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The Effects of Trade Liberalization on Productivity and Welfare: The Role of Firm Heterogeneity, R&D and Market Structure

by Ngo Van Long, Horst Raff and Frank Stähler
The Effects of Trade Liberalization on Productivity and Welfare: The Role of Firm Heterogeneity, R&D and Market Structure

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Abstract

This paper develops an oligopolistic model of international trade with heterogeneous firms and endogenous R&D to examine how trade liberalization affects firm and industry productivity, as well as social welfare. We identify four effects of trade liberalization on productivity: (i) a direct effect through changes in R&D investment; (ii) a scale effect due to changes in firm size; (iii) a selection effect due to inefficient firms leaving the market; and (iv) a market-share reallocation effect as efficient firms expand and inefficient firms reduce their output. We show how these effects operate in the short run when market structure is fixed, and in the long run when market structure is endogenous. Among the robust results that hold for any market structure are that trade liberalization (i) increases (decreases) aggregate R&D for low (high) trade costs; (ii) increases expected firm size if trade costs are high; and (iii) raises expected social welfare if trade costs are low.

JEL classification: F12, F15
Keywords: international trade, firm heterogeneity, R&D, productivity, market structure


1 Introduction

How trade liberalization affects the productivity of firms and industries has been one of the key questions in both the academic and political debate about trade policy. The idea, popular in the 1960s and 70s especially among governments of less-developed countries, that firms would raise their productivity if only one protected them from foreign competition, has since been supplanted by the diametrically opposite view: namely that firms would increase their productivity, if they were exposed to foreign competition. Among the popular arguments offered to support this view are that import competition would force firms to become more efficient if they wanted to survive in tougher markets, or that greater access to foreign markets would expose firms to foreign technology and management techniques. This alleged productivity enhancing effect of trade is often portrayed as one of the main reasons why trade liberalization may raise social welfare.

The purpose of the current paper is to examine more rigorously some of the possible channels through which trade liberalization might affect firm and industry productivity, as well as social welfare. Specifically we want to study how trade policy affects the incentives of firms to invest in cost-reducing R&D when the outcome of this investment is stochastic. For this purpose we develop an oligopoly model of international trade, in which we can study the effects of trade liberalization on R&D, as well as domestic and foreign sales both for an exogenous and for an endogenous market structure. The case of an exogenous market structure can be interpreted either as a short-run scenario or as a model of an industry facing large sunk entry or exit costs. In both interpretations, firms adjust to trade liberalization by adjusting domestic and foreign sales (possibly to zero) and R&D investment, but there is no entry of new firms. The case of an endogenous market structure may serve as a long-run scenario, in which profits are driven to zero by free entry and exit of firms. Alternatively, we may interpret this case as representing an industry in which sunk entry or exit costs are small. Firms may still respond to trade liberalization by adjusting output and R&D expenditure. However,
part of the adjustment will be in the form of entry and exit. We are especially interested in identifying trade liberalization effects that hold across different market structures and can therefore be expected to occur in a wide range of industries.

Is there any evidence that trade liberalization leads to higher productivity at the firm and industry level? The vast empirical literature on the relationship between trade and productivity offers conflicting answers. Some of the studies seeking a direct link between trade liberalization and firm productivity find a positive effect of lower trade barriers on productivity, while others show no or even a negative effect.\(^1\) The only apparently robust result found by recent empirical studies using firm- and plant-level data, namely that there is a positive correlation between the productivity of firms and their export-market participation, also offers little help. In particular, while there is ample evidence to suggest that only the most productive firms in an industry become exporters, these studies offer only limited evidence that exporting makes firms more productive (see Bernard and Jensen, 1999, Tybout, 2003, and Wagner, 2007, for surveys of this literature).

But why should exporting make firms more productive? What are the precise channels through which trade policy affects firm and industry productivity? We know from homogeneous-firm models of monopolistic competition (Krugman, 1979) and reciprocal dumping (Brander and Krugman, 1983) that trade liberalization may raise productivity due to a firm-scale effect: firms become larger, allowing them to spread their fixed costs over a larger output. Models accounting for firm heterogeneity, in which firms draw their marginal costs from a probability distribution,\(^2\) add two additional positive effects on industry productivity: a selection effect, whereby

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\(^2\)See Bernard et al. (2003), Melitz (2003), Melitz and Ottaviano (2004), Baldwin (2005); Greenaway and Kneller (2005) provide a recent survey of the literature.
trade liberalization forces firms that have drawn a high marginal cost to exit the market; and a market-share reallocation effect, whereby low-cost firms gain market share at the expense of high-cost firms.

This cannot be the whole story, however, because firm productivity is itself endogenous: firms directly influence their productivity. In particular, firms may take action to lower their costs with a view to becoming exporters. Self-selection into export markets would then be a result of a "conscious process" rather than just the result of a lucky draw. That firms do indeed pursue strategies to boost their productivity to increase their chances of entering export markets is suggested by Alvarez and López (2005) and others who find empirical evidence to that effect.\footnote{See also Hallward-Driermeier et al. (2002), and Emami-Namini and Lopez (2006).}

In the current paper we try to shed new light on the trade policy-productivity relationship by examining one particular way in which firms may influence their productivity, namely by investing in cost-reducing (i.e., process) R&D. We let the outcome of this investment be stochastic in that a firm’s marginal production cost is a random variable. An increase in R&D simply raises the probability of drawing a low marginal cost. This framework allows us to study a number of interesting questions. First, what effect does trade liberalization have on R&D investment? This is an important question, not least because investment in innovation is one of the key determinants of economic growth. Second, how does the R&D channel interact with other possible mechanisms through which firms adapt to a more liberal trading environment?

We are able to incorporate R&D and firm heterogeneity in a surprisingly simple model of international trade. Our model is a variant of the reciprocal dumping model (Brander and Krugman, 1983), in which firms first decide on market entry and investment in R&D to increase the likelihood of drawing a low marginal cost, then individually learn their marginal cost, and finally play a Bayesian Cournot game to determine their domestic and foreign sales. The model allows us to derive comparative static effects of a
reduction in trade costs on R&D, domestic output, exports, mark-ups, critical values of marginal cost below which firm sell domestically and below which they export. Moreover, we are able to determine how trade liberalization affects aggregate industry productivity and social welfare, both under an exogenous and an endogenous market structure. We are able to prove the following robust results that hold independent of market structure: trade liberalization (i) raises (reduces) aggregate R&D spending if trade costs are low (high); (ii) raises expected exports and, provided that trade costs are high, reduces expected local sales; (iii) increases expected firm size provided that trade costs are high; (iv) forces the least efficient firms to exit the market; (v) leads to a reallocation of market share from less to more efficient firms; and (v) raises social welfare if trade costs are sufficiently low. The effect of trade liberalization on productivity generally depends on market structure. However, we are able to derive sufficient conditions under which this effect is positive: first, trade liberalization raises productivity under a fixed market structure, if trade costs are sufficiently low; second, a reduction in trade costs raises productivity under an endogenous market structure if these costs are sufficiently high.

Our paper contributes to at least two strands of literature. First, it extends the already mentioned literature on trade with heterogeneous firms. In addition to introducing R&D and thereby endogenizing firm productivity, our paper offers a novel approach to modelling firm heterogeneity. In particular, we allow firms to interact strategically in an oligopolistic market instead of relying on monopolistic competition. An important benefit of our approach is that it explicitly reproduces output and mark-up adjustments by firms that are among the most robust empirical regularities of international trade, but are typically absent in monopolistic competition models (see Melitz and Ottaviano, 2004, for an exception).

Second, the paper is directly related to several papers that explicitly examine the link between trade policy and innovation activity. Navas and Sala (2006) introduce process innovation into the Melitz (2003) models by adding
another type of firm, namely an exporter/innovator. However, in the open-economy version of their paper the amount of innovation investment is held constant. Ederington and McCalman (2007) study the effect of trade liberalization on the incentives of firms to adopt a more productive foreign technology. They show both in a theoretical model with heterogeneous firms and in an empirical study of Colombian firms that the effect of trade policy on the speed of technology adoption depends on firm and industry-specific factors. Miyagiwa and Ohno (1995) also examine the impact of trade policy on the speed of technology adoption, and find that it depends on the type of trade policy used. Atkeson and Burstein (2006) develop a dynamic general equilibrium model in which firms may invest "managerial time" to improve their technology. Their main result is that a marginal decrease in trade costs has no effect on firms' incentives to innovate. Gustafson and Segerstrom (2006) provide a version of the Melitz (2003) model in which R&D is carried out in an innovation sector that uses labor to develop new product varieties. The effect of trade liberalization on productivity, economic growth and ultimately welfare is shown to depend crucially on the presence of intertemporal knowledge spillovers in the innovation sector. Pires (2006) uses an oligopoly model to examine how differences in country size lead to cross-country differences in R&D investment and hence serve as a basis for international specialization in production. Funk (2003) examines empirically how R&D spending by firms adjusts to exchange-rate movements, showing that purely domestic firms react differently than firms that export.\footnote{There is also a sizeable literature on the link between strategic R&D and trade policy in oligopolistic markets. See, for instance, Bagwell and Staiger (1994). Haaland and Kind (2004) employ a model in which R&D and output are determined simultaneously, after the government has set R&D subsidies.}

The remainder of the paper is structured as follows. Section 2 introduces the model. Section 3 contains the results both in the case of a fixed market structure and in the case of an endogenous market structure. Section 4 concludes. The Appendix contains proofs.
2 The Model

We consider an oligopolistic trade model with two segmented markets: the home and the foreign market. The oligopolists produce a homogeneous good. Consumers in each market have quadratic quasi-linear preferences over this good (and a numeraire good) that give rise to a linear inverse demand function,

\[ p_j = A - Q_j, \]  

(1)

where \( p_j \) and \( Q_j \) denote price and total sales in market \( j \). Labor is the only factor of production and comes in fixed supply. Assuming that the numeraire good is produced under constant returns to scale at unit cost and traded freely on a competitive world market, the equilibrium wage in each country is equal to one, and trade is always balanced.

Let \( n \) denote the number of active oligopolists in each market. Firms in the oligopoly industry produce under constant (but ex-ante unknown) marginal cost \( c \) (equal to the unit labor requirement). We assume that the marginal cost is firm-specific and is revealed to the firm (as private information) only after it has incurred a set-up cost \( f > 0 \). The probability that a firm’s marginal cost is less than or equal to \( c \) is given by the ex-ante cumulative distribution \( F(c) \); the support of the density function \( f(c) \) is the interval \([0, \bar{c}]\). The per-unit trade cost on shipments between countries is denoted by \( t \).

A firm may invest an amount \( r \geq 0 \) in R&D to increase its chances to become a lower-cost firm. Let \( G(c) \) denote the corresponding after-R&D cumulative probability distribution. We assume R&D increases this probability such that

\[ G(c) = g(r)F(c), g(0) = 1, g' > 0, g'' \leq 0. \]  

(2)

Obviously, expression (2) holds true only as long as \( G(c) \leq 1.5 \) The cost of R&D is given by

\[^5\text{Precisely, } G(c) = \min(g(r)F(c), 1).\]
\( \rho(r) : \rho(0) = 0, \rho' > 0, \rho'' \geq 0. \quad (3) \)

Consider home firm \( i \). It has incurred the entry cost \( f \) and the R&D cost \( \rho(r) \). Upon learning its cost \( c_i \), its output decision will be \( y(c_i) \) for the home market and \( x(c_i) \) for the foreign market. This output decision will depend on the expected output of all rival firms. Note that output decisions have to be made under asymmetric information as marginal costs will be revealed only to the individual firm and individual output decisions have to be based on expectations about the rivals’ output. The home firm will face \( n - 1 \) domestic rivals, each expected to produce and sell \( \hat{y} \) units in the home market, and \( n \) foreign rivals, each expected to sell \( \hat{x} \) units in the home market. Define

\[ Q_{-i} \equiv (n - 1)\hat{y} + n\hat{x}. \]

The home firm’s first-order condition for its domestic sales \( y(c_i) \) is

\[ p(y(c_i) + Q_{-i}) + y(c_i)p'(y(c_i) + Q_{-i}) - c_i \leq 0, (= 0 \text{ if } y(c_i) > 0) \quad (4) \]

Let us define the critical marginal cost for which \( y(c_i) \) becomes zero:

\[ \tilde{c}_y \equiv A - (n - 1)\hat{y} - n\hat{x}. \quad (5) \]

Then the first-order conditions give rise to the decision rule\(^6\)

\[ y(c_i) = \begin{cases} 0 & \text{if } c_i \geq \tilde{c}_y, \\ \frac{1}{2} \left( \tilde{c}_y - c_i \right) & \text{if } c_i < \tilde{c}_y, \end{cases} \quad (6) \]

and the profit in the home market is equal to

\[ \pi(c_i) = \begin{cases} 0 & \text{if } c_i \geq \tilde{c}_y, \\ \frac{1}{4} (\tilde{c}_y - c_i)^2 & \text{if } c_i < \tilde{c}_y. \end{cases} \quad (7) \]

Similarly in the foreign market, the home firm faces \( n \) foreign rivals, each supplying \( \hat{y}^* \) units, and \( n - 1 \) domestic rivals, each exporting \( \hat{x} \) units. Firm \( i \)'s first-order condition for its exports \( x(c_i) \) is

\[ p(x(c_i) + Q^*_{-i}) + x(c_i)p'(x(c_i) + Q^*_{-i}) - t - c_i \leq 0, (= 0 \text{ if } x(c_i) > 0), \quad (8) \]

\(^6\)See also Cramton and Palfrey (1990), Lemma 5 (p. 26 and pp. 41-2).
where

\[ Q^*_{-i} \equiv n\hat{y}^* + (n - 1)\hat{x}. \]

The critical marginal cost for which \( x(c_i) \) becomes zero is given by:

\[ \tilde{c}_x \equiv A - (n - 1)\hat{x} - n\hat{y}^* - t. \] (9)

Hence the quantity of exports is

\[ x(c_i) = \begin{cases} 0 & \text{if } c_i \geq \tilde{c}_x, \\ \frac{1}{2} (\tilde{c}_x - c_i) & \text{if } c_i < \tilde{c}_x. \end{cases} \] (10)

and the export profit is

\[ \pi^*(c_i) = \begin{cases} 0 & \text{if } c_i \geq \tilde{c}_x, \\ \frac{1}{4} (\tilde{c}_x - c_i)^2 & \text{if } c_i < \tilde{c}_x. \end{cases} \] (11)

Prior to learning its marginal cost, the home firm forms expectations about its sales levels. This expectation coincides with the expected sales levels of all rivals. In what follows, we set \( \hat{y}^* = \hat{y} \) and \( \hat{x}^* = \hat{x} \), because the two countries are identical. The following Lemma shows that the expected local and export sales of a firm are determined by a system of only two equations:

**Lemma 1** Expected sales are

\[ \hat{y} = \frac{g(r)}{2} \int_{0}^{\tilde{c}_y} F(c) dc, \] (12)

\[ \hat{x} = \frac{g(r)}{2} \int_{0}^{\tilde{c}_x} F(c) dc. \] (13)

Proof: See Appendix A.1. \( \square \)

Using (7) and (11) we may write the total expected profit of a firm as

\[ \hat{\Pi} = \frac{g(r)}{4} \Omega - (f + \rho(r)), \] (14)

where

\[ \Omega \equiv \int_{0}^{A-(n-1)\hat{y}-n\hat{x}} [A - (n - 1)\hat{y} - n\hat{x} - c]^2 dF(c) + \]

\[ \int_{0}^{A-(n-1)\hat{x}-n\hat{y}-t} [A - (n - 1)\hat{x} - n\hat{y} - t - c]^2 dF(c). \] (15)
Prior to learning its marginal cost, each entrant chooses its R&D level according to the following first-order condition:

\[ \frac{\partial \hat{\Pi}}{\partial r} = g'(r) \frac{\Omega}{4} - \rho'(r) = 0. \]  (16)

We assume that \( \hat{\Pi}_{rr} \equiv g''(r) \Omega - 4 \rho'(r) < 0 \). For future convenience, let us denote the optimal level of R&D by \( \hat{r} \), where

\[ g'(\hat{r}) \Omega - 4 \rho'(\hat{r}) = 0. \]  (17)

The following assumption guarantees that \( \hat{r} > 0 \), i.e., that the optimal R&D level is non-zero:

**Assumption 1**

\[ \Omega > 4 \rho'(0). \]

### 3 The Effects of Trade Liberalization

In this section we examine how trade liberalization in the form of a marginal reduction in \( t \) affects the equilibrium of the model. We start with the case of a fixed market structure. That is, we determine how trade liberalization affects expected local sales, expected exports and R&D, holding fixed the number of firms. One may interpret this as a short-run scenario, in which the number of firms has not yet had time to adjust. We then turn to the case of endogenous market structure, where market entry and exit occur until expected profits are equal to zero. In this case we want to know how trade liberalization affects expected local sales, expected exports, R&D, as well as the equilibrium number of firms.

#### 3.1 Fixed Market Structure

In the case of a fixed market structure the equilibrium \( \hat{y}, \hat{x} \) and \( \hat{r} \) are determined by equations (12), (13) and (17). To derive the comparative static effects of a reduction in \( t \) we totally differentiate these equilibrium conditions.
A formal analysis is presented in Appendix A.2. Here we want to focus on building intuition for the results. For this purpose it is useful to first consider the effect of trade liberalization on the threshold values of the marginal cost, $\tilde{c}_y$ and $\tilde{c}_x$. For $t = 0$ we obviously have $\tilde{c}_y = \tilde{c}_x$: there is only one critical value such that firms with marginal cost draws below this value are active on the integrated home and foreign markets, whereas firms with higher marginal costs do not produce any output. For $t > 0$, we must have $\tilde{c}_y > \tilde{c}_x$: only the most efficient firms export, whereas firms with cost draws between $\tilde{c}_y$ and $\tilde{c}_x$ only sell on the domestic market. To see how $\tilde{c}_y$ and $\tilde{c}_x$ change with $t$, we can use (5) and (9) to obtain:

$$\frac{d\tilde{c}_y}{dt} = -(n - 1) \frac{d\hat{y}}{dt} - n \frac{d\hat{x}}{dt},$$

$$\frac{d\tilde{c}_x}{dt} = -(n - 1) \frac{d\hat{x}}{dt} - n \frac{d\hat{y}}{dt} - 1.$$  

We can prove the following result:

**Proposition 1** If the number of firms is fixed, $\frac{d\tilde{c}_y}{dt} > 0$ and $\frac{d\tilde{c}_x}{dt} < 0$.

Proof: see Appendix A.2. □

This result implies that as trade costs decline, the threshold cost level $\tilde{c}_x$ becomes higher, so that firms with marginal cost draws just above the old export threshold level will now be able to export. On the other hand, the threshold cost level $\tilde{c}_y$ falls, meaning that firms that were just efficient enough to sell on their local market are now forced to exit the market altogether.

We are now able to explain the following comparative static results:

**Proposition 2** If the number of firms is fixed, trade liberalization (i) increases expected exports; (ii) decreases expected local sales if trade costs are high, and has an ambiguous effect on local sales if trade costs are low; (iii) increases the expected total output of each firm; and (iv) increases (decreases) R&D if trade costs are low (high).

Proof: see Appendix A.2. □
Consider first how trade liberalization affects a firm’s expected sales holding fixed the level of R&D expenditure. Expected export sales rise, since trade liberalization raises the probability that any given firm will be efficient enough to be able to export ($\tilde{c}_x$ falls), and allows those firms that do export to increase their shipments abroad. Expected domestic sales decrease, since firms respond to import competition by reducing local sales. In addition, as $\tilde{c}_y$ rises, the likelihood that a given firm will be able to sell on its local market falls. These arguments explain the increase in export sales (part (i) of the proposition), but not why domestic sales might increase if trade costs are low (see part (ii)). This ambiguity has to come from changes in R&D spending. Specifically, expected domestic sales can only rise after trade liberalization, if increased R&D leads to a big enough downward shift in the marginal cost distribution. The effect of trade liberalization on total sales of a firm is unambiguously positive (part (iii)), as the expected increase in exports more than compensates even an expected decrease in domestic sales.

How does R&D respond to a reduction in trade costs? A firm selling only on the domestic market would want to reduce its R&D spending, since tougher import decreases its output and hence also the marginal benefit from R&D. An exporter would want to increase R&D, since the increase in its export sales more than compensates the decrease in local market share, meaning that it has a greater incentive to invest in cost-reducing R&D. If $t$ is sufficiently close to the prohibitive level, the probability of being an exporter is very low ($\tilde{c}_x$ is small), whereas there is a large probability of selling only on the domestic market ($\tilde{c}_y$ and $(\tilde{c}_y - \tilde{c}_x)$ are big). This implies that for high trade cost, R&D spending falls as trade is liberalized. If $t$ is close to zero, almost all active firms will have access to the export market and therefore be able to expand output as trade is liberalized. Hence for low trade costs, R&D spending increases with trade liberalization. This explains the U-shaped relationship between trade costs and R&D in part (iv).

Propositions 1 and 2 indicate that trade liberalization has four effects on industry productivity: (i) a scale effect: as firm size increases the fixed cost
is spread over higher expected output; (ii) a selection effect: the least efficient firms exit; (iii) a market-share reallocation effect: more efficient firms gain access to the export market and raise their output at the expense of less efficient firms; and (iv) a direct effect due to changes in R&D investment. Trade liberalization induces greater R&D spending and hence has an unambiguously positive effect on industry productivity when trade costs are low.

Finally, consider how trade liberalization affects consumer surplus and social welfare. Since expected output increases with trade liberalization, it follows that consumer surplus must rise. To determine how social welfare is affected, we have to take into account the change in the domestic firms’ expected profits. If \( t \) is close enough to zero, the usual pro-competitive effect of trade liberalization dominates, meaning that the increase in consumer surplus caused by tougher competition more than compensates for the decline in industry profit. If \( t \) is near the prohibitive level, this pro-competitive effect may be offset by the fact that exporters have to bear high trade costs so that profits fall by more than consumer surplus rises. These effects are similar to those in the reciprocal dumping model. However, in our model the welfare effect of trade liberalization may be positive for close to prohibitive trade costs. The reason for this is that as trade costs are lowered from the prohibitive level only the most efficient firms are able to export, whereas the least efficient active firms are forced to exit the market. In other words, the selection effect of trade liberalization provides an additional boost to productivity and hence welfare that is not present in the conventional reciprocal dumping model. The following Proposition summarizes these results:

**Proposition 3** If the number of firms is fixed, trade liberalization raises (i) industry productivity, if the trade cost is sufficiently low; (ii) expected consumer surplus; and (iii) expected social welfare provided that the trade cost is sufficiently low.

Proof: see Appendix A.2. □
3.2 Endogenous Market Structure

Now consider the case of an endogenous market structure. Free entry and exit of firms ensures that expected profits (14) are zero, which implies that

$$\frac{\Omega}{4} = \frac{\rho(r) + f}{g(r)}.$$  \hspace{1cm} (19)

Using (19), we may rewrite the first-order condition for R&D (16) as:

$$\frac{g'(\hat{r})}{g(\hat{r})} = \frac{\rho'(\hat{r})}{\rho(\hat{r}) + f}.$$ \hspace{1cm} (20)

Expression (20) clearly shows that the optimal R&D level per firm depends only on \( g \) and \( \rho \). This has the following consequence:

**Proposition 4** Firm-level R&D does not depend on trade costs if market structure is endogenous.

According to Proposition 4, firm entry and exit eliminates any effect of trade liberalization on R&D per firm. This does not, however, mean that trade liberalization has no effect on aggregate R&D, since the equilibrium number of firms may change. To determine the effects of trade liberalization, we may treat R&D expenditures as a fixed cost and use equations (12), (13) and (19) to solve for the remaining endogenous variables \((n, \bar{x}, \bar{y})\). We may rewrite these equations as

$$2\bar{y} - \int_{0}^{A-(n-1)\bar{y} - n\bar{x}} G(c) dc = 0, \hspace{1cm} (21)$$

$$2\bar{x} - \int_{0}^{A-(n-1)\bar{y} - n\bar{x} - t} G(c) dc = 0, \hspace{1cm} (22)$$

$$\int_{0}^{A-(n-1)\bar{y} - n\bar{x}} [A - (n - 1)\bar{y} - n\bar{x} - c]^2 dG(c) + \int_{0}^{A-(n-1)\bar{y} - n\bar{x} - t} [A - (n - 1)\bar{x} - n\bar{y} - t - c]^2 dG(c) - 4(f + \rho(r^*)) = 0. \hspace{1cm} (23)$$
Before turning to the comparative static effects of trade liberalization, let us verify that the selection effect operates in the same way as under a fixed market structure. We obviously still have \( \tilde{c}_y = \tilde{c}_x \) for \( t = 0 \), and \( \tilde{c}_y > \tilde{c}_x \) for \( t > 0 \). In the derivatives of \( \tilde{c}_y \) and \( \tilde{c}_x \) with respect to \( t \) we obtain an additional effect, since the number of firms changes:

\[
\begin{align*}
\frac{d\tilde{c}_y}{dt} & = -(n-1)\frac{d\hat{y}}{dt} - n \frac{d\hat{x}}{dt} - (\hat{y} + \hat{x}) \frac{dn}{dt}, \\
\frac{d\tilde{c}_x}{dt} & = -(n-1)\frac{d\hat{x}}{dt} - n \frac{d\hat{y}}{dt} - (\hat{y} + \hat{x}) \frac{dn}{dt} - 1.
\end{align*}
\]

Still we can prove that the selection effect is equivalent to the one in the fixed market structure case. That is, trade liberalization eliminates the least efficient firms from the domestic market, whereas more efficient firms are able to export.

**Proposition 5** If the number of firms is endogenous, \( \frac{d\tilde{c}_y}{dt} > 0 \) and \( \frac{d\tilde{c}_x}{dt} < 0 \).

Proof: see Appendix A.3. □

Total differentiation of (21), (22) and (23) yields the following comparative static results:

**Proposition 6** If market structure is endogenous, trade liberalization (i) increases expected exports and decreases expected home production of each firm; (ii) increases the expected output of each firm if trade costs are high; and (iii) increases (decreases) the number of firms and hence aggregate R&D if trade costs are low (high).

Proof: see Appendix A.3. □

To gain intuition for part (i) recall that with a fixed market structure the effect of trade liberalization on local sales was ambiguous for low trade costs, because trade liberalization induced firms to raise their R&D spending. Since this effect is absent here, the impact of trade liberalization is straightforward: the probability that a given firm exports rises as do sales of each exporting firm abroad. Increased competition from abroad reduces local sales,
as does the selection effect. The intuition for parts (ii) and (iii) is straightforward when trade costs are high. Trade liberalization increases competition and hence makes each firm’s residual demand more elastic. Firms hence are forced to raise their output to make up on volume what they lose on price so that the expected profit remains zero. As a result, the number of firms has to fall. For low trade costs, we observe a different effect. Consider an infinitesimal increase in the trade cost starting at free trade. This leaves output of the firm unchanged. However, since firms now have to pay a transportation cost, expected profit has to fall and the number of firms has to decrease so as to keep expected profit equal to zero. Hence at free trade, and by continuity sufficiently close to it, trade liberalization will raise the number of firms and therefore also industry-level R&D. A sufficient condition for the number of firms to increase with trade liberalization is states in the following Proposition:

**Proposition 7**  *If the expected output of each firm decreases with trade liberalization, the number of firms will increase.*

Proof: see Appendix A.3. □

Finally consider the effects of trade liberalization on industry productivity and social welfare. The selection and market-share reallocation effects, ceteris paribus, unambiguously raise industry productivity when trade is liberalized. The scale effect goes in the same direction provided that trade costs are high, since in this case we obtain fewer but larger firms. Since R&D per firm remains unchanged, trade liberalization hence raises industry productivity at least for high trade costs.

The effect of trade liberalization on social welfare is equal to the effect on consumer surplus, since expected profits are zero due to free entry. We are able to prove that welfare unambiguously increases with trade liberalization. These results are summarized in the following Proposition:

**Proposition 8**  *If market structure is endogenous, trade liberalization raises (i) expected social welfare; and (ii) industry productivity provided that trade costs are high.*
4 Conclusions

In this paper we developed a model of international trade with oligopolistic competition to explore the effects of trade liberalization on R&D, firm and industry productivity, production patterns and social welfare. We were able to identify a number of robust results concerning the effects of trade liberalization—robust in the sense that we can identify sufficient conditions under which these results hold for both fixed and endogenous market structures and hence should be observed across different industries independent of whether their entry cost is large or small. Specifically, we find that trade liberalization (i) raises (reduces) aggregate R&D spending if trade costs are low (high); (ii) raises expected exports, and lowers firms’ local sales if trade costs are high; (iii) increases expected firm size provided that trade costs are high; (iv) forces the least efficient firms to leave the market; (v) reallocates market share from less to more efficient firms; and (vi) raises expected social welfare if trade costs are sufficiently low. The productivity effect of trade liberalization is shown to depend on market structure. If there is no entry of firms, a sufficient condition for trade liberalization to increase industry productivity and welfare is that trade costs are low. However, if firms adjust to trade liberalization through entry and exit, a sufficient condition for productivity to increase is that trade costs are high.

The dependency of productivity effects on market structure may explain the fact, mentioned in the Introduction, that empirical studies come to such conflicting conclusions about the link between trade policy and productivity. In particular, our paper suggests that one should control for both market structure and the level of protection when analyzing this link. Similarly, our paper suggests that the scale effect that figures prominently in older studies of trade liberalization (see, for instance, Cox and Harris, 1985) will not necessarily be observed, especially in industries in which entry and exit costs are low. This provides a potential explanation why empirical studies

Proof: see Appendix A.3. □
of trade liberalization seem to fail to find a scale effect (see Head and Ries, 1999).

Our model also makes a methodological contribution to the literature. Specifically it shows how one can model firm heterogeneity in a simple way without resorting to the assumption of monopolistic competition, and how one can endogenize firm productivity by allowing for R&D. This approach has the advantage that it matches quite well the key stylized facts of trade liberalization summarized by Tybout (2003) and Wagner (2007). In particular, it reproduces the stylized facts that trade liberalization (i) reduces price-cost margins; (ii) lowers domestic sales of import-competing firms (at least provided that trade costs are high, or market structure is endogenous); (iii) expands markets for very efficient firms; (iv) increases efficiency at the plant level (at least for low trade costs, or endogenous market structure). In addition, (v) exporters tend to be larger and more productive than firms that do not export.

Appendix

A.1 Proof of Lemma 1

Expected output for the home market is

$$E[y(c)] = \hat{y} = g(r) \int_{0}^{\bar{c}_y} y(c) dF(c) = \frac{g(r)}{2} \int_{0}^{\bar{c}_y} [\bar{c}_y - c] dF(c)$$  \tag{A.1}$$

and expected exports to the foreign market are

$$E[x(c)] = \hat{x} = g(r) \int_{0}^{\bar{c}_x} x(c) dF(c) = \frac{g(r)}{2} \int_{0}^{\bar{c}_x} [\bar{c}_x - c] dF(c).$$  \tag{A.2}$$
Evaluating the integral on the right-hand side of (A.1) by parts, and defining \( \phi(c) \equiv [\tilde{c}_y - c] \), we have

\[
\int_0^{\tilde{c}_y} [\tilde{c}_y - c] dF(c) = \int_0^{\tilde{c}_y} \phi(c) F'(c) dc
\]

\[
= [\phi(\tilde{c}_y) F(\tilde{c}_y) - \phi(0) F(0)] - \int_0^{\tilde{c}_y} \phi'(c) F(c) dc
\]

\[
= \int_0^{\tilde{c}_y} F(c) dc,
\]

because \( \phi(\tilde{c}_y) = F(0) = 0 \) and \( \phi'(c) = -1 \). A similar derivation leads to the expected export level. □

A.2 Proofs of Propositions 1 to 3

Differentiating (12), (13) and (17) totally, we obtain

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33}
\end{bmatrix}
\begin{bmatrix}
\frac{dr}{dt} \\
\frac{d\tilde{x}}{d\tilde{y}}
\end{bmatrix}
= \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
\]

where

\[
\alpha_{11} \equiv -\frac{2g'\tilde{y}}{g}, \quad \alpha_{12} \equiv gnF(\tilde{c}_y), \quad \alpha_{13} \equiv 2 + g(n - 1)F(\tilde{c}_y),
\]

\[
\alpha_{21} \equiv -\frac{2g'\tilde{x}}{g}, \quad \alpha_{22} \equiv 2 + g(n - 1)F(\tilde{c}_x), \quad \alpha_{23} \equiv gnF(\tilde{c}_x),
\]

\[
\alpha_{31} \equiv \tilde{\Pi}_{rr}, \quad \alpha_{32} = -\frac{4g'}{g}((n - 1)\tilde{x} + n\tilde{y}), \quad \alpha_{33} = -\frac{4g'}{g}((n - 1)\tilde{y} + n\tilde{x}),
\]

\[
\beta_1 = 0, \quad \beta_2 = -gF(\tilde{c}_x), \quad \beta_3 = \frac{4g'}{g} \tilde{x}.
\]

Expanding along the first column yields the determinant

\[
\Delta = \frac{8g'}{g^2} \left( \tilde{x}^2[(2n - 1)(1 - gF(\tilde{c}_y)) - 1] + \tilde{y}^2[(2n - 1)(1 - gF(\tilde{c}_y)) - 1] + 4n\tilde{x}\tilde{y} \right)_{\equiv \Delta_1}
\]

\[
+ \tilde{\Pi}_{rr} \left( g^2 n^2 F(\tilde{c}_x) F(\tilde{c}_y) - (2 + g(n - 1)F(\tilde{c}_y))(2 + g(n - 1)F(\tilde{c}_x)) \right)_{\equiv \Delta_2}
\]

We first establish that \( \Delta > 0 \). Since \( gnF(\tilde{c}_x) < 2 + g(n - 1)F(\tilde{c}_x) \) and \( gnF(\tilde{c}_y) < 2 + g(n - 1)F(\tilde{c}_y) \), \( \Delta_2 < 0 \) and hence \( \tilde{\Pi}_{rr}\Delta_2 > 0 \). Thus, \( \Delta > 0 \)

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will hold true if we can show that $\Delta_1 > 0$. We will show that $\Delta_1 > 0$ by contradiction. We observe first that $\Delta_1 > 0$ if $(2n - 1)(1 - gF(\bar{c}_y)) - 1 \geq 0$ and $(2n - 1)(1 - gF(\bar{c}_x)) - 1 \geq 0$. Thus, $\Delta_1 < 0$ requires that $(2n - 1)(1 - gF(\bar{c}_y)) - 1 < 0$ and/or $(2n - 1)(1 - gF(\bar{c}_x)) - 1 < 0$. Since $gF(\bar{c}_y) \geq gF(\bar{c}_x)$, $(2n - 1)(1 - gF(\bar{c}_x)) - 1 \geq (2n - 1)(1 - gF(\bar{c}_y)) - 1$, and we have to consider two possible cases:

**Case 1:** $(2n - 1)(1 - gF(\bar{c}_x)) - 1 > 0$, $(2n - 1)(1 - gF(\bar{c}_y)) - 1 < 0$

In this case,

$$\Delta_1 > \bar{x}^2[(2n - 1)(1-gF(\bar{c}_y)) - 1] + 4n\bar{x}\hat{y} = \bar{x}(\bar{x}[(2n - 1)(1-gF(\bar{c}_y)) - 1] + 4n\hat{y}) > 0$$

because $\hat{y} > \bar{x}$ and $4n > -(2n - 1)(1 - gF(\bar{c}_y)) + 1$.

**Case 2:** $(2n - 1)(1 - gF(\bar{c}_x)) - 1 < 0$, $(2n - 1)(1 - gF(\bar{c}_y)) - 1 < 0$

First observe that for zero trade costs, $\hat{x} = \hat{y}$, $F(\bar{c}_x) = F(\bar{c}_y)$ and

$$\Delta_1 = 2\hat{y}^2(2n - 1)(2 - gF(\bar{c}_y)) > 0$$

Hence, $\Delta_1 < 0$ warrants the existence of a critical $\bar{x} < \hat{y}$ such that

$$\bar{x}^2[(2n - 1)(1 - gF(\bar{c}_y)) - 1] + \hat{y}^2[(2n - 1)(1 - gF(\bar{c}_x)) - 1] + 4n\bar{x}\hat{y} = 0.$$ 

Solving for quadratic equation yields the two solutions

$$\bar{x}_{1,2} = \frac{-4n\hat{y} \pm \sqrt{8n^2\hat{y}^2 - 4((2n - 1)(1 - gF(\bar{c}_y)) - 1)[(2n - 1)(1 - gF(\bar{c}_x)) - 1]\hat{y}^2}}{(2n - 1)(1 - gF(\bar{c}_y)) - 1}$$

Note carefully that $(2n - 1)(1 - gF(\bar{c}_y)) - 1 \in [-1, 0]$ so that $\bar{x}$ is larger than the numerator in absolute terms. The negative solution is irrelevant as it implied $\bar{x} > 4n\hat{y}$ which violates $\bar{x} < \hat{y}$. The positive solution fulfills $\bar{x} < \hat{y}$ only if

$$\sqrt{8n^2\hat{y}^2 - 4((2n - 1)(1 - gF(\bar{c}_y)) - 1)[(2n - 1)(1 - gF(\bar{c}_x)) - 1]\hat{y}^2} > (4n - 1)\hat{y}.$$
However,
\[
\sqrt{n^2 \hat{g}^2 - 4[(2n - 1)(1 - gF(\bar{c}_x)) - 1][(2n - 1)(1 - gF(\bar{c}_y)) - 1]\hat{g}^2} < \sqrt{n^2 \hat{g}^2} = 2\sqrt{2n}\hat{g} < (4n - 1)\hat{g},
\]
so that no solution exists in the relevant range and \(\Delta_1 > 0\) holds also for that case. This proves that \(\Delta > 0\).

We can now derive the comparative-static effects:
\[
\frac{dr}{dt} = \frac{8g'}{g \Delta}(gn(\hat{y}F(\bar{c}_x) - \hat{\chi}F(\bar{c}_y)) - \hat{\chi}(2 - gF(\bar{c}_y))),
\]
\[
\frac{dr}{dt} < 0 \text{ at } t = 0 \iff \hat{\chi} = \hat{y} \iff F(\bar{c}_x) = F(\bar{c}_y), \frac{dr}{dt} > 0 \text{ at } x = 0,
\]
\[
\frac{d\hat{\chi}}{dt} = -\frac{8g'^2}{g^2 \Delta}(2 - g(n\hat{y} - \hat{\chi})\hat{y}gF(\bar{c}_x) + \hat{\Pi}_{rr}gF(\bar{c}_x)(2 + g(n\hat{y} - \hat{\chi})) < 0,
\]
\[
\frac{d\hat{y}}{dt} = \frac{8g'^2}{g^2 \Delta}(F(\bar{c}_x)[(n - 1)\hat{y}g^2F(\bar{c}_x) + g\hat{\chi}(n\hat{\chi}F(\bar{c}_y) + \hat{\chi}F(\bar{c}_x)) - 2\hat{\chi}\hat{y}) - \frac{\hat{\Pi}_{rr}g^2nF(\bar{c}_x)F(\bar{c}_y)},
\]
\[
\frac{d\hat{y}}{dt}(\hat{\chi} = 0) = \frac{8g'^2}{g^2 \Delta}(n - 1)\hat{y}g^2F(\bar{c}_x) - \frac{\hat{\Pi}_{rr}g^2nF(\bar{c}_x)F(\bar{c}_y)} > 0,
\]
\[
\frac{d\hat{y}}{dt}(\hat{\chi} = \hat{y}) = -2\hat{y}^2(2 - ng(F(\bar{c}_x) + F(\bar{c}_y)))\frac{8g'^2}{g^2 \Delta} - \frac{\hat{\Pi}_{rr}g^2nF(\bar{c}_x)F(\bar{c}_y)} \text{ is ambiguous.}
\]

However, we can sign the change in total expected output per firm \(\hat{\varphi} \equiv \hat{\chi} + \hat{\chi}^2,\)
\[
\frac{d\hat{\varphi}}{dt} = -\frac{8g'^2}{g^2 \Delta}(2\hat{\varphi}g(1 - gF(\bar{c}_x)) - \hat{\chi}^2(2 - gF(\bar{c}_y)) - \hat{\varphi}gF(\bar{c}_x)) + \hat{\Pi}_{rr}(g(2 - gF(\bar{c}_y))) < 0,
\]
and therefore in expected industry output \((Q = n\hat{\varphi})\):
\[
\frac{dQ}{dt} = n\frac{d\hat{\varphi}}{dt} < 0.
\]
As for the critical values of marginal costs, \(d\hat{c}_v/dt\) can be rewritten as
\[
\frac{d\hat{c}_v}{dt} = -(n - 1)\frac{d\hat{\varphi}}{dt} - \frac{d\hat{\chi}}{dt} > 0.
\]
Differentiating \(d\hat{c}_x/dt\) yields
\[
\frac{d\hat{c}_x}{dt} = \frac{2}{g^2}\left(2g^2\hat{\Pi}_{rr} + g^3(n - 1)\hat{\Pi}_{rr}F(\bar{c}_y) - 8g^2\hat{g}(n\hat{\chi} + (n - 1)\hat{y})\right) < 0.
\]
The welfare effect of integration consists of the effect on aggregate expected profits and consumer surplus. The change in expected profit (14) is

\[
\frac{d\hat{\Pi}}{dt} = \frac{g(r)}{4} \left( \frac{\partial \Omega}{\partial \hat{y}} \frac{d\hat{y}}{dt} + \frac{\partial \Omega}{\partial \hat{x}} \frac{d\hat{x}}{dt} \right)
\]

\[
= - (n - 1) \frac{d\hat{q}}{dt} \hat{q} + \frac{d\hat{y}}{dt} \hat{x} - \frac{d\hat{x}}{dt} \hat{y} - \hat{x},
\]

taking into account that \(\partial \hat{\Pi}/\partial r = 0\). Let \(\hat{CS} \equiv (n\hat{q})^2/2\) denote expected consumer surplus. Its change with \(t\) is

\[
\frac{d\hat{CS}}{dt} = n^2 \hat{q} \frac{d\hat{q}}{dt} < 0,
\]

since \(d\hat{q}/dt < 0\). The total expected welfare change is determined as

\[
\frac{d\hat{W}}{dt} = \frac{d\hat{CS}}{dt} + n \frac{d\hat{\Pi}}{dt} = n \left( \frac{d\hat{q}}{dt} \hat{q} + \frac{d\hat{y}}{dt} \hat{x} - \frac{d\hat{x}}{dt} \hat{y} - \hat{x} \right)
\]

For \(t = 0 \Leftrightarrow \hat{y} = \hat{x} \Leftrightarrow d\hat{y}/dt = d\hat{x}/dt\), we find

\[
\frac{d\hat{W}}{dt} (t = 0) = n \left( \frac{d\hat{q}}{dt} \hat{q} - \hat{x} \right) < 0,
\]

whereas the marginal welfare effect at the prohibitive trade cost level, \(i.e.,\) for \(\hat{x} = 0\), is ambiguous:

\[
\frac{d\hat{W}}{dt} (\hat{x} = 0) = n \left( \frac{d\hat{q}}{dt} \hat{q} - \frac{d\hat{x}}{dt} \hat{y} \right).
\]

### A.3 Proofs of Propositions 4 to 8

Differentiating (21), (22) and (23) totally, we get

\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  \frac{dn}{dt} \\
  \frac{d\hat{x}}{dt} \\
  \frac{d\hat{y}}{dt}
\end{bmatrix}
= \begin{bmatrix}
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix} dt,
\]
where
\[
\begin{align*}
a_{11} &\equiv (\hat{x} + \hat{y})G(\bar{c}_y), & a_{12} &\equiv nG(\bar{c}_y), & a_{13} &\equiv 2 + (n - 1)G(\bar{c}_y), \\
a_{21} &\equiv (\hat{x} + \hat{y})G(\bar{c}_x), & a_{22} &\equiv 2 + (n - 1)G(\bar{c}_x), & a_{23} &\equiv nG(\bar{c}_x), \\
a_{31} &\equiv -4(\hat{x} + \hat{y})^2, & a_{32} &\equiv -4((n - 1)\hat{x} + n\hat{y}), & a_{33} &\equiv -4((n - 1)\hat{y} + n\hat{x}), \\
b_1 &\equiv 0, & b_2 &\equiv -G(\bar{c}_x), & b_3 &\equiv 4\hat{x}.
\end{align*}
\]

The determinant is
\[
\Delta = 8(\hat{x} + \hat{y})[\hat{x}(2 - G(\bar{c}_y)) + \hat{y}(2 - G(\bar{c}_y))]> 0.
\]

The comparative-static effects are given by
\[
\frac{dn}{dt} = \frac{n(\hat{y}G(\bar{c}_x) - \hat{x}G(\bar{c}_y)) - (2 - G(\bar{c}_y))\hat{x}}{\Delta},
\]
where
\[
\frac{dn}{dt} < 0 \text{ at } t = 0 \Leftrightarrow \hat{x} = \hat{y} \Leftrightarrow G(\bar{c}_y) = G(\bar{c}_x), \quad \frac{dn}{dt} > 0 \text{ at } \hat{x} = 0;
\]
\[
\frac{d\hat{x}}{dt} = -\frac{8\hat{y}(\hat{x} + \hat{y})G(\bar{c}_x)}{\Delta} < 0,
\]
\[
\frac{d\hat{y}}{dt} = \frac{8\hat{x}(\hat{x} + \hat{y})G(\bar{c}_y)}{\Delta} > 0,
\]
\[
\frac{d\hat{q}}{dt} = \frac{(\hat{x} + \hat{y})(\hat{x}G(\bar{c}_y) - \hat{y}G(\bar{c}_x))}{\Delta},
\]
\[
\frac{d\hat{q}}{dt} = 0 \text{ at } t = 0 \Leftrightarrow x = y \Leftrightarrow G(\bar{c}_y) = G(\bar{c}_x), \quad \frac{d\hat{q}}{dt} < 0 \text{ at } \hat{x} = 0.
\]

In addition
\[
\frac{d\hat{q}}{dt} > 0 \Leftrightarrow \hat{y}G(\bar{c}_x) - \hat{x}G(\bar{c}_y) < 0 \Rightarrow \frac{dn}{dt} < 0.
\]

The effect on consumption is
\[
\frac{dQ}{dt} = -\frac{8\hat{x}(\hat{x} + \hat{y})(2 - G(\bar{c}_y))}{\Delta} = -\frac{\hat{x}(2 - G(\bar{c}_y))}{\hat{x}(2 - G(\bar{c}_y)) + \hat{y}(2 - G(\bar{c}_y))} < 0.
\]

Furthermore, using these results for (24) yields
\[
\frac{d\bar{c}_y}{dt} = \frac{2\hat{y}}{2(\hat{x} + \hat{y}) - \hat{x}G(\bar{c}_y) - \hat{y}G(\bar{c}_x)} > 0,
\]
\[
\frac{d\bar{c}_x}{dt} = -\frac{2\hat{x}}{2(\hat{x} + \hat{y}) - \hat{x}G(\bar{c}_y) - \hat{y}G(\bar{c}_x)} < 0.
\]
References


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