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An Interactive Package of Programs for Unconstrained Optimization Purposes UCNLP

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DISCUSSION PAPER NO. 171

AN INTERACTIVE PACKAGE OF PROGRAMS FOR
UNCONSTRAINED OPTIMIZATION PURPOSES - UCNLP

by

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September 26, 1975

Preface

Development of the following interactive package would not have been possible without the help of the IBM Corporation which provided the funds and the support which made it feasible for me to attend the intensive APL course in UCLA in the Summer of 1975. Special thanks are due to Ephraim McLean, Ken Siler and Jim Schneck from UCLA who introduced me to the APL language and helped me develop UCNLP.

I am also indebted to Northwestern University's Graduate School of Management for covering my transportation expenses from Chicago to Los Angeles and awarding me a summer grant which enabled me to leave the Northwestern Campus for the period of the APL intensive seminar.

I. Background Information

UCNLP (Unconstrained Nonlinear Programming) is a package of computer programs coded in APL and designed for users who are mainly interested in experiments and tests involving statistical comparisons of nonlinear functions minimized under different conditions. The user provides the computer with information regarding the mathematical expressions of the function to be minimized. The number of iterations per one-one dimensional search. The starting point, and the name of the method to be used. The optional methods are:

1. Steepest Descent [5]
2. Newton's Method [5]
3. The Fletcher-Reeves Conjugate Gradient [4,5]
4. Perry's Modified Conjugate Gradient [6]
5. Davidon -Fletcher-Powell Variable Metric [33]
6. Broyden's Rank 1 Method [1, 5]

and the user may select the one he wishes to apply.

All the above methods are also known as gradient methods. They all require information regarding the mathematical expression of the gradient of the function in question. The only method which requires information regarding the mathematical expression of the hessian matrix is Newton's Method.

Gradient methods are procedures for minimizing a given differentiable unconstrained nonlinear function. The common denominator of all these methods is the fact that they all depend very heavily on the efficiency and the accuracy of the line-search used in the process. The main principle behind these methods is the fact that the function is evaluated at a given point, a direction of descent is then determined, and a search procedure locating

the function minimum along that direction is performed. The new point is then used as the starting point for the next iteration. The process terminates at a local minimum where no direction of descent can be found.

While different gradient methods (in this package) vary with respect to the direction used in the process, they all use the same line search procedure. This procedure is divided into two major stages. In the first stage the program searches for the best unimodal region along the given direction. This is done by simultaneously evaluating the function at 10 arbitrary points along the given direction. If a unimodal region is determined, the interval of uncertainty is reduced to 1/9th of its original size, and the search moves on to its second stage. In the event that the simultaneous search fails to secure a unimodal region the process repeats itself until a unimodal region is located. Once a unimodal interval is located along the search direction, a sequential Fibonacci search [7] is performed. The accuracy of the search is determined by the user who specifies the number of sequential steps in the one-dimensional search. The final error in determining the minimum point along the original interval of uncertainty is $\pm 1/F_N$ where F_N is the Nth Fibonacci number specified by the user.

II. The Computer Application

After loading UCNLP the user is ready to start.

User: OPTIMIZE

Computer: ENTER FUNCTION TO BE MINIMIZED

User: The function is entered here. User should be familiar with APL arithmetic rules according to which the function is expressed. The variables in the function should be of the type $X[J]$ where $J = 1, 2, \dots$

Computer: ENTER GRADIENT OF FUNCTION

User: The gradient vector is entered here according to the same rules as above. If some elements of the gradient vector are mathematical expressions, they should be separated from each other by means of placing them within parentheses and separating them by a comma.

Computer: ENTER INITIAL STARTING POINT

User: The user may enter a numerical vector or a predefined variable name containing that vector.

Computer: ENTER NUMBER OF ITERATIONS PER ONE-ONE DIMENSIONAL SEARCH.

User: Enters the number which is used in the sequential part of the search after a unimodal region is determined.

Computer: DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO.

User: If user enters "YES" output will be printed at the end of every one dimensional search. The word "iteration" refers to a step consisting of a direction fixing and the line search along that direction. If user enters "NO" the only output consists of two statements regarding the optimal value of the function and the optimal point.

Computer: ENTER NAME OF ALGORITHM TO BE EXECUTED: CHOOSE AMONG: STEEP, NEW, CONGRD, DFPVM, MODCG, RANK.

User: The user must enter the full name (described above) of the algorithm he wishes to use.

STEEP = The Method of Steepest Descent
NEW = Newton's Method
CONGRD = Fletcher-Reeves: Conjugate Gradient Method
DFPVM = Davidon -Fletcher-Powell: Variable Metric Method
MODCG = Perry: Modified Conjugate Gradient Method
RANK = Broyden's: Rank-One Method

The computer now calculates the minimum value of the function and the corresponding n-dimensional point. After completing this assignment the user may restart by responding positively to the question:

Computer: DO YOU WISH TO RESTART? YES OR NO

The computer will then go on asking questions until all the required information is given and the process repeats itself. Upon a negative response to the above question the computer closes the "OPTIMIZE" function and gets ready for any new assignment.

Note that if the user wishes to use Newton's Method he must provide the computer with information on the hessian matrix. Therefore, upon responding "NEW" to the question concerning the algorithm's name the computer responds by first requesting that information. The Hessian matrix must be entered row-wise for example. Let $A = \{a_{ij}\}$ be the hessian of f then entering A is done in the following way: $a_{11}, a_{12}, \dots, a_{1n}, a_{21}, \dots, a_{2n}, \dots, a_{nn}$. (There is no need to dimension the matrix) if a_{ij} is a mathematical expression, it must be entered according to APL rules.

In the Appendix we provide the listing of the APL source programs followed by examples.

References

- [1] Broyden, C. G., "Quasi-Newton Methods and Their Application to Function Minimization," Math. Comp., 21, 1967, pp. 368-381.
- [2] Davidon, W. G., "Variable Metric Method for Minimization," Research and Development Report ANL-5990 (Ref.) U.S. Atomic Energy Commission, Argonne National Laboratories, 1959.
- [3] Fletcher, R. and Powell, M. J. D., "A Rapidly Convergent Descent Method for Minimization," Computer J., 6, 1963, pp. 163-168.
- [4] Fletcher, R. and Reeves, C. M., "Function Minimization by Conjugate Gradients," Computer J., 7, 1964, pp. 149-159.
- [5] Himmelblau, D. M., "Applied Nonlinear Programming," McGraw-Hill, 1972, pp. 63-140.
- [6] Perry, A., "Duality in Unconstrained Nonlinear Optimization," The Center for Mathematical Studies in Economics and Management Science, Northwestern University (forthcoming).
- [7] Wilde, D. J. and Beightler, C. S., Foundations of Optimization, Prentice-Hall, 1967.

A P P E N D I C E S

Appendix 1

```

&CGM[#1]&
& XBAR$IS CGM X;CØUNT;Z;DØ;GØ;Y;X1;G1;Q;P;S1;SØ;D1
[1]  CØUNT$IS 0
[2]  START:Z$IS FUN X
[3]  DØ$IS-GØ$IS GR X
[4]  SØ$IS($IØ@GØ)$NL.= $IØ@GØ
[5]  Y$ISX+DØ
[6]  L1:X1$IS X FIBSCH Y
[7]  CØUNT$ISCØUNT+1
[8]  G1$IS GR X1
[9]  Q$IS((@G1),1)@(G1-GØ)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$ISSØ+(P+.$ML($TP(P-Q)))$DV($TP)+.$MLQ
[12] D1$IS-S1+.$MLG1
[13] Y$ISX1+D1
[14] X$ISX1
[15] DØ$ISD1
[16] GØ$ISG1
[17] $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[18] 'NØ. ØF ITERATIØNS = ';CØUNT
[19] 'MIN. FUNCTION VALUE = ';FUN X
[20] 'ØPTIMUM PØINT = ';X
&

```

```

&DFP[#1]&
& XBAR$IS DFP X;CØUNT;Z;DØ;GØ;Y;X1;G1;Q;P;S1;SØ;D1
[1]  CØUNT$ISØ
[2]  START:Z$IS FUN X
[3]  DØ$IS-GØ$ISGR X
[4]  SØ$IS($IØ@GØ)$NL.= $IØ@GØ
[5]  Y$ISX+DØ
[6]  L1:X1$ISXBAR$IS X FIBSCH Y
[7]  CØUNT$IS CØUNT+1
[8]  G1$IS GR X1
[9]  Q$IS((@G1),1)@(G1-GØ)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$IS(SØ-((SØ+.$MLQ+.$ML($TPQ)+.$MLSO)$DV((STPQ)+.$MLSO+.$MLQ)))+(P
.$ML($TP)+.$MLQ)$DV((STP)+.
      $MLQ)
[12] D1$IS-S1+.$MLG1
[13] Y$ISX1+,D1
[14] X$ISX1
[15] DØ$ISD1
[16] GØ$ISG1
[17] SØ$ISS1
[18] $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[19] ' NØ. ØF ITERATIØNS = ';CØUNT
[20] 'MIN. FUNCTION VALUE = ';FUN X
[21] ' ØPTIMUM PØINT = ';X
&

```

```

&FIBNUM[ # ]&
& FNSIS FIBNUM K;R
[1] FNSIS 1 1
[2] END:$GØ(K>@FNSISFN,+/'2$TAFN)/END
&

```

```

&FIBSCH[ # ]&
& PØINT$ISX FIBSCH Y;CØUNT;MI;FALPHA;C1;C2;X1;Y1;R;RN;FN;D;ND;RD;K;F;X

O;NI
[1] L20:D$ISY-X
[2] L10:X1$ISX+EPS$MLD
[3] $GØ(($MN/((FUN X),FUN X1))=((FUN X),FUN X1))/(L11,L12)
[4] L11:Y$ISX+D$IS-GR X
[5] $GØ L10
[6] L12:ND$IS(D+.$MLD)*.5
[7] RDSIS$DV$IO9
[8] X$IS(10,@D)@X,(,((9,@D)@X)+RD[$UGRD]$NL.$MLD)
[9] K$IS1
[10] F$ISSIØØ
[11] L21:F$ISF,FUN X[K;]
[12] K$ISK+1
[13] $GØL21$ML$IØK$LEIØ
[14] $GØ L22$ML$IØ(1$TASUGF)=10
[15] $GØ L25$ML$IØ(1$TASUGF)=1
[16] X1$IS(EPS$MLD)+,XØ$ISX[1$TASUGF;]
[17] $GØ(((FUN X1)$GEFUN ,XØ),(FUN X1)<FUN ,XØ)/L23,L24
[18] L22:X$IS,X[9;]
[19] Y$ISY+D
[20] $GØ L20
[21] L23:Y$ISX1
[22] X$IS,X[(1$TASUGF)-1;]
[23] $GØLØ
[24] L24:Y$IS,X[(1$TASUGF)+1;]
[25] X$IS,XØ
[26] $GØLØ
[27] L25:Y$IS,X[2;]
[28] X$IS,X[1;]
[29] $GØ L20
[30] LO:NI$ISN
[31] X1$IS,X
[32] Y1$IS,Y
[33] CØUNT$IS 2
[34] D$ISY-X
[35] L1:FNSIS FIBNUM NI
[36] R$IS+1$DRFN$DV+1$TAFN
[37] C2$ISX+(+1$STAR)$MLD
[38] C1$ISY-(+1$STAR)$MLD
[39] R$IS+2$DRR
[40] LØØP:$GØ(((FUN C1)>FUN C2),((FUN C1)=FUN C2),((FUN C1)<FUN C2))/(L2,L

```

```

4,L3)
[41] L2:X$IS C1
[42] C1$ISC2
[43] C2$IS Y-(+1$STAR)$MLD
[44] $G0 RESTART
[45] L4:N1$ISN1-COUNT
[46] X$IS C1
[47] Y$ISC2
[48] D$IS Y-X
[49] COUNT$IS2
[50] $G0L1$ML$I0(N1>COUNT)
[51] $G0 RESTART
[52] L3:Y$ISC2
[53] C2$ISC1
[54] C1$ISX+(+1$STAR)$MLD
[55] RESTART:R$IS+1$DRR
[56] COUNT$ISCOUNT+1
[57] $G0L00P$ML$I0(N1>COUNT+1)
[58] C1$ISC1-EPS$MLY-X
[59] $G0L00P$ML$I0(N1=COUNT+1)
[60] FALPHA$IS($MN/(FUN X),(FUN C1),(FUN C2),FUN Y)=(FUN X),(FUN C1),(FUN
C

```

```

2),FUN Y
[61] M1$IS(4,@,X)@(X,C1,C2,Y)
[62] P0INT$ISM1[FALPHA$I01;]
[63] $G0L15$ML0UTPT
[64] L15:'THE FUNCTION EVALUATED AT THE MIN. IS ';FUN P0INT
[65] ' THE P0INT MINIMIZING THE FUNCTION IS ';P0INT

```

&

```

&FRCG[#]&
& XBAR$IS FRCG X;COUNT;D0;D1;G0;G1;Z
[1] COUNT$IS 1
[2] START:Z$IS FUN X
[3] D0$IS-G0$ISGR X
[4] Y$ISX+D0
[5] X$IS X FIBSCH Y
[6] G1$IS GR X
[7] L1:COUNT$ISCOUNT+1
[8] D1$IS(-G1)+((G1+.$MLG1)$DV(G0+.$MLG0))$MLD0
[9] Y$ISX+D1
[10] X$IS X FIBSCH Y
[11] G0$ISG1
[12] D0$ISD1
[13] G1$ISGR X
[14] $G0 L1$ML($I0($ND/($MDG1)$LE EPS))
[15] ' N0. OF ITERATIONS = ';COUNT
[16] ' MIN. FUNCTION VALUE = '; FUN X
[17] 'OPTIMUM P0INT = ';X

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&

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      &FUNC[#]&
& F$IS FUN X
[1] F$IS$EVFX
&
      &GR[#]&
& G$IS GR X
[1] G$IS$EVGX
&
      &HES[#]&
& H$IS HES X
[1] F$IS((@X),@X)@$EVHX
&

```

```

      &NEWTON[#]&
& XBAR$IS NEWTON X;Z;G;D;H;Y;COUNT
[1] COUNT$IS 0
[2] START:Z$ISFUN X
[3] G$IS((@X),1)@GR X
[4] $GØ LI$ML($IØ($AN/($MD,G)<EPS))
[5] COUNT$ISCOUNT+1
[6] H$IS HES X
[7] D$IS-($XDH)+.$MLG
[8] Y$ISX+,D
[9] X$ISX FIBSCH Y
[10] $GØ START
[11] LI:'NO. ØF ITERATIONS = ';COUNT
[12] 'MIN. FUNCTION VALUE = ';FUN X
[13] 'ØPTIMUM POINT = ';X
&

```

```

&OPTIMIZE[#]&
& OPTIMIZE
[1] L1:'ENTER FUNCTION TO BE MINIMIZED'
[2] FX$ISSQP
[3] L2:'ENTER GRADIENT VECTOR OF THE ABOVE FUNCTION'
[4] GX$ISSQP
[5] L3:'ENTER INITIAL STARTING POINT'
[6] A$IS#
[7] L4:'ENTER NUMBER OF ITERATIONS PER ONE ONE DIMENSIONAL SEARCH'
[8] N$IS#
[9] L6:'DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO'
[10] OUTPT$IS(1$TASQP)='Y'
[11] L5:'ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:'
[12] 'STEEP, NEW, CONGRD, DFPVM, M0DCG, RANK'
[13] $G0SEV$QP
[14] STEEP:SDT A
[15] $G0 FIN
[16] NEW:'ENTER HESSIAN MATRIX OF THE ABOVE FUNCTION'
[17] HX$ISSQP
[18] NEWTON A
[19] $G0 FIN
[20] CONGRD:FRCG A
[21] $G0 FIN
[22] M0DCG:CGM A
[23] $G0 FIN
[24] RANK:RANK1 A
[25] $G0 FIN
[26] DFPVM:DFP A
[27] FIN:'DO YOU WISH TO RESTART? ANSWER YES OR NO'
[28] ANSS$ISI$TASQP
[29] $G0 START$MLANS='Y'
[30] START:'DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO'
[31] ANSS$ISI$TASQP
[32] $G0 L1$ML$IOANS='Y'
[33] 'DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO'
[34] ANSS$ISI$TASQP
[35] $G0 L3$ML$IOANS='Y'
[36] 'DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL
L SEARCH? YES OR NO'
[37] ANSS$ISI$TASQP
[38] $G0 L4$ML$IOANS='Y'
[39] 'DO YOU WISH TO CHANGE ALGORITHM? YES OR NO'
[40] ANSS$ISI$TASQP
[41] $G0 L5$MLANS='Y'

```

&

```

&RANKI[#]&
& XBAR$IS RANKI X;CØUNT;Z;DØ;D1;GØ;Y;X1;G1;Q;P;S1;SØ
[1] CØUNT$ISO
[2] START:Z$IS FUN X
[3] DØ$IS-GØ$ISGR X
[4] SØ$IS($IØ@GØ)$NL.= $IØ@GØ
[5] Y$ISX+DØ
[6] L1:X1$ISXBAR$ISX FIBSCH Y
[7] CØUNT$ISCØUNT+1
[8] G1$IS GR X1
[9] Q$IS((@G1),1)@(G1-GØ)
[10] P$IS((@G1),1)@(X1-X)
[11] S1$IS SØ+((P-SØ+.$MLQ)+.$ML($TP(P-SØ+.$MLQ)))$DV($TP(P-SØ+.$MLQ))+.$
LQ M
[12] D1$IS-S1+.$MLG1
[13] Y$ISX1+,D1
[14] X$ISX1
[15] DØ$ISD1
[16] GØ$ISG1
[17] SØ$ISS1
[18] $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[19] ' NØ. ØF ITERATIONS = ';CØUNT
[20] ' MIN. FUNCTION VALUE = ';FUN X
[21] ' ØPTIMUM PØINT = ';X
&

```

```

&SDT[#]&
& XBAR$IS SDT X;Z;D;Y;CØUNT
[1] CØUNT$ISO
[2] START:Z$IS FUN X
[3] D$IS GR X
[4] $GØ L1$ML($IØ($AN/($MDD)$LEEPS))
[5] Y$IS X-D
[6] X$IS X FIBSCH Y
[7] CØUNT$IS CØUNT+1
[8] $GØ START
[9] L1:'NØ. ØF ITERATIONS = ';CØUNT
[10] 'MIN. FUNCTION VALUE = ';Z
[11] 'ØPTIMUM PØINT ';X
&

```

```

) VARS
A      ANS      EPS      FRI      FX      GRI      GX      HRI      HX
N      ØUTPT
      EPS
0.000001
      FRI
(100$ML(X[2]-X[1]*2)*2)+(1-X[1])*2
      FX
$EVFRI
      GRI
(((400$MLX[1]*3)-(400$MLX[1]$MLX[2]))+(2$MLX[1])-2),(200$ML(X[2]-X[1]*2))
      GX
$EVGRI
      HRI
(((1200$MLX[1]*2)-400$MLX[2])+2),(-400$MLX[1]),(-400$MLX[1]),200
      HX
$EVHRI

```


OPTIMIZE
 ENTER FUNCTION TO BE MINIMIZED
 \$EVFR1

ENTER GRADIENT VECTOR OF THE ABOVE FUNCTION
 \$EVGR1

ENTER INITIAL STARTING POINT

#:

+1.2 1

ENTER NUMBER OF ITERATIONS PER ONE ONE DIMENSIONAL SEARCH

#:

20

DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO

YES

ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEEP, NEW, CONGRD, DFPVM, MDCG, RANK

NEW

ENTER HESSIAN MATRIX OF THE ABOVE FUNCTION

\$EVHR1

THE FUNCTION EVALUATED AT THE MIN. IS 4.73154706

THE POINT MINIMIZING THE FUNCTION IS +1.175177674 1.382263817

THE FUNCTION EVALUATED AT THE MIN. IS 4.045639354

THE POINT MINIMIZING THE FUNCTION IS +0.9595197027 0.3752994427

THE FUNCTION EVALUATED AT THE MIN. IS 3.160046149

THE POINT MINIMIZING THE FUNCTION IS +0.6897928596 0.4206194213

THE FUNCTION EVALUATED AT THE MIN. IS 1.954889045

THE POINT MINIMIZING THE FUNCTION IS +0.333958907 0.06964267338

THE FUNCTION EVALUATED AT THE MIN. IS 1.25388344

THE POINT MINIMIZING THE FUNCTION IS +0.05087007794 +0.03608464387

THE FUNCTION EVALUATED AT THE MIN. IS 0.6662845319

THE POINT MINIMIZING THE FUNCTION IS 0.2388743903 0.02756990143

THE FUNCTION EVALUATED AT THE MIN. IS 0.3241042274

THE POINT MINIMIZING THE FUNCTION IS 0.4820798993 0.2087656752

THE FUNCTION EVALUATED AT THE MIN. IS 0.10968867

THE POINT MINIMIZING THE FUNCTION IS 0.7072517458 0.484717253

THE FUNCTION EVALUATED AT THE MIN. IS 0.02053228579

THE POINT MINIMIZING THE FUNCTION IS 0.8842157548 0.7733957628

THE FUNCTION EVALUATED AT THE MIN. IS 0.00006242121203

THE POINT MINIMIZING THE FUNCTION IS 1.0030349 1.006808466

THE FUNCTION EVALUATED AT THE MIN. IS 1.723432357E+7

THE POINT MINIMIZING THE FUNCTION IS 0.9996244692 0.9992667769

THE FUNCTION EVALUATED AT THE MIN. IS 5.004856024E+13

THE POINT MINIMIZING THE FUNCTION IS 0.999999698 0.9999993321

THE FUNCTION EVALUATED AT THE MIN. IS 5.63422129E+22

THE POINT MINIMIZING THE FUNCTION IS 1 1

NO. OF ITERATIONS = 13

MIN. FUNCTION VALUE = 5.63422129E+22

OPTIMUM POINT = 1 1

DO YOU WISH TO RESTART? ANSWER YES OR NO

YES

DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO

NO

DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO

NO

DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL SEARCH?
H? YES OR NO

NO

DO YOU WISH TO CHANGE ALGORITHM? YES OR NO

YES

ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEEP, NEW, CONGRD, DFPVM, MDCG, RANK

CONGRD

THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277

THE POINT MINIMIZING THE FUNCTION IS +1.030025571 1.069377318

THE FUNCTION EVALUATED AT THE MIN. IS 4.010183434

THE POINT MINIMIZING THE FUNCTION IS +0.9876896931 0.9511848589

THE FUNCTION EVALUATED AT THE MIN. IS 3.601940313

THE POINT MINIMIZING THE FUNCTION IS +0.8211936621 0.6209554766

THE FUNCTION EVALUATED AT THE MIN. IS 3.386261918

THE POINT MINIMIZING THE FUNCTION IS +0.7070239084 0.4311564444

THE FUNCTION EVALUATED AT THE MIN. IS 3.238243177

THE POINT MINIMIZING THE FUNCTION IS +0.6166182964 0.301174562

THE FUNCTION EVALUATED AT THE MIN. IS 3.126287847

THE POINT MINIMIZING THE FUNCTION IS +0.5406214117 0.2055089893

THE FUNCTION EVALUATED AT THE MIN. IS 3.036978248

THE POINT MINIMIZING THE FUNCTION IS +0.4745132746 0.1322764198

THE FUNCTION EVALUATED AT THE MIN. IS 2.96319986

THE POINT MINIMIZING THE FUNCTION IS +0.4156882953 0.07486685993

THE FUNCTION EVALUATED AT THE MIN. IS 2.900652678

THE POINT MINIMIZING THE FUNCTION IS +0.3624651504 0.02918796243

THE FUNCTION EVALUATED AT THE MIN. IS 2.846511903

THE POINT MINIMIZING THE FUNCTION IS +0.3136710074 +0.007477414943

THE FUNCTION EVALUATED AT THE MIN. IS 2.798815827

THE POINT MINIMIZING THE FUNCTION IS +0.2684463449 +0.03701725006

THE FUNCTION EVALUATED AT THE MIN. IS 2.756135641

THE POINT MINIMIZING THE FUNCTION IS +0.2261347296 +0.06078847288

THE FUNCTION EVALUATED AT THE MIN. IS 2.717387403

THE POINT MINIMIZING THE FUNCTION IS +0.1861986648 +0.07979927274

THE FUNCTION EVALUATED AT THE MIN. IS 2.681754192

THE POINT MINIMIZING THE FUNCTION IS +0.148229187 +0.0947895712

THE FUNCTION EVALUATED AT THE MIN. IS 2.64855315

THE POINT MINIMIZING THE FUNCTION IS +0.1118561848 +0.1063296418

THE FUNCTION EVALUATED AT THE MIN. IS 2.617246695

THE POINT MINIMIZING THE FUNCTION IS +0.07677704204 +0.1148445875

THE FUNCTION EVALUATED AT THE MIN. IS 2.58738084

THE POINT MINIMIZING THE FUNCTION IS +0.04272125981 +0.120654003

THE FUNCTION EVALUATED AT THE MIN. IS 2.558574948

THE POINT MINIMIZING THE FUNCTION IS +0.009466284078 +0.1239891052

THE FUNCTION EVALUATED AT THE MIN. IS 2.530473758

THE POINT MINIMIZING THE FUNCTION IS 0.02321349277 +0.1250143845

THE FUNCTION EVALUATED AT THE MIN. IS 2.502780236

THE POINT MINIMIZING THE FUNCTION IS 0.05550318465 +0.1238329892

THE FUNCTION EVALUATED AT THE MIN. IS 2.475206843

THE POINT MINIMIZING THE FUNCTION IS 0.08758582501 +0.1204968635

THE FUNCTION EVALUATED AT THE MIN. IS 2.447487845

THE POINT MINIMIZING THE FUNCTION IS 0.1196329015 +0.1150108832

THE FUNCTION EVALUATED AT THE MIN. IS 2.419371266

THE POINT MINIMIZING THE FUNCTION IS 1.493878832 2.234517067
THE FUNCTION EVALUATED AT THE MIN. IS 0.000006527763908
THE POINT MINIMIZING THE FUNCTION IS 1.002513586 1.004987702
THE FUNCTION EVALUATED AT THE MIN. IS 0.000006206209242
THE POINT MINIMIZING THE FUNCTION IS 1.002488856 1.004994771
THE FUNCTION EVALUATED AT THE MIN. IS 0.000005927883814
THE POINT MINIMIZING THE FUNCTION IS 1.002399737 1.004764107
THE FUNCTION EVALUATED AT THE MIN. IS 1.895894375E+9
THE POINT MINIMIZING THE FUNCTION IS 1.000007092 1.00001848
THE FUNCTION EVALUATED AT THE MIN. IS 6.514123359E+11
THE POINT MINIMIZING THE FUNCTION IS 1.000008069 1.000016156
THE FUNCTION EVALUATED AT THE MIN. IS 6.503300759E+11
THE POINT MINIMIZING THE FUNCTION IS 1.000008048 1.000016147
THE FUNCTION EVALUATED AT THE MIN. IS 1.211397662E+12
THE POINT MINIMIZING THE FUNCTION IS 1.00000022 1.000000332
THE FUNCTION EVALUATED AT THE MIN. IS 7.364359448E+16
THE POINT MINIMIZING THE FUNCTION IS 1.000000027 1.000000054
NO. OF ITERATIONS = 58
MIN. FUNCTION VALUE = 7.364359448E+16
OPTIMUM POINT = 1.000000027 1.000000054

THE POINT MINIMIZING THE FUNCTION IS	0.1517942861	†0.107339411
THE FUNCTION EVALUATED AT THE MIN. IS	2.39058343	
THE POINT MINIMIZING THE FUNCTION IS	0.184231729	†0.0974021342
THE FUNCTION EVALUATED AT THE MIN. IS	2.360878633	
THE POINT MINIMIZING THE FUNCTION IS	0.2170924165	†0.08508035092
THE FUNCTION EVALUATED AT THE MIN. IS	2.32996354	
THE POINT MINIMIZING THE FUNCTION IS	0.2505357894	†0.07020801672
THE FUNCTION EVALUATED AT THE MIN. IS	2.297567044	
THE POINT MINIMIZING THE FUNCTION IS	0.2846951892	†0.05258644599
THE FUNCTION EVALUATED AT THE MIN. IS	2.263357725	
THE POINT MINIMIZING THE FUNCTION IS	0.3197355306	†0.03195555258
THE FUNCTION EVALUATED AT THE MIN. IS	2.226996313	
THE POINT MINIMIZING THE FUNCTION IS	0.3558126825	†0.008008590538
THE FUNCTION EVALUATED AT THE MIN. IS	2.188100793	
THE POINT MINIMIZING THE FUNCTION IS	0.3930932344	0.01962362634
THE FUNCTION EVALUATED AT THE MIN. IS	2.146242483	
THE POINT MINIMIZING THE FUNCTION IS	0.4317526786	0.05137936184
THE FUNCTION EVALUATED AT THE MIN. IS	2.100929744	
THE POINT MINIMIZING THE FUNCTION IS	0.4719823617	0.08778116097
THE FUNCTION EVALUATED AT THE MIN. IS	2.051595151	
THE POINT MINIMIZING THE FUNCTION IS	0.5139949322	0.1294542242
THE FUNCTION EVALUATED AT THE MIN. IS	1.997580631	
THE POINT MINIMIZING THE FUNCTION IS	0.5580249705	0.1771443834
THE FUNCTION EVALUATED AT THE MIN. IS	1.938102693	
THE POINT MINIMIZING THE FUNCTION IS	0.6043449384	0.231757722
THE FUNCTION EVALUATED AT THE MIN. IS	1.872227326	
THE POINT MINIMIZING THE FUNCTION IS	0.6532646528	0.294391497
THE FUNCTION EVALUATED AT THE MIN. IS	1.798798831	
THE POINT MINIMIZING THE FUNCTION IS	0.7051751092	0.3664332128
THE FUNCTION EVALUATED AT THE MIN. IS	1.7164265	
THE POINT MINIMIZING THE FUNCTION IS	0.7605098747	0.4495703402
THE FUNCTION EVALUATED AT THE MIN. IS	1.623251309	
THE POINT MINIMIZING THE FUNCTION IS	0.8198908302	0.5460935777
THE FUNCTION EVALUATED AT THE MIN. IS	1.516990607	
THE POINT MINIMIZING THE FUNCTION IS	0.8340456825	0.6589176347
THE FUNCTION EVALUATED AT THE MIN. IS	1.394443629	
THE POINT MINIMIZING THE FUNCTION IS	0.9540163456	0.7921501906
THE FUNCTION EVALUATED AT THE MIN. IS	1.251157984	
THE POINT MINIMIZING THE FUNCTION IS	1.03127862	0.9517241611
THE FUNCTION EVALUATED AT THE MIN. IS	1.080561686	
THE POINT MINIMIZING THE FUNCTION IS	1.118119649	1.146914764
THE FUNCTION EVALUATED AT THE MIN. IS	0.872629344	
THE POINT MINIMIZING THE FUNCTION IS	1.218322423	1.39348196
THE FUNCTION EVALUATED AT THE MIN. IS	0.6128575212	
THE POINT MINIMIZING THE FUNCTION IS	1.33862715	1.72134013
THE FUNCTION EVALUATED AT THE MIN. IS	0.3184557171	
THE POINT MINIMIZING THE FUNCTION IS	1.475597928	2.147014532
THE FUNCTION EVALUATED AT THE MIN. IS	0.2469437958	
THE POINT MINIMIZING THE FUNCTION IS	1.495507449	2.232779336
THE FUNCTION EVALUATED AT THE MIN. IS	0.2449650529	
THE POINT MINIMIZING THE FUNCTION IS	1.494892075	2.235387057
THE FUNCTION EVALUATED AT THE MIN. IS	0.2448859153	
THE POINT MINIMIZING THE FUNCTION IS	1.494609096	2.235430381
THE FUNCTION EVALUATED AT THE MIN. IS	0.2447246239	

DO YOU WISH TO RESTART? ANSWER YES OR NO

YES

DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO

NO

DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO

NO

DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL SEARCH?
H? YES OR NO

NO

DO YOU WISH TO CHANGE ALGORITHM? YES OR NO

YES

ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEEP, NEW, CONGRD, DFPVM, MDCG, RANK

DFPVM

THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277

THE POINT MINIMIZING THE FUNCTION IS +1.030025571 1.069377318

THE FUNCTION EVALUATED AT THE MIN. IS 3.849850871

THE POINT MINIMIZING THE FUNCTION IS +0.9292133286 0.8276621666

THE FUNCTION EVALUATED AT THE MIN. IS 3.532289332

THE POINT MINIMIZING THE FUNCTION IS +0.7851689728 0.5577143794

THE FUNCTION EVALUATED AT THE MIN. IS 1.881189587

THE POINT MINIMIZING THE FUNCTION IS +0.3303949877 0.07580837214

THE FUNCTION EVALUATED AT THE MIN. IS 1.805750468

THE POINT MINIMIZING THE FUNCTION IS +0.3424666381 0.1113388264

THE FUNCTION EVALUATED AT THE MIN. IS 1.454761675

THE POINT MINIMIZING THE FUNCTION IS +0.1565763472 +0.009702676838

THE FUNCTION EVALUATED AT THE MIN. IS 1.118841237

THE POINT MINIMIZING THE FUNCTION IS 0.03019715963 +0.04131652107

THE FUNCTION EVALUATED AT THE MIN. IS 0.8928892843

THE POINT MINIMIZING THE FUNCTION IS 0.05656563833 0.008510876165

THE FUNCTION EVALUATED AT THE MIN. IS 0.6918518279

THE POINT MINIMIZING THE FUNCTION IS 0.2015731731 0.01731513417

THE FUNCTION EVALUATED AT THE MIN. IS 0.6008832811

THE POINT MINIMIZING THE FUNCTION IS 0.3028501993 0.0578264394

THE FUNCTION EVALUATED AT THE MIN. IS 0.3604238432

THE POINT MINIMIZING THE FUNCTION IS 0.4351964485 0.2097480473

THE FUNCTION EVALUATED AT THE MIN. IS 0.2225880912

THE POINT MINIMIZING THE FUNCTION IS 0.5420599557 0.2824804243

THE FUNCTION EVALUATED AT THE MIN. IS 0.2025620443

THE POINT MINIMIZING THE FUNCTION IS 0.5943418833 0.3337477784

THE FUNCTION EVALUATED AT THE MIN. IS 0.07082108686

THE POINT MINIMIZING THE FUNCTION IS 0.7765607936 0.5885912146

THE FUNCTION EVALUATED AT THE MIN. IS 0.05347983179

THE POINT MINIMIZING THE FUNCTION IS 0.7691162362 0.5902263184

THE FUNCTION EVALUATED AT THE MIN. IS 0.0252599519

THE POINT MINIMIZING THE FUNCTION IS 0.8636421911 0.7377129706

THE FUNCTION EVALUATED AT THE MIN. IS 0.00052548464

THE POINT MINIMIZING THE FUNCTION IS 0.9866367256 0.9715894828

THE FUNCTION EVALUATED AT THE MIN. IS 0.0002786455954

THE POINT MINIMIZING THE FUNCTION IS 0.9836218938 0.9671894895

THE FUNCTION EVALUATED AT THE MIN. IS 0.000002263217993

THE POINT MINIMIZING THE FUNCTION IS 0.9999141729 0.9996781582

THE FUNCTION EVALUATED AT THE MIN. IS 5.472959736E+8

THE POINT MINIMIZING THE FUNCTION IS 0.999767203 0.9995321469

THE FUNCTION EVALUATED AT THE MIN. IS 2.200698406E+14

THE POINT MINIMIZING THE FUNCTION IS 0.9999999829 0.9999999806

THE FUNCTION EVALUATED AT THE MIN. IS 1.10206391E+19

THE POINT MINIMIZING THE FUNCTION IS 1 1.000000001

NO. OF ITERATIONS = 22

MIN. FUNCTION VALUE = 1.10206391E+19

OPTIMUM POINT = 1 1.000000001

1 1.000000001

DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ

YES

DØ YØU WISH TØ CHANGE FUNCTION? ANSWER YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIONS PER ØNE ØNE-DIMENSIONAL SEARCH?
H? YES ØR NØ

NØ

DØ YØU WISH TØ CHANGE ALGORITHM? YES ØR NØ

YES

ENTER NAME ØF ALGORITHM TØ BE EXECUTED, CHØØSE AMØNG:

STEEP, NEW, CONGRD, DFPVM, MØDCG, RANK

MØDCG

THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277

THE POINT MINIMIZING THE FUNCTION IS +1.030025571 1.069377318

THE FUNCTION EVALUATED AT THE MIN. IS 3.849850782

THE POINT MINIMIZING THE FUNCTION IS +0.9292113248 0.8276573746

THE FUNCTION EVALUATED AT THE MIN. IS 3.532275169

THE POINT MINIMIZING THE FUNCTION IS +0.7851616052 0.5577017769

THE FUNCTION EVALUATED AT THE MIN. IS 1.880950734

THE POINT MINIMIZING THE FUNCTION IS +0.3303426877 0.07578876472

THE FUNCTION EVALUATED AT THE MIN. IS 1.805621297

THE POINT MINIMIZING THE FUNCTION IS +0.3424164357 0.1112997209

THE FUNCTION EVALUATED AT THE MIN. IS 1.454661692

THE POINT MINIMIZING THE FUNCTION IS +0.1565297751 +0.009718390248

THE FUNCTION EVALUATED AT THE MIN. IS 1.118604278

THE POINT MINIMIZING THE FUNCTION IS 0.03028794762 +0.04130382137

THE FUNCTION EVALUATED AT THE MIN. IS 0.892787072

THE POINT MINIMIZING THE FUNCTION IS 0.0566124391 0.008503064048

THE FUNCTION EVALUATED AT THE MIN. IS 0.6917714814

THE POINT MINIMIZING THE FUNCTION IS 0.2016323047 0.0173359584

THE FUNCTION EVALUATED AT THE MIN. IS 0.6007337278

THE POINT MINIMIZING THE FUNCTION IS 0.3029645676 0.05789352772

THE FUNCTION EVALUATED AT THE MIN. IS 0.04833889252

THE POINT MINIMIZING THE FUNCTION IS 0.7860151782 0.6127707134

THE FUNCTION EVALUATED AT THE MIN. IS 0.04718751008

THE POINT MINIMIZING THE FUNCTION IS 0.7834156684 0.6120705665

THE FUNCTION EVALUATED AT THE MIN. IS 0.02301899187

THE POINT MINIMIZING THE FUNCTION IS 0.8712791102 0.7510961458

THE FUNCTION EVALUATED AT THE MIN. IS 0.00005305329812

THE POINT MINIMIZING THE FUNCTION IS 0.9960114992 0.9914294383

THE FUNCTION EVALUATED AT THE MIN. IS 0.00002753941573

THE POINT MINIMIZING THE FUNCTION IS 0.9948852046 0.98967917

THE FUNCTION EVALUATED AT THE MIN. IS 1.458011723E+8

THE POINT MINIMIZING THE FUNCTION IS 1.000018323 1.000024711

THE FUNCTION EVALUATED AT THE MIN. IS 3.05479106E+11

THE POINT MINIMIZING THE FUNCTION IS 0.9999945787 0.9999890499

THE FUNCTION EVALUATED AT THE MIN. IS 1.905018297E+16

THE POINT MINIMIZING THE FUNCTION IS 0.9999999972 0.9999999957

NØ. ØF ITERATIONS = 18

MIN. FUNCTION VALUE = 1.905018297E+16

OPTIMUM PØINT = 0.9999999972 0.9999999957

DØ YOU WISH TØ RESTART? ANSWER YES ØR NØ

YES

DØ YOU WISH TØ CHANGE FUNCTION? ANSWER YES ØR NØ

NØ

DØ YOU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ

NØ

DØ YOU WISH TØ CHANGE NUMBER ØF ITERATIONS PER ØNE ØNE-DIMENSIONAL SEARCH?
H? YES ØR NØ

NØ

DØ YOU WISH TØ CHANGE ALGORITHM? YES ØR NØ

YES

ENTER NAME ØF ALGORITHM TØ BE EXECUTED, CHØØSE AMØNG:

STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK

RANK

THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277

THE PØINT MINIMIZING THE FUNCTION IS +1.030025571 1.069377318

THE FUNCTION EVALUATED AT THE MIN. IS 3.849850871

THE PØINT MINIMIZING THE FUNCTION IS +0.9292133332 0.8276621778

THE FUNCTION EVALUATED AT THE MIN. IS 3.532289685

THE PØINT MINIMIZING THE FUNCTION IS +0.7851666627 0.5577100201

THE FUNCTION EVALUATED AT THE MIN. IS 3.079141875

THE PØINT MINIMIZING THE FUNCTION IS +0.7526418437 0.5750653501

THE FUNCTION EVALUATED AT THE MIN. IS 2.594523342

THE PØINT MINIMIZING THE FUNCTION IS +0.5687657336 0.2869570933

THE FUNCTION EVALUATED AT THE MIN. IS 2.395497261

THE PØINT MINIMIZING THE FUNCTION IS +0.4497160099 0.1480392545

THE FUNCTION EVALUATED AT THE MIN. IS 0.07551420072

THE PØINT MINIMIZING THE FUNCTION IS 1.274319794 1.625512206

THE FUNCTION EVALUATED AT THE MIN. IS 0.07543834475

THE PØINT MINIMIZING THE FUNCTION IS 1.274557857 1.62524825

THE FUNCTION EVALUATED AT THE MIN. IS 0.03933280092

THE PØINT MINIMIZING THE FUNCTION IS 1.176880394 1.376077441

THE FUNCTION EVALUATED AT THE MIN. IS 0.01635171155

THE PØINT MINIMIZING THE FUNCTION IS 1.087379081 1.173056988

THE FUNCTION EVALUATED AT THE MIN. IS 0.007012927705

THE PØINT MINIMIZING THE FUNCTION IS 1.08368169 1.17468698

THE FUNCTION EVALUATED AT THE MIN. IS 0.001874578518

THE PØINT MINIMIZING THE FUNCTION IS 1.032472849 1.063136459

THE FUNCTION EVALUATED AT THE MIN. IS 0.00005159673666

THE PØINT MINIMIZING THE FUNCTION IS 1.004959463 1.010463141

THE FUNCTION EVALUATED AT THE MIN. IS 6.733153324E+7

THE PØINT MINIMIZING THE FUNCTION IS 1.000661884 1.001275706

THE FUNCTION EVALUATED AT THE MIN. IS 1.14356833E+9

THE PØINT MINIMIZING THE FUNCTION IS 0.9999312977 0.9999597784

THE FUNCTION EVALUATED AT THE MIN. IS 1.32057555E+16

THE PØINT MINIMIZING THE FUNCTION IS 1.000000009 1.000000018

NØ. ØF ITERATIONS = 16

MIN. FUNCTION VALUE = 1.32057555E+16

ØPTIMUM PØINT = 1.000000009 1.000000018

1.000000009 1.000000018

DØ YOU WISH TØ RESTART? ANSWER YES ØR NØ

NØ

In Appendix 2 we provide examples of the UCNLP package in action. The function used for the testing is the well-known "banana-shaped" Rosenbrock [5] function. The procedure of entering the function, its gradient and hessian was somewhat different from the one described earlier. Here, instead of entering the function directly we first stored its alpha-numeric image in the variable names FR1, GR1, HR1 (function, gradient, hessian respectively) and then used the APL Evaluate operator with that variable name. This procedure is equivalent to entering the alpha-numeric image of the function directly. The user may elect to do it either way.

Ending Statement of

Appendix 3

Also note that several symbols on teletype 33 are used in place of the above Mneomonics.

<u>Graphic</u>	<u>Mneomonics</u>	<u>Meaning</u>
@	\$RO	Rho
#	\$QD	Quad
&	\$DL	Del
↑	\$NG	Neg