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An Interactive Package of Programs for Unconstrained Optimization Purposes UCNLP

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DISCUSSION PAPER NO. 171

AN INTERACTIVE PACKAGE OF PROGRAMS FOR
UNCONSTRAINED OPTIMIZATION PURPOSES - UCNLP

by

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Preface

Development of the following interactive package would not have been possible without the help of the IBM Corporation which provided the funds and the support which made it feasible for me to attend the intensive APL course in UCLA in the Summer of 1975. Special thanks are due to Ephraim McLean, Ken Siler and Jim Schneck from UCLA who introduced me to the APL language and helped me develop UCNLP.

I am also indebted to Northwestern University's Graduate School of Management for covering my transportation expenses from Chicago to Los Angeles and awarding me a summer grant which enabled me to leave the Northwestern Campus for the period of the APL intensive seminar.

I. Background Information

UCNLP (Unconstrained Nonlinear Programming) is a package of computer programs coded in APL and designed for users who are mainly interested in experiments and tests involving statistical comparisons of nonlinear functions minimized under different conditions. The user provides the computer with information regarding the mathematical expressions of the function to be minimized. The number of iterations per one-one dimensional search. The starting point, and the name of the method to be used. The optional methods are:

1. Steepest Descent [5]
2. Newton's Method [5]
3. The Fletcher-Reeves Conjugate Gradient [4,5]
4. Perry's Modified Conjugate Gradient [6]
5. Davidon -Fletcher-Powell Variable Metric [33]
6. Broyden's Rank 1 Method [1, 5]

and the user may select the one he wishes to apply.

All the above methods are also known as gradient methods. They all require information regarding the mathematical expression of the gradient of the function in question. The only method which requires information regarding the mathematical expression of the hessian matrix is Newton's Method.

Gradient methods are procedures for minimizing a given differentiable unconstrained nonlinear function. The common denominator of all these methods is the fact that they all depend very heavily on the efficiency and the accuracy of the line-search used in the process. The main principle behind these methods is the fact that the function is evaluated at a given point, a direction of descent is then determined, and a search procedure locating

the function minimum along that direction is performed. The new point is then used as the starting point for the next iteration. The process terminates at a local minimum where no direction of descent can be found.

While different gradient methods (in this package) vary with respect to the direction used in the process, they all use the same line search procedure. This procedure is divided into two major stages. In the first stage the program searches for the best unimodal region along the given direction. This is done by simultaneously evaluating the function at 10 arbitrary points along the given direction. If a unimodal region is determined, the interval of uncertainty is reduced to 1/9th of its original size, and the search moves on to its second stage. In the event that the simultaneous search fails to secure a unimodal region the process repeats itself until a unimodal region is located. Once a unimodal interval is located along the search direction, a sequential Fibonacci search [7] is performed. The accuracy of the search is determined by the user who specifies the number of sequential steps in the one-dimensional search. The final error in determining the minimum point along the original interval of uncertainty is $\pm 1/F_N$ where F_N is the Nth Fibonacci number specified by the user.

II. The Computer Application

After loading UCNLP the user is ready to start.

User: OPTIMIZE

Computer: ENTER FUNCTION TO BE MINIMIZED

User: The function is entered here. User should be familiar with APL arithmetic rules according to which the function is expressed. The variables in the function should be of the type X[J] where J = 1,2,...

Computer: ENTER GRADIENT OF FUNCTION

User: The gradient vector is entered here according to the same rules as above. If some elements of the gradient vector are mathematical expressions, they should be separated from each other by means of placing them within parentheses and separating them by a comma.

Computer: ENTER INITIAL STARTING POINT

User: The user may enter a numerical vector or a predefined variable name containing that vector.

Computer: ENTER NUMBER OF ITERATIONS PER ONE-ONE DIMENSIONAL SEARCH.

User: Enters the number which is used in the sequential part of the search after a unimodal region is determined.

Computer: DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO.

User: If user enters "YES" output will be printed at the end of every one dimensional search. The word "iteration" refers to a step consisting of a direction fixing and the line search along that direction. If user enters "NO" the only output consists of two statements regarding the optimal value of the function and the optimal point.

Computer: ENTER NAME OF ALGORITHM TO BE EXECUTED: CHOOSE AMONG: STEEP,
NEW, CONGRD, DFPVM, MODCG, RANK.

User: The user must enter the full name (described above) of the algorithm he wishes to use.

STEEP = The Method of Steepest Descent

NEW = Newton's Method

CONGRD = Fletcher-Reeves: Conjugate Gradient Method

DFPVM = Davidon -Fletcher-Powell: Variable Metric Method

MODCG = Perry: Modified Conjugate Gradient Method

RANK = Broyden's: Rank-One Method

The computer now calculates the minimum value of the function and the corresponding n-dimensional point. After completing this assignment the user may restart by responding positively to the question:

Computer: DO YOU WISH TO RESTART? YES OR NO

The computer will then go on asking questions until all the required information is given and the process repeats itself. Upon a negative response to the above question the computer closes the "OPTIMIZE" function and gets ready for any new assignment.

Note that if the user wishes to use Newton's Method he must provide the computer with information on the hessian matrix. Therefore, upon responding "NEW" to the question concerning the algorithm's name the computer responds by first requesting that information. The Hessian matrix must be entered row-wise for example. Let $A = \{a_{ij}\}$ be the hessian of f then entering A is done in the following way: $a_{11}, a_{12}, \dots, a_{1n}, a_{21}, \dots, a_{2n}, \dots, a_{n1}, \dots, a_{nn}$. (There is no need to dimension the matrix) if a_{ij} is a mathematical expression, it must be entered according to APL rules.

In the Appendix we provide the listing of the APL source programs followed by examples.

References

- [1] Broyden, C. G., "Quasi-Newton Methods and Their Application to Function Minimization," Math. Comp., 21, 1967, pp. 368-381.
- [2] Davidon, W. C., "Variable Metric Method for Minimization," Research and Development Report ANL-5990 (Ref.) U.S. Atomic Energy Commission, Argonne National Laboratories, 1959.
- [3] Fletcher, R. and Powell, M. J. D., "A Rapidly Convergent Descent Method for Minimization," Computer J., 6, 1963, pp. 163-168.
- [4] Fletcher, R. and Reeves, C. M., "Function Minimization by Conjugate Gradients," Computer J., 7, 1964, pp. 149-159.
- [5] Himmelblau, D. M., "Applied Nonlinear Programming," McGraw-Hill, 1972, pp. 63-140.
- [6] Perry, A., "Duality in Unconstrained Nonlinear Optimization," The Center for Mathematical Studies in Economics and Management Science, Northwestern University (forthcoming).
- [7] Wilde, D. J. and Beightler, C. S., Foundations of Optimization, Prentice-Hall, 1967.

A P P E N D I C E S

Appendix 1

```
&CGM[#]&
& XBAR$IS CGM X;C0UNT;Z;D0;G0;Y;X1;G1;Q;P;S1;S0;D1
[1] C0UNT$IS 0
[2] START:Z$IS FUN X
[3] D0$IS-G0$IS GR X
[4] S0$IS($I0@G0)$NL.= $I0@G0
[5] Y$ISX+D0
[6] L1:X1$IS X FIBSCH Y
[7] C0UNT$IS C0UNT+1
[8] G1$IS GR X1
[9] Q$IS((@G1),1)@(G1-G0)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$ISS0+(P+.SML($TP(P-Q)))$DV($TPP)+.SMLQ
[12] D1$IS-S1+.SMLG1
[13] Y$ISX1+D1
[14] X$ISX1
[15] D0$ISDI
[16] G0$ISG1
[17] $G0 L1$ML($I0($0R/($MDG1)>EPS))
[18] 'NO. OF ITERATIONS = ';C0UNT
[19] 'MIN. FUNCTION VALUE = ';FUN X
[20] 'OPTIMUM POINT = ';X
```

```
& &DFP[#]&
& XBAR$IS DFP X;C0UNT;Z;D0;G0;Y;X1;G1;Q;P;S1;S0;D1
[1] C0UNT$ISO
[2] START:Z$IS FUN X
[3] D0$IS-G0$ISGR X
[4] S0$IS($I0@G0)$NL.= $I0@G0
[5] Y$ISX+D0
[6] L1:X1$ISXBAR$IS X FIBSCH Y
[7] C0UNT$IS C0UNT+1
[8] G1$IS GR X1
[9] Q$IS((@G1),1)@(G1-G0)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$IS(S0-((S0+.SMLQ+.SML($TPQ)+.SMLS0)$DV((STPQ)+.SMLS0+.SMLQ)))+((P
. .SML($TPP))$DV((STPQ)+.
$MLQ))
[12] D1$IS-S1+.SMLG1
[13] Y$ISX1+,D1
[14] X$ISX1
[15] D0$ISDI
[16] G0$ISG1
[17] S0$ISS1
[18] $G0 L1$ML($I0($0R/($MDG1)>EPS))
[19] ' NO. OF ITERATIONS = ';C0UNT
[20] ' MIN. FUNCTION VALUE = ';FUN X
[21] ' OPTIMUM POINT = ';X
```

```

&FIBNUM[#]&
& FN$IS FIBNUM K;R
[1] FN$IS 1 1
[2] END:$GØ(K>@FN$ISFN,+/*2$TAFN)/END
&

&FIBSCH[#]&
& PØINT$ISX FIBSCH Y;CØUNT$M1;FALPHA;C1;C2;X1;Y1;R;RN;FN;D;ND;RD;K;F;X

O;N1
[1] L20:D$ISY-X
[2] L10:X1$ISX+EPSSMLD
[3] $GØ((MN/((FUN X),FUN X1))=((FUN X),FUN X1))/(L11,L12)
[4] L11:Y$ISX+D$IS-GR X
[5] SGØ L10
[6] L12:ND$IS(D+.MLD)*.5
[7] RDSIS$DV$109
[8] X$IS(10,@D)@X,(,((9,@D)@X)+RDC$UGRD)$NL.$MLD)
[9] K$IS1
[10] F$ISSI00
[11] L21:F$ISF,FUN X[K;]
[12] K$ISK+1
[13] $GØL21$ML$10K$LE10
[14] $GØ L22$ML$10(1$TA$UGF)=10
[15] $GØ L25$ML$10(1$TA$UGF)=1
[16] X$IS(EPSSMLD)+,X0$ISX[1$TA$UGF;]
[17] $GØ(((FUN X1)$GEFUN ,X0),(FUN X1)<FUN ,X0)/L23,L24
[18] L22:X$IS,X[9;]
[19] Y$ISY+D
[20] $GØ L20
[21] L23:Y$ISX1
[22] X$IS,X[(1$TA$UGF)-1;]
[23] $GØL0
[24] L24:Y$IS,X[(1$TA$UGF)+1;]
[25] X$IS,X0
[26] $GØL0
[27] L25:Y$IS,X[2;]
[28] X$IS,X[1;]
[29] $GØ L20
[30] LO:N1$ISN
[31] X1$IS,X
[32] Y1$IS,Y
[33] CØUNT$IS 2
[34] D$ISY-X
[35] L1:FN$IS FIBNUM N1
[36] R$IS+1$DRFN$DV+1$TAFN
[37] C2$ISX+(+1$STAR)$MLD
[38] C1$ISY-(+1$STAR)$MLD
[39] R$IS+2$DRR
[40] L00P:$GØ(((FUN C1)>FUN C2),((FUN C1)=FUN C2),((FUN C1)<FUN C2))/(L2,L

```

```

4,L3)
[41] L2:X$IS C1
[42] C1$ISC2
[43] C2$IS Y-(↑1$STAR)$MLD
[44] $GØ RESTART
[45] L4:N1$ISN1-COUNT
[46] X$IS C1
[47] Y$ISC2
[48] D$IS Y-X
[49] COUNT$IS2
[50] $GØL1$ML$IØ(N1>COUNT)
[51] $GØ RESTART
[52] L3:Y$ISC2
[53] C2$ISC1
[54] C1$ISX+(↑1$STAR)$MLD
[55] RESTART:R$IS↑1$DRR
[56] COUNT$ISCOUNT+1
[57] $GØLØOP$ML$IØ(N1>COUNT+1)
[58] C1$ISC1-EPS$MLY-X
[59] $GØLØOP$ML$IØ(N1=COUNT+1)
[60] FALPHASIS($MN/(FUN X),(FUN C1),(FUN C2),FUN Y)=(FUN X),(FUN C1),(FUN
C
    2),FUN Y
[61] M1$IS(4,@,X)@(X,C1,C2,Y)
[62] PØINT$ISM1[FALPHASIS1;]
[63] $GØL15$MLØUTPT
[64] L15:'THE FUNCTION EVALUATED AT THE MIN. IS ' ; FUN PØINT
[65] ' THE PØINT MINIMIZING THE FUNCTION IS ' ; PØINT

```

&

```

&FRCG[#]&
& XBARSIS FRCG X;COUNT;DO;D1;GØ;G1;Z
[1] COUNT$IS 1
[2] START:Z$IS FUN X
[3] DO$IS-GØ$ISGR X
[4] Y$ISX+DO
[5] X$IS X FIBSCH Y
[6] G1$IS GR X
[7] L1:COUNT$ISCOUNT+1
[8] DISIS(-G1)+((G1+. $MLG1)$DV(G0+. $MLG0))$MLDO
[9] Y$ISX+D1
[10] X$IS X FIBSCH Y
[11] GØ$ISG1
[12] DO$ISD1
[13] G1$ISGR X
[14] $GØ L1$ML($IØ($ND/($MDG1)$LE EPS))
[15] ' NO. OF ITERATIONS = ' ; COUNT
[16] ' MIN. FUNCTION VALUE = ' ; FUN X
[17] ' OPTIMUM PØINT = ' ; X

```

&

```
&FUN[#]&
& F$IS FUN X
[1]  F$IS$EVFX
&
&GR[#]&
& G$IS GR X
[1]  G$IS$EVGX
&
&HES[#]&
& F$IS HES X
[1]  F$IS((@X),@X)@$EVHX
&

&NEWTON[#]&
& XBAR$IS NEWTON X;Z;G;D;H;Y;COUNT
[1]  COUNT$IS 0
[2]  START:Z$ISFUN X
[3]  G$IS((@X),1)@GR X
[4]  SGØ L1$ML($IØ($AN/($MD,G)<EPS))
[5]  COUNT$ISCOUNT+1
[6]  H$IS HES X
[7]  D$IS-( $XDH)+. $MLG
[8]  Y$ISX+,D
[9]  X$ISX FIBSCH Y
[10]  SGØ START
[11] L1:'NØ. ØF ITERATIONS = ' ;COUNT
[12] 'MIN. FUNCTION VALUE = ' ;FUN X
[13] 'OPTIMUM POINT = ' ;X
&
```

```
&OPTIMIZE[#]&
& OPTIMIZE
[1] L1:'ENTER FUNCTION TO BE MINIMIZED'
[2] FX$IS$QP
[3] L2:'ENTER GRADIENT VECTOR OF THE ABOVE FUNCTION'
[4] GX$IS$QP
[5] L3:'ENTER INITIAL STARTING POINT'
[6] A$IS#
[7] L4:'ENTER NUMBER OF ITERATIONS PER ONE ONE DIMENSIONAL SEARCH'
[8] N$IS#
[9] L6:'DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO'
[10] OUTPT$IS(I$TAS$QP)='Y'
[11] L5:'ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:'
[12] 'STEEP, NEW, CNGRD , DFPVM, MDCG, RANK'
[13] SG$SEV$QP
[14] STEEP:SDT A
[15] $GØ FIN
[16] NEW:'ENTER HESSIAN MATRIX OF THE ABOVE FUNCTION'
[17] HX$IS$QP
[18] NEWTON A
[19] $GØ FIN
[20] CNGRD:FRCG A
[21] $GØ FIN
[22] MDCG:CGM A
[23] $GØ FIN
[24] RANK:RANK1 A
[25] $GØ FIN
[26] DFPVM:DFP A
[27] FIN:'DO YOU WISH TO RESTART? ANSWER YES OR NO'
[28] ANS$ISI$TAS$QP
[29] $GØ START$MLANS='Y'
[30] START:'DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO'
[31] ANS$ISI$TAS$QP
[32] $GØ L1$ML$IOANS='Y'
[33] 'DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO'
[34] ANS$ISI$TAS$QP
[35] $GØL3$ML$IOANS='Y'
[36] 'DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL SEARCH? YES OR NO'
[37] ANS$ISI$TAS$QP
[38] $GØ L4$ML$IOANS='Y'
[39] 'DO YOU WISH TO CHANGE ALGORITHM? YES OR NO'
[40] ANS$ISI$TAS$QP
[41] $GØ L5$MLANS='Y'
```

&

```

&RANK1[#]&
& XBAR$IS RANK1 X;C0UNT;Z;DO;D1;G0;Y;X1;G1;Q;P;S1;S0
[1] C0UNT$ISO
[2] START:Z$IS FUN X
[3] DO$IS-G0$ISGR X
[4] SO$IS($I0@G0)$NL.= $I0@G0
[5] Y$ISX+DO
[6] L1:X1$ISXBAR$ISX FIBSCH Y
[7] C0UNT$IS C0UNT+1
[8] G1$IS GR X1
[9] Q$IS((@G1),1)@(G1-G0)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$IS SO+((P-S0+.SMLQ)+.SML($TP(P-S0+.SMLQ)))$DV($TP(P-S0+.SMLQ))+.S
LQ M
[12] D1$IS-S1+.SMLG1
[13] Y$ISX1+,D1
[14] X$ISX1
[15] DO$ISD1
[16] GO$ISG1
[17] SO$ISS1
[18] $G0 L1$ML($I0($0R/($MDG1)>EPS))
[19] ' NO. OF ITERATIONS = ' ;C0UNT
[20] ' MIN. FUNCTION VALUE = ' ;FUN X
[21] ' OPTIMUM POINT = ' ;X

```

```

&SDT[#]&
& XBAR$IS SDT X;Z;D;Y;COUNT
[1] COUNT$ISO
[2] START:Z$IS FUN X
[3] D$IS GR X
[4] $GØ L1$ML($IØ($AN/($MDD)$LEEPS))
[5] Y$IS X-D
[6] X$IS X FIBSCH Y
[7] COUNT$IS COUNT+1
[8] $GØ START
[9] L1:'NØ. ØF ITERATIONS = ';COUNT
[10] 'MIN. FUNCTION VALUE = ';Z
[11] 'OPTIMUM POINT 'X

```

```

) VARS
A      ANS      EPS      FR1      FX      GR1      GX      HR1      HX
N      OUTPT
EPS
0.000001
FR1
(100$ML(X[2]-X[1]*2)*2)+(1-X[1])*2
FX
$EVFR1
GR1
(((400$MLX[1]*3)-(400$MLX[1]$MLX[2]))+(2$MLX[1])-2),(200$ML(X[2]-X[1]*2))

GX
$EVGR1
HR1
(((1200$MLX[1]*2)-400$MLX[2])+2),(-400$MLX[1]),(-400$MLX[1]),200
HX
$EVHBI

```

Appendix 2

OPTIMIZE

ENTER FUNCTION TO BE MINIMIZED
\$EVFRI

ENTER GRADIENT VECTOR OF THE ABOVE FUNCTION

\$EVGR1

ENTER INITIAL STARTING POINT

#:

1.2 1

ENTER NUMBER OF ITERATIONS PER ONE ONE DIMENSIONAL SEARCH

#:

20

DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO
YES

ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEEP, NEW, CONGRD, DFPVM, MDCG, RANK

NEW

ENTER HESSIAN MATRIX OF THE ABOVE FUNCTION

\$EVHRI

THE FUNCTION EVALUATED AT THE MIN. IS 4.73154706

THE POINT MINIMIZING THE FUNCTION IS 1.175177674 1.382263817

THE FUNCTION EVALUATED AT THE MIN. IS 4.045639354

THE POINT MINIMIZING THE FUNCTION IS 10.9595197027 0.8752994427

THE FUNCTION EVALUATED AT THE MIN. IS 3.160046149

THE POINT MINIMIZING THE FUNCTION IS 10.6897928596 0.4206194213

THE FUNCTION EVALUATED AT THE MIN. IS 1.954889045

THE POINT MINIMIZING THE FUNCTION IS 10.333958907 0.06964267338

THE FUNCTION EVALUATED AT THE MIN. IS 1.25388344

THE POINT MINIMIZING THE FUNCTION IS 10.05087007794 10.03608464387

THE FUNCTION EVALUATED AT THE MIN. IS 0.6662845319

THE POINT MINIMIZING THE FUNCTION IS 0.2388743903 0.02756990143

THE FUNCTION EVALUATED AT THE MIN. IS 0.3241042274

THE POINT MINIMIZING THE FUNCTION IS 0.4820798993 0.2087656752

THE FUNCTION EVALUATED AT THE MIN. IS 0.10968867

THE POINT MINIMIZING THE FUNCTION IS 0.7072517458 0.484717253

THE FUNCTION EVALUATED AT THE MIN. IS 0.02053228579

THE POINT MINIMIZING THE FUNCTION IS 0.8842157543 0.7733957628

THE FUNCTION EVALUATED AT THE MIN. IS 0.00006242121203

THE POINT MINIMIZING THE FUNCTION IS 1.0030349 1.006803466

THE FUNCTION EVALUATED AT THE MIN. IS 1.723432357E+7

THE POINT MINIMIZING THE FUNCTION IS 0.9996244692 0.9992667769

THE FUNCTION EVALUATED AT THE MIN. IS 5.004856024E+13

THE POINT MINIMIZING THE FUNCTION IS 0.999999698 0.9999993321

THE FUNCTION EVALUATED AT THE MIN. IS 5.63422129E+22

THE POINT MINIMIZING THE FUNCTION IS 1 1

NO. OF ITERATIONS = 13

MIN. FUNCTION VALUE = 5.63422129E+22

OPTIMUM POINT = 1 1

DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIONS PER ØNE-DIMENSIØNAL SEARC
H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØOSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
CØNGRD
THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277
THE PØINT MINIMIZING THE FUNCTØN IS 1.030025571 1.069377318
THE FUNCTØN EVALUATED AT THE MIN. IS 4.010183434
THE PØINT MINIMIZING THE FUNCTØN IS 10.9876896931 0.9511848589
THE FUNCTØN EVALUATED AT THE MIN. IS 3.601940313
THE PØINT MINIMIZING THE FUNCTØN IS 10.8211936621 0.6209554766
THE FUNCTØN EVALUATED AT THE MIN. IS 3.386261918
THE PØINT MINIMIZING THE FUNCTØN IS 10.7070239084 0.4311564444
THE FUNCTØN EVALUATED AT THE MIN. IS 3.238243177
THE PØINT MINIMIZING THE FUNCTØN IS 10.6166182964 0.301174562
THE FUNCTØN EVALUATED AT THE MIN. IS 3.126287847
THE PØINT MINIMIZING THE FUNCTØN IS 10.5406214117 0.2055089893
THE FUNCTØN EVALUATED AT THE MIN. IS 3.036978248
THE PØINT MINIMIZING THE FUNCTØN IS 10.4745132746 0.1322764198
THE FUNCTØN EVALUATED AT THE MIN. IS 2.96319986
THE PØINT MINIMIZING THE FUNCTØN IS 10.4156882953 0.07486685993
THE FUNCTØN EVALUATED AT THE MIN. IS 2.900652678
THE PØINT MINIMIZING THE FUNCTØN IS 10.3624651504 0.02918796243
THE FUNCTØN EVALUATED AT THE MIN. IS 2.846511903
THE PØINT MINIMIZING THE FUNCTØN IS 10.3136710074 10.007477414943
THE FUNCTØN EVALUATED AT THE MIN. IS 2.798815827
THE PØINT MINIMIZING THE FUNCTØN IS 10.2684463449 10.03701725006
THE FUNCTØN EVALUATED AT THE MIN. IS 2.756135641
THE PØINT MINIMIZING THE FUNCTØN IS 10.2261347296 10.06078847288
THE FUNCTØN EVALUATED AT THE MIN. IS 2.717387403
THE PØINT MINIMIZING THE FUNCTØN IS 10.1861986648 10.07979927274
THE FUNCTØN EVALUATED AT THE MIN. IS 2.681754192
THE PØINT MINIMIZING THE FUNCTØN IS 10.148229187 10.0947895712
THE FUNCTØN EVALUATED AT THE MIN. IS 2.64855315
THE PØINT MINIMIZING THE FUNCTØN IS 10.1118561848 10.1063296418
THE FUNCTØN EVALUATED AT THE MIN. IS 2.617246695
THE PØINT MINIMIZING THE FUNCTØN IS 10.07677704204 10.1148445875
THE FUNCTØN EVALUATED AT THE MIN. IS 2.58738084
THE PØINT MINIMIZING THE FUNCTØN IS 10.04272125981 10.120654003
THE FUNCTØN EVALUATED AT THE MIN. IS 2.558574948
THE PØINT MINIMIZING THE FUNCTØN IS 10.009466284078 10.1239891052
THE FUNCTØN EVALUATED AT THE MIN. IS 2.530473758
THE PØINT MINIMIZING THE FUNCTØN IS 0.02321349277 10.1250143845
THE FUNCTØN EVALUATED AT THE MIN. IS 2.502780236
THE PØINT MINIMIZING THE FUNCTØN IS 0.05550318465 10.1238329892
THE FUNCTØN EVALUATED AT THE MIN. IS 2.475206843
THE PØINT MINIMIZING THE FUNCTØN IS 0.08758582501 10.1204968635
THE FUNCTØN EVALUATED AT THE MIN. IS 2.447487845
THE PØINT MINIMIZING THE FUNCTØN IS 0.1196329015 10.1150108832
THE FUNCTØN EVALUATED AT THE MIN. IS 2.419371266

THE PØINT MINIMIZING THE FUNCTØN IS 1.493878832 2.234517067
THE FUNCTØN EVALUATED AT THE MIN. IS 0.000006527763908

THE PØINT MINIMIZING THE FUNCTØN IS 1.002513586 1.004987702
THE FUNCTØN EVALUATED AT THE MIN. IS 0.000006206209242

THE PØINT MINIMIZING THE FUNCTØN IS 1.002488856 1.004994771
THE FUNCTØN EVALUATED AT THE MIN. IS 0.000005927883814

THE PØINT MINIMIZING THE FUNCTØN IS 1.002399737 1.004764107
THE FUNCTØN EVALUATED AT THE MIN. IS 1.895894375E⁹

THE PØINT MINIMIZING THE FUNCTØN IS 1.000007092 1.00001848
THE FUNCTØN EVALUATED AT THE MIN. IS 6.514123359E¹¹

THE PØINT MINIMIZING THE FUNCTØN IS 1.000008069 1.000016156
THE FUNCTØN EVALUATED AT THE MIN. IS 6.503300759E¹¹

THE PØINT MINIMIZING THE FUNCTØN IS 1.000008048 1.000016147
THE FUNCTØN EVALUATED AT THE MIN. IS 1.211397662E¹²

THE PØINT MINIMIZING THE FUNCTØN IS 1.00000022 1.000000332
THE FUNCTØN EVALUATED AT THE MIN. IS 7.364359448E¹⁶

THE PØINT MINIMIZING THE FUNCTØN IS 1.000000027 1.000000054
NØ. ØF ITERATØNS = 58
MIN. FUNCTØN VALUE = 7.364359448E¹⁶
OPTIMUM PØINT = 1.000000027 1.000000054

THE PØINT MINIMIZING THE FUNCTØN IS	0.1517942861	10.107339411
THE FUNCTØN EVALUATED AT THE MIN. IS	2.39058343	
THE PØINT MINIMIZING THE FUNCTØN IS	0.184231729	10.0974021342
THE FUNCTØN EVALUATED AT THE MIN. IS	2.360878633	
THE PØINT MINIMIZING THE FUNCTØN IS	0.2170924165	10.08508035092
THE FUNCTØN EVALUATED AT THE MIN. IS	2.32996354	
THE PØINT MINIMIZING THE FUNCTØN IS	0.2505357894	10.07020801672
THE FUNCTØN EVALUATED AT THE MIN. IS	2.297567044	
THE PØINT MINIMIZING THE FUNCTØN IS	0.2846951892	10.05258644599
THE FUNCTØN EVALUATED AT THE MIN. IS	2.263357725	
THE PØINT MINIMIZING THE FUNCTØN IS	0.3197355306	10.03195555258
THE FUNCTØN EVALUATED AT THE MIN. IS	2.226996313	
THE PØINT MINIMIZING THE FUNCTØN IS	0.3558126825	10.008008590538
THE FUNCTØN EVALUATED AT THE MIN. IS	2.188100793	
THE PØINT MINIMIZING THE FUNCTØN IS	0.3930932344	0.01962362634
THE FUNCTØN EVALUATED AT THE MIN. IS	2.146242483	
THE PØINT MINIMIZING THE FUNCTØN IS	0.4317526786	0.05137936184
THE FUNCTØN EVALUATED AT THE MIN. IS	2.100929744	
THE PØINT MINIMIZING THE FUNCTØN IS	0.4719823617	0.08778116097
THE FUNCTØN EVALUATED AT THE MIN. IS	2.051595151	
THE PØINT MINIMIZING THE FUNCTØN IS	0.5139949322	0.1294542242
THE FUNCTØN EVALUATED AT THE MIN. IS	1.997580631	
THE PØINT MINIMIZING THE FUNCTØN IS	0.5580249705	0.1771443834
THE FUNCTØN EVALUATED AT THE MIN. IS	1.938102693	
THE PØINT MINIMIZING THE FUNCTØN IS	0.6043449384	0.231757722
THE FUNCTØN EVALUATED AT THE MIN. IS	1.872227326	
THE PØINT MINIMIZING THE FUNCTØN IS	0.6532646528	0.294391497
THE FUNCTØN EVALUATED AT THE MIN. IS	1.798798831	
THE PØINT MINIMIZING THE FUNCTØN IS	0.7051751092	0.3664332128
THE FUNCTØN EVALUATED AT THE MIN. IS	1.7164265	
THE PØINT MINIMIZING THE FUNCTØN IS	0.7605098747	0.4495703402
THE FUNCTØN EVALUATED AT THE MIN. IS	1.623251309	
THE PØINT MINIMIZING THE FUNCTØN IS	0.8198908302	0.5460935777
THE FUNCTØN EVALUATED AT THE MIN. IS	1.516990607	
THE PØINT MINIMIZING THE FUNCTØN IS	0.8840456825	0.6589176347
THE FUNCTØN EVALUATED AT THE MIN. IS	1.394443629	
THE PØINT MINIMIZING THE FUNCTØN IS	0.9540163456	0.7921501906
THE FUNCTØN EVALUATED AT THE MIN. IS	1.251157984	
THE PØINT MINIMIZING THE FUNCTØN IS	1.03127862	0.9517241611
THE FUNCTØN EVALUATED AT THE MIN. IS	1.080561686	
THE PØINT MINIMIZING THE FUNCTØN IS	1.118119649	1.146914764
THE FUNCTØN EVALUATED AT THE MIN. IS	0.872629344	
THE PØINT MINIMIZING THE FUNCTØN IS	1.218322423	1.39348196
THE FUNCTØN EVALUATED AT THE MIN. IS	0.6128575212	
THE PØINT MINIMIZING THE FUNCTØN IS	1.33862715	1.72134013
THE FUNCTØN EVALUATED AT THE MIN. IS	0.3184557171	
THE PØINT MINIMIZING THE FUNCTØN IS	1.475597928	2.147014532
THE FUNCTØN EVALUATED AT THE MIN. IS	0.2469437958	
THE PØINT MINIMIZING THE FUNCTØN IS	1.495507449	2.232779336
THE FUNCTØN EVALUATED AT THE MIN. IS	0.2449650529	
THE PØINT MINIMIZING THE FUNCTØN IS	1.494892075	2.235387057
THE FUNCTØN EVALUATED AT THE MIN. IS	0.2448859153	
THE PØINT MINIMIZING THE FUNCTØN IS	1.494609096	2.235430381
THE FUNCTØN EVALUATED AT THE MIN. IS	0.2447246239	

DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATØNS PER ØNE ØNE-DIMENSIØNAL SEARC
H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNRD , DFPVM, MØDCG, RANK
DFPVM
THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277
THE PØINT MINIMIZING THE FUNCTØN IS 1.030025571 1.069377318
THE FUNCTION EVALUATED AT THE MIN. IS 3.849850871
THE PØINT MINIMIZING THE FUNCTØN IS 10.9292133286 0.8276621666
THE FUNCTION EVALUATED AT THE MIN. IS 3.532289332
THE PØINT MINIMIZING THE FUNCTØN IS 10.7851689728 0.5577143794
THE FUNCTION EVALUATED AT THE MIN. IS 1.881189587
THE PØINT MINIMIZING THE FUNCTØN IS 10.3303949877 0.07580837214
THE FUNCTION EVALUATED AT THE MIN. IS 1.805750468
THE PØINT MINIMIZING THE FUNCTØN IS 10.3424666381 0.1113388264
THE FUNCTION EVALUATED AT THE MIN. IS 1.454761675
THE PØINT MINIMIZING THE FUNCTØN IS 10.1565763472 10.009702676838
THE FUNCTION EVALUATED AT THE MIN. IS 1.118841237
THE PØINT MINIMIZING THE FUNCTØN IS 0.03019715963 10.04131652107
THE FUNCTION EVALUATED AT THE MIN. IS 0.8928892843
THE PØINT MINIMIZING THE FUNCTØN IS 0.05656563833 0.008510876165
THE FUNCTION EVALUATED AT THE MIN. IS 0.6918518279
THE PØINT MINIMIZING THE FUNCTØN IS 0.2015731731 0.01731513417
THE FUNCTION EVALUATED AT THE MIN. IS 0.6008832811
THE PØINT MINIMIZING THE FUNCTØN IS 0.3028501993 0.0578264394
THE FUNCTION EVALUATED AT THE MIN. IS 0.3604238432
THE PØINT MINIMIZING THE FUNCTØN IS 0.4351964485 0.2097480473
THE FUNCTION EVALUATED AT THE MIN. IS 0.2225880912
THE PØINT MINIMIZING THE FUNCTØN IS 0.5420599557 0.2824804243
THE FUNCTION EVALUATED AT THE MIN. IS 0.2025620443
THE PØINT MINIMIZING THE FUNCTØN IS 0.5943418833 0.3337477784
THE FUNCTION EVALUATED AT THE MIN. IS 0.07082108686
THE PØINT MINIMIZING THE FUNCTØN IS 0.7765607936 0.5885912146
THE FUNCTION EVALUATED AT THE MIN. IS 0.05347983179
THE PØINT MINIMIZING THE FUNCTØN IS 0.7691162362 0.5902263184
THE FUNCTION EVALUATED AT THE MIN. IS 0.0252599519
THE PØINT MINIMIZING THE FUNCTØN IS 0.8636421911 0.7377129706
THE FUNCTION EVALUATED AT THE MIN. IS 0.00052548464
THE PØINT MINIMIZING THE FUNCTØN IS 0.9866367256 0.9715894828
THE FUNCTION EVALUATED AT THE MIN. IS 0.0002786455954
THE PØINT MINIMIZING THE FUNCTØN IS 0.9836218938 0.9671894895
THE FUNCTION EVALUATED AT THE MIN. IS 0.000002263217993
THE PØINT MINIMIZING THE FUNCTØN IS 0.9999141729 0.9996781582
THE FUNCTION EVALUATED AT THE MIN. IS 5.472959736E¹⁸
THE PØINT MINIMIZING THE FUNCTØN IS 0.999767203 0.9995321469
THE FUNCTION EVALUATED AT THE MIN. IS 2.200698406E¹⁴
THE PØINT MINIMIZING THE FUNCTØN IS 0.9999999829 0.9999999806
THE FUNCTION EVALUATED AT THE MIN. IS 1.10206391E¹⁹
THE PØINT MINIMIZING THE FUNCTØN IS 1 1.000000001
NØ. ØF ITERATØNS = 22
MIN. FUNCTØN VALUE = 1.10206391E¹⁹
OPTIMUM PØINT = 1 1.000000001
1 1.000000001

DO YOU WISH TO RESTART? ANSWER YES OR NO

YES

DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO

NO

DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO

NO

DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE-DIMENSIONAL SEARCH? YES OR NO

NO

DO YOU WISH TO CHANGE ALGORITHM? YES OR NO

YES

ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEEP, NEW, CONGRD, DFPVM, MDCG, RANK

MDCG

THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277

THE POINT MINIMIZING THE FUNCTION IS 1.030025571 1.069377318

THE FUNCTION EVALUATED AT THE MIN. IS 3.849850782

THE POINT MINIMIZING THE FUNCTION IS 1.9292113248 0.8276573746

THE FUNCTION EVALUATED AT THE MIN. IS 3.532275169

THE POINT MINIMIZING THE FUNCTION IS 0.7851616052 0.5577017769

THE FUNCTION EVALUATED AT THE MIN. IS 1.880950734

THE POINT MINIMIZING THE FUNCTION IS 0.3303426877 0.07578876472

THE FUNCTION EVALUATED AT THE MIN. IS 1.805621297

THE POINT MINIMIZING THE FUNCTION IS 0.3424164357 0.1112997209

THE FUNCTION EVALUATED AT THE MIN. IS 1.454661692

THE POINT MINIMIZING THE FUNCTION IS 0.1565297751 0.009718390248

THE FUNCTION EVALUATED AT THE MIN. IS 1.118604278

THE POINT MINIMIZING THE FUNCTION IS 0.03028794762 0.04130382137

THE FUNCTION EVALUATED AT THE MIN. IS 0.892787072

THE POINT MINIMIZING THE FUNCTION IS 0.0566124391 0.008503064048

THE FUNCTION EVALUATED AT THE MIN. IS 0.6917714814

THE POINT MINIMIZING THE FUNCTION IS 0.2016323047 0.0173359584

THE FUNCTION EVALUATED AT THE MIN. IS 0.6007387278

THE POINT MINIMIZING THE FUNCTION IS 0.3029645676 0.05789352772

THE FUNCTION EVALUATED AT THE MIN. IS 0.04833889252

THE POINT MINIMIZING THE FUNCTION IS 0.7860151782 0.6127707134

THE FUNCTION EVALUATED AT THE MIN. IS 0.04718751008

THE POINT MINIMIZING THE FUNCTION IS 0.7834156684 0.6120705665

THE FUNCTION EVALUATED AT THE MIN. IS 0.02301899187

THE POINT MINIMIZING THE FUNCTION IS 0.8712791102 0.7510961458

THE FUNCTION EVALUATED AT THE MIN. IS 0.00005305329812

THE POINT MINIMIZING THE FUNCTION IS 0.9960114992 0.9914294383

THE FUNCTION EVALUATED AT THE MIN. IS 0.00002753941573

THE POINT MINIMIZING THE FUNCTION IS 0.9948852046 0.98967917

THE FUNCTION EVALUATED AT THE MIN. IS 1.458011723E+8

THE POINT MINIMIZING THE FUNCTION IS 1.000018323 1.000024711

THE FUNCTION EVALUATED AT THE MIN. IS 3.05479106E+11

THE POINT MINIMIZING THE FUNCTION IS 0.9999945787 0.9999890499

THE FUNCTION EVALUATED AT THE MIN. IS 1.905018297E+16

THE POINT MINIMIZING THE FUNCTION IS 0.9999999972 0.9999999957

NO. OF ITERATIONS = 18

MIN. FUNCTION VALUE = 1.905018297E+16

OPTIMUM POINT = 0.9999999972 0.9999999957

DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ

YES

DØ YØU WISH TØ CHANGE FUNCTØN? ANSWER YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE NUMBER ØF ITERATØNS PER ØNE ØNE-DIMENSIØNAL SEARC
H? YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ

YES

ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:

STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK

RANK

THE FUNCTØN EVALUATED AT THE MIN. IS 4.128101277

THE PØINT MINIMIZING THE FUNCTØN IS 1.030025571 1.069377318

THE FUNCTØN EVALUATED AT THE MIN. IS 3.849850871

THE PØINT MINIMIZING THE FUNCTØN IS 1.9292133332 0.8276621778

THE FUNCTØN EVALUATED AT THE MIN. IS 3.532289685

THE PØINT MINIMIZING THE FUNCTØN IS 0.7851666627 0.5577100201

THE FUNCTØN EVALUATED AT THE MIN. IS 3.079141875

THE PØINT MINIMIZING THE FUNCTØN IS 0.7526418437 0.5750653501

THE FUNCTØN EVALUATED AT THE MIN. IS 2.594523842

THE PØINT MINIMIZING THE FUNCTØN IS 0.5687657336 0.2869570933

THE FUNCTØN EVALUATED AT THE MIN. IS 2.395497261

THE PØINT MINIMIZING THE FUNCTØN IS 0.4497160099 0.1480392545

THE FUNCTØN EVALUATED AT THE MIN. IS 0.07551420072

THE PØINT MINIMIZING THE FUNCTØN IS 1.274319794 1.625512206

THE FUNCTØN EVALUATED AT THE MIN. IS 0.07543834475

THE PØINT MINIMIZING THE FUNCTØN IS 1.274557857 1.62524825

THE FUNCTØN EVALUATED AT THE MIN. IS 0.03933280092

THE PØINT MINIMIZING THE FUNCTØN IS 1.176880394 1.376077441

THE FUNCTØN EVALUATED AT THE MIN. IS 0.01635171155

THE PØINT MINIMIZING THE FUNCTØN IS 1.087379081 1.173056988

THE FUNCTØN EVALUATED AT THE MIN. IS 0.007012927705

THE PØINT MINIMIZING THE FUNCTØN IS 1.08368169 1.17468698

THE FUNCTØN EVALUATED AT THE MIN. IS 0.001874578518

THE PØINT MINIMIZING THE FUNCTØN IS 1.032472849 1.063136459

THE FUNCTØN EVALUATED AT THE MIN. IS 0.00005159673666

THE PØINT MINIMIZING THE FUNCTØN IS 1.004959463 1.010463141

THE FUNCTØN EVALUATED AT THE MIN. IS 6.733153324E¹⁷

THE PØINT MINIMIZING THE FUNCTØN IS 1.000661884 1.001275706

THE FUNCTØN EVALUATED AT THE MIN. IS 1.14356833E¹⁹

THE PØINT MINIMIZING THE FUNCTØN IS 0.9999312977 0.9999597784

THE FUNCTØN EVALUATED AT THE MIN. IS 1.32057555E¹⁶

THE PØINT MINIMIZING THE FUNCTØN IS 1.000000009 1.000000018

NØ. ØF ITERATØNS = 16

MIN. FUNCTØN VALUE = 1.32057555E¹⁶

OPTIMUM PØINT = 1.000000009 1.000000018

1.000000009 1.000000018

DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ

NØ

In Appendix 2 we provide examples of the UCNLP package in action. The function used for the testing is the well-known "banana-shaped" Rosenbrock [5] function. The procedure of entering the function, its gradient and hessian was somewhat different from the one described earlier. Here, instead of entering the function directly we first stored its alpha-numeric image in the variable names FR1, GR1, HR1 (function, gradient, hessian respectively) and then used the APL Evaluate operator with that variable name. This procedure is equivalent to entering the alpha-numeric image of the function directly. The user may elect to do it either way.

Ending Statement of

Appendix 3

Also note that several symbols on teletype 33 are used in place of the above Mnemonics.

<u>Graphic</u>	<u>Mnemonics</u>	<u>Meaning</u>
@	\$RO	Rho
#	\$QD	Quad
&	\$DL	Del
↑	\$NG	Neg