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**Working Paper**

## Quantifying urban centrality: A simple index proposal and international comparison

Discussion Paper, No. 189

**Provided in Cooperation with:**

Institute of Applied Economic Research (ipea), Brasília

*Suggested Citation:* Pereira, Rafael Henrique Moraes; Nadalin, Vanessa Gapiotti; Monasteiro, Leonardo; Albuquerque, Pedro H. (2015) : Quantifying urban centrality: A simple index proposal and international comparison, Discussion Paper, No. 189, Institute for Applied Economic Research (ipea), Brasília

This Version is available at:

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# 189

**DISCUSSION PAPER**

Originally published by Ipea in January 2012 as number 1675a of the series Texto para Discussão.

## **QUANTIFYING URBAN CENTRALITY: A SIMPLE INDEX PROPOSAL AND INTERNATIONAL COMPARISON**

**Rafael Henrique Moraes Pereira  
Vanessa Nadalin  
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Brasília, January 2015

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Discussion paper / Institute for Applied Economic

Research.- Brasília : Rio de Janeiro : Ipea, 1990-

ISSN 1415-4765

1. Brazil. 2. Economic Aspects. 3. Social Aspects.  
I. Institute for Applied Economic Research.

CDD 330.908

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JEL: R10, R12, R14, N90.

# SUMMARY

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ABSTRACT

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## ABSTRACT

This study introduces a new measure of urban centrality. It identifies distinct urban structures from different spatial patterns of jobs and resident population. The proposed urban centrality index constitutes an extension of the spatial separation index (MIDELFART-KNARVIK *et al.*, 2000). It is suggested that urban structure should be more accurately analyzed by considering a centrality scale (varying from extreme monocentricity to extreme polycentricity) rather than a binary variable (monocentric or polycentric). The proposed index controls for differences in size and shape of the geographic areas for which data is available, and can be calculated using different variables, such as employment and population densities and trip generation rates. The properties of the index are illustrated in simulated artificial data sets. Simulation results for hypothesized urban forms are compared to other similar measures proposed by previous literature. The index is then applied to the urban structure of four different metropolitan areas: Pittsburgh and Los Angeles in the United States; São Paulo, Brazil; and Paris, France. The index is compared to other traditional spatial agglomeration measures, such as global and local Moran's I, and density gradient estimations.





## 1 INTRODUCTION<sup>1</sup>

Each city in each moment of its history has its own spatial pattern. Would it be possible to summarize one of its most salient features, namely its degree of centrality, in one single index number? This is the challenge of the present paper.

A long list of studies relates urban centrality to other very important urban issues: efficiency of the transport system and commuting patterns (LEVTN SON, KUMAR 1997; GIULIANO, NARAYAN, 2003; BERTA UD, 2004; SCHWANEN *et al.* 2004; AGUILERA, 2005), pollution (BERTAUD *et al.*, 2009), energy consumption and urban structure in general (SHIM *et al.*, 2006). A proper measure of urban centrality is essential to make empirical claims about such matters. However, when reviewing these studies it is clear that they do not use similar ways of quantifying urban spatial structure.

The problem seems to be that this concept does not have a widely accepted definition and measure. There is still room for improvement, particularly in measuring the degree of centrality of urban agglomerations. Several studies within a more traditional strand of research present methods for estimating monocentric and polycentric density functions, as well as methods for identifying potential subcenters. Few studies, though, present a summary measure for quantifying the centrality degree of urban agglomerations. Such a measure could be important for the comparison of cities, linking their urban form to their performance regarding the urban issues mentioned above.

This seeks to fill this gap, introducing a new measure of urban centrality. It is an extension of the spatial separation index (MIDELFART-KNARVIK *et al.*, 2000) originally proposed to measure spatial distribution of industries in the European Union. The proposed *Urban Centrality Index* (UCI) identifies distinct urban structures from different spatial patterns of employment activities, regarding the degree of

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1. The authors would like to thank Anne Aguilera, Olivier Bonin and the team from LVTM (Laboratoire Ville Mobilité Transport), who calculated the statistics and made the maps for Paris.

monocentricity or polycentricity that an urban area can assume. It is suggested that urban structure should be more accurately analyzed by considering a centrality scale (varying from extreme monocentricity to extreme polycentricity) rather than a binary variable (monocentric or polycentric). The proposed index can be applied to urban areas of different shapes and sizes, making it feasible to compare them.

The paper presents a review of the related literature and defines a concept of urban centrality in section 2. In section 3, the Urban Centrality Index (UCI) is presented, and its properties are illustrated in artificial data sets. Results for simulated urban forms are compared to other similar measures proposed by previous literature. The index is then applied, in section 4, to the urban structure of four different metropolitan areas: Pittsburgh and Los Angeles in the United States; São Paulo, Brazil; and Paris, France. The results are also compared to other traditional spatial agglomeration measures, such as global and local Moran's I and density gradient estimations. Section 5 follows with the concluding remarks.

## 2 THEORETICAL BACKGROUND: MONO AND POLYCENTRICITY

Within the framework of urban studies, the issue of urban centrality has been addressed mainly through the analysis of the spatial patterns of jobs and population location. These patterns have been traditionally captured by density functions (ANAS, ARNOTT, SMALL, 1998). These functions collapse the available information into only two variables: the density and the distance of different values of densities from the central business district (CBD).

Clark (1951) was one of the first authors to introduce density functions to urban populations, though without presenting a structured model. He estimated a negative exponential function such as:

$$D(k) = D_0 e^{-D_1 k} \tag{1}$$

Where  $D(K)$  is the density of the unit of analysis located  $k$  units of distance away from the CBD, is the density at the CBD and the density gradient.<sup>2</sup> In terms of empirical estimates of density functions with the goal of identifying distinct urban forms, the econometric models used are not necessarily linked to theoretical models. When the estimation includes only the distance to the CBD, the density function can be derived from the monocentric city model of Alonso (1964), if certain conditions are met (BAUMONT *et al.*, 2004).

Further on, the spatial pattern of a polycentric city can be captured through the inclusion of distances to subcenters in the estimated density functions. Estimating these functions is a two-step procedure. First, it is necessary to identify potential subcenters. Once identified, these are included in the estimated equation. The ones with significant coefficients are then considered as subcenters. This procedure stands on the understanding that a subcenter has influence on the global organization of employment or population.

Another feature of a subcenter is that the amount of employment or population in the unit of analysis is higher than its neighbors. To determine how high is high enough, Giuliano and Small (1991) establish a cutoff point of density. According to the authors, all spatial units that comprise a subcenter should have in total at least 10,000 employees per acre. Each spatial unit should have a minimum density of 10 employees per acre. Anas, Arnott and Small (1998) warn that the cutoff points may be arbitrary. MacMillen (2001) also adds that this method relies too much on local knowledge.

Another way of identifying subcenters is by using spatial statistics. Baumont *et al.* (2004), for instance, applied exploratory spatial data analysis to deal with the spatial autocorrelation and spatial heterogeneity of the data. They use the local Moran statistic to detect clusters of employment and employment density. Paez *et al.* (2001) uses the local version of the G statistic applied to property values. Griffith and Wong (2007) also uses the local G statistic, but applied it to employment densities.

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2. This functional form has been modified by many other studies trying to reach a better match to actual spatial distribution of densities. Anderson (1982) uses cubic splines and McMillen (2001) uses a flexible Fourier form, while Griffith and Wong (2007) improve the function using Minkowskian distance, instead of Euclidean distance, and use a negative power function.

Griffith and Wong (2007) argue that the estimation of density functions should not ignore spatial dependence of the densities. Real cities have neighborhoods and a much richer spatial pattern, information which could not be simplified with the single variable distance to the CBD. Baumont *et al.* (2004) had already detected spatial autocorrelation in the error terms of the monocentric and polycentric estimation of employment density functions, proposing that they should be estimated using spatial econometric methods.

This strand of research, though, does not aim to find a summary measure for quantifying the degree of centrality of urban agglomerations. In spite of that, the issue of urban polycentricity has been gathering more attention within other methodological approaches (RILEY, DRAVITZKI; 2004). These studies usually break down the concept of urban form into different dimensions, which can be measured by different statistics.

## 2.1 DIMENSIONS AND QUANTITATIVE MEASURES OF URBAN CENTRALITY

The studies reviewed in this section separate urban morphology into different dimensions according to the key for understanding the specific phenomena they analyze (ANAS *et al.*, 1998; GALSTER *et al.* 2001; TSAI 2005; LEE, 2006). For the purpose of this article, we are going to focus exclusively on the dimensions that relate to the proximity level of employment activities across space and its unequal distribution, which are most related to monocentric and polycentric urban patterns, as we will argue. The morphological dimensions are summarized in Table 1.

TABLE 1  
Morphological Dimensions related to urban centrality

Dimension	Authors	Definitions	Measures
Unequal distribution/ Concentration	Tsai (2005)	"The degree to which development is concentrated in a few parts of a metropolitan area, regardless of high-density sub-areas being clustered or sparsely scattered" (pg. 143)	Gini coefficient
	Lee (2006)	"How disproportionately jobs are clustered in a few locations" (pg. 11)	Gini coefficient and delta index
	Galster <i>et al.</i> (2001)	"The degree to which housing units or jobs are disproportionately located in a relatively few areas or spread evenly in the urban area." (pg. 700)	Delta/dissimilarity index (difference between employment share and the units area share)

(Continues)

(Continued)

Dimension	Authors	Definitions	Measures
Centrality	Galster <i>et al.</i> (2001)	"The degree to which observations of a given urban land use are located close to the CBD of a urban area." (pg. 701)	Average distance of a land use from CBD (weighted by the number of observations). Centralization Index
	Anas <i>et al.</i> (1998)	"At the city-wide level, activity may be relatively centralized or decentralized depending on how concentrated it is near a CBD" (pg. 1431)	Monocentric density gradient
	Lee (2006)	"Centrality is the extent to which employment is concentrated with reference to the CBD" (pg. 11)	Modified Wheaton Index. Area Based Centralization Index. Ratio of weighted distance from the CBD to urban radius.
Clustering	Tsai (2005)	"the degree to which high-density sub-areas are clustered or randomly distributed" (pg. 146)	Global Moran index
	Galster <i>et al.</i> (2001)	"The degree to which development within any one-mile-square is clustered within one of the four one-half-mile squares contained within (as opposed to spread evenly throughout)." (pg. 701)	"The average for all one-mile squares of the standard deviations of the density of a particular land use (e.g. housing units or employees) among the four squares of each one-mile grid with developable land, standardized by the average density across m-scale grids" (pg 701)
	Anas <i>et al.</i> (1998)	"At a more local level, activities may be clustered in a polycentric pattern or dispersed in a more regular pattern" (pg. 1431)	Three different approaches: point pattern analysis; fractals; subcenters identification (polycentric density function, minimum density)

Source: Tsai (2005), Lee (2006), Galster *et al.* (2001), Anas, Arnott e Small (1998).

The morphological dimension of unequal distribution, also named as concentration dimension, "[...] is defined as the degree to which development is concentrated in a few parts of a metropolitan area, regardless of high-density sub-areas being clustered or sparsely scattered" (TSAI, 2005). Tsai (2005) has chosen the Gini coefficient to quantitatively characterize the degree of inequality in the distribution of population or employment along the spatial units in a metropolitan area. Galster *et al.* (2001) apply the delta index<sup>3</sup> to residential spatial distribution in order to measure urban concentration.

Both measures, however, are non-spatial measures and therefore do not provide information about how densities are distributed along space (TSAI, 2005; RILEY; DRAVITZKI, 2004). Thus, the construction of an urban centrality index characterized solely by the Gini coefficient would result in a poor indicator.

3. It compares the share of the phenomenon in each spatial unit to the share of the area of the spatial unit with respect to the total of the entire urbanized area.

Anas, Arnott, and Small (1998), Galster *et al.* (2001) and Lee (2006) also suggest two different morphology dimensions to address urban structure that help us better understand monocentricity and polycentricity patterns: Centrality (Centralized vs. Decentralized) and Clustering (Clustered vs. Dispersed). According to the authors, the Centrality dimension is the extent to which employment is located close to the Central Business District (CBD). As mentioned in the previous section, centralization has traditionally been measured by estimating monocentric density gradients. Galster *et al.* (2001) and Lee (2006) use the same intuitive procedure to capture centrality on an empirical basis. Both try to measure how fast metropolitan employment accumulates along the way from the CBD to the urban edge.

The Clustering dimension, on the other hand, is the degree to which employment activities are clustered in a few areas or more dispersed in a more regular pattern. While Tsai (2005) uses the global Moran index to gauge this dimension, Anas, Arnott and Small (1998) emphasize that empirical economists have traditionally used three different approaches to measure it: point pattern analysis, fractals analysis and subcenters identification.

As can be seen from table 1, the conceptual distinction between the morphological dimensions of centrality and clustering proposed by the authors is vague. To some extent, one possible reason is that the proposed morphological dimensions are not fully independent. For instance, in extreme cases, an urban agglomeration with a very high centrality level (i.e. with a large number of employment activities located close to the CBD) would necessarily present a high cluster level depicted in monocentric pattern. On the other hand, an urban agglomeration presenting a very low cluster level (with jobs spread evenly through space would necessarily present a low centrality level depicted in an acentric pattern. In that sense, Anas, Arnott and Small (1998) and Lee (2006, p.11) state that “*polycentric urban structure is a combined outcome of metro-wide decentralization and local level clustering*”.

To deal with this conceptual overlap, we decided to treat these two dimensions as being one single dimension, named as Proximity. It also addresses how clustered or dispersed jobs are in space. In other words, the Proximity factor is the degree to which employment activities are close to or far apart from each other. We decided to use this term, Proximity, because it is a natural interpretation of the spatial separation

index originally proposed by Midelfart-Knarvik *et al.* (2000), upon which our Urban Centrality Index is based (to be presented at the next section of this article).

On important lesson that can be derived from the reviewed authors is that centrality should be more accurately analyzed by considering a continuum scale. Low levels represent more monocentric urban structures and high levels represent more polycentric ones. The need to understand urban centrality in this flexible way can be illustrated by Bertaud when he states that “*No city is ever 100% monocentric, and it is seldom 100% polycentric (i.e., with no discernable “downtown”). Some cities are dominantly monocentric, others are dominantly polycentric and many are in between.*” (BERTAUD, 2004, p.9).

Thus, we understand that urban centrality is better characterized by the spatial arrangement of employment activities. We argue that urban centrality captures the degree of monocentricity or polycentricity an urban structure can assume, and that these spatial patterns are appropriately described by two factors: its unequal distribution and its proximity level.

### 3 URBAN CENTRALITY INDEX

The starting point of our proposed index is the widely known location coefficient (*LC*) introduced by Florence (1948, p. 34), and here introduced to measure the unequal distribution factor of employment activities distributions within an urban area.

$$LC = \frac{1}{2} \sum_i^n \left| s_i - \frac{1}{n} \right| \quad (2)$$

Where:

$n$  = number of areas;

$S_i = E_i / E$ , i.e. the share of employment in area  $i$  ( $E_i$ ) in the total employment ( $E$ ) of the city;

$E$  = total number of jobs in the city.



The  $LC$  varies in a range between 0 and  $1-1/n$ . If  $LC$  equals 0, then economic activity is evenly distributed, while values close to  $(1-1/n)$  indicate that employment is concentrated in a few areas. It is worth pointing out that this coefficient captures only the non-spatial inequality of the distribution. In other words,  $LC$  takes no account of distance or spatial patterns; therefore, cities with similar values of  $LC$  may have completely different spatial profiles (as illustrated in Figure number 1 in the next subsection). The location coefficient has the same mathematical intuition behind the Gini coefficient. We chose to use the  $LC$  because it is easier to calculate and it is widely accepted in regional science studies (HOOVER; GIARRATANI, 1999).

The second term of our Urban Centrality Index is based on the Spatial Separation Index originally proposed by Midelfart-Knarvik *et al.* (2000, 2002) to evaluate changes in the spatial distribution of economic activity across European regions. The Spatial Separation Index, aka “Venables index”, is calculated as follows (SOUSA, 2002):

$$V = S^t \times D \times S \quad (3)$$

Where:

$V$  = Spatial Separation Index;

$S$  = column vector of ;

$D$  = distance matrix. is the distance between the centroids of areas  $i$  and  $j$ . In the simplest version, the main diagonal of  $D$  is equal to 0.<sup>4</sup>

When all employment activity is concentrated in just one spatial unit, the minimum value of the Venables index is reached, i.e. zero (no matter where this spatial unit is). However, the index has no maximum value and therefore, it cannot be compared across different spatial settings. In order to overcome this limitation, it is

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4. As suggested by Crafts and Mulatu (2005), we decided to use the “self-distance”, approximated by  $d_{ii} = (\text{area}/\pi)^{1/2}$ , in the distance matrix, since this would control for the variation in the size of the polygons.

necessary to calculate the maximum attainable value of the Venables index (this issue will be explored ahead).

The created Proximity Index ( $P$ ) solves the normalization issue with the Venables index and changes its interpretation to suit our needs:

$$P = 1 - \frac{V}{V_{\max}} \quad (4)$$

$V_{\max}$  = Maximum attainable value of the Spatial Separation Index

Obviously, the interpretation of  $P$  is the opposite of the Venables index, with the difference that its theoretical range is  $[0, 1]$ . Values of  $P$  closer to 1 mean that employment is clustered in one single center. (This economic center does not necessarily match the geometric center). If  $P=0$ , the employment is as spatially separated as possible. That means that employment activities would be distributed in a way that maximizes the distances between them.<sup>5</sup>

Our proposed urban centrality index (UCI) is just the product of the location coefficient and the proximity index:

$$UCI = LC \times P \quad (5)$$

The application of UCI to hypothesized spatial patterns and actual data set will show the advantages of our proposed measure in comparison to other traditional measures of urban centrality.

The estimation of  $P$  is not trivial because there is no closed-form solution. In the very simple square grids presented in the next section,  $P$  is obtained when each corner has one-fourth of total employment. In a perfect circle shape, the maximum value of  $V$  would be obtained when all employment is evenly distributed along the external edge.

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5. If the employment activities are evenly distributed across space, however, the  $P$  index will not be equal to zero. The next section will explore this issue.

In real cities, however, can only be reached through a constrained optimization algorithm that depends on the distance matrix ( $D$ ). We still have not coded this, but an approximation of can be estimated by analogy to the circle shape. Thus, we have chosen to consider the ‘opposite of maximum proximity’ as a homogeneous distribution of values along the edge of the map. Although this solution is not the global maximum of the Venables index, it was considered a satisfactory solution for two reasons: a) intuitively, it is the opposite of a completely monocentric city with all employment in the center; and b) it is easy to calculate and does not require a specific algorithm. This normalization procedure makes it possible compare urban areas of different shapes and sizes.

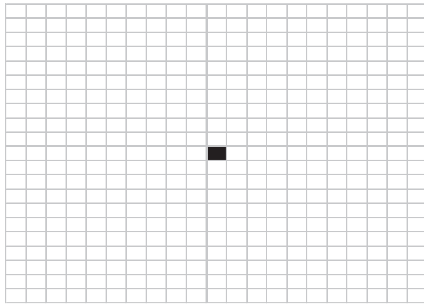
It is true that UCI does not follow all the criteria set up by Combes and Overman (2004). It is subject to MAUP (HAINING, 2003), it has no direct connection with theory, and there is no statistical test of hypothesis (yet). However, these shortcomings are shared among other measures of urban form, such as the rank-order tests and measures of employment distribution.

### 3.1 EXPERIMENTS USING ARTIFICIAL DATA SETS OF HYPOTHESIZED METROPOLITAN FORMS

A few experiments using artificial data sets of hypothesized metropolitan forms are presented in this session. The aim with this simulation is to verify if the Urban Centrality Index (UCI) here proposed distinguishes between theoretical urban structures along the centrality scale. The simulation results are compared to other similar measures proposed by previous literature. All calculations were done using R (R DEVELOPMENT CORE TEAM, 2011) and spdep (BIVAND, 2011). In the theoretical examples that follow, the chart tones represent the number of jobs in each cell on a 21 x 21 cell territory. The total number of jobs is fixed at 441 jobs for every hypothesized metropolitan form, thus leading to a constant urban density at the metropolitan scale.

The first urban structure (Figure 1) represents the highest possible monocentricity level, consistent with our definition of centrality and the two dimensions describing it. In this situation, there is simultaneously the highest possible level of (a) inequality in the distribution of employment among spatial units and of (b) proximity in the localization of jobs, since its spatial pattern is fully concentrated at a single center. The UCI assumes its maximum value equal to 1 in this structure.

FIGURE 1  
Highest possible monocentricity level

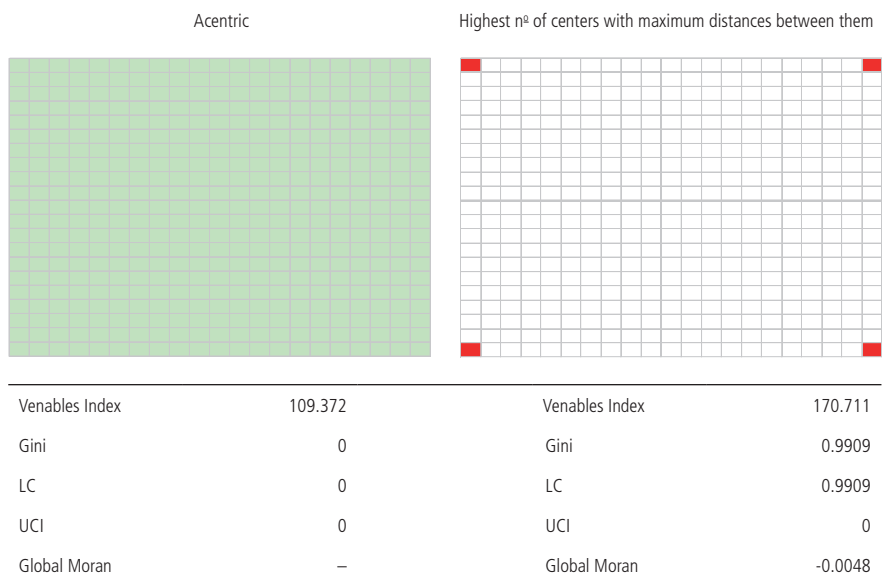


Venables Index	0
Gini	0.9977
LC	0.9977
UCI	0.9977
Global Moran	-0.0023

Source: authors' elaboration.

On the other hand, the definition of the highest level of polycentricity is not so straightforward. Which is the most polycentric city? One possible answer would be the urban structure where the number of centers would be as high as possible. In a city whose territory is divided into discrete polygons, it would occur with the number of centers being equal to the number of polygons. The result would be an acentric urban structure with a perfectly even spatial distribution of jobs (Figure 2-A). But this structure could also be classified as the least monocentric city, because there are no centers at all. The second possibility is the urban structure with the highest number of centers that maximize the distances between them. In the case of a regular squared grid shape, this is obtained when each corner has one-fourth of total employment (Figure 2-B). In both situations the UCI assumes its lowest value (equal to 0).

**FIGURE 2**  
**Highest possible polycentricity levels**

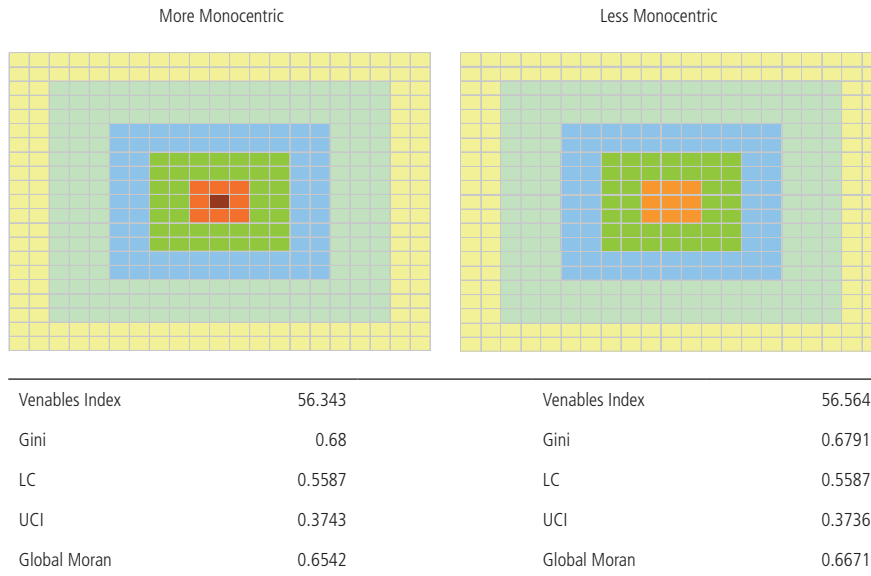


Source: authors' elaboration.

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

The UCI calculated for all other urban structure patterns will vary between these two extreme monocentric and polycentric patterns. Thus, one advantage of the UCI is its ability to distinguish among varying degrees of urban mono or polycentricity in situations where other indices would not. In Figure 3, for instance, there are two notably monocentric urban areas. Analyzing in more detail, however, it is possible to see that the first figure exhibits a steeper monocentric slope, while the second hypothetical urban form demonstrates a more flattened slope. In these examples, we changed the employment distribution for the nine central cells, causing small changes in the calculated values of the Gini and Locational coefficients. These changes decreased the relative distance between jobs, affecting the proximity index level of the distribution of jobs more severely.

FIGURE 3  
Varying degrees of monocentricity

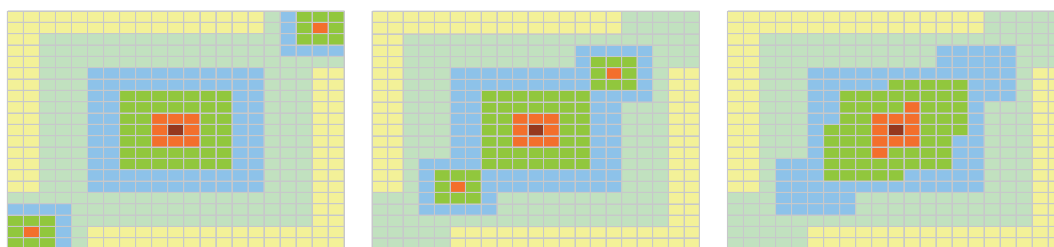


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In another example, Figure 4 shows two urban areas easily recognized as polycentric and a third one easily recognized as monocentric. One of the polycentric structures has closer subcenters to the CBD than the other. The closer the subcenters are, the more the urban structure approximates a monocentric pattern. In the limit that the distances approach zero, all subcenters would merge into one big CBD as illustrated in the third figure. Despite the fact that they share the same distribution of jobs among their spatial units, they are clearly different concerning their spatial distribution. Thus, although the Gini and location coefficients remain the same, the UCI and Moran index do not. The UCI index rises monotonically as the subcenters get closer to the CBD. On the other hand, the Moran Index does not vary in a comprehensible way. Initially, it falls when subcenters get closer to the CBD. Later, however, it increases when subcenters merge into one big CBD.

**FIGURE 4**  
**Varying degrees of polycentricity**



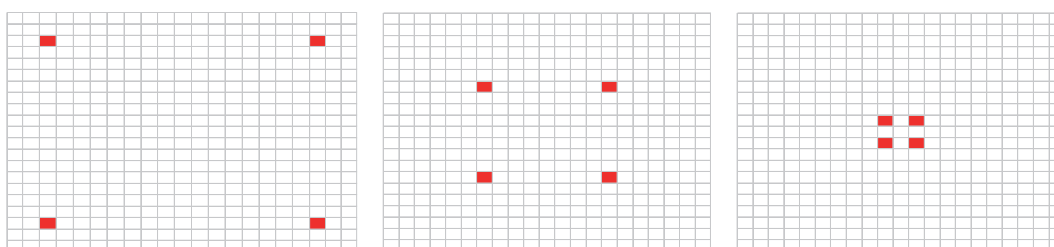
Venables Index	84.234	Venables Index	70.287	Venables Index	65.260
Gini	0.6219	Gini	0.6219	Gini	0.6219
LC	0.5039	LC	0.5039	LC	0.5039
UCI	0.2552	UCI	0.2964	UCI	0.3112
Global Moran	0.7025	Global Moran	0.6851	Global Moran	0.7309

Source: authors' elaboration.

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

One of the positive properties of the UCI index is its ability to distinguish monocentric and polycentric patterns in situations where other indices would not. Figure 5 illustrates one of these situations. The presented spatial pattern consists of employment highly concentrated in four cells. The Venables index varies with dispersion of the centers. The location coefficient and the global Moran index remain the same.

**FIGURE 5**  
**Polycentricity converging to monocentricity**



Venables Index	136.569	Venables Index	68.284	Venables Index	17.071
Gini	0.9909	Gini	0.9909	Gini	0.9909
LC	0.9909	LC	0.9909	LC	0.9909
UCI	0.1982	UCI	0.5946	UCI	0.8918
Moran	-0.0092	Moran	-0.0092	Moran	-0.0092

Source: authors' elaboration.

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

## 4 CASE STUDIES: PITTSBURGH, LOS ANGELES, SÃO PAULO AND PARIS

In this section, we apply the Urban Centrality Index to different urban areas, comparing their degree of centrality. We also present other measures of urban centrality, such as local Moran's I and estimated density gradients, which are less simple to compare among cities, but can add information on their spatial pattern of jobs.

We have chosen cities in the U.S. (Pittsburgh and Los Angeles), France (Paris) and Brazil (São Paulo), that represent four different urbanization processes. It is well known that Paris is the oldest of the four cities we examine and that it did not have a car-oriented development. Furthermore, São Paulo is located in a less developed country, with very poor mass transportation and urban infrastructure. We also chose two cities in the U.S. to measure urban centrality. On the one hand, Los Angeles is widely recognized as a leading example of a decentralized, sprawled city (GIULIANO; SMALL, 1991). On the other hand, Pittsburgh is a former industrial city with one clear CBD (QUINLAN, 2006), and presumably features a more monocentric city.

We will use employment density data for metropolitan areas. For the U.S. cities, we used data set from Census tracts for the year 2000, obtained at the NHGIS website.<sup>6</sup> For São Paulo Metropolitan Area, we used data for the year 1997 from the Origin-Destination survey (Pesquisa Origem-Destino), produced by the firm that runs the subway system of São Paulo (Companhia do Metropolitano de São Paulo – Metrô). For Paris, we used data from the population census of 1999 available at the website of INSEE (France's National Institute of Statistics and Economic Studies).<sup>7</sup>

Table 2 shows some descriptive statistics of the metropolitan areas under study. Los Angeles, Pittsburgh and Paris are quite similar in terms of total area. In total population and employment, however, Pittsburgh is smaller. That is also why it has the

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6. National Historical Geographic Information System. <<http://www.nhgis.org/>>.

7. We thank Anne Aguilera, Olivier Bonin and the team from LVTM (Laboratoire Ville Mobilité Transport), who calculated the statistics and made the maps for Paris.



smallest average employment density. São Paulo has the smallest area and the highest total employment, leading to the highest average employment density, which is almost twice the density of Los Angeles and Paris.

Map tracts vary in number and average area. For São Paulo's map, the spatial information is not very spatially refined; it has a small number of polygons with a high average area. The information per polygon shows us how Paris and São Paulo have much higher densities and total employment, indicating a greater concentration of jobs. The comparison between Los Angeles and Pittsburgh polygons show similar distribution of total number of jobs, but a considerably higher job density for Los Angeles, a surprising fact, since Los Angeles is recognized as being a sprawled area. Another surprising fact to highlight is that Los Angeles has an average employment density per polygon (1,746 per km<sup>2</sup>) greater than Paris (819 per km<sup>2</sup>).

TABLE 2  
Metropolitan Areas Features

Features	Metropolitan Area				
	Los Angeles*	Pittsburgh*	São Paulo**	Paris ***	
	2000	2000	1997	1999	
<b>Total area (km<sup>2</sup>)</b>	<b>12,317</b>	<b>13,839</b>	<b>7,954</b>	<b>12,069</b>	
<b>Total employment</b>	<b>5,169,266</b>	<b>1,086,842</b>	<b>6,959,394</b>	<b>5,038,730</b>	
Population	12,365,627	2,358,695	16,792,406	10,947,510	
Average Employment Density	420	79	875	417	
Number of polygons	2,629	721	389	1,300	
Average area of polygons (km <sup>2</sup> )	4,68	19,19	20,45	9,28	
Number of jobs per polygon	maximum	5,649	4,711	108,059	170,748
	average	1,966	1,507	17,890	3,876
	Std, Dev,	846	819,92	16,371	12,613
Employment Density per polygon (km <sup>2</sup> )	maximum	16,466	5,544	131,922	62,586
	average	1,746	605	5,854	819
	Std, Dev,	1,462	689	11,508	3,582

Source: \* NHGIS/population census of 2000.

\*\* Metrô/Origin-Destination Survey of 1997.

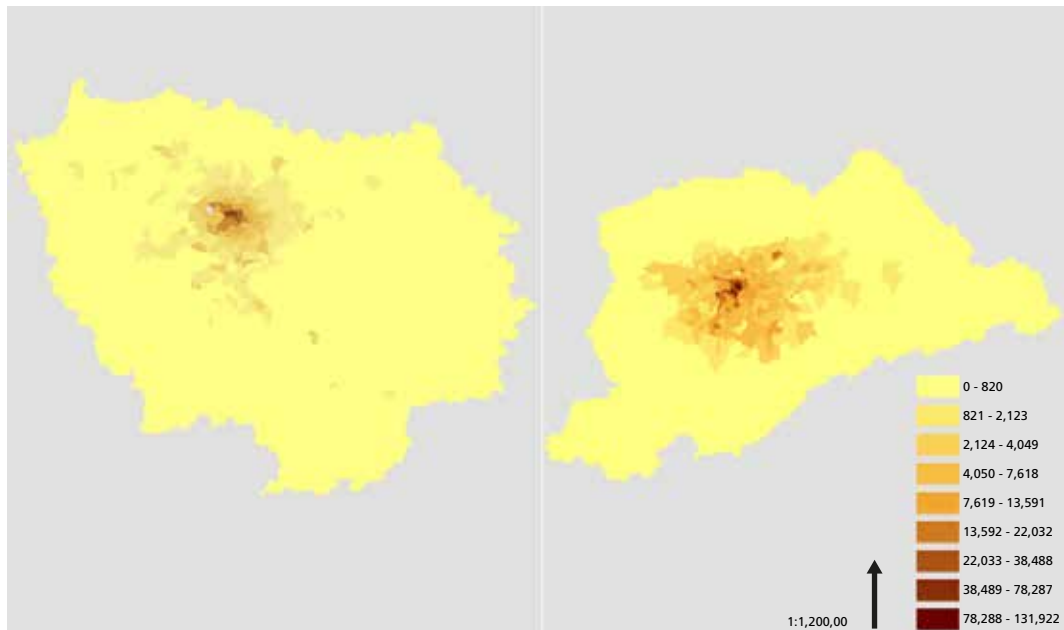
\*\*\* INSEE/population census of 1999.

These first descriptive statistics show us a complex and wide range of spatial patterns of employment distributions. We prepared employment density maps for the

four cities, displayed in figures 6 and 7. To do so, we used the natural breaks (Jenks) method for generating nine classes of employment densities. The color classes were maintained fixed for all cities to visually compare densities across them. We chose a high number of classes to capture the great variability of densities.

We can roughly identify that São Paulo and Paris present similar patterns, where just a few polygons have very high densities. On the other hand, the American cities have few, if any, polygons in the upper-density classes. The distribution of jobs in Los Angeles looks visually spread out through the territory. Even though Pittsburgh presents relatively low employment densities, they do look much more spatially concentrated.

FIGURE 6  
Employment density maps – Paris (left) and São Paulo (right)

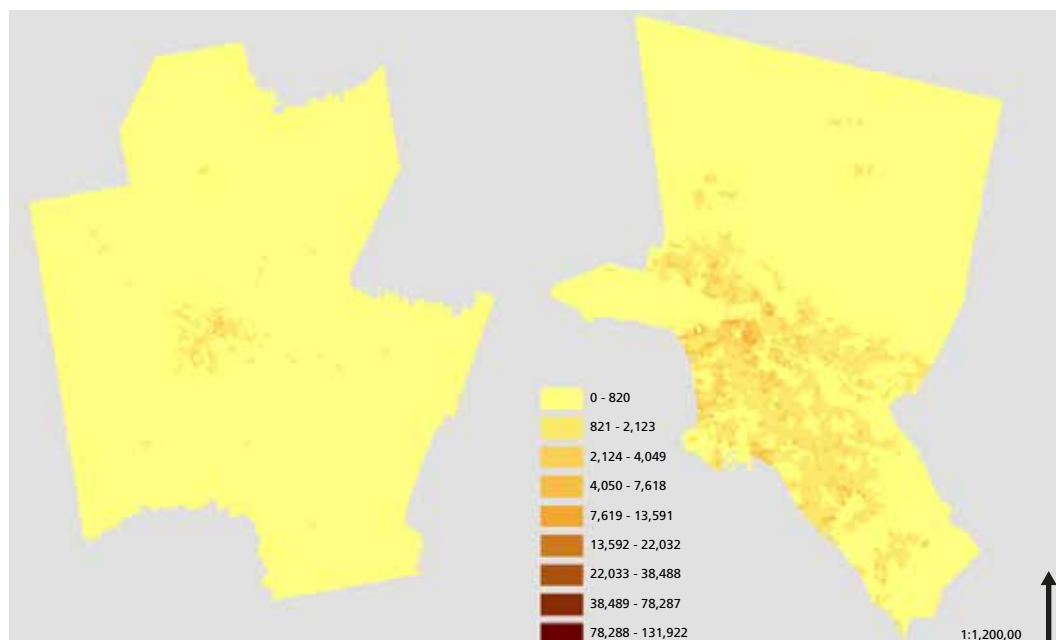


Source: Paris – Census, 1999 (INSEE), LVTM (Laboratoire Ville Mobilité Transport). São Paulo – Origin-Destination survey, 1997 (Metrô/SP).

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

The figure has been reproduced in accordance with the original file provided by the authors, whose characteristics did not permit improvement for printing purposes.

FIGURE 7  
**Employment density maps – Pittsburgh (left) and Los Angeles (right)**



Source: U. S. Census 2000 (NHGIS).

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

The figure has been reproduced in accordance with the original file provided by the authors, whose characteristics did not permit improvement for printing purposes.

Another way of visualizing the patterns of spatial distribution of jobs in cities is to use employment density charts, presented in figure 8. Again, it is clear how São Paulo and Paris have similar patterns that are very different from those of the North American cities. Densities near the CBD are extremely high, and drop sharply up to 10 to 20 km away from the CBD. Comparing Los Angeles and Pittsburgh, we can see visually how much higher L.A. densities are. At the same time, we can see that in Pittsburgh, all high density tracts are located near the CBD, while in Los Angeles there are high density areas far away from the CBD.

To investigate further these spatial patterns, we estimated the density gradient for each city.<sup>8</sup> The estimated densities are also showed in figure 8, and the gradients

8. Since the estimation of density functions was not the purpose of this paper, we estimated a simple monocentric employment density function. We did not try to find the best functional form or control for spatial dependence in the data.

in table 3. The regressions (exponential functions) fit better to São Paulo, Paris and Pittsburgh cases. But the densities at the CBD are in all cases very underestimated. Focusing on the gradient, São Paulo and Paris present the two highest density gradients. This is as expected, since jobs are much more concentrated in these two cities.

**TABLE 3  
Density Gradients Estimation**

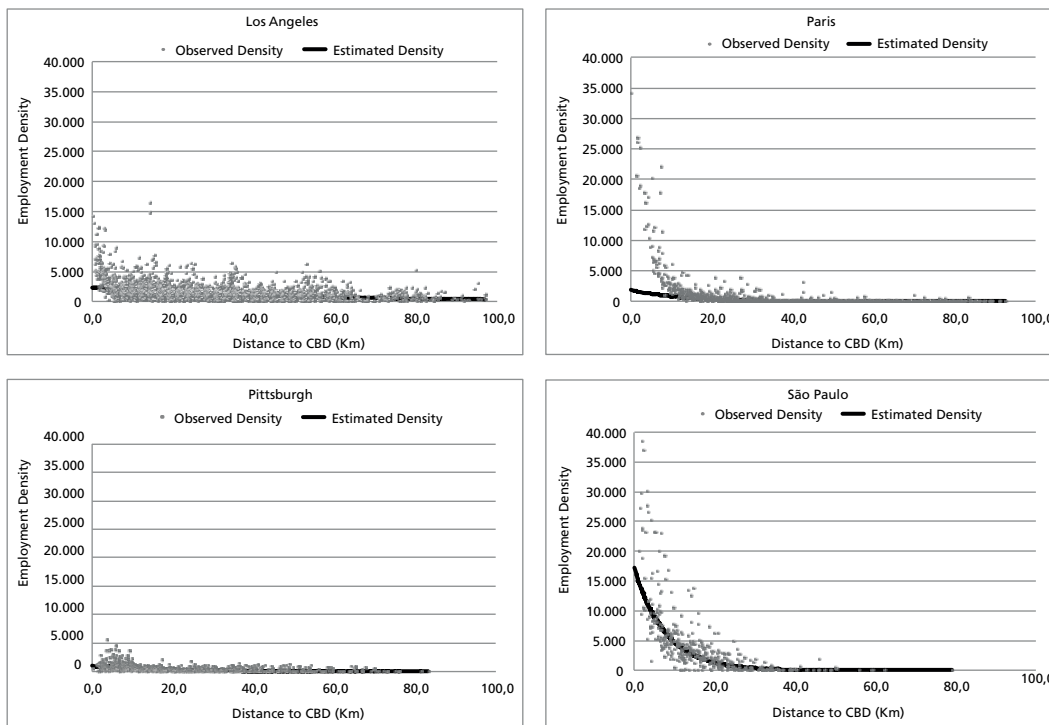
Metro Area	Density		Density at CBD	R <sup>2</sup>
	gradient			
	coeff	t-stat		
Los Angeles	-0.02	-16.46	2,314	0.15
Pittsburgh	-0.05	-19.44	979	0.4
Paris	-0.08	-38.67	1,840	0.59
São Paulo	-0.13	-18.18	17,265	0.62

Elaboração dos autores com colaboração do LVMT.

Further on, we applied local Moran's I statistic (ANSELIN, 1995; KELEIJAN, PRUCHA, 2001) to the employment density data to take a closer look at the analyzed urban areas.<sup>9</sup> The Local Moran's I cluster maps are shown in figures 9 and 10.

9. We use a first-order queen's contiguity matrix at 5% of significance level in GeoDa software.

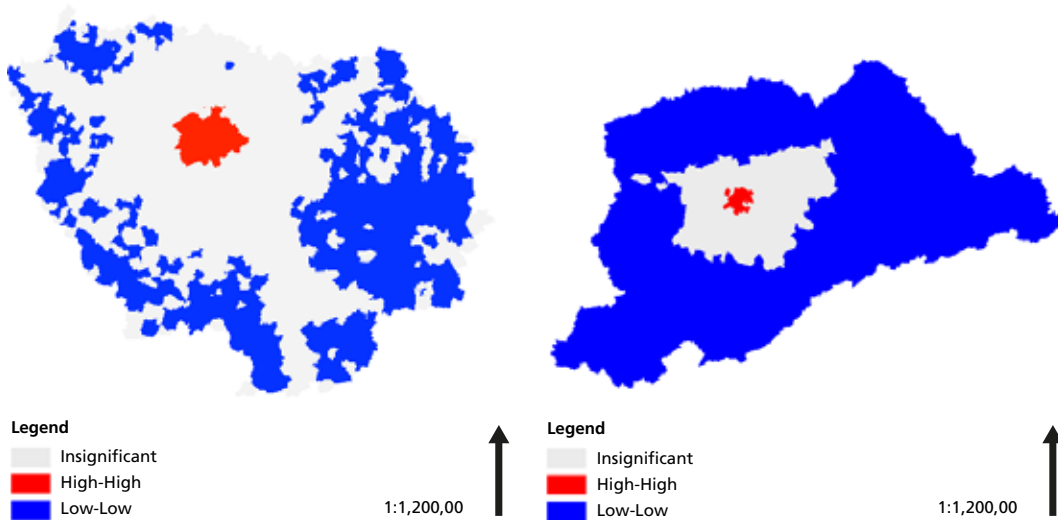
FIGURE 8  
Employment density charts with estimated density functions



Source: Paris – Census, 1999 (INSEE), LVTM (Laboratoire Ville Mobilité Transport). São Paulo – Origin-Destination survey, 1997 (Metrô/SP). Pittsburgh and Los Angeles – U.S. Census, 2000 (NHGIS).

Obs.: The figure has been reproduced in accordance with the original file provided by the authors, whose characteristics did not permit improvement for printing purposes.

FIGURE 9  
Local Moran's I clusters map for employment densities – Paris (left) and São Paulo (right)

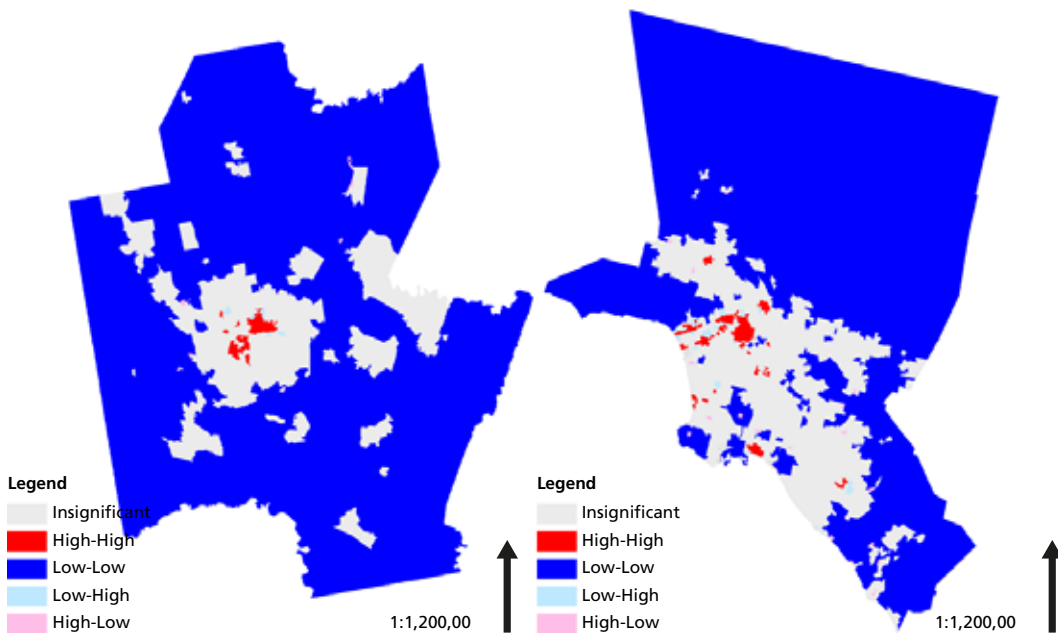


Source: Paris – Census, 1999 (INSEE), LVTM (Laboratoire Ville Mobilité Transport). São Paulo – Origin-Destination survey, 1997 (Metrô/SP).

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

The figure has been reproduced in accordance with the original file provided by the authors, whose characteristics did not permit improvement for printing purposes.

FIGURE 10  
Local Moran's I clusters map for employment densities – Pittsburgh (left) and Los Angeles (right)



Source: U.S. Census 2000 (NHGIS).

Obs.: To see it in color, please access: <[http://www.ipea.gov.br/005/00502001.jsp?ttCD\\_CHAVE=533](http://www.ipea.gov.br/005/00502001.jsp?ttCD_CHAVE=533)>.

The figure has been reproduced in accordance with the original file provided by the authors, whose characteristics did not permit improvement for printing purposes.

Again, São Paulo and Paris have a more similar spatial pattern, presenting a large contiguous central area with high employment concentration. In Pittsburgh, we see the central area with two main clusters of high employment densities and a few more employment clusters nearby. In L.A. there is an even larger number of clusters, although they are more fragmented and dispersed through the metropolitan area. The bigger cluster (in terms of area) corresponds to the CBD region, but clearly there are many other clusters which do not seem to orbit around it. These patterns of clusters support a description of São Paulo and Paris as more “monocentric” urban structures, and Pittsburgh and L.A. as more “polycentric” ones.

In section 3, we proposed that Urban Centrality Index would be composed by the proximity index and the location coefficient. Table 4 displays a comparison of these indices with Moran’s I statistic and UCI. All indices are consistent with the same ranking of cities, from more monocentric to more policentric: Paris, São Paulo, Pittsburgh and Los Angeles.

TABLE 4  
Concentration and Spatial Pattern Indexes

	Location Coefficient	Proximity Index	Global Moran “i”	Urban Centrality Index
Paris	0.708	0.725	0.76	0.514
São Paulo	0.319	0.628	0.67	0.201
Pittsburgh	0.3	0.48	0.56	0.11
Los Angeles	0.171	0.261	0.55	0.045

Source: \* NHGIS/Censo Populacional de 2000.

\*\* Metrô/Pesquisa Origem-Destino de 1997.

\*\*\* INSEE/Censo Populacional de 1999.

Elaboração dos autores com colaboração do LVMT.

## 5 FINAL REMARKS

This paper contributes to the literature on the measurement of the degree of centrality in urban agglomerations by proposing a very simple index that overcomes many of the limitations of established attempts to quantify this phenomenon. The Urban Centrality Index (UCI) is closely related to the intuitive notion of monocentricity, and it measures changes in urban spatial structure that other indices could not capture. Furthermore, it is a normalized index that can be compared across different cities and through time,

despite different shapes and sizes. The application of the index to hypothesized urban forms and to four case studies has shown its ability to measure the centrality of extreme cases, as well as empirical data set from real urban forms.

The UCI makes clear the definition of the two extremes of the centrality scale: maximum monocentricity with all jobs concentrated in one single spatial unit, and minimal monocentricity with (a) density of jobs distributed evenly throughout the territory, or (b) the largest number of dense centers that maximizes the distance between them.

UCI is so simple that users can apply it directly using their software of choice. Nevertheless, we intend to include its estimation as part of IPEAGEO (IPEA, 2010), a free geostatistical software, and as a function in R (R DEVELOPMENT CORE TEAM, 2011). We also plan to calculate the maximum value of the Venables index using constrained optimization routines instead of our approximation. Finally, following Combes and Overman (2004), it will be necessary to develop a statistical hypothesis testing of UCI.

Obviously, the UCI is not an end in itself. We hope that it might help in studies focusing on the relationship between urban spatial structure and commuting, and even to the economic and environmental performance of cities.

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