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# Monetary and Fiscal Policy in a Large Asymmetric Monetary Union - A Dynamic Three-Country Analysis

by Volker Clausen and Hans-Werner Wohltmann

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# MONETARY AND FISCAL POLICY IN A LARGE ASYMMETRIC MONETARY UNION – A DYNAMIC THREE-COUNTRY ANALYSIS

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## **Abstract**

This paper analyzes the dynamic effects of anticipated monetary and fiscal policies in a large monetary union, which is characterized by asymmetric interest rate transmission. We explicitly solve the asymmetric three-country model using the decomposition methods of Aoki (1981) and Fukuda (1993). Anticipated monetary and fiscal expansions lead to negative international spillovers and to intertemporal reversals in the relative effectiveness of policy on member country outputs. Intertemporal international coordination of monetary policies between Euroland and the US is able to stabilize the output adjustment processes induced by an anticipated unilateral fiscal expansion.

*JEL classification:* E58, F41

*Keywords:* Monetary Union, Fiscal Policy, Monetary Policy, Policy Coordination

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# 1 Introduction

This paper analyzes the dynamic effects of anticipated monetary and fiscal policies in a large asymmetric monetary union. A three-country framework serves to illustrate the dynamic spillover effects of monetary and fiscal policies and the potential benefits of international policy coordination. We investigate international transmission within the monetary union (called internal) and between the monetary union and the rest of the world (external).

The motivation for our analysis is the establishment of the European Monetary Union (EMU or Euroland) in 1999. Viewed in a global context, the EMU can already now, and even more after the upcoming EU-enlargement, no longer be interpreted as a *small* country. In addition, there is considerable evidence that the EMU member countries, and even the core countries Germany, France and Italy, are characterized by asymmetric macroeconomic structures. It is debated, however, to which extent these asymmetries in the macroeconomic structures actually translate into perceptible and robust differences in the strength and the timing of ECB policy transmission. In this context, it is fair to say that most attention focuses on differences in the strength of interest rate transmission within the EMU.<sup>1</sup>

Against this background, our paper is the first to discuss the effects of monetary and fiscal policy in a *dynamic* context of a *large* and *asymmetric* monetary union.<sup>2</sup> The two union member countries are assumed to a large extent to be completely symmetric, except for the strength of the interest rate channel in aggregate demand. Furthermore, we allow the non-member or third country to have preferences for home goods and import goods different from those in the union member countries. Our analysis applies the decomposition method by Aoki (1981) and its generalization to the N-country case by Fukuda (1993). While Fukuda (1993) assumes in an N-country setting complete symmetry of countries we restrict our attention to the three-country case but are able to introduce several empirically relevant macroeconomic asymmetries.

The paper is organized as follows: Section 2 describes the model, applies the decomposition method to the asymmetric three-country context and derives the solutions to the various respective sub-systems. Section 3 analyzes the dynamic effects of anticipated monetary and fiscal policies in the monetary union on the member countries and the third country. Moreover, section 3 discusses the benefits of international monetary policy coordination with respect to the absorption of anticipated fiscal policies. Section 4 summarizes the main results.

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<sup>1</sup>See e.g. Angeloni et al. (2001), Clausen (2001), De Grauwe (2000), Dornbusch et al. (1998) and Cecchetti (1999).

<sup>2</sup>Dynamic models for a *small* asymmetric monetary union are provided in Wohltmann and Clausen (2002(a), 2002(b), 2001) and Clausen and Wohltmann (2004).

## 2 A Large Asymmetric Monetary Union

### 2.1 Model equations

We consider three large open economies of identical size. Two economies ( $U_1$  and  $U_2$ ) form a monetary union. The common monetary policy is assumed to be conducted by the ECB. As in reality, the union has flexible exchange rates with respect to the third country, the US. The two member countries are taken to be largely symmetric except for selected asymmetries: As in Wohltmann and Clausen (2004), we consider an asymmetry in the strength of the interest rate channel in aggregate demand. Furthermore, both member countries are allowed here to have asymmetries in their relations to the large third country on the demand as well as on the supply side. This assumption is justified by the presence of intra-European asymmetries in the respective degrees of openness toward the rest of the world (Clausen, 2001). As these are not primarily linked to the monetary policy regime they are likely to persist despite the establishment of a common monetary policy.

The demand side of the model is specified in the tradition of Mundell/Fleming-models, i.e., consists of IS- and LM-equations and uncovered interest parity. The supply side is represented by expectations-augmented Phillips-curves. The price adjustment equations assume rational expectations and contain a long-run supply function.

The notation is as follows:  $y$  = real output,  $i$  = short-term nominal interest rate,  $i - \dot{p}^c$  = real interest rate,  $p^c$  = consumer price index,  $g$  = real government consumption,  $p$  = price level of the domestically produced good,  $e$  = common external nominal exchange rate,  $\tau$  = external terms of trade,  $m$  = money supply. The variables of the member country  $U_1$  ( $U_2$ ) are indexed as „1“ („2“). Variables of the third country are denoted by a „\*“. A dot on top of a variable denotes the right-hand side derivative with respect to time  $t$ . A bar on top represents the long-run or steady state level of the respective variable. All variables are expressed in logarithms, except for the interest rates  $i_1$ ,  $i_2$  and  $i^*$ . All parameters are specified to be positive and are interpreted as elasticities or semi-elasticities.

The complete model consists of the following equations:

*IS equations:*

$$y_1 = (a_{01} + a_1 y_1 - a_{21}(i_1 - \dot{p}_1^c)) + g_1 + (b_0 - b_1 y_1 + b_2 y_2 - b_3(p_1 - p_2)) + (c_0 - c_1 y_1 + c_2 y^* - c_3 \tau_1) \quad (1)$$

$$y_2 = (a_{02} + a_1 y_2 - a_{22}(i_2 - \dot{p}_2^c)) + g_2 - (b_0 - b_1 y_1 + b_2 y_2 - b_3(p_1 - p_2)) + (c_0 - c_1 y_2 + c_2 y^* - c_3 \tau_2) \quad (2)$$

$$y^* = (a_0 + a_1 y^* - a_2(i^* - \dot{p}^{*c})) + g^* - (c_0 - c_1 y_1 + c_2 y^* - c_3 \tau_1) - (c_0 - c_1 y_2 + c_2 y^* - c_3 \tau_2) \quad (3)$$

where

$$\tau_1 = p_1 - (p^* + e), \quad \tau_2 = p_2 - (p^* + e) \quad (4)$$

$$p_1^c = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3(p^* + e) \quad (5)$$

$$p_2^c = \alpha_1 p_2 + \alpha_2 p_1 + \alpha_3(p^* + e) \quad (6)$$

$$p^{*c} = \alpha_1^* p^* + \alpha_2^*(p_1 - e) + \alpha_3^*(p_2 - e) \quad (7)$$

and making the following assumptions:

$$a_{21} > a_{22}, \quad b_1 = b_2, \quad b_1 > c_1 \quad (8)$$

$$a_2 = \frac{1}{2}(a_{21} + a_{22}) \quad (9)$$

$$\alpha_1 = \alpha_2 > \alpha_3, \quad \alpha_2^* = \alpha_3^* < \alpha_1^* \quad (10)$$

*LM equations:*

$$m = m_1 + m_2 = (p_1^c + l_0 + l_1 y_1 - l_2 i_1) + (p_2^c + l_0 + l_1 y_2 - l_2 i_2) \quad (11)$$

$$m^* = p^{*c} + l_0 + l_1 y^* - l_2 i^* \quad (12)$$

where the interest rates  $i_1, i_2, i^*$  are linked by uncovered interest parity:

$$i_1 = i_2 = i^* + \dot{e} \quad (13)$$

*Price adjustment equations:*

$$\dot{p}_1 = \dot{p}_1^c + \delta(y_1 - \bar{y}_1) \quad (14)$$

$$\dot{p}_2 = \dot{p}_2^c + \delta(y_2 - \bar{y}_2) \quad (15)$$

$$\dot{p}^* = \dot{p}^{*c} + \delta(y^* - \bar{y}^*) \quad (16)$$

where the steady-state-outputs  $\bar{y}_1, \bar{y}_2, \bar{y}^*$  are determined by the following long-run

supply functions:

$$\bar{y}_1 = f_0 + f_1\bar{\tau}_1 + f_2(\overline{p_1 - p_2}) \quad (17)$$

$$\bar{y}_2 = f_0 + f_1\bar{\tau}_2 + f_2(\overline{p_2 - p_1}) \quad (18)$$

$$\bar{y}^* = f_0^* - f_1^*\bar{\tau}_1 - f_2^*\bar{\tau}_2 \quad (19)$$

with

$$f_1^* = f_2^* \quad (20)$$

The equations (1), (2) and (3) are standard IS-curves to model aggregate demand in the three countries. Aggregate demand consists of real private absorption, real government consumption and the aggregate trade balance. Real private absorption, the first expression in brackets, depends on real output and on real interest rates. The aggregate trade balance, the second and third expression in brackets, depends on output developments and on relative prices. This holds within the monetary union (with internal relative prices or terms of trade) as well as between the respective union country and the rest of the world (with external relative prices or terms of trade).<sup>3</sup> We specify the IS-equations (1), (2) of the union member countries  $U_1$  and  $U_2$  to be structurally symmetric except for an asymmetry in the semi-interest elasticity of private absorption. Real private absorption in  $U_1$  is assumed to react more strongly to real interest rates than in  $U_2$  ( $a_{21} > a_{22}$ ).<sup>4</sup> As regards to the trade balances of  $U_1$  and  $U_2$  it is assumed that international trade between the union member countries is characterized by identical marginal import and export propensities ( $b_1 = b_2$ ). The propensity to import is taken to be higher within the monetary union than with respect to the rest of the world ( $b_1 > c_1$ ). In other words, households have a stronger preference for imports from the respective partner country than from the third country.

In order to solve our asymmetric three-country-model with the aid of the decomposition methods by Aoki (1981) and Fukuda (1993), we need to assume in the definitions (5), (6) of the consumer price indices  $p_1^c$  and  $p_2^c$  that member country households have the same preference for their home goods and for imports from the other member country ( $\alpha_1 = \alpha_2$ ). This assumption requires a highly integrated monetary union. It ensures that the member country price indices ( $p_j^c$ ) as well as real interest rates ( $i_j - \dot{p}_j^c$ ;  $j = 1, 2$ ) are identical across the monetary union (Clausen and Wohltmann (2004)). This allows us to decompose the overall system in *three*

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<sup>3</sup>The internal bilateral trade balances *exactly* mirror each other if the bilateral trade balance is initially in equilibrium and if purchasing power parity holds. The third expression in brackets in (1) (or (2)) has the opposite sign of the second (or third) expression in brackets in (3).

<sup>4</sup>Given that both member countries are of identical size in the initial equilibrium, i.e.,  $(\bar{y}_1)_0 = (\bar{y}_2)_0$ , we need to assume that the constants  $a_{01}$  and  $a_{02}$  in (1) and (2) are *not* identical:  $a_{01} \neq a_{02}$ . It is straightforward to compute the necessary numerical difference between the constants (see Clausen and Wohltmann (2004)).

sub-systems - one aggregate system and two difference systems - despite the assumed asymmetry in the strength of interest rate transmission in (1) and (2).

The two member countries  $U_1$  and  $U_2$  are aggregated to an *average* member country by considering the arithmetic mean of respective variables and equations ( $\frac{1}{2}(U_1+U_2)$ ) and compared in their behaviour with the non-member country (*NMC*). The large third country *NMC* is assumed to have an IS-equation (3) being fully symmetric with the *average* IS-equation of  $U_1$  and  $U_2$ . This case prevails if the interest sensitivity of private absorption in the *NMC* ( $a_2$ ) equals the arithmetic mean of the semi-interest elasticities  $a_{21}$  and  $a_{22}$  in  $U_1$  and  $U_2$  (cf.(9)). Moreover, *NMC* households have the same preference for imports from both union member countries  $U_1$  and  $U_2$  ( $\alpha_2^* = \alpha_3^*$ ).

The expressions (11) and (12) reflect standard money market equilibrium conditions in both, the monetary union and in the third country *NMC*.<sup>5</sup> The monetary union has a common money market in which the aggregate money stock  $m$  is assumed exogenous while its distribution across the member countries is determined endogenously by the respective national money demand functions. Interest rates affect money demand and are linked internationally by uncovered interest parity (see 13)). We assume rational expectations.<sup>6</sup> The price adjustment equations (14), (15), (16) are expectations-augmented Phillips-curves with rational expectations concerning consumer price inflation. The rate of inflation depends positively on the output gap, i.e., on the difference between actual and long-run equilibrium output. Long-run equilibrium or natural output is determined within a long-run supply function by the internal and external equilibrium terms of trade.<sup>7</sup> In the third country, natural output is analogously affected by the equilibrium terms of trade  $\bar{\tau}_1$  and  $\bar{\tau}_2$  with  $f_1^* = f_2^*$ .

## 2.2 Solution method

Our asymmetric dynamic three country model can be solved using the decomposition methods pioneered by Aoki (1981) and Fukuda (1993). The overall system is transformed into three sub-systems - two difference systems and one aggregate system. The *first difference system* ( $\frac{1}{2}(U_1 + U_2) - NMC$ ) describes the differential developments between the average union member country and the third country.

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<sup>5</sup>As in Turnovsky (1986) and Fukuda (1993), the real money stock is derived by deflating the nominal money stock by the respective consumer price index, not by the price level of the domestically produced good.

<sup>6</sup>In this deterministic context, rational expectations concerning the exchange rate is equivalent to assuming perfect foresight. Denoting  $E$  as the expectations operator this means  $E(\dot{e}) = \dot{e}$ . Analogous equations are assumed to hold for consumer price inflation  $\dot{p}_1^c$ ,  $\dot{p}_2^c$  and  $\dot{p}^{*c}$  in the IS- and Phillips-curves.

<sup>7</sup>Microeconomic foundations of this specification are provided in Devereux and Purvis (1990) and in Clausen and Wohltmann (2004).



It is derived by subtracting from the arithmetic mean of the IS-, LM- and price equations in the union the respective equations of the third country. The dynamics of this difference system are described by a two-dimensional differential equation system in the state variables

$$\tau^a = \frac{1}{2}(\tau_1 + \tau_2), \quad l^d = (m^a - p^a) - (m^* - p^*) \quad (21)$$

with

$$m^a - p^a = \frac{1}{2}(m_1 - p_1 + m_2 - p_2) = \frac{m}{2} - \frac{1}{2}(p_1 + p_2). \quad (22)$$

$\tau^a$  represents the arithmetic mean of the external terms of trade of both member countries,  $l^d$  the difference between the average real money stock within the monetary union and the real money stock in the third country. This system exhibits saddle path stability. Its solution is completely independent from the other two sub-systems ( $U_1 - U_2$ ,  $\frac{1}{2}(U_1 + U_2) + NMC$ ).

The *second difference system* ( $U_1 - U_2$ ) describes within the monetary union the differential developments of both member countries  $U_1$  and  $U_2$ . It is derived by subtracting corresponding equations of the two member countries  $U_1$  and  $U_2$  from each other. The dynamics of this difference system  $U_1 - U_2$  are governed by a (non-homogeneous) differential equation in the internal price differential  $p_1 - p_2$ . This differential equation contains the rate of change of the average external terms of trade  $\dot{\tau}^a$  as input, which is a component of the first difference system  $\frac{1}{2}(U_1 + U_2) - NMC$ . It implies that the second difference system cannot be solved independently from the first difference system.<sup>8</sup> However, as in the case of the first difference system, it does not depend on the aggregate system  $\frac{1}{2}(U_1 + U_2) + NMC$ .

The *aggregate system*  $\frac{1}{2}(U_1 + U_2) + NMC$  can be derived by adding the equations for the average member country and the respective equations of the third country. Its dynamics are completely captured by a single differential equation in the aggregate variable

$$l^s = (m^a - p^a) + (m^* - p^*). \quad (23)$$

Like the difference system  $U_1 - U_2$ , the aggregate system  $\frac{1}{2}(U_1 + U_2) + NMC$  *cannot* be solved independently from the first difference system  $\frac{1}{2}(U_1 + U_2) - NMC$  because the differential equation in the aggregate variable  $l^s$  contains the rate of change of the average external terms of trade  $\dot{\tau}^a$  as a time-varying driving factor.

The solutions to the three sub-systems allow us to recover by suitable transformations the solution paths of all variables of interest in the original three-country model. Output developments in the individual countries  $U_1$ ,  $U_2$  and  $NMC$  are sol-

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<sup>8</sup>This unidirectional dependence between the two difference systems directly follows from the assumed asymmetry on the demand side in the two union countries. In the case of symmetry ( $a_{21} = a_{22}$ ), the solution to the second difference system would be completely independent from the other two sub-systems.

ved only on the basis of the solution paths for the two difference systems. They are completely independent from the aggregate system. In contrast, the determination of the average real money stocks  $m^a - p^a$  and  $m^* - p^*$  and of nominal interest rates  $i_1$ ,  $i_2$  and  $i^*$  require the solution to the aggregate system.

The following subsections present the solution to the asymmetric three country model on the basis of the above decomposition. We start with the two difference systems  $\frac{1}{2}(U_1 + U_2) - NMC$  and  $U_1 - U_2$  and continue with the aggregate system  $\frac{1}{2}(U_1 + U_2) + NMC$ . It is assumed that all countries start in  $t = 0$  from an original steady state, which is assumed to be identical for all three countries. Furthermore, the impulse is assumed to be a policy announcement in  $t = 0$  for an implementation in  $T > 0$ . The private sector has rational expectations, regards the policy announcement as credible and anticipates the policy effects already before implementation. More concretely, we investigate the dynamic effects of an anticipated expansionary monetary policy taking the form of an increase in the rate of growth of the union money stock ( $d\dot{m} > 0$ ) and of an anticipated symmetric fiscal expansion in the monetary union ( $dg_1 = dg_2 > 0$ ).

### 2.3 The difference system $\frac{1}{2}(U_1 + U_2) - NMC$

The first difference system is based on the following equation system, which can be derived from the complete model (1)–(20):

$$\lambda_1 y^a - \lambda_2 y^* = a_0 - \frac{1}{2}(a_{01} + a_{02}) - c_0 + g^a - g^* - 3c_3 \tau^a + 2a_2(\alpha_1 - \alpha_3^*) \dot{\tau}^a \quad (24)$$

$$(m^a - p^a) - (m^* - p^*) = -(\alpha_3 + 2\alpha_3^*) \tau^a + l_1(y^a - y^*) + l_2 \dot{\tau}^a + l_2((\dot{m}^a - \dot{p}^a) - (\dot{m}^* - \dot{p}^*)) - l_2(\dot{m}^a - \dot{m}^*) \quad (25)$$

$$y^a = \bar{y}^a + \frac{\alpha_3}{\delta} \dot{\tau}^a \quad (26)$$

$$y^* = \bar{y}^* - \frac{2\alpha_3^*}{\delta} \dot{\tau}^a \quad (27)$$

$$\bar{y}^a = f_0 + f_1 \bar{\tau}^a \quad (28)$$

$$\bar{y}^* = f_0 - 2f_1^* \bar{\tau}^a \quad (29)$$

with

$$y^a = \frac{1}{2}(y_1 + y_2), \quad g^a = \frac{1}{2}(g_1 + g_2) \quad (30)$$

$$\lambda_1 = 1 - a_1 + 3c_1, \quad \lambda_2 = 1 - a_1 + 3c_2 \quad (31)$$

The dynamics are governed by a system using the state variables  $\tau^a$  and  $l^d$  as defined in (21):

$$\begin{pmatrix} \dot{\tau}^a \\ \dot{l}^d \end{pmatrix} = \begin{pmatrix} r_1 & 0 \\ \gamma & r_2 \end{pmatrix} \begin{pmatrix} \tau^a - \bar{\tau}^a \\ l^d - \bar{l}^d \end{pmatrix} \quad (32)$$

with

$$r_1 = \frac{-3c_3\delta}{\lambda_1\alpha_3 + 2\lambda_2\alpha_3^* - 2a_2\delta(\alpha_1 - \alpha_3^*)}, \quad r_2 = \frac{1}{l_2} \quad (33)$$

$$\gamma = \frac{1}{l_2} \left( \alpha_3 + 2\alpha_3^* - \left( l_2 + \frac{l_1}{\delta}(\alpha_3 + 2\alpha_3^*) \right) r_1 \right) \quad (34)$$

The terms of trade  $\tau^a$  are considered as a jump variable because they contain the flexible and forward-looking nominal exchange rate  $e$ . In contrast, the difference in the real money stocks  $l^d$  is taken as a predetermined variable as long as the levels of the nominal money stock  $m^a$  and  $m^*$  are constant and given that the difference between the average price level developments in the union  $p^a$  and the price level in the third country  $p^*$  remains continuous at every point in time  $t$ .<sup>9</sup>

Under the weak assumption

$$\lambda_1\alpha_3 + 2\lambda_2\alpha_3^* - 2a_2\delta(\alpha_1 - \alpha_3^*) > 0 \quad (35)$$

the characteristic root  $r_1$  is unambiguously negative.<sup>10</sup> Then, due to  $r_2 > 0$ , the determinant of the system matrix (32) is always negative. It follows that the dynamic system (32) is characterized by saddle-path stability. The non-explosive, convergent solution to (32) implies the following time paths for the state variables  $\tau^a$  and  $l^d$ :

$$\begin{pmatrix} \tau^a \\ l^d \end{pmatrix} = \begin{pmatrix} \bar{\tau}_0^a \\ \bar{l}_0^d \end{pmatrix} + A_1 h_1 e^{r_1 t} + A_2 h_2 e^{r_2 t} \quad \text{for } 0 < t < T \quad (36)$$

$$\begin{pmatrix} \tau^a \\ l^d \end{pmatrix} = \begin{pmatrix} \bar{\tau}_1^a \\ \bar{l}_1^d \end{pmatrix} + \tilde{A}_1 h_1 e^{r_1 t} \quad \text{for } t > T \quad (37)$$

where  $e^{r_j t} := \exp(r_j t)$  and the characteristic vectors  $h_1$  and  $h_2$  and the constants

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<sup>9</sup>The interpretation of  $\tau^a$  as a jump variable and of  $l^d$  as a predetermined variable follows Buiter and Miller (1982) and Turnovsky (1986).

<sup>10</sup>See Buiter and Miller (1982) for an economic interpretation of the stability condition (35).

$A_1$ ,  $A_2$  and  $\tilde{A}_1$  are defined as follows (see Turnovsky 1986, 2000):

$$h_1 = \begin{pmatrix} h_{11} \\ 1 \end{pmatrix}, \quad h_{11} = \frac{r_1 - r_2}{\gamma} < 0, \quad h_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (38)$$

$$A_1 = -A_2 = -\left(d\bar{l}^d - \frac{1}{h_{11}}d\bar{\tau}^a\right)e^{-r_2T} \quad (39)$$

$$\begin{aligned} \tilde{A}_1 &= A_1 - \frac{1}{h_{11}}d\bar{\tau}^ae^{-r_1T} \\ &= -d\bar{l}^de^{-r_2T} + \frac{1}{h_{11}}d\bar{\tau}^a(e^{-r_2T} - e^{-r_1T}) \end{aligned} \quad (40)$$

The expressions  $d\bar{l}^d := \bar{l}_1^d - \bar{l}_0^d$  and  $d\bar{\tau}^a := \bar{\tau}_1^a - \bar{\tau}_0^a$  denote the steady state effects of exogenous shocks on the state variables.<sup>11</sup> In the case of monetary and fiscal policy taking the forms  $d(\dot{m}^a - \dot{m}^*) > 0$  and  $d(g^a - g^*) > 0$ , it follows from the steady-state-condition  $\dot{l}^d = 0 = \dot{\tau}^a$  and (24) - (29):

$$d\bar{\tau}^a = \frac{1}{\phi}d(g^a - g^*) + 0 \cdot d(\dot{m}^a - \dot{m}^*) \quad (41)$$

with

$$\phi = f_1(1 - a_1 + 3c_1) + 2f_1^*(1 - a_1 + 3c_2) + 3c_3 > 0 \quad (42)$$

$$\begin{aligned} d\bar{l}^d &= -k \cdot d\bar{\tau}^a - l_2d(\dot{m}^a - \dot{m}^*) \\ &= \frac{-k}{\phi}d(g^a - g^*) - l_2d(\dot{m}^a - \dot{m}^*) \end{aligned} \quad (43)$$

with

$$k = (\alpha_3 - l_1f_1) + 2(\alpha_3^* - l_1f_1^*) \quad (44)$$

Using (26) and (27), we find the following solution paths for the average union output level  $y^a$  and the output level in the third country  $y^*$ :

$$y^a = \begin{cases} \bar{y}_0^a + \frac{\alpha_3}{\delta}A_1h_{11}r_1e^{r_1t} & \text{for } 0 < t < T \\ \bar{y}_1^a + \frac{\alpha_3}{\delta}\tilde{A}_1h_{11}r_1e^{r_1t} & \text{for } t > T \end{cases} \quad (45)$$

$$y^* = \begin{cases} \bar{y}_0^* - \frac{2\alpha_3^*}{\delta}A_1h_{11}r_1e^{r_1t} & \text{for } 0 < t < T \\ \bar{y}_1^* - \frac{2\alpha_3^*}{\delta}\tilde{A}_1h_{11}r_1e^{r_1t} & \text{for } t > T \end{cases} \quad (46)$$

<sup>11</sup>The expressions  $\bar{\tau}_0^a$  (and  $\bar{l}_0^d$ ) denote the initial steady state values of  $\tau^a$  ( $l^d$ ) and  $\bar{\tau}_1^a$  ( $\bar{l}_1^d$ ) the values in the new steady state.

with

$$\bar{y}_1^a = \bar{y}_0^a + d\bar{y}^a, \quad d\bar{y}^a = f_1 d\bar{\tau}^a \quad (47)$$

$$\bar{y}_1^* = \bar{y}_0^* + d\bar{y}^*, \quad d\bar{y}^* = -2f_1^* d\bar{\tau}^a \quad (48)$$

## 2.4 Difference system $U_1 - U_2$

The second difference system describes differential macro-economic developments within the monetary union. Defining the internal differences in output and price developments as:

$$y^d = y_1 - y_2, \quad p^d = p_1 - p_2 \quad (49)$$

the IS and price adjustment equations imply:

$$(1 - a_1 + 2b_1 + c_1)y^d = a_{01} - a_{02} + 2b_0 + g_1 - g_2 \quad (50)$$

$$- (a_{21} - a_{22})(i_1 - \dot{p}_1^c) - (2b_3 + c_3)p^d$$

$$y^d = \bar{y}^d + \frac{1}{\delta}\dot{p}^d \quad (51)$$

$$\bar{y}^d = (f_1 + 2f_2)\bar{p}^d \quad (52)$$

Due to  $a_{21} > a_{22}$ , the differential output developments within the union depend on the behaviour of the (common) real interest rate. The union real interest rate can be expressed as the arithmetic mean of the aggregate (world) real interest rate

$$i_r^s = (i_1 - \dot{p}_1^c) + (i^* - \dot{p}^{*c}) \quad (53)$$

and the differential in real interest rates

$$i_r^d = (i_1 - \dot{p}_1^c) - (i^* - \dot{p}^{*c}) = -2(\alpha_1 - \alpha_3^*)\dot{\tau}^a \quad (54)$$

as:

$$(i_1 - \dot{p}_1^c) = \frac{1}{2}(i_r^s + i_r^d) \quad (55)$$

The aggregated real interest rate  $i_r^s$  follows from the aggregation of the IS equations (1), (2), (3) and considering (26), (27):

$$i_r^s = -\frac{\chi}{a_2}\dot{\tau}^a + \frac{c_3}{a_2}\tau^a - \frac{1}{a_2}\left\{(1 - a_1 - c_1)\bar{y}^a + (1 - a_1 + c_2)\bar{y}^* \right. \\ \left. - a_0 - \frac{1}{2}(a_{01} + a_{02}) + c_0 - (g^a + g^*)\right\} \quad (56)$$

with

$$\chi = (1 - a_1 - c_1)\frac{\alpha_3}{\delta} - 2(1 - a_1 + c_2)\frac{\alpha_3^*}{\delta} \quad (57)$$

The system (50) to (56) can be reduced in conjunction with the state equation for  $\tau^a$  (eq. (32)) to a single stable differential equation governing the internal price differential  $p^d$ :

$$\dot{p}^d = r_3(p^d - \bar{p}^d) + \mu \cdot \dot{\tau}^a \quad (58)$$

with

$$r_3 = -\frac{(2b_3 + c_3)\delta}{1 - a_1 + 2b_1 + c_1} < 0 \quad (59)$$

$$\begin{aligned} \mu &= -\frac{\delta\tilde{a}_2}{1 - a_1 + 2b_1 + c_1} \left( 2(\alpha_3^* - \alpha_1) - \frac{\chi}{a_2} + \frac{c_3}{a_2 r_1} \right) \\ &= \frac{4}{3} \frac{\delta\tilde{a}_2}{1 - a_1 + 2b_1 + c_1} \left( (\alpha_1 - \alpha_3^*) + \frac{1 - a_1}{a_2 \delta} (\alpha_3 - \alpha_3^*) \right) \end{aligned} \quad (60)$$

$$\tilde{a}_2 = \frac{1}{2}(a_{21} - a_{22}) > 0 \quad (61)$$

The differential equation (58) cannot be solved independently from the first difference system because the constant  $\mu$  differs from zero due to  $a_{21} > a_{22}$ .<sup>12</sup> (58) then contains the rate of change of the average terms of trade  $\dot{\tau}^a$  as a variable input. The solution path for  $\dot{\tau}^a$  follows from (36), (37) by differentiation with respect to time  $t$ ; for the internal price differential  $p^d$  we find the following continuous solution path:

$$p^d = \bar{p}_0^d + \mu h_{11} r_1 A_1 \frac{1}{r_1 - r_3} (e^{r_1 t} - e^{r_3 t}) \quad \text{for } 0 \leq t \leq T \quad (62)$$

$$\begin{aligned} p^d &= \bar{p}_1^d - d\bar{p}^d e^{r_3(t-T)} + \mu h_{11} r_1 \frac{1}{r_1 - r_3} (A_1 - \tilde{A}_1) e^{(r_1 - r_3)T} e^{r_3 t} \\ &\quad + \mu h_{11} r_1 \frac{1}{r_1 - r_3} \left( -A_1 e^{r_3 t} + \tilde{A}_1 e^{r_1 t} \right) \quad \text{for } t \geq T \end{aligned} \quad (63)$$

We use

$$d\bar{p}^d = \bar{p}_1^d - \bar{p}_0^d = \frac{1}{\psi} (d(g_1 - g_2) - \tilde{a}_2 d\bar{i}_r^s) \quad (64)$$

with

$$\psi = (1 - a_1 + 2b_1 + c_1)(f_1 + 2f_2) + 2b_3 + c_3 > 0 \quad (65)$$

$$d\bar{i}_r^s = \frac{1}{a_2} \left( (c_3 - f_1(1 - a_1 - c_1) + 2f_1^*(1 - a_1 + c_2)) d\bar{\tau}^a + d(g^a + g^*) \right) \quad (66)$$

The time path of the output differential  $y^d$  is derived from (51) by differentiating the solution path for  $p^d$  with respect to time  $t$  and inserting the result in (51). Assuming

<sup>12</sup>In the following, the weights  $\alpha_3$  and  $\alpha_3^*$  in (5), (6), (7) are assumed to satisfy  $\alpha_3^* \leq \alpha_3 \leq 2\alpha_3^*$ . The assumption  $\alpha_3 \leq 2\alpha_3^*$  is according to (10) equivalent to  $\alpha_1 + \alpha_2 \geq \alpha_1^*$  and implies  $\chi < 0$  in (57). The assumption  $\alpha_3 \geq \alpha_3^*$  is - given (60) - sufficient with respect to  $\mu > 0$ .

identical initial values for  $y_1$  and  $y_2$  we arrive due to  $\bar{y}_0^d = 0$  at

$$y^d = \frac{1}{\delta} \mu h_{11} r_1 A_1 \frac{1}{r_1 - r_3} (r_1 e^{r_1 t} - r_3 e^{r_3 t}) \quad \text{for } 0 < t < T \quad (67)$$

$$y^d = \bar{y}_1^d - \frac{1}{\delta} r_3 d\bar{p}^d e^{r_3(t-T)} + \frac{1}{\delta} \mu h_{11} r_1 r_3 \frac{1}{r_1 - r_3} (A_1 - \tilde{A}_1) e^{(r_1 - r_3)T} \cdot e^{r_3 t} \quad (68)$$

$$+ \frac{1}{\delta} \mu h_{11} r_1 \frac{1}{r_1 - r_3} (-A_1 r_3 e^{r_3 t} + \tilde{A}_1 r_1 e^{r_1 t}) \quad \text{for } t > T$$

with

$$\bar{y}_1^d = d\bar{y}^d = (f_1 + 2f_2) d\bar{p}^d \quad (69)$$

If the solution path of the output differential shows within the anticipation period  $0 < t < T$  a point of reversal  $t^*$  (i.e.,  $y^d(t^*) = 0$ ), we find from (67) by setting the expression in brackets equal to zero:

$$t^* = \frac{1}{r_3 - r_1} \ln \left( \frac{r_1}{r_3} \right) \quad (70)$$

If a point in time  $t^*$  exists in the anticipation period  $(0, T)$  in which the output levels are identical across the union, it will exclusively depend on the stable characteristic roots of the two difference systems. Therefore,  $t^*$  will be the same for all exogenous shocks and will not depend on the length of the anticipation period  $T$  between the announcement in  $t = 0$  and the implementation in  $t = T$ .

The solution paths for the average output level  $y^a$  and for the output differential  $y^d$  serve to derive the output developments in the individual member countries by using<sup>13</sup>

$$y_1 = y^a + \frac{1}{2} y^d, \quad y_2 = y^a - \frac{1}{2} y^d \quad (71)$$

## 2.5 Aggregate system $\frac{1}{2}(U_1 + U_2) + NMC$

The aggregate system only serves to determine the evolution of the real money stocks  $m^a - p^a$ ,  $m^* - p^*$  and of nominal interest rates  $i_1$ ,  $i_2$  and  $i^*$ . The aggregation of the

<sup>13</sup>An analogous expression holds for the external terms of trade  $\tau_1$  and  $\tau_2$  where  $\tau^d = \tau_1 - \tau_2 = p_1 - p_2 = p^d$ . The determination of real interest rates  $i_1 - p_1^c$  and  $i^* - p^{*c}$  does not require the solution paths for the internal output and price differentials  $y^d$  and  $p^d$  as they only depend on the rate of change of the average external terms of trade  $\dot{\tau}^a$ .

IS and LM relationships yields the following equation system<sup>14</sup>

$$(1 - a_1 - c_1)y^a + (1 - a_1 + c_2)y^* = \frac{1}{2}(a_{01} + a_{02}) + a_0 - c_0 + g^a + g^* \quad (72)$$

$$+ c_3\tau^a - a_2(2i^* + \dot{e}) + a_2(\dot{p}_1^c + \dot{p}^{*c})$$

$$(m^a - p_1^c + m^* - p^{*c}) = 2l_0 + l_1(y^a + y^*) - l_2(2i^* + \dot{e}) \quad (73)$$

In conjunction with (26), (27) we arrive at the following differential equation describing the development of the (consumer price index based) aggregate real money stock

$$l_c^s = (m^a - p^{c^a}) + (m^* - p^{*c}) \quad (74)$$

which fully captures the dynamic behaviour of the aggregate systems  $\frac{1}{2}(U_1 + U_2) + NMC$ :

$$\dot{l}_c^s = \frac{1}{l_2}(l_c^s - \bar{l}_c^s) + \frac{c_3}{a_2}(\tau^a - \bar{\tau}^a) + \frac{1}{l_2 a_2 \delta} \varepsilon \dot{\tau}^a \quad (75)$$

with

$$\varepsilon = 2\alpha_3^*((1 - a_1 + c_2)l_2 + l_1 a_2) - \alpha_3((1 - a_1 - c_1)l_2 + l_1 a_2) > 0 \quad (76)$$

The real money stock  $l_c^s$  is a jump variable as it contains the forward-looking and flexible nominal exchange rate  $e$ . The corresponding unstable (positive) characteristic root is  $r_2 = 1/l_2$ . As in the case of the differential equation (58) of the difference system  $U_1 - U_2$  we find in the differential equation (75) of the aggregate system again the average terms of trade as a variable input. The dynamic behaviour of the aggregate system can be expressed alternatively via the monetary aggregate  $l^s$  as defined in (23). Due to

$$l_c^s = l^s + (\alpha_3 - 2\alpha_3^*)\tau^a \quad (77)$$

it follows from (75) and (32)

$$\dot{l}^s = r_2(l^s - \bar{l}^s) + \nu \cdot \dot{\tau}^a \quad (78)$$

with

$$\nu = \left( \frac{1}{l_2}(\alpha_3 - 2\alpha_3^*) + \frac{c_3}{a_2} \right) \frac{1}{r_1} + 2\alpha_3^* - \alpha_3 + \frac{1}{l_2 a_2 \delta} \varepsilon \quad (79)$$

As a result of  $r_2 > 0$ , a non-explosive (convergent) solution to (78) requires  $l^s$  to be classified as a jump variable.<sup>15</sup>

<sup>14</sup>Due to  $\alpha_1 = \alpha_2$  it follows  $p_1^c = p_2^c = \frac{1}{2}(p_1^c + p_2^c) = p^{c^a}$ . In contrast with the aggregate systems in Fukuda (1993), p. 594, and in Turnovsky (1986), p. 141, our system (72), (73) does *not* represent a closed economy as it depends on the external terms of trade.

<sup>15</sup>See Turnovsky (2000), p. 147 (Proposition 5.3). This assumption can be justified economically - in particular in the special case  $\alpha_3 = 2\alpha_3^*$  - because this implies via (26) and (27) that the aggregate



On economic grounds, it is reasonable to assume that  $l^s$  remains continuous at the anticipation date and is only allowed to adjust discontinuously in the presence of a policy shock at the implementation date  $T$ . For  $l^s$ , this implies the following general solution:

$$l^s = \bar{l}_0^s + \int_0^t e^{r_2(t-s)} \nu \dot{\tau}^a(s) ds \quad (80)$$

$$= \bar{l}_0^s + \nu h_{11} r_1 A_1 \frac{1}{r_1 - r_2} (e^{r_1 t} - e^{r_2 t})$$

for  $0 \leq t < T$

$$l^s = \bar{l}_1^s + c \cdot e^{r_2 t} + \int_T^t e^{r_2(t-s)} \nu \dot{\tau}^a(s) ds \quad (81)$$

$$= \bar{l}_1^s + \nu h_{11} r_1 \tilde{A}_1 \frac{1}{r_1 - r_2} e^{r_1 t} + \left( c - \nu h_{11} r_1 \tilde{A}_1 \frac{1}{r_1 - r_2} e^{(r_1 - r_2)T} \right) e^{r_2 t}$$

for  $t > T$  and arbitrary  $c \in \mathbb{R}$ .

The convergent solution is given by

$$c = \nu h_{11} r_1 \tilde{A}_1 \frac{1}{r_1 - r_2} e^{(r_1 - r_2)T} \quad (82)$$

(81) then implies for  $t > T$  the solution time path

$$l^s = \bar{l}_1^s + \nu h_{11} r_1 \tilde{A}_1 \frac{1}{r_1 - r_2} e^{r_1 t} \quad (t > T) \quad (83)$$

with  $\bar{l}_1^s = \bar{l}_0^s + d\bar{l}^s$  and

$$d\bar{l}^s = (2\alpha_3^* - \alpha_3 + l_1(f_1 - 2f_1^*)) d\bar{\tau}^a - l_2(2d\bar{i}^* + d\bar{e}) \quad (84)$$

$$d\bar{i}^* = d(\overline{i^* - \bar{p}^{*c}}) + d\bar{p}^{*c} = \frac{1}{2} d\bar{i}_r^s + d\bar{m}^* \quad (85)$$

$$d\bar{e} = d\bar{m}^a - d\bar{m}^* \quad (86)$$

The real money stocks  $m^a - p^a$  and  $m^* - p^*$  can be inferred from the arithmetic mean of  $l^s$  and  $l^d$  (rsp.  $-l^d$ ). As  $l^d$  remains continuous at every point in time,  $m^a - p^a$  and  $m^* - p^*$  jump by an equal amount at the implementation date  $T$ . The size of the change at  $T$  equals half of the change of the aggregate variable  $l^s$  at  $T$ . For  $m^a - p^a$  and  $m^* - p^*$ , we yield the following solutions:<sup>16</sup>

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output level  $y^a + y^*$  equals its long-run equilibrium level  $\bar{y}^a + \bar{y}^*$  at *every* point in time. As long as even temporary deviations from the steady state-output are ruled out, the neoclassical paradigm requires that adjustments have to be borne by the perfectly flexible price level  $p^a + p^*$ . As the nominal money stock  $m^a + m^*$  is given at a point in time this implies that  $l^s$  is a jump variable.

<sup>16</sup>The behaviour of nominal interest rates  $i_1 (= i_2)$  and  $i^*$  is derived by means of the LM equations

$$i_1 - \bar{i}_1 = \frac{1}{l_2} \left\{ -((m^a - p^a) - (\overline{m^a - p^a})) - \alpha_3(\tau^a - \bar{\tau}^a) + l_1(y^a - \bar{y}^a) \right\}$$

For  $0 \leq t < T$  this implies:

$$m^a - p^a = (\overline{m^a - p^a})_0 + \frac{1}{2} \left( \nu h_{11} r_1 \frac{1}{r_1 - r_2} + 1 \right) A_1 (e^{r_1 t} - e^{r_2 t}) \quad (87)$$

$$m^* - p^* = (\overline{m^* - p^*})_0 + \frac{1}{2} \left( \nu h_{11} r_1 \frac{1}{r_1 - r_2} - 1 \right) A_1 (e^{r_1 t} - e^{r_2 t}) \quad (88)$$

For  $t > T$ :

$$m^a - p^a = (\overline{m^a - p^a})_1 + \frac{1}{2} \left( \nu h_{11} r_1 \frac{1}{r_1 - r_2} + 1 \right) \tilde{A}_1 e^{r_1 t} \quad (89)$$

$$m^* - p^* = (\overline{m^* - p^*})_1 + \frac{1}{2} \left( \nu h_{11} r_1 \frac{1}{r_1 - r_2} - 1 \right) \tilde{A}_1 e^{r_1 t} \quad (90)$$

with

$$\begin{aligned} (\overline{m^a - p^a})_1 &= (\overline{m^a - p^a})_0 + d(\overline{m^a - p^a}) \\ &= (\overline{m^a - p^a})_0 + \frac{1}{2} \left( d\bar{l}^s + d\bar{l}^d \right) \end{aligned} \quad (91)$$

$$\begin{aligned} (\overline{m^* - p^*})_1 &= (\overline{m^* - p^*})_0 + d(\overline{m^* - p^*}) \\ &= (\overline{m^* - p^*})_0 + \frac{1}{2} \left( d\bar{l}^s - d\bar{l}^d \right) \end{aligned} \quad (92)$$

### 3 Effects of anticipated monetary and fiscal policies

In the following, we analyze the dynamic effects of monetary and fiscal policies in the union on the member countries  $U_1$  and  $U_2$  and on the third country  $NMC$ . We consider an anticipated increase in the rate of monetary growth in the union ( $d\dot{m} > 0$ ) and a coordinated symmetric fiscal expansion ( $dg_1 = dg_2 > 0$ ). All policies are credibly announced at  $t = 0$  to be implemented at a later date  $T > 0$ .

#### 3.1 Monetary expansion in the union

An isolated increase in the rate of growth of the union money stock  $\dot{m}$  while holding for simplicity monetary growth in the third country constant ( $d\dot{m}^* = 0$ ) leaves in the long run all output and terms of trade variables as well as the real interest rates unchanged. The flexible nominal exchange rate fully insulates the third country from the monetary expansion in the union. Within the union, we find in the long run a

and

$$i^* - \bar{i}^* = \frac{1}{l_2} \left\{ -((m^* - p^*) - (\overline{m^* - p^*})) + 2\alpha_3^*(\tau^a - \bar{\tau}^a) + l_1(y^* - \bar{y}^*) \right\}.$$

proportional change in all nominal variables and due to (43), (84) a permanent decline in the real money stock:

$$dm^a = \frac{1}{2}d\bar{m} = d\bar{p}^a = d\bar{p}_1^c = d\bar{e} = d\bar{i}_1 \quad (93)$$

$$d(\overline{m^a - p^a}) = d\bar{l}^d = d\bar{l}^s = -l_2 dm^a, \quad d(\overline{m^* - p^*}) = 0 \quad (94)$$

Figure 1 illustrates the dynamic effects of an anticipated increase in  $\dot{m}$ . Figure 1(a) shows the phase diagram for the first difference system  $\frac{1}{2}(U_1 - U_2) - NMC$ .<sup>17</sup>  $Q_0$  denotes the initial and  $Q_1$  the final equilibrium. The adjustment process of the state vector  $(\tau^a, l^d)'$ , which is given formally by (36) with (37) and  $A_1 = \tilde{A}_1 > 0$ , starts with a discontinuous jump from  $Q_0$  to  $B$ . It is followed by a movement along a trajectory, which belongs to the unstable arm  $I_0$  of the saddle point  $Q_0$ . From the date of implementation  $T$  onwards, the system moves along the new convergent saddle path  $S_1$  from  $C$  to  $Q_1$ .<sup>18</sup> After the immediate real depreciation ( $\tau^a(0+) < \bar{\tau}_0^a$ ) the economy experiences over time a real appreciation ( $\dot{\tau}^a > 0$ ). This implies according to (54) that the common union-wide level of real interest rates lies below those in the third country. In other words, the real interest differential between the union and the rest of the world remains negative throughout the overall adjustment process ( $i_r^d < 0$ ).

This behaviour of the real interest differential over time is illustrated in Figure 1(c).<sup>19</sup>

Furthermore, (26) and (27) imply a positive output differential viewed from the perspective of the monetary union ( $y^a > y^*$ ). This follows from the results that the average output level in the monetary union  $y^a$  remains continuously above the steady state level  $\bar{y}^a$  and that the output level in the third country experiences a negative spillover effect ( $y^* < \bar{y}^*$ ; see Figure 1(b)).<sup>20</sup> The temporary increase in  $y^a$  is

<sup>17</sup>In the case of a *small* monetary union we find an analogous adjustment process in the  $\tau^a - (m^a - p^a)$  dimension. See Clausen and Wohltmann (2004). The negative slope of the stable arm  $S_0$  (or  $S_1$ ) belonging to the saddle point  $Q_0$  (or  $Q_1$ ) in Figure 1(a) equals  $h_{11}$  from (38).

<sup>18</sup>In the special case  $T = 0$ , i.e., a sudden and therefore unanticipated increase in  $\dot{m}$ , the state vector moves instantaneously and vertically onto the saddle path  $S_1$  (Point  $B'$ ).

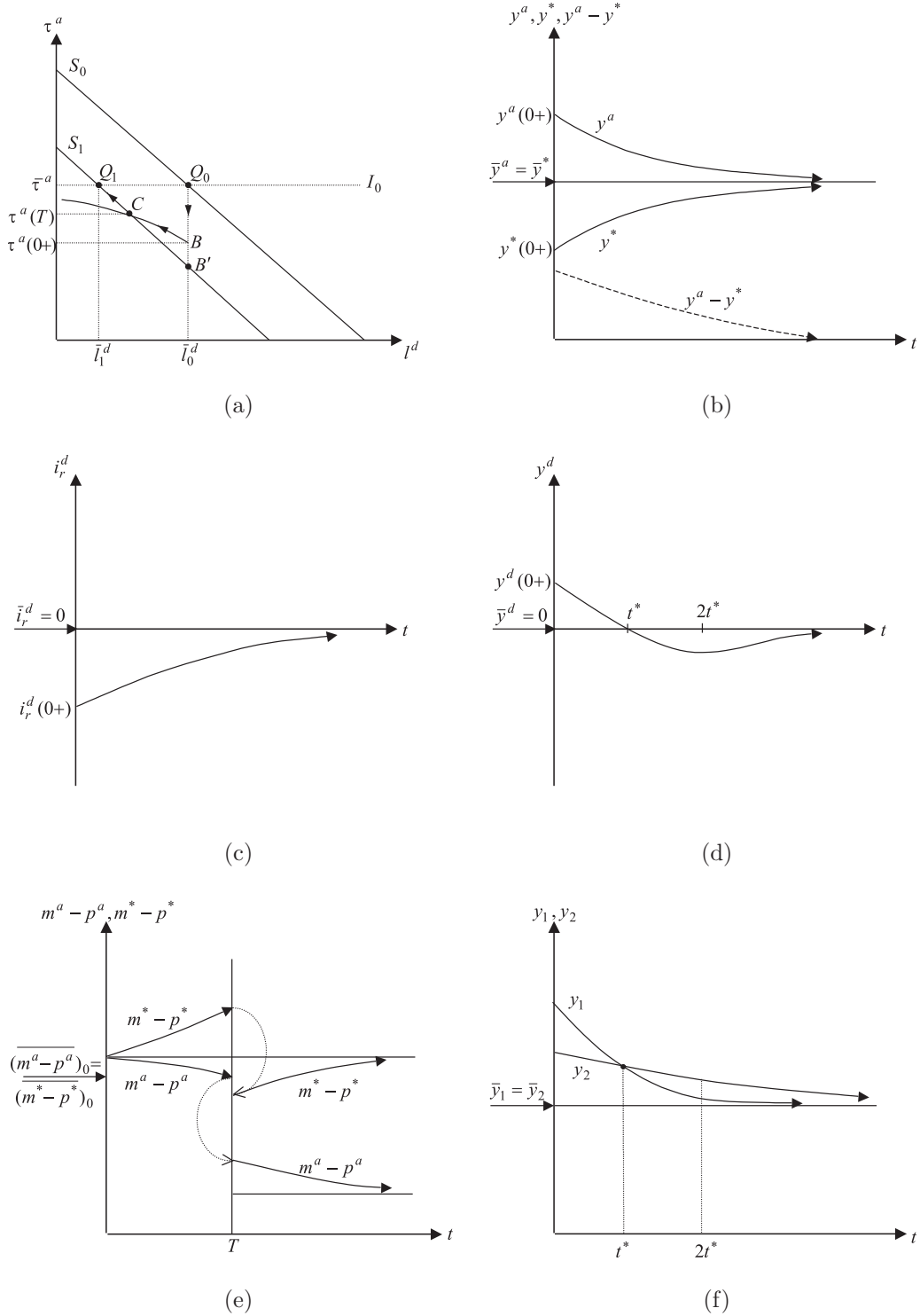
<sup>19</sup>In contrast, under the weak condition  $h_{11} > -1$  the differential in *nominal* interest rates  $i_1 - i^*$  is *positive* throughout the adjustment process, i.e., for  $t < T$  and for  $t > T$ . The following relationship holds:

$$(i_1 - i^*) - (\overline{i_1 - i^*}) = \dot{e} - \bar{e} = -(\dot{\tau}^a + \dot{l}^d)$$

with

$$\dot{\tau}^a + \dot{l}^d = \begin{cases} A_1(r_1(1 + h_{11})e^{r_1 t} - r_2 e^{r_2 t}) & \text{for } 0 < t < T \\ \tilde{A}_1 r_1 (1 + h_{11}) e^{r_1 t} & \text{for } t > T \end{cases}$$

<sup>20</sup>Due to the contraction of output in the NMC, world output  $y_1 + y_2 + y^* = 2y^a + y^* = \bar{y}_1 + \bar{y}_2 + \bar{y}^* + \frac{2}{\delta}(\alpha_3 - \alpha_3^*)\dot{\tau}^a$  moves for all  $t > 0$  either above (for  $\alpha_3 > \alpha_3^*$ ) or below ( $\alpha_3 < \alpha_3^*$ ) the steady state level. This statement holds as  $\dot{\tau}^a > 0$  for all  $t > 0$ .



**Figure 1:** Effects of an anticipated monetary expansion in the union

caused by the real depreciation of the common currency, which improves the trade balance with respect to the third country, and, secondly, by the fall in real interest rates within the union, which normally occurs and boosts real private absorption. In the case that real interest rates *within* the monetary union  $i_1 - \dot{p}_1^c$  fall on impact we find that output in country  $U_1$ , which is characterized by the relatively higher interest sensitivity of private absorption ( $a_{21} > a_{22}$ ), receives initially a stronger boost than output in  $U_2$ . Therefore, the output differential  $y^d = y_1 - y_2$  is initially positive ( $y^d(0+) > 0$ ; see Figure 1(d)).<sup>21</sup>

The size and direction of this asymmetric monetary transmission within the union changes over time. The cyclical asymmetries lead to dynamic adjustments of relative prices within the monetary union and with respect to the third country. These, in turn, start to dampen the initial asymmetries within the union over time and lead in the further course of the adjustment process even to a reversal in the direction of cyclical asymmetries (Figures 1(d), 1(f)). At the date of announcement, we find on impact a higher inflationary pressure in country  $U_1$  than in  $U_2$ . This positive inflation differential ( $\dot{p}^d(0+) > 0$ ) leads to an increase in the internal and external terms of trade  $p_1 - p_2$  and  $\tau_1$  and, over time, to a relatively stronger worsening of the trade balance and reduction in output in  $U_1$  than in  $U_2$ . For a sufficiently large  $t$ , the output differential  $y^d$  changes its sign. This point in time is defined as  $t^*$  (cf. (70)), which can be computed by setting (67) equal to zero.<sup>22</sup> While the diffe-

<sup>21</sup>Using (51), (58):

$$y^d(0+) = \frac{1}{\delta} \dot{p}^d(0+) = \frac{\mu}{\delta} \dot{\tau}^a(0+) > 0 \Leftrightarrow \mu > 0 \Leftrightarrow \alpha_1 - \alpha_3^* + \frac{1-a_1}{a_2\delta}(\alpha_3 - \alpha_3^*) > 0 \quad (\text{vgl. (60)}).$$

According to (53)-(56) this (weak) condition is equivalent to the statement that the real interest rate  $i_1 - \dot{p}_1^c$  immediately falls: In the case  $dm^a > 0$  it follows for all  $t > 0$

$$(i_1 - \dot{p}_1^c) = \frac{1}{2}(\overline{i_r^s})_0 - \frac{2}{3} \left\{ (\alpha_1 - \alpha_3^*) + \frac{1-a_1}{a_2\delta}(\alpha_3 - \alpha_3^*) \right\} A_1 h_{11} r_1 e^{r_1 t}$$

As long as the expression in brackets is positive, for which  $\alpha_3 \geq \alpha_3^*$  constitutes a sufficient condition, we find

$$i_1 - \dot{p}_1^c < \frac{1}{2}(\overline{i_r^s})_0 = (\overline{i_1 - \dot{p}_1^c})_0$$

The solution for the real interest rate in the third country is

$$(i^* - \dot{p}^{*c}) = \frac{1}{2}(\overline{i_r^s})_0 - \frac{4}{3} \left\{ -2(\alpha_1 - \alpha_3^*) + \frac{1-a_1}{a_2\delta}(\alpha_3 - \alpha_3^*) \right\} A_1 h_{11} r_1 e^{r_1 t}$$

Under the assumption that the expression in brackets is negative it follows that

$$i^* - \dot{p}^{*c} > \frac{1}{2}(\overline{i_r^s})_0 = (\overline{i^* - \dot{p}^{*c}})_0 \quad \text{for all } t > 0.$$

<sup>22</sup>In the case of an expansionary monetary policy  $dm > 0$ , we find due to  $A_1 = \tilde{A}_1$  and  $d\tilde{p}^d = 0$  the same value for  $t^*$  by setting in (68)  $y^d$  for  $t > T$  equal to zero. The overall adjustment process in the case of monetary policy is characterized by a single point of reversal in the output differential  $y^d$ .

rence variable  $y^a - y^*$  stays positive throughout the adjustment process, we find the internal output differential  $y^d$  for  $t > t^*$  to be negative. This means that the output  $y_2$ , which is smaller than  $y_1$  up to the point of cyclical reversal  $t^*$ , exceeds the value of  $y_1$  for  $t > t^*$ . In the new steady state, both outputs are equal again and return to their initial values.

Figure 1(e) displays the evolution of the real money stocks  $m^a - p^a$  and  $m^* - p^*$ .<sup>23</sup> In the anticipation period  $0 < t < T$  we derive a continuous fall of  $m^a - p^a$ , which results from the increase in the average rate of inflation in the union ( $\dot{p}^a > \dot{m}_0^a$  for  $0 < t < T$ ). In contrast, the negative output-spillover with respect to the third country causes there a fall in the rate of inflation ( $\dot{p}^* < \dot{m}_0^*$ ). In consequence, the global economy experiences during the anticipation period an inflation differential between the union and the third country. At the date of implementation  $T$ , both real money stocks  $m^a - p^a$  and  $m^* - p^*$  fall by the same amount, which, taken individually, equal half of the decline of the aggregate real money stock  $l^s$  in  $T$ .<sup>24</sup> The union continues to experience an overshooting of the average rate of inflation even after the implementation of the monetary expansion ( $\dot{p}^a > \dot{m}_1^a$  for  $t > T$ ). At the end of the adjustment process average inflation falls back to the the new rate of monetary growth in the union ( $\overline{\dot{p}}_1^a = \dot{m}_1^a$ ). In contrast, inflation in the third country returns in the new steady state to the initial level ( $\overline{\dot{p}}_1^* = \overline{\dot{p}}_0^* = \dot{m}_0^*$ ). The union faces according to (93) a lasting increase in nominal interest rates  $i_1$  and  $i_2$ . This incurs a permanent fall in the real money stock  $m^a - p^a$  in order to maintain an equilibrium in the money market. In contrast, the third country does not experience in the long run a change in  $i^*$  and  $m^* - p^*$ .

### 3.2 Fiscal Expansion in the Union

A simultaneous and identical increase in union government spending while keeping fiscal policy in the third country unchanged (i.e.,  $dg_1 = dg_2 > 0$ ,  $dg^* = 0$ ) leads according to (41), (47), (48) to a lasting increase in the average terms of trade and in the average union output level. In the long run, the third country clearly experiences a negative output-spillover effect ( $d\bar{\tau}^a > 0$ ,  $d\bar{y}^a > 0$ ,  $d\bar{y}^* < 0$ ).<sup>25</sup> A unilateral fiscal expansion in the union induces therefore a permanent cyclical differential in favor of the union relative to the third country ( $d(\bar{y}^a - \bar{y}^*) > 0$ ). The same holds within

<sup>23</sup>These adjustment paths are based on the solution formulas (87)-(90) in conjunction with the realistic assumptions  $\nu > 0$  and  $1 + \nu h_{11} r_1 \frac{1}{r_1 - r_2} > 0$ .

<sup>24</sup>The jump of  $l^s$  in  $T$  satisfies:

$$l^s(T+) - l^s(T-) = d\bar{l}^s - \nu \frac{r_1}{\gamma} d\bar{l}^d < 0 \quad (\text{in the case } \nu > 0).$$

<sup>25</sup>The change in world output in the steady state may be positive or negative:  $d\bar{y}_1 + d\bar{y}_2 + d\bar{y}^* = 2(f_1 - f_1^*)d\bar{\tau}^a \gtrless 0$  for  $f_1 \gtrless f_1^*$ .

the union where we find, despite the symmetric fiscal impulse in both countries, a permanent cyclical differential in favor of the member country with the relatively lower interest sensitivity of private aggregate demand ( $U_2$ ):<sup>26</sup>

$$d\bar{y}^d = (f_1 + 2f_2)d\bar{p}^d = -\frac{1}{\psi}(f_1 + 2f_2)\tilde{a}_2\bar{d}\bar{i}_r^s < 0 \quad (95)$$

The internal price differential between  $U_1$  and  $U_2$  experiences a lasting decline ( $d\bar{p}^d < 0$ ), which improves the internal trade balance to the benefit of  $U_1$ . Furthermore, we derive from (66) a permanent rise in the aggregate real interest rate ( $\bar{d}\bar{i}_r^s > 0$ ). Both member countries experience an identical increase in real and nominal interest rates, which equals due to  $\bar{d}\bar{i}_r^d = 0$  the change in the corresponding rates in the third country:<sup>27</sup>

$$d(\bar{i}_j - \bar{p}_j^c) = \bar{d}\bar{i}_j = \bar{d}\bar{i}^* = d(\bar{i}^* - \bar{p}^{*c}) > 0 \quad (j = 1, 2) \quad (96)$$

The member country  $U_2$  faces an increase in the internal and external terms of trade ( $d(\bar{p}_2 - \bar{p}_1) > 0$ ,  $d\bar{\tau}_2 = d\bar{\tau}^a - \frac{1}{2}d\bar{p}^d > 0$ ). This unambiguously leads according to the long-run supply function (18) to a higher output level in the steady state while country  $U_1$  may under some circumstances also experience a reduction in  $\bar{y}_1$ .<sup>28</sup>

The different reactions of the output variables in the steady state  $\bar{y}^a$  and  $\bar{y}^*$  correspond with the ultimate changes in the real money stocks  $m^a - p^a$  and  $m^* - p^*$ . The long-run differential in the real money stocks amounts to

$$\frac{d\bar{l}^d}{dg^a} = -[(\alpha_3 - l_1f_1) + 2(\alpha_3^* - l_1f_1^*)] \cdot d\bar{\tau}^a \quad (97)$$

The conditions  $\alpha_3 > l_1f_1$  and  $\alpha_3^* > l_1f_1^*$ , which receive empirical support, are sufficient for  $d\bar{l}^d < 0$ . Furthermore, the union experiences a lasting decline in the average real money stock.<sup>29</sup>

The third country faces according (92) a relatively smaller decline in  $(\bar{m}^* - \bar{p}^*)$ , while as a theoretical possibility a steady state *increase* in the real money stock

<sup>26</sup>This follows from the impact of the large monetary union on world interest rates. In the case of a small monetary union, we find  $d\bar{y}^d = 0$  because world real interest rates remain exogenous and ultimately determine the level of real interest rates within the union (Clausen and Wohltmann (2004)).

<sup>27</sup>Assuming  $2f_1^* \geq f_1$ , we find from (66) the unambiguous result  $\bar{d}\bar{i}_r^s > 0$  given that  $dg^a > 0$ . From (28), (29) it then follows that  $d(\bar{y}^a - \bar{y}^*) = (f_1 - 2f_1^*)d\bar{\tau}^a < 0$ .

<sup>28</sup>According to the long-run supply function (17),  $d\bar{p}^d/dg^a < 0$  and  $d\bar{\tau}_1/dg^a < 0$  are jointly sufficient for  $d\bar{y}_1/dg^a < 0$ . The condition  $f_1a_2[(1 - a_1 + 2b_1 + c_1)(f_1 + 2f_2) + 2b_3 + c_3] < 2\tilde{a}_2(f_1 + 2f_2)[f_1c_1 + f_1^*(1 - a_1) + 2f_1^*c_2 + c_3]$  is necessary and sufficient for a decline in  $\bar{y}_1$ . This condition will be fulfilled for a sufficiently large value of  $f_1^*$  or of  $c_2$ . In these cases, we also have  $d\bar{\tau}_1/dg^a < 0$ .

<sup>29</sup>We assume here  $d\bar{l}^s < 0$ . Using (84), (85), (66) it follows that:

$$\frac{d\bar{l}^s}{d(g^a + g^*)} = \frac{1}{\phi} \left\{ 2(\alpha_3^* - l_1f_1^*) - (\alpha_3 - l_1f_1) - \frac{4l_2}{a_2}(f_1c_1 + f_1^*(1 - a_1) + 2f_1^*c_2 + c_3) \right\}$$

cannot be ruled out.

Figure 2 describes the dynamic effects of an anticipated symmetric fiscal expansion in the union. The phase diagram in Figure 2(a) illustrates the adjustment process as  $Q_0 \rightarrow B \rightarrow C \rightarrow Q_1$ . We assume here the case of a permanent decline in the real money stock differential ( $d\bar{l}^d < 0$ ).<sup>30</sup> After the real appreciation on impact the union experiences over time a real depreciation up to the date of implementation  $T$ , which is accompanied by a continuous increase in  $l^d$ . After  $T$ , the adjustment process moves along the new convergent saddle path  $S_1$  referring to the new steady state. For  $t > T$ , we find a real appreciation and a gradual decline in the differential in real money stocks. In view of (54), the real interest differential  $i_r^d$  moves opposite to the rate of change of the average external terms of trade  $\dot{\tau}^a$  (Figure 2(c)). Up to the date of implementation  $T$ , the union experiences a positive real interest differential relative to the third country ( $i_r^d > 0$  for  $t < T$ ); after  $T$  a negative one. Real interest rates in the union  $i_1 - \dot{p}_1^c$  exceed for  $t < T$  the third country level  $i^* - \dot{p}^{*c}$ ; the opposite holds for  $t > T$ . The steady state in the world economy is again characterized by real interest parity  $i_1 - \dot{p}_1^c = i^* - \dot{p}^{*c}$ .

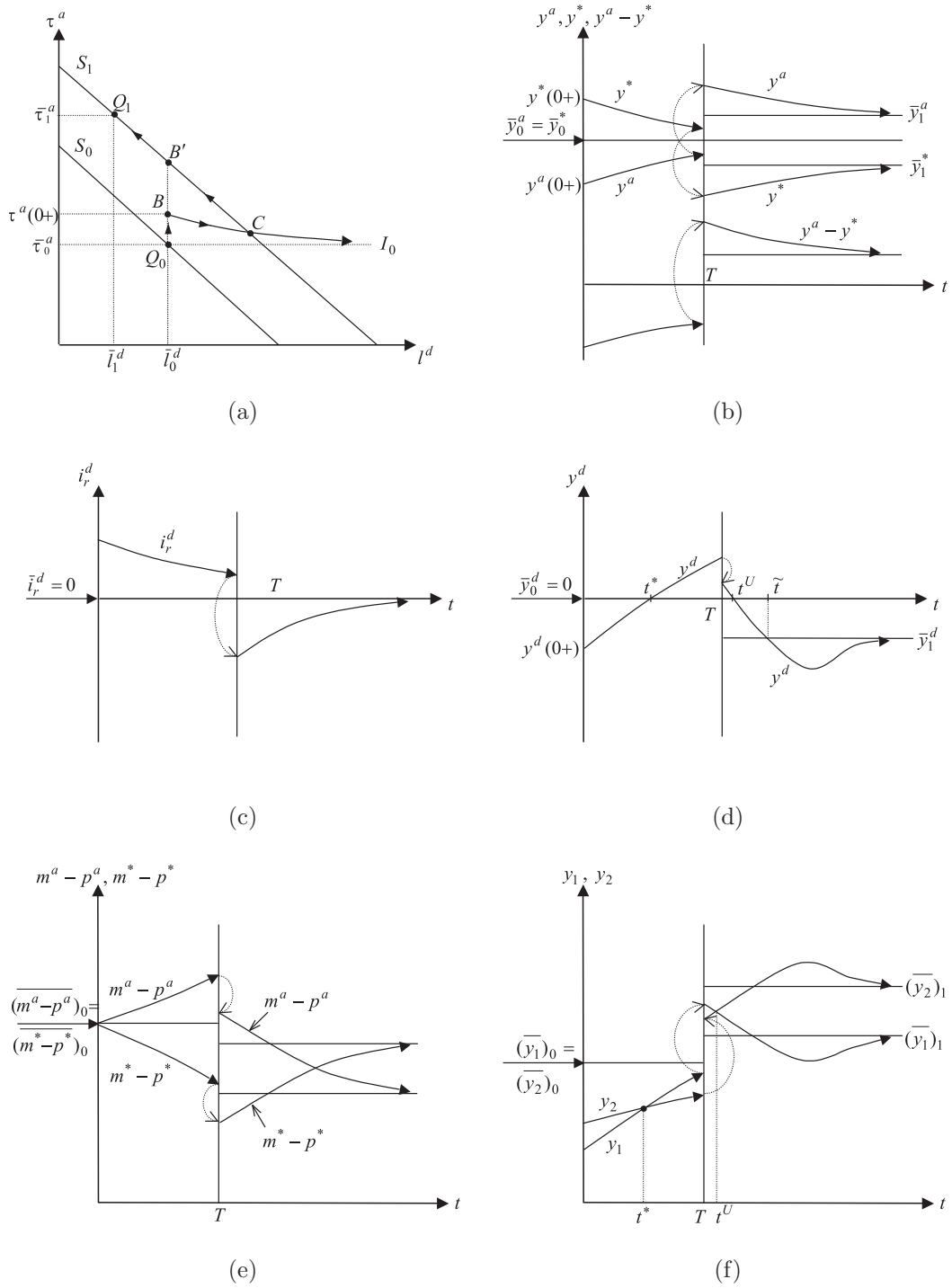
Figure 2(b) describes the output developments  $y^a$  and  $y^*$ . In the anticipation period  $0 < t < T$ , average union output remains below the original equilibrium level ( $y^a < \bar{y}_0^a$ ). This is caused by the initial real appreciation of the common currency and by the likely increase in real interest rates  $i_1 - \dot{p}_1^c$ .<sup>31</sup> At the date of implementation  $T$ , we find a jump increase in  $y^a$ . Afterwards output converges according to (26) and (45) from above to the new steady state level  $\bar{y}_1^a$ . Output in the third country experiences in the anticipation period a positive spillover effect ( $y^* > \bar{y}_0^*$  for  $0 < t < T$ ). At the implementation date  $T$ , we find despite the increase in  $y^a$  a discontinuous fall  $y^*$ , which is caused by an increase in real interest rates  $i^* - \dot{p}^{*c}$ . In the period after implementation  $t > T$  and governed by (27) and (46), output  $y^*$  approaches from below its new and lower steady state level ( $\bar{y}_1^* < \bar{y}_0^*$ ). Regarding the global cyclical differential between the union and the third country, which is technically represented by  $y^a - y^*$ , we find it to be negative up to  $T$ . In contrast, for  $t > T$  and as a result of the unilateral symmetric fiscal expansion, the union experiences a permanent increase in output relative to the rest of the world.<sup>32</sup> moves in the case  $\alpha_3 > \alpha_3^*$  similarly to  $y^a - y^*$ , i.e., for  $t < T$  we find a contraction and for  $t > T$  a development above the new steady state level. In the case  $\alpha_3 < \alpha_3^*$ , the adjustment process mirrors the previous evolution.

<sup>30</sup>Note that in the solution form (36), (37) belonging to the state vector  $(\tau^a, l^d)'$  we have  $\tilde{A}_1 > 0 > A_1$ . In the special case of an unanticipated symmetric fiscal expansion (i.e.,  $T = 0$ ), the relevant constant is  $\tilde{A}_1 = -d\bar{l}^d$  and the adjustment process is given by the time path  $Q_0 \rightarrow B' \rightarrow Q_1$ .

<sup>31</sup> $i_1 - \dot{p}_1^c$  necessarily rises as long as the constant  $\mu$  in (60) is positive. A sufficient condition for this scenario is  $\alpha_3 \geq \alpha_3^*$ .

<sup>32</sup>The world output level  $y_1 + y_2 + y^* = 2y^a + y^* = \bar{y}_1 + \bar{y}_2 + \bar{y}^* + \frac{2}{\delta}(\alpha_3 - \alpha_3^*)\dot{\tau}^a$





**Figure 2:** Anticipated symmetric fiscal expansion in the union

A permanent real output differential also happens within the monetary union. Under the realistic assumption that real interest rates increase on impact, member country  $U_1$  faces due to  $a_{21} > a_{22}$  a relatively stronger recession than  $U_2$  ( $y^d(0+) < 0$ ).<sup>33</sup> Correspondingly, (51), (58) imply a negative inflation differential ( $\dot{p}^d(0+) < 0$ ) such that  $U_1$  faces a fall in its internal and external terms of trade  $p_1 - p_2$  and  $\tau_1$ . As a result,  $y_1$  and the internal output differential  $y^d$  increase. In the course of the adjustment process  $y^d$  may change its sign and turn positive. In contrast with the case of monetary policy, a reversal in the internal cyclical differential only occurs as long as the period between the announcement and the implementation of the symmetric fiscal expansion is sufficiently long. In this case, the date of cyclical reversal  $t^*$  given by (70) satisfies  $t^* < T$ . This case is illustrated in the Figures 2(d) and 2(f).<sup>34</sup> At the date of implementation  $T$ , we usually find a fall of  $y^d$  as - under regular circumstances - the real interest rate  $i_1 - \dot{p}_1^c$  increases. This dampens the stimulating effects from the symmetric fiscal impulse in  $T$  and leads to a weaker expansion in  $y_1$  relative to  $y_2$ . In the case  $t^* < T$ , the internal output differential  $y^d$  necessarily runs through another sign reversal in the further course of the adjustment process ( $t^U \geq T$ ) as the new steady state level  $\bar{y}_1^d$  is negative. For  $t > t^U$ , we find  $y^d < 0$ , as in the beginning of the adjustment process  $0 < t < t^*$ . For a sufficiently large  $t$  (i.e., for  $t > \tilde{t}$  ( $\geq t^U$ ) in Figure 2(d))  $y^d$  approaches  $\bar{y}_1^d$  from below.<sup>35</sup> This implies according to (51) that the inflation differential  $\dot{p}^d$ , which remains positive over  $t^* < t < \tilde{t}$ , turns negative toward the end of the adjustment process.

The negative inflation differential reduces over time the internal terms of trade  $p^d$ , which implies an increase in  $y^d$  and therefore a convergence toward the new steady state value  $\bar{y}_1^d$  from below. The behavior of the output differential implies that output  $y_1$  in  $U_1$  remains above the level in  $U_2$  only over the period  $t^* < t < t^U$ . Figure 2(f) summarizes the output developments  $y_1$  and  $y_2$  over time and shows that the symmetric fiscal impulse leads to asymmetric cyclical developments across the monetary union.

The development of the real money stocks  $m^a - p^a$  and  $m^* - p^*$  is provided in Figure 2(e).<sup>36</sup> The anticipation period  $0 < t < T$  is characterized by a gradual decline in the average rate of inflation in the union ( $\dot{p}^a < \dot{m}_0^a$ ) and by a gradual increase in inflation in the third country ( $\dot{p}^* > \dot{m}_0^*$ ). This can be rationalized by the recession in the union ( $y^a < \bar{y}_0^a$ ) and by the nevertheless positive output spillover to the third country during this period ( $y^* > \bar{y}_0^*$ ). Correspondingly, we find a continuous increase in  $m^a - p^a$  and a fall in  $m^* - p^*$ . At the date of implementation  $T$ , both

<sup>33</sup>This result also holds in the case of a small monetary union (Clausen and Wohltmann 2004).

<sup>34</sup>In the case  $T < t^*$ , i.e., with a fairly brief anticipation period, we find in most circumstances that  $y^d$  does not experience a sign reversal because it falls discontinuously at  $T$  and  $\bar{y}_1^d < 0$ .

<sup>35</sup>In the special case  $\tilde{t} = t^U = T$  we find  $y^d < \bar{y}_1^d$  for all  $t > T$  and  $y_2 > (\bar{y}_2)_1 > y_1$  for all  $t > T$ .

<sup>36</sup>We continue to assume in (87)–(90)  $\nu > 0$  and  $1 + \nu h_{11} r_1 \frac{1}{r_1 - r_2} > 0$ .

real money stocks fall by the same amount.<sup>37</sup>

For  $t > T$  we derive a continuous decline in  $m^a - p^a$ , which results from the increase in the rate of inflation in the union ( $\dot{p}^a > \dot{m}_0^a$ ). In the third country,  $m^* - p^*$  gradually increases for  $t > T$  because the fall in  $y^*$  generates a reduction in the rate of inflation *NMC* ( $\dot{p}^* < \dot{m}_0^*$ ). The new steady state is characterized by a lasting reduction in  $m^a - p^a$  compared with the initial equilibrium value. In comparison, the fall in  $m^* - p^*$  in the third country is quantitatively smaller as its output  $\bar{y}^*$  and its related long-run money demand decline.

### 3.3 Intertemporal international coordination of monetary policy

The previous section demonstrated that a unilateral symmetric fiscal expansion in the union of the form  $dg_1 = dg_2 > 0$  causes asymmetric output developments across the world economy for  $y^a$  and  $y^*$  (see Figure 2). Moreover, this coordinated symmetric fiscal expansion also causes cyclical asymmetries across the monetary union. This raises the question whether some form of international coordination of *monetary* policies might be able to dampen or even neutralize the asymmetric dynamics in the world economy as well as within the union. More specifically, we analyze whether the asymmetric output dynamics arising from anticipated demand shocks can be encountered by an appropriate design of monetary policies in the union and in the third country such that the system is stabilized at the output levels in the respective steady states.

The primary goal of monetary policy is assumed to be price stability. Price stability is operationalized in our current framework by the requirement that consumer price inflation rates  $\dot{p}_1^c (= \dot{p}_1^{c^a})$  and  $\dot{p}^{*c}$  are not allowed to exceed the initial equilibrium levels  $(\bar{p}_1^c)_0$  and  $(\bar{p}^{*c})_0$ , which are determined in turn by the respective original rates of monetary growth  $\dot{m}_0^a$  and  $\dot{m}_0^*$ :

$$\dot{p}_1^c \leq (\bar{p}_1^c)_0 = \dot{m}_0^a, \quad \dot{p}^{*c} \leq (\bar{p}^{*c})_0 = \dot{m}_0^* \quad (98)$$

The current announcement of future fiscal policies does not only cause effects from its ultimate implementation but also already in the anticipation period  $0 < t < T$ , as long as the announcement is considered as credible by the private sector. A complete stabilization in both, the anticipation period  $0 < t < T$  and in the period after implementation  $t > T$ , requires a design of monetary policy, which is

<sup>37</sup>We assume  $d\bar{l}^s/dg^a < 0$ . Then, the jump of  $l^s$  in  $T$  turns out to be unambiguously negative:

$$l^s(T+) - l^s(T-) = d\bar{l}^s - \nu \frac{r_1}{\gamma} d\bar{l}^d < 0$$

able to neutralize the announcement and implementation effects of anticipated fiscal policies.

More formally, complete stabilization of the output variables  $y^a$  and  $y^*$  by use of the monetary policy instruments  $\dot{m}^a$  and  $\dot{m}^*$  will only be achieved if the constants  $A_1$ ,  $A_2$  and  $\tilde{A}_1$  in the solution to the difference system  $\frac{1}{2}(U_1 + U_2) - NMC$  given by (39), (40) are set equal to zero. In this case there are no dynamic adjustment processes. The state variables  $\tau^a$  and  $l^d$  as well as the output variables  $y^a$  and  $y^*$  remain tied to their initial equilibrium levels  $(\bar{\tau}_0^a, \bar{l}_0^d, \bar{y}_0^a, \bar{y}_0^*)$  up to the implementation date  $T$  and jump at the implementation date  $T$  into the new steady state  $(\bar{\tau}_1^a, \bar{l}_1^d, \bar{y}_1^a, \bar{y}_1^*)$ . In the long run, monetary policy can only affect the variable  $\bar{l}^d$ . The steady state values of the real variables  $\tau^a$ ,  $y^a$  and  $y^*$  do not depend on  $\dot{m}^a$  and  $\dot{m}^*$ . It follows immediately that monetary policy cannot neutralize the steady state effects of fiscal policy on  $\bar{\tau}^a$ ,  $\bar{y}^a$  and  $\bar{y}^*$ .

The condition  $A_1 = A_2 = 0$  is necessary and sufficient to fully offset the announcement effects of fiscal policy. Due to  $A_1 = -A_2$  it follows in the case  $dg_1 = dg_2 > 0$  and  $dg^* = 0$  from (39), (41), (43):

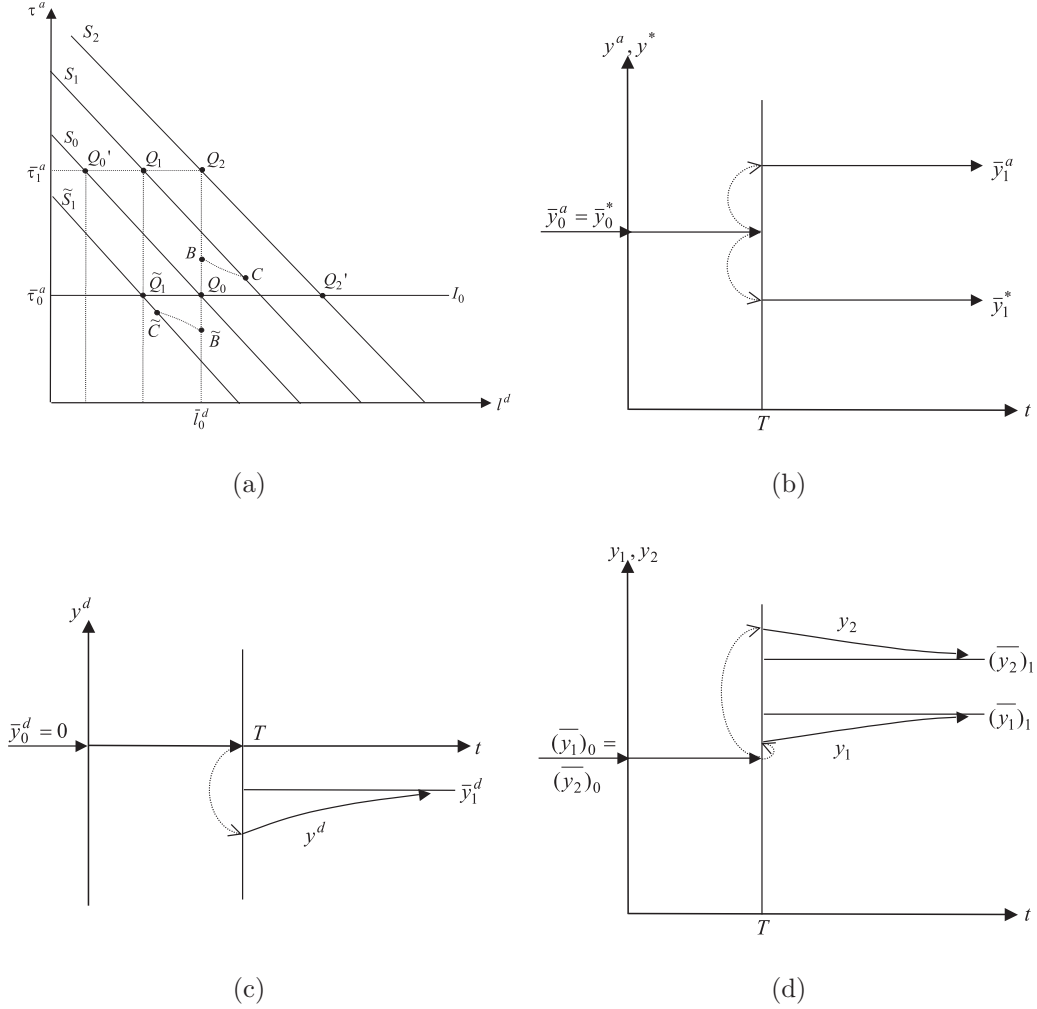
$$d(\dot{m}^a - \dot{m}^*) = -\frac{1}{l_2\phi} \left( k + \frac{1}{h_{11}} \right) dg^a > 0 \quad (99)$$

The constants  $h_{11}$ ,  $\phi$  and  $k$  are defined in (38), (42) and (44). We find  $k + 1/h_{11} < 0$  irrespective of the sign of  $k$ ; in the case  $dg^a > 0$  the right hand side of (99) follows to be clearly positive.

The primary goal of price stability (98) is assumed to only allow for a contractionary or a passive monetary policy in the union and in the third country (i.e.,  $d\dot{m}^a \leq 0$ ,  $d\dot{m}^* \leq 0$ ) in their efforts to dampen the cyclical effects of fiscal policy at home and abroad. With this side condition, there remains only one way to counteract the announcement effects from anticipated fiscal policy in the union. The central bank of the third country has to announce in  $t = 0$  that it will lower its rate of growth of the money stock  $\dot{m}^*$  at the implementation date  $T$ :

$$d\dot{m}^{*ann} = \frac{1}{l_2\phi} \left( k + \frac{1}{h_{11}} \right) dg^a < 0 \quad (100)$$

If this reaction function of monetary policy in the third country is considered by the private sector in all three countries  $U_1$ ,  $U_2$  and  $NMC$  as credible it will lead to anticipation effects. It implies that both difference systems as well as the aggregate system remain in their initial steady states up to the implementation date  $T$ . In particular, all output variables and rates of inflation remain constant up to  $T$ . In the  $\tau^a - l^d$ -phase diagram in Figure 3(a) the initial announcement and final implementation of the monetary policy rule (100) lead in conjunction with the



**Figure 3:** Stabilization by intertemporal international coordination of monetary policy

symmetric fiscal expansion in the union  $dg_1 = dg_2 > 0$  and an unchanged monetary policy in the union ( $dm^a = 0$ ) to the following adjustment: the state vector  $(\tau^a, l^d)'$  remains up to  $T$  in the original steady state  $Q_0$  and converges subsequently along the stable saddle path, which belongs to the original equilibrium  $S_0$ , to the long-run equilibrium  $Q'_0$ .<sup>38</sup> In comparison with the new steady state  $Q_1$  with a passive monetary policy ( $dm^* = dm^a = 0$ ), we find here a larger decline in the difference in real money stocks  $\bar{l}^d$ . Moreover, the implementation of the contractionary monetary policy (100) implies for  $t > T$  that the output fluctuations from fiscal policy, i.e., the deviations of the output variables  $y^a$  and  $y^*$  from their new steady state values  $\bar{y}_1^a$  and  $\bar{y}_1^*$ , are *magnified*. The central bank faces an intertemporal tradeoff in its stabilization policies. The monetary policy rule (100), which stabilizes the system

<sup>38</sup>In isolation, the symmetric fiscal expansion leads in the  $\tau^a - l^d$ -phase diagram to the adjustment process  $Q_0BCQ_1$  (where we assume  $d\bar{l}^d/dg^a < 0$ , i.e.,  $k > 0$ ). The stabilizing monetary policy (100) causes in isolation an adjustment process up to  $T$ , which represents a mirror image ( $Q_0\tilde{B}\tilde{C}$ ) and for  $t \geq T$  from  $\tilde{C}$  to  $\tilde{Q}_1$  following  $\tilde{S}_1$ . The simultaneous consideration of fiscal and monetary policy immediately leads the system to the equilibrium  $Q'_0$ .

during the anticipation period  $t < T$ , *destabilizes* output developments for  $t > T$ .<sup>39</sup>

However, a complete stabilization of output in all countries can still be achieved despite the monetary policy rule (100) if the union central bank adopts unexpectedly, and therefore unanticipated by the private sector, a contractionary stance ( $d\dot{m}^a < 0$ ) in  $T$ . The appropriate size of the unanticipated reduction in  $\dot{m}^a$  can be derived from the stability condition  $d\bar{l}^d = 0$ :<sup>40</sup>

$$d\dot{m}^{a\text{unant}} = \frac{1}{l_2} \left( -\frac{k}{\phi} dg^a + l_2 d\dot{m}^{*\text{ann}} \right) = \frac{1}{l_2 \phi h_{11}} dg^a < d\dot{m}^{*\text{ann}} \quad (101)$$

These results can be illustrated in the  $\tau^a - l^d$ -diagram where the unanticipated fall of  $\dot{m}^a$  causes in isolation the adjustment process  $Q_0 Q_2 Q'_2$  (Figure 3(a)). The increase of  $\bar{l}^d$  (the distance  $\overline{Q_0 Q'_2}$ ) is identical with the fall of  $\bar{l}^d$ , which follows from the reduction of  $\dot{m}^*$  and the joint fiscal expansion in the union (distance  $\overline{Q_2 Q'_0}$ ). As all three policy measures are carried out simultaneously at  $T$ , the state vector  $(\tau^a, l^d)'$  jumps instantaneously and vertically into the new long run equilibrium  $Q_2$ . The difference system  $\frac{1}{2}(U_1 + U_2) - NMC$  does not experience any further adjustment processes, neither for  $t < T$  nor for  $t > T$  (Figure 3(b)).<sup>41</sup>

In consequence, an intertemporal international coordination of monetary policies allows for a complete stabilization of the difference system  $\frac{1}{2}(U_1 + U_2) - NMC$ : In the case of an anticipated unilateral fiscal expansion in the union being announced in  $t = 0$  for implementation in  $T > 0$ , cyclical dynamics between the monetary union and the third country can be completely avoided if (1) monetary policy in the third country announces in  $t = 0$  for  $T > 0$  a reduction of its rate of growth of the money stock  $\dot{m}^*$  and (2) the union central bank (ECB) adopts unexpectedly in  $T$  a contractionary stance of monetary policy. The reduction of monetary growth in the union needs to be larger than the reduction of  $\dot{m}^*$ . Furthermore, the reduction of  $\dot{m}^a$  has to be unanticipated.<sup>42</sup> Both monetary policies lead to a permanent decline in

<sup>39</sup>The value of the constant  $\tilde{A}_1$ , which is defined by (40) and positive in the case  $dg^a > 0$ , will rise due to  $A_1 = 0$ . From (45), (46) we then find for  $t > T$  that the positive difference  $y^a - \bar{y}_1^a$  increases in comparison with the case of a passive monetary policy while the for  $t > T$  negative difference  $y^* - \bar{y}_1^*$  is smaller.

<sup>40</sup>In the case of a sudden and therefore unanticipated policy it is necessary to set the date of implementation  $T$  in (40) equal to zero. The stability condition for  $t > T$ , i.e.,  $\tilde{A}_1 = 0$ , is then equivalent to  $d\bar{l}^d = 0$ . The policy rule for  $\dot{m}^a$  then follows from (43) and (100).

<sup>41</sup>The implementation of the monetary policy rules (100), (101) simultaneously ensures the complete stabilization of the aggregate system  $\frac{1}{2}(U_1 + U_2) + NMC$ , i.e., prevents dynamic adjustment processes in the real money stocks  $m^a - p^a$  and  $m^* - p^*$ .

<sup>42</sup>In the case that the anticipated fiscal expansion only occurs in the third country ( $dg^* > 0$ ,  $dg_1 = dg_2 = 0$ ), the expression  $dg^a$  in (99) has to be replaced by  $-dg^*$ . In this case, complete stabilization of the difference system  $\frac{1}{2}(U_1 + U_2) - NMC$  can be achieved by switching the roles of the two central banks ECB and NMC (i.e., a previously announced and therefore anticipated reduction of  $\dot{m}^a$  and an unanticipated reduction in  $\dot{m}^*$  being larger than the decline of  $\dot{m}^a$ ). This result also holds in the case of an anticipated fiscal contraction in the union ( $dg^a < 0$ ,  $dg^* = 0$ ).

consumer price inflation and are therefore consistent with the primary goal of price stability.<sup>43</sup>

The monetary policies (100), (101) completely stabilize the difference system  $\frac{1}{2}(U_1 + U_2) - NMC$  and, furthermore, dampen the output fluctuations within the monetary union (Figure 3(d)). As a result of  $A_1 = 0 = \tilde{A}_1$  and in conjunction with (62), (63) as well as (67), (68), we find for the solution paths  $p^d$  and  $y^d$  of the difference system  $U_1 - U_2$  that the difference variables  $p^d$  and  $y^d$  remain tied to their initial equilibrium values  $\bar{p}_0^d$  and  $\bar{y}_0^d$  up to the implementation date  $T$ . For  $t > T$ , both variables converge monotonically toward their new steady state values  $\bar{p}_1^d$  and  $\bar{y}_1^d$ :

$$p^d = \bar{p}_1^d - d\bar{p}^d e^{r_3(t-T)} \quad \text{for } t > T \quad (102)$$

$$y^d = \bar{y}_1^d - \frac{1}{\delta} r_3 d\bar{p}^d e^{r_3(t-T)} \quad \text{for } t > T \quad (103)$$

At  $T$ ,  $y^d$  falls discontinuously. Afterwards it converges monotonically from below toward the new, lower steady state value  $\bar{y}_1^d$  (Figure 3(c)). The difference variable  $y^d$  does *not* experience a sign reversal anymore ( $y^d < 0$  for all  $t > T$ ). This corresponds with monotonic adjustment paths of the output variables  $y_1$  and  $y_2$  after the implementation date  $T$ :

$$y_1 = (\bar{y}_1)_1 - \frac{1}{\delta} r_3 d\bar{p}^d e^{r_3(t-T)} \quad \text{for } t > T \quad (104)$$

$$y_2 = (\bar{y}_2)_1 + \frac{1}{\delta} r_3 d\bar{p}^d e^{r_3(t-T)} \quad \text{for } t > T \quad (105)$$

After the jump at  $T$ ,  $y_2$  converges from above to its new, higher level  $(\bar{y}_2)_1$  while  $y_1$  converges from below. Figure 3(d) assumes an increase in  $\bar{y}_1$ , while the effect is theoretically ambiguous. Due to  $d\bar{y}^d < 0$ , this increase must be smaller than the corresponding increase in  $\bar{y}_2$ .

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<sup>43</sup>Note that  $d\bar{p}_1^c = dm^a < d\bar{p}_1^{*c} = dm^* < 0$ . The asymmetric reductions in the rates of monetary growth dampen according to (93) and (96) to different degrees the symmetric increase in nominal interest rates  $\bar{i}_1$  and  $\bar{i}^*$  following from the fiscal expansion. This results in a permanent differential in nominal (but not in real) interest rates between the union and the third country:

$$d\bar{i}_1 - d\bar{i}^* = d\bar{e} = dm^a - dm^* = -\frac{1}{l_2\phi} dg^a < 0$$

Note that

$$d\bar{i}_1 = d(\overline{i_1 - \bar{p}_1^c}) + d\bar{p}_1^c = \frac{1}{2} d\bar{i}_r^s + dm^a, \quad d\bar{i}^* = \frac{1}{2} d\bar{i}_r^s + dm^*$$

## 4 Summary

This paper analyzes the dynamic effects of anticipated monetary and fiscal policies in a large asymmetric monetary union. We develop a macroeconomic three-country model with sluggish price adjustment in the tradition of the expectations augmented Phillips curve. Expectations formation concerning inflation and the exchange rate is assumed rational. Two countries form a large monetary union, which is characterized on the demand side by asymmetric interest rate transmission. The solution methodology draws upon the decomposition method by Aoki (1981) and its generalization in Fukuda (1993). The complete and somewhat complex model is disaggregated by suitable transformations into three sub-systems - two difference systems and one aggregate system. The first difference system has a saddle path equilibrium and describes the difference between the average development in the union and the macroeconomic developments in the third country. It can be solved independently from the other two sub-systems and generates the solution paths for the average external terms of trade, the average output level in the union and for output in the third country. The second difference system models the differential macroeconomic developments within the monetary union. It is again independent from the aggregate system. However, as it contains the rate of change of the average terms of trade as a variable input, it is not independent from the first difference system. The second difference system determines the solution paths for the internal price and output differential. Output developments in both member countries are recovered by means of the solution paths for the output variables in both difference systems. The aggregate system is derived by the aggregation of the average developments in the union and the corresponding developments in the third country. Its solution is independent from the second difference system. However, it also contains the rate of change of the average external terms of trade as an input variable. The aggregate system generates the solution paths for the real money stocks and for nominal interest rates in all three countries.

An anticipated increase in monetary growth in the union leads to a real depreciation of the common currency, a decline in real interest rates and a temporary expansion of average output in the union. The international spillover effect is negative - the third country experiences a temporary contraction in output. Furthermore, we find asymmetric output developments within the monetary union, which result from real interest rate changes and different interest sensitivities of real private absorption. The boost in member country outputs turns out to be relatively stronger in country 1 where the interest sensitivity of aggregate demand is by assumption relatively higher. In the course of the adjustment process, differential cyclical developments within the union cause differential relative price developments, which lead over time to a reversal in the relative effectiveness of monetary policy on member



country outputs. It can be shown analytically that there exists only one point of reversal in the relative effectiveness of monetary policy on member country outputs, i.e., where the common monetary policy starts to be more effective in the country with the relatively smaller interest sensitivity of aggregate demand. This point of reversal only depends on the stable eigenvalues of the system. In the long run, output and relative prices in all three countries return to their initial equilibrium values. The monetary union merely experiences corresponding adjustments in the inflation rates and a lasting fall in the real money stock, while all real and nominal variables in the third country remain unchanged.

An anticipated symmetric fiscal expansion in the union leads during the anticipation period to a real appreciation and an increase in real interest rates, which trigger a temporary contraction of average output in the union. Output in the third country rises initially. After the implementation of fiscal policy, average union output rises permanently beyond its initial equilibrium level while output in the third country experiences a lasting contraction. We also find asymmetric output developments within the monetary union. In the short as well as in the long run, output in the member country with the lower interest sensitivity of aggregate demand is higher than in country 1. As long as the time span between the anticipation and the implementation of the symmetric fiscal expansion is sufficiently long, differential inflation developments worsen the internal and external terms of trade of the member country with the relatively higher interest sensitivity of aggregate demand such that by improvement of its international competitiveness its output is temporarily higher than in the other member country.

An intertemporal international coordination of monetary policy, which is required to be consistent with the primary goal of price stability, allows to prevent output fluctuations and differential macroeconomic developments resulting from the anticipated symmetric fiscal expansion. The average output level in the union and in the third country can be continuously kept with the following policy mix at their respective steady state levels: the central bank of the third country credibly announces in  $t = 0$  for  $T > 0$  a reduction of its rate of monetary growth and implements this policy jointly with the fiscal expansion in the union. Moreover, the union central bank simultaneously conducts in  $T > 0$  an unanticipated contraction of its rate of monetary growth, which goes beyond the respective reduction in the third country. This policy mix removes all anticipation effects and keeps therefore up to the implementation date the output levels in all three countries at their initial equilibrium levels. At the date of implementation of all fiscal and monetary policy measures, average output in the union and in the third country immediately jump to their new steady state values. Within the union, both member country outputs move in a monotonic fashion to their respective new steady state levels.

In conclusion, our paper extends the previous literature in various dimensions:

First, we apply the decomposition methodology pioneered by Aoki (1981) and applied to the symmetric N-country monetary union by Fukuda (1993) to the case of an asymmetric monetary union with different interest rate transmission. Second, while most previous work considers a small monetary union we explicitly solve the model of a *large* monetary union. We investigate both, the macroeconomic dynamics from anticipated fiscal shocks as well as the macroeconomic control options and reaction functions of monetary policy. We show that a problem of *national* policy coordination - how national monetary policy ought to respond to anticipated national fiscal policy shocks - can be partly solved by an *intertemporal international* policy coordination. Policy coordination has to be *international* because it can be shown on the basis of explicit solution of our model that national monetary policy alone is unable to absorb the anticipated fiscal shock. Policy coordination has to be *intertemporal* as it needs to refer to actions at the announcement date  $t$  and the implementation date  $T$  in order to counteract the effects from both, the announcement *and* from the implementation of fiscal policy. Interestingly, the implications for the optimal monetary policies at home and abroad are very different and also depend on the direction of the fiscal impulse: given an anticipated fiscal **expansion** at home, national monetary policy has to follow an *unanticipated* expansion in  $T$  while the foreign country needs to conduct a specifically designed *anticipated* monetary policy with an announcement at  $t = 0$  for an implementation in  $T > 0$ . Given an anticipated fiscal **contraction** in the home country, national monetary policy has to make an announcement at  $t = 0$  for a contraction in  $T > 0$  while foreign monetary policy is required to follow an unanticipated contraction in  $T$ .

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