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142

DISCUSSION PAPER

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Elcyon Caidado Rocha Lima



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THE INTERPRETATION OF COEFFICIENTS OF THE VECTOR AUTOREGRESSIVE MODEL¹

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SINOPSE

Em Johansen (2002) é sugerido um “desenho de experimento” (*design of experiment*), que pode ser implementado no modelo de auto-regressão vetorial, com o objetivo de se interpretar os coeficientes numa relação de co-integração identificada.

Neste artigo propõe-se um “desenho de experimento” alternativo que, ao contrário do de Johansen, não parte da dicotomia entre o curto e o longo prazos. O experimento permite interpretar os coeficientes em uma relação de co-integração identificada. Partimos da idéia de que os coeficientes, e determinadas operações com eles, são previsões condicionadas — em diversos horizontes — a certos valores das variáveis do modelo e dos choques exógenos nos erros das equações estruturais do VAR. A dinâmica do modelo pode ser utilizada para testar se esses valores podem ser gerados por choques exógenos nesses erros. Pode-se também construir [ver, a esse respeito, Doan, Litterman e Sims (1984)] um índice de plausibilidade desses choques exógenos.

A análise das previsões condicionais de curto e longo prazos pode ser tão útil quanto a inspeção dos sinais e significância dos coeficientes da matriz com as relações contemporâneas entre as variáveis em um VAR estrutural. Ela pode ser um complemento importante da análise das funções de resposta a impulsos.

ABSTRACT

Johansen (2002) suggests a counterfactual experiment that can be implemented in the vector autoregressive model to interpret the coefficients of an identified cointegrating relation.

This article proposes an alternative counterfactual experiment (“design of experiment”) that, contrary to the one suggested by Johansen, does not imply a dichotomy of short run and long run values. The experiment interprets the coefficients of an identified cointegrating relation. It is based on the idea that the coefficients, and some operations with them, are projections—at different horizons—conditional on paths of the variables of the model and on exogenous shocks in the error terms of the equations of a structural VAR. The model dynamics can be used to test if these values can be generated by exogenous shocks in these error terms. It is also feasible to construct, as was shown by Doan, Litterman and Sims (1984), a plausibility index for these exogenous shocks.

The analysis of the proposed conditional projections can be as useful as checking coefficients, of the matrix with the contemporaneous correlations among variables, for the correct sign and significance in a structural VAR. It can be an important complement to the impulse response function analysis.

1 INTRODUCTION

The use of counterfactuals to interpret regression coefficients is well known in statistics and econometrics [Haavelmo (1944), Rubin (1974) and Holland (1986)]. Johansen (2002) suggests a counterfactual experiment that can be implemented in the vector autoregressive model to interpret the coefficients of an identified cointegrating relation. He makes a distinction between what happens in the long run and in the short run. In his approach there is a dichotomy of current and long run values and the concept of elasticity is very different from the standard definition.

This article proposes an alternative counterfactual experiment (“design of experiment”) that, contrary to the one suggested by Johansen, does not imply a dichotomy of short run and long run values. The experiment interprets the coefficients of an identified cointegrating relation. It is based on the idea that the coefficients, and some operations with them, are projections—at different horizons—conditional on paths of the variables of the model and on exogenous shocks in the error terms of the equations of a structural VAR. The model dynamics can be used to test if these values can be generated by exogenous shocks in this error terms. It is also feasible to construct, as was shown by Doan, Litterman and Sims (1984), a plausibility index for these exogenous shocks.

The paper is organized as follows: in Section 2 we describe the model and the standard definition of elasticities; in Section 3 we present the proposed counterfactual experiment; in Section 4 we show that the long run elasticities do not violate the long run properties of the data; in Section 5 we conclude.

2 THE MODEL AND THE STANDARD DEFINITION OF ELASTICITY

2.1 THE STRUCTURAL VAR REPRESENTATION AND THE STANDARD DEFINITION OF ELASTICITY

The structural VAR model, in n dimensions, can be represented by:

$$\phi(L) y_t = \mu + \xi D_t + v_t, \quad t = 1, 2, \dots, T \quad (1)$$

$$v_t \sim N(0, \Omega)$$

v_t is i.i.d. and Ω is diagonal.

where y_t is a $n \times 1$ column vector; D_t is a column vector with deterministic variables; $\phi(L) = \phi_0 - \phi_1 L - \dots - \phi_p L^p$ (is a polynomial in the lag operator L); μ is the vector with the equation’s constant terms.

Consider equation i of the structural VAR:

$$y_{it} = \mu_i + \xi_i D_t - \sum_{s \neq i} \phi_{so}^i y_{st} + \sum_{s=1}^n \phi_s^i(L) y_{st} + v_{it} \quad (2)$$

where y_{st} is the observed value of variable s , at period t ; ϕ_{sk}^i is the coefficient of variable s , with lag k , in equation i ; and $\phi_s^i(L) = \sum_{k=1}^p \phi_{sk}^i L^k$.

Equation (2) can be estimated by different methods and, if the values of the variables are measured by their logarithms, the elasticities for different steps-ahead, of variable i with respect to variable s , can be computed. In order to compute the elasticities it is necessary to impose certain restrictions that are presented in the next section.

Let:

$\varepsilon(i,s,h)$ = variable i 's elasticity with respect to variable s when variable i has h quarters to respond.

$$\theta(1) = 1 \text{ and } \theta(n) = \sum_{i=2}^n \left\{ \sum_{k=1}^{\min\{i-1,p\}} [\phi_{ik}^i \cdot \theta(i-k)] \right\}; n = 2, 3, \dots \quad (3)$$

Then,

$$\varepsilon(i,s,0) = \phi_{so}^i$$

$$\varepsilon(i,s,h) = \sum_{n=1}^h \sum_{k=1}^n \sum_{m=0}^{\min\{k-1,p\}} \phi_{sm}^i \cdot \theta(k-m) \quad (4)$$

$$h = 1, 2, \dots \text{ and } s \neq i.$$

Suppose, for instance, that variable i measures energy consumption. After estimating the parameters of equation (2) we can apply formulas (3) and (4) to obtain the elasticity of energy consumption with respect to variable s , when a certain period of time, measured in quarters, is given for the response. That is, equations (3) and (4) allow for the calculation of the energy consumption response, h quarters ahead, to a persistent increase of 100% in variable s that is not followed by any increase in other explanatory variables. Monte Carlo simulations easily calculate the elasticity's confidence intervals, if the distribution of the estimators of the parameters of equation (1) is known.

The standard concept of elasticity is interpreted by a thought experiment that does not have an immediate meaning in the VAR model where there is no dichotomy of endogenous and exogenous variables. Lütkepohl (1994) points out that it is not correct to interpret as elasticities the coefficients of variables, measured by their logarithms, of a cointegrating relation.

3 THE IMPLEMENTATION OF AN EXPERIMENT IN A VAR MODEL TO INTERPRET SHORT AND LONG RUN ELASTICITIES

The observed forecast error, h steps ahead, of y using observations up to period t (η_{t+h}), is equal to the difference between the observed y_{t+h} and the expected y_{t+h} conditional on observations up to t ($E_t y_{t+h}$). This error is given by the following equation:

$$\eta_{t+h} = y_{t+h} - E_t y_{t+h} = \sum_{k=0}^{h-1} G_k v_{t+h-k} \quad (5)$$

where:

$$G_0 = \phi_0^{-1} \Omega^{1/2}; G_k = \sum_{j=1}^{\min\{k,p\}} \Omega^{-1/2} \phi_j G_{k-j}; k = 1, 2, \dots$$

Let $\eta_{j,t+h}$ be the observed forecast error, using the observed values of the variables until period t , of variable j for h steps-ahead.

Equation (5) gives the relation that has to be satisfied when the observed forecast errors are substituted by forecast errors conditional on future paths or values of some of the variables or shocks that belong, respectively, to y_{t+h} or v_{t+h} . Note that, for $h > 0$, restricting the value of a variable that belongs to y_{t+h} , since $E_t y_{t+h}$ is given at t , is equivalent to restricting η_{t+h} .

3.1 SHORT RUN ELASTICITIES

The coefficients of a regression are usually interpreted with the help of a thought experiment that does not have an immediate meaning in the VAR model where there is no dichotomy of endogenous and exogenous variables. Lütkepohl (1994) points out that it is not correct to interpret as elasticities the coefficients of variables, measured by their logarithms, of a cointegrating relation. Proposition 1, presented below, suggests an experiment that interprets a certain type of conditional projection as a short run elasticity. Let r be the number of cointegrating relations in the VAR.

Proposition 1

Let $\{\xi_{t+k}\}_{k=1}^b$ be the sequence of conditional forecast errors, one to b steps-ahead, of y with observed data up to period t , when the following restrictions are imposed: $\{\eta_{i,t+k}\}_{k=1}^b = \{\varepsilon(i,s,k)\}_{k=1}^b$, $\{\eta_{s,t+k}\}_{k=1}^b = \{1\}_1^b$, $\{v_{i,t+k}\}_{k=1}^b = \{0\}_1^b$ and $\{\eta_{j,t+k}\}_{k=1}^b = \{0\}_1^b$, for every $j \in J$ ($j \in J$ if $j \neq i$, $j \neq s$ and j is one of the q variables, $q = \max\{0, n-r-2\}$, that have to be selected to show zero forecast errors). If $R_- \cdot R_-$ —defined below—has full rank then the impact of change, of variables that do not belong to J , add up to zero and the change in variable i is caused only by past changes in itself and in variable s .

Proof.

Our goal is to obtain $\{v_{t+k}\}_{k=1}^b$ conditional on $\{\eta_{i,t+k}\}_{k=1}^b = \{\varepsilon(i,s,k)\}_{k=1}^b$, $\{\eta_{s,t+k}\}_{k=1}^b = \{1\}_1^b$, $\{v_{i,t+k}\}_{k=1}^b = \{0\}_1^b$ and $\{\eta_{j,t+k}\}_{k=1}^b = \{0\}_1^b$, for $j \in J$. Let $v_b^* = \text{vec}[v_{t+1}, v_{t+2}, \dots, v_{t+b}]'$ be the column vector built from the conditional and structural forecast errors. Given v_b^* , from equation (5) we obtain $\{\xi_{t+k}\}_{k=1}^b$.

To make explicit the procedure adopted to compute v_b^* we define, below, some matrices and vectors.

Let G_{jk}^m be the element of row m , column j , of matrix G_k and let $G_{ik}^m = 0$ for every m and k (i is our equation of interest); $G_k^i = [G_{1k}^i, G_{2k}^i, \dots, G_{nk}^i]$; $R_{ib}^i = \text{vec}[G_b^i, G_{b-1}^i, \dots, G_1^i]$, $R_{ik}^i = \text{vec}[G_k^i, G_{k-1}^i, \dots, G_1^i, 0_{b-k}]$ for k such that $b > k \geq 1$; $R_k^i = [R_{1k}^i, R_{2k}^i, \dots, R_{nk}^i]$, $R^i = [R_1^i, R_2^i, \dots, R_b^i]$ and $P^i = [\xi_{t+1}^i, \xi_{t+2}^i, \dots, \xi_{t+b}^i]$. The vector ξ_{t-}^i is built by taking rows i , s and all rows j , $j \in J$, of ξ_t^i . Matrix R_{t-}^i is obtained in a similar fashion from R_k^i . Let us also define $R_-^i = [R_{1-}^i, R_{2-}^i, \dots, R_{b-}^i]$ and $P_-^i = [\xi_{t+1}^i, \xi_{t+2}^i, \dots, \xi_{t+b}^i]$. The column vector y_{t-}^i is built from rows i , s and all rows j , $j \in J$, of y_t^i and S_k^i is a column vector, with the same dimension of y_{t-}^i , with the element correspondent to variable s in y_{t-}^i equal to 1, with the element correspondent to variable i equal to $\varepsilon(i,s,k)$ and with the element

correspondent to all variable j , such that $j \in J$, equal to zero. Then, $P'_- = [S'_1, \dots, S'_b]$, P'_- is a column vector with dimension $[(2+q)*b] \times 1$.

The conditional v_b^* solves the following minimization problem.¹

$$\text{Min}_{v_b^*} v_b^{*'} \cdot v_b^*$$

Subject to:

$$R_- \cdot v_b^* = P_-$$

If $R_- \cdot R'_-$ has full rank, there is a solution given by $v_b^* = R'_-(R_- \cdot R'_-)^{-1} P'_-$. Note that, given the restrictions, the changes in variable i result, exclusively, from changes in it and from the change equal to 1 in the value of variable s . The added impact of changes of other variables on variable i is zero.

The short run elasticity defined in Proposition 1 differs from the usual interpretation because not only variables i and s are changing their values with respect to the expected ones. All structural equations, with the exception of equation i , show structural errors that can be different from zero. Furthermore, all variables that do not belong to J , and are different from i and s , can alter their values if the added impact of these changes on variable i is zero.

Nevertheless, the changes in variable i are, exclusively, its response to a permanent change equal to 1 in variable s . All structural errors and changes in the values of the variables follow a pattern that is consistent with the data generating process, given by the VAR. That is, all variables are taken as endogenous.

3.2 LONG RUN ELASTICITIES

Next we present the definition and properties of long run elasticities that are implied by the interpretation of short run elasticities given by Proposition 1.

Let, $\phi_{is}(1)$ = element of row i , column s , of matrix $\phi(1)$, defined by $\phi(1) = \phi_0 - \sum_{k=1}^p \phi_k$. If $\phi_{ij}(1) > 0$ then we define $\varphi_{is}^j(1) \equiv \phi_{is}(1)/\phi_{ij}(1)$ and $\varphi_i^j(1) = (\varphi_{i1}^j(1) \dots \varphi_{in}^j(1))$.

Proposition 2

If $\phi_{ii}(1) > 0$ then $\lim_{k \rightarrow \infty} \varepsilon(i, s, k) = -\varphi_{is}^i(1)$, $s \neq i$. Therefore, by definition ($\xi_{i,t+k}$ defined by Proposition 1), $\lim_{k \rightarrow \infty} \xi_{i,t+k} \equiv \xi_{i,t+\infty} = -\varphi_{is}^i(1)$, $s \neq i$.

Proof.

The proof of this proposition is trivial and can be easily verified considering the definitions, of $-\varphi_{is}^i(1)$ and $\xi_{i,t+k}$, and inspecting equations (3) and (4).

1. A detailed explanation can be found in Doan, Litterman and Sims (1984).

4 COINTEGRATION AND LONG RUN ELASTICITIES

4.1 VEC AND COINTEGRATED VAR REPRESENTATION

From now on we assume that all variables are $I(1)$. The number of cointegrating relations can be higher than 1. Therefore, the model has a VEC representation. The VEC model can be estimated using the methods proposed by Johansen (1991) and Johansen and Juselius (1988 and 1990).

4.1.1 VEC representation

$$H(L) \Delta y_t = \rho + \beta D_t - \gamma \alpha' y_{t-1} + u_t \quad (6)$$

$$u_t \sim N(0, \Sigma) \quad (7)$$

$$H(1) = H_0 - \sum_{k=1}^{p-1} H_k$$

where $H_0=I$; α =cointegrating vector; $\alpha' y_{t-1}$ = vector of residual of the cointegrating equations; the matrices γ and α have dimension $n \times r$, where r is equal to the number of cointegrating relations between the variables of the model.

4.1.2 Cointegrated VAR representation—reduced form

$$B(L) y_t = \rho + \beta D_t + u_t \quad (8)$$

where:

$$B(L) = H(L) (1-L) + B(1)L \quad (9)$$

$$B(1) = \gamma \alpha' = \phi_0^{-1} \phi(1) = I - \sum_{k=1}^p B_k \quad (10)$$

$$B(L) = \phi_0^{-1} \phi(L); \rho = \phi_0^{-1} \mu; \beta = \phi_0^{-1} \xi \quad (11)$$

$$u_t = \phi_0^{-1} V_t \text{ and } \text{var}(v_t) = \Omega = \phi_0 \Sigma \phi_0' \quad (12)$$

After estimating the VEC model [equation (6)] we obtain the parameters of the cointegrated VAR in its reduced form. The relationship between the VAR and VEC representations can be verified inspecting equations (9) and (11). Given the cointegrated reduced form VAR parameters it is possible to arrive at the parameters of the cointegrated structural form VAR parameters [equation (1)] if the parameters of ϕ_0 are known [this can be verified inspecting equations (10), (11) and (12)]. The cointegrated structural VAR model estimated through this procedure takes into account the restrictions imposed by de cointegrating relations.

The expected difference, at $t+p$, between the long run value of y (y_∞) and its expected value at t ($E_t(y_\infty)$), is given by [Johansen (2002)]:

$$Y_t^p = E_{t+p}(y_\infty - E_t(y_\infty)) = C(1)(\eta_{t+p} + \sum_{i=1}^{p-1} H_i \eta_{t+p-i}), p \geq 0 \quad (13)$$

$$C(1) = \alpha_\perp (\gamma_\perp' H(1) \alpha_\perp)^{-1} \gamma_\perp'$$

Proposition 3²

If $\{\eta_{t+k}\}_{k=1}^p = \{b\}_1^p$ then $Y_t^p = C(1)H(1)h \in sp(\alpha_\perp)$. If $\eta_{t+p} = b$ and $\{\eta_{t+k}\}_{k=1}^{p-1} = \{0\}_1^{p-1}$ then $Y_t^p = C(1)h \in sp(\alpha_\perp)$. Alternatively, $Y_t^p = k \in sp(\alpha_\perp)$ can be obtained by two alternative ways: letting $\{\eta_{t+k}\}_{k=1}^p = \{k\}_1^p$ or letting $\eta_{t+p} = H(1)k$ and $\{\eta_{t+k}\}_{k=1}^{p-1} = \{0\}_1^{p-1}$.

Proof.

The proof of this proposition is in Johansen (2002).

Proposition 4 shows that Proposition 1's definition of short run elasticity implies a definition of long run elasticity that satisfies the cointegrating properties of the data.

Proposition 4

There is a unique vector $\xi_{t+\infty} \in sp(\alpha_\perp)$ that takes the following values: $\varepsilon(i, s, \infty)$ in row i , 1 in row s and zero in q rows, $q = \max\{0, n-r-2\}$. If Proposition 1 hypothesis are satisfied, then $\lim_{k \rightarrow \infty} \xi_{t+k} = \xi_{t+\infty}$ (ξ_{t+k} defined in Proposition 1). Furthermore, the columns of $\phi(1) \in sp(\alpha)$ and if $\phi_{ii}^i(1) > 0$ then $\phi_i^i(1) \in sp(\alpha)$.

Proof.

If $\xi_{t+\infty} \in sp(\alpha_\perp)$ then $\phi(1) \xi_{t+\infty} = 0$. The matrix $\phi(1)$ is of rank r and the last system of equations can be represented by $\phi_-(1) \xi_{t+\infty} = 0$, where $\phi_-(1)$ is a $r \times n$ matrix of rank r . Fixing the values of $\xi_{t+\infty}$, at $\varepsilon(i, s, \infty)$ and 1 for, respectively, rows i and s , and at 0 for q rows, it is possible to obtain the values of $\xi_{t+\infty}$ for the rest of the rows solving the set of equations, $\phi_-(1) \xi_{t+\infty} = 0$. There is a unique $\xi_{t+\infty}$ that satisfies these restrictions.

If $h \in sp(\alpha_\perp)$ and $\{\eta_{t+i}\}_{i=1}^p = \{h\}_1^p$ then $Y_t^p = h$ with $E_{t+p}(v_s) = 0$ for $s > t+p$ (Proposition 3). Since $\lim_{k \rightarrow \infty} \varepsilon(i, s, k) = \varepsilon(i, s, \infty)$ then, under the hypothesis and restrictions of Proposition 1, it is possible to construct a sequence $\{\eta_{t+k}\}_{k=1}^\infty$ such that $\lim_{k \rightarrow \infty} \eta_{t+k} = \xi_{t+\infty}$ and $\lim_{k \rightarrow \infty} v_k = 0$. Therefore, any alternative sequence $\{\xi_{t+k}\}_{k=1}^\infty$ for which $\lim_{k \rightarrow \infty} v_k \neq 0$ cannot solve the minimization problem of Proposition 1. If $\lim_{k \rightarrow \infty} v_k = 0$ then, from equations (5) and (13), the sequence $\{\xi_{t+k}\}_{k=1}^\infty$ must converge to a $h \in sp(\alpha_\perp)$ and we have shown that this h must be equal to $\xi_{t+\infty}$.

Inspecting equation (10) it is easy to verify that the columns of $\phi(1) \in sp(\alpha)$ and that $\phi_i^i(1) \in sp(\alpha)$.

If there is only one cointegrating relation, $\xi_{t+\infty}^i = (0, \dots, 0, \varepsilon(i, s, \infty), 0, \dots, 0, 1, 0, \dots, 0)$ and, from Proposition 2, we obtain that $\xi_{t+\infty}^i = (0, \dots, 0, -\phi_{is}^i(1), 0, \dots, 0, 1, 0, \dots, 0)$. Since, $\phi_{is}^i(1) = (\phi_{i1}^i(1), \dots, \phi_{i,i-1}^i(1), 1, \phi_{i,i+1}^i(1), \dots, \phi_{i,s-1}^i(1), \phi_{i,s}^i(1), \phi_{i,s+1}^i(1), \dots, \phi_{i,n}^i(1))$ it is easy to verify that $\phi_i^i(1) \xi_{t+\infty}^i = 0$ and that all variables different from s and i have to show zero forecast errors [if all elements of $\phi_i^i(1)$ are different from zero].

Proposition 5³

Let α_1 be a cointegrating vector of the form $\alpha_1' = (1, \alpha_{21}, \dots, \alpha_{m1}, 0, \dots, 0)$. The counterfactual experiment for implementing the definition of $-\alpha_{21}$ as the long-run

2. Adapted version of Johansen (2002, Proposition 1).

3. Johansen (2002, Proposition 2).

effect of variable 2 on variable 1, keeping variables from 3 to m fixed, and using variables $m+1$ to n as instruments, can be performed using a shift in the value of the variables at period t , if and only if α_{21} is identified by the zero restrictions.

Since $\xi_{t+\infty} \in sp(\alpha_{\perp})$ then $Y_t^p = \xi_{t+\infty}$ if $\eta_t = H(1)$ $\xi_{t+\infty}$ and $\{\eta_{t-k}\}_{k=1}^{p-1} = \{0\}_1^{p-1}$. The counterfactual experiment proposed by Proposition 5 is not consistent with the usual short-run elasticity concept since the change in variable 2 (our variable s) only occurs at period t and other variables, different from variable 2, may have to be altered at t and have an impact on variable i .

4.2 LONG RUN ELASTICITIES WITH ONLY ONE COINTEGRATING RELATION

To compute the short run elasticities, for the VAR model, it is always necessary to impose restrictions that allow the identification of the model. When there is only one cointegrating equation it is not necessary to identify the full model to arrive at the long run elasticities. It is only required that the variables with parameters different from zero in the equation of interest (equation i) and in the cointegrating relation are the same. In this case, if $\phi_{ii}(1) > 0$ then $\alpha_i \neq 0$ and the coefficients of the unique cointegrating equation can be normalized so that the coefficient of variable i is equal to one (α_i^* , α_i^* is obtained normalizing α such that the coefficient for variable i is equal to one).

The vectors $B_j^i(1)$ and $\varphi_j^i(1)$ are defined in a similar fashion and $B_j^i(1)$ can be obtained from $B(L)$ using the same computations that allow $\varphi_j^i(1)$ to be obtained from $\phi(L)$.

Proposition 6

For $j=1, \dots, n$, if $B_{jj}(1) > 0$, $\phi_{jj}(1) > 0$ and there is only one cointegrating relation in which the parameter of variable s are significant if and only if $B_{js}(1) \neq 0$ and $\phi_{js}(1) \neq 0$ ($s = 1, \dots, n$), then, $\alpha_i \neq 0$ and $\alpha_i^* = \varphi_j^i(1) = B_j^i(1)$.

If the hypothesis of this proposition applies to $j = i$ then $\alpha_i^* = \varphi_i^i(1)$.

Proof:

From equation (10), we obtain: $\phi(1) = \phi_0 \gamma \alpha'$. If there is only one cointegrating relation then $\phi_0 \gamma$ is a $n \times 1$ matrix. That is, every row k of $\phi(1)$ is obtained multiplying α' by a scalar that can be different and is equal to the element of row k of the $\phi_0 \gamma$ matrix. To arrive at $\varphi_j^i(1)$ and $B_j^i(1)$, every element of row j of $\phi(1)$ is divided by the element of row j and column i of this last matrix and, this last element, is equal to the element of matrix $\phi_0 \gamma$ at row j times α_i . Therefore, given the hypothesis of Proposition 6, matrix $\phi_0 \gamma$ is irrelevant in the determination of $\varphi_j^i(1)$ and $B_j^i(1)$.

The short run identification of the model is not necessary, with only one cointegrating relation, to arrive at the long run elasticities of equation i that satisfy the hypothesis of Proposition 6. Furthermore, if the hypothesis of Proposition 6 apply to equation i then $\alpha_i^* = \varphi_i^i(1)$.

4.3 LONG RUN ELASTICITIES WITH MORE THAN ONE COINTEGRATING RELATION

To compute the long run elasticities, when there is more than one cointegrating relation, it is necessary to impose restrictions that allow for the identification of the equation of interest (equation \hat{i}). In this case, it is still true that $\gamma\alpha' = B(1)$ and that $\phi_0^{-1} \gamma\alpha' = \phi(1)$, but in general $\phi_j^i(1) \neq \phi_k^i(1)$, when both exist and for $j \neq k$. Furthermore, frequently $\phi_s^i(1) \neq B_s^i(1)$ and to arrive at $\phi_i^i(1)$ we need to estimate the cointegrated structural VAR. To identify row i of matrix $\phi(1)$, so that $\phi_i^i(1)$ gives us the long run elasticities of equation i , it is necessary to identify the model. These long run elasticities will exist if $\phi_{ii}(1) > 0$.

5 CONCLUSIONS

Johansen (2002) suggests a design of experiment, counterfactual experiment that can be implemented in the vector autoregressive model to interpret the coefficients of an identified cointegrating relation.

This article proposes an alternative counterfactual experiment (“design of experiment”) that, contrary to the one suggested by Johansen, does not imply a dichotomy of short run and long run values. The experiment interprets the coefficients of an identified cointegrating relation. It is based on the idea that the coefficients, and some operations with them, are projections—at different horizons—conditional on paths of the variables of the model and on exogenous shocks in the error terms of the equations of a structural VAR. The model dynamics can be used to test if these values can be generated by exogenous shocks in this error terms. It is also feasible to construct, as was shown by Doan, Litterman and Sims (1984), a plausibility index for these exogenous shocks.

The analysis of the proposed conditional projections can be as useful as checking coefficients, of the matrix of contemporaneous correlations among variables, for the correct sign and significance in a structural VAR. It can be an important complement to the impulse response function analysis.

BIBLIOGRAPHY

- DOAN, T., LITTERMAN, R., SIMS, C. Forecasting and conditional projection using realistic prior distributions. *Econometric Review*, v. 3, p. 1-100, 1984.
- HAAVELMO, T. The probability approach in econometrics. *Econometrica*, v. 12, Issue Supplement, p. 1-115, 1944.
- HOLLAND, P. Statistics and causal inference. *Journal of American Statistical Association*, v. 81, p. 945-970, 1986.
- JOHANSEN, S. Estimating and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica*, v. 59, p. 1.551-1.580, 1991.
- _____. *The interpretation of cointegrating coefficients in the cointegrated vector autoregressive model*. University of Copenhagen, 2002 (preprint 14/2002).

- JOHANSEN, S., JUSELIUS, K. Statistical analysis of cointegrating vectors. *Journal of Economic Dynamics and Control*, v. 12, p. 231-254, 1988.
- _____. Maximum likelihood estimation and inference on cointegration, with application to the demand for money. *Oxford Bulletin of Economics and Statistics*, v. 52, p. 169-210, 1990.
- LÜTKEPOHL, H. Interpretation of cointegrating relations. *Econometric Reviews*, v. 13, p. 391-394, 1994.
- RUBIN, D. B. Estimating causal effects of treatment in randomized and non randomized studies. *Journal of Educational Psychology*, v. 66, p. 688-701, 1974.

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