly compromise the rights of majority citizens. From this perspective, public policy should steadfastly endeavor to assure equality of opportunity for all. However, we should avoid stronger programs because they limit the opportunities of some in order to enhance the prospects of others.

The former individuals have less faith than the latter that a laissez-faire system, devoid of overt discrimination, can eliminate differences in earnings between racial groups. They suggest intervention on behalf of minorities. The latter individuals, believing the market essentially sound, insist upon adherence to a set of racially unbiased rules in the allocation of opportunities.⁴

To evaluate these opposing points of view we must seek an understanding of how relative incomes change over time. It is particularly important to know what happens when we move from a regime of overt discrimination to one of equality of opportunity.

There are also differences in values implicit in the arguments paraphrased above. Such differences can not be resolved by scientific inquiry. We cannot determine from a priori considerations exactly how much consideration should be given to minority families who find themselves significantly less well off by virtue of the historical practice of racial discrimination. This observation does not render the analyst impotent, however. Policy options may be evaluated by establishing a minimal set of requirements which all can agree that any acceptable policy must satisfy. We may then subject alternative options to critical review to see if they meet the necessary requirements. The history of social criticism provides ample precedent for such an approach.

In the present essay, this method will be employed to explore the problem of racial inequality. We begin with the premise that the current aggregate income differences among blacks and whites are the consequence of an historical legacy of discrimination and denial of opportunity. We further posit that these historical acts were "ethically illegitimate". These we take as statements in which reasonable men may find agreement. It is natural then to require of whatever policy adopted that it imply the attenuation and eventual elimination of racial economic inequality. That is, any policy option which perpetuates into the indefinite future the consequences of ethically unacceptable historical practices should, we suggest, be deemed inappropriate. It is the major thesis of this paper that a laissez-faire policy of equal opportunity (precisely defined in the sequel) will in general fail to meet this minimal requirement.

We realize, of course, that this result is significantly at variance with the conventional wisdom which currently prevails concerning the efficacy of equal opportunity. It is widely held that the elimination of racial discrimination will result in the eventual elimination of racial economic inequality.⁵ This view derives from the traditional economic analysis of labor markets and racial income differences. We shall briefly examine this theory in the next section, and shall argue that it does not take adequate account of the effect of an individual's family and community background on his acquisition of skills. To the extent that the low education and earnings of blacks in today's labor market inhibit the ability of their offspring to convert their natural talents into characteristics valued by the market, racial income differences will be observed tomorrow as well.

Section III attempts an extension of the traditional theory of the labor market, which incorporates these background effects explicitly into the analysis. The theory presented differs from the standard conception in its treatment of the skill acquisition process of workers. The effect of parental economic success on an offspring's opportunities to acquire skills is represented as an intergenerational external economy.⁶ In Section IV it is shown that this classical market failure not only vitiates the efficiency properties of equilibrium in the competitive labor market, but may also render equal opportunity ineffective as a tool for assuring equal results in the long run. The paper concludes with an assessment of the implications of this discussion for the economic theory

of income distribution and for the formulation of public policy to treat the problem of racial income differences.

II. Conventional Theory and Its Problems

Conventional economic analysis has appealed to supply and demand factors to explain black-white income disparities. Arguments focusing on the supply side of the labor market center on the characteristics of black workers (the quality and quantity of their education and work experience), which are on average below those of their white counterparts. Thus, even in the absence of discrimination, black earnings would be lower as a result of a lower investment in human capital. These factors are, however, insufficient to account for the entire differential. Even when the quality and quantity of human capital are controlled, blacks still earn considerably less than whites.⁷ To explain this differential economists have hypothesized that white employers or workers may harbor a distaste for associations with blacks. The market implication of these tastes can be differential returns to otherwise identical black and white workers.⁸ The racial differences in incomes may thus be attributed to differences in the supply of market valued characteristics (human capital) and differences in the demand for workers due to a "taste for discrimination" against blacks.

This framework suggests two approaches from which to attack racial income differences. One attempts to close the earnings gap by prohibiting the expression of discriminatory tastes, or at least neutralizing the deleterious effects of discriminatory preferences. Second, if the racial difference in the acquisition

of market-valued characteristics can be narrowed,⁹ then further progress toward the elimination of the income disparity will be made.

Important steps in both these directions have been taken in recent years. Particularly noteworthy are the ethical judgements implicit in the Supreme Court's interpretations of the Fourteenth Amendment's "equal protection" clause and the enactment of the Civil Rights Act of 1964. These judicial and legislative actions embody the view that the expression of neither private nor public discriminatory preferences can be permitted if the consequence is to limit the educational or employment opportunity of minorities. This view may be termed the Equal Opportunity Doctrine. The Doctrine has been broadly accepted in the American Society and has become the law of the land.¹¹

In classical liberal fashion the Doctrine's motivation is to assure each individual the opportunity to develop to the fullest of his or her abilities. If effectively enforced the Doctrine would eliminate the exercise of discriminatory preferences as a factor in generating racial income differences. Assuming (as we shall throughout this study) that the distribution of native ability among blacks and whites is the same, over time one might expect racial differences in the supply of market valued characteristics to diminish as well. Thus, this traditional analysis suggests that once established, the Equal Opportunity Doctrine would lead to the eventual elimination of racial income differences. This notion has gained widespread acceptance in the social science community.¹²

The problem of differences in the acquisition of market-valued characteristics, however, is quite complex. While economists of

the traditional school have analyzed in some detail the effect of human capital on earnings, the socioeconomic process underlying its acquisition has generally been ignored.¹³ Understanding this process is fundamental to understanding persistent racial inequality. So long as the Equal Opportunity Doctrine permits the social class and racial background of an individual to influence the process by which he or she acquires marketable skills, group differences in the supply of market-valued characteristics will tend to persist across generations. Since any reasonable interpretation of the Doctrine must necessarily allow such effects,¹⁴ this effect is likely to be evident even in the presence of equal opportunity. Thus, the prevailing belief in the ability of the Equal Opportunity Doctrine to guarantee (eventual) racial economic justice must be shown to be valid in spite of this tendency before it is accepted.

The growing sociological literature on occupational mobility sheds some light on this issue.¹⁵ Of particular interest are the recursive, life-cycle models of individual achievement which have been developed. These models enable the analyst to focus successively on (1) the impact of family background variables (usually father's occupation and education) on educational achievement; (2) the effect of background and education on occupation, and (3) the combined effect of background, education and occupation on income.¹⁶ Empirical tests have revealed several important facts. Family background has been found to have a significant direct effect on the educational and occupational achievement of both blacks and whites. Yet, the effect of father's occupation and educational attainment on the occupation of the offspring and

the impact of current occupational status on earnings have been found to differ appreciably between blacks and whites.¹⁷ Blacks suffer a relative disadvantage in converting favorable social origins into occupational achievement. Moreover, blacks tend to earn less than whites in the same occupations.¹⁸

Implementation of the Doctrine of Equal Opportunity would lead to a world in which occupational achievement is determined solely by the qualifications of the worker. Similarly, earnings differences between equally well educated blacks and whites in the same occupation would also be eliminated with a vigorous enforcement of the Doctrine. However, the fact that parental achievement in education and occupation influences less favorably the qualifications of black children than white belies a more subtle racial bias than that which the Doctrine is intended to rule out. If the social process by which parental status affects offspring's achievement works differently for blacks and whites, who can be held at fault? In a racially stratified society where individuals, of their own volition, socially group themselves along racial lines, we may expect the intergenerational status transmission mechanism to differ for families of different racial groups. However, racial differences of this sort are not accounted for by individuals' "tastes for discrimination",¹⁹ since they derive from the social relations of the racial groups themselves. These group relations, so important in the transfer of racial economic inequality from generation to generation, have received no serious attention by economists in their studies of the subject.

The foregoing analysis leads us to conclude that the framework which has traditionally been used to study racial income differences is inadequate for forecasting the long run consequences of particular policy alternatives. This inadequacy stems from two major deficiencies. First, the theory does not take account of the intertemporal consequences of racial discrimination which arise because parents' economic status influences the opportunities of their offspring. Second, and perhaps more critically, the theory is an individualistic one. That is, discrimination is conceived as an act which one individual perpetrates against another. As such, the traditional theory views race relations in individual terms rather than in terms of interaction between social groups.

There are many reasons why the opportunities of a child to acquire skills will vary with the economic success of his parents. For example, the quality of schooling varies considerably across communities, tending to be higher in the suburbs than in the central city.²⁰ When there is housing segregation based on income and the quality of schools in a neighborhood is positively correlated with the community's wealth,²¹ we can expect a child's educational opportunities to vary with parental economic achievement. Furthermore, the absence of a perfect capital market for educational loans renders the opportunities for higher education and the quality of that education sensitive to one's economic background.²²

The information about career opportunities and job requirements available to young people will also depend on the socioeconomic status of their parents.²³ Word of mouth referrals and informal contacts have always played an important role in the job allocation

process. Prospective workers with high income backgrounds are no doubt "better connected" than their low income counterparts. Thus, for example, the quality of information as well as the quality of education will vary with parental status. These effects will exist for blacks and whites alike.

Considerations such as these indicate that a careful analysis of racial economic differences must take account of the ongoing effects of past discrimination. The problem is analogous to that of stocks and flows in dynamic macroeconomics.²⁴ The traditional theory of racial income differences is a theory of flow equilibrium. It determines incomes differences in the market today, but regards the pre-existing stock of inequality as given. No attempt is made to explain the evolution of the stock over time, or to understand how a change in the stock might affect the flow equilibrium at any point in time. Hence the traditional view does not provide an adequate framework for the evaluation of long term policy. The dynamic extension of the theory offered here attempts to remedy this problem.

As the static equilibrium character of the traditional view obscures important intergenerational effects, so too does the image of society as a set of individuals interacting only through the market veil the role played by race in the perpetuation of racial economic differences. Previous theory has considered race only insofar as some economic agents exercise a taste for discrimination against others of a different ethnic origin. Received doctrine attempts to analyze racial income differences without

acknowledging the existence of racial groups as entities affecting individual behavior. Yet long after the Equal Opportunity Doctrine has eliminated overt discrimination as a factor in sustaining group earnings differences, the social relations which broadly obtain between the races will still be important in determining the relative economic position of blacks.

This point is best illustrated by example. Suppose that all people were either Baptist or Episcopalian, and that religious preferences were the only means of grouping individuals. Imagine further that there is equal opportunity in the sense that no one is permitted to discriminate in employment or education against anyone else because of religion. Now if Baptists and Episcopalians were quite tolerant of the others' religious differences, and felt that an individual's beliefs were a personal affair not to be interfered with, then we might expect to find quite a bit of interaction between them. Their children would probably go to school together, play together, and perhaps even marry. They might belong to the same secular clubs and organizations, and exchange information freely without distrust. In government, industry or the university community, a person's religion would be an unimportant characteristic and might not even be known to his associates. In such a world it would be hard to imagine that glaring economic differences between the groups could persist indefinitely.

If, on the other hand, each religion taught that only its adherents were decent individuals worthy of respect, and all non-believers were to be shunned, the world would be quite different

indeed. Since discrimination in public education would be outlawed, parochial schools would no doubt proliferate. There would be extensive housing segregation by religion, and most young people would probably not know individuals of the other faith until early adulthood. By then they would have learned not to trust such individuals, and certainly would not consider marrying "one of them". If the basic institutions of society were controlled by one group, then we might imagine that the other would have a rather difficult time. Even though they would not be overtly discriminated against, tension and antagonism would characterize most inter-religious activities. Anti-discrimination laws might not be enforced as vigorously as possible. The subordinate group would not benefit from free social interaction with the dominant group. One might easily envision long lasting economic differences being sustained in this situation.

This example is not meant to be realistic, but only intended to suggest that the social milieu within which economic activity takes place can affect the outcome of the economic process.²⁶ This social environment will change slowly over time, in a manner that is little understood, and can be expected to respond only marginally to the legal resolutions which exemplify the Equal Opportunity Doctrine. The failure of blacks to find open to them the path to assimilation which other ethnic minorities have traveled is indicative of the pervasiveness of racial stratification in our society. The total economic impact of this state of affairs cannot, in our view, be adequately accounted for by the market transactions of individual economic agents.²⁷

A number of writers on this subject have considered the possibility that whites might find it in their economic interest to act collectively against blacks.²⁸ Where group behavior has been considered, it is viewed as the outcome of rational coalition formation on the part of individuals. However, this approach cannot explain why the coalitions form along racial dimensions rather than some other lines. If collusive behavior for group gain were the only motive for discrimination, then there would be many possible criteria which could be used to partition society into competing groups. The fact of the matter is that some coalitions are exceedingly more likely than others.

A more subtle point concerning the nature of race relations as a group phenomenon should also be made. The previous authors cited above appear to have missed the distinction between collusion and consensus. Divisions between racial groups are not the consequence of some implicit social contract with the participating parties pursuing self-interest, but are rather the result of customary behavior which individuals learn during the process of social maturation.²⁹ Psychologists have long recognized that in a racially stratified society, an individual's self-image as well as his image in the eyes of those around him, is basically conditioned by his racial identity.³⁰ Furthermore, some sociologists have emphasized that race prejudice finds its existence in the sense of group position adopted by the prejudiced individual, rather than his feelings toward persons of another race as individuals.³¹ As Blumen has observed, "...the locus of race prejudice

is not in the area of individual feeling, but in the definition of the respective positions of the racial groups."³² These observations are noteworthy because they imply that the social milieu which an individual faces in pursuing his life's goals will depend upon his group identity and will reflect the underlying relations of racial groups in the society.³³ This will be so in all major institutions in the society, especially in the educational system and in the work place.

Thus, we conclude that a careful analysis of racial income differences must consider the effects on individual achievement of both parental economic status and the broader social relations which obtain among racial groups. Below, a simplified model of income determination is presented, which, apparently for the first time, explicitly incorporates these effects. Subsequent analysis of this model reveals, contrary to conventional belief, that the Equal Opportunity Doctrine cannot be relied upon to eliminate economic differences between the races, even in the long run.

III. A Socioeconomic Model of Income Determination

A. Preliminaries. This section and the next will be devoted to the specification and analysis of a highly stylized model of racial income differences. The model abstracts from all but the bare essentials of the problem and hence should not be viewed as an attempt to realistically describe the job allocation process. The effort is not without purpose or fruit, however. By removing many of the complicating real world factors, we may cast into sharp relief the roles of the limited number of remaining forces which, as was argued above, are important in determining how the income differences evolve over time.

Assume an individual's economic life consists of three stages: a primary socialization phase, where the principle interactions occur within the family; an educational stage where basic characteristics and behavioral traits requisite for productive and satisfying employment are acquired; and finally, the stage of employment when the individual joins productive activity.³⁴ We may suppose that the hypothetical economic agent is born with an endowment of innate capabilities such as intelligence and certain physical characteristics. The agent also possesses a particular socioeconomic background determined by race and parental income. Two racial groups (blacks and whites) are assumed to exist. Thus, an individual in the model is completely characterized at the beginning of life by his innate endowment, parental income, and race. The latter two characteristics will define the individual's socioeconomic background.

Temporally speaking it is assumed that life occurs in two periods of equal duration, youth and maturity. The initial period of life (youth) encompasses the first two stages of the life cycle. Thus youth is a time of socialization and education while maturity is characterized by employment activity. The particular demographic structure of this model is designed for simplicity and plays no substantive role. Specifically it is assumed that the population size is stationary, that only men participate in economic activity,³⁵ that each family consists of two parents (but only one breadwinner) and two children (one male and one female), and that mating occurs randomly among the young at the end of their first period of life.³⁶

The offspring of a couple are assumed to "appear" immediately after mating, i.e. at the onset of maturity. Two further assumptions must be added. First, there is no interracial marriage, a phenomenon of minute empirical significance.³⁷ Second, and more crucially, the socio-economic background of the non-breadwinning parent is irrelevant in the determination of the family's social class, which depends only on the breadwinner's income. This assumption is strong, but necessary for simplicity.

The acquisition of productive characteristics by a young person is modeled as a social process. That is, through complementary interaction with his home and community environment and an educational institution, a young person is able to convert his endowment of innate capabilities into a bundle of marketable characteristics. This specification captures the fact, documented in the previous section, that an individual's opportunities for achievement depend on his socioeconomic background. The employment opportunities of a mature individual are determined by the characteristics acquired through this social process during youth.

The social structure of this economy may exhibit stratification along both racial and income dimensions. We assume that mature individuals tend to group themselves and their families together, both residentially and in terms of their informal social contacts. Such groupings will be referred to hereafter as "communities".³⁸ Young individuals belonging to the same community will tend to have similar socioeconomic backgrounds only to the extent that the society is stratified along racial and income lines. They will, however, attend the same educational institution provided by the

mature individuals of that community. To account for the resource flows involved in the maintenance of this institution one may imagine a poll tax levied on all parents in each period as the means by which the process is supported.

B. Equal Opportunity and Racialism. In what follows we shall want to analyze the implications of the Equal Opportunity Doctrine for the long term development of racial income differences. Before the argument can proceed it will be necessary to specify just what is meant by the Doctrine in this context. Since society is composed of heterogeneous individuals with different innate capabilities, the Doctrine will not require that the scope for activity which a person faces be the same for all individuals. Hence we must determine the characteristics which people possess in varying quantities that justify differences in individual opportunity. Call these the "critical characteristics." We can then define equality of opportunity as the state of affairs in which any two individuals with identical holdings of critical characteristics face the same set of possibilities for action.

Denote by " α " a young individual's endowment of innate capacities, and let " x " represent the bundle of productive characteristics possessed by an arbitrary mature individual. We assume that a young individual exercises some discretion in choosing the productive characteristics actually acquired, though in general the array of possibilities from which he can choose depends on his innate endowment, his home environment, and the community environment (including the educational institution) which he faces in his youth. Note that we distinguish conceptually between genetic

(i.e. innate) and environmental effects on individual achievement, though empirically this separation remains a serious problem.

We may now offer two distinct conceptions of equal opportunity, depending on whether α or x are taken as critical characteristics. In the former instance we say that equal opportunity exists if any two people with the same innate endowment face the same set of possible productive characteristics from which to choose, with the reward structure for productive characteristics identical for all individuals. Under this definition, equal opportunity does not permit socioeconomic background (i.e. family and community environment) to affect achievement independently of innate ability.

Alternatively, if productive characteristics are taken to be critical then equal opportunity implies that any persons with the same bundle of such characteristics face the same array of employment opportunities and rewards in the labor market. This latter definition of equal opportunity is implied by but does not imply that given earlier. In particular, equal opportunity with x as critical characteristics is consistent with family background effects on earnings, so long as those effects occur at the level of skill acquisition and not in the labor market.

It is apparent that the current policy thrust toward equal opportunity is best characterized by the latter of the two definitions offered above. The work of The Equal Employment Opportunity Commission is limited to enforcing the laws against employment discrimination. While there has been much discussion of equal educational opportunity, the varying quality of public education

across communities is widely acknowledged.³⁹ Furthermore, so long as parents have the ability to allocate resources (including their time) so as to affect the quality of a child's home and community environment, we may expect parental income and education to condition the opportunities of their offspring. This will be true for individuals of either racial group. For these reasons we shall interpret the Equal Opportunity Doctrine in this model to mean that there is equal opportunity, using the definition given above, taking productive characteristics x as critical.

We assumed above that both the home and community environment of a young individual would affect his acquisition of productive characteristics, and hence his earnings. The quality of home (family) environment was indexed by parents' income. If there is social stratification by income, then parents' income may serve as a proxy for the quality of the community environment as well.

Suppose that in addition there is social stratification by race. In this instance, the racial composition of communities, while not necessary completely homogeneous, will tend to be somewhat concentrated. Hence the community environment of an individual will depend on his racial group's economic position as well as that of his family. Here again, a history of discrimination against a particular group will have an impact on the earning opportunities of young people in that group. Note that in this instance every person in the group will be affected, not just those from families with low income. This is because if a person belongs to a racial group which has been discriminated against, then even

though his parents may have been successful, the average income of his community is lowered by the past discrimination.⁴⁰

We shall say that racialism exists whenever the community environment of individuals with the same family environment differs for people of different racial groups. No normative connotation is intended by use of the word "racialism". It simply means that people tend to socially group themselves along racial lines, and that this tendency has consequences for the opportunities of their offspring. In particular it should be noted that the Equal Opportunity Doctrine (as interpreted here) is perfectly consistent with the notion of racialism. However, we shall see that the long run efficacy of the Doctrine for eliminating group economic differences depends crucially on whether or not racialism prevails.

C. Market Valued Characteristics and Earnings. Let us turn now to a more detailed specification of how individuals' earnings are determined.⁴¹ In all that follows we shall assume that innate capability, α , may be measured as a non-negative number. This capability will vary among individuals, one person being "more able" than another if the former's innate endowment is larger numerically than that of the latter. We assume that the distribution of innate capacity among the young people of each generation is identical to, though independent of that which prevailed in the previous generation. Furthermore, the distribution of innate talent is the same for each racial group.⁴² Thus, the innate endowment of an individual is assumed independent of his socio-economic background.

For simplicity we shall also assume that the bundle of market-valued characteristics which an individual acquires in the first period of life may be represented by a pair of non-negative numbers, $X = (X_1, X_2)$. That is, there are effectively only two types of characteristics, and the quantities of these acquired by an individual are represented by the numbers X_1 and X_2 . The acquisition of characteristics is imagined to be an abstract process of interaction with home and community environments and an educational institution, which occurs during youth. Each young individual may decide, within certain limits, what the outcome of this process is to be. The limits on this decision are determined by the innate endowment of the individual and the nature of the social environment and educational institution which he faces.

We may express these constraints by supposing that for each individual there is a set of attainable characteristic bundles among which he may choose. Two such sets, representing the opportunities of two different individuals, are illustrated in Figure 1. The first individual, say individual a , may choose among all characteristic pairs (X_1, X_2) which lie on or below the locus AB . The other individual, a' , can select any pair which does not lie above $A'B'$. It is apparent that the opportunities of a' are broader than those of a . This may occur for several reasons. First, a and a' , though facing identical social environments and educational institutions, may differ in their innate capacities (i.e. $\alpha_{a'} > \alpha_a$). A more favorable innate endowment means that an individual has wider latitude in choosing the benefits

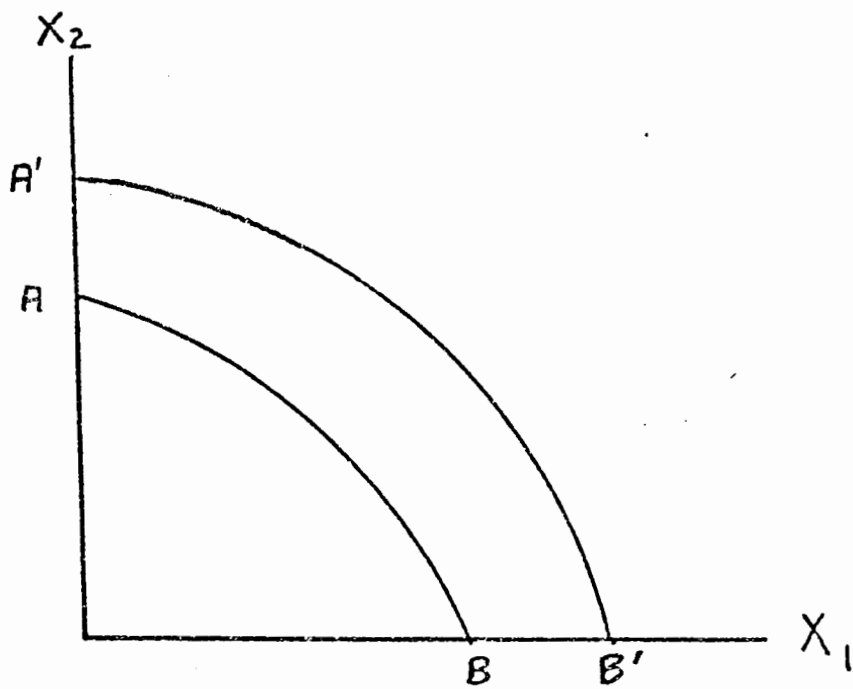


FIGURE 1

he will derive from the education-socialization process. In addition, even if a and a' have the same innate endowment and are a part of the same community, a' may have a "better" home environment than a. That is, if one's family income is increased then an expansion of opportunities occurs. Finally, opportunities may vary for young individuals of different communities. Thus, a' may come from a community with a higher quality educational institution or more favorable environmental influences than the community to which a belongs.

Social stratification is necessary for this last effect to be operative. This is true because in the absence of stratification, the composition of each community would mirror the composition of the population as a whole. Social stratification by income leads to community associations of families with similar incomes. This would tend to exacerbate the influence on opportunities of parental income, since greater family income would mean a more favorable community environment as well as a better home environment. If in addition there were social stratification by race, i.e. racialism, then the community environment, while sensitive to parental income, would also depend on the average income of the individual's racial group. Under racialism, two individuals of different racial groups but otherwise identical would face different opportunities, unless the economic positions of their respective groups were the same.⁴³ The extent to which these opportunities would diverge (e.g. the "distance" between AB and A'B' in Figure 1) would depend directly on the magnitude of existing racial income differences. For the rest of this discussion the extent of racial income differences will

be indexed by the ratio of mean black income to mean white income, denoted by r . We assume, for the sake of historical realism, that r lies between zero and one. We remind the reader that the Equal Opportunity Doctrine, as interpreted above, is assumed to apply throughout this discussion.

The demand side of the labor market will be specified in the following manner. We assume that there are a large number of identical competitive firms producing a homogeneous output under conditions of constant returns to scale by employing only skilled and unskilled labor. Output is perishable and there is no accumulation of capital in the model.⁴⁴ A mature worker is considered skilled if he has the "right" characteristics. The set of all characteristic bundles which enable an individual to gain employment as a skilled worker is called the acceptance set, and is denoted by A . The acceptance set is like a rule which enables employers to determine whether or not a given employee can perform skilled tasks. If the employee has characteristics $x \in A$ then he is acceptable as a skilled worker. If, on the other hand, $x \notin A$ then the individual can find only unskilled employment. The acceptance set is assumed to be time invariant and known to all firms and workers alike.

We assume that the factor markets are competitive and that workers are paid their marginal products. Skilled employees earn more than unskilled employees, but wages are the same for all workers within a given occupational class. Let \bar{w} denote the wage of a skilled worker and \underline{w} the wage of an unskilled

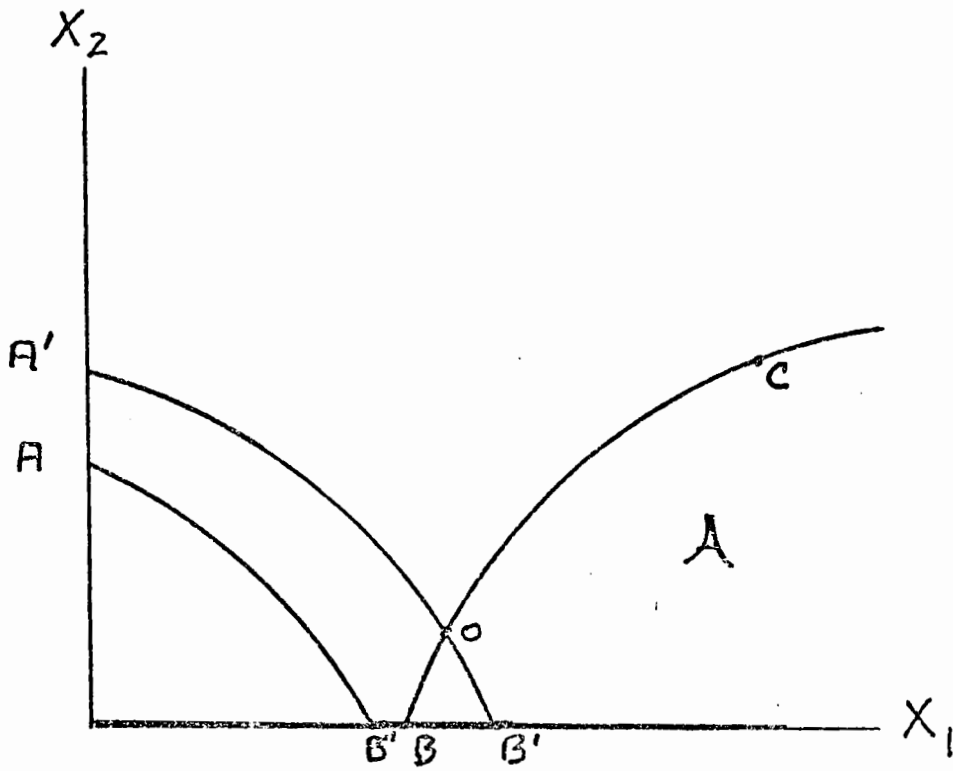


FIGURE 2

employee. Further, let $w = \bar{w} - \underline{w}$ represent the wage differential. The technology of production for all firms is assumed to exhibit constant returns to scale and diminishing returns to each factor. In Figure 2 the acceptance set A is given by the collection of all characteristic pairs being on or below the locus BC . In this instance firms consider X_1 to be a useful characteristic for skilled work and X_2 a nuisance. This may be inferred by observing that the minimal level of the first characteristic necessary to qualify for skilled employment is an increasing function of the quantity of the second characteristic possessed by the worker.

Whether or not a mature individual gains skilled employment depends on the characteristics acquired during youth. Figure 2 illustrates the situation where the individual faced with possibilities AB cannot obtain acceptable characteristics for skilled employment, while the person with opportunities $A'B'$ may become skilled by choosing a pair in the "triangle" BOB' . Since the array of possible characteristics which a young individual faces varies with his innate endowment and socioeconomic background, it follows that these factors will affect his chances of becoming a skilled worker and hence his earnings. To determine which people become skilled workers requires a more detailed consideration of the criterion used by young individuals in selecting characteristics. This will be given momentarily. We note in passing the simplicity gained by the assumption of only two occupational categories. Since an individual's socioeconomic background is defined by his parent's income and race, among any generation of young people only four

different backgrounds are possible. This will enable us to analyze precisely how the distribution of economic advantage evolves over time.⁴⁵

Each individual in the society is assumed to possess a set of preferences by which he evaluates his state of well being. It is further assumed that these preferences are identical for all individuals.⁴⁶ A person's well being is determined by two factors: the bundle of characteristics acquired in the first period of life and the level of income obtained in the second period of life. We assume that each characteristic bundle may be assigned a value which represents its dollar equivalent to all individuals. An agent's well being may then be measured in dollar terms, being the sum of the value of characteristics acquired during youth plus the wages earned in employment during maturity. An individual decides upon the characteristics to acquire during youth by choosing a bundle which maximizes his state of well being among all bundles which are attainable by him.

This choice may be described with the aid of Figure 3. The set of attainable characteristic pairs for an arbitrary agent is given by all points on or below the locus A'B'. The curve BC bounds from above the acceptance set A. The agent's preferences over characteristics may be exhibited in the diagram by a collection of indifference curves. A representative indifference curve is the locus UU depicted in the figure. Any two characteristic pairs on an indifference curve have the same value to an individual.

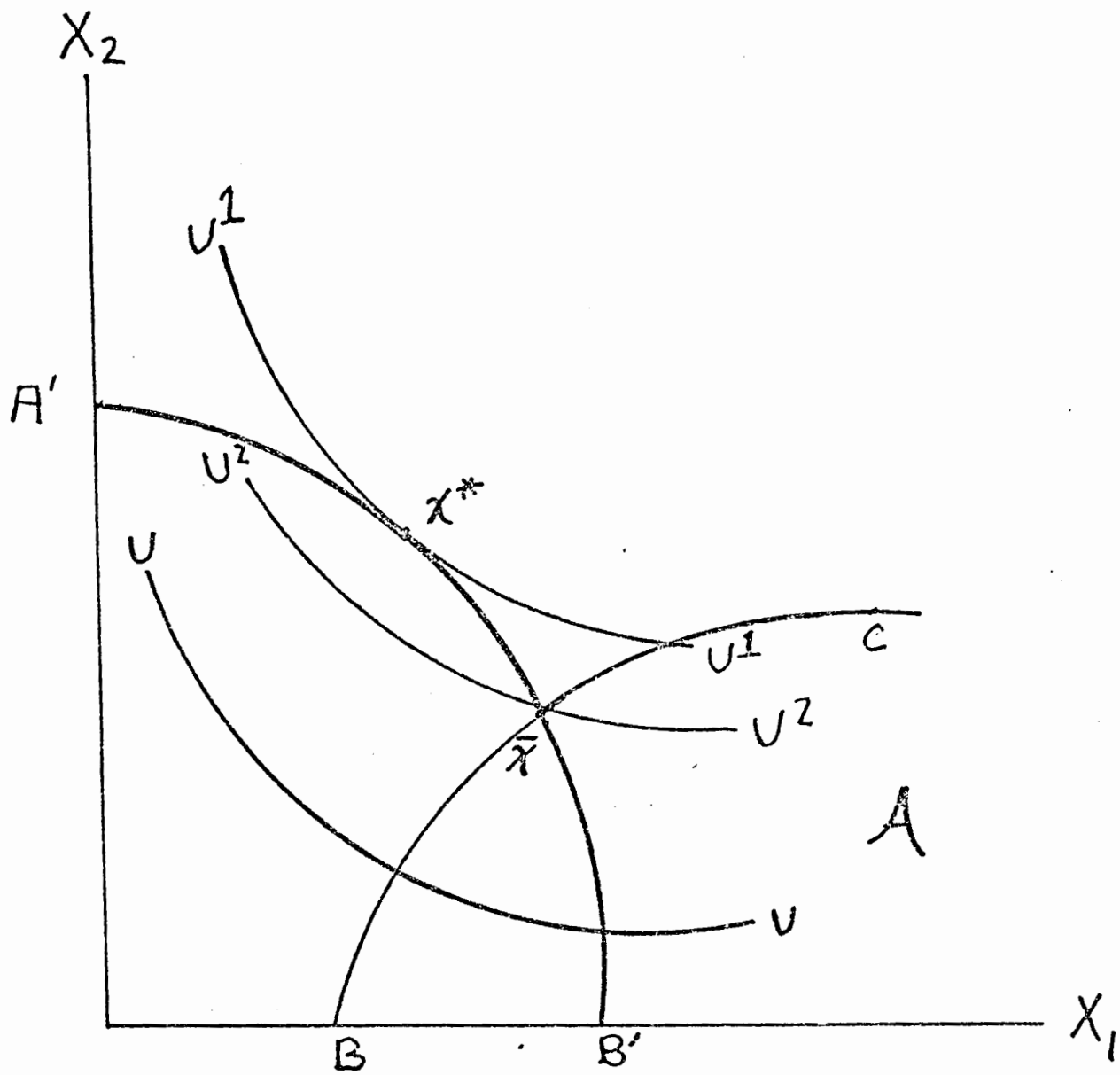


FIGURE 3

If one indifference curve lies above another (as U^2U^2 lies above UU in the diagram) then any characteristic bundle on the higher curve is more valuable than a bundle on the lower curve. Now we may imagine a young agent making his choice in two stages: He must decide whether or not to become skilled. Given this decision, he must choose an appropriate bundle of characteristics.

Let us consider this latter choice first. Given the set of attainable characteristics, the agent considers the bundle of characteristics whose dollar value to him is greatest of those bundles attainable. This is the point X^* in Figure 3. X^* lies on the indifference curve U^1U^1 , which is the highest indifference curve which still intersects the set of points on or below $A'B'$. If X^* is in A , then the agent will select X^* , since this characteristic bundle gives the greatest income during maturity as well as the greatest value during youth. In general, however, these characteristics will be insufficient to qualify him for skilled employment. In this instance the individual will select X^* only if he decides to enter unskilled employment. Next the individual considers the most desirable characteristic bundle which is both attainable and consistent with becoming a skilled worker. This assumes, of course, that such points exist. If not, then the agent has no occupational decision to make. The set of all such points for our hypothetical agent in Figure 3 is bounded by the curve $B\bar{x}B'$. U^2U^2 is the highest indifference curve which intersects this set and the most desirable bundle is \bar{x} . Thus, if the agent decides to become skilled, \bar{x} is the bundle he will choose. The difference between the value of the bundle X^* and the bundle

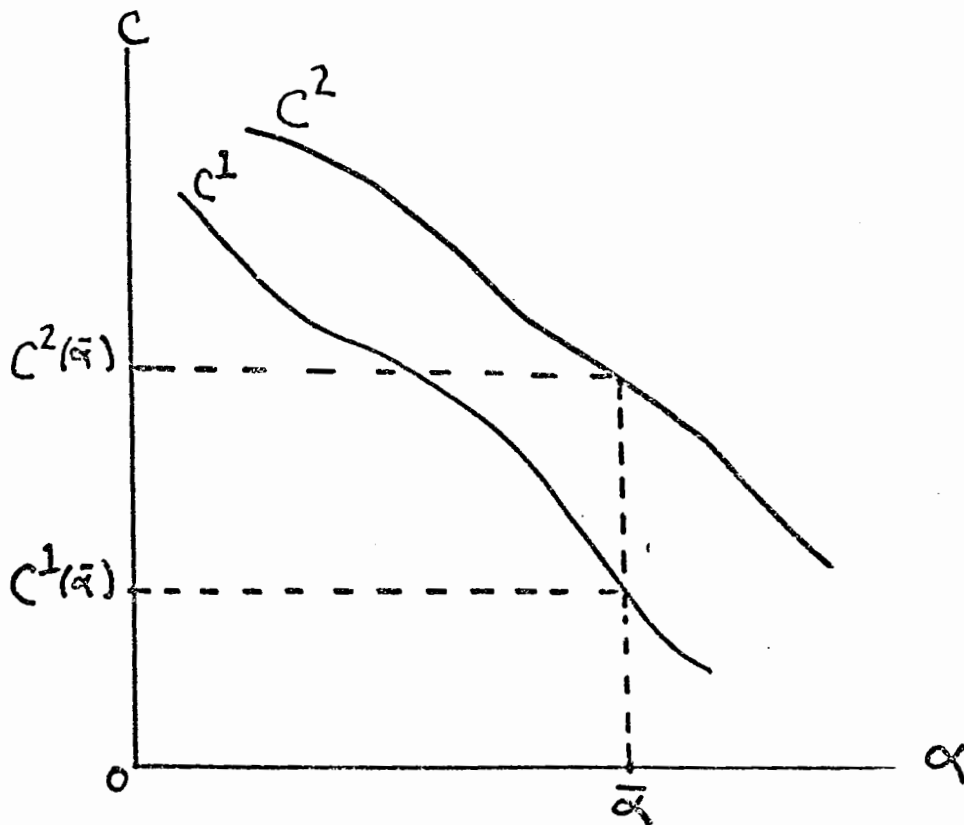


FIGURE 4

so long as there is social stratification by race. Indeed, the greater is the degree of racialism, the more will the cost of becoming skilled increase for black worker with a given decrease in r .

Figure 4 depicts cost as a function of innate capacities for individuals of two different socioeconomic backgrounds. Each curve holds socioeconomic background constant and considers the effect of α on costs. The diagram illustrates the assumption that more capable individuals may acquire the characteristics of skilled workers at a lower expense than the less capable. It is readily seen that the socioeconomic background corresponding to curve C^1 is more favorable for young people than that represented by C^2 . Assuming the degree of social stratification constant, this difference could reflect either (1) the advantage of having greater parental income; (2) the advantage of being white rather than black - where mature whites earn more on average and there is racialism; or (3) the relative advantage to being black in a racialistic society when the extent of racial income differences has been lessened (i.e. r has increased). In either event, a more favorable socioeconomic background will imply a decrease in cost for equally capable individuals. This is shown in Figure 4 by the fact that $C^1(\alpha) < C^2(\alpha)$.

It is now possible to determine exactly when an individual will choose to become a skilled worker. As pointed out above, an individual with infinite costs cannot gain skilled employment, and someone with zero costs will always be employed as a skilled worker.

We will assume that these cases are not the norm, however.⁴⁷ Most people may gain skilled employment if they are willing to make the necessary sacrifice during their youth. The extent of sacrifice necessary for any individual depends on his innate endowment and socioeconomic background, and is measured in monetary terms by the cost of becoming skilled. Suppose that at the beginning of each period of time (i.e. each generation) firms announce the wages they will pay to skilled and unskilled workers (\bar{w} and \underline{w} respectively) in the subsequent period. The wage differential, w , is the payoff to a young person for incurring the cost of becoming skilled. Since people choose characteristics to maximize their well being, they will become skilled workers if and only if the payoff to doing so exceeds the cost.

We may then summarize a young person's choice of characteristics as follow: First he considers the most valuable of all characteristic bundles attainable by him. He then considers the best bundle he can acquire which also suffices to gain him skilled employment. The cost of becoming skilled is the difference between the values of these two bundles. Only if the extra wages which he could earn by becoming skilled exceed this cost, will he choose that bundle which qualifies him for skilled employment. This situation is depicted in Figure 5. With cost measured on the upward vertical axis and capability on the horizontal axis, the figure depicts the cost curves C^1 and C^2 representative of two distinct socioeconomic backgrounds. The wage differential announced by firms for next period's employment is given as w . Since cost declines

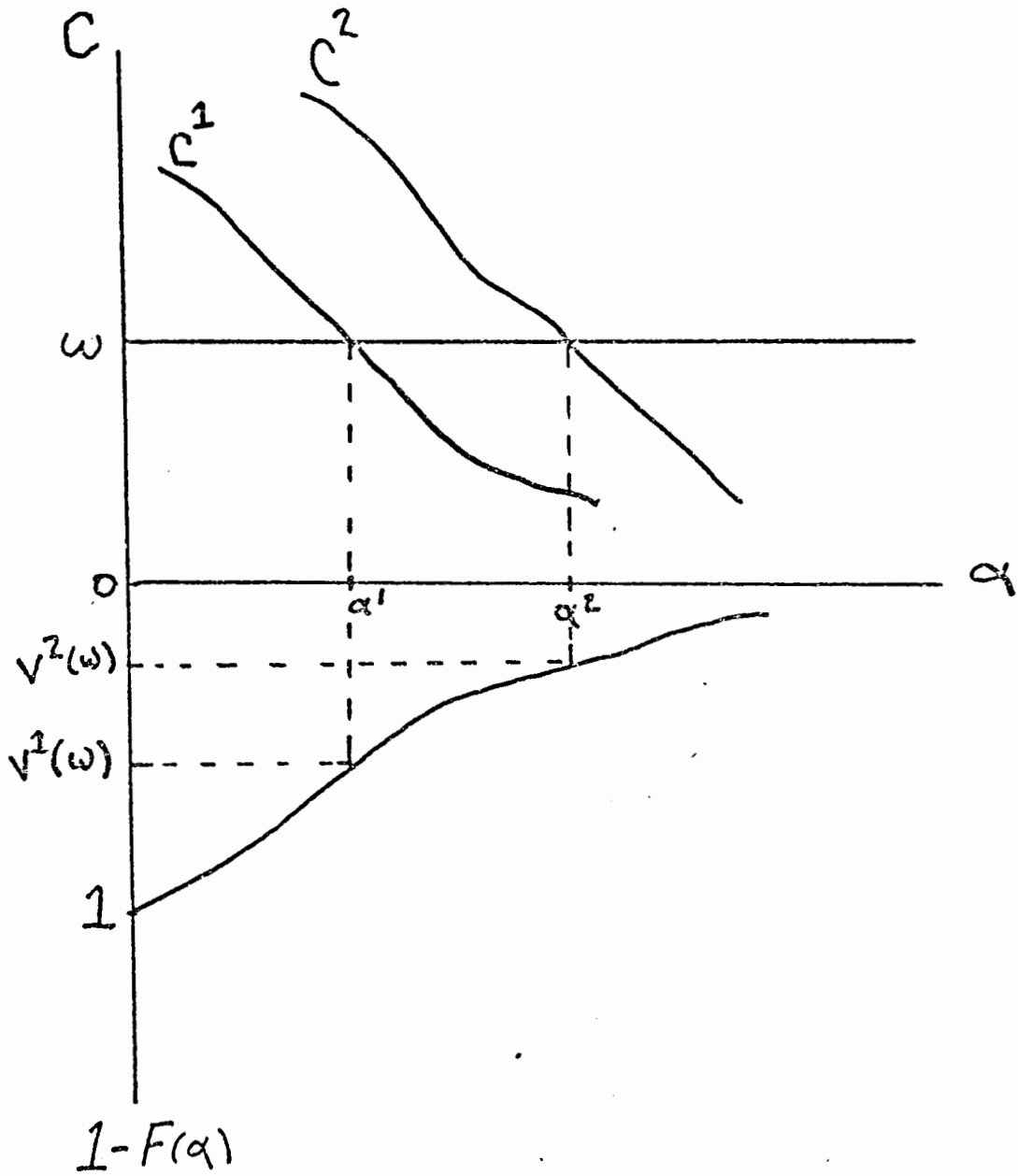


FIGURE 5

with increasing innate capability, to each socioeconomic background there corresponds a critical level of innate ability with the property that anyone of that background with capability greater than this critical level will become skilled.

Obviously this critical level of capability is determined by the requirement that the cost of becoming skilled to a person of the given socioeconomic background endowed with the critical level of innate capability be equal to the wage differential offered by firms. α^1 and α^2 satisfy this requirement for the socioeconomic backgrounds represented by cost functions C^1 and C^2 respectively in Figure 5. Hence in each generation, given an offered wage differential, any group of young individuals with the same socioeconomic background (i.e. race and parent's income) will divide according to innate endowments into those who become skilled and those who do not. The dividing point (i.e. critical level of capability) will be known, once the cost function for this group and the wage differential are known. Since the distribution of innate capabilities in the population is the same for each generation, race and social class, the exact number of individuals from this group who become skilled workers may be determined.

This process is also illustrated in Figure 5. Let $F(\alpha)$ be the cumulative distribution function of innate capability. Then $1-F(\alpha)$, measured on the downward vertical axis in the figure, represents the fraction of the population with innate capability greater than α . By our independence assumptions, this will also be the fraction of young individuals of a given socioeconomic background with innate

endowment greater than α . Thus, for the socioeconomic backgrounds represented by C^1 and C^2 in the figure, we may calculate the fractions of these groups that acquire skilled characteristics when the wage differential to skilled employment is w . These fractions depend on w and are given as $v^1(w)$ and $v^2(w)$ respectively in Figure 5. It is clear from the figure that v^1 and v^2 are increasing functions of w , and that for every w , $v^1(w) > v^2(w)$. That is, higher wage differentials for skilled workers will induce more young people to acquire the characteristics of skilled employees. Furthermore, one group with a more favorable socioeconomic background than another will always have a larger fraction of its young people qualified for skilled employment.

D. Static Equilibrium. It is now possible to completely characterize the static labor market equilibrium within a generation. An equilibrium occurs when supply is equal to demand for both skilled and unskilled workers. Despite the fact that two kinds of labor are involved, our assumptions enable us to analyze equilibrium in the simple supply-demand framework pictured in Figure 6. In this diagram the wage differential between skilled and unskilled workers is measured on the vertical axis, while the aggregate ratio of skilled to unskilled employment, denoted by ℓ , is given on the horizontal axis. The assumptions of competitive factor markets and constant returns to scale imply the downward sloping demand relation D depicted in the figure. Firms can be on their demand curves for both types of labor if and only if the

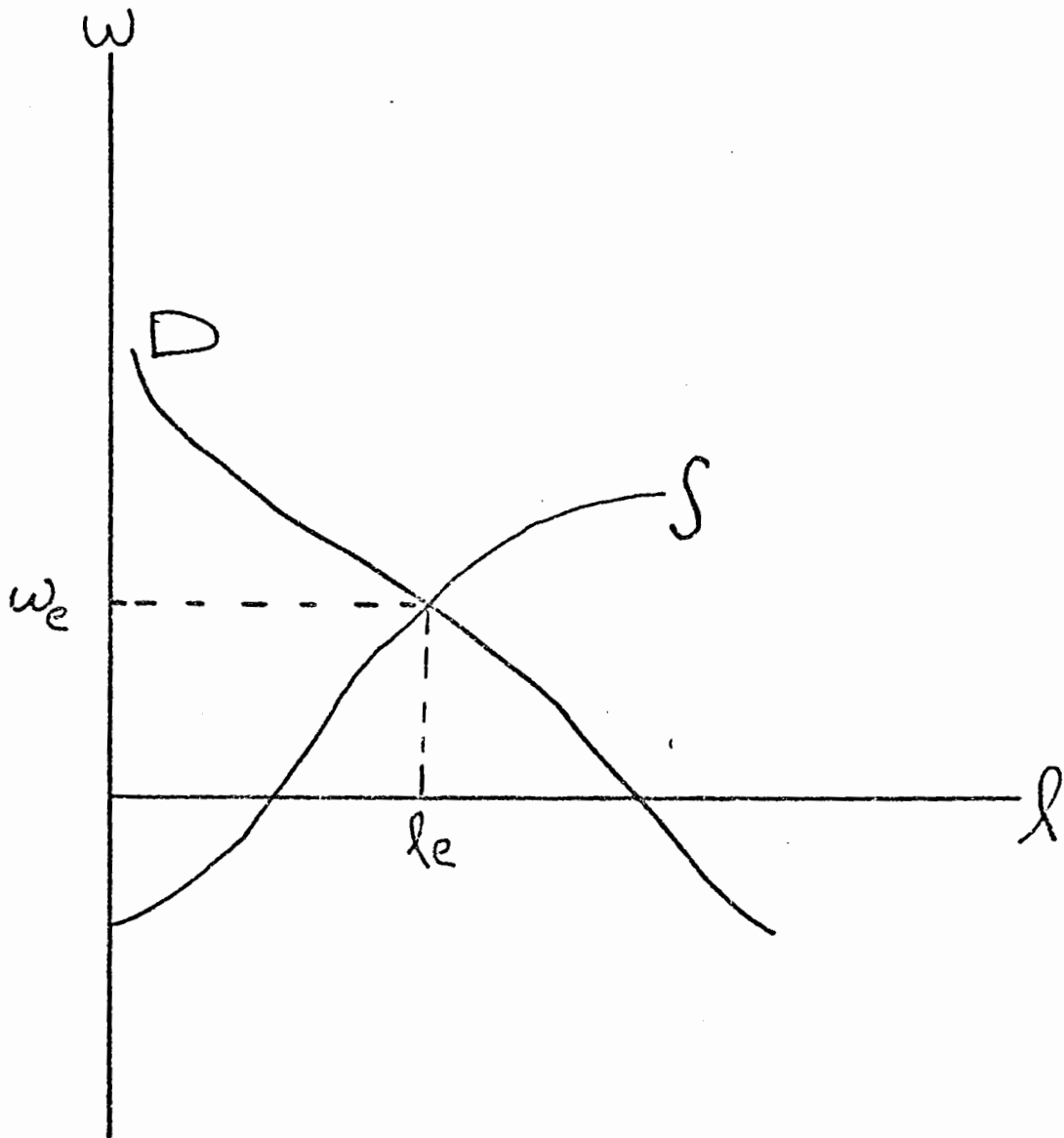


FIGURE 6

corresponding wage differential and employment ratio is on the curve D.

Since the decision of an individual to acquire skilled characteristic depends on his socioeconomic background, it is apparent that the supply of workers to skilled occupations will depend upon the nature of the labor market equilibrium which occurred in the previous period (generation).⁴⁸ This is the result of the intergenerational externalities in the model. Accordingly, the static labor market equilibrium for any generation must always be conditional on that equilibrium which obtained for the preceding generation. In the next section we shall analyze the path which these equilibria follow over time. However, for now we need only observe that in order to specify the supply of skilled workers as a function of the wage differential (the S curve in Figure 6), two facts about the previous equilibrium must be known. What we must know are the fractions of the black and white mature population who acquired skilled characteristics in the previous period. This information is sufficient to determine the socioeconomic backgrounds of all of young individuals in the economy.

Assume that the extent of social stratification by income and race remains unchanged over time. Then we may determine the cost to becoming skilled as a function of innate ability for people of each of the four possible socioeconomic backgrounds. Recall from the previous discussion that this cost function depends only on parental income for whites, and that for blacks it depends on parental income and (when there is racialism) the degree of racial

income differences. We shall show (in section VI) how the wages mature workers in skilled and unskilled employment as well as the degree of racial income differences may be deduced, once the fraction of each racial group to have acquired skilled characteristics in the previous period is known. Combining the knowledge of young individuals' cost functions with the methods employed in the analysis of Figure 5, one may construct the locus S given in Figure 6. This supply curve may be traced out by considering the fraction of young people with each of the four socioeconomic backgrounds who will acquire skilled characteristics at a given wage differential, and then varying the wage differential.

Thus we will have equilibrium, depicted by the intersection of demand and supply in Figure 6, when the fraction of young people who want to become skilled at a given wage differential equals the fraction of its work force which each firm desires to have as skilled employees at that wage differentials. Given the equilibrium wage differential, we may compute the fractions of the black and white young people who will be employed in skilled occupations in the subsequent period. By exactly the method employed above, knowledge of these fractions enables us to determine the labor market equilibrium among the next generation of young individuals. In this way we may generate a sequence of market equilibria and associated distributions of economic advantage, starting from any historically given initial situation.

To summarize the developments of this section, we have constructed an economic model of individual earnings determination in

which the structure of social organization directly effects the economic outcome. In preparing themselves for employment, individuals weigh the costs and benefits of alternative actions, choosing that which maximizes their well being. Their costs are directly influenced by their innate capability and their socioeconomic background. The nature of the impact of socioeconomic background on achievement is strongly conditioned by the degree of stratification in the society along income and racial lines. If community associations tend to divide sharply along these dimensions, then the absence of economic success of a young person's parent and or racial group becomes a serious liability to that individual's achievement. The more representative is the composition of each community, the greater is the weight placed on an individual's innate capacities in determining his degree of success. In any event, the distribution of economic advantage within any generation depends on the distribution which obtained in the preceding generation. We now turn to an analysis of the dynamic implications of these observations.

IV. Dynamic Analysis of the Model

A. Equal Opportunity and Racial Justice. We are now in a position to shed some light on the question: "What is the appropriate public policy with regard to the elimination of racial economic differences?" A complete answer to this question hinges crucially on what one intends by use of the word "appropriate". We observed in the introduction to this essay that this is in large part a value judgement about which reasonable people may differ.

However, we suggested as compelling the following minimal requirement which any "appropriate" policy must meet: Any public posture toward racial income differences must not permit the effects of past discrimination to be reflected in a permanent earnings gap between blacks and whites. We shall refer to this requirement as the Weak Criterion of Racial Justice. Our purpose in this section is to determine whether, and under what circumstances, the Equal Opportunity Doctrine satisfies this reasonable, minimal criterion.⁵⁰

We envisage the following scenario: Historically there has been discrimination against blacks and denial of equal opportunity as defined above. Society then reaches an enlightened moral state where these practices are deemed reprehensible and no longer permitted. The Equal Opportunity Doctrine is then adopted. However there remains an historical legacy of past discriminatory acts in the form of current earnings differences between the groups. Since equal opportunity allows home and community environment to affect a young person's opportunities, the historical practice of discrimination will impact on the opportunities and hence subsequent earnings of young blacks. The central question then becomes whether or not this intergeneration effect would enable the historically generated group earnings differences to be sustained indefinitely. That is, we wish to determine whether or not equal opportunity, in the sense employed here, necessarily implies equal results in the long run. Of course, failure to satisfy the Weak Criterion of Racial Justice does not mean that the Equal Opportunity Doctrine should be abandoned. It does imply, however, that appropriate public

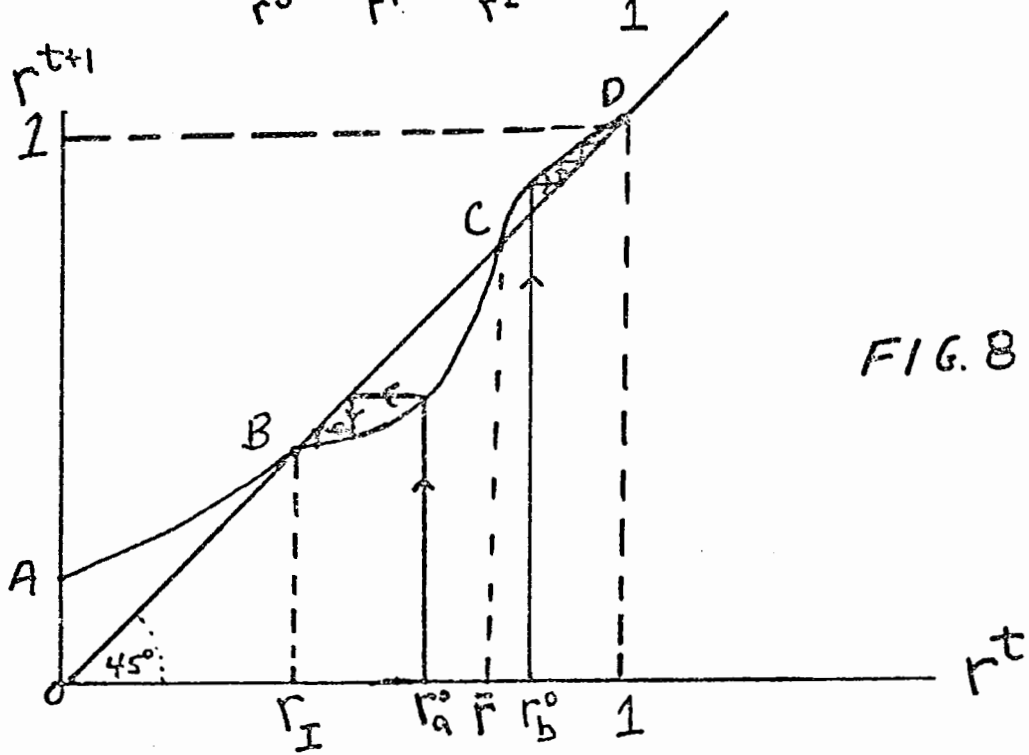
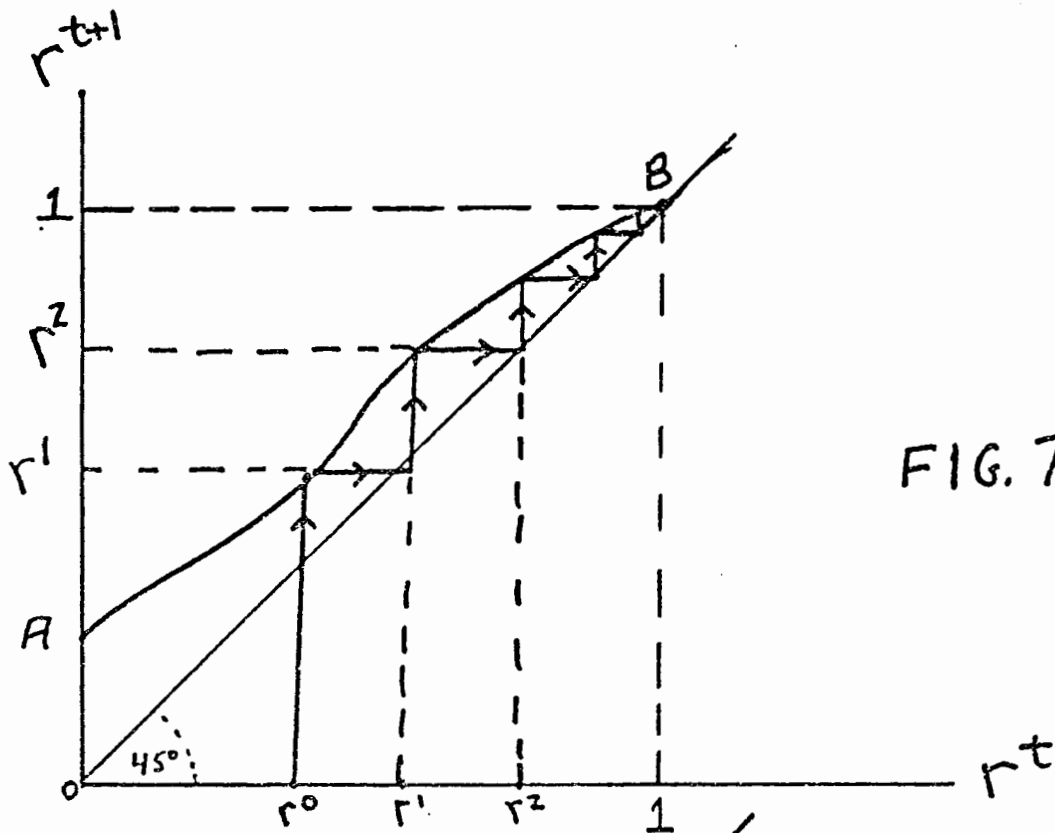
policy must supplement the Doctrine in some fashion to overcome its shortcomings.

In the previous section we saw how the equilibrium wages and employment levels of skilled and unskilled workers could be determined for any generation. The only knowledge necessary for that determination, we discovered, is awareness of the fractions of black and white workers who were employed in skilled occupations in the previous generation. Our analysis of equilibrium under the Equal Opportunity Doctrine revealed that a dynamic relation could be defined, which would enable us to determine the fractions of blacks and whites in skilled employment in any subsequent generation, knowing only the state in which the economy started.

Using these facts we may address our central question in the following manner. We suppose that the currently observed earnings differences between the races may be represented in the model as a smaller initial fraction of blacks than whites employed in skilled occupations. Given this initial data, we trace out the future path of the black and white economic position, employing the methods described above. In each subsequent generation (say the t^{th}) we note the value taken by the index of racial income differences, r^t . (We remind the reader that r^t represents the ratio of the average income of mature blacks to the average earnings of mature whites in the t^{th} generation. This ratio may be determined once the fractions of blacks and whites employed in skilled occupations in that generation is known.) Since blacks have been discriminated against in the past, r^0 will be less than one. The Weak Criterion of Racial Justice is satisfied only if r^t approaches one, for t sufficiently large. This would mean that racial economic differences eventually become negligible.

We may illustrate this mode of analysis with the aid of Figure 7. This figure summarizes the dynamic relation of labor market equilibria across generations. The horizontal axis represents the index of racial income differences in an arbitrary generation (say the t^{th}). The vertical axis measures the extent of racial economic disparity in the immediately subsequent generation (the $(t+1)^{\text{st}}$). The locus AB summarizes the relationship between these two quantities. Notice that in order for this graphical analysis to be valid, this relationship must not change over time. This reflects the assumption, which we shall be making throughout, that the social structure (i.e. extent of income and racial stratification) remains unchanged over time. Thus, what follows is an analysis of the effects of a given set of social relations on the evolution of racial economic positions. The study of changing social attitudes remains beyond the scope of this effort.

Suppose that the current racial income differences may be represented by the point r^0 on the horizontal axis in Figure 7. By following a vertical line upward from r^0 to the curve AB, we see that r^1 , given on the vertical axis, will be the index of racial differences in the following generation. If one then traces a horizontal line from the point r^1 on the vertical axis to the 45° line, it becomes possible to see that the extent of racial income differences will be two generations hence. This is given as r^2 in the figure. Continuing in this way, the entire future evolution of relative economic positions may be determined. The



FIGURES 7 & 8

path starting from r^0 is indicated in the figure. It is apparent that this path leads to the eventual elimination of racial income differences, since r^t will eventually become negligibly different from one. Moreover, inspection of the diagram will show that any initial position will determine a path with the same long run consequence. That is, no matter how great the initial disadvantage of the black population, the dynamic process of income determination illustrated by AB in Figure 7 will lead to an eventual equalization of racial economic positions. Whenever this is the case, the Weak Criterion of Racial Justice will be satisfied.

Another possibility is illustrated in Figure 8. Here the intergenerational relation of relative economic positions is depicted by the locus AD. Inspection of the diagram reveals that the long run evolution of racial income differences depends critically upon the starting position. If past discrimination has not been too severe, so that the initial index of racial earnings disparity is greater than \bar{r} , then eventual equality may be expected. A representative path, beginning at r_b^0 , in the figure, illustrates this point. On the other hand, if history has been less charitable, leading to an initial earnings gap such as that represented by r_a^0 in the diagram, then black-white income differences will persist indefinitely and may even become exacerbated over time. It is readily seen that any initial earnings ratio less than \bar{r} will, in the long run, lead to the ratio r_I , which represents permanent inequality. In this instance the Weak Criterion is not satisfied.

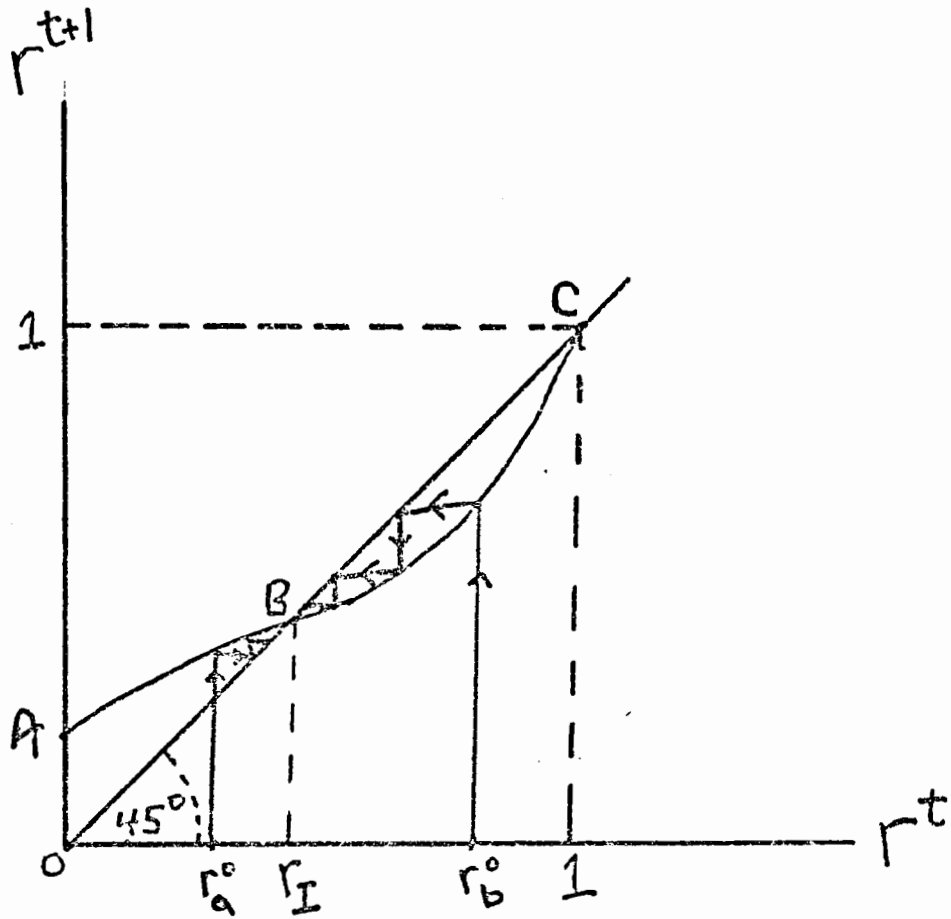


FIGURE 9

Racial justice is seen to depend on the exigencies of historical development.

An extreme example of this kind of failure of the Equal Opportunity Doctrine is illustrated in Figure 9. In the unlikely event that the locus AC should characterize the relation of relative economic positions across a generation, the slightest degree of initial inequality is sufficient to guarantee a permanent earnings gap. The social structure underlying the relationship depicted in the figure exhibits an inherent tendency toward inequality.

B. The Limits of the Equal Opportunity Doctrine. What are the reasons for these drastic differences in the long run performance of a laissez-faire economy devoid of racial discrimination? What factors determine whether a benign structure such as that illustrated in Figure 7, or an inequality preserving relation as in Figure 9, will come to pass? These questions are answered in the propositions presented below. Before stating these, however, it is possible to gain some insight into the forces at work here. Recall from the discussion of section III-C that there are three major influences on the ability of a young person to become a skilled worker. These are the effects of his family background, his community environment, and his endowment of innate capabilities. We have assumed that innate ability is identically distributed among blacks and whites. Hence any persistence of racial income differences in the face of equal opportunity must result from the first two effects.

When family background affects achievement, the fact that more black than white youngsters inherit poor family backgrounds due to past discrimination against their parents means that fewer blacks than whites will achieve the earnings of skilled occupations in the next generation. However, each generation's advancement (if indeed there is advancement) enables the next generation to start with less of a relative disadvantage. The cumulative effect of this process could be the elimination of differences in the average earnings of the two groups.

Community effects will only be important when there is some degree of social stratification. When there is stratification by income, having poor parents represents an even greater handicap than that discussed above. Historical discrimination implies that blacks will face this impediment more frequently than whites. Social stratification by race leads to racially homogeneous communities. In this case if there has been discrimination, the community effects for black and white youngsters, even with the same parental income, will differ. Consequently black parents who have succeeded will be less able than whites to assure the success of their children. Again, however, if the racial earnings gap continues to narrow under the Equal Opportunity Doctrine, then this effect will diminish over time.

Thus, there seem to be two separate though related elements which work to distinguish the kinds of dynamic relations of earnings over generations which can arise. The first is the extent of social stratification by income and race, which works to

determine the strength of the bond between parent's socioeconomic status and offspring's achievement. The other element is the ability of each generation of black workers to make progress on the position of the preceding generation, allowing over time a diminution of the handicap of historical discrimination. The significance of the social structure is illustrated in the following proposition:

Proposition I: Suppose that there is no racialism in the society, so that social stratification occurs only along income lines. Assume that whenever a parent's income is increased by a dollar, the cost to his offspring of acquiring skilled characteristics is reduced by less than a dollar. Assume further that the greater is a parent's income, the less will a dollar increment to that income reduce the offspring's cost of becoming skilled. Under these conditions a policy of enforcement of the Doctrine of Equal Opportunity will satisfy the Weak Criterion of Racial Justice. That is, any historically generated differences in earnings between blacks and whites will diminish and tend to zero as time recedes indefinitely.

The implication of this proposition is that in a society in which one's race is socially irrelevant, and in which the practice of racial discrimination in the labor market is forbidden, differences in the economic positions of the races cannot persist. Before one concludes that it is transparent, note that conditions under which it is true. First it is required that parental economic

position not be so important that a given rise in parent's earnings leads to an even greater ultimate monetary benefit to the offspring. This seems quite a weak requirement if family environment is the only vehicle for the intergenerational external effect. If the rise in parental income causes a shift in community as well, and if the society is highly stratified by income, then this condition could be violated.

The second condition requires that the marginal benefit to young people of their parent's income not increase as parental earnings increase. This is a stronger condition and there is some (less than conclusive) evidence that it may not hold.⁵² Yet it is not altogether implausible. Examples may be constructed to show that there must exist a sufficiently strong range of "intergenerational increasing returns" before a group which starts with a large fraction of its members in unskilled employment may fail to catch up with a group which starts in a more favorable position. It should also be borne in mind that these conditions are sufficient, but not necessary for long run equality. Even if these conditions do not hold, the Weak Criterion may be satisfied. Thus, in the absence of racialism it is likely that equal opportunity will lead to the elimination of racial income differences. Faith in the free market is not without foundation.

Proposition I also provides a rigorous treatment of a question raised in the sociology literature of the late sixties. The question concerned whether or not the "inheritance of poverty", which blacks face more frequently than whites, could cause persistent racial inequality. By considering only linear

models, in which the assumptions of Proposition I are constrained to hold, the writers of this period answered with an unqualified "no".⁵³ As stated, the proposition above gives a precise set of conditions under which this answer is correct.

The assumption of no racialism in Proposition I is very strong, however. Racial stratification in our society is readily apparent to even a casual observer. The true test of the efficacy of the Equal Opportunity Doctrine is how it stands up in the presence of antagonistic social relations among racial groups. In order to isolate the impact of racialism, the following proposition considers an economy without income stratification. Moreover, family background does not affect offspring opportunities. It is assumed that community effects remain operative. We have already seen that under certain conditions, parental income effects alone cannot sustain racial economic differences. Unfortunately, for those who would restrict public action to enforcement of the Doctrine of Equal Opportunity, the consequences of racialism are not so benign. This is shown by the following proposition:

Proposition II: Suppose that there is no social stratification by income, and that family environment does not affect a young person's opportunities. Imagine, however, that social stratification by race is prevalent and that community external influences are also present. In such a situation, the Equal Opportunity Doctrine need not insure that any initial difference in group earnings will eventually become negligible. As such, the Weak Criterion of Racial Justice will, in

general, not be satisfied. Furthermore, eventual equality will result from establishing the Doctrine only if the relative economic position of blacks improves continually over time.

The first result of Proposition II is a negative one. It states that the presence of racialism is sufficient to obviate any necessary connection between equal opportunity (as defined in section III) and eventual equality for blacks. While the possibility cannot be ruled out that the favorable situation of Figure 7 in fact obtains, no assurance of this circumstances can be given when there is racialism. What ultimately happens will depend on the strength of community external effects (the importance of school quality and job market information, for example), and the extent of social stratification by race.

The final statement in the proposition yields further insights. It gives a specific test by which one can determine whether or not an observed economy satisfies the Weak Criterion. If, through the normal operation of the competitive labor market under equal opportunity, the immizerization of the relative economic position of blacks should ever occur, then there exist an historical disparity of sufficient magnitude that blacks will never gain equality if they start at any greater disadvantage. Given the simplicity of the model, however, this result is only suggestive of the more complex conditions under which the Doctrine may fail in reality. Particularly troublesome is the absence of unemployment and cyclical effects. Nonetheless, it would appear that the comfortable long run conclusions of the traditional liberal view are called fundamentally into question.

V. Conclusions

Several preliminary conclusions about the process of personal income determination and its regulation may be drawn from this socioeconomic analysis. This discussion has considered the problem of income distribution in an explicitly intertemporal framework. By doing so we have learned that, even in the absence of transfers of physical wealth within families, the economic advantages of an individual will only partially reflect his innate productive capacities. The facts that generations overlap and that individual development is influenced by the prevailing external environment imply that the pattern of ownership of resources today will influence the distribution of productive capacities among tomorrow's workers.

It follows that the creation of a viable work force is necessarily a social process. The meritocratic notion that in a free society each individual will rise to the level justified by his competence must be tempered with the observation that no one travels that road entirely on his own. The social context within which individual maturation occurs strongly conditions what equally competent individuals can achieve. These facts imply that absolute equality of opportunity, where a person's chance to succeed depends only on his innate capabilities, is an ideal which cannot be achieved. We have shown here that in at least one instance, the limited version of equal opportunity which is attainable does not have the desirable properties of the impossible ideal.

Traditional economic theory teaches that earnings differences among workers may be understood on the basis of individual differences

in the amounts of education, and work experience which workers possess. The notion of "human capital" has been invented to summarize investments such as these which are made in individuals. This focus on objective determinants of earnings disparities, while providing a convenient rationale for existing inequality, ignores the process by which such investments are made. Thus, human capital theorists can accurately predict the consequence of dropping out of high school on lifetime earnings, but have not analyzed why a given per capita expenditure yields a lower caliber education in the ghetto than in more affluent communities of the same school district.

An individual's social origin has an obvious and important effect on the amount of resources which are ultimately invested in his development. It may thus be useful to employ a concept of "social capital" to represent the consequences of social position in facilitating individual acquisition of (say) the standard human capital characteristics. While measurement problems abound, this idea does have the advantage of forcing the analyst to consider the extent to which individual earnings are accounted for by social forces outside the individual's control. However, for precisely this reason such analysis is unlikely to develop within the confines of traditional neoclassical theory.

We began this essay with a consideration of the current policy debate concerning the role of government in addressing the historically generated difference in living standard between majority and minority Americans. We have proposed as reasonable the requirement that any policy option considered must, at a minimum, satisfy the

Weak Criterion of Racial Justice. This criterion is wholly consistent with the Liberal principles which underlie the Equal Opportunity Doctrine. Nonetheless, we have discovered that in a racially stratified society, public policy which relies solely on the Doctrine will generally not be racially just.

Our results also imply that the widespread belief in the efficacy of equal opportunity to secure equal results for minority groups must not be adopted as conventional wisdom. Propositions I and II have demonstrated that this belief is only valid in a social context devoid of racialism. Melting pot theorists notwithstanding, this is unlikely to be an accurate characterization of the American social scene for some time to come.⁵⁴ Accordingly, the politically appealing practice of evening up the game now, so that all players have a fair chance, will not guarantee racial economic equality. This is hardly surprising when one considers that we've been playing with a stacked deck for the past several centuries.

Any successful policy must take explicit recognition of the systemic nature of the impediments to minority progress. We cannot rely on the eradication of racial income differences coming about as the result of ten million black Horatio Alger's "making it" in an unchanged American society. Compensatory efforts are suggested, within both the educational sphere and the world of work. It must be recognized that continued racial economic disparities, no matter how well they are accounted for by "objective" factors, reflect the social and economic consequences of historical inequity. It follows therefore that public responsibility does not end with the acknowledgement that racism is unjust or discrimination illegal. No one

committed to justice can be satisfied until we no longer live with the legacy of past discrimination.

VI. Mathematical Addendum*

A. A Formal Socioeconomic Model of Income Determination

Endowment: Each agent begins life with a random innate endowment.

Q is the endowment set, taken to be a subset of an arbitrary, finite dimensional Euclidean space. The stochastic assignment of innate endowments follows a probability law characterized by the probability measure ν on Q. The following assumptions on ν and Q are adopted.

A1: Q is a compact, convex subset of n-dimensional Euclidean space, E^n .

A2: ν is a positive measure defined on the Borel sets of E^n , absolutely continuous with respect to the Lebesgue measure μ on E^n with the properties:

$$\nu(Q) = 1; \text{ and } A \subset C, \mu(A) > 0 \Rightarrow \nu(A) > 0 .$$

We assume that each generation consists of an indefinitely large number of agents with innate endowments distributed in Q according to the law ν .

Remark: Note that the probability law followed by an agent's endowment is independent of his race and the economic status of his parents.

Characteristics: Young agents, in the process of education and socialization, acquire a bundle of characteristics, x. The set of all possible characteristics is called the characteristic set and is denoted by ζ . We make the following assumption on ζ :

A3: ζ is a convex subset of E^m .

* This section should be viewed as a formal development and rigorous justification of the ideas set out heuristically in sections III and IV above.

Technology of Characteristic Acquisition: Each young agent faces a restrictive set of possible characteristic bundles which he may acquire, given his innate endowment and socio-economic background. For socio-economic background fixed (see "Social Process" below) the technical possibilities are summarized by a technology \bar{T} ,* containing all feasible endowment-characteristic bundle pairs. Specifically we assume:

A4: A technology \bar{T} is a compact, convex subset of $Q \times \zeta$.

Social Process: The operative relationship between socio-economic background and technology is called a social process. A social process is a mapping T satisfying (in the absence of racialism)

$$T: R_+ \rightarrow \tau \quad \text{where } \tau \equiv \{ \bar{T} \mid \bar{T} \subset Q \times \zeta, \bar{T} \text{ compact, convex} \}.$$

Concerning T we assume the following:

A5: T is a convex, compact valued mapping, continuous, satisfying

$$y_1 > y_2 \quad \text{iff} \quad T(y_1) \supseteq T(y_2) \quad \text{and} \quad \text{Proj}_Q T(y) \equiv Q, \quad \forall y.$$

Here $\text{Proj}_Q T(y) \equiv \{ \alpha \in Q \mid (\alpha, x) \in T(y), \text{ for some } x \in \zeta \}$. Also $A \subseteq B$ means that A is a proper subset of B .

* See Figure 1 in section III above, which shows a section of a technology, for fixed innate endowment, as the area bounded by the axes and the locus AB.

Remark: T associates with any level of parent's income the technology of characteristic acquisition relevant to a young agent.

Racialism: In a society in which race matters the social process will differ for groups B and W (blacks and whites), because community associations reflect the underlying racial stratification in the society. We shall assume that the extent of the divergence between social processes for the two groups depends only on the relative deprivation of the subordinate group.

That is

A5': Let $\tilde{T}: R_+ \times [0,1] \rightarrow \tau$, with \tilde{T} continuous in both arguments. Then the social process for group W is time invariant and given by

$$T(y) = \tilde{T}(y,1) ,$$

while the social process for group B in any period depends on the relative income of group B mature agents in that period and is given by

$$T^B(y) = \tilde{T}(y,r)$$

where r is the ratio of group B to group W per capita income, assumed to lie in $[0,1]$. $T(y,r)$ satisfies A5 $\forall r \in [0,1]$ and furthermore

$$\tilde{T}(y,r_1) \subset \tilde{T}(y,r_2) \quad \text{if } r_1 \leq r_2 .$$

Define: Racialism is said to exist if whenever $0 \leq r_1 < r_2 \leq 1$, then

$$\tilde{T}(y,r_1) \subset \tilde{T}(y,r_2)$$

for all y.

Preferences: An agent's well being depends on his acquired characteristics x and his income from working I . We assume that the agent's preferences may be represented as follows:

A6: $W(x,I) = U(x) + I$ gives the lifetime utility of an agent with characteristics x and income I . $U(\cdot)$ is a continuous, real valued function on ζ .

Remark: We assume constant (unitary) marginal utility of income. $U(x)$ measures the "dollar equivalent" of a bundle of characteristics x . The entire analysis may be carried through with a more general utility function. However, little is gained thereby and the model becomes much less intuitive.

Technology of Production: Firms face a neoclassical production function $Y = F(L_1, L_2)$ where Y is total output and L_1 (L_2) is the number of skilled (unskilled) workers employed. Furthermore an employee is designated skilled if his characteristic bundle is an element of the acceptance set $\bar{A}\zeta$.

Concerning technology we assume:

A7: $F(\cdot, \cdot)$ is a first degree homogeneous, real valued, twice continuously differentiable function with the properties $F_1 > 0$, $F_2 > 0$, $F_{11} < 0$, $F_{22} < 0$, and $F_{11}F_{22} > F_{12}^2$.

A8: $\bar{A}\zeta$ is a closed, convex set.

Denote $F(\lambda) = F(\frac{L_1}{L_2}, 1)$ as output per unskilled worker, where λ represents the skilled-unskilled ratio in employment. Assumed competitive factor markets imply:

A9: $\bar{w} = f'(\ell)$ is the skilled wage when $\frac{L_1}{L_2} = \ell$, while $\underline{w} = f(\ell) - \ell f'(\ell)$ is the unskilled wage. Let $w \equiv \bar{w} - \underline{w}$

denote the income differential. Assume $f'(0) = \infty$, $f'(\infty) = 0$, and

$$\lim_{\ell \rightarrow \infty} f(\ell) = \infty.$$

Remark: The Equal Opportunity Doctrine is readily interpreted in this context to mean that the occupation and income of an individual is completely determined by his bundle of characteristics, independent of social group. Note also that output is taken as numeraire, with its price set at unity. Denote $A \equiv Q \times \bar{A}$.

Choice of Characteristic Bundle: In general an agent's choice of characteristics will depend upon his innate endowment, his parent's income and his race. The race distinction is suppressed here. Define

$$C(\alpha, y) \equiv [\text{Max}_{(\alpha, x) \in T(y)} U(x) - \text{Max}_{(\alpha, x) \in T(y) \cap A} U(x)].$$

The function is well defined by virtue of A5, A6, and A8. Note the identity of tastes. (See note 46). α and y act solely through the technology. The following lemma is trivially obvious.

Lemma 1: In any period an agent will qualify as skilled if

$C(\alpha, y) \leq \bar{w} - \underline{w} \equiv w$. (Here we assume that an indifferent agent becomes skilled.)

Remark: $C(\alpha, y)$ may be interpreted as the (psychic) cost to an agent with endowment α and parent's income y of acquiring skilled qualifications. The lemma says that he will do so only if the benefits outweigh the costs. Note that in general $C(\cdot, \cdot)$ will be different for groups B and W since $T(\cdot)$ is. To capture the notion that as parent's income increases obtaining the qualifications of a manager becomes less onerous, we assume:

A10: $C(\alpha, y)$ is strictly decreasing in y .

A further important property of $C(\cdot, \cdot)$ is given in the following:

Lemma 2: $C(\alpha, y)$ is continuous individually in α and y .

Proof:

Define

$$Z^1(\alpha, y) \equiv \text{Max}_{(\alpha, x) \in T(y)} U(X)$$

$$\text{and } Z^2(\alpha, y) \equiv \text{Max}_{(\alpha, x) \in T(y) \cap A} U(X)$$

We shall show $Z^i(\alpha, y)$ continuous in α and y , $i = 1, 2$.

That $Z^1(\alpha, y)$ is continuous in y follows from the continuity of the correspondence $T(y)$, the fact that if $T(y)$ is continuous in y , then

$\bar{T}(y) \equiv T(y) \cap A$ is a continuous correspondence when A is closed^{*}, and the well known result that the maximum value of a continuous function over a set which varies continuously with respect to some parameter is itself a continuous function of that parameter (Debreu [48], thm 1.8 k, p. 19).

Slightly more work is required to show continuity in α .

Define

$$H(\alpha, y) \equiv \{x \mid (\alpha, x) \in T(y)\}.$$

Fix $y = \bar{y}$. Then we show $H(\alpha, \bar{y})$ is a continuous correspondence in α . The result then follows from the remarks immediately above. We shall show $H(\alpha, \bar{y})$ is upper and lower hemicontinuous in α .

upper hemicontinuity: Let $\alpha_n \rightarrow \alpha$, $x_n \in H(\alpha_n, \bar{y})$ and $x_n \rightarrow x$. Then $(\alpha_n, x_n) \rightarrow (\alpha, x)$ and $(\alpha_n, x_n) \in T(\bar{y}) \forall n$. But $T(\bar{y})$ compact $\Rightarrow (\alpha, x) \in T(\bar{y}) \Rightarrow x \in H(\alpha, \bar{y})$.

lower hemicontinuity: Let $\alpha_n \rightarrow \alpha$, $x \in H(\alpha, \bar{y})$. Let $\lambda_n > 0$, $\lambda_n \rightarrow 0$, where $\lambda_n \in \mathbb{R} \forall n$. By A5 $\exists x_n \in \zeta \exists (\alpha_n, x_n) \in T(\bar{y}) \forall n$. Now define $x'_n = \lambda_n x_n + (1 - \lambda_n)x$; $\alpha'_n = \lambda_n \alpha_n + (1 - \lambda_n)\alpha$. Then convexity (A4) implies $(\alpha'_n, x'_n) \in T(\bar{y})$, and $\alpha'_n \rightarrow \alpha$, $x'_n \rightarrow x$.

Since T is closed, $(\alpha, x) \in T$ or $x \in H(\alpha, \bar{y})$. Thus $H(\alpha, \bar{y})$ is lower hemicontinuous in α . Hence $H(\alpha, \bar{y})$ is continuous in α for fixed \bar{y} , and the lemma is proved. Q.E.D.

* It follows from A11 that $T(y) \cap A \neq \emptyset \forall y \in \mathbb{R}_+$.

Define: $C^{-1}(w,y) \equiv \{\alpha \in Q \mid C(\alpha,y) \leq w\}$. Note that Lemma 2 implies along with A2 that $C^{-1}(w,y)$ is a v -measurable subset of Q for all (w,y) .

Define: $v(w,y) \equiv v(C^{-1}(w,y))$.

Remark: Clearly $v(w,y)$ is the fraction of a social group (either W or B) with parent's income y who will become skilled if they face the wage differential w .

Lemma 3: $v(w,y)$ is continuous and strictly increasing in both arguments.

Proof :

$v(w,y) \equiv v(C^{-1}(w,y))$ where $C^{-1}(w,y) \equiv \{\alpha \in Q \mid C(\alpha,y) \leq w\}$

obviously $w_1 > w_2 \Rightarrow C^{-1}(w_1,y) \supseteq C^{-1}(w_2,y)$, hence by A2 we have strict monotonicity in w . Furthermore A10 $\Rightarrow [y_1 > y_2 \Rightarrow C^{-1}(w,y_1) \supseteq C^{-1}(w,y_2)]$,

hence A2 \Rightarrow strict monotonicity in y . Below we show continuity in w .

A5 and exactly the same technique can be used to show continuity in y .

Let $w_n \rightarrow w$. For $y = \bar{y}$. Define

$$\Lambda_n \equiv C^{-1}(w,\bar{y}) \cap [C^{-1}(w_n,\bar{y})]^c \quad w_n \leq w$$

$$\equiv [C^{-1}(w,\bar{y})]^c \cap C^{-1}(w_n,\bar{y}) \quad w_n > w$$

where X^c is the complement of X . Now the continuity of $C(\alpha, \bar{y})$ in α (Lemma 2) implies $\mu(A_n) \rightarrow 0$ (μ is the Lebesgue measure). Then $\nu(A^n) \rightarrow 0$ by absolute continuity (A2) (see Friedman [49], Thm 2.12.2, p. 68). But

$$C^{-1}(w, \bar{y}) = C^{-1}(w_n, \bar{y}) \cup A^n, \quad w_n \leq w, \text{ and}$$

$$C^{-1}(w, \bar{y}) \cup A^n = C^{-1}(w_n, \bar{y}), \quad w_n > w.$$

Hence

$$\nu(C^{-1}(w_n, \bar{y})) \leq \nu(C^{-1}(w, \bar{y})) = \nu(C^{-1}(w_n, \bar{y})) + \nu(A^n), \quad w_n \leq w,$$

and

$$\nu(C^{-1}(w, \bar{y})) < \nu(C^{-1}(w_n, \bar{y})) = \nu(C^{-1}(w, \bar{y})) + \nu(A^n), \quad w_n > w.$$

Taking $n \rightarrow \infty$, $\nu(A^n) \rightarrow 0$. Hence $w_n \rightarrow w \Rightarrow \nu(w_n, \bar{y}) \rightarrow \nu(w, \bar{y})$.

The lemma is proved.

Q.E.D.

Concerning $v(.,.)$ we adopt the following assumption:

All: 1) $\forall (w,y) \in \text{Int } R_+^2, v(w,y) > 0$

2) $\lim_{w \rightarrow 0} v(w,y) = 0, \forall y; \lim_{w \rightarrow \infty} v(w,y) = 1, \forall y.$

Thus we assume that someone will always choose to be skilled if there is a positive wage differential, that no one will so choose if the wage differential is zero, and that everyone has his price! Note that this implies $\forall y$ that $T(y) \cap A \neq \emptyset$.

Remark: Clearly when racialism obtains the cost function for group B agents will depend on r . That is

$$C^B = C^B(\alpha, y, r) \equiv [\text{Max}_{(\alpha, x) \in T(y, r)} U(x) - \text{Max}_{(\alpha, x) \in T(y, r) \setminus A} U(x)].$$

Assume:

A12: $C^B(\alpha, y, r)$ is decreasing in r , when there is racism. Furthermore $\forall r \in [0, 1], C^B(\alpha, y, r)$ satisfies A_{10} .

Define: $v^B(w, y, r) = v(C^{-1}(w, y, r))$. For r fixed, v^B satisfies All.

Lemma 4: For fixed $r \in [0, 1], C^B(\alpha, y, r)$ and $v^B(w, y, r)$ as functions of α, y , and w satisfy Lemmas 1-3. Furthermore, $C^B(\alpha, y, r)$ is continuous in r , and $v^B(w, y, r)$ is continuous and (under racialism) increasing in r .

Pf: The first statement is an immediate consequence of the provision of $A5'$ that $T(y, r)$ satisfy $A5$ as a function of $y, \forall r \in [0, 1]$. The second

statement follows from the proofs of Lemmas 2 and 3, and the symmetry in r and y of assumptions on \tilde{T} . Q.E.D.

Notation: m_B = fraction of group B mature agents in managerial class

m_W = similarly for group W.

b = fraction which group B represents in the population

$r = \frac{wm_B + w}{wm_W + w}$ = ratio of B income to W income.

a superscript "t" denotes time period t.

Define: The state of the economy in any period t is the pair

$$(m_B^t, m_W^t) \in [0,1]^2.$$

Remark: It should be apparent that all other relevant variables in the model are given, once the state is known. The following relations summarize this fact:

$$\ell^t = \frac{bm_B^t + (1-b)m_W^t}{1 - [bm_B^t + (1-b)m_W^t]}$$

$$\bar{w}^t = f'(\ell^t); \quad \underline{w}^t = f(\ell^t) - \ell^t f'(\ell^t); \quad w^t = \bar{w}^t - \underline{w}^t$$

$$r^t = \frac{w^t m_B^t + \bar{w}^t}{w^t m_W^t + \bar{w}^t}$$

Equilibrium: Conditional on the state of the economy in period $t-1$ (m_B^{t-1}, m_W^{t-1}) , an equilibrium for the economy in period t is a state $(m_B^t, m_W^t) \in [0,1]^2$ which satisfies:

$$(1) \quad \ell^t = \frac{bm_B^t + (1-b)m_W^t}{1 - (bm_B^t + (1-b)m_W^t)}$$

$$(2) \quad w^t = (1 + \ell^t)f'(\ell^t) - f(\ell^t)$$

$$(3) \quad m_B^t = m_B^{t-1} v^B(w^t, \underline{w}^{t-1}, r^{t-1}) + (1 - m_B^{t-1}) v^B(w^t; \underline{w}^{t-1}, r^{t-1})$$

$$(4) \quad m_W^t = m_W^{t-1} V(w^t, \underline{w}^{t-1}) + (1 - m_W^{t-1}) V(w^t, \underline{w}^{t-1}),$$

Remark: Note that the above are implicit relations for (m_B^t, m_W^t) to be an equilibrium. Also, when there is no racism, the function v^B is independent of r and identical to the function v . We assume the economy is perpetually in equilibrium. Existence of equilibrium is addressed below.

We are now ready to examine the dynamics of the model. Our focus will be on specifying conditions under which B and W incomes will equalize as time recedes indefinitely. For this purpose we need the following definitions:

Define: An economy is said to be characterized by global asymptotic equality (GAE) if $\forall (m_B^0, m_W^0)$,

$$\lim_{t \rightarrow \infty} (m_B^t, m_W^t) = (m, m), \text{ for some } m \in (0, 1).$$

Thus GAE implies racial income differences eventually become negligible. Its relation to the Weak Criterion of Racial Justice is immediate. It is obvious that if $r^0 < 1$, then $r^t < 1 \forall t$. Thus the subordinate group (B) always remains behind if it starts behind, though the discrepancy may become arbitrarily small. We shall assume throughout that $r^0 < 1$, i.e. $m_B^0 < m_W^0$. This initial discrepancy may be viewed as a remnant of the less enlightened days before the adoption of the Equal Opportunity Doctrine.

B. Theorems

We may now state and prove the formal results underlying Propositions I and II of section IV above.

Theorem I: Under assumptions A1-A12 given any state $(m_B^{t-1}, m_W^{t-1}) \in (0,1)^2$ there exists a state $(m_B^t, m_W^t) \in (0,1)^2$ such that (m_B^t, m_W^t) is the unique equilibrium for the economy in period t , conditional on (m_B^{t-1}, m_W^{t-1}) . Furthermore, if $m_B^{t-1} < m_W^{t-1}$, then $m_B^t < m_W^t$.

Proof:

Let $(m_B^{t-1}, m_W^{t-1}) \in (0,1)^2$ be given. Then so too are r^{t-1} , \bar{w}^{t-1} , \underline{w}^{t-1} , and w^{t-1} . Consider now eqs. (3) and (4) from the definition of equilibrium:

$$(3) \quad m_B^t = m_B^{t-1} v^B(w^t, \bar{w}^{t-1}, r^{t-1}) + (1 - m_B^{t-1}) v^B(w^t, \underline{w}^{t-1}, r^{t-1}) \equiv m_B(w^t; m_B^{t-1}, m_W^{t-1})$$

$$(4) \quad m_W^t = m_W^{t-1} v^W(w^t, \bar{w}^{t-1}) + (1 - m_W^{t-1}) v^W(w^t, \underline{w}^{t-1}) \equiv m_W(w^t; m_B^{t-1}, m_W^{t-1})$$

Now All, Lemma 3 and Lemma 4 imply $\forall (m_B^{t-1}, m_W^{t-1}) \in (0,1)^2$ that $m_{B,W}^t(w; m_B^{t-1}, m_W^{t-1})$ is continuous, strictly increasing, and satisfies

$$\lim_{w \rightarrow \infty} m_{B,W}^t(w; m_B^{t-1}, m_W^{t-1}) = 1, \quad \text{and} \quad \lim_{w \rightarrow 0} m_{B,W}^t(w; m_B^{t-1}, m_W^{t-1}) = 0,$$

where $m_{B,W}$ refers to either of m_B or m_W .

Let us consider then the set of all $(w, \ell) \in \mathbb{R}_+^2$ for which eqs. (1), (3) and (4) of the definition of equilibrium hold. Such (w, ℓ) satisfy the following equation (suppressing (m_B^{t-1}, m_W^{t-1})):

$$\ell = \frac{b m_B(w) + (1-b) m_W(w)}{1 - (b m_B(w) + (1-b) m_W(w))} \equiv S(w),$$

where $S(\cdot)$ is continuous, strictly increasing and satisfies

$$\lim_{w \rightarrow 0} S(w) = 0 \quad \text{and} \quad \lim_{w \rightarrow \infty} S(w) = \infty.$$

Similarly we may consider the locus of all $(w, \ell) \in \mathbb{R}_+^2$ satisfying eq. (2) of the definition of equilibrium. This requires

$$w = (1+\ell)f'(\ell) - f(\ell) \equiv D^{-1}(\ell).$$

Now $D^{-1}(\cdot)$ is continuous, strictly decreasing in ℓ , with

$$\lim_{\ell \rightarrow 0} D^{-1}(\ell) = \infty, \quad \text{and} \quad \lim_{\ell \rightarrow \infty} D^{-1}(\ell) = -\infty.$$

by virtue of A9. Hence $D^{-1}(\cdot)$ has an inverse, $D(\cdot)$, with the properties that: (2) holds if $\lambda = D(\omega)$; $D(\cdot)$ is continuous and strictly decreasing; and

$$\lim_{\omega \rightarrow \infty} D(\omega) = \infty, \quad \lim_{\omega \rightarrow 0} D(\omega) = \bar{\lambda} > 0.$$

It follows from the preceding that there exists a unique $(\omega^*, \lambda^*) \in \text{Int}(\mathbb{R}_+^2)$ for which

$$\lambda^* = D(\omega^*) = S(\omega^*).$$

It is clear by construction that the above equation holds when and only when the economy is in equilibrium. But then m_B^t and m_W^t are given uniquely by

$$m_B^t = m_B(\omega^*; m_B^{t-1}, m_W^{t-1}) \quad \text{and} \quad m_W^t = m_W(\omega^*; m_B^{t-1}, m_W^{t-1}).$$

Furthermore, it is apparent that for any $(m_B^{t-1}, m_W^{t-1}) \in (0,1)^2$ with $m_B^{t-1} < m_W^{t-1}$ we have $v^B \leq v$. Hence (3) and (4) \Rightarrow

$$\{m_B^{t-1} < m_W^{t-1} \Rightarrow m_B^t < m_W^t\}. \quad \text{Q.E.D.}$$

Remark: This proposition assures that starting from any initial state (m_B^0, m_W^0) , the economy will follow a unique, deterministic equilibrium path for the rest of time. Note that our focus throughout is on aggregates, and not the incomes of specific individuals, which are stochastic ex ante (by virtue of the random endowments). Below we characterize the asymptotics of the economy's equilibrium path under a variety of conditions.

Define: A social process is said to be well behaved if it satisfies the following two conditions:

- a) $\forall \Delta y > 0, \forall \alpha \in Q, \forall y_1 > 0$,
 $C(\alpha, y) - C(\alpha, y + \Delta y) < \Delta y$, and
- b) $\forall \alpha \in Q, \forall \Delta y > 0, \forall y_1, y_2 > 0$
 $(y_1 - y_2) [(C(\alpha, y_1) - C(\alpha, y_1 + \Delta y)) - (C(\alpha, y_2) - C(\alpha, y_2 + \Delta y))] \leq 0$

Notice that (a) implies a dollar increase in parent's income reduces any young agent's cost of becoming skilled by less than a dollar. (b) is equivalent to assuming that the payoff to a young agent of a given increase in parent's income is less, the greater is parent's income initially.

Theorem 2 : Suppose that assumptions A1-A12 hold, and that the social process of the economy is well behaved. Then in the absence of racialism the economy exhibits GAE.

Proof :

Let (m_B^0, m_W^0) be given. Consider the equilibrium path for the economy $\{(m_B^t, m_W^t)\}_{t=1}^{\infty}$ conditional on the given initial state. Define $x^t = bm_B^t + (1-b)m_W^t$, $\forall t$. Observe that the definition of equilibrium implies that $\{x^t\}_{t=1}^{\infty}$ satisfies the following implicit difference relation (in the absence of racialism):

$$(a1) \quad x^t = v(w(x^t), \bar{w}(x^{t-1})) \cdot x^{t-1} + v(w(x^t), \underline{w}(x^{t-1})) \cdot (1-x^{t-1})$$

where

$$\bar{w}(x) \equiv f' \left(\frac{x}{1-x} \right)$$

$$\underline{w}(x) \equiv f \left(\frac{x}{1-x} \right) - \left(\frac{x}{1-x} \right) f' \left(\frac{x}{1-x} \right), \text{ and}$$

$$w(x) = \bar{w}(x) - \underline{w}(x).$$

First we show $\exists x^* \in (0, 1) \ni \lim_{t \rightarrow \infty} x^t = x^*, \forall x^0$.

Notice that (a1) defines x^t as a function, say $h(\cdot)$, of x^{t-1} .

Thus $x^t = h(x^{t-1})$. Now Lemma 3 and A9 imply that $\lim_{x \rightarrow 0} h(x) = \bar{x}$, where \bar{x} is the root of $x = V(w(x), 0)$, and hence is strictly positive. Similarly let \hat{x} be the root of $w(\hat{x}) = 0$. Then $\lim_{x \rightarrow \hat{x}} h(x) < \hat{x}$. Since $h(\cdot)$ is continuous, $\exists x^* \in (0, \hat{x}) \ni h(x^*) = x^*$. We shall show that x^* is approached by all paths $\{x^t\}$ generated by (a1).

First we note that Lemma 3 implies $v(\cdot, \cdot)$ possesses first partial derivatives almost everywhere in R_+^2 . Thus we lose little generality by taking it to be differentiable.* Using the implicit function theorem we may calculate the derivative of $h(\cdot)$ at a stationary point x^* :

$$(a2) \quad \frac{dh}{dx}|_{x^*} = \frac{(\bar{v}-v) + x^*(\bar{v}_2 \frac{d\bar{v}}{dx}|_{x^*}) + (1-x^*)(v_2 \frac{dv}{dx}|_{x^*})}{1 - \frac{dw}{dx}|_{x^*} (x^* \bar{v}_1 + (1-x^*)v_1)}$$

(For notational convenience we denote by subscript the partial derivative of V with respect to the indicated argument. Also a "bar" above (below) the function V or any of its derivatives indicates the function is to be evaluated in the point (w, \bar{w}) ((w, \underline{w}) .) If this derivative is always less than one at a stationary point, then there can be only one stationary point. (Proof: $\frac{d}{dx}(x-h(x))|_{x^*} = [1 - \frac{d}{dx}h(x)]|_{x^*} > 0$. Hence $x = h(x)$ has only one root.) But for a well behaved social process this derivative is always less than 1. This may be seen by comparing terms in the numerator and denominator of the RHS of (a2). First it is clear that $\bar{v}-v < 1$, since $v \in [0,1]$ by definition. Furthermore,

$$\frac{d\bar{w}}{dx}|_{x^*} \cdot x^* \bar{v}_2 < 0 < - \frac{dw}{dx}|_{x^*} \cdot x^* v_1, \text{ so that } \frac{dh}{dx}|_{x^*} < 1$$

$$\text{if } (1-x^*)v_2 \frac{dv}{dx}|_{x^*} < - (1-x^*)v_1 \frac{dw}{dx}|_{x^*}. \text{ Clearly } - \frac{dw}{dx} > \frac{dv}{dx},$$

so that a sufficient condition for $\frac{dh}{dx}|_{x^*} < 1$ is that $v_2 < v_1$. That is, a sufficient condition for uniqueness of the stationary point x^* is that the fraction of young agents who qualify as skilled but have unskilled

* The proof goes through by considering left and right hand derivatives at points where $v(\cdot, \cdot)$ is not differentiable, but we avoid this tedium here.

parents increases less when parents' income is raised one unit than it increases when the wage differential is increased one unit (or equivalently, the cost of qualifying is reduced one unit). Now it is easy to see that this requirement follows from (a) of the definition of a well behaved social process. Hence x^* is unique. Notice that none of the above argument depended on the fact that x^* is a stationary point. Hence we conclude additionally that $h'(x) < 1$, $\forall x \in (0, \hat{x})$. The above considerations also establish that $\forall x \in (0, x^*)$, $h(x) > x$, while $\forall x \in (x^*, \hat{x})$, $h(x) < x$.

Consider the interval $(0, x^*)$. We shall now show that $h(\cdot)$ is strictly increasing on this interval. To see this, consider the derivative (a2) at an arbitrary point x in this interval. The denominator is clearly positive. The numerator will be positive if

$$-x\bar{v}_2 \frac{d\bar{w}}{dx}|_x < (1-x) \frac{v_2}{v_2} \frac{dw}{dx}|_x$$

or equivalent by

$$-\frac{x}{1-x} \frac{dw/dx}{d\bar{w}/dx}|_x < \frac{v_2}{v_2}$$

Readers familiar with capital theory will recognize that $\frac{dw/dx}{d\bar{w}/dx}$ is just the slope of the factor price frontier at x and hence equals the negative of the factor intensity. But this latter is precisely $\frac{x}{1-x}$. Thus the LHS of the above inequality is unity. Hence $h'(x) > 0$, $x \in (0, \hat{x})$ if

$$\bar{v}_2 < v_2$$

That is, if increasing all parents' incomes by one unit leads to the fraction of workers' offspring who qualify as managers increasing by more than the fraction of managers offspring who so qualify in the subsequent period. But this result is an immediate consequence of (b) of the definition of a well behaved social process. This together with the argument of the previous paragraph shows that

$$0 < h'(x) < 1, \quad \forall x \in (0, \hat{x}) .$$

Now suppose $x^t \in (0, x^*)$. Then it follows from all of the above that $x^t < x^{t+1} = h(x^t) = h(0) + \int_0^{x^t} h'(x) dx < h(0) + \int_0^{x^*} h'(x) dx = h(x^*) = x^*$. Hence, $x^0 \in (0, x^*) \Rightarrow \{x^t\}$ is monotonically increasing and bounded above by x^* . Hence it converges to some point \tilde{x} . Now the continuity of $h(\cdot)$ implies

$$\tilde{x} = \lim_{t \rightarrow \infty} x^{t+1} = \lim_{t \rightarrow \infty} h(x^t) = h(\lim_{t \rightarrow \infty} x^t) = h(\tilde{x}).$$

Hence \tilde{x} is a stationary point. By uniqueness $\tilde{x} = x^*$. The identical argument establishes that if $x^0 \in (x^*, \hat{x})$ then $\lim_{t \rightarrow \infty} x^t = x^*$. Hence, the system possesses global asymptotic stability and x^* is the unique attractor.

Recall the definition $x^t \equiv b m_B^t + (1-b) m_W^t$. That is, x^t is the fraction of the mature agents who are skilled workers in any period. Furthermore, $\bar{w}(x^t)$ is the wage of the skilled in period t and $\underline{w}(x^t)$ is the income of unskilled workers in the same period. Using either equation (3) or (4)

of the definition of equilibrium implies that $m_{B,W}^t$ must satisfy the following difference equation:

$$(a3) \quad m_{B,W}^t = m_{B,W}^{t-1} v(w(x^t), \bar{w}(x^{t-1})) + (1 - m_{B,W}^t) v(w(x^t), \underline{w}(x^{t-1})).$$

Note that this equation has variable (though convergent) coefficients.

Suppressing the group subscript, it is sufficient to show that for any m^0 ,

$$\lim_{t \rightarrow \infty} m^t = m^*$$

for the solution path for (a3), in order to establish GAE. This result is shown below.

$$\text{Define } a^t \equiv v(w(x^t), \bar{w}(x^{t-1})) - v(w(x^t), \underline{w}(x^{t-1}))$$

$$b^t = v(w(x^t), \underline{w}(x^{t-1})) \quad t = 1, 2, \dots$$

Notice that (a3) becomes

$$(a3') \quad m^t = a^t m^{t-1} + b^t$$

$$\text{and } \lim_{t \rightarrow \infty} a^t = a^* \equiv v(w(x^*), \bar{w}(x^*)) - v(w(x^*), \underline{w}(x^*))$$

$$\lim_{t \rightarrow \infty} b^t = b^* \equiv v(w(x^*), \underline{w}(x^*))$$

Furthermore, define

$$a_{\min}^t \equiv \inf_{t' \geq t} \{a^{t'}\}, \quad a_{\max}^t \equiv \sup_{t' \geq t} \{a^{t'}\}$$

and

$$b_{\min}^t \equiv \inf_{t' \geq t} \{b^{t'}\}, \quad b_{\max}^t \equiv \sup_{t' \geq t} \{b^{t'}\}.$$

Clearly $\lim_{t \rightarrow \infty} a_{\min}^t = \lim_{t \rightarrow \infty} a_{\max}^t = a^*$, and similarly $\lim_{t \rightarrow \infty} b_{\min}^t = \lim_{t \rightarrow \infty} b_{\max}^t = b^*$.

Now iterating (a3') yields $m^{t+\tau} = (\prod_{k=0}^{\tau} a^{t+k}) m^{t-1} + \sum_{k=0}^{\tau} (b^{t+k} \prod_{j=1}^{\tau-k} a^{t+j}) m^{t-1-k}$, $\tau=0,1,2,\dots$,

where $\prod_{j=1}^0 a^{t+j} \equiv 1$. The following bounds on $m^{t+\tau}$ are thus suggested:

$$(a4) \quad (a_{\min}^t)^{\tau+1} m^{t-1} + b_{\min}^t \left(\sum_{k=0}^{\tau} [a_{\min}^t]^k \right) < m^{t+\tau} < (a_{\max}^t)^{\tau+1} m^{t-1} + b_{\max}^t \left(\sum_{k=0}^{\tau} [a_{\max}^t]^k \right).$$

Letting $\tau \rightarrow \infty$ in (a4) gives

$$b_{\min}^t (1 - a_{\min}^t)^{-1} < \lim_{\tau \rightarrow \infty} m^{t+\tau} < b_{\max}^t (1 - a_{\max}^t)^{-1}.$$

If we let $t \rightarrow \infty$ above we find

$$\lim_{t \rightarrow \infty} m^t = b^* (1 - a^*)^{-1}$$

using the convergence of b_{\min}^t , b_{\max}^t , a_{\min}^t and a_{\max}^t . This result is independent of initial conditions, as inspection of the proof will show. Thus the fraction of either group holding managers' jobs asymptotically approaches a number depending only upon x^* . Hence the economy exhibits GAE and the theorem is proved.

Q.E.D.

Remark: This result supports Proposition I of section IV.

Define: \bar{Y}_B^t is defined as the mean income of group B mature agents in period t . \bar{Y}_W^t is defined analogously for group w . η_B^t is the elasticity of \bar{Y}_B^t with respect to r^{t-1} , and η_W^t is the elasticity of \bar{Y}_W^t with respect to r^{t-1} .

In keeping with the development of section IV, we now assume that there is no social stratification by income, and that the effects of parent's background are negligible. This enables us to focus on the effects of race.

A13: $\tilde{T}(y, r) \equiv T(r)$, $\forall y \in R_+$ is the social process for group B. $T(1)$ is the social process for group W. Furthermore $T(r_1) \subseteq T(r_2)$ whenever $0 \leq r_1 < r_2 \leq 1$. $T(\cdot)$ is assumed to satisfy all applicable assumptions adopted on $T(\cdot, \cdot)$ earlier.

We may now state

Theorem 3: The Equal Opportunity Doctrine may not be sufficient to insure GAE, when A1-A12 hold. Asymptotic equality will result from the application of the Doctrine if and only if the relative income of the subordinate group improves continually over time. A sufficient condition for racial justice when there is racialism is

$$\text{Condition 1: } |\eta_B^t| + |\eta_W^t| < 1, \text{ always.}$$

Proof:

We have assumed that parent's income does not affect offspring's opportunities. In this case we may rewrite the equilibrium conditions as follows:

$$(1') \quad x^t = bv^B(w(x^t), r^{t-1}) + (1-b)v(w(x^t))$$

$$(2') \quad w(x^t) \equiv \left(1 + \frac{x^t}{1-x^t}\right) f'\left(\frac{x^t}{1-x^t}\right) - f\left(\frac{x^t}{1-x^t}\right)$$

$$(3') \quad m_B^t = v^B(w(x^t), r^{t-1})$$

$$(4') \quad m_W^t = v(w(x^t))$$

and

$$(5) \quad r^t = \frac{w(x^t)v^B(w(x^t), r^{t-1}) + \underline{w}(x^t)}{w(x^t)v(w(x^t)) + \underline{w}(x^t)}$$

where $\underline{w}(x)$ is defined as in the proof of Proposition II. Now (1') and (2') define x^t as a function of r^{t-1} , say $x^t = g(r^{t-1})$. It is readily shown that $g: [0,1] \rightarrow [0,1]$, $g(0) > 0$, $g(1) < 1$, and (assuming again differentiability for v) $g'(r) > 0$ on $[0,1]$. Hence (5) may be rewritten as a difference equation in r :

$$(5') \quad r^t = \frac{w(g(r^{t-1}))v^B(w(g(r^{t-1})), r^{t-1}) + \underline{w}(g(r^{t-1}))}{w(g(r^{t-1}))v(w(g(r^{t-1}))) + \underline{w}(g(r^{t-1}))} \equiv H(r^{t-1})$$

Now assumption A5' implies that $H(1) = 1$. That is 1 is a stationary point of (5'), or equivalently, if racial parity in incomes were ever attained it would be sustained indefinitely. The following necessary and sufficient condition for GAE then follows immediately:

$$(a5) \quad \text{GAE} \Leftrightarrow [H(r) > r \quad \forall r \in [0,1]]$$

Condition (a5) follows from the following observation: $H(\bar{r}) = \bar{r}$ for $\bar{r} \in [0,1]$ implies the existence of at least one [not necessarily stable] stationary point for (5') in the interval $[0, \bar{r}]$, since $H(0) > 0$. Thus GAE is equivalent to the uninterrupted improvement in the relative income of the subordinate group over time. Moreover, it is clear from the proof of Theorem 2 that $H'(\bar{r}) < 1$ at any stationary point \bar{r} implies GAE, given that 1 is also a stationary point. Now

$$H'(r^{t-1}) = \frac{d}{dr^{t-1}} (\bar{Y}_B^t / \bar{Y}_W^t) = \frac{\bar{Y}_B^t}{\bar{Y}_W^t} \left(\frac{d}{dr^{t-1}} \bar{Y}_B^t / \bar{Y}_B^t \right)$$

$$- \frac{\bar{Y}_B^t}{\bar{Y}_W^t} \left(\frac{d}{dr^{t-1}} \bar{Y}_W^t / \bar{Y}_W^t \right)$$

Now if $r^{t-1} = \bar{r}$ is a stationary point, then $\bar{Y}_B^t / \bar{Y}_W^t = \bar{r}$, and

$$H'(\bar{r}) = \frac{\bar{r}}{\bar{Y}_B^t} \frac{d}{d\bar{r}} \bar{Y}_B^t - \frac{\bar{r}}{\bar{Y}_W^t} \frac{d}{d\bar{r}} \bar{Y}_W^t = |\eta_B^t| + |\eta_W^t| .$$

Thus, Condition 1 implies GAE.

Q.E.D.

Remark: This result supports Proposition II of section IV above. We see that equal opportunity can bring about racial equality when the cumulative effect of the relative economic position of mature blacks on the prospective incomes of young blacks and whites is sufficiently small. Such a condition would have to be shown to hold before any presumption of the satisfaction of the Weak Criterion could be valid.

C. An Example

In order to examine more concretely the conditions under which racial income differences may persist, we turn now to a particularly simplified version of the model. The analysis which follows, culminating in Theorem 4, should be viewed as metaphorical. The objective here is to determine the social and technological considerations upon which the asymptotic behavior of group incomes turn. In so doing it is hoped that empirically answerable questions will arise, lending further justification to our theoretical exercise.

We begin by respecifying the basic elements of the model given in section IIIb above. An agent's innate endowment is taken to lie in the unit interval, i.e. $Q = [0,1]$. Thus, $\alpha \in Q$ is simply a number which may be taken to represent the agent's innate ability or intelligence. We assume that each agent is assigned an ability at the beginning of life according to a random drawing from $[0,1]$, and that the random α is uniformly distributed on the interval. In this case, ν is the measure induced by the uniform distribution on the unit interval.

The characteristic set ζ is taken to be the first quadrant of the Euclidean plane. There are two characteristics, "work skill" and "artistic sensitivity" for example. Denote by $x = (x_1, x_2)$, the quantity of the two characteristics possessed by an agent. A technology is thus a subset of $[0,1] \times \mathbb{R}_+^2$ which, for any $\alpha \in [0,1]$, gives the set of attainable "skill-sensitivity" pairs. We shall assume that there is racialism, but abstract from the effects of social class (see Theorem 2). Thus, the social process will specify the technology faced

by B young agents when their parents' relative income is r . Specifically,

$$(5) \quad T(r) = \{(\alpha, x) \mid x_1 + x_2 \leq \underline{x}_1 + e_1 \alpha + e_2 r\}, \quad \underline{x}, e_1, e_2 > 0, \quad r \in [0, 1]$$

is the technology for young B agents conditional on parents' relative income. $T(1)$ is the technology faced by young W agents. Here \underline{x} is a positive constant, and e_1 and e_2 are parameters representing the effects of ability and background on opportunities.

We shall take "skill" and "sensitivity" to be perfect complements from the point of view of individuals. That is, their preferences are given by $U(x) = \min(x_1, x_2)$. An agent beginning with equal amounts of x_1 and x_2 will "enjoy" an increment to either characteristic if and only if it is accompanied by an increment to the other. Firms, on the other hand, like their employees to be skilled, but find "artistic sensitivity" to be a nuisance. As such, the acceptance set is specified so that an agent of no "sensitivity" requires some minimal account of "work skill" to become a skilled employee, but the higher his level of "artistic sensitivity," the higher becomes the minimal skill requirement. Thus:

$$(6) \quad \bar{A} = \{(x_1, x_2) \in \mathbb{R}_+^2 \mid x_1 \geq \underline{x}_1 + \gamma x_2\}; \quad \bar{A} \in [0, 1] \times \bar{A},$$

where $\gamma > 0$ measures the firm's aversion to "sensitivity," and \underline{x}_1 is the same constant as in the specification of the social process above.

We will retain the general representation of the firm's production possibilities given above in A7-A9. \underline{x}_1 is chosen to be the same in

(5) and (6) in order to satisfy All(1). Similarly it may be shown that All(2) implies that $\gamma \geq \frac{e_1 + e_2 - \underline{x}_1}{e_1 + e_2 + \underline{x}_1}$. Let $e = e_1 + e_2$. We shall assume

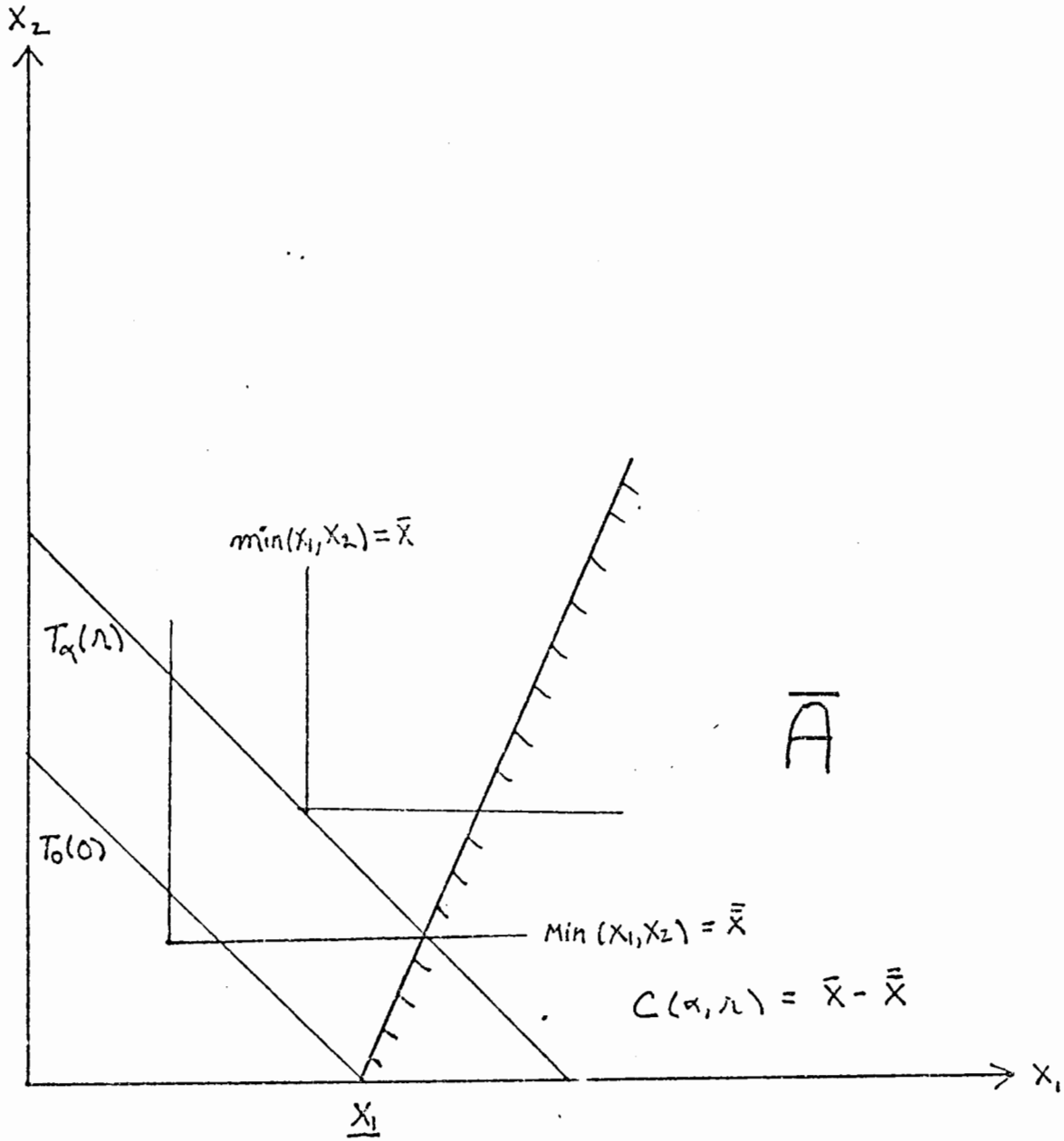


FIGURE 10

$\gamma = \frac{e^{-x_1}}{e^{+x_1}}$. The technology, and acceptance set faced by an agent are depicted in Figure 10.

Our specifications above imply that the cost function for a B young agent when B mature agents' relative income is r may be written:

$$(7) \quad C(\alpha, r) = \underset{(\alpha, x) \in T(r)}{\text{Max}} (\text{Min}(x_1, x_2)) - \underset{(\alpha, x) \in T(r) \cap A}{\text{Max}} (\text{Min}(x_1, x_2))$$

$$= \frac{x_1}{2} [1 - \hat{e}_1 \alpha - \hat{e}_2 r], \text{ where } \hat{e}_i = \frac{e_i}{e_1 + e_2}, \quad i = 1, 2.$$

A W young agent has cost function $C(\alpha, 1)$. Straightforward calculations also enable one to obtain the function relating the fraction of B young agents who become skilled to the prevailing wage differential and the relative income of B mature agents:

$$(8) \quad v(w, r) \equiv v(C^{-1}(w, r)) = \frac{1}{\hat{e}_1} \left[\frac{2w}{x_1} - \hat{e}_2(1-r) \right].$$

Again, the fraction of W young agents qualifying is simply $v(w, 1)$.

Determination of the function $v(\cdot)$ is shown in Figure 11.

It is now possible to specify the dynamics of the B-W ratio of mean incomes, r , as is done in the proof of Theorem

3 above. If the relative B income is known in period $t-1$ (r^{t-1}), then the fraction of the total mature population in skilled jobs in period t (x^t) is given implicitly by *

* Henceforth the notation "x" shall be used in this fashion, and not to denote agent characteristics. No ambiguity should arise.

$$(9) \quad x^t = bV(w(x^t), r^{t-1}) \div (1-b)V(w(x^t), 1),$$

where

$$(10) \quad w(x^t) = (1-x^t)^{-1} f' \left(\frac{x^t}{1-x^t} \right) - f \left(\frac{x^t}{1-x^t} \right)$$

gives the wage differential (w) which prevails when the fraction of the work force in the skilled jobs is x^t . Furthermore, if $\underline{w}(x^t)$ represents the earnings of unskilled workers when x^t is the fraction of the labor force in skilled employment, then the relative income of B workers in period t (r^t) is given by

$$(11) \quad r^t = \frac{w(x^t)V(w(x^t), r^{t-1}) + \underline{w}(x^t)}{w(x^t)V(w(x^t)) + \underline{w}(x^t)}$$

$$= 1 - \left[\frac{V(w(x^t)) - V(w(x^t), r^{t-1})}{V(w(x^t)) + \underline{w}(x^t)/w(x^t)} \right] \dots$$

Using (8) we may rewrite (9) and (11) for the special case under consideration as:

$$(9') \quad x^t = \frac{2w(x^t)}{\hat{e}_1 x_1} - \frac{b\hat{e}_2}{\hat{e}_1} (1-r^{t-1}), \quad \text{and}$$

$$(11') \quad r^t = 1 - \frac{\hat{e}_2}{\hat{e}_1} (1-r^{t-1}) \left[\frac{2w(x^t)}{\hat{e}_1 x_1} + \frac{w(x^t)}{w(x^t)} \right]^{-1}.$$

These two equations imply the kind of relationship between r^{t-1} and r^t illustrated in Figures 7-9. The last equation may be further simplified:

$$(11'') \quad \frac{1-r^{t-1}}{1-r^t} = \frac{\hat{e}_1}{\hat{e}_2} \left[\frac{2w(x^t)}{\hat{e}_1 x_1} + \frac{w(x^t)}{w(x^t)} \right].$$

It should be noted that the RHS of (11'') depends on r^{t-1} through (9').

Theorem 3 is then seen to imply that GAE obtains in this case if and only if the RHS of (11'') exceeds unity for all $r^{t-1} \in [0,1]$.

Consider the bracketed term on the RHS of (11''). Clearly this term is unbounded as $x \rightarrow 0$, or as $x \rightarrow \bar{x}$, where \bar{x} is the root of $w(x)=0$. Thus it attains a minimum as r^{t-1} varies over $[0,1]$ (and consequently x^t varies in the interior of $[0,\bar{x}]$). Denote the bracketed term by K^t , and its minimum by K_{\min} . Let \hat{x} be the point where the (RHS) of 11'' is least. Then we have the following result.

Lemma 5: K_{\min} exists, is unique, and is given by

$$(12) \quad K_{\min} = \left[\frac{2}{(\bar{w}(\hat{x})/w(\hat{x})-1)} + \hat{x} \right].$$

Proof: We have demonstrated the existence of a minimum above.

Recall

$$K^t = \frac{2w(x^t)}{e_{1x_1}} + \frac{w(x^t)}{w(x^t)}$$

Differentiate with respect to x to get:

$$\frac{dK^t}{dx} = \frac{2}{e_{1x_1}} \frac{dw}{dx} + [w(x)]^{-2} (w(x) \frac{dw}{dx} - \frac{w(x)}{dx} \frac{dw}{dx})$$

Now

$$\frac{dK^t}{dx} = 0 \quad \text{if} \quad \frac{dw}{dx} \left[\frac{2w(x)}{e_{1x_1}} - \frac{w(x)}{w(x)} \right] + \frac{dw}{dx} = 0$$

That is, if

$$\left[\frac{2w(x)}{e_{1x_1}} + \frac{w(x)}{w(x)} \right] = \frac{2w(x)}{w(x)} - \frac{dw/dx}{dw/dx}$$

Recall from the proof of Theorem 2 that

$$\begin{aligned} \frac{dw/dx}{dw/dx} &= \frac{dw/dx}{\overline{dw/dx} - dw/dx} = \frac{1}{\frac{\overline{dw/dx}}{dw/dx} - 1} \\ &= \frac{1}{-\left(\frac{x}{1-x}\right)^{-1} - 1} = -x \end{aligned}$$

Thus, we have $\frac{dK^t}{dx} = 0$ if

$$K^t = \left[\frac{2}{\bar{w}(x)/\underline{w}(x) - 1} + x \right].$$

That is, K_{\min} is given implicitly by

$$K_{\min}(x) = \left[\frac{2}{\bar{w}(x)/\underline{w}(x) - 1} + x \right].$$

Note that the above analysis also implies:

$$\frac{dK^t}{dx} = 0 \text{ if } x = -\frac{\bar{w}(x)}{w(x)} + \frac{2w(x)}{e_1 - 1}.$$

It is easily seen that this last equation has one and only one root for $x \in [0, \bar{x}]$. (\bar{x} is the root of $w(x) = 0$.) Hence K^t achieves a unique minimum on this interval. Q.E.D.

Equation (12) is an implicit relation which could (in principle) be solved for the value of x at which the RHS of (11'') is minimum. In any event, (12) may be used to give a lower bound on the RHS of (11''), and hence a sufficient condition for GAE.

It follows from (11'') and (12) that whether or not GAE obtains depends on three factors: (a) the fraction of the work force employed as skilled workers; (b) the earnings of skilled relative to unskilled workers; and (c) the ratio of the "marginal product" of the innate endowment to the "marginal product" of parents' relative economic position in the acquisition of characteristics by young agents of the subordinate group. Values for factors (a) and (b) are taken at the point where the total B-W wage differential as a fraction of W income improves least across a generation. This result may be summarized as:

Theorem 4 : Let \hat{x} be the solution to (12). Then, in the model developed above, any initial income discrepancy between groups will disappear asymptotically if and only if the following condition holds:

$$\frac{2w(\hat{x})}{w(\hat{x})} + \hat{x} > \frac{\hat{e}_2}{\hat{e}_1} .$$

FOOTNOTES

1. See, for example, Freeman (21), p. 67.
2. Though this conclusion is broadly accepted, its interpretation is the subject of much disagreement. While Freeman [21] has proclaimed a "virtual collapse" of traditional discrimination against blacks, Vroman [42], Wallace and Anderson [44] and the Manpower Policy Task Force [31] have adopted a more cautious view. Hall and Kasten [24] have shown that occupational discrimination against young black males was substantially reduced during the 60's.
3. I have seen no conclusive evidence on either side of this debate. Freeman [22] takes the view that the gains are the direct result of government action, while Ashenfelter, in his comment on the Freeman paper [22], disagrees. I have argued elsewhere (see comments on the Vroman paper [42]) that the evidence is consistent with alternative interpretations. A recent study by Ashenfelter and Heckman [3] finds very little impact of federal antidiscriminatory action.
4. Ironically, this represents a recent conversion for many who hold this view.
5. Consider: "...it appears that the absence of racial discrimination in the job market would not eliminate racial differences in occupations immediately, since there are broad societal processes operating to the disadvantage of Negroes....Several generations would be necessary before parity was reached," Lieberman and Fuguitt [28], p. 188; or "But if there were remedies for all these forms of discrimination, so that only the handicap of family background remained, that handicap would be materially diminished in the next generation. It would be further attenuated in successive generations...and... would tend to disappear of its own accord," Duncan [18], p. 102 (emphasis added); or finally, "In other words, if we could eliminate the inheritance of race, in the sense of the exposure to discrimination experienced by Negroes, the inheritance of poverty in this group would take care of itself," Duncan [18], p. 103.
6. Intergenerational externalities have been studied in a partial equilibrium context by Ishikawa [25] and Lazear [27]. However, these authors do not investigate the distributional implications of these effects.
7. See, e.g. Welch [46], or Weiss [45].

8. This is, of course, Becker's idea [5]. Whether or not differential returns will actually occur depends on how costly it is for employers to turn over their work force (Arrow [2]) and on the ability of non-discriminating entrepreneurs to expand their numbers and scale of production (as emphasized by Freeman [22]). To the extent that labor turnover costs are high, or growth of the non-discriminating sector (non-existent in the U.S. economy during the first half of this century) is limited, the theory predicts protracted black-white wage differentials.
9. The term "market-valued characteristics" shall be used interchangeably with the more common term "human capital" in the sequel. This is done to avoid the (possibly erroneous) association of these characteristics with some objective notion of productivity. The empirical results of human capital theory are apparently consistent with the assumption that education per se does not appreciably affect job performance. See, e.g., Thurow [39], or the "informational" literature cited in note 13 below.
10. The historical development of this policy since 1964 has been recently summarized in Wallace [43].
11. An interesting legal analysis of the Doctrine's limitations on the exercise of discriminatory public preferences is contained in Baeza [4].
12. See note 5 above for support of this statement.
13. Possible exceptions to this statement are the recent literature on job market signalling, (see Spence [34], Starrett [35], and Stiglitz [36]), and from a totally different perspective, the "radical" literature on education (see, e.g. Bowles and Gintis [12], [13]).
14. This point will be further elaborated in Section III.
15. See especially Blau and Duncan [6], Duncan et al. [19], Duncan [18] and Lieberman and Fuguitt [28]. For a review of the intragenerational mobility literature see McFarland [30]. An interesting analysis of the intergenerational dynamic interplay of social mobility and educational opportunity is Boudon [10]. A recent comparative study of intragenerational occupational mobility among young black and white males is Hall and Kasten [24].
16. This is the analytic method of path analysis applied to life-cycle models of socio-economic achievement. For a discussion of the development of this methodology see Duncan, et al. [19], esp. Chapters 1 and 2.
17. This fact was established early in the course of this research by sociologists. See Duncan [18], or Duncan, et al. [19], chapter 4 for example. The problem is well summarized by

Duncan [18], p. 95-96, "...the (relatively few) Negroes who do have favorable social origins cannot, as readily as whites, convert this advantage into occupational achievement and monetary returns...The Negro Family...is relatively less able than the white to pass on to the next generation any advantage that may accrue to substantial status achievement in the present generation."

18. This effect may be an artifact of the broad definition of occupation used in these studies. A more recent study of occupational achievement by Stolzenberg [37], using much finer occupational categories, finds negligible intra-occupational earnings differences between blacks and whites, once education is controlled. However, he also finds that blacks tend to be concentrated in the lower paying occupations of each broader occupational category.
19. The term is Becker's, [5] p. 6.
20. See, e.g., U.S. Commission on Civil Rights [40], Vol. I and II.
21. That this is the case has been firmly established. See, e.g., Feldstein [20], Table 1, p. 82, or Wise [48], Table 2, p. 125.
22. In their provocative book [13] Bowles and Gintis give data which supports this argument. For example, in 1971 the ratio of the number of students with family income over \$20,000 to the number of students with family income under \$8,000 was nine times greater in private universities than in public two-year colleges, (Fig. 8-1, p. 210.)
23. The importance of imperfect information in determining the distribution of income is also emphasized by Starret [35].
24. This very omission in macroeconomics, it has recently been argued, is at the heart of the differences between Monetarists and Keynesians over the long term effects of fiscal policy. See Blinder and Solow [7].
25. See, e.g., Duncan et al. [19], Table 4.3, p. 59.
26. Bowles [11] is keenly aware of this possibility when he states "The legitimation of the hierarchical division of labor, as well as the smooth day-to-day control over the work process, requires the authority structure of the enterprise... (to) respect the wider society's ascriptive and symbolic distinctions. In particular, socially acceptable relations of domination and subordination must be respected: white over black; male over female; old over young; and schooled over unschooled." (p. 352)
27. The persistent view to the contrary espoused by traditional economists may be traced back to the seminal work of Becker: "If an individual has a 'taste for discrimination', he must

act as if he were willing to pay something...to be associated with some persons instead of others. When actual discrimination occurs, he must, in fact, either pay or forfeit income for this privilege. This simple way of looking at the matter gets at the essence of prejudice and discrimination." ([5]), p. 6) It is simple indeed, but hardly the essence! Yet we can find similar views in the current literature. Writing 17 years after Becker, Richard Freeman states: "Despite a group 'norm' for discrimination, adhered to by, say 85% of the population, prejudice need not have economic consequences." ([22], p. 516-17).

28. See, for example, Arrow [1] p. 98-100, Freeman [22] p. 525-529, or Marshall [32].
29. This should in no way be interpreted to mean that economic gain has no role in the socio-historical analysis of race relations. We have left unaddressed the question of why customary racial attitudes have come to be such as they are. The coincidental rise of race prejudice among Europeans with the industrial revolution and the development of modern capitalism cannot be ignored. The economic role of slavery in nineteenth century U.S. needs no elaboration. In fact, it is the major thesis of a recent work by social theorist Wilson [49] that racial stratification finds its *raison d'etre* in the economic subjugation and exploitation of minority group members. However, a study of the economic origins of racism in this sense would seem to require an analysis in the Marxian tradition of historical materialism beyond the scope of this essay. For the substantial beginnings of such an analysis see Cox [14].
30. See, e.g., Bloom [8], especially Chapter 3.
31. This point is made forcefully by Blumen [9]; also relevant is the discussion in Metzger [33] and van den Berghe [44].
32. Blumen [9], p. 3.
33. Doeringer and Piore [16] implicitly acknowledge this point in their discussion of how the natural workings of the internal labor market could impede the attainment of equal employment opportunities for minorities.
34. This temporal sequence is merely Duncan's socio-economic life-cycle mentioned earlier. See Duncan et al. [19].
35. In order to consider the dynamic implications of racial discrimination we must (regrettably) neglect the problems of sexism. The economic consequences of the interaction of these two important social forces provides a formidable agenda for future research.

36. We neglect here the possibility of assortative mating which could slow considerably the convergence of black and white incomes.
37. We know of no national study estimating the frequency of interracial marriage. Casual empiricism suggest the incidence of the phenomenon, while limited, has been increasing in recent years.
38. The term "community" is being used here in a generalized sense. Its connotation is intended to be broader than the ordinary notion of a residential neighborhood.
39. See note 21, supra. It must be acknowledged that recent judicial efforts toward school desegregation exemplify how the Doctrine might be used to limit some of the effects of social stratification by race on young people's opportunities. Note, however, the hesitancy of the courts to consider this issue along class as well as racial lines.
40. One instance of this phenomenon is that in many large cities the residential areas of middle and lower class blacks are located contiguously, with the obvious spillover effects. Recall also the broad definition of community employed here (see note 38, Supra).
41. For ease and clarity of exposition many technical details of the specification of the theory are omitted here. Furthermore, in this and the following section no attempt is made at mathematical rigor in the arguments. Readers interested in a more general and completely rigorous derivation of results cited here are referred to section VI.
42. Thus we are explicitly disregarding the arguments of Herrnstein, Jensen et al. that the heritability of IQ has a major role in sustaining racial income differences. This omission may be justified on two grounds. First, nearly all of the racial differences in performance on IQ tests may be accounted for by the difference in family environments (see Gordon [23]) and hence are already considered here. Second, the relationship between IQ and earnings is at best extremely tenuous (see Bowles and Gintis [13] Chapter 4), and thus could hardly explain the magnitude of observed racial earnings differences.
43. Observe the subtle point that the Equal Opportunity Doctrine does not imply complete racial equality of opportunity unless either (a) an individual's ability to acquire characteristics may be rendered independent of his community environment; or (b) the economic status of blacks and whites are on the whole equalized. Apparently condition (a) requires a great deal more than integrated education.

44. We abstract from physical capital and the existence of a propertied class. Racial income inequality derives primarily from the relatively poor position of the black worker. Consequently nothing fundamental is lost by our assumption. We point out, however, that a full investigation of the determination of the social relations which obtain among various groups (and which we have taken as given here) would not countenance a similar abstraction. See note 29, supra.
45. The method employed here could be used as the basis for a large scale simulation effort with considerably more detail allowed in the characterization of the occupational structure and the family and community background. We restrict ourselves to this simple framework only to gain insight into the qualitative properties of the system. Such insight, we believe, will prove useful in the formulation of policy, and in the eventual construction of more elaborate empirically based models.
46. Thus, Moynihan-Banfield "culture of poverty" effects are neglected to the extent that they require the poor to have a greater preference for liesure, or to be more "present oriented". We seek to show that even under the most favorable of conditions the Equal Opportunity Doctrine may fail to achieve its goal. Hence the assumption.
47. Suppose that almost all people had zero costs. Then nearly everyone would become skilled. If both types of labor were necessary to production then diminishing marginal productivity would eventually lead to the unskilled wage rising above the skilled wage. Since the initial designation of occupational categories as "skilled" and "unskilled" was arbitrary, this assumption is effectively innocuous. See Figure 6 and the accompanying discussion below.

48. We are implicitly imagining that the labor market for any generation meets, and wages are decided upon, the period before these agents become employed (i.e. while they are still young).
49. For the justification of this and other claims made here, see the mathematical development of section VI.
50. We wish to emphasize that this is a very weak criterion indeed. We have made no mention of how quickly the earnings gaps between blacks and whites should disappear. Thus, the criterion employed here may be used to reject certain prescriptions as unsatisfactory, but can hardly be viewed as endorsing those which satisfy its requirements.
51. In keeping with the heuristic development of this section, this and the next proposition are stated here without proofs. A completely rigorous formal treatment of these results may be found in section VI.
52. Consider the analysis of the Survey of Educational Opportunity data conducted by the United States Civil Rights Commission in the late 1960's ([40], vol. 1, pp. 80-85). The Commission found that grade level performance of 12th grade students varied significantly by the individual student's social class, as well as by the social class level of the school. Middle and upper class students did consistently better than lower class students. However, there were some interesting differences in the patterns between blacks and whites. While white gains from increasing student's social class diminished as one moved first from lower to middle and then from middle to upper class (Figure 1, p. 80). Blacks gained little in moving from lower to middle class, but made quite significant gains when background was advanced to upper class (Figure 1, p. 80 and Figure 3, p. 85). Similar nonconvexities for blacks in the effect of social class on achievement on I.Q. tests have been uncovered by Gordon [23]. In his work, piecewise linear regression of I.Q. performance on socioeconomic background variables reveals significantly greater marginal effects for parent's income in the range \$7,500 to \$10,000 than for either lower or higher income classes.
53. See Duncan [18] and Lieberman and Fugitt [28]. See also the passages from these works quoted in note 5.
54. It is obvious that blacks have not entered the mainstream of American society with the relative ease of European ethnic assimilation. It is considerably less obvious that this is a goal toward which black people should strive. While our analysis indicates some of the deleterious consequences of racial stratification when blacks carry with them the effects of past racial discrimination, we should not be interpreted as endorsing the "integrationist" strategy of black development.

An informative review of the sociological debate on this question is contained in Metzger [33]. It is clear from his discussion that, even in discussion among white academics, the traditional sociological view of black assimilation as inevitable and desirable is under serious attack.

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