

Bläsi, Albrecht; Requate, Till

Working Paper

Learning-by-Doing with Spillovers in Competitive Industries, Free Entry, and Regulatory Policy

Economics Working Paper, No. 2005-09

Provided in Cooperation with:

Christian-Albrechts-University of Kiel, Department of Economics

Suggested Citation: Bläsi, Albrecht; Requate, Till (2005) : Learning-by-Doing with Spillovers in Competitive Industries, Free Entry, and Regulatory Policy, Economics Working Paper, No. 2005-09, Kiel University, Department of Economics, Kiel

This Version is available at:

<https://hdl.handle.net/10419/21997>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Learning-by-Doing with Spillovers in Competitive Industries, Free Entry, and Regulatory Policy

by Albrecht Bläsi and Till Requate



Christian-Albrechts-Universität Kiel

Department of Economics

Economics Working Paper

No 2005-09



Learning-by-Doing with Spillovers in Competitive Industries, Free Entry, and Regulatory Policy

by

Albrecht Bläsi and Till Requate*

Abstract

We study the impact of learning-by-doing with spillovers in competitive markets with free market entry. Within a two period model, we consider first the case where fixed costs are incurred only once, and entry is once and for all. In the second case fixed costs are incurred in each period, and both market exit after the first period and late entry in the second period is possible. For the first case first best allocations can only be decentralized by subsidizing output in the first period and additionally paying an entry premium. If exit and late entry are possible and if market exit by some firms is socially optimal, the optimal policy scheme requires a nonlinear output subsidy which serves to discriminate between exiting and staying firms. We further investigate the comparative statics effects of the different policy instruments.

JEL classification: H23, L11

Keywords: learning-by-doing, spillovers, regulatory policy

*Department of Economics, University of Kiel, Olshausenstrasse 40, 24118 Kiel, Germany, email: blaesi@bwl.uni-kiel.de, requate@bwl.uni-kiel.de

Contents

1	Introduction	3
2	Entry Once and for All	6
2.1	The Basic Model	6
2.2	The Social Optimum	7
2.3	The Optimal Policy in a Competitive Industry	8
2.4	Comparative Statics	10
3	Fix Costs in Both Periods - Early Exit and Late Entry	13
3.1	Extension of the Model	13
3.2	Optimal Policies if Late Entry or Early Exit Occurs	15
3.2.1	Late Entry	15
3.2.2	Early Exit	16
3.2.3	Complete Learning Spillovers	20
3.2.4	Laissez-faire and Welfare Effects through Blocking Entry . . .	22
3.3	Comparative Statics	24
4	Conclusions	26
A	Appendix	28

1 Introduction

Learning-by-doing has been identified as an important strategic device in the theories of both industrial organization and international trade. The bulk of literature has investigated the strategic effects of learning-by-doing, and several researchers have identified market failure through learning. In the political debate it is often argued that there might be too little learning, and hence production of innovative industries should be subsidized in order to spur learning. Therefore, Petrakis, Rasmusen and Roy (1997) have asked the most natural question whether learning induces market failure in competitive markets with free entry. For the case of merely private learning, their answer is a clear "no". Put differently, they show that there is no need for a regulator to intervene in the market since firms fully internalize the effects of learning today on their cost structure tomorrow, and hence are willing to cope with negative profits in the infant stage of production, anticipating positive profits in the mature stage.

In this paper we extend the work of Petrakis et al. by assuming that in addition to private learning there are also positive learning spillovers among the firms. This assumption is supported by empirical evidence from several industries (see e.g. Argote and Epple (1990), Isoard and Soria (2001), Lieberman (1984), and Lloyd (1979)). If we refer to pure *private learning* we mean that cost reductions in the mature stage of production depend only on each firm's own level of output in the early stage, while with *learning spillovers* firms also benefit from other firms' previous volumes of production. If firms enjoy learning spillovers, it is not surprising that they will not engage in sufficient learning compared to the social optimum. What, however, comes out as a surprise is that the regulator is not able to reach a first best allocation by simply subsidizing output. In order to characterize optimal policy schemes we study two different cases. In the first case, fixed costs are incurred only once and entry is once and for all. In the second case fixed costs occur in each period and both early exit and late entry in the market are possible. We find that if entry is once and for all, the regulator needs two separate instruments in order to implement both the socially efficient levels of learning and market entry under

decentralized, competitive decision making, a subsidy per unit of output in the first stage and a premium on entry (a lump sum subsidy). If in the second case with fixed costs in each period it is socially optimal that some firms leave the market, the regulator needs to employ a non-linear subsidy scheme on output in the first period in order to discriminate between staying and exiting firms.

We also investigate the comparative statics effects of those instruments. If entry is once and for all, we show that raising the entry premium unambiguously leads to an increase of the equilibrium number of firms, and to a decrease in firm output in the first period. By contrast, the effects of increasing the output subsidy are less clear: the number of firms still rises under certain conditions, but the effect on firms output is ambiguous for both periods. If fix costs occur in each period, we get more crisp results. If either the entry premium or the output subsidy is raised, prices cannot increase and total output cannot decrease.

As mentioned above most researchers who were interested in learning effects have emphasized the strategic aspects and, therefore, have studied either oligopoly models or imperfect competition with endogenous market structure. Arrow (1962) was the first to emphasize that learning-by-doing may cause spillovers and hence can be a public good. Bardhan (1971) was the first to ask whether or not the existence of learning spillovers may cause market failure. Within a trade model he shows that in the presence of learning a subsidy on output unilaterally improves welfare of the home country. Rauch (1992) extends this model and offers a rule of thumb for policy makers for the optimal subsidy. A second branch of literature focuses on learning-by-doing effects in models of imperfect competition. Fudenberg and Tirole (1983) study the implications of (mainly) private learning-by-doing for market conduct. Under the assumption of constant instantaneous marginal costs they show that no price taking equilibrium exists and, henceforth, they study an oligopoly model where learning is a strategic variable. For this case they predict excessive learning. Hence, in their model, the regulator can improve welfare by taxing output in the first period and subsidizing it in the second one. Dasgupta and Stiglitz (1988) draw attention to the point that Fudenberg and Tirole (and also Spence (1981)) assume identical firms and exogenous market structure. They argue that if learning is at

least partly private and one firm has an initial cost of any size, this advantage will increase and the industry will experience growing concentration. Cabral and Riordan (1994) adopt the assumption of different firms. They show that in a price-setting, differentiated duopoly with only private learning and constant instantaneous marginal costs an initial market dominance of one firm can be increasingly self-enforcing. They also show that learning can lead to an equilibrium where firms are worse off with learning than without learning, i.e. profits are lower, although learning is socially desirable. In other words, learning increases competition too much from the firm's perspective. Leahy and Neary (1999) combine the issues of imperfect competition and trade by examining optimal (trade) policy if firms engage in Cournot competition on international markets and the domestic firm incurs cost cuts in the second period through learning-by-doing in the first one. They show that, if intertemporal pre-commitments are possible, the optimal subsidy is increasing in the rate of learning but is decreasing if pre-commitment is not possible.

Miravate (2003) studies the optimal time-consistent tariff which the regulator should set when a national, learning monopolist competes with a cost efficient foreign firm that produces a substitute to the monopolists' good. Under certain assumptions he concludes that the tariff and the domestic price will decline with the rate of learning. Jin, Perote-Pena and Troege (2004) take a different approach by considering Bertrand competition in differentiated products and non-strategic learning, i.e. firms do not take into account the cost reduction in future periods caused through production in the present. They argue that firms act boundedly rational either because the learning process is too complex to be subject of optimization, or because managers want to maximize profits in the short run. Under these assumptions they show that there might be a social loss due to high market concentration. To minimize this loss the regulator should enhance the spillovers, for example by encouraging information flow among firms.

Our paper is organized as follows. In section 2 we first set up the basic model in 2.1. In subsection 2.2, we characterize socially optimal allocations for the case where fixed costs are incurred only once, and in subsection 2.3, we study the optimal policy of the regulator. We investigate the comparative statics effects with respect to

changes in the subsidy levels in subsection 2.4, where we also present some examples. In section 3, we investigate the case where fixed costs are incurred in each period, and market exit and late entry is possible. We present the extension of the model in subsection 3.1, characterize the optimal policies in subsection 3.2, look at the comparative statics effects in subsection 3.3 and present examples in subsection 3.2.4. Some concluding remarks follow in section 4.

2 Entry Once and for All

2.1 The Basic Model

We consider a closed economy with a competitive industry and free entry. Firms incur learning effects by both their own level of output in the early phase of production (private learning), but also through the level of output by the other firms (learning spillovers). In order to model this we have to consider at least two periods $t = 1, 2$. We denote by n the number of firms and by y_t the output of a typical (symmetric) firm in period t . Total output in period t is written as $Y_t = ny_t$. Further we denote by $C^1(y_1)$ and $C^2(y_2; L)$ the variable production cost in period 1 (without experience) and period 2, respectively, where $L = y_1 + \epsilon(n-1)\tilde{y}_1$ represents the total level of learning, and $(n-1)\tilde{y}_1$ is the output of the remaining $(n-1)$ firms.¹ The extent of learning spillovers is reflected by the parameter ϵ with $0 \leq \epsilon \leq 1$, where $\epsilon = 0$ represents the case of pure private learning and $\epsilon = 1$ the case of complete spillovers. In the latter case, it does not matter for the cost reduction whether some output is produced in the own firm or by some other firm.

We assume that the cost functions C^t satisfy the following properties: $C_{y_t}^t > 0$ and $C_{y_t y_t}^t > 0$, i.e. we have positive and increasing marginal costs in each period. Moreover, $C_L^2 < 0$ and $C_{y_2 L}^2 < 0$, which means that learning through production (by

¹Note that $n \geq 1$. Allowing also for firm numbers $0 \geq n \geq 1$ as it is often done - see the examples in Petrakis et al. (1997)- does not make sense here, because learning spillover effects would be negative.

the own or by the other firms) in period 1 decreases both cost and marginal cost in period 2. Furthermore, $C_{LL}^2 > 0$, stating that the marginal effect of learning is decreasing. For technical reasons, in particular second order conditions, we assume overall convexity of C^2 , implying $C_{LL}^2 C_{y_2 y_2}^2 - [C_{Ly_2}^2]^2 > 0$ for $y_2, L > 0$. Finally, we assume $C^2(y_2; 0) = C^1(y_1)$, i.e. in the absence of learning the cost functions are identical in both periods. Since we allow for free entry, we assume that there is also a fixed cost F which in this section the firms incur only once as an entry cost as is also the case in the model of Fudenberg and Tirole (1983). In section 3 we also study the case where firms incur a fixed cost in both periods. A fixed cost incurred in only one period implies that the firms will either produce or stay out in both periods. The reason is that with strictly convex costs the first few units in the second period are produced almost for free but sold at a positive price. Therefore, competitive firms will always want to produce in the second period until the marginal costs are equal to the price.

Demand for the industry's output is given by an inverse demand function $p_t = P_t(Y_t)$ which, as usual, is downward sloping, i.e. $P'_t(Y_t) < 0$. Since firms are symmetric and incur increasing marginal costs, an optimal allocation must also be symmetric. Hence we can define welfare by

$$W = \int_0^{ny_1} P_1(Y) dY - nC^1(y_1) - nF + \delta \left[\int_0^{ny_2} P_2(Y) dY - nC^2(y_2; L) \right] \quad (1)$$

where δ is the social and private discount factor.²

2.2 The Social Optimum

The social planner maximizes welfare with respect to y_1, y_2 and the optimal number of firms n . The first-order conditions are given by the following equations, where

²In general, the social and the private discount factor need not coincide. We abstract from those differences since they are not important for the focus of this paper.

W_{y_1} denotes the partial derivative $\frac{\partial W}{\partial y_1}$ and so on:

$$W_{y_1} = P_1(Y_1) - C_{y_1}^1(y_1) + \delta[-C_L^2(y_2; L)[1 + \epsilon(n-1)]] = 0 \quad (2)$$

$$W_{y_2} = P_2(Y_2) - C_{y_2}^2(y_2; L) = 0 \quad (3)$$

$$W_n = P_1(Y_1)y_1 - C^1(y_1) - F + \delta[P_2(Y_2)y_2 - C^2(y_2; L) - nC_L^2(y_2; L)\epsilon y_1] = 0 \quad (4)$$

The first condition states that a typical firm's marginal cost in the first period should be equal to the consumers' marginal willingness to pay for the good plus the social marginal benefit incurred by a cost cut in the second period. The second equation is the usual text book condition stating that the marginal willingness to pay equals marginal costs. Finally, condition (4) determines (together with conditions (2) and (3)) the optimal number of firms. Interestingly, this condition also contains a learning effect. This is so because the marginal firm which enters the market imposes a positive externality on the other firms by creating a new source of learning.

2.3 The Optimal Policy in a Competitive Industry

In the decentralized setting we assume that firms are small and behave as price takers. They maximize their intertemporal profit taking into account only the marginal learning effect of their own production but not the positive externality which their production imposes on the other firms. As we will see below, a laissez-faire environment does not lead to a first best allocation. The existing literature remains - as far as we know - silent about optimal subsidy regimes which account for learning spillovers in competitive industries with free entry. Anticipating the optimal policy, we allow the regulator to pay both a per unit subsidy on output s_{out} and a premium on market entry s_{ent} . Given these instruments a typical firm's profit can be written as

$$[p_1 + s_{out}]y_1 - C^1(y_1) - F + s_{ent} + \delta[p_2 y_2 - C^2(y_2; L)]$$

The first order conditions of a price taking firm are then given by

$$\pi_{y_1} = [p_1 + s_{out}] - C_{y_1}^1(y_1) + \delta[-C_L^2(y_2; L)] = 0 \quad (5)$$

$$\pi_{y_2} = p_2 - C_{y_2}^2(y_2; L) = 0 \quad (6)$$

In addition, free entry leads to the following zero-profit condition:

$$\pi = [p_1 + s_{out}]y_1 - C^1(y_1) - F + s_{ent} + \delta[p_2y_2 - C^2(y_2; L)] = 0 \quad (7)$$

Equating (2) - (4) with (5) - (7) reveals that the optimal subsidy on output must be just equal to the marginal spillover of each firm to all the other $(n - 1)$ firms, i.e.

$$s_{out} = -\delta\epsilon(n^* - 1)C_L^2(y_2^*; L^*) \quad (8)$$

while the optimal premium for market entry is given by

$$s_{ent} = -\delta\epsilon C_L^2(y_2^*; L^*)y_1^*, \quad (9)$$

where y_1^* , y_2^* , L^* , n^* are the efficient output levels, amount of learning, and the number of firms, respectively.

The intuitive explanation of why besides the output subsidy the entry premium is necessary to reach a first best allocation is as follows. While the output subsidy s_{out} accounts for the fact that there is too little output of each firm in the first period, we need the entry premium s_{ent} to take into account that there is also too little market entry. This is maybe best explained at hand of the following example. Assume that only two firms are active without an entry premium but three firms operate with the optimal entry premium s_{ent} . The two active firms get a per unit subsidy for the spillovers they cause on each other. If the third firm enters, it receives a subsidy for the spillovers it renders to the other two firms. But in its profit function the third firm does not account for the effect that the spillovers of the first two firms are now spilled over to itself and are thus of more value. This increase in value is exactly represented by the term $\delta\epsilon C_L^2(y_2^*; L^*)y_1^*$.

We can summarize our findings in the following result:

Proposition 1 *In a competitive economy with free entry and both private learning-by-doing effects and learning spillovers, a per unit subsidy $s_{out} = -\delta\epsilon(n^* - 1)C_L^2(y_2^*; L^*)$ on first period output and an entry premium $s_{ent} = -\delta\epsilon C_L^2(y_2^*; L^*)y_1^*$ for each active firm lead to the first best allocation.*

From this it follows immediately that a laissez-faire policy induces market failure.

In order to make sense of the subsidy policy, we assume that market equilibrium is unique and stable. The stability conditions are given in the appendix.

2.4 Comparative Statics

To study the effects of a change in both the output subsidy and the entry premium we totally differentiate (5) - (7) and solve for dy_1/s_{ent} , dy_2/s_{ent} and dn/s_{ent} keeping s_{out} constant, and vice versa. We look also for the comparative static effects on total output Y_1 and Y_2 . Some effects follow directly from our assumptions. Other effects require additional assumptions on the cost functions. For this purpose consider the following set of conditions:

$$-P'_2 y_2 + \epsilon C_{y_2 L}^2 y_1 \geq 0 \quad (10)$$

$$C_{y_1 y_1}^1 y_1 + \delta C_{y_2 L}^2 y_2 \geq 0 \quad (11)$$

$$C_{y_2 y_2}^2 y_2 + C_{y_2 L}^2 y_1 \geq 0 \quad (12)$$

The effects on an increase of the entry premium can now be summarized by the following result:

Proposition 2 *Let our assumptions on cost and demand hold, and assume that the competitive equilibrium with market entry is stable.*

- i) Then $\partial n / \partial s_{ent} > 0$ and $\partial y_1 / \partial s_{ent} < 0$.*
- ii) If (10) holds, then $\partial y_2 / \partial s_{ent} < 0$.*
- iii) If (11) holds, then $\partial Y_1 / \partial s_{ent} > 0$.*
- iv) If (12) holds, then $\partial Y_2 / \partial s_{ent} > 0$.*

The proof is given in the appendix. The intuition is as follows: an entry subsidy or premium has the same effect as a decrease of fixed costs and thus lowers the (minimum) average cost. In the standard textbook static model of partial equilibrium

with free entry this results in more entry and less output per firm. The same effect can be observed here with respect to both the number of firms and output per firm in the first period. The effect on second period output per firm is ambiguous in general, but output decreases if condition (10) holds. The reason for this ambiguity is that after the first period has passed, the number of firms can be considered as fixed since no further entry is possible by assumption. With learning and spillovers, more output in the first period lowers the second period variable cost. In the standard partial equilibrium model with a fixed number of firms, however, lower costs raise the single firm's level of output. Hence in our model two offsetting effects occur where under condition (10) the output contraction effect through market entry dominates the output expansion effect through lower costs.

The effect on increased total output is intuitive and consistent with the results of the textbook model without both learning and spillovers. Although we need additional conditions such as (11) and (12) to prove the effect, we did not find counterexamples where the entry premium causes total output to fall in at least one of the two periods.

Let us next consider the comparative statics effects of an increasing *output subsidy*. To derive unambiguous effects, we need to impose, however, some further conditions on the cost functions. Besides (10) - (12) consider the following two conditions:

$$\delta\epsilon C_L^2 + C_{y_1 y_1}^1 \geq 0 \quad (13)$$

$$C_{LL}^2 y_1 + C_{y_2 L}^2 y_2 \geq 0 \quad (14)$$

Note that conditions (11) - (14) all have in common that the cut in both cost and marginal cost through learning is not too large. We obtain the following results:

Proposition 3 *i) If condition (12) holds and ϵ is sufficiently small, then*

$$\partial y_1 / \partial s_{out} > 0.$$

ii) If the conditions (10) and (14) hold, then $\partial y_2 / \partial s_{out} < 0$.

iii) If condition (14) holds, then $\partial n / \partial s_{out} > 0$.

iv) If the conditions (12) and (13) hold, $\partial Y_1/\partial s_{out} > 0$ and $\partial Y_2/\partial s_{out} > 0$.

The proof is given in the appendix. We call the effects stated in the proposition the *normal* effects. Thus, as one would expect, a subsidy on first period output increases the individual firm's output in the same period. It seems to be surprising, however, that normally a firm's second period output decreases. The intuition is that the subsidy makes it comparatively more attractive for the firm to produce in the first period. Since market entry eliminates all profits the firm shifts production from the second to the first period. This is true although the costs of the second period decrease with a higher level of learning. If, however, some of the conditions, stated above, are violated, it can happen that raising a subsidy causes output to decrease in the first period and to increase in the second period, as example 1 shows.

Example 1 ³ Let $P_t = (100 - Y_t)/2$. Further let $C^1(y_1) = 20y_1^2$, $C^2(y_2, L) = \frac{20y_2^2}{(1+L)}$, $\delta = 0.0001$, $\epsilon = 1$ and $F = 10$. Increasing the output subsidy from $s_{out} = 0.2$ to $s_{out} = 1$, causes y_1 to fall from 1.22463 to 1.22458 and y_2 to increase from 3.956 to 5.215.

Note that in the example the conditions (10) and (14) are both violated. Condition (12) holds, but ϵ is not small enough to cause $\partial y_1/\partial s_{out} > 0$.

Due to the increase of the output subsidy the number of firms can decrease if condition (14) is violated as is shown in example 2.

Example 2 Let $\Pi = p_1y_1 - 6.5y_1^2 - 10 + \delta[p_2y_2 - 0.5(3y_2 - 0.1L)^2 - 2y_2^2] + s_{out}y_1$ with $\delta = 1$, $\epsilon = 0.9$. We first set $s_{out} = 0$ and then $s_{out} = 1$. Further let $P_t = 100 - Y_t$. Then we get the results of table 1.

³The reader might notice that the cost functions chosen in this and the following examples do not satisfy $C_{yy}C_{LL} - C_{yL}^2 > 0$ as assumed in the theoretical part but yield $C_{yy}C_{LL} - C_{yL}^2 = 0$, i.e. only the necessary but not the sufficient condition for overall convexity holds. However, we could easily get overall convexity by adding some ηy^2 with $\eta \rightarrow 0$ to the cost function without changing the trend of the results of the examples. We did not, because the computation time for the examples almost exploded.

Table 1: The number of firms can decrease.

	$s_{out} = 0$		$s_{out} = 1$	
	1. Period	2. Period	1.Period	2.Period
y	1.49	1.82	1.51	1.83
n	53.87	53.87	53.75	53.75
Y	80.42	98.09	81.18	98.21

The number of firms decreases from 53.87 to 53.75, while individual and total output increases in both periods. Two offsetting effects influence the number of firms. On the one hand, the output subsidy leads to increasing total output produced by more firms. On the other hand, increasing total output induces more learning and lower costs in the second period implying the possibility that the market can sustain less firms in the second period. Here, the second effect dominates.

3 Fix Costs in Both Periods - Early Exit and Late Entry

In the preceding section we have assumed that fixed costs occur only once, i.e. they are sunk after the first period. Petrakis et al. (1997), by contrast, allow for fixed costs in both periods and also for learning in fixed costs. They show that with pure private learning under certain conditions market exit is possible which means that there are some firms which produce only in the first period. In this section we also assume that firms incur fixed costs in both periods and thus we allow for both market exit and late entry.

3.1 Extension of the Model

For simplicity we assume that fixed costs are not affected by learning and are denoted by F_1 and F_2 with $F_1 = F_2 = F$. In this case three types of firms are possible: Firms

which exit early after the first period (X-firms), firms which stay and produce in both periods (S-firms), and finally firms which enter lately and produce only in the second period (E-firms). For this analysis we denote by n_s, n_x , and n_e the number of S-firms, X-firms, and E-firms, respectively. Further, we denote by y_1^s and y_2^s the output of the S-firms in period 1 and 2, respectively, by y_x the output of the X-firms in the first period, and by y_e the output of the E-firms in the second period. Assuming symmetry of staying, exiting and lately entering firms, we can write $Y_1 = n_s y_1^s + n_x y_x$ as total output in the first period and $Y_2 = n_s y_2^s + n_e y_e$ as total output in the second period. Thus welfare can now be written as:

$$W = \int_0^{Y_1} P_1(Y) dY - n_s C^1(y_1^s) - n_x C^1(y_x) - (n_s + n_x)F + \delta \left[\int_0^{Y_2} P_2(Y) dY - n_s C^2(y_2^s; L_s) - n_e C^2(y_e; L_e) - (n_s + n_e)F \right] \quad (15)$$

Two things are important to emphasize. First, in line with the results of Petrakis et al. (1997), it is easy to see that, if $\epsilon < 1$, i.e. if there is some private learning, it can never be optimal that some firms leave the market while at the same time new firms enter the market in order to produce in the second period. The argument is that, given such an allocation, we could simply improve welfare by substituting firms entering lately by exiting firms because the potentially exiting firms have learned already and hence, by staying, would incur lower costs in the second period compared to those firms entering lately. Secondly, for cases where some firms enter lately, the learning parameter L is different for staying and for entering firms, denoted by $L_s = y_1^s + \epsilon[(n_s - 1)y_1^s]$, and $L_e = \epsilon(n_s y_1^s)$, respectively. We can summarize the first observation in the following proposition.⁴

Proposition 4 *Assume that spillovers are incomplete, i.e. $\epsilon < 1$, firms incur fixed costs in both periods and both exit and late entry are possible.*

Then it is socially optimal that either some firms leave the market after the first period and no new firms enter, or no firm leaves the market, and some (or no) firms

⁴This proposition is similar to Baumol's et al. (1982) result for multiproduct firms.

enter lately. It can never be optimal that some firms leave the market and at the same time some new firms enter the market.

3.2 Optimal Policies if Late Entry or Early Exit Occurs

Petrakis et al.(1997) show that with convex costs late entry is never optimal, but that early exit can be optimal. In the presence of learning spillovers, however, both early exit and late entry are possible under both regimes, laissez-faire and in the social optimum. This we will demonstrate in the examples 3 and 4 below.

3.2.1 Late Entry

If late entry is optimal, the stocks of learning incurred by the staying and lately entering firms, L_s and L_e respectively, are different. This does not cause a problem though, and the first best allocation can be achieved by the same instruments as in the case where there is entry only once and for all. This result can be stated as follows:

Proposition 5 *If late entry of some firms is optimal, then the regulator can achieve the first best allocation by offering an output subsidy and an entry premium according to the following rule:*

$$s_{out} = -\delta\epsilon[(n_s^* - 1)C_L^2(y_2^{s*}, L_s^*) + n_e^*C_L^2(y_e^*, L_e^*)] \quad (16)$$

$$s_{ent} = -\delta\epsilon C_L^2(y_2^{s*}, L_s^*)y_1^{s*} \quad (17)$$

where y_1^{s*} , y_2^{s*} and y_e^* are the efficient output levels of the first and the second period, L_s^* and L_e^* the optimal amounts of learning capital of staying and late entering firms, respectively, and n_s^* and n_e^* the optimal number of firms.

For the proof see the appendix.

The following example 3 shows that it can indeed be optimal that some firms enter lately.

Example 3 [Late entry in the social optimum]

Let $\epsilon = 0.9$, $\delta = 1$, $F = 10$, $P_t = (100 - Y_t)/9$, $C^1(y_1) = 2y_1^2$, $C^{2,i}(y_{2,i}, L_i) = (y_{2,i}^2 - \alpha L_i)^2 + y_{2,i}^2$ with $i = e, s$, and $\alpha = 1/15$. Then we will observe late entry in the social optimum with the results given in table 2. E-firms produce less than the

Table 2: Late entry in the social optimum, $\epsilon = 0.9$, $\delta = 1$

	S-firms		E-firms			
	y	n	y	n	p	Y
1. Period	2.24	14.05	0	0	7.62	31.42
2. Period	2.61	14.05	2.60	1.36	6.64	40.22

S-firms in the second period, the market price decreases and total output increases.

3.2.2 Early Exit

If, by contrast, it is socially optimal that some firms cease production after the first period, matters are more tricky. In this case the size of the externality created by staying firms and by exiting firms is different. The reason is that the staying firms internalize their marginal private learning effect of size $\delta C_L^2(y_2^s, L_s)$ and impose a positive externality on the other $(n_s - 1)$ firms of size $\delta \epsilon (n_s - 1) C_L^2(y_2^s, L_s)$, whereas the exiting firms create a positive externality of size $\delta \epsilon n_s C_L^2(y_2^s, L_s)$ for the staying firms. Hence, a regulator cannot achieve the first best allocation by paying both a uniform subsidy and a uniform entry premium because the firms decisions are not identical ex-post. We first describe the optimal policy given that the regulator can actually discriminate between firms before we drop this assumption and characterize the optimal subsidy scheme if discrimination is not possible.

If we assume that the regulator can ex ante discriminate between the firms or - if this is possible - pays the entry premium and the output subsidies after the first period, i.e. after exit has occurred, we can establish the following result.⁵ Here, we denote by y_1^{s*} , y_x^* the socially optimal output levels in the first period of the staying

⁵A problem with this kind of policy may be that the first period in the model may last several

and the exiting firms, respectively. Accordingly, we denote by y_2^{s*} the optimal level of output of the staying firms in the second period, and by $L_s^* = (1 + \epsilon(n_s^* - 1))y_1^{s*} + \epsilon n_x^* y_x^*$ the optimal level of learning.

Proposition 6 [Optimal policy with discrimination in the case of early exit]

If exit of some firms is optimal and the regulator is able to discriminate between staying and exiting firms, he can achieve the first best allocation by paying an output subsidy s_{out}^x to those firms which plan to leave the market after their first period, and both an output subsidy s_{out}^s and an entry premium s_{ent}^s to the staying firms according to the following rule:

$$s_{out}^x = -\delta \epsilon n_s^* C_L^2(y_2^{s*}, L_s^*) \quad (18)$$

$$s_{out}^s = -\delta \epsilon (n_s^* - 1) C_L^2(y_2^{s*}, L_s^*) \quad (19)$$

$$s_{ent}^s = -\delta \epsilon C_L^2 y_1^{s*} \quad (20)$$

For the proof see the appendix.

Discrimination like this may not be legally feasible though. If we drop this assumption, the regulator can still implement the first best allocation by using a concave output subsidy function $S_{out}(y)$ which adopts the values $S'_{out}(y_1^{s*}) = \delta \epsilon (n_s^* - 1) C_L^2(y_2^{s*}, L_s^*)$ and $S'_{out}(y_x^*) = \delta \epsilon (n_s^*) C_L^2(y_2^{s*}, L_s^*)$ if firms choose the optimal levels y_1^{s*} and y_x^* , respectively. Since these are the only requirements on $S_{out}(y)$, this subsidy function is not uniquely determined. To achieve the optimal number of firms, the regulator has to pay either an entry premium or to charge an entry fee. These premia or fees may be different for staying and exiting firms. This may cause a problem if the regulator cannot ex ante differentiate or discriminate between exiting and staying firms. Hence, it is necessary to construct a mechanism consisting of a subsidy function and entry premia/fees in such a way that the staying firms receive higher premia (or pay lower entry fees) than exiting firms. In this case the premia

years in real time. Therefore, the commitment of the regulator to pay subsidies after exit has occurred might not be credible because policies can change with a new and even with the same government. That said, for short real time periods this result might be appealing because it seems easy to implement.

can be split into two parts, a uniform premium paid to (or a uniform fee charged from) both firms in the first period, and a premium paid to S-firms in the second period. Indeed we establish in the next result that such a mechanism is feasible. The regulator can choose a simple concave quadratic subsidy function, combined with a uniform entry fee to be paid by the firms in the first period and a premium to be paid to those firms which stay in the second period.

Proposition 7 [Optimal policy without discrimination in the case of early exit]

If exit of some firms is optimal, the regulator can achieve the first best allocation by subsidizing output of both firms in the first period through the quadratic concave output subsidy function

$$S_{out}(y_1) = ay_1 - \frac{b}{2}y_1^2,$$

where

$$a = -\delta\epsilon C_L^2(y_2^{s*}, L_s^*) \frac{n_s^* y_1^{s*} - (n_s^* - 1)y_x^*}{y_1^{s*} - y_x^*}, \quad (21)$$

$$b = \frac{-\delta\epsilon C_L^2(y_2^{s*}, L_s^*)}{y_1^{s*} - y_x^*}, \quad (22)$$

by charging an entry tax in the first period given by

$$T_{ent} = \frac{-\delta\epsilon C_L^2(y_2^{s*}, L_s^*) \cdot (y_x^*)^2}{2(y_1^{s*} - y_x^*)}, \quad (23)$$

and by paying a premium for staying and producing in the second period of size

$$P_{stay} = \frac{-\epsilon C_L^2(y_2^{s*}, L_s^*) \cdot (y_1^{s*} - y_x^*)}{2}. \quad (24)$$

Moreover, the marginal output subsidy function takes the following values at the first best output levels:

$$S'_{out}(y_1^{s*}) = -\delta\epsilon(n_s^* - 1)C_L^2(y_2^{s*}, L_s^*) \quad (25)$$

$$S'_{out}(y_x^*) = -\delta\epsilon n_s^* C_L^2(y_2^{s*}, L_s^*) \quad (26)$$

For the proof see the appendix.

Note that the concavity of the subsidy function guarantees that the firms' second-order conditions are satisfied. This also guarantees that exiting firms receive a higher

marginal subsidy since E-firms are not interested in private learning. Note that in total the entry tax plus the premium for staying can be positive or negative.

With the help of figure 1 one can easily see the difference between the two regulatory policies, the discriminatory linear scheme and the concave scheme. If the

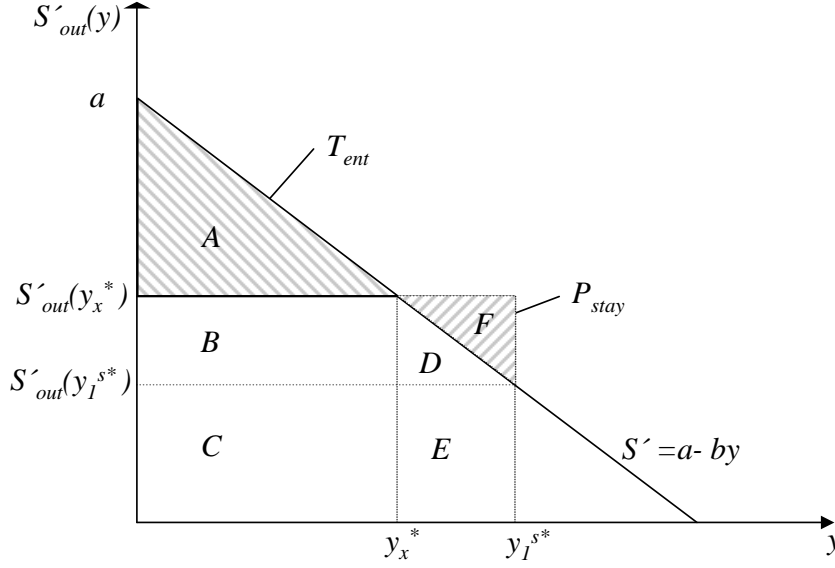


Figure 1: Comparison between the mechanism with an output subsidy function and the constant output subsidy regime

regulator chooses the mechanism stated in proposition 7, both X-firms and S-firms have to pay the entry tax $T_{ent} = A = \frac{1}{2}(a - S'_{out})y_x^*$. In the optimum X-firms receive output subsidies of size $A + B + C$ and S-firms those of size $A + B + C + D + E$ according to the marginal output function $S'_{out} = a - by$. Thus X-firms receive total subsidies of $A + B + C - T_{ent} = B + C$, whereas S-firms receive total subsidies of $A + B + C + D + E - T_{ent} + P_{stay}$ with $P_{stay} = F = \frac{1}{2}[S'_{out}(y_x^*) - S'_{out}(y_I^{s*})][y_I^{s*} - y_x^*]$. If the regulator can commit himself to pay discriminating constant output subsidies and an entry premium for the S-firms after the first period, X-firms receive in the optimum total output subsidies of $B + C = S'_{out}y_x^*$ whereas S-firms get output subsidies of $C + E = S'_{out}y_I^{s*}$ plus the entry premium $B + D + F = S'_{out}y_x^* - S'_{out}y_I^{s*} = -\delta\epsilon C_L^2 y_I^{s*}$. Of course, the effective allocation resulting from the two regimes is the same. The advantage of the mechanism with a concave output subsidy function is that the regulator does not need to know of which type a firm is. If the regulator introduced the

constant subsidies and could not discriminate between the firms, S-firms would opt for the higher output subsidies s_{out}^x and produce too much in the first period leading to a suboptimal allocation. Note that even with the concave subsidy function scheme we have a commitment problem since S-firms must believe that the premium for staying P_{stay} is paid in the second period.

It is obvious that situations exist where early exit of some firms is socially optimal. Petrakis et al. (1997) have shown that this is possible for $\epsilon = 0$. In subsection 3.2.3 we will show that this is also possible for $\epsilon = 1$. It is not difficult to find parameters for $0 < \epsilon < 1$, where early exit is socially optimal. For example, if we choose $P_t = 1000 - Y_t$, $C^1(y_{1,i}) = 2y_{1,i}^2$ with $i = x, s$, and $C^2(y_2, L_s) = (y_2 - 5/6L_s)^2 + y_2^2$ and $\epsilon = 1/1000$, early exit is optimal.

3.2.3 Complete Learning Spillovers

Here, we consider the special case where learning spillovers are complete. This means, there are no private learning effects. All experience gathered by any firm immediately spills over to all other firms. In our model this boils down to $\epsilon = 1$ which immediately implies $L_S = L_E$. Hence, the S-firms have no cost advantage over the E-firms and thus they are not able to make any profits in the second period to cover their losses of the first one. Therefore, the first period market price is determined by $p_X = [C(y^X) + F]/y^X$, the second period price by $p_E = [C^2(y^E, L_E) + F]/y^E$. Moreover all firms make zero profit in each period. For the case of complete learning spillovers we can state explicitly the conditions when late entry is optimal:

Proposition 8 *If learning spillovers are complete, i.e. $\epsilon = 1$, late entry is optimal if $-C_{\tilde{L}}^2 C_{y_2 y_2}^2 + C_{\tilde{L}}^2 n_2 P_2' - C_{y_2 \tilde{L}}^2 n_2 P_2' > 0$ and δ sufficiently small.*

For the proof see the appendix.

The following example verifies that both late entry and early exit are indeed possible for complete learning spillovers both without regulation and in the social

optimum.⁶

Example 4 Let $\epsilon = 1$, $P_t(Y_t) = (100 - Y_t)/\beta$, $C^1(y_1) = 20y_1^2$, $C^2(y_2, L) = 20y_2^2/(1 + L)$. and $F = 10$. Further let $\beta = 2.5$ or $\beta = 3.3$. Then we get the results of Table 3.

Table 3: Entry and exit of firms is possible, $\epsilon = 1$

	Laissez-faire				Social Optimum			
	$\beta = 2.5$		$\beta = 3.3$		$\beta = 3.3$		$\beta = 7$	
Period	1	2	1	2	1	2	1	2
p	28.28	5.14	28.28	10.22	21.34	5.11	12.83	8.47
y	0.71	3.89	0.71	1.96	0.71	3.91	0.71	2.36
n	41.42	22.39	9.42	33.86	41.85	21.25	14.37	17.24
Y	29.29	87.15	6.66	66.27	29.59	83.12	10.16	40.74

We see that for $\beta = 2.5$ (demand is relatively strong) and laissez-faire, some firms leave the market after the first period. By virtue of the relatively strong demand, learning effects are strong such that in the second period, both marginal costs and prices are so low that only few firms with a high output can survive in the market. Total output in the second period is also higher than in the first one.

For $\beta = 3.3$ demand is relatively weak such that the market can sustain only a few firms that can learn. Nevertheless, learning leads to reduced variable costs and the market is able to sustain more firms in the second period in case of laissez-faire. Note that in the social optimum with $\beta = 3.3$ there is early exit. Choosing $\beta = 7$ we can also verify late entry in the social optimum.

Note that due to complete learning spillovers, incumbent firms do not have a cost advantage over late entering firms. Hence, in contrast to the case $\epsilon < 1$, it is now in

⁶Note that in the examples 4, 5, and 6 demand is always the same in the first and the second period. Therefore exit and entry is not induced by a surging or collapsing consumption in the second period.

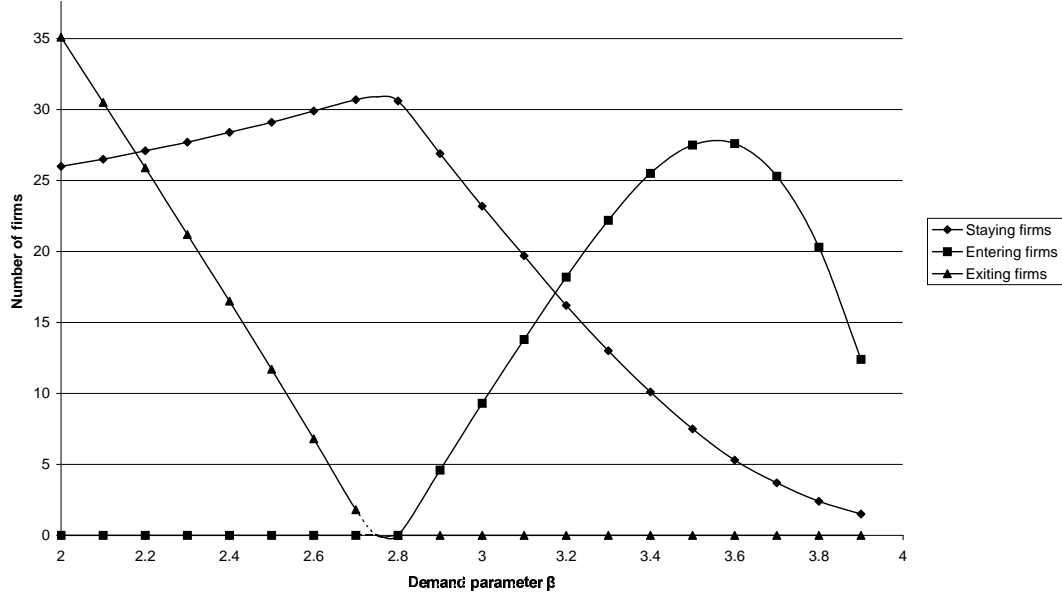


Figure 2: Number of S-, X-, and E-firms for different demand parameters

principle possible that some firms leave the market at the end of period 1 and new firms enter lately at the same time.

3.2.4 Laissez-faire and Welfare Effects through Blocking Entry

To conclude this section we show that also under laissez-faire both cases may arise: Some firms may exit early without taking advantage of their stock of knowledge. If parameters are different some firms may decide to enter lately, exploiting the experience of the pioneers. The following example verifies this claim.

Example 5 [Late entry and early exit without regulation]

Let $\epsilon = 0.5$ and $P_t = (100 - Y_t)/\beta$. Further let $C^1(y_1) = 20y_1^2$, $C^2(y_2, L) = \frac{20y_2^2}{(1+L)}$, $\delta = 1$, and $F = 10$. If we increase the demand parameter β from 2 to 3.9 we obtain the graph in figure 2 which plots the equilibrium number of firms as a function of the demand parameter β . Whereas for high demand (β small) we observe both X-firms and S-firms, the number of X-firms decreases with increasing β . There is a small

Table 4: Late entry and early exit of firms without regulation, $\epsilon = 0.5$, $\delta = 1$

	$\beta = 3.3$		$\beta = 2$	
	1. Period	2. Period	1. Period	2. Period
p	27.43	11.80	28.28	5.89
y, S-firms	0.73	1.80	0.72	4.40
y, X-firms	0	0	0.71	0
y, E-firms	0	1.69	0	0
n, S-firms	13.01	13.01	25.97	25.97
n, X-firms	0	0	35.05	0
n, E-firms	0	22.19	0	0
Y	9.49	61.06	43.43	88.22
II, S-firms	-0.634	+0.634	-0.002	+0.002

range for β around 2.8 where there are only S-firms.⁷ For higher β we have both S-firms and E-firms in the market. In table 4 we provide the results for all variables for $\beta = 3.3$ and $\beta = 2$.

Moreover, it is worth to mention that without a subsidy policy the market outcome with late entry can be worse than without entry. If a regulator protects the market by forbidding late entry, we may get the surprising result that prices may be lower and total output can be higher compared to the scenario when entry is allowed. This can be verified by means of the following example:

Example 6 [Increasing welfare through blocking entry]

Using the same demand and cost functions as in example 5 we obtain the results shown in table 5. Comparing the first and second period results of the two cases we

⁷This is different to Baumols et al. (1982) result for two-product firms. They show that we observe only for one particular parameter constellation an equilibrium with only two-product firms in the market. For all other parameters we always observe both two-product and one-product firms. There, the outcome which types of firms are in the market hinges only on total costs. The reason for this difference are the learning spillovers in our model: Firm costs are also influenced by demand because costs depend on the number of firms and the number of firms clearly depends on demand.

see that prices increase from 25.55 to 28.28 in the first and from 7.54 to 10.22 in the second period. Total output decreases from 15.69 to 6.66 in the first period and from 75.13 to 66.27 in the second period.

Table 5: $\epsilon = 1, \beta = 3.3$

	Entry forbidden		Entry allowed	
	1. Period	2. Period	1. Period	2. Period
p	25.55	7.54	28.28	10.22
y	0.66	3.14	0.71	1.96
n	23.90	23.90	22.39	33.86
Y	15.69	75.13	6.66	66.27

The intuition for example 6 is that without entry more of the benefits of learning can be reaped by the S-firms which invested in learning. Thus the incentive to learn is higher. Under such circumstances and if subsidies are not feasible it can be a second best policy for the regulator to regulate or even forbid late market entry.

3.3 Comparative Statics

To derive the marginal comparative static effects in the case of late entry and early exit resulting from a small increase of the subsidy or the entry premium, we have to distinguish between the three possible cases with S-firms only, with S-firms and X-firms, or with S-firms and E-firms. While for the first case the results are the same as in section 2.4,⁸ the results are different for the two other cases.

Proposition 9 *Consider a regime where the regulator pays an entry premium and (uniform or differentiated) subsidies on output. Assume that in a competitive equilibrium with entry premia and output subsidies there are X-firms and S-firms in the market, i.e. some firms leave the market after the first period.*

⁸For the comparative statics result, the additional fix costs in period 2 do not affect the results.

i) Then in both regimes (with uniform or differentiated subsidies on output), the effect of increasing the entry premium is as follows:

$$\begin{aligned}\partial Y_1 / \partial s_{ent}^s &= 0, \partial Y_2 / \partial s_{ent}^s > 0, \partial p_1 / \partial s_{ent}^s = 0, \partial p_2 / \partial s_{ent}^s < 0, \\ \partial y_1 / \partial s_{ent}^s < 0, \partial y_2 / \partial s_{ent}^s < 0, \partial n_s / \partial s_{ent}^s > 0, \partial y_x / \partial s_{ent}^s &= 0,\end{aligned}$$

ii) If the regulator can discriminate between exiting and staying firms, then the effects of increasing the output subsidy for staying firms, denoted by s_{out}^s , yields

$$\partial Y_1 / \partial s_{out}^s = 0, \partial p_1 / \partial s_{out}^s = 0,$$

iii) whereas an increase of the output subsidy for exiting firms, denoted by s_{out}^x , yields

$$\partial Y_1 / \partial s_{out}^x > 0, \partial p_1 / \partial s_{out}^x < 0.$$

iv) If the regulator pays a uniform subsidy $s_{out}^{s,x}$ for S-firms and X-firms, then an increase of the subsidy yields:

$$\partial Y_1 / \partial s_{out}^{s,x} > 0, \partial Y_2 / \partial s_{out}^{s,x} > 0, \partial p_1 / \partial s_{out}^{s,x} < 0, \partial p_2 / \partial s_{out}^{s,x} < 0.$$

Compared to the comparative static results without early exit or late entry, we get clear effects for more variables. An increase of the entry subsidy for S-firms (be it output subsidies for both firms or an entry subsidy for the S-firms) increases total output and thus lowers the price in both periods for sure. While the entry subsidy leaves the output of X-firms unaffected, the output of each S-firm decreases in both periods while the number of firms increases. For the output subsidies we get unambiguous results for less variables. The uniform subsidy increases total output in both periods while prices decrease. For the different output subsidies we see that first period output does not decrease, and the price does not increase.

Proposition 10 *Consider a regime where the regulator pays an entry premium for S-firms and subsidies on first period output. Assume that in a competitive equilibrium with entry premium and output subsidy there are E-firms and S-firms in the market, i.e. some firms enter the market after the first period. Then the effects of an increase of the entry subsidy s_{ent} for S-firms are as follows:*

$$\partial Y_1 / \partial s_{ent} > 0, \partial y_1 / \partial s_{ent} < 0, \partial n_s / \partial s_{ent} > 0, \partial p_1 / \partial s_{ent} < 0,$$

$$\partial Y_2 / \partial s_{ent} > 0, \partial p_2 / \partial s_{ent} < 0$$

An entry subsidy for S-firms increases total output and decreases prices in both periods. The number of S-firms increases while the first period output decreases. Unfortunately, the effects resulting from an increase of an output subsidy on first period output are ambiguous.

The proofs for propositions 9 and 10 are very similar to the proofs of propositions 2 and 3 in subsection 2.4 and can be obtained by the authors on request.

4 Conclusions

In this paper we have shown that in a model with competitive firms, free entry and learning spillovers, unlike the case of purely private learning-by-doing effects, efficient allocations do not emerge through a laissez-faire regime. By virtue of the positive externalities, this does not seem to be surprising per se. It is surprising, however, that the regulator needs (at least) two instruments to internalize the spillover effects. The reason is that under laissez-faire there is both too little production and too little market entry. If market entry is once and for all, we have shown that the learning spillovers require both a subsidy on output and a premium for starting a business.

If early exit and late entry are possible, matters are more complicated. First of all, both cases can occur: It can be optimal that more firms enter lately and take advantage of the experience made by the pioneers. By contrast, it can also be optimal that some firms leave the market and do not harvest the fruit from learning, but nevertheless let the staying firms benefit from their experience. In the first case, where late entry is optimal, the optimal policy scheme is basically the same as for the case where there is entry once and for all and no exit. If exit is optimal for a certain number of firms, the regulatory scheme is more complicated. The regulator needs to employ a concave subsidy scheme, combined with an entry fee for the first

period, and a premium for staying in the second period.

Since the policy scheme is quite different for the two cases, it is difficult to propose a clear general policy recommendation. For the regulator the informational requirement is quite high in order to implement an optimal policy scheme. Nevertheless, we think that our results are important to give more insight in the kind of externalities arising from learning spillovers. Even without imperfect competition which was the main focus of the previous literature in the presence of learning and learning spillovers markets with free entry lead to complex market imperfections and thus would in principle require complex regulation schemes. Hence, we see necessity for further research into two directions, one theoretical, and one empirical. From a theoretical point of view it would be interesting to know how the regulatory schemes developed in this paper can be extended for cases of imperfect information on the size of the learning effects, the spillovers, and the elasticity of market demand. The question is whether, similarly to Rauch's (1992) contribution, there is at least a robust rule of thumb for a second best optimal scheme of subsidies and entry premia. The second branch of research is empirical. We need to know more about the quantity of learning effects and learning spillovers. Of course, this is difficult too since learning effects are largest for new products and new production processes for which we usually do not have sufficient data.

Even though it is difficult to determine optimal regulation schemes, we think that our results shed some new light on the discussion and criticism on start-up premia and subsidies for new industries. Those may even be optimal if market conditions are seemingly perfect, i.e. markets are competitive and no entry barriers (apart from a lack of experience) exist.

A Appendix

Stability Conditions:

In the following we write for short: $P'_1 = P'(ny_1)$ and $P'_2 = P'(ny_2)$. Totally differentiating equations (5)-(7) for s_{out} and s_{ent} yields the following systems:

$$M * \begin{pmatrix} \partial y_1 / \partial s_{out} \\ \partial y_2 / \partial s_{out} \\ \partial n / \partial s_{out} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -y_1 \end{pmatrix} \quad (27)$$

$$M * \begin{pmatrix} \partial y_1 / \partial s_{ent} \\ \partial y_2 / \partial s_{ent} \\ \partial n / \partial s_{ent} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \quad (28)$$

where M is a 3 x 3 - matrix given by:

$$\begin{pmatrix} nP'_1 - C_{y_1 y_1}^1 - \delta(1 + \epsilon(n-1))C_{LL}^2 & -\delta C_{y_2 L}^2 & P'_1 y_1 - \delta \epsilon C_{LL}^2 y_1 \\ -(1 + \epsilon(n-1))C_{y_2 L}^2 & nP'_2 - C_{y_2 y_2}^2 & P'_2 y_2 - \epsilon C_{y_2 L}^2 y_1 \\ nP'_1 y_1 - \delta \epsilon(n-1)C_L^2 & \delta n P'_2 y_2 & P'_1 y_1^2 + \delta P'_2 y_2^2 - \delta \epsilon C_L^2 y_1 \end{pmatrix}$$

For stability M must be negative definite. This requires that the three principal minors satisfy the following conditions: $M_1 < 0$, $M_2 > 0$, $M_3 < 0$. Note that $M_1 < 0$ and $M_2 > 0$ follows directly from our assumptions. M_3 follows from our assumption on stability of equilibrium.

Proof of proposition 2:

Solving (28) for $\partial y_1 / \partial s_{ent}$, $\partial y_2 / \partial s_{ent}$, and $\partial n / \partial s_{ent}$, and for the effect on total

output $\partial Y_1/\partial s_{ent}$, $\partial Y_2/\partial s_{ent}$, keeping s_{out} constant, yields:

$$\begin{aligned}
\partial y_1/\partial s_{ent} &= \frac{1}{|M|} [\delta \varepsilon y_1 [C_{y_2 y_2}^2 C_{LL}^2 - (C_{y_2 L}^2)^2] - \delta \varepsilon n y_1 C_{LL}^2 P_2' \\
&\quad - P_1' C_{y_2 y_2}^2 y_1 + n P_1' P_2' y_1 + \delta C_{y_2 L}^2 P_2' y_2] < 0 \\
\partial y_2/\partial s_{ent} &= \frac{1}{|M|} [(1 - \varepsilon) C_{y_2 L}^2 P_1' y_1 - \delta (1 + \varepsilon (n - 1)) C_{LL}^2 P_2' y_2 \\
&\quad + n P_1' P_2' y_2 - \underbrace{C_{y_1 y_1}^1 [P_2' y_2 - \varepsilon C_{y_2 L}^2 y_1]}_{\text{Condition(10)}}] < 0 \\
\partial n/\partial s_{ent} &= \frac{1}{|M|} [\delta (1 + \varepsilon (n - 1)) (C_{LL}^2 n P_2' - C_{y_2 y_2}^2 C_{LL}^2 + (C_{y_2 L}^2)^2) \\
&\quad + (C_{y_1 y_1}^1 - n P_1') (n P_2' - C_{y_2 y_2}^2)] > 0 \\
\partial Y_1/\partial s_{ent} &= \frac{1}{|M|} [n P_2' \overbrace{(C_{y_1 y_1}^1 y_1 + \delta C_{y_2 L}^2 y_2)}^{\text{Condition(11)}} - C_{y_1 y_1}^1 C_{y_2 y_2}^2 y_1 \\
&\quad + \delta (1 - \varepsilon) y_1 ((C_{y_2 L}^2)^2 - C_{LL}^2 C_{y_2 y_2}^2 + C_{LL}^2 n P_2')] > 0 \\
\partial Y_2/\partial s_{ent} &= \frac{1}{|M|} [n P_1' \overbrace{(C_{y_2 y_2}^2 y_2 + (1 - \varepsilon) C_{y_2 L}^2 y_1)}^A - C_{y_1 y_1}^1 (C_{y_2 y_2}^2 y_2 - n C_{y_2 L}^2 y_1) \\
&\quad + \delta (1 + \varepsilon (n - 1)) y_2 ((C_{y_2 L}^2)^2 - C_{LL}^2 C_{y_2 y_2}^2)] > 0
\end{aligned}$$

If condition (10) is satisfied, we obtain $\partial y_1/\partial s_{ent} < 0$, $\partial y_2/\partial s_{ent} < 0$, and $\partial n/\partial s_{ent} > 0$. If condition (11) is satisfied, $\partial Y_1/\partial s_{ent} > 0$ holds. A is positive for all ε if condition (12) is satisfied. Then $\partial Y_2/\partial s_{ent} > 0$.

Proof of proposition 3:

Next solving (27) for $\partial y_1/\partial s_{out}$, $\partial y_2/\partial s_{out}$, $\partial n/\partial s_{out}$, $\partial Y_1/\partial s_{out}$, and $\partial Y_2/\partial s_{out}$

keeping s_{ent} constant and using conditions (12)-(13), yields:

$$\begin{aligned}
\partial y_1 / \partial s_{out} &= \frac{1}{|M|} \delta \{ \varepsilon C_L^2 [n P_2' - C_{y_2 y_2}^2] y_1 + [C_{y_2 y_2}^2 C_{LL}^2 - (C_{y_2 L}^2)^2] \varepsilon y_1^2 \\
&\quad - \varepsilon n y_1 P_2' \underbrace{[C_{LL}^2 y_1 + C_{y_2 L}^2 y_2]}_{\text{Condition(14)}} + P_2' y_2 \underbrace{[C_{y_2 y_2}^2 y_2 + C_{y_2 L}^2 y_1]}_B \} \\
\partial y_2 / \partial s_{out} &= \frac{1}{|M|} [-\delta(1 + \varepsilon(n - 1)) P_2' y_2 \underbrace{[C_{LL}^2 y_1 + C_{y_2 L}^2 y_2]}_{\text{Condition(14)}} \\
&\quad + C_{y_1 y_1}^1 \underbrace{[C_{y_2 L}^2 \varepsilon y_1 - P_2' y_2]}_{\text{Condition(10)}} + \delta \varepsilon C_L^2 [C_{y_2 L}^2 y_1 + (n - 1) P_2' y_2] < 0 \\
\partial n / \partial s_{out} &= \frac{1}{|M|} \{ [\delta \varepsilon(n - 1) C_L^2 - C_{y_1 y_1}^1 y_1] [C_{y_2 y_2}^2 - n P_2'] \\
&\quad + \delta [1 + \varepsilon(n - 1)] [-C_{y_2 y_2}^2 C_{LL}^2 + (C_{y_2 L}^2)^2] y_1 \\
&\quad + n P_2' \underbrace{[C_{LL}^2 y_1 + C_{y_2 L}^2 y_2]}_{\text{Condition(14)}} \} > 0 \\
\partial Y_1 / \partial s_{out} &= \frac{1}{|M|} \{ \delta [(1 - \epsilon) [(C_{y_2 L}^2)^2 - C_{LL}^2 C_{y_2 y_2}^2] y_1^2 \\
&\quad + n P_2' \underbrace{[(1 - \epsilon) C_{LL}^2 y_1^2 + (2 - \epsilon) C_{y_2 L}^2 y_1 y_2 + C_{y_2 y_2}^2 y_2^2]}_D \\
&\quad - (C_{y_2 y_2}^2 - n P_2') y_1 \underbrace{(\delta \epsilon C_L^2 + C_{y_1 y_1}^1 y_1)}_{\text{Condition(13)}} \} > 0 \\
\partial Y_2 / \partial s_{out} &= \frac{1}{|M|} \{ \delta \epsilon n C_L^2 \underbrace{[C_{y_2 L}^2 y_1 + C_{y_2 y_2}^2 y_2]}_{\text{Condition(12)}} \\
&\quad + \delta (1 + \epsilon(n - 1)) y_1 y_2 [(C_{y_2 L}^2)^2 - C_{LL}^2 C_{y_2 y_2}^2] \\
&\quad - C_{y_2 y_2}^2 y_2 \underbrace{(\delta \epsilon C_L^2 + C_{y_1 y_1}^1 y_1)}_{\text{Condition(13)}} \} > 0
\end{aligned}$$

If ϵ is small and B satisfies condition (12) the numerator is negative and $\partial y_1 / \partial s_{out} > 0$. With increasing ϵ it is possible that $\partial y_1 / \partial s_{out} < 0$, because all other terms of the numerator are positive if condition (14) holds. If conditions (14) and (10) hold, all terms of the numerator of $\partial y_2 / \partial s_{out}$ are positive and therefore $\partial y_1 / \partial s_{out} < 0$. If condition (14) is satisfied all terms of the numerator of $\partial n / \partial s_{out}$ are negative and $\partial n / \partial s_{out} > 0$. C is positive for all ε if condition (12) is satisfied and overall convexity holds. If additionally condition (13) holds, we obtain $\partial Y_1 / \partial s_{out} > 0$. Finally, with conditons (12) and (13) satisfied $\partial Y_2 / \partial s_{out} > 0$.

Proof of proposition 5:

- i. has already been proved.
- ii. Assume that late entry is optimal. Then the f.o.c. for the social optimum with respect to y_1^s , the output of a staying firm in period 1, is given by

$$P_1(y_1^s) - C_{y_1^s}^1(y_1^s) - \delta[[1 + \epsilon(n_s - 1)]C_L^2(y_2^s, L_s) + \epsilon n_e C_L^2(y_e, L_e)] = 0 \quad (29)$$

while the first order condition with respect to n_s (the number of staying firms) is given by

$$\begin{aligned} P_1(y_1^s)y_1^s - C^1(y_1^s) - F - \delta\epsilon[n_s C_L^2(y_2^s, L_s) + n_e C_L^2(y_e, L_e)]y_1^s \\ + \delta[P_2(y_2^s)y_2^s - C^2(y_2^s, L_s) - F] = 0 \end{aligned} \quad (30)$$

Both, the first order conditions with respect to output in the second period, y_2^s and y_e , and with respect to the number of lately entering firms, n_e , are straightforward and can be omitted since they do not contain terms which create externalities. In the decentralized economy, the zero profit condition and the first order condition of a staying firm which is subject to both an entry premium and an output subsidy in the first period are given by:

$$\begin{aligned} P_1(y_1^s)y_1^s - C^1(y_1^s) - F + s_{ent} + s_{out}y_1^s \\ + \delta[P_2(y_2^s)y_2^s - C^2(y_2^s, L_s) - F] = 0 \end{aligned} \quad (31)$$

$$P_1(y_1^s) + s_{out} - C_{y_1^s}^1(y_1^s) - \delta C_L^2(y_2^s, L_s) = 0 \quad (32)$$

Comparing these two equations to (31) and (29) we obtain (16) and (17). The late entering firms do not cause any externality. Hence no regulation is necessary.

Proof of proposition 6: Assume that exit is optimal. By Proposition 5 i) n_e is zero in (15). The first order conditions for the social optimum with respect to n_s (the staying firms) and n_x (the exiting firms) and output are given in proposition 7. The first order condition with respect to y_2^s can be omitted. In the decentralized economy with an entry premium for the S-firms and output subsidies s_{out}^s and s_{out}^x

the zero profit conditions and the first order conditions with respect to output in the first period of staying and the exiting firms, respectively are given by:

$$P_1(y_1^s)y_1^s - C^1(y_1^s) - F + s_{out}^s y_1^s + s_{ent}^s \quad (33)$$

$$+ \delta[P_2(y_2^s)y_2^s - C^2(y_2^s, L_s) - F] = 0$$

$$P_1(y_x)y_x - C^1(y_x) - F + s_{out}^x y_x = 0 \quad (34)$$

$$P_1(y_1^s) - C_{y_1^s}^1(y_1^s) + s_{out}^s - \delta C_L^2(y_2^s, L_s) = 0 \quad (35)$$

$$P_1(y_x) - C_{y_x}^1(y_x) - s_{out}^x = 0 \quad (36)$$

where s_{out}^s is given by (19), s_{ent}^s by (20), and s_{out}^x by (18). Substituting those expressions into (33) - (36), rearranging, and noting that the equilibrium is unique, we obtain the first order conditions for the socially optimal allocation (37) - (40).

Proof of proposition 7: Assume that exit is optimal. By Proposition 5 a) n_e is zero in (15). Differentiating (15), the first order conditions for social optimum with respect to n_s (the staying firms) and n_x (the exiting firms) are given by

$$P_1(y_1^s)y_1^s - C^1(y_1^s) - F - \delta \epsilon n_s C_L^2(y_2^s, L_s)y_1^s \quad (37)$$

$$+ \delta[P_2(y_2^s)y_2^s - C^2(y_2^s, L_s) - F] = 0$$

$$P_1(y_x)y_x - C^1(y_x) - \delta \epsilon n_s C_L^2(y_2^s, L_s)y_x - F = 0 \quad (38)$$

The first order conditions with respect to output are given by

$$P_1(y_1^s) - C_{y_1^s}^1(y_1^s) - \delta[1 + \epsilon(n_s - 1)]C_L^2(y_2^s, L_s) = 0 \quad (39)$$

$$P_1(y_x) - C_{y_x}^1(y_x) - \delta \epsilon n_s C_L^2(y_2^s, L_s) = 0 \quad (40)$$

The first order condition with respect to y_2^s can be omitted. In the decentralized economy with entry premia a non-linear output subsidy function $s_{out}(y_1)$ in the first period, the zero profit conditions and the first order conditions with respect to output in the first period of staying and the exiting firms, respectively are given by:

$$P_1(y_1^s)y_1^s + ay_1^s - \frac{b}{2}(y_1^s)^2 - C^1(y_1^s) - T_{ent} - F \quad (41)$$

$$+ \delta[P_2(y_2^s)y_2^s - C^2(y_2^s, L_s) + P_{stay} - F] = 0$$

$$P_1(y_x)y_x + ay_x - \frac{b}{2}(y_x)^2 - C^1(y_x) - T_{ent} - F = 0 \quad (42)$$

$$P_1(y_1^s) + a - by_1^s - C_{y_1^s}^1(y_1^s) - \delta C_L^2(y_2^s, L_s) = 0 \quad (43)$$

$$P_1(y_x) + a - by_x - C_{y_x}^1(y_x) = 0 \quad (44)$$

where a is given by (21), b by (22), T_{ent} by (23), and P_{stay} by (24). Substituting those expressions into (41) - (44), rearranging, and noting that the equilibrium is unique, we obtain the first order conditions for the socially optimal allocation (37) - (40). Since the subsidy function $S_{out}(y_1)$ is concave, the second order conditions of the firms are satisfied.

Proof of proposition 8: Without loss of generality let us define $\tilde{L}_i = \gamma L_i$ with $i = e, s$ and $\gamma \geq 0$. γ is a parameter reflecting the strength of total learning (private or public). If $\gamma = 0$ there is no learning at all. If $\epsilon = 1$, i.e. spillovers are complete, we have $L_e = L_s$. Thus late entry firms face the same costs as S-firms. The social planner does not need to distinguish between the two types of firms. The same holds for the first period: With complete learning spillovers there is no real difference between X-firms and S-firms. Therefore, we only need to look at the total number of firms, n_1 and n_2 , in either periods. If $n_1 < n_2$ there is late entry and vice versa. First suppose that $\delta = 0$ and that $\gamma = 0$, i.e. the regulator is extremely myopic and there is no learning (public or private) at all. Then it is obvious that $n_1 = n_2$ if demand is the same in both periods. The comparative static result for an increase of γ if $\delta = 0$ yields:

$$\frac{\partial n_1}{\partial \gamma} = 0 \quad (45)$$

$$\frac{\partial n_2}{\partial \gamma} = \frac{-C_{\tilde{L}}^2 C_{y_2 y_2}^2 + C_{\tilde{L}}^2 n_2 P_2' - C_{y_2 \tilde{L}}^2 n_2 P_2'}{-C_{y_2 y_2}^2 P_2' y_2} \begin{matrix} \geq \\ < \end{matrix} 0. \quad (46)$$

If $-C_{\tilde{L}}^2 C_{y_2 y_2}^2 + C_{\tilde{L}}^2 n_2 P_2' - C_{y_2 \tilde{L}}^2 n_2 P_2' > 0$ holds, the number of firms increases with increasing total learning, i.e. we have late entry. The comparative static result for an increase of δ shows that n_1 is increasing and n_2 can go in either direction. But as the functions are continuous we can be sure that for δ sufficiently small we observe more firms in the second period.

References

- ARGOTE, L. AND EPPLER, D. "Learning Curves in Manufacturing." *Science*, Vol. 247 (1990), pp. 920-924.
- ARROW, K. "The Economic Implication of Learning-by-Doing." *Review of Economic Studies*, Vol. 29 (1962), pp. 155-173.
- BARDHAN, P.K. "On Optimum Subsidy to a Learning Industry: An Aspect of the Theory of Infant-Industry Protection." *International Economic Review*, Vol. 12 (1971), pp. 54-70.
- BAUMOL, W.J., PANZAR, J.C. AND WILLIG, R.D. *Contestable Markets and the Theory of Industry Structure*, New York: Harcourt Brace Jovanovich, Inc., (1982)
- CABRAL, L. AND RIORDAN, M. "The Learning Curve, Market Dominance, and Predatory Pricing", *Econometrica*, Vol. 62 (1994), pp. 1115-1140.
- DASGUPTA, P. AND STIGLITZ, J. "Learning-by-Doing, Market Structure and Industrial and Trade Policies", *Oxford Economic Papers*, Vol. 40 (1988), pp. 246-268.
- FUDENBERG, D. AND TIROLE, J. "Learning-by-Doing and Market Performance", *The Bell Journal of Economics*, Vol. 14 (1983), pp. 522-530.
- ISOARD, S. AND SORIA, A. "Technical Change Dynamics: Evidence from the Emerging Renewable Energy Technologies", *Energy Economics*, Vol. 23 (2001), pp. 619-636.
- JIN, J.Y., PEROTE-PENA, J., AND TROEGE, M. "Learning-by-Doing, Spillovers and Shakeouts", *Journal of Evolutionary Economics*, Vol. 14 (2004), pp. 85-98.
- LEAHY, D. AND NEARY, P.J. "Learning-by-Doing, Precommitment and Infant-Industry Promotion", *Review of Economic Studies*, Vol. 66 (1999), pp. 447-474.
- LIEBERMAN, M.B. "The Learning Curve and Pricing in the Chemical Processing Industries", *RAND Journal of Economics*, Vol. 15 (1984), pp. 213-228.

- LLOYD, R. A. "Experience Curve Analysis", *Applied Economics*, Vol. 11 (1979), pp. 221-234.
- MIRAVETE, E.J. "Time-Consistent Protection with Learning-by-Doing", *European Economic Review*, Vol. 47 (2003), pp. 761-790.
- PETRAKIS, E., RASMUSEN, E. AND ROY, S. "The Learning Curve in a Competitive Industry", *RAND Journal of Economics*, Vol. 28 (1997), pp. 248-268.
- RAUCH, J.E. "A Note on the Optimum Subsidy to a Learning Industry", *Journal of Development Economics*, Vol. 38 (1992), pp. 233-243.
- SPENCE, M.A. "The Learning Curve and Competition", *The Bell Journal of Economics*, Vol. 12 (1981), pp. 49-70.