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## Short-Run Elasticity of Substitution - Error Correction Model

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**Short-Run Elasticity of Substitution – Error Correction  
Model**

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## Short-Run Elasticity of Substitution – Error Correction Model<sup>1</sup>

**JEL Classification:** C13; E23; E24

**Keywords:** *short-run and long-run elasticity of substitution, aggregate and sectoral estimations, vector error correction model, labour demand of the profit maximizing firm*

### Abstract

**Research background:** The value of the elasticity of the substitution has been a subject of the research around the world in last decades. It affects the qualitative and quantitative answers to a host of economic questions.

**Purpose of the article:** We suggest the co-integration estimation form to estimate short-run elasticity of substitution. Using U.S. NIPA aggregate time series we estimate aggregate short-run elasticity of substitution. In comparison with estimations in economic literature, we confirm theoretical assumptions described in the research background.

**Methodology/methods:** Different econometric estimation forms are used to estimate elasticity of the substitution coefficient. One possibility is a constant elasticity of substitution production function linearization. Others come from the first-order conditions of a representative firm expressing factor demand functions. Error correction models are natural and elegant way to estimate the forms with non-stationary data. However, the use of error correction models in the factor demand econometric forms is useless for estimating a long-run elasticity of substitution coefficient. The co-integration relationship is given by the theoretical assumption of the labour share constancy in the long-run or by other underlying processes. Though, we can use this co-integration relationship to correct error term in the short-run estimation form. To estimate the short-run elasticity of substitution, we use Stock and Watson's estimation form. Stability, stationarity and serial correlation of residuals are tested by the relevant econometric tests.

**Findings:** The value of aggregate short-run elasticity of substitution is closed to one. In comparison with other relevant theoretical and empirical papers, our results incline to the Cobb-Douglas aggregate production function in U.S. economy.

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## Introduction

There are many ways to estimate the elasticity of substitution. Chirinko (2008) and Klump, McAdam and Willman (2012) provide rich literature survey of elasticity of input substitution estimation problem. We focus to the co-integration analysis of the factor prices. Caballero (1994) measures long-run values by exploiting the co-integration relations between the capital/output ratio and the user cost of capital. As argued in Chirinko and Mallick (2011), this estimation strategy faces some econometric difficulties in recovering production function parameters. In this paper we use similar analysis of labour/output. We prefer labour demand analysis to the capital one, because there are large data series consisting of labour, output and prices in the U.S. NIPA data sources. The large observation set is needed for the co-integration analysis. We use Chirinko's and Mallick's (2011) suggestion to form and estimate a co-integration econometric specification suitable to quantify short-run values of the elasticity of substitution.

$$\Delta(y_t - l_t) = \alpha_0 + \beta_1 \Delta(w_t - p_t) + \lambda [(y_{t-1} - l_{t-1}) - \gamma_0 - \gamma_1 (w_{t-1} - p_{t-1})] + u_t \quad (1)$$

where  $y_t$ ,  $l_t$ ,  $p_t$  and  $w_t$  are the natural logarithms of output  $y$ , labour  $l$  and their prices,  $u_t$  is a white-noise stochastic term. Coefficients  $\beta_1$  and  $\gamma_1$  are estimations (suggested by Caballero, 1994) of long-run and short-run elasticity of substitution and  $-1 \leq \lambda \leq 0$  is a co-integration adjustment coefficient. Chirinko and Mallick (2011) argue that neoclassical growth theory assumes the constancy of the factor share  $w_t + l_t - p_t - y_t$ .

However, after substituting the factor share to the co-integration form (1), "the constancy holds if and only if the influence of relative prices is eliminated. In this case coefficient  $\gamma_1$  must equal 1" (Chirinko and Mallick, 2011, p. 206) and the coefficient is not a measure of the long-run elasticity of substitution. We argue that the estimation form (1) is suitable for estimating the short-run elasticity of substitution  $\beta_1$ .

According to Chirinko and Mallick (2011), three cases consistent with a general economic knowledge may exhibit the co-integration form (1). Firstly, co-integration relation holds. This may be reasonable according to the neoclassical growth theory, if labour is the factor. Then  $\gamma_1 = 1$ . Secondly, co-integration relation does not hold. This may be reasonable according to the theory, if capital is the factor. Finally, co-integration relation does not hold, but variables are driven by different underlying co-integration processes. Considering labour demand estimation form, we can estimate co-integration form with  $\gamma_1 = 1$ . To estimate all coefficients in one step we

rewrite the co-integration relation into the form suggested by Stock and Watson (1993).

$$\Delta(y_t - l_t) = \beta_0 + \beta_1 \Delta(w_t - p_t) + \lambda(y_{t-1} - l_{t-1}) + \delta_1(w_{t-1} - p_{t-1}) + u_t \quad (2)$$

where  $\delta_1 = -\lambda\gamma_1$ . Considering the mentioned restriction  $\gamma_1 = 1$ , we gain a specification:

$$\Delta(y_t - l_t) = \alpha_0 + \beta_1 \Delta(w_t - p_t) + \lambda[(y_{t-1} - l_{t-1}) - (w_{t-1} - p_{t-1})] + u_t \quad (3)$$

Szomolányi, Lukáčik and Lukáčiková (2015) showed that the both co-integration form (2) and (3) are consistent with the normalised constant elasticity of substitution production function suggested by De La Grandville (1989) and Klump, McAdam and Willman (2012).

The purpose of the article is to verify the suggested co-integration estimation forms for labour demand and estimate the short-run elasticity of substitution using U.S. aggregate data.

## Data and Method of the Research

To estimate the coefficients of the forms (2) and (3) we use yearly data of logarithms of average labour product in constant prices,  $y_t - l_t$ , and its price,  $w_t - p_t$ , in the period 1929 – 2015 obtained from NIPA tables of U.S. Bureau of Economic Analysis portal<sup>2</sup>. Deriving the data we follow Gollin (2002) and Klump, McAdam and Willman (2007).

Gollin (2002) refers an inconsistency between a theory and observed values of labour share. This inconsistency comes from incorrect calculation of labour share. Compensation to employees is not suitable indicator for labour income because they exclude proprietors (self-employed) labour income. It is unclear how the income of self-employed workers should be categorized in the labour-capital dichotomy.

We consider two approaches. Following Krueger (1999) and Antràs (2004) we add two thirds of self-employed workers' income to the compensations of employees. We denote this approach by the symbol (*a*).

Blanchard's Nordhaus's and Phelps's (1997), Gollin's (2002) and Ben-tolila's and Saint-Paul's (2003) approach (*b*) is to use compensation per employee as a shadow price of labor of self-employed workers, i.e. labour income in extensive form,  $l_t w_t$ , is:

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<sup>2</sup> <https://www.bea.gov/>

$$labour\ income = \left(1 + \frac{self\ employed}{total\ employment}\right) \cdot compensation\ to\ employees \quad (4)$$

Gollin (2002) also introduced two more ways to modify data for correct labour share calculation, but as he stated, these two ways are not suitable for the U.S. economy.

We consider *GDP* for output. We can use employment or number of hours worked as a labour indicator. For a long-run analysis, we consider the employment to be satisfactory measure of the labour.

In the first look on data we focus to the stationarity tests. Both augmented Dickey-Fuller and Phillips-Perron tests (see Lukáčik and Lukáčiková, 2008) imply stationarity in the data series of the average product and its price measured by both ways (a) and (b), if trend and intercept are not included in the test specification. However, the correlogram of the all data series imply unit roots. The first-order serial correlation is closed to one and autocorrelation values are slowly decreasing with time. Differencing the data series both test procedures as well as correlograms imply non-stationarity. Therefore we need to use their first differences in the estimation forms. Both (2) and (3) forms use the first differences of average factor products. The least square method is used to estimate the coefficients. The autocorrelation of residuals is tested by the Breusch-Godfrey serial correlation LM test.

Using the (b) measure of labour, the price residuals are serial correlated. In the case of serial correlation, we compute the standard errors with procedure of Newey and West (1994). The stationarity of residuals is tested using the same procedure as the data series. The normality of residuals is tested using the Jarque-Bera test. For testing of the co-integration adjustment coefficients  $\lambda$ , tables suggested by Banerjee, Dolado and Mestre (1998) are used. The coefficient restriction tests used  $\chi^2$  distributed Wald statistics which is preferred when the restriction is not linear as in our case.

## Results

The estimations of (2) specification coefficients are in the Table 1. Using the (a) measure of the labour price, the estimated value of the short-run elasticity of substitution is 0.91. Using the Banerjee, Dolado and Mestre (1998) tables, the co-integration adjustment coefficient  $\lambda$  is statistically significant at 5 % significance level. We computed the coefficient by  $\gamma_1 = -\delta_1/\lambda$ . The estimation of the coefficient is closed to 1 (precisely 0.932), however we do reject the hypothesis  $\gamma_1 = 1$  using  $\chi^2$  distributed statistics.

The standard errors of estimated coefficients of (2) using the (b) measure of the labour price (in the last column) are computed with the Newey-West (1994) procedure. The corresponding elasticity of substitution estimation is 1.074. The co-integration adjustment coefficient  $\lambda$  is statistically significant at 5 % significance level. The estimation of the  $\gamma_1$  coefficient is 1.005 and we do not reject the hypothesis  $\gamma_1 = 1$  using  $\chi^2$  distributed statistics.

**Table 1.** The estimations of the (2) specification coefficients

Coefficient	Data Set (a)		Data Set (b)	
	Value	Standard Error	Value	Standard Error
$\beta_1$	0.910	0.018	1.074	0.061
$\lambda$	-0.248	0.065	-0.412	0.095
$\delta_1$	0.231	0.060	0.414	0.100

Source: own processing

Even if we reject the unity of the  $\gamma_1$  coefficient in the (a) case, both estimations are closed to 1, confirming the theory. Using both datasets, we estimated the restricted estimation form (3) implying  $\gamma_1 = 1$ . The results are in the Table 2. The short-run elasticity of substitution estimations are consistent with the estimations corresponding to the (2) specification in the Table 1. Using the (a) measure of the labour price, the estimated value of the short-run elasticity of substitution is 0.906. However, using the Banerjee, Dolado and Mestre (1998) tables, the co-integration adjustment coefficient  $\lambda$  is not statistically significant.

The standard errors of estimated coefficients of (2) using the (b) measure of the labour price (in the last column) are computed with the Newey-West (1994) procedure. The corresponding elasticity of substitution estimation is 1.072. The co-integration adjustment coefficient  $\lambda$  is statistically significant at 1 % significance level.

**Table 2.** The estimations of the (3) specification coefficients

Coefficient	Data Set (a)		Data Set (b)	
	Value	Standard Error	Value	Standard Error
$\beta_1$	0.906	0.019	1.072	0.059
$\lambda$	-0.105	0.044	-0.405	0.087

Source: own processing

Our short-run elasticity of substitution estimation is closed to 1 in all cases, implying the Cobb-Douglas production function. Therefore we tested Cobb-Douglas restriction hypothesis  $\beta_1 = 1$ . Using  $\chi^2$  distributed Wald

statistics, we reject the hypothesis with the estimations based on the (a) dataset, but we do not reject the hypothesis with the estimations based on the (b) dataset. Note that estimations based on (a) dataset do not fit the considered theory. Non-unity of  $\gamma_1$  coefficient implies the non-constancy of the labour share or other underlying co-integration processes.

## Conclusions

The most recent studies of Chirinko and Mallick (2014) or Klump, McAdam and Willman (2007) suggest the elasticity of substitution markedly lower than 1. The co-integration analysis of the average labour and its relative price relationship, estimating the long-run elasticity of substitution suggested by Caballero (1994), has been criticised by Chirinko and Mallick (2011). The neoclassical growth theory that comes from the long-run constancy of the factor share implies the studied relationship independent on the elasticity of substitution. Our study return to the co-integration analysis and it considers the Chirinko's and Mallick's (2011) suggestions. Using co-integration form suggested by Stock and Watson (1993), estimating the short-run and long-run coefficients in one step, we estimate the short-run elasticity of substitution closed to 1.

## References

- Antràs, P. (2004). Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *The B.E. Journal of Macroeconomics*, 4(1). DOI: <http://dx.doi.org/10.2202/1534-6005.1161>
- Banerjee, A., Dolado, J. J., & Mestre, R. (1998). Error-Correction Mechanism Tests for Cointegration in a Single-Equation Framework. *Journal of Time Series Analysis*, 19(3). DOI: <http://dx.doi.org/10.1111/1467-9892.00091>
- Bentolila, S., & Saint-Paul, G. (2003). Explaining Movements in the Labor Share. *Contributions in Macroeconomics*, 3(1). DOI: <http://dx.doi.org/10.2202/1534-6005.1103>
- Blanchard, O. J., Nordhaus, W. D., & Phelps, E. S. (1997). The Medium Run. *Brookings Papers on Economic Activity*, 1997(2). DOI: <http://dx.doi.org/10.2307/2534687>
- Chirinko, R. S. (2008).  $\sigma$ : The long and short of it. *Journal of Macroeconomics*, 30(2). DOI: <http://dx.doi.org/10.1016/j.jmacro.2007.10.010>
- Caballero, R. J. (1994). Small Sample Bias and Adjustment Costs. *The Review of Economics and Statistics*, 76(1). DOI: <http://dx.doi.org/10.2307/2109825>
- Chirinko, R. S., & Mallick, D. (2011). Cointegration, factor shares and production function parameters. *Economics Letters*, 112(2). DOI: <http://dx.doi.org/10.1016/j.econlet.2011.04.002>



- Chirinko, R. S., & Mallick, D. (2014). The Substitution Elasticity, Factor Shares, Long-Run Growth, and The Low-Frequency Panel Model. *CESifo Working Paper Series* 4895, CESifo Group Munich. Retrieved from <http://as.vanderbilt.edu/econ/sem papers/Chirinko.pdf> (24.03.2017)
- De La Grandville, O. (1989). In Quest of the Slutsky Diamond. *American Economic Review*, 79(3).
- Gollin, D. (2002). Getting Income Shares Right. *Journal of Political Economy*, 110(2). DOI: <http://dx.doi.org/10.1086/338747>
- Klump, R., McAdam, P., & Willman, A. (2007). Factor Substitution and Factor Augmenting Technical Progress in the US. *Review of Economics and Statistics*, 89(1). DOI: <http://dx.doi.org/10.1162/rest.89.1.183>
- Klump, R., McAdam, P., & Willman, A. (2012). The Normalized CES Production Function: Theory and Empirics. *Journal of Economic Surveys*, 26(5). DOI: <http://dx.doi.org/10.1111/j.1467-6419.2012.00730.x>
- Krueger, A. B. (1999). Measuring Labor's Share. *American Economic Review*, 89(2). DOI: <http://dx.doi.org/10.1257/aer.89.2.45>
- Lukáčik, M., & Lukáčiková, A. (2008). Význam testovania stacionarity v ekonometrii. *Ekonomika a informatika*, 6(1).
- Newey, W. K., & West, K. D. (1987). A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55(3). DOI: <http://dx.doi.org/10.2307/1913610>
- Stock, J. H., & Watson, M. W. (1993). A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems. *Econometrica*, 61(4). DOI: <http://dx.doi.org/10.2307/2951763>
- Szomolányi, K., Lukáčik, M., & Lukáčiková, A. (2015). Long-Run Elasticity of Substitution. *Proceedings of 33rd international conference Mathematical methods in economics*. Plzeň: Faculty of Economics, University of West Bohemia.