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## The term structure of sovereign spreads in emerging markets: A calibration approach for structural models

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# 135

**DISCUSSION PAPER**

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## **THE TERM STRUCTURE OF SOVEREIGN SPREADS IN EMERGING MARKETS: A CALIBRATION APPROACH FOR STRUCTURAL MODELS**

**Katia Rocha**  
**Francisco A. Alcaraz Garcia**





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Brasília, January 2015

## **THE TERM STRUCTURE OF SOVEREIGN SPREADS IN EMERGING MARKETS: A CALIBRATION APPROACH FOR STRUCTURAL MODELS<sup>1</sup>**

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## SINOPSE

Este trabalho apresenta um modelo estrutural para estimar a estrutura a termo do *spread* soberano e a probabilidade implícita de *default* em um grupo de países emergentes que compõe mais do que 50% do índice EMBIG do JP Morgan. A dinâmica da taxa real de câmbio evolui de acordo com um processo de difusão simples, e representa a variável indicativa do evento de *default*. Relaxando-se a hipótese de mercado completo, o modelo calibrado reproduz a estrutura a termo dos *spreads* de forma consistente com a observada no mercado, gerando desvios absolutos menores que 30 (México, Rússia e Turquia) ou 60 (Brasil) pontos-base. O modelo proposto é robusto e, portanto, a crítica a respeito dos modelos estruturais subestimando a magnitude dos *spreads* deve ser reconsiderada.

Nossos resultados revelam que o mercado está sobreestimando os *spreads* para o Brasil, enquanto para México, Turquia e Rússia o modelo reproduz o comportamento do mercado.

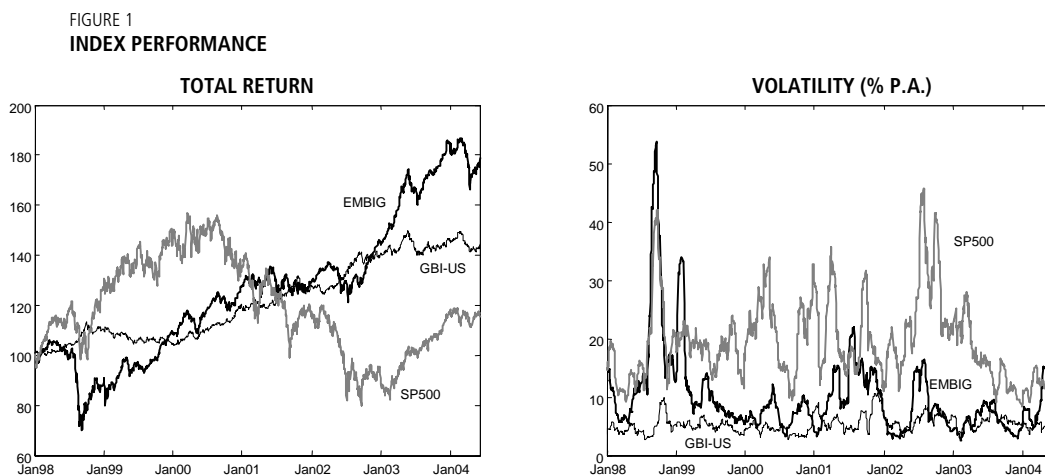
## ABSTRACT

This paper proposes a simple structural model to estimate the term structure of sovereign spreads and the implied default probability of a selected group of emerging countries, which accounts for more than 50% of the J. P. Morgan EMBIG index. The real exchange rate dynamics, modeled as a pure diffusion process, are assumed to trigger default event. By relaxing the hypothesis of market completeness, the calibrated model generates sovereign spread curves consistent with market data, giving average deviations below 30 (Mexico, Russia and Turkey) or 60 (Brazil) basis points over time. We show the robustness of the model and argue that the criticism of structural models for underestimating the magnitude of market spreads should be reconsidered. The results suggest that the market tends to overprice the spreads for Brazil, whereas for Mexico, Russia and Turkey the model reproduces the market behavior.

# 1 INTRODUCTION

The strong performance of investment returns in emerging markets during recent years seems to have consolidated the role of emerging markets in international investment portfolios, despite of the crises occurring during the second half of the 1990s. As noted in the *Global Financial Stability Report* published by the International Monetary Fund (2004), the strong risk-adjusted returns in emerging securities, especially in sovereign bonds, have led many institutional investors to make strategic portfolio allocations in emerging markets.

This reallocation has been further increased by the improvement in the emerging markets' fundamentals and also by the exceptionally low short-term interest rates in the major financial centers. These facts have created a scenario of excess of liquidity since 2001, especially in 2003, when the emerging market sovereign spread fell from historical high levels. Figure 1 shows the total return and the annualized daily volatility of EMBI Global, S&P500 and GBI-US from January 1998 to July 2004.



Credit spread, defined as the yield difference between a risky and a riskless bond with similar characteristics, is related to the implied default probability and credit risk analysis of the issuer. Implied default probabilities are crucial for credit portfolio risk management and for pricing credit derivatives such as credit default swaps (CDSs).<sup>1</sup>

There are two broad financial approaches to assess, price, and manage credit risk: the structural and reduced-form models.

In structural models, initially proposed by Merton (1974), the option-based approach is adopted. The risky bond is modeled as a contingent claim over some measure related to the economic or financial conditions of the debtor that triggers the default event – defined as when such a measure crosses a critical barrier. By making assumptions about the recovery of capital and interest rate models, the default

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1. According to British Bankers' Association — Credit Derivatives Report (2001/2002), the credit derivatives market is the fastest-growing segment of the OTC derivatives market, especially after the Asian and Russian crises. This market grew from US\$ 40 billion outstanding notional value in 1996 to an estimated US\$ 4.8 trillion by the end of 2004. In 2002, CDSs accounted for roughly 45% of the overall credit derivative market while sovereign CDSs represented around 8% of the CDS market.



probability is derived endogenously along with the term-structure of credit spread. Relevant extensions of Merton's model include Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Zhou (1997), and Saá-Requejo and Santa-Clara (1999).

In contrast, reduced-form models, presented by Duffie and Singleton (1999) among others, take the default as an unpredictable event governed by a hazard rate process, where the credit spread is not explicitly related to the financial state or economic conditions of the bond issuer.

The discussion of the appropriate model to evaluate credit spread is highly controversial, tending to state that structural models are better for explaining and reduced models for forecasting. Sarig and Warga (1989) empirically investigate the term-structure of corporate credit spreads and found it appears to conform to the existing theoretical results of Merton's model. Collin-Dufresne, Goldstein, and Martin (2001) suggest that liquidity proxies drive credit-spread changes more than the structural variables. Huang and Huang (2002) show that the class of structural models explains about 60-80% of the spread on corporate bonds rated by Moody's as high-risk obligations (Ba), and roughly 100% for those rated as B, i.e., speculative-grade ratings. Hund (2002) points out the difficulty of reconciling the behavior implied by the structural models with the realities observed in the credit spread market, and Delianedis and Geske (2002) attribute this empirical finding to market incompleteness.

The literature on credit risk models applied to sovereign risk is not straightforward. Cantor and Packer (1996) find that sovereign ratings are broadly consistent with macroeconomic fundamentals and spreads. Martins (1997) uses the default risk-premium obtained from U.S. speculative long-term corporate bonds to price Brady bonds. Lehrbass (1999) develops a structural model based on an equity index of the borrower country expressed in the lender's foreign currency to analyze DM-Eurobonds issued by emerging economies. Wiggers (2002) adopts a structural model following the optimal endogenous default approach, where the domestic output in lender's foreign currency represents the default index. Hui and Lo (2002) present a structural model based on foreign exchange rate to explain the sovereign spreads in South Korea and Brazil. Duffie, Pedersen, and Singleton (2002) construct a reduced-form model for pricing sovereign debt with empirical evidence for Russia. Xu and Ghezzi (2002) develop a model that relates the term structure of sovereign spreads in emerging markets to the country's fiscal dynamics; and Moreira and Rocha (2004) introduce a two-factor structural model based on macroeconomic fundamentals and time-varying risk premiums to forecast Brazilian sovereign risk.

This paper implements a simple calibrated structural model to estimate the term structure of sovereign spreads and implied default probabilities of a selected group of emerging countries comprising more than 50% of the EMBIG index.

The indicator triggering default is considered to be the real exchange rate of each sovereign with respect to the U.S. dollar. Although real exchange rate does not directly represent the country's solvency or liquidity, it has the advantage of being a daily market variable promptly reflecting and capturing changes in daily market

spreads, unlike the lower-frequency fundamentals (monthly or even quarterly) usually seen in structural models.

The assumption that real exchange rate is interpreted as an indicator of default can be supported by the reasoning that the depreciation in the domestic exchange rate of the sovereign issuer against the denominated sovereign bond currency (usually U.S. dollar) puts pressure on the ability of the sovereign issuer to pay its liabilities, increasing the country risk. Such an argument is in agreement with Reinhart's (2002) results, where 84% of the emerging market defaults are associated with currency crises, mainly due to the considerable dollar denominated debt in such economies. Therefore, currency devaluation may exacerbate fiscal problems when the economy has an open capital account but a relatively small tradable sector. Moreover, according to Kaminsky, Lizondo, and Reinhart (1998), real exchange rate is one particularly useful indicator in anticipating currency crises.

The paper is organized as follows. The next section introduces the model, Section 3 continues with the data, Section 4 presents the results, and the last section discusses the main conclusions.

## 2 THE MODEL

Let  $S$  be the dynamics of the real exchange rate as of July 2004, in the Martingale equivalent measure, described by the stochastic process of equation (1); where  $dz$  is the Wiener increment,  $\sigma_t$  is the volatility parameter, and  $\lambda_t$  is the risk-neutral time-varying drift.

$$\frac{dS}{S} = \lambda_t dt + \sigma_t dz^* \quad (1)$$

In the absence of arbitrage opportunities, the complete market assumption implies that the risk-neutral drift of equation (1) equals the short-term interest rate differential between the sovereign issuer and the U.S. dollar market [see Neftci (2000)]. This relation, known as the covered interest rate parity in international finance, is fairly correct for developed countries according to Frankel (1993). However, empirical evidence indicates it fails for emerging economies due to the existence of country risk that cannot be hedged. Therefore, we assume market incompleteness in emerging economies and estimate the risk-neutral time-varying drift parameter  $\lambda_t$  by calibration with market data. Bates (1991) used a similar approach in order to explain the 1987 stock market crash.

The default event is triggered the first time the real exchange rate variable  $S$  crosses the default barrier  $\alpha$ , i.e., when the exchange rate reaches a value that makes the debt's repayment unlikely. The moment of default is uncertain and has a probability distribution function (first hitting time) shown in Appendix.

The price of a default risky zero coupon bond  $B(t, T)$  with a principal of \$1 maturing at time  $T$  is given by equation (2), where  $P(t, T)$  is the price of a default riskless zero coupon bond with the same characteristics,  $w$  is the writedown in case of

default,<sup>2</sup>  $1_{\{\tau < T\}}$  is the indicator function in case a default event occurs prior to maturity,  $r$  is the default riskless instantaneous rate, and the expectation is taken with respect to the equivalent Martingale measure  $Q$ .

$$B(t, T) = P(t, T) - E_t^Q \left[ w 1_{\{\tau < T\}} e^{-\int_t^T r(u) du} \right] \quad (2)$$

Equation (2) can be written as equation (3), where  $F_t(\tau < T)$  is the risk-neutral default cumulative probability function of a default event occurring before time  $T$ .

$$B(t, T) = P(t, T) [1 - w F_t(\tau < T)] \quad (3)$$

Let the spread  $s(t, T)$  be the difference in yield between the risky and riskless bond. Hence, the sovereign spread is given by equation (4).

$$s(t, T) = -\frac{1}{(T-t)} \ln \left( \frac{B(t, T)}{P(t, T)} \right) = -\frac{1}{(T-t)} \ln(1 - w F_t(\tau < T)) \quad (4)$$

### 3 THE DATA

An important benchmark for the analysis of risk and returns of worldwide emerging markets appeared with the introduction of the J.P. Morgan Emerging Markets Bond Index (EMBI). The EMBI is a total-return index for traded U.S. dollar-denominated Brady bonds in the emerging markets that satisfy some restrictive liquidity criteria. The J.P. Morgan EMBI Plus (EMBI+) relaxed the liquidity criterion of EMBI by incorporating more instruments in its composition.

The J.P. Morgan EMBI Global (EMBIG) contains U.S.-dollar-denominated Brady bonds, Eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities; establishing a different criterion for eligible countries to be included in the index and admitting less liquid instrument than its predecessor EMBI+.

In order to make the country risk consistent with the assumptions of the model and be able to compare the risk measures among the countries, we should use the sovereign spread implicit in the EMBIG of each country. Sovereign spread is the yield (stripped yield) difference in basis points between a risky and a risk-free instrument with similar characteristics, where the present value of the flows from the collateral has been removed since the collateral is not subject to sovereign risk.

Similarly, the collateral also affects the duration of the EMBIG. The appropriate duration for a sovereign spread that measures just the remaining risk after stripping away the collateral is the spread duration on sovereign-risk. Sovereign spread

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2. Writedown is assumed to be constant and we use the "Recovery of Treasury" formulations as in Jarrow and Turnbull (1995).

duration is defined as the percentage price change per basis-point change in the sovereign spread, and can also be interpreted as an average maturity of the index (without collateral). Thus, we have a series of sovereign spreads for every emerging country and their corresponding sovereign spread durations, from which we calibrate the model.

To test the performance of our analysis, we choose the EMBIG for Brazil, Mexico, Russia, and Turkey, which correspond roughly to 58% of the EMBIG composite on July 15, 2004. The sovereign spreads of the selected emerging economies since January 2000 are shown in Figure 2, and the composition by country of the EMBIG as of July 15, 2004 in Figure 3.

FIGURE 2  
SOVEREIGN SPREADS

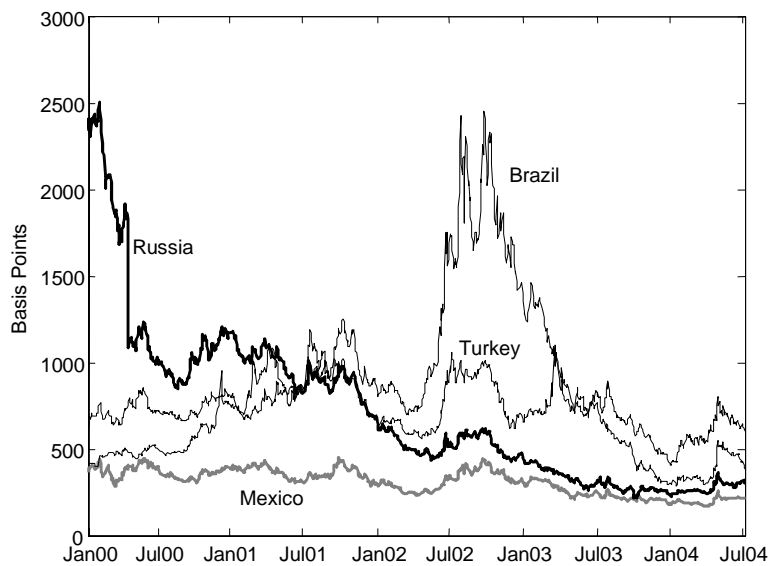
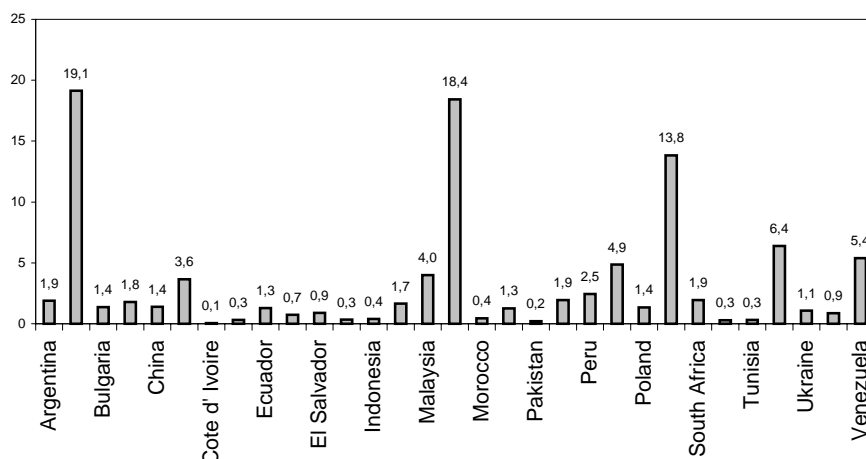


FIGURE 3  
EMBIG INDEX COUNTRY WEIGHTS, %



The average defaulted debt recovery rate for sovereign bonds is taken from Moody's Special Comment (2003), giving an average value of roughly 40%. Hence, the corresponding writedown value  $w$  is 60%.

Nominal exchange rate data for each emerging country were converted into real exchange rate with the consumer price indices available from the IMF's International Finance Statistics. Figure 4 shows the real exchange rate of the selected countries since January 1995, in U.S. dollars of July 2004, and the maximum rate achieved.

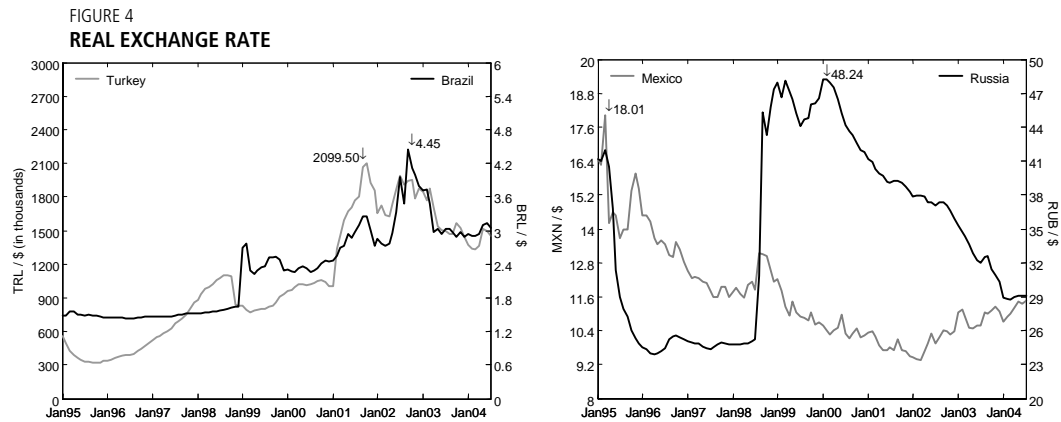
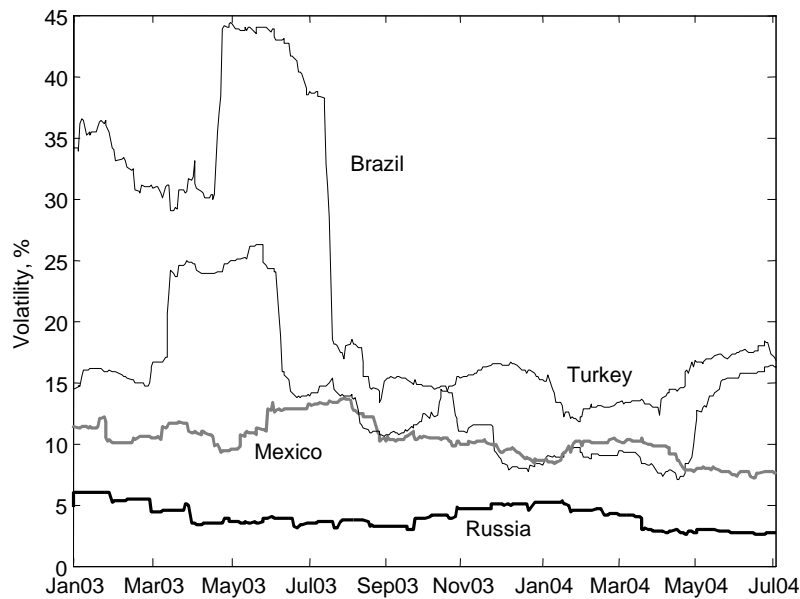


Figure 5 shows the historical annualized daily volatility parameter  $\sigma_t$ , of equation (1), estimated in a running window of 60 days since January 2003.

FIGURE 5  
ANNUAL VOLATILITY



## 4 RESULTS

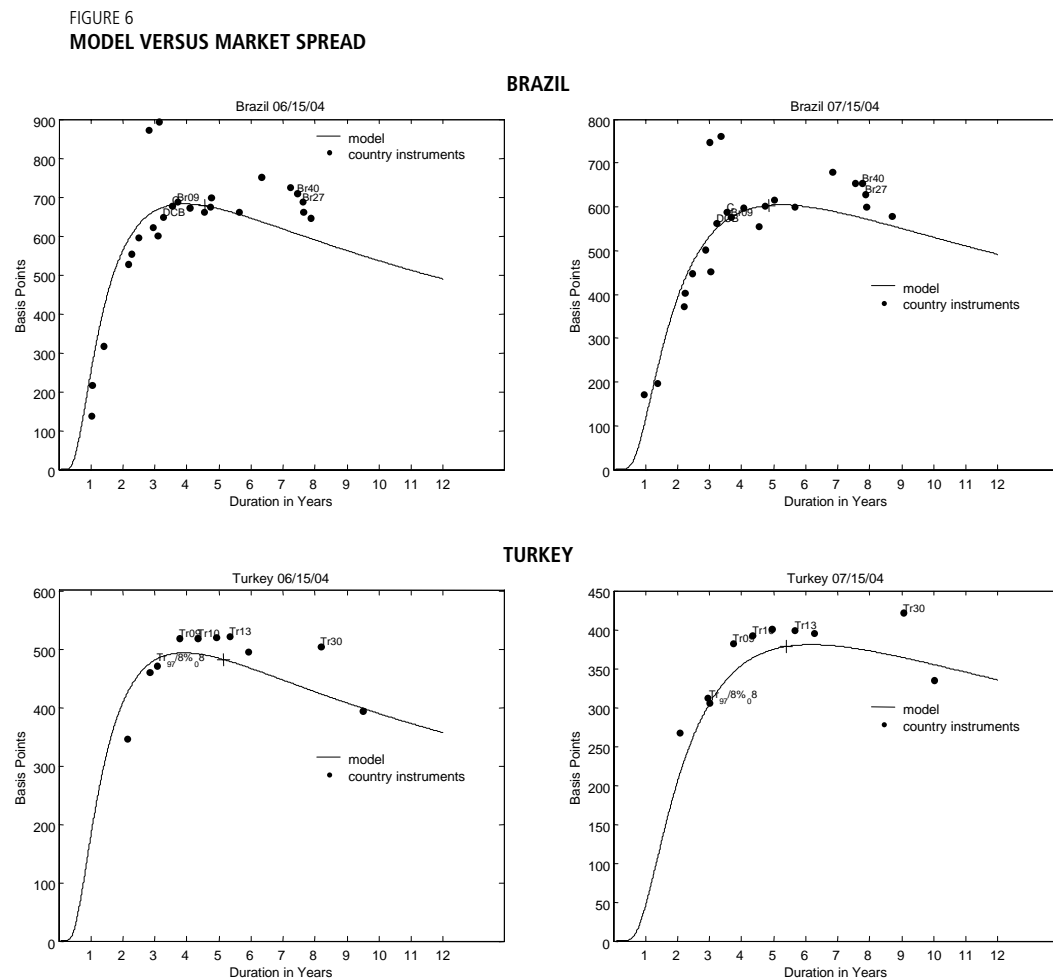
The calibration process employed in this study is similar to that used by market practitioners. The risk-neutral time-varying drift parameter  $\lambda_t$  is calibrated with the most liquid instrument (in our case the benchmark EMBIG of each country) and

then used as an input for pricing the less liquid ones (the real instruments included in the country's EMBIG). By constantly updating the calibrated parameter, we are able to incorporate all market information available up to date.

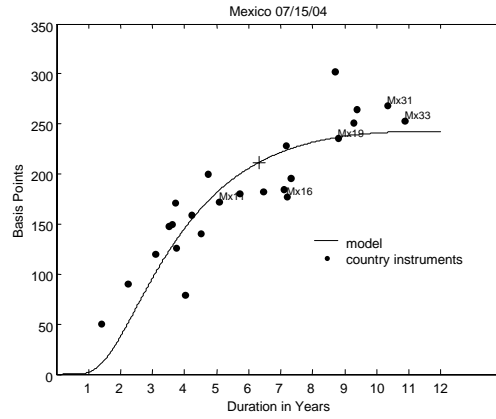
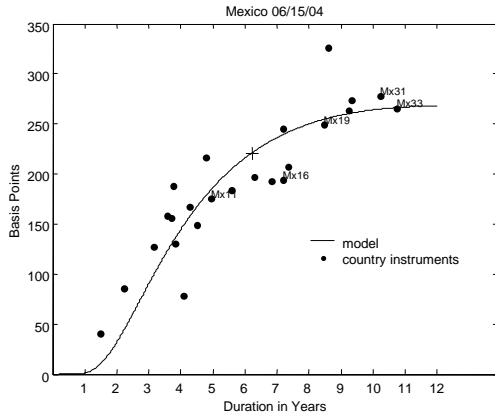
The period of our study starts in September 2003 up to July 2004. The time series of the EMBIG sovereign spread and duration are available on a daily basis since, however, the real instruments used for the estimation of the default barrier are taken on a weekly basis due to data access limitations.

The default barrier is estimated by minimizing the mean square error between the sovereign spreads generated by the model and those of the real instruments. While the estimated barrier is updated weekly and kept constant over the next week, the risk-neutral time-varying drift is calibrated with the daily EMBIG sovereign spread and duration for every country. The calibrated parameter is used as an input to equation (4) to calculate the term structure of sovereign spreads, which is compared to the one observed in the market.

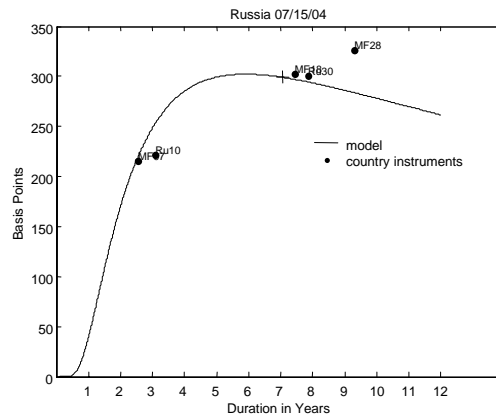
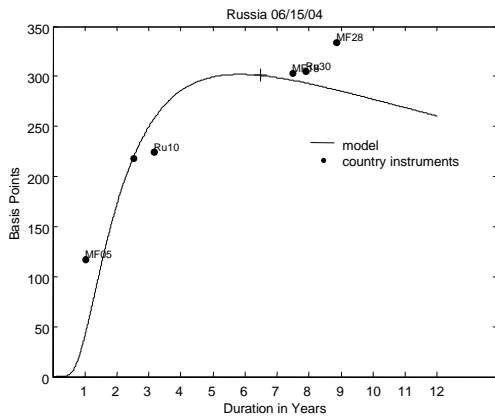
To verify how the model fits the actual market data, Figure 6 plots the term structure of sovereign spreads generated by the model versus that observed in the market for the last two months of our study. The cross-marks in the graphs represents the country's EMBIG used in the sovereign term structure calculation.



## MEXICO



## RUSSIA

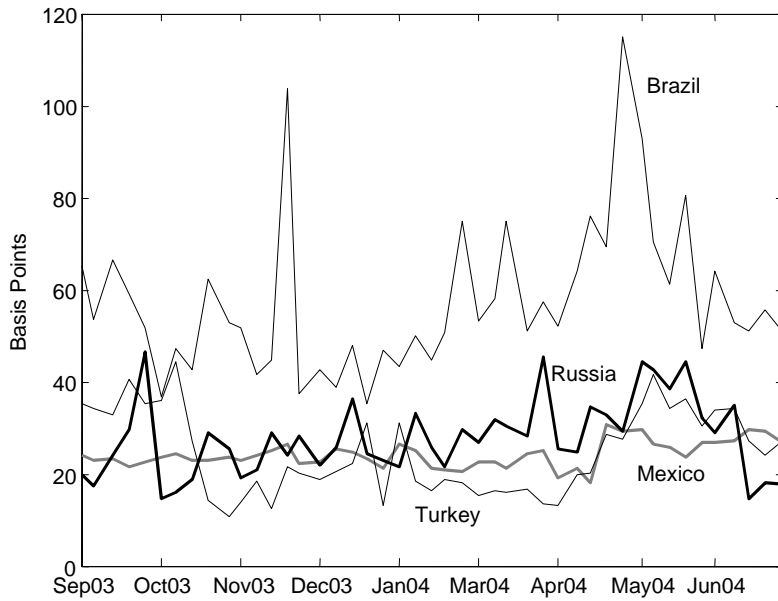


The term structure of the sovereign spreads obtained is in conformity to empirical findings of corporate credit spreads, suggesting a hump-shaped term structure for speculative-grade sovereigns (Brazil and Turkey) and an upward sloping one for investment-grade sovereigns (Mexico and Russia).

Figure 7 shows the average of absolute deviations in basis points of the model compared to the market data over time. This measure is quite stable during the study period and indicates a more accurate fit for Mexico, Russia and Turkey than for Brazil, probably due to the higher volatility of the Brazilian currency (the *Real*) as shown in Figure 5. Considering the deviations for the short-, medium- and long-term sovereign spreads, set to less than three years, from three to five years, and over five years duration respectively, we found no strong evidence of a systematic underpricing bias in the short-term spreads generated by the model.

Deviations are on average less than 30 basis points for Mexico, Russia and Turkey, and less than 60 basis points for Brazil, thus supporting the evidence for the model's consistency. Moreover, we find Russia and Turkey with more than 80% on average of their instruments within a deviation of less than 50 basis points, Mexico with more than 90%, while the proportion for Brazil is nearly 60%.

FIGURE 7  
AVERAGE ABSOLUTE DEVIATIONS



If we calculate the percentage of instruments with positive deviations over time, we can identify trends in overpricing spreads. We find that, on average, nearly 60% of the instruments are overpriced for Brazil, 50% for Mexico and roughly 70% for Russia and Turkey. Nevertheless, this information is not enough to determine the magnitude of that trend. The percentage for Brazil accounts for roughly 70 basis points on average (the highest overpriced spread); whereas Russia and Turkey have average deviations of just 30 basis points. Mexico, with a difference around 25 basis points, does not give evidence of over or underpricing spreads over time. Such results indicate that the market tends to overprice Brazilian spreads compared to the other emerging economies considered.

The percentage of instruments with overpriced spreads for Brazil and the magnitude of the overpricing for all countries are presented in Figure 8. Figure 9 shows the daily-implicit default probability for three and five years.

FIGURE 8  
OVERPRICING SPREADS

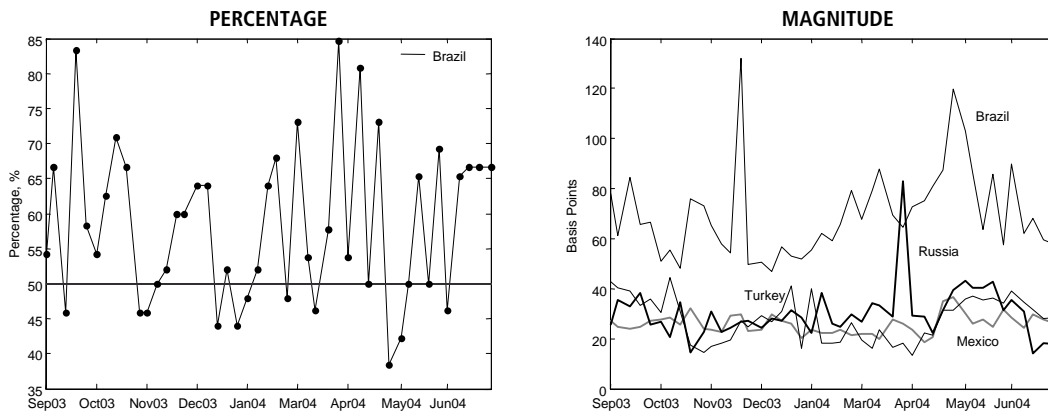
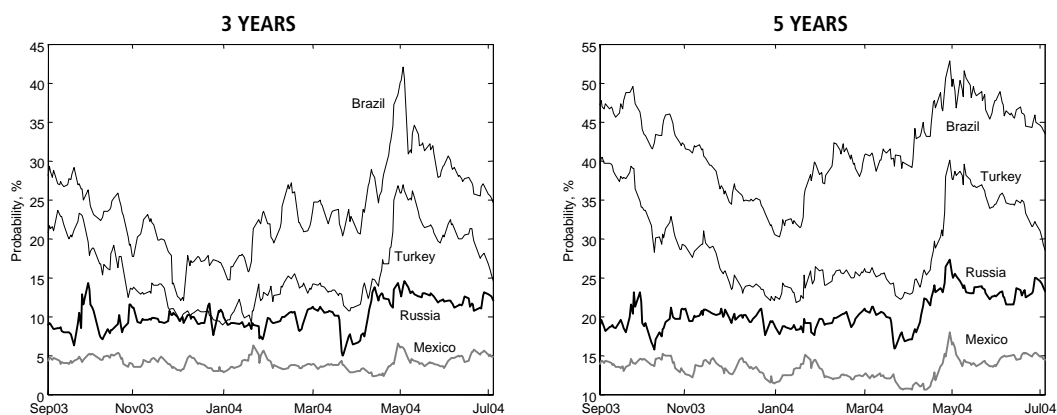




FIGURE 9  
IMPLIED DEFAULT PROBABILITY



The implied default probability distribution decreases sharply in the autumn of 2003, after the turmoil of the Argentina crisis in 2002 and the Brazilian elections in October 2002. Such a performance was achieved via improvements in the country-specific fundamentals combined with the high global liquidity in 2003. This favorable external environment changed in the beginning of 2004 with the expectation of an increase in U.S. interest rates and rising oil prices, especially in May 2004.

The table below shows the estimates of the implied (risk-neutral) cumulative default probability for the last day in the sample (July 15, 2004) and the issuer-weighted cumulative sovereign default rates available in Moody's Special Comment (2003).

As expected, the implied (risk-neutral) default probabilities are higher than the historical ones estimated by the credit rating agency.

**CUMULATIVE DEFAULT PROBABILITIES**  
[%]

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Implied default probabilities										
Mexico Baa2	0.03	1.26	4.74	9.43	14.43	19.32	23.93	28.19	32.10	35.68
Russia Baa3	0.66	5.58	11.98	17.96	23.16	27.64	31.51	34.88	37.84	40.46
Turkey B1	0.75	6.68	14.57	21.99	28.45	34.01	38.80	42.96	46.60	49.81
Brazil B2	1.88	12.55	24.58	34.93	43.43	50.40	56.18	61.01	65.10	68.59
Moody's data on sovereign default rate (January 1985–December 2002)										
Baa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	7.89	14.25	18.33	18.33	22.22	27.08	32.69	38.81	45.61	53.38
Investment grade	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Speculative grade	3.87	7.87	10.62	14.19	16.59	19.74	23.75	28.67	35.47	45.39

## 5 SUMMARY AND CONCLUSIONS

This paper proposes a structural model for estimating the term structure of sovereign spreads and implied default probabilities in emerging countries that account for more than 50% of the EMBIG index. The real exchange rate is assumed to trigger the default event, and since it is a daily market variable, it captures changes in daily spread sooner than do low-frequency fundamentals (monthly or quarterly).

Once the model is calibrated, the results show that the term structure of sovereign spreads agrees with empirical findings, thus supporting the structural approach. They suggest a hump-shaped term structure for speculative-grade sovereigns (Brazil and Turkey) and an upward sloping one for investment-grade countries (Mexico and Russia).

The fit of the model over time indicates robustness, with greater accuracy for Mexico, Russia and Turkey than for Brazil, probably due to the higher volatility of Brazil's currency. The small average of absolute deviations over time of less than 30 basis points for Mexico, Russia and Turkey, and less than 60 basis points for Brazil, supports the evidence in favor to the model.

According to the model, the market tends to overprice the spreads for Brazil, whereas it replicates the market behavior Mexico, Russia and Turkey. As expected, implied (risk-neutral) default probabilities are higher than the historical ones available from Moody's.

The use of other proxies triggering default, such as sovereign equity indices and the application of CDS sovereign spreads instead of bonds are left for future research.

## APPENDIX

### FIRST HITTING TIME DISTRIBUTION

Let the following be a stochastic process with  $dz$  as a Wiener process.

$$dx = \lambda_t dt + \sigma_t dz \quad (A1)$$

Following Karatzas and Shreve (1991), the first passage time density of  $x$  evaluated at  $\tau > t$ , i.e.,  $\tau = \inf \{t \geq 0, x(t) > 0\}$ , is given by equation (A2).

$$\pi(\tau | x_t, \lambda_t, \sigma_t) = \frac{|x_t|}{\sigma_t \sqrt{2\pi(\tau-t)^3}} \exp\left[-\frac{(x_t + \lambda_t(\tau-t))^2}{2\sigma_t^2(\tau-t)}\right] \quad (A2)$$

Through Ito's Lemma, we have that if  $S$  follows equation (1), then  $x_t = \ln(S_t/\alpha)$  follows the following stochastic differential equation:

$$dx = (\lambda_t - 0.5\sigma_t^2) dt + \sigma_t dz \quad (A3)$$

Hence, the first passage time density of  $S$  evaluated at  $\tau > t$ , i.e.,  $\tau = \inf \{t \geq 0, S(t) \geq \alpha\}$  is given by equation (A4).

$$\pi(\tau | S_t, \lambda_t, \sigma_t, \alpha) = \frac{|\ln(S_t/\alpha)|}{\sigma_t \sqrt{2\pi(\tau-t)^3}} \exp \left[ -\frac{(\ln(S_t/\alpha) + (\lambda_t - 0.5\sigma_t^2) \cdot (\tau-t))^2}{2\sigma_t^2(\tau-t)} \right] \quad (\text{A4})$$

The cumulative distribution function is given by equation (A5), where “ $\phi(\cdot)$ ” is the cumulative normal distribution function.

$$F_t(\tau < T) = \left\{ \begin{array}{l} 1 - \phi \left( \frac{\left| \ln \left( \frac{S_t}{\alpha} \right) \right| - (\lambda_t - 0.5\sigma_t^2) \cdot (T-t)}{\sigma_t \sqrt{T-t}} \right) + \\ + e^{\left( \frac{2 \cdot (\lambda_t - 0.5\sigma_t^2) \cdot \left| \ln \left( \frac{S_t}{\alpha} \right) \right|}{\sigma_t^2} \right)} \cdot \phi \left( \frac{-\left| \ln \left( \frac{S_t}{\alpha} \right) \right| - (\lambda_t - 0.5\sigma_t^2) \cdot (T-t)}{\sigma_t \sqrt{T-t}} \right) \end{array} \right. \quad (\text{A5})$$

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