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A Heterogenous Agents Model Usable for the Analysis of Currency Transaction Taxes

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Abstract: We extend the model by DeGrauwe and Grimaldi (2006, *EER*) by currency transaction taxes. This model explains the exchange rate behavior by the interaction of heterogeneous traders who display either trend chasing behavior or rely on a return of the exchange rate back to its arbitrage free fundamental value. Within this model framework we can show analytically that the steady-state of the original model is unaffected by the transaction tax rate. We inferred from numerical simulations that the transaction tax is able to reduce the number of speculative equilibria to zero. Moreover, we show that the tax will lead to a faster convergence of the system back to its fundamental steady state.

Key words: *Currency Transaction Taxes, Exchange Rates, Financial Market Volatility, Heterogenous Agents Model, Numerical Simulation*

JEL Classification: *C15, F31, F32, G15, G18*

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Contents

1 Introduction

Risks and high volatility in foreign exchange markets lead also to additional risk for national economies because a lot of their transactions are involved in foreign exchange. Academics and the popular press often claim that these risks are caused by destabilizing speculators relying on trends in financial prices who move exchange rates away from their fundamental values. In this context the following proposition is often heard:

Transaction taxes stabilize the exchange rate because they harm noise traders more than traders who rely on economic fundamentals.

The analysis of this proposition can be conducted by analyzing if

(i) the number of non-fundamental equilibria is reduced through the currency transaction tax,

(ii) during the out of equilibrium dynamics the market is dominated by stabilizing traders, while noise traders are crowded out.

For this purpose we generalized the nonlinear exchange rate model by De-Grauwe and Grimaldi (2006) for currency transaction taxes. This Brock-Hommes-style model[1] assumes two types of traders: a stabilizing fundamental based trader type and a destabilizing trend chasing one. Traders are allowed to change their trading rule according to the past success of their trading strategy. Therefore the market can be dominated by one group of traders for some time periods. We analyze the out-of equilibrium dynamics of the model using impulse response analysis by shocking the system such that the exchange rate deviates from the fundamental steady state so that we can study its way of convergence back to this equilibrium. For the study

[1] See Brock and Hommes (1997) for a description of this model class.

of the possible reduction of speculative equilibria through currency transaction taxes we rely on numerical bifurcation analysis where we plot the equilibria emerging from the model against varying values of the tax rate.

Similar models of this area of research are those used in Westerhoff (2003) and Ehrenstein, Westerhoff and Stauffer (2003) who also study the effectiveness of currency transaction taxes. Westerhoff (2003) finds that small transaction taxes reduce exchange rate volatility while high transaction taxes increase the volatility. In his study this is due to the fact that low tax rates crowd out destabilizing traders while high tax rates crowd out stabilizing traders.

The following results emerge. After a shock hits the system in the baseline simulation without taxes the exchange rate and the population fractions of traders converged back to their fundamental steady state values. After the impulse the exchange rate overshoots but a then a trend reversal occurs with a convergence back to the fundamental equilibrium. After the shock hit the system the number of fundamental based traders rise sharply to 100%, while after the trend reversal trend chasing traders dominated the market. Because this dominance occurs after the trend reversal back to the fundamental value these traders do not lead to a destabilization. Variations of the currency transaction tax rate reveal that a positive tax rate helps the system to converge faster to the steady state. The bifurcation analysis reveals that the model displays a fundamental equilibrium and multiple speculative equilibria. We find out that a transaction tax larger than 2% leads to diminishing speculative equilibria, such that the fundamental steady-state is the only remaining equilibrium. A sensitivity analysis shows that this result is robust for a wide range of values of the behavioral parameters of the model. A stochastic simulation of the model shows that it is able to pro-

duce realistic exchange rate time series which makes is usable for economic policy analysis.

The remainder of the paper is structured as follows. The next section introduces the economic model. Section three contains the solution to this model, while section four contains the numerical analysis of the proposition above with the help of the model. Section five concludes.

2 The Model's Building Blocks

The model used for economic policy analysis in this paper is a generalized version of the model developed by DeGrauwe and Grimaldi (2006) by currency transaction taxes. If the transaction tax rate is zero then our model collapses to their model. This nonlinear exchange rate model consists of the following building blocks:

(i) agents' optimal portfolio decision within a mean-variance utility framework,

(ii) agents' forecasts of the future exchange rate based upon simple rules of thumb,

(iii) evaluation of these trading rules based upon a comparison of their risk-adjusted profitability,

(iv) a policy maker who sets the transaction tax rate.

In the next subsections we will describe these building blocks in more detail.

2.1 Demand and Supply of Foreign Assets

Our financial market is populated by agents with the identification number $1, ..., i-1, i, i+1, ..., N$. We assume agents to be heterogeneous in their expectations about their future wealth W^i_{t+1} and the future risk of their wealth. The individual agent's preferences towards risk can be represented by the following utility function displaying constant absolute risk aversion

$$U(W^i_{t+1}) = E^i_t(W^i_{t+1}) - \frac{1}{2}\mu Var^i_t(W^i_{t+1}), \tag{1}$$

where μ is the coefficient of absolute risk aversion, $E^i_t(W^i_{t+1})$ is agent i's conditional expectation of his future wealth, while $Var^i_t(W^i_{t+1})$ is his portfolio variance.

The evolution of the individual trader's wealth is specified as follows

$$W^i_{t+1} = (1+r^*)(1-\tau)s_{t+1}d_{i,t} + (1+r)(W^i_t - s_t d_{i,t}), \tag{2}$$

where r and r^* are the domestic and foreign interest rates which are assumed to be constant over time, while s_t is the exchange rate between the domestic and foreign country, $d_{i,t}$ is the trader's holdings of foreign assets held at time t, while τ is the transaction tax rate which will be levied if the agent wants to invest into foreign assets. Thus, $(1+r^*)(1-\tau)s_{t+1}d_{i,t}$ represents the value of the foreign portfolio denominated in domestic currency at time $t+1$, while $(1+r)(W^i_t - s_t d_{i,t})$ represents the value of the domestic portfolio at time $t+1$.

By maximizing equation (1) with respect to the budget constraint (2) we get the following demand function for foreign assets as the solution to this portfolio allocation problem:

$$d_{i,t} = \frac{(1+r^*)(1-\tau)E_t^i(s_{t+1}) - (1+r)s_t}{\mu\sigma_{i,t}^2}, \tag{3}$$

where

$$\sigma_{i,t}^2 = (1+r^*)^2(1-\tau)^2 Var_t^i(s_{t+1}). \tag{4}$$

Thus, the demand for foreign assets rises, if the foreign interest rate r^* rises or if the domestic interest rate r falls. It also rises if the future exchange rate is expected to rise. The demand will decrease if the risk of the future exchange rate is expected to rise or if the transaction tax rate τ rises.

The market demand for foreign assets D_t is the sum over all N individual demands

$$\sum_{i=1}^{N} n_{i,t} d_{i,t} = D_t, \tag{5}$$

where $n_{i,t}$ is the number of agents of type i.

Following DeGrauwe and Grimaldi (2006) the market supply for foreign assets X_t is assumed to be exogenous and determined by the net current account and by the sales and purchases of foreign currency by the domestic and foreign central banks. Market equilibrium is given if market demand equals market supply

$$X_t = D_t. \tag{6}$$

The market clearing exchange rate can be calculated by substituting the optimal holdings for foreign currency into the market equilibrium equation

$$X_t = \sum_{i=1}^{N} n_{i,t} \cdot \frac{(1+r^*)(1-\tau)E_t^i(s_{t+1}) - (1+r)s_t}{\mu\sigma_{i,t}^2} \tag{7}$$

and solving for the exchange rate s_t. This we will do by dividing both sides

by $\sum_{i=1}^{N} n_{i,t}$, while rearranging yields

$$\frac{\mu X_t}{\sum_{i=1}^{N} n_{i,t}} = \sum_{i=1}^{N} \frac{n_{i,t}}{\sum_{i=1}^{N} n_{i,t}} \cdot \left\{ (1+r^*)(1-\tau) \frac{E_t^i(s_{t+1})}{\sigma_{i,t}^2} - \frac{(1+r)s_t}{\sigma_{i,t}^2} \right\}. \quad (8)$$

By defining

$$w_{i,t} = \frac{n_{i,t}}{\sum_{j=1}^{N} n_{j,t}} \quad (9)$$

as the percentage fraction of agents using trading rule i at time t and rearranging we get

$$(1+r)s_t \sum_{i=1}^{N} \frac{w_{i,t}}{\sigma_{i,t}^2} = (1+r^*)(1-\tau) \sum_{i=1}^{N} \frac{w_{i,t}}{\sigma_{i,t}^2} E_t^i(s_{t+1}) - \frac{\mu X_t}{\sum_{i=1}^{N} n_{i,t}}. \quad (10)$$

By defining

$$\Omega_t = \frac{\mu}{(1+r^*) \sum_{i=1}^{N} n_{i,t}} \quad (11)$$

and rearranging again we get the market clearing exchange rate at time t

$$s_t = \left(\frac{1+r^*}{1+r} \right) \frac{1}{\sum_{i=1}^{N} \frac{w_{i,t}}{\sigma_{i,t}^2}} \left[\sum_{i=1}^{N} w_{i,t} \frac{(1-\tau) E_t^i(s_{t+1})}{\sigma_{i,t}^2} - \Omega_t X_t \right]. \quad (12)$$

From this equation we get the information that the equilibrium exchange rate depends on the exogenous supply of foreign assets X_t, the domestic and foreign interest rates and the weighted sum of the traders' forecasts of the future exchange rate which are characterized by behavioral heterogeneity. In the following subsection we will model these behavioral heterogeneity in more detail.

2.2 Forecasting Models and Trading Rules

Following the literature of heterogeneous agents model of financial markets[2] we assume that the market is populated by two types of traders[3]. The first group of traders is called fundamental traders or arbitrageurs in the literature. This trader types searches for assets which are over- or undervalued with respect to a fundamental value. Let us denote this fundamental exchange rate by s_t^* and the difference between the realized exchange rate and the fundamental exchange rate $s_t - s_t^*$ as the misalignment.

The arbitrageurs' forecasting rule expects the exchange rate to rise, if the realized exchange rate is smaller than the fundamental exchange rate and he expects the exchange rate to fall back to its fundamental value if the realized exchange rate lies above its fundamental value. Thus the arbitrageurs' forecasting rule can be specified as follows

$$E_t^f(\Delta s_{t+1}) = -\psi(s_{t-1} - s_{t-1}^*), \tag{13}$$

where $\Delta s_{t+1} = s_{t+1} - s_t$. Thus, this trader type assumes that $\psi \cdot 100\%$ of the misalignment will be corrected by the future exchange rate change.

Following DeGrauwe and Grimaldi (2006) we assume that fundamental traders behave differently depending on whether the exchange rate lies within or outside a transaction cost band of width C. This changes their

[2] See Brock and Hommes (1997), Chiarella and He (2002) and Westerhoff (2003) to get an overview over this model class.

[3] This assumption is based on the empirical finding of Taylor and Allen (1992) who conducted a survey at the London Foreign Exchange about the trading rules of traders. See also Menkhoff (1997) for a similar study.

forecasting model to

$$E_t^f(\Delta s_{t+1}) = \begin{cases} -\phi(s_{t-1} - s_{t-1}^*), & |s_{t-1} - s_{t-1}^*| > C; \\ 0, & |s_{t-1} - s_{t-1}^*| < C. \end{cases} \tag{14}$$

Thus, if the exchange rate lies outside the transaction cost band, arbitrageurs believe the misalignment to be corrected by the future exchange rate change, while they believe that the exchange rate will not change if it lies within the transaction cost band. The rationale behind this is that arbitrage will not function within the transaction cost band. So there will be no mechanism to correct the misalignment.

The chartist traders or technical traders how they are often denoted in the literature bet on lasting trends in the exchange rate. Thus, this trader type computes a moving average of past exchange rate changes and extrapolates them into the future, where the degree of extrapolation is given by the parameter β

$$E_t^c(\Delta s_{t+1}) = \beta \sum_{i=1}^{T} \alpha_i \Delta s_{t+i}. \tag{15}$$

If we plug these forecasting models into the demand functions for foreign currency developed in the former section, we can derive the following trading rules. The arbitrageurs' trading rule will be

$$d_{f,t} = \frac{(1+r^*)(1-\tau)(s_t - \phi(s_{t-1} - s_{t-1}^*)) - (1+r)s_t}{\mu\sigma_{i,t}^2}, \tag{16}$$

while the technical trading rule will be

$$d_{c,t} = \frac{(1+r^*)(1-\tau)(s_t + \beta\sum_{i=1}^{T} \alpha_i \Delta s_{t+i}) - (1+r)s_t}{\mu\sigma_{i,t}^2}. \tag{17}$$

Thus, fundamental based traders purchase or sell currency, when a misalign-

ment arises, while technical traders purchase or sell currency when trends in the exchange rate arise. The first trading strategy stabilizes the exchange rate, while the second one destabilizes by amplifying trends.

In the next subsection we will elaborate on the agents' portfolio risk evaluation which is also characterized by behavioral heterogeneity.

2.3 Risk Evaluation

Following DeGrauwe and Grimaldi (2006) agents evaluate their portfolio risk by a weighted average of squared past forecast errors

$$\sigma^2_{i,t+1} = \sum_{k=1}^{\infty} \theta_k \big(E^i_{t-k}(s_{t-k+1}) - s_{t-k+1}\big)^2. \tag{18}$$

The weights can be computed as $\theta_k = \theta(1-\theta)^k$. Following these authors arbitrageurs take the deviation of the market exchange rate from the fundamental value into account in addition to the forecast error. This changes the arbitrageurs' risk evaluation to

$$\sigma^2_{f,t+1} = \frac{\sum_{k=1}^{\infty} \theta_k \big(E^f_{t-k}(s_{t-k+1}) - s_{t-k+1}\big)^2}{1 + (s_t - s^*_t)^2} \tag{19}$$

So if the misalignment $s_t - s^*_t$ increases arbitrageurs become more confident that the exchange rate will convert back to the fundamental value, thus their risk perception declines. If the market displays higher volatility risk averse traders reduce their demand for foreign currency, while they increase it if the market is in a period of low volatility.

Now we finished the agents' portfolio decision problem and the involved components forecasting rules and risk evaluation. What remains is to intro-

duce an evolutionary mechanism that tells us which trading rule the agent prefers most. This problem will be tackled in the next subsection.

2.4 Evaluation of Trading Rules

The evaluation of the two trading rules follows the idea of Brock and Hommes (1997) who claim to use discrete choice probabilities to compute the fraction of agents using a particular trading rule. Following DeGrauwe and Grimaldi (2006) the chartist weight is calculated as

$$w_{c,t} = \frac{\exp\left\{\gamma\pi'_{c,t-1}\right\}}{\exp\left\{\gamma\pi'_{c,t-1}\right\} + \exp ig\{\gamma\pi'_{f,t-1}\right\}}, \tag{20}$$

where $\pi'_{c,t}$ is the risk-adjusted realized profit of the technical trading rule, where the parameter γ can be interpreted as the intensity of choice, which measures how strong agents react to changes in their profits. The risk-adjusted profits are calculated as follows

$$\pi'_{i,t-1} = \pi_{i,t-1} - \mu\sigma^2_{i,t-1}, \tag{21}$$

where $\pi_{i,t-1}$ is agent i's realized past profit, μ is his degree of risk-aversion, while $\sigma^2_{i,t-1}$ measures his portfolio risk.

Analogue, the fundamental trader' forecasting rule can be calculated in a similar way

$$w_{f,t} = \frac{\exp\left\{\gamma\pi'_{f,t}\right\}}{\exp\left\{\gamma\pi'_{c,t}\right\} + \exp\left\{\gamma\pi'_{f,t}\right\}}. \tag{22}$$

In a similar way the realized profits of fundamental based trading are cal-

culated as

$$\pi_{i,t} = \big(s_t(1+r^*)(1-\tau)-s_{t-1}(1+r)\big)\text{sign}\big\{E^i_{t-2}(s_{t-1}(1+r^*)(1-\tau)-s_{t-2}(1+r))\big\}, \tag{23}$$

where

$$\text{sign}(x) = \begin{cases} 1, & \text{for } x > 0; \\ 0, & \text{for } x = 0; \\ -1, & \text{for } x < 0. \end{cases} \tag{24}$$

From this equation we can infer that the transaction tax rate not only has an effect on the traders' demand for foreign currency by also has a direct effect on the evaluation and choice of trading rules. Thus, a high transaction tax may agents' prevent to use a trading rule which might be very profitable with no tax levied.

These are the building blocks the model consists of. The next section will present the solution to this model and the calculation of the model's equilibria.

3 Solution of the Model

For solving the model we can restrict it without loss of generality to a simpler one. In this section we calculate the model's equilibria and check if the transaction tax changes them. The most interesting point to analyze is if the currency transaction tax changes the fundamental steady-state. After solving for the fundamental steady-state we are prepared for a detailed analysis of the policy effects of the transaction tax which will be done in the chapter thereafter.

3.1 A Simplified Version of the Model

Following DeGrauwe and Grimaldi (2006) we use a simplified version of the model presented in order to get some analytical results. We simplify the model by assuming zero transaction cost in the goods market $C = 0$. Interest rates in the domestic and foreign country can be normalized to zero $r = r^* = 0$, while also the fundamental interest rate is assumed to be constant in time and normalized to zero $s_t^* = 0$ without loss of generality. Moreover, we restrict our analysis on only two types of traders: chartists and fundamental based traders.

The assumptions above simplify the market clearing exchange rate to

$$\begin{aligned} s_t &= \frac{w_{c,t}/\sigma_{c,t}^2}{(w_{c,t}/\sigma_{c,t}^2)+(w_{f,t}/\sigma_{f,t}^2)}(1-\tau)E_t^c s_{t+1} \\ &+ \frac{w_{f,t}/\sigma_{f,t}^2}{(w_{c,t}/\sigma_{c,t}^2)+(w_{f,t}/\sigma_{f,t}^2)}(1-\tau)E_t^f s_{t+1} \end{aligned} \tag{25}$$

By defining

$$\Theta_{f,t} := \frac{w_{f,t}/\sigma_{f,t}^2}{(w_{f,t}/\sigma_{f,t}^2)+(w_{c,t}/\sigma_{c,t)}^2} \tag{26}$$

and

$$\Theta_{c,t} := \frac{w_{c,t}/\sigma_{c,t}^2}{(w_{f,t}/\sigma_{f,t}^2)+(w_{c,t}/\sigma_{c,t}^2)} \tag{27}$$

as the new population fractions of agents and assuming the following simplified forecasting rules for chartists

$$E_t^c s_{t+1} = s_t + \beta(s_{t-1} - s_{t-2}) \tag{28}$$

and arbitrageurs

$$E_t^f s_{t+1} = (1-\psi)s_{t-1} \tag{29}$$

we can simplify the closed form solution for the market clearing exchange rate to

$$s_t = (1-\tau)\Big(s_{t-1} + \Theta_{c,t}\beta(s_{t-1} - s_{t-2}) - \Theta_{f,t}\psi s_{t-1}\Big). \tag{30}$$

This equation can be interpreted as follows. In econometric language the exchange rate equation consists of two parts: an autoregressive part $\Theta_{c,t}\beta(s_{t-1} - s_{t-2})$ and a mean-reverting part $-\Theta_{f,t}\psi s_{t-1}$. If the weight of the autoregressive part $\Theta_{c,t}$ is large then the exchange rate will diverge from its fundamental value which we normalized to zero. This is the case if the number of trend-chasing traders is large compared to the fundamental traders. If the weight of the mean-reverting part $\Theta_{f,t}$ is large then the exchange rate will be close to its fundamental value. This is the case if if the number of fundamental traders is large compared to the number of chartist traders. This fundamental traders have a stabilizing effect on the exchange rate while chartist traders have a destabilizing effect. If a tax levied on foreign exchange transactions should stabilize the exchange rate these effects have to be taken into account. If the tax harms fundamental traders more than chartist traders the effect of the transaction tax might be destabilizing, while it might be stabilizing if it harms technical trading rules and favors fundamental-based trading rules.

Under these simplifying assumptions chartists' risk perception collapses to

$$\begin{aligned} \sigma^2_{c,t} &= (1-\theta)\sigma^2_{c,t-1} + \theta\big(E^c_{t-2}(s_{t-1}) - s_{t-1}\big)^2 && (31)\\ &= (1-\theta)\sigma^2_{c,t-1} + \theta\big((1+\beta)x_{t-1} - \beta z_{t-1} - s_{t-1}\big)^2, && (32) \end{aligned}$$

while the arbitrageurs' risk perception simplifies to

$$\sigma^2_{f,t} = (1-\theta)\sigma^2_{f,t-1} + \frac{\theta\big(E^f_{t-2}(s_{t-1}) - s_{t-1}\big)^2}{1+(s_{t-1})^2} \tag{33}$$

$$= (1-\theta)\sigma^2_{f,t-1} + \frac{\theta\big((1-\psi)x_{t-1} - s_{t-1}\big)^2}{1+(s_{t-1})^2}, \tag{34}$$

where u_t, x_t and z_t are defined as

$$u_t := s_{t-1} \tag{35}$$

$$x_t := u_{t-1} \tag{36}$$

$$z_t := x_{t-1}. \tag{37}$$

The fundamental traders' realized profits simplify to

$$\pi_{f,t-1} = \big(s_{t-1}(1-\tau) - s_{t-2}\big)\text{sign}\big\{E^f_{t-2}(s_{t-1}(1-\tau)) - s_{t-2}\big\} \tag{38}$$

$$= \big(s_{t-1}(1-\tau) - u_{t-1}\big)\text{sign}\big\{(1-\phi)x_{t-1}(1-\tau) - u_{t-1}\big\},$$

while the technical traders' profits simplify to

$$\pi_{c,t-1} = \big(s_{t-1}(1-\tau) - s_{t-2}\big)\text{sign}\big\{E^c_{t-2}(s_{t-1}(1-\tau)) - s_{t-2}\big\} \tag{39}$$

$$= \big(s_{t-1}(1-\tau) - u_{t-1}\big)\text{sign}\big\{(x_{t-1} + \beta(x_{t-1} - z_{t-1})(1-\tau) - u_{t-1}\big\}.$$

Thus, the dynamical system can be expressed as

$$
\begin{aligned}
s_t &= (1-\tau)[1+\beta-\Theta_{f,t}(\psi+\beta)]s_{t-1}-(1-\Theta_{f,t}\beta)u_{t-1} \qquad (40)\\
u_t &= s_{t-1}\\
x_t &= u_{t-1}\\
z_t &= x_{t-1}\\
\sigma^2_{c,t} &= (1-\theta)\sigma^2_{c,t-1}+\theta\big((1+\beta)x_{t-1}-\beta z_{t-1}-s_{t-1}\big)^2\\
\sigma^2_{f,t} &= (1-\theta)\sigma^2_{f,t-1}+\theta\frac{[(1-\psi)x_{t-1}-s_{t-1}]^2}{1+(s_{t-1})^2}\\
\Theta_{f,t} &= \frac{w_{f,t}/\sigma^2_{f,t}}{(w_{f,t}/\sigma^2_{f,t})+(w_{c,t}/\sigma^2_{c,t})}\\
w_{f,t} &= \frac{\exp\{\gamma(\pi_{f,t-1}-\mu\sigma^2_{f,t-1})\}}{\exp\{\gamma(\pi_{f,t-1}-\mu\sigma^2_{f,t-1})\}+\exp\{\gamma(\pi_{c,t-1}-\mu\sigma^2_{c,t-1})\}}\\
\pi_{f,t-1} &= (s_{t-1}(1-\tau)-u_{t-1})\mathrm{sign}\big\{(1-\psi)x_{t-1}(1-\tau)-u_{t-1}\big\}\\
\pi_{c,t-1} &= (s_{t-1}(1-\tau)-u_{t-1})\mathrm{sign}\big\{(x_{t-1}+\beta(x_{t-1}z_{t-1}))(1-\tau)-u_{t-1}\big\}
\end{aligned}
$$

In the following subsection we will use these equations to calculate the model's fundamental steady-state. Moreover, these equations can be used later on for the numerical analysis of the model.

3.2 The Fundamental Steady-State

For the system to be in the steady-state all variables should remain constant in time. Thus, the following condition has to be fulfilled

$$
\begin{aligned}
\big(s_{t-1},u_{t-1},x_{t-1},z_{t-1},\sigma^2_{f,t-1},\sigma^2_{c,t-1}\big) &= \big(s_t,u_t,x_t,z_t,\sigma^2_{f,t},\sigma^2_{c,t}\big) \qquad (41)\\
&= \big(\overline{s},\overline{u},\overline{x},\overline{z},\overline{\sigma^2_f},\overline{\sigma^2_c}\big).
\end{aligned}
$$

This means, that in the steady-state all variables are constant and equal to their long-run equilibrium values $\overline{s}$, $\overline{u}$, $\overline{x}$, $\overline{z}$, $\overline{\sigma_f^2}$, and $\overline{\sigma_c^2}$.

In line with DeGrauwe and Grimaldi's (2006) model there is a unique fundamental steady-state where

$$\left(\overline{s}, \overline{u}, \overline{x}, \overline{z}, \overline{\sigma_f^2}, \overline{\sigma_c^2}\right) = \left(0, 0, 0, 0, 0, 0\right). \tag{42}$$

One property of this steady-state is that

$$\overline{w_c} = \frac{1}{2}, \overline{w_f} = \frac{1}{2}, \overline{\pi_c} = 0, \overline{\pi_f} = 0. \tag{43}$$

Thus, in the steady-state 50% of all traders are fundamental value traders, 50% are chartists, and all profits are zero because there are no arbitrage opportunities left. Moreover, it can be seen that changes in the transaction tax rate τ have no influence on the fundamental steady-state of the system. The economic interpretation of this is that transaction taxes do not change the long-run average returns on holding foreign currency. They only change the transitory out-of-equilibrium behavior of the model.

Because in DeGrauwe and Grimaldi's (2006) model, which is a special case of our model, one is not able to perform a local stability analysis based on the eigenvalues of the Jacobian matrix evaluated at the steady-state, we are also not able do this for our model. The reason is that the nonlinear map is not differentiable at the steady-state. Therefore, we have to rely on numerical methods to analyze the properties of the model and to conduct the economic policy analysis. This we will do in the following section.

4 Numerical Analysis

In this part we check the behavior of the model by numerical simulations. At first we analyze how the deterministic skeleton of the model behaves after the model is shocked by a deviation of the exchange rate from the fundamental steady-state. The behavior of the fundamental steady-state and the speculative steady states will be analyzed later on using bifurcation analysis. After that we study the behavior of a stochastic simulation of the model.

4.1 Impulse Response Analysis

The deterministic skeleton of the model is given by the equations given above, while the fundamental steady state is given by

$$\left(\overline{s}, \overline{u}, \overline{x}, \overline{z}, \overline{\sigma_f^2}, \overline{\sigma_c^2}\right) = \left(0, 0, 0, 0, 0, 0\right) \tag{44}$$

and displays the following property

$$\overline{w_c} = \frac{1}{2}, \overline{w_f} = \frac{1}{2}, \overline{\pi_c} = 0, \overline{\pi_f} = 0. \tag{45}$$

We shock the system by introducing a deviation of the exchange rate from this steady-state and study how the systems trajectories return back to this equilibrium. With the help of this impulse response analysis we can get information about the stability of the system and about the time length of disequilibria. Our research strategy is to perform a baseline simulation without transaction taxes and to compare its result to simulations with different transaction tax rates. The following figures represent impulse response se-

ries from the models variables after the model is shocked by a deviation of the exchange rate from the fundamental steady state. Thus, all impulse responses a measured in deviations from the steady-state which is given by the red brocken lines.

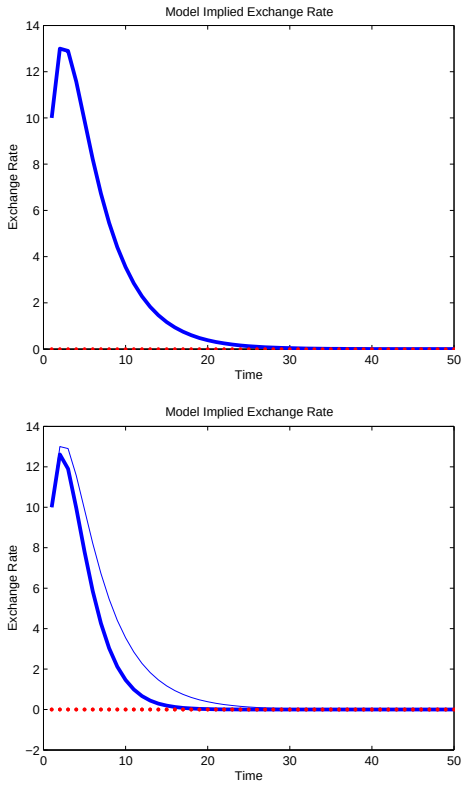

Fig. 1: Impulse Responses of the Exchange Rate

The impulse responses in the upper subfigure represent the baseline case with a transaction tax rate $\tau = 0\%$*, while the impulse responses lower subfigure are based on transaction tax rates* $\tau = 3\%$*. The remaining parameter values are* $\psi = 0.5$*,* $\beta = 0.5$*,* $\theta = 0.5$*,* $\gamma = 1$*, and* $\mu = 1$*.*

This first subfigure of **Fig. 1** shows us the hump-shaped response of the exchange rate to a disturbance to the system. As we can see, after the shock the exchange rate overshoots for 2.5 degrees of measure while trend reversal occurs after that. The exchange rate then returns back to its steady-state after 25 periods of time. The subfigure below shows the same response as a thin line and the response of the exchange rate after the same shock

but under a transaction tax of 3% represented by the thick line. As we can inspect, the overshooting is much smaller and convergence to the equilibrium is faster. In this simulation the fundamental steady-state is reached after 15 periods.

The following figure shows the response of the population of stabilizing fundamental traders after the same shock.

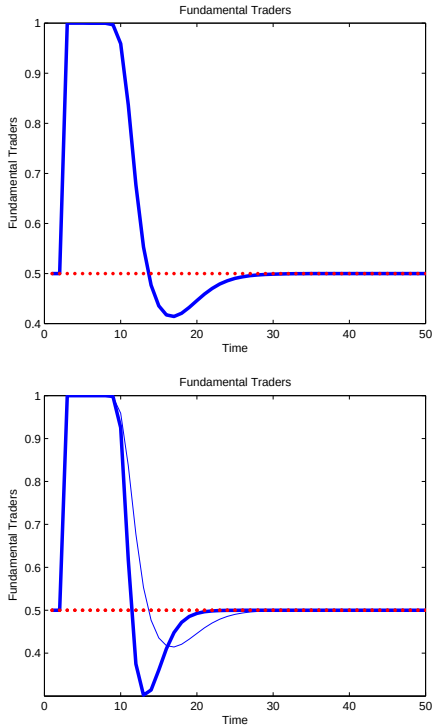

Fig. 2: Impulse Responses of the Population of Fundamental Traders

The impulse responses in the upper subfigure represent the baseline case with a transaction tax rate $\tau = 0\%$, while the impulse responses lower subfigure are based on transaction tax rates $\tau = 3\%$. The remaining parameter values are $\psi = 0.5$, $\beta = 0.5$, $\theta = 0.5$, $\gamma = 1$, and $\mu = 1$.

Again, the subfigure above represents the baseline case with no transaction tax levied on sells and purchases of foreign currency, while the subfigures below represent the responses of the population of stabilizing fundamental traders under transaction tax rates of 3% and 5%. The dotted red line at 1/2 represents the population fraction when the system is in the fundamental

equilibrium. When the system is shocked the population of fundamentalist traders is rising to 100%, because the initialized deviation from steady-state rises their demand for foreign currency. Thus, they are trading against this deviation. This stabilizes the exchange rate as we saw in the figure before. The exchange rate is converging back to its fundamental value. After ten periods of dominance in the market their number is decreasing sharply to a value below its steady-state value of 1/2. This means that now a period of dominance of chartist traders begins. Because fundamental traders are trading against the misalignment this creates a trend of the exchange rate towards its fundamental value. This rises the chartists' demand for foreign currency. After 26 periods the population fractions are back at their steady-state value where they remain. Under a transaction tax rate of 3% we reach a faster convergence of the exchange rate towards its fundamental steady-state as we saw in the figure before. This faster convergence corresponds with the following finding. Under a 3% transaction tax rate the decline of the fundamental traders' population is much more abrupt and also larger. This you can see by comparing the thick and the thin line where the thin line represents the baseline case. Thus, under a positive transaction tax rate the number of chartist traders rises much more after the fundamental traders purchases of domestic currency leads to a trend reversal back to the steady-state. Because the trend is much stronger than in the baseline case, more trend chasers rise their demand for foreign currency. This reaction leads to the fact, that the convergence of the exchange rate and the convergence of the population towards their steady-state values is much faster under a positive tax rate. Thus, the currency transaction tax uses the chartists' trend chasing behavior in a positive way to stabilize the exchange rate.

Thus, with the help of the impulse response analysis we could show that

the system near the steady state is stable but that transaction taxes help to eliminate misalignments much faster. Our findings verify the proposition that transaction taxes stabilize the exchange rate by crowding out trend chasing traders in favor of stabilizing fundamental based traders, because the dominance of fundamental based trades stabilizes the exchange rate. But we also find that after fundamentalist trades lead to a reversal of the trend towards the fundamental value chartist traders do not harm the system anymore but help to reach a faster convergence. Remind that these results are only valid if we start from the fundamental steady state and when the shocks are small. If a shock is large enough the exchange rate can also converge to one of the numerous speculative equilibria.

After this local analysis of the behavior near the steady state the following part analyzes the qualitative properties of the model by analyzing how the number of speculative equilibria changes under a variation of the transaction tax rate.

4.2 Sensitivity Analysis via Bifurcation Diagrams

In this section we analyze how the model changes its qualitative results due to parameter variations with the help of bifurcation diagrams. In this diagrams we plot the fundamental and speculative steady-states against different parameter values, where we will put most weight on our policy variable which is the transaction tax rate. Remind, that this section does not analyze the dynamics of the system but only the occurrence of equilibria under different parameter values.

The simulation tool we use for our analysis of equilibria is the bifurcation diagram in which equilibria are plotted against values of the parameter of

interest. In our case this will be the behavioral parameters of fundamental and trend chasing traders and our policy variable the currency transaction tax rate. We construct these diagrams by Monte-Carlo simulation techniques. The simulation consists of the following steps

(i) we draw a starting value for the exchange rate s_0 from a probability distribution,

(ii) then we simulate the deterministic part of the model until the steady-state is reached,

(iii) we save this equilibrium value,

(iv) steps (i) to (iii) are repeated several times in order to calculate all equilibria of the model,

(v) we save all equilibrium values for the parameter value we have used,

(vi) we repeat steps (i) to (v) for different values of the parameter we want to analyze,

(vii) we plot equilibria against parameter values.

Fig. 3 shows a bifurcation diagram where the model's equilibria are plotted against our policy variable, the currency transaction tax rate. Again, the fundamental equilibrium is normalized to zero.

As you can see the baseline case without transaction taxes is given at the left of the x-axis of this diagram, where the transaction tax rate is zero. For this parameter value a lot of equilibria emerge. Because this is a stochastic simulation only those equilibria are plotted which occur most frequently. This means the fundamental steady state is not reached, because the speculative equilibria occur more frequently. These speculative equilibria are distributed in a range of ± 0.5 around the fundamental steady state. When

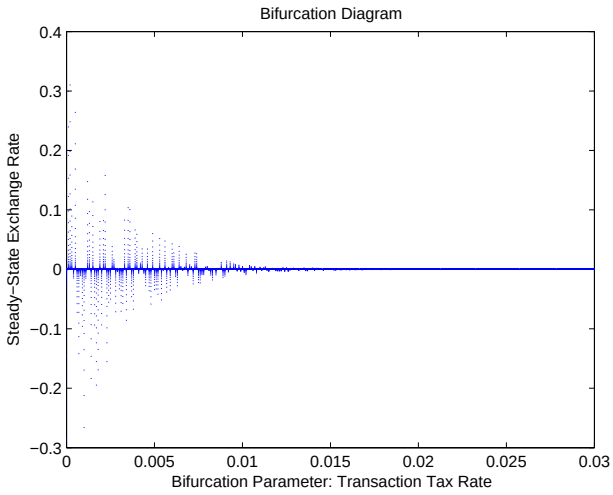

Fig. 3: Bifurcation Analysis: Change of the Transaction Tax Rate

This figure plots the models equilibria for different values of the transaction tax rate. The remaining parameter values are $\psi = 0.5$, $\beta = 0.5$, $\theta = 0.5$, $\gamma = 1$, *and* $\mu = 1$.

we rise the transaction tax rate this range gets symmetrically smaller and the number of speculative equilibria is declining. For a transaction tax rate of 2% the fundamental steady-state is the only equilibrium point of the model. Thus, in this case, our system will converge back to the fundamental equilibrium after a shock which leads to a temporary disequilibrium. Remind that for tax rates smaller than 2% the system can also rest in an speculative equilibrium after a shock. The result of this analysis is that positive transaction taxes larger than 2% lead to a stabilization of the system by reducing the number of speculative equilibria to zero.

Figure ?? shows a bifurcation diagram at the point, where the transaction tax is 2% but where we vary the fundamentalist forecasting parameter ϕ between zero and one in order to robust how robust our results above are.

From this figure we can infer that our results are robust for a wide range of fundamentalist error correction parameters. For all parameter values in the interval $]0, 1]$ we have only the fundamental equilibrium despite the case

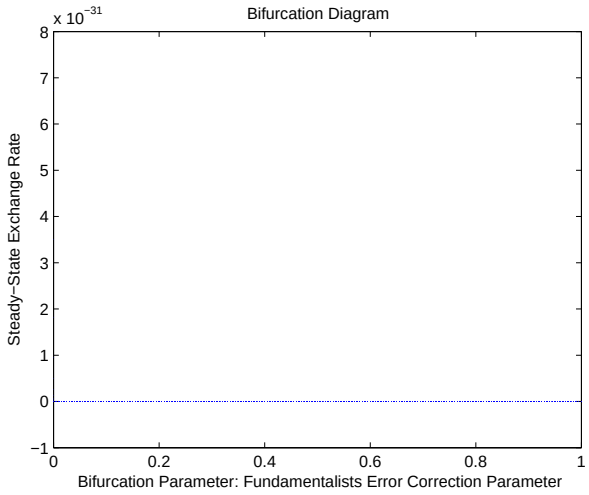

Fig. 4: Bifurcation Analysis: Change of the Fundamentalist Forecasting Parameter

This figure plots the models equilibria for different values of the fundamentalist forecasting parameter ϕ. The remaining parameter values are $\tau = 0.02$, $\beta = 0.5$, $\theta = 0.5$, $\gamma = 1$, and $\mu = 1$.

$\phi = 0$, where fundamentalist expect the exchange rate to follow a random walk. In this special case we again get multiple equilibria.

In Figure ?? we use the bifurcation diagram at $\tau = 2\%$ but we vary the technical traders trend extrapolation parameter β here in order to check the robustness of our results above.

From this figure we can infer that our findings are also robust for a wide range of values of the chartists' extrapolation parameter β. For values of this parameter between 0.1 and 1 the fundamental equilibrium is the only possible equilibrium point while for values between 0 and 0.1 we have multiple equilibria although the transaction tax is 2%.

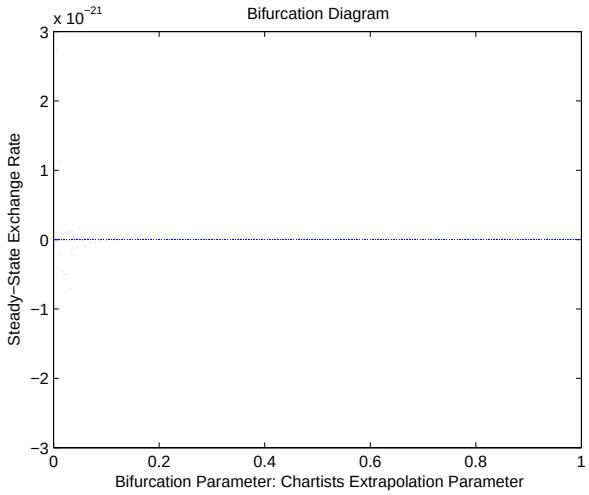

Fig. 5: Bifurcation Analysis: Change of the Technical Traders Trend Extrapolation Parameter

This figure plots the models equilibria for different values of the chartist forecasting parameter β. The remaining parameter values are $\tau = 0.02$, $\phi = 0.5$, $\theta = 0.5$, $\gamma = 1$, and $\mu = 1$.

4.3 Stochastic Simulation

Following DeGrauwe and Grimaldi (2006) in line with authors like Brock and Hommes (1997), Chiarella and He (2002) and Westerhoff (2003) we assume the fundamental exchange rate to follow a random walk

$$s_t^* = s_{t-1}^* + \varepsilon_t, \tag{46}$$

where ε_t is a normally distributed mean zero random variable. In contrast to the deterministic case, were we set the fundamental exchange rate to zero, here we have to put it into the system of equations.

In this case the market clearing exchange rate equation will be extended to

$$\begin{aligned} s_t &= (1-\tau)\Big[\Theta_{c,t}(s_{t-1}+\beta(s_{t-1}-s_{t-2}))+\Theta_{f,t}(s_{t-1}-\psi(s_{t-1}-s^*_{t-1}))\Big] \\ &= (1-\tau)\Big[s_{t-1}+\Theta_{c,t}(\beta(s_{t-1}-s_{t-2}))+\Theta_{f,t}(-\psi(s_{t-1}-s^*_{t-1}))\Big] \\ &= (1-\tau)\Big[(1+\beta-\Theta_{f,t}(\psi+\beta))s_{t-1}-(1-\Theta_{f,t})\beta u_{t-1}+\Theta_{f,t}\psi s^*_{t-1})\Big]. \end{aligned} \quad (47)$$

Moreover we have to change the risk evaluation of the fundamental traders to

$$\sigma^2_{f,t} = (1-\theta)\sigma^2_{f,t-1} + \theta\frac{[(1-\psi)x_{t-1}+\psi s^*_{t-3}-s_{t-1}]^2}{1+(s_{t-1}-s^*_{t-1})^2}. \quad (48)$$

In the case of stochastic fundamental values, the realized profits of fundamentalists change to

$$\pi_{f,t-1} = (s_{t-1}(1-\tau)-u_{t-1})\text{sign}[((1-\psi)x_{t-1}+\psi s^*_{t-3})(1-\tau)-u_{t-1}]. \quad (49)$$

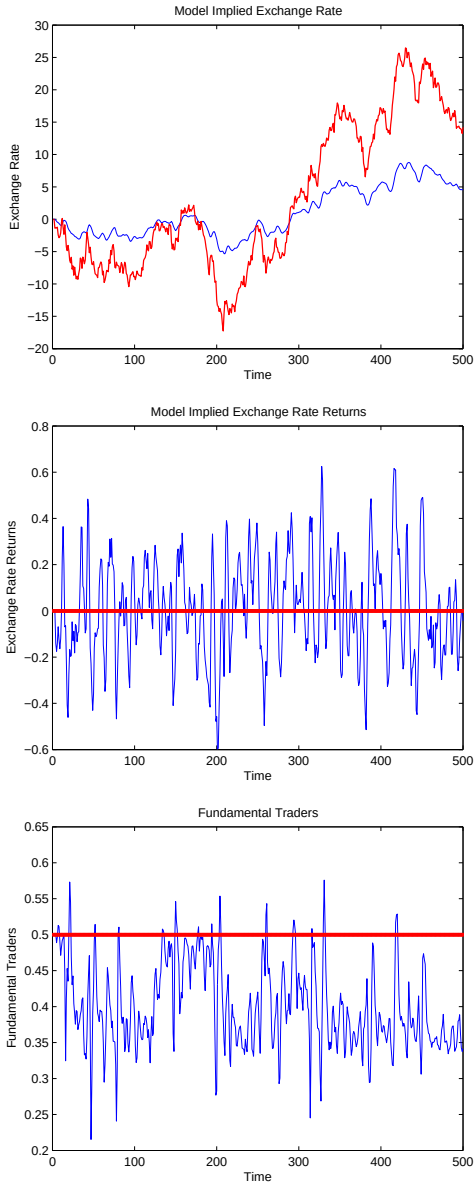

Fig. 6: Stochastic Simulation of the Model

The model is simulated with a transaction tax rate $\tau = 0\%$. *The remaining parameter values are* $\psi = 0.2$, $\beta = 0.8$, $\theta = 0.6$, $\gamma = 1$, *and* $\mu = 1$. *The standard deviation of the fundamental shock is* $\sigma = 1$. *Red lines represent fundamental equilibrium values.*

From the stochastic simulation in Figure ?? we can see that the model is able to reproduce realistic time series behavior of the exchange rate because it follows a random walk like behavior which looks similar than empirical time series. As you can see the exchange rate displays periods where it

deviates from its fundamental variable and periods where it is attracted from its fundamental value. As we have seen before, this corresponds to periods where the market is either dominated by trend chasing traders or dominated by fundamental based traders as can be inferred from the subfigure at the bottom and as explained the section before. The exchange rate returns do not really display volatility clustering like in empirical return time series. This is not problematic because we do not want to reproduce all stylized facts[4] but our aim was to conduct economic policy analysis. All in all, the model is able to reproduce some features of empirical exchange rate time series which makes it usable for economic policy analysis.

A conclusion for this section is that the currency transaction tax rate is able to stabilize the exchange rate by leading to a faster elimination of temporary disequilibria after small shocks and near the steady-state. Remind that the exchange rate can also switch to a speculative equilibrium if the shock is large enough. Moreover, we found out that the tax is able to reduce the number of speculative equilibria to zero. Thus, the currency transaction tax might be an effective policy tool for stabilizing exchange rates.

5 Conclusion

In this study we wanted to analyze the following proposition:

Transaction taxes stabilize the exchange rate because they harm noise traders more than traders who rely on economic fundamentals

which is often part of academic research and also often to be read in the popular press. From this proposition we get the following points to analyze

[4]For a model that reproduces all stylized facts of financial time series see Lux and Marchesi (****).

(i) the number of non-fundamental equilibria is reduced through the currency transaction tax,

(ii) during the out of equilibrium dynamics the market is dominated by stabilizing traders, while noise traders are crowded out.

For this purpose we generalized the nonlinear exchange rate model by De-Grauwe and Grimaldi (2005) for currency transaction taxes. This Brock-Hommes-style model assumes two types of traders: a stabilizing fundamental based trader type and a destabilizing trend chasing one. Traders are allowed to change their trading rule according to the past success of their trading strategy. Therefore the market can be dominated by one group of traders for some time periods. We analyze the out-of equilibrium dynamics of the model using impulse response analysis by shocking the system such that the exchange rate deviates from the fundamental steady state and studied its convergence back to this equilibrium. For the study of the possible reduction of speculative equilibria through transaction taxes we relied on numerical bifurcation analysis where we plot the equilibria emerging from the model against varying parameter values. The parameter we were most interested in was our policy variable.

The following results emerged. After a shock to the system the exchange rate and the population fractions of traders converged back to their fundamental steady state values. After the impulse the exchange rate overshot but the a trend reversal occurred with a convergence back to the fundamental equilibrium. After the shock the number of fundamental based traders rise sharply to 100%, after the trend reversal trend chasing traders dominated the market. Because this domination was after the trend reversal back to the fundamental value these traders did not lead to a destabilization. Summing

up this point we found that a positive transaction tax helped the system to converge faster to the steady state. The bifurcation analysis revealed that the model displays a fundamental equilibrium and multiple speculative equilibria. A transaction tax larger than 2% leads to diminishing speculative equilibria, so that the fundamental steady-state is the only remaining equilibrium. A sensitivity analysis showed that this result is robust for a wide range of values of the model's behavioral parameters. A stochastic simulation of the model shows that it is able to produce realistic exchange rate time series which makes is usable for economic policy analysis.

Summing up, the model shows that stabilization policy is not necessary after small shocks because the system is stable near the fundamental equilibrium. But positive transaction taxes help to eliminate misalignments faster. Moreover, the model shows that positive transaction taxes are necessary if shocks are large because then the economy might stuck in one of the multiple speculative equilibria. Our second result was that these equilibria can be eliminated if the transaction tax is high enough.

Future research should find out how large these shocks are and how often speculative equilibria are reached. This should give us more hints about the necessity of currency transaction taxes.

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