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## Hill-Climbing Algorithm for Robust Emergency System Design with Return Preventing Constraints

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**Hill-Climbing Algorithm for Robust Emergency System  
Design with Return Preventing Constraints**

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## Hill-Climbing Algorithm for Robust Emergency System Design with Return Preventing Constraints

**JEL Classification:** C61; C63

**Keywords:** *Emergency system design; robustness; iterative algorithm; convex combination; return preventing constraints*

### Abstract

**Research background:** This paper deals with smart design of robust emergency service system. The robustness means here resistance of the system to various detrimental events, which can randomly occur in the associated transportation network. Consequences of the detrimental events are formalized by establishing a set of detrimental scenarios. A robust emergency service system is usually designed so that the deployment of given number of service centers minimizes the maximal value of objective functions corresponding with the specified scenarios. The original approach to the system design using means of mathematical programming faces computational difficulties caused by the link-up constraints.

**Purpose of the article:** The main purpose of our research is to overcome the computational burden of the branch-and-bound method caused by the min-max constraints in the model. We suggest an iterative hill climbing algorithm, which outperforms the original approach in both computational time and computer memory demand.

**Methodology/methods:** The methodology consists in approximation of the maximum of original objective functions by a suitable convex combination of the objective functions. The previously developed hill climbing algorithm is extended by return preventing constraints and their influence on computational effectiveness is studied within this paper. Especially, we focus on finding the most suitable form of the return preventing constraints and strategy of their implementation.

**Findings & Value added:** We present here a comparison of the suggested algorithm to the original approach and the Lagrangean relaxation of the original approach. We found that the suggested algorithm outperforms the original exact approach as concerns the computational time with maximal two percent deviation from the optimal solution. In addition, the algorithm outperforms the Lagrangean approach in both computational time and the deviation.

## Introduction

Emergency service systems are established to provide users by necessary service in emergency cases like accidents, fire or similar severe events endangering life or property. As serviced population of users is spread over a geographical area comprising big number of dwelling places and the considered kind of service can be provided only from limited number of service centers, the center deployment in the given area represents the main part of the emergency system design. The optimal deployment of a given number of centers in a set of possible center locations is searched with the objective to minimize a sum of weighted time-distances from users to the nearest located center. The weight corresponds with population at the user location (Avella et al., 2007, pp. 89-114; Current et al., 2002, pp. 81-118; Marianov & Serra, 2002, pp. 119-150). Each designer's dream is to suggest such a system, which is resistant to randomly occurring events in the transportation network used for the service transport. To make the suggested system robust, a finite set of the most damaging scenarios is specified and the center deployment is designed so that the objective function value connected with the worst scenario is minimal (Correia & Saldanha da Gama, 2015, pp. 177-203; Janáček, 2015, pp. 595-606; Pan et al., 2014, pp. 164-172; Scaparra & Church, 2015, pp. 623-642). The associated linear programming model minimizes maximum of several objective functions and thus the model must contain so-called link-up constraints. The associated solving computational process is then very slow due to the link-up constraints and the size of the model. To overcome this drawback, we suggested an iterative algorithm based on processing a convex combination of the scenarios and iterative adjustment of the associated multipliers. The algorithm was designed in the form of "hill-climbing" algorithm. When testing its preliminary version we found that the computational process had been often locked in a short cycle formed by two or three solutions. Within this paper, we focus on such algorithm arrangement, which prevents the computational process from the deadlock. We present a study of the arrangement impact on computational efficiency and give a recommendation for a choice of solving tool for the robust emergency system design.

The remainder of the paper is organized as follows. The next section is devoted to a concise explanation of a radial formulation of the emergency system design problem (García et al., 2011, pp. 546-556; Janáček & Kvet, 2013, pp. 332 – 337). The third section introduces the approach to the robust emergency system design based on the convex combination of the detrimental scenarios. The fourth section comprises results of numerical experiments. The fifth section summarizes the achieved results and findings.

## Radial formulation of emergency system design problem

To describe the emergency system design problem with  $p$  located centers in the radial form (García et al., 2011, pp. 546-556; Janáček & Kvet, 2013, pp. 332 – 337), we denote  $I$  a set of possible center locations and  $J$  the set of possible users' locations. The symbol  $b_j$  denotes population of users located at  $j \in J$ . The symbol  $d_{ij}$  denotes the integer distance between a user at  $j$  and the possible center location  $i$ . To model the decision on locating or not locating a center at the particular location  $i \in I$ , we introduce a zero-one variable  $y_i \in \{0, 1\}$ , which takes the value of 1, if a center should be located at the location  $i$ , and it takes the value of 0 otherwise. An integer time-distance between a user location  $j$  and the nearest located center is expressed by the sum  $x_{j0} + x_{j1} + x_{j2} + \dots + x_{jv}$  of auxiliary zero-one variables  $x_{js}$  for  $s = 0, \dots, v$ . The variable  $x_{js}$  takes the value of 1, if the distance of the user at  $j \in J$  from the nearest located center is greater than  $s$  and it takes the value of 0 otherwise. We assume that the maximal considered time distance between user and possible center locations is  $v+1$ . Further we introduce a zero-one constant  $a_{ij}^s$  for each  $i \in I, j \in J$  and  $s = 0, \dots, v$ . The constant  $a_{ij}^s$  is equal to 1, if the time-distance  $d_{ij}$  between the user location  $j$  and the possible center location  $i$  is less than or equal to  $s$ , otherwise  $a_{ij}^s = 0$ . Then the radial model can be formulated as follows:

$$\text{Minimize} \quad \sum_{j \in J} b_j \sum_{s=0}^v x_{js} \quad (1)$$

$$\text{Subject to:} \quad x_{js} + \sum_{i \in I} a_{ij}^s y_i \geq 1 \quad j \in J, \quad s = 0, \dots, v \quad (2)$$

$$\sum_{i \in I} y_i \leq p \quad (3)$$

$$x_{js} \geq 0 \quad j \in J, \quad s = 0, \dots, v \quad (4)$$

$$y_i \in \{0, 1\} \quad i \in I \quad (5)$$

The objective function (1) gives the sum of weighted time-distance values. The constraints (2) ensure that the variables  $x_{js}$  are allowed to take the value of 0, if there is at least one center located in radius  $s$  from the user location  $j$ . The constraint (3) limits the number of located centers by the number  $p$ . Even if only zero-one values of variable  $x_{js}$  are meaningful in the above model, constraints (4) are sufficient to ensure it due to partial integrality property of the model.

## An iterative approach to the robust emergency system design

The robust emergency system design problem is formulated with use of a set of possible failure scenarios denoted by  $U$ . The time distance between locations  $i$  and  $j$  under a specific scenario  $u \in U$  is denoted by  $d_{iju}$ . The suggested approach is based on a convex combination of the objective functions associated with the individual scenarios. The objective function a solution  $\mathbf{y}$  associated with a scenario  $u$  of is denoted as  $f^u(\mathbf{y})$ . We distinguish scenario  $b \in U$ , which is the basic scenario corresponding to the standard conditions. Let  $\mathbf{y}^u$  minimize the objective function  $f^u(\mathbf{y})$ . The convex combination of the scenario objective function for the given set of nonnegative multipliers  $\lambda_u \geq 0$  for  $u \in U$  satisfying the condition that their sum equals to one can be defined using convex combination of considered matrices  $\{d_{iju}\}$  for  $u \in U$ . The distance between  $i$  and  $j$  according to the convex combination is defined by (6).

$$d_{ij}(\lambda) = \sum_{u \in U} \lambda_u d_{iju} \quad i \in I, j \in J \quad (6)$$

If we define a zero-one constant  $a_{ij}^s(\square)$  which is equal to 1 in the case that the time distance  $d_{ij}(\square)$  is less or equal to  $s$ , otherwise  $a_{ij}^s(\square)$  equals to 0, then we can easily obtain the optimal solution for the convex combination objective functions by solving problem (1)-(5). Here, the incidental coefficients  $a_{ij}^s(\square)$  are used instead of the original coefficients  $a_{ij}^s$ . For a given current solution  $\mathbf{y}$ , values of  $f^u(\mathbf{y})$  can be easily found. Then the multiplier values can be computed from a current solution  $\mathbf{y}$  for a positive parameter  $\square$  according to (7).

$$\lambda_u(\mathbf{y}) = \frac{(f^u(\mathbf{y}) - f^b(\mathbf{y}^b))^\beta}{\sum_{u \in U} (f^u(\mathbf{y}) - f^b(\mathbf{y}^b))^\beta} \quad u \in U \quad (7)$$

The iterative process with the return preventing constraints is described by the following steps:

- 0) Compute the optimal solution  $\mathbf{y}^b$  for the objective function  $f^b$  and the multipliers  $\lambda_u$  associated with  $\mathbf{y}^b$ . Set  $\mathbf{y} = \mathbf{y}^b$ . Initialize the best found solution  $\mathbf{y}^* = \mathbf{y}^b$ . Set noFails=0. Set  $k=0$  and produce list  $I(k) = \{i \in I, y_i^b = 1\}$ .
- 1) Solve the problem (1) - (5), (8) for the current  $a_{ij}^s(\square)$  and obtain current solution  $\mathbf{y}$ . Set  $k=k+1$  and produce list  $I(k) = \{i \in I, y_i = 1\}$ .
- 2) If  $f^r(\mathbf{y}) < f^r(\mathbf{y}^*)$ , then update  $\mathbf{y}^*$ , otherwise set noFails= noFails +1.

- 3) Determine new values of  $\alpha_u$  for the current solution  $y$  according to (7).
- 4) If  $\text{noFails} > \text{MaxFails}$ , then terminate, otherwise go to 1).

We have to comment on  $\alpha$  adjustment in the above algorithm. In the case of updating  $y^*$ , we set  $\alpha$  at the value of 1 and in the opposite case, we used the strategy of incrementing current value of  $\alpha$  by 1.2, if the worst scenario was the same as in the previous step and we decremented the value of  $\alpha$  by 0.2, when the worst scenarios differed.

The set of return preventing constraints applied at the  $k$  th run of the algorithm has the form of (8).

$$\sum_{i \in I(t)} y_i \leq p - r \quad t = 0, \dots, k - 1 \quad (8)$$

The set of constraints (8) ensures that the current solution of the problem (1) - (5), (8) differs from the solutions obtained in the previous  $k$  runs of the algorithm at least in  $r$  locations.

## Methodology of the associated research

Our effort in this field was focused on complying curse of dimensionality in robust emergency system design. This associated model contains multiply more decision variables than the model with only one scenario. The same situation occurs in the number of constraints and furthermore, the model of robust system design must be enlarged by link-up constraints. These min-max link-up constraints and the cardinality of the scenario set represent an undesirable burden in any solving proves used for design solution. To overcome these obstacles, we aimed our research at finding such properties of models and algorithms, which enable to reduce the problem size, to simplify its structure and to accelerate the associated computational process.

The methodology of our research can be divided into two parts. The first one consists of constituting a convex combination of the scenarios and studying the characteristics of the associated iterative hill-climbing process, which is performed with the problem, which size is considerably reduced in comparison with the original robust emergency system design problem. The associated synthesis has led to creating a solving iterative algorithm, which characteristics were obtained by performing numerical experiments and analyzing the

results. As an accompanying phenomenon of the hill-climbing algorithm is the situation when the process gets locked in a short cycle formed by two or three solutions (see Figure 1).

The second part seeks for such a measure, which prevents the computational process from the deadlock. Research in this part was focused on construction of proper forms of return preventing constraints. We have suggested a general form of return preventing constraints and searched for a sufficient setting of constraint parameters.

## Numerical experiments

This chapter is devoted to the results of numerical experiments. To get the results, the optimization software FICO Xpress 8.0 (64-bit, release 2016) was used and the experiments were run on a PC equipped with the Intel® Core™ i7 5500U processor with the parameters: 2.4 GHz and 16 GB RAM. The used benchmarks were derived from the road network of Slovakia for eight self-governing regions, i.e. Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA). All cities and villages with corresponding number  $b_j$  of inhabitants were taken into account. The coefficients  $b_j$  were rounded to hundreds. In the benchmarks, the set of communities represents both the set  $J$  of users' locations and also the set  $I$  of possible service center locations. The cardinalities of these sets vary from 87 to 664. The number  $p$  of located centers was derived from real emergency health care system, which has been originally implemented in studied regions and it varies from 9 to 67. The matrix  $\{d_{ij}\}$  of the time-distances from possible center locations to individual users' locations obtained from the associated road network represents so-called basic scenario in each solved instance. To study the suggested algorithm for robust emergency system design, several scenarios are needed. Due to the lack of common benchmarks for study of robustness, the scenarios used in our computational study were created in the following way. We selected 25 percent of matrix rows so that these rows correspond to the biggest cities concerning the number of system users. Then we chose randomly from 5 to 15 rows from the selected ones and the associated time-distances in the chosen rows were multiplied by the randomly chosen constant from the range 2, 3 and 4. The rows, which were not chosen by this random process, stay unchanged (they are multiplied by the value of 1). This way, 10 different scenarios were generated for each self-governing region. The scenarios represent the consequence of fatal detrimental events, when some time-distances are several times elongated.

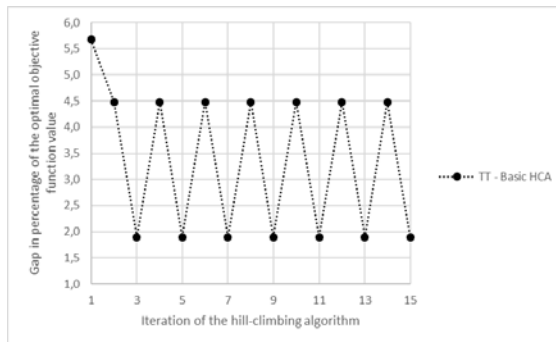


An individual experiment was organized so that the optimal solution  $y$  of the robust emergency system design problem was obtained using the radial formulation. Here, the maximal objective function value over all scenarios is minimized. This criterion can be formulated as follows.

$$h = \max \left\{ \sum_{j \in J} b_j \min \{ d_{iju} : i \in I, y_i = 1 \} : u \in U \right\} \quad (9)$$

After getting the optimal solution, the basic version of the hill-climbing algorithm with no return preventing constraints was applied on the same problem to evaluate its time demands and the results accuracy. For this computational study, the hill-climbing algorithm was restricted to 15 iterations. While analyzing the algorithm performance, it was found, that the iterative process might get stuck at a local minimum or alternate between two or three solutions. Such situation is demonstrated in the Figure 1, which reports the computational process of the hill-climbing algorithm for the self-governing region of Trnava (TT). In each iteration, the objective function (9) for the resulting vector  $y$  of location variables  $y_i$  was computed. Instead of its absolute value, the relative gap is depicted. Here, the gap is defined as the difference between the resulting objective function value (9) and the optimal objective function value in percentage of the optimal objective function value.

**Figure 1.** Analysis of the computational process of suggested hill-climbing algorithm for the self-governing region of Trnava with  $|I| = 249$  and  $p = 25$ .



Source: Own calculation based on performed numerical experiments

Based on the observed obstacle consisting in possible computational process deadlock, the concept of return preventing constraints was introduced to prevent the algorithm from the above-mentioned drawback.

The achieved results are summarized in the following Table 1, in which three different approaches to the robust emergency system design are compared. Each benchmark was solved by the exact approach to get the optimal objective function value. Then, the hill-climbing algorithm was applied and finally, the problem was solved by the hill-climbing algorithm with embedded return preventing constraints. For each approach, the computational time in seconds is reported in the columns denoted by CT. The columns denoted by  $h$  contain the objective function value computed according to (9) for particular resulting solution. To evaluate the accuracy of the suggested algorithm, the value of  $Dif$  is reported. It is defined as difference between the optimal and the resulting solution objective function value given in percentage of the optimal objective function value. We can observe that the resulting accuracy is very satisfactory.

**Table 1.** Comparison of two versions of the hill-climbing algorithm suggested for robust emergency system design to the original approach.

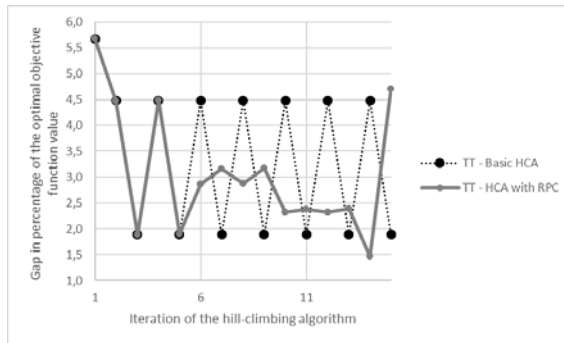
Region	$ I $	$p$	Standard approach		Basic HCA (without RPC)			HCA with RPC		
			CT	$h$	CT	$h$	$Dif$	CT	$h$	$Dif$
BA	87	9	28.7	25417	2.1	26102	2.7	13.1	25417	0.0
BB	515	52	1063.1	18549	50.3	19056	2.7	53.6	19056	2.7
KE	460	46	1284.2	21286	32.5	21756	2.2	36.5	21717	2.0
NR	350	35	2017.6	24193	22.4	24402	0.9	53.8	24372	0.7
PO	664	67	1180.4	21298	77.8	21607	1.5	123.2	21590	1.4
TN	276	28	264.1	17524	11.4	17605	0.5	28.1	17605	0.5
TT	249	25	433.8	20558	10.9	20947	1.9	29.2	20859	1.5
ZA	315	32	1229.7	23004	11.9	23224	1.0	39.5	23156	0.7

Source: The results of performed computational study

The impact of the return preventing constraints on the algorithm performance is shown on the Figure 2, where the benchmark of Trnava (TT) was used. We can observe that the return preventing constraints may significantly change the computational process and bring better results than the original hill-climbing algorithm provided.

It must be noted that we have studied many other settings of the return preventing constraints, which differed in the parameter  $r$ . Its value defines the minimal number of located centers, in which the solutions must differ. Our research did not bring better results than reported above. Thus, the difference in at least one located service center is sufficient.

**Figure 2.** Impact of the return preventing constraints in the hill-climbing algorithm for the self-governing region of Trnava with  $|I| = 249$  and  $p = 25$ .



Source: Own calculation based on performed numerical experiments

## Conclusions

The main goal of this paper was to introduce an effective solving technique for robust emergency system design. The robustness follows the idea that the designed system should be resistant to various detrimental events occurring randomly. We have presented an iterative approach, which can considerably outperform the exact approach as concerns the computational time, whereas the resulting objective function value of the iterative approach does not differ from the exact value by more than two percent on average. Since the computational process may get locked in a short cycle, we have introduced the concept of return preventing constraints to avoid this obstacle. Based on achieved results we can conclude that we have constructed an efficient algorithm for robust emergency system designing.

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