

Pietrzak, Michal Bernard

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**Application of TOPSIS Method in Condition of the Presence
of Spatial Dependence**

Michał Bernard Pietrzak

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Application of TOPSIS Method in Condition of the Presence of Spatial Dependence

Abstract: The theme of the paper is focused on the application TOPSIS method in condition of the presence of spatial dependence. The occurrence of positive spatial dependence is embedded in the majority of economic phenomena. According to Tobler's first law of geography, one of the key issues in doing the regional research is considering spatial location. Failure to consider the existing spatial dependence for the analysed phenomena can lead to cognitive errors. Therefore the purpose of this article is to take into account the issue of the consideration of spatial dependence within the TOPSIS method. The realisation of the objective set in the present article allowed the development of the procedure for constructing the spatial taxonomic measure of development sTMD using the TOPSIS method. Spatial dependence will be taken into account by using the potential strength of the interaction between the regions. The spatial taxonomic measure of development (sTMD) calculated by means of the modified TOPSIS method allows to determine the trend in the level of the development of the phenomenon under study, assuming the impact of the spatial mechanisms.

The spatial taxonomic measure of development (sTMD) defined in that way has been applied in the analysis of the situation on the labour market in Poland. The study will be carried out for the territorial composition of 66 subregions (NUTS 3 level) as of 2013. The research allowed us to assess the situation in 2013 and to identify the trend in the development of the labour market. The analysis appointed to the usefulness of the proposed measure sTMD, which is complementary to the use of the others taxonomic measure in the process of explaining the economic reality. The use of the spatial taxonomic measure of development allows expanding the results by conclusions concerning the trend in development of the analysed phenomenon.

JEL classification: C21,J01, O10

Keywords: numeric taxonomy, TOPSIS, Taxonomic Measure of Development, spatial econometrics, spatial dependence

INTRODUCTION

The economic research has increasingly dealt with the problem of the occurrence of spatial dependence and with the consideration of this dependence in analyses being conducted. On the one hand, this fact results from the popularisation of the methods applied in spatial econometrics as well as in spatial statistics; on the other hand, it is also impacted significantly by the availability of software that allows carrying out tests on spatial autocorrelation and estimating parameters of spatial econometric models (see: Haining, 2003; Arbia, 2006; Bivand et al., 2008; Ward and Kristian, 2008; LeSage and Pace, 2009; Fischer and Getis, 2010; Gelfand et al., 2010; Griffith and Paelinck, 2011; Plant, 2012, Pietrzak et al., 2014).

The presence of positive spatial dependence is embedded in the majority of economic phenomena resulting from the nature of the functioning of economic systems. This property is expressed as Tobler's first law of geography, where it is assumed that the level of interaction between regions decreases along with the increase in the distance between them (see: Tobler, 1970). The problem of spatial dependence is crucial, since it means the possibility of the formation of the level of phenomena depending upon which spatial location is being considered. Therefore, failure to consider the existing spatial dependence for the analysed phenomena while conducting economic research can lead to cognitive errors (see: Cliff and Ord, 1973; Paelinck and Nijkamp, 1975, Paelinck and Klaassen, 1979; Ripley, 1981; Anselin, 1988; Griffith, 1988; Arbia, 1989; Haining, 1990; Cressie, 1993).

The composite index also known as 'taxonomic measure of development' (see: Hellwig, 1972; Hwang and Yoon, 1981) is a commonly used tool in economic research. Taxonomic measure of development (TDM) allowing for evaluation and ranking of objects due to the level

of the development of the analysed phenomenon (Reiff et al., 2016; Renigier-Biłozor and Biłozor, 2015). In order to use this composite index, the examined economic phenomenon is broken down to a group of economic aspects and each of them describes a different part of the phenomenon. Then, for each aspect a set of variables is selected, characterizing the aspect and allowing its description. In the last step, based on the accepted variables, the synthetic index is calculated, which takes into account the effect of all the economic determinants of the phenomenon under study and allows a synthetic assessment of the level of its development to be made (see: Balcerzak, 2009, 2015; Wilk et al., 2013; Jantoń-Drozdowska and Majewska, 2015; 2016; Łyszczarz, 2016; Małkowska and Głuszak, 2016). The TOPSIS method is universal because we can study every economic phenomenon, for example the phenomena from labour markets (Gajdos and Suchecki, 2005, Gajdos et al., 2005; Müller-Fraćzek and Pietrzak, 2011; Gajdos, 2012; Gajdos and Żmurkow-Poteralska, 2012; Gajdos et al., 2013; Gajdos et al., 2014; Gajdos, 2015; Gajdos and Kusideł, 2015; Balcerzak and Pietrzak, 2016a, 2016b, 2016c, 2017; Pietrzak and Balcerzak, 2016a; 2016b).

In the case where the objects evaluated are regions, economic phenomena are generally characterized by the presence of spatial dependence. Therefore, it seems necessary to take into account this dependence in the construction of the taxonomic measure of development.

The spatial taxonomic measure of development (sTMD) extended with spatial properties would allow considering the impact of spatial mechanisms while assessing regions. These mechanisms are responsible for mutual interactions between regions. Thus they contribute to the maintaining of the current situation of regions, or they affect significantly changes in their situation.

Therefore, the proposal to use the spatial taxonomic measure of development in research is complementary to the use of TMD. The use of the taxonomic measure of development allows the assessment of the current situation of the analysed system of regions. The use of sTMD, however, gives the possibility to extend the results with the conclusions on the trend in the level of the development of regions relative to the phenomenon under study.

The purpose of this article is to take into account the issue of the consideration of spatial dependence within the TOPSIS method. The outcome of the implementation of the proposal made will be the construction of the spatial taxonomic measure of development (sTMD). The measure proposed in the article will be applied in the analysis of the situation on the labour market in Poland. The study will be carried out for the territorial composition of 66 subregions (NUTS 3 level) as of 2013. According to the nomenclature of the European Union in Poland, as a member state, the effective administrative division is NUTS (Nomenclature of Territorial Units for Statistics). There are three regional levels, NUTS1 (regions), NUTS 2 (voivodships) and NUTS 3 (subregions). Then two levels are distinguished at the local level. The territorial composition NUTS 4 (districts and city districts) and NUTS 5 (municipalities).

1. THE CONSTRUCTION OF THE SPATIAL TAXONOMIC MEASURE OF DEVELOPMENT WITH THE USE OF THE TOPSIS METHOD

The concept of the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) was presented in a work by Hwang and Yoon (1981). This method is used for solving a multiple criteria decision making problem (MADM). This means that TOPSIS is the method for order alternatives by similarity to an ideal solution. The TOPSIS method is the extension of the taxonomic method proposed by Hellwig (1972). The method proposed by Hellwig was a tool for the evaluation of objects relative to the level of the development of the phenomenon under study. In the case of Hellwig's method for calculating the value of the composite index (taxonomic measure of development), only the distance of objects from the pattern of development is considered. However, the TOPSIS method takes into account the distance of objects both

from the pattern and from the anti-pattern (see Balcerzak, 2016; Balcerzak and Pietrzak, 2016).

In the present article the TOPSIS method will be considered in the context of determining the taxonomic measure of development for the needs of the assessment of economic objects relative to the level of development of the phenomenon under study. Therefore, the use of the TOPSIS method will allow the evaluation of objects, working out the ranking of objects and, then, grouping these objects into classes.

Identification of spatial dependence for selected variables compels their consideration while constructing the taxonomic measure of development. Since each variable represents a different aspect of the economic reality, therefore, it is essential that the level of spatial dependence, which occurs with varying intensity, should be examined separately for each of them. In order to identify and assess the strength of spatial dependence, the spatial autoregressive model SAR can be applied (see: Anselin, 1988; LeSage and Pace, 2009). In addition to the impact of explanatory variables, this model takes into account also the spatial lag of the dependent variable \mathbf{WY} . Spatial lag describes the average impact of the neighbouring regions on the value of the dependent variable in the selected region. The neighbourhood of regions is expressed by means of a spatial weight matrix \mathbf{W} . However, a standardized spatial contiguity matrix is most commonly used while doing economic research. In the case of this matrix, the neighbourhood is determined on the basis of the criterion of having a common border between the regions (see: Anselin, 1988). In addition, the matrix is standardized in the way that the row elements sum to one.

The SAR model with one explanatory variable X is defined by the following formula (1):

$$\mathbf{Y} = \rho \mathbf{WY} + \beta_1 \mathbf{X} + \varepsilon. \quad (1)$$

$$\mathbf{V}(\mathbf{W}) = (\mathbf{I} - \rho \mathbf{W})^{-1}, \quad (2)$$

$$\mathbf{Y} = \mathbf{V}(\mathbf{W})\beta_1 \mathbf{X} + \mathbf{V}(\mathbf{W})\varepsilon, \quad (3)$$

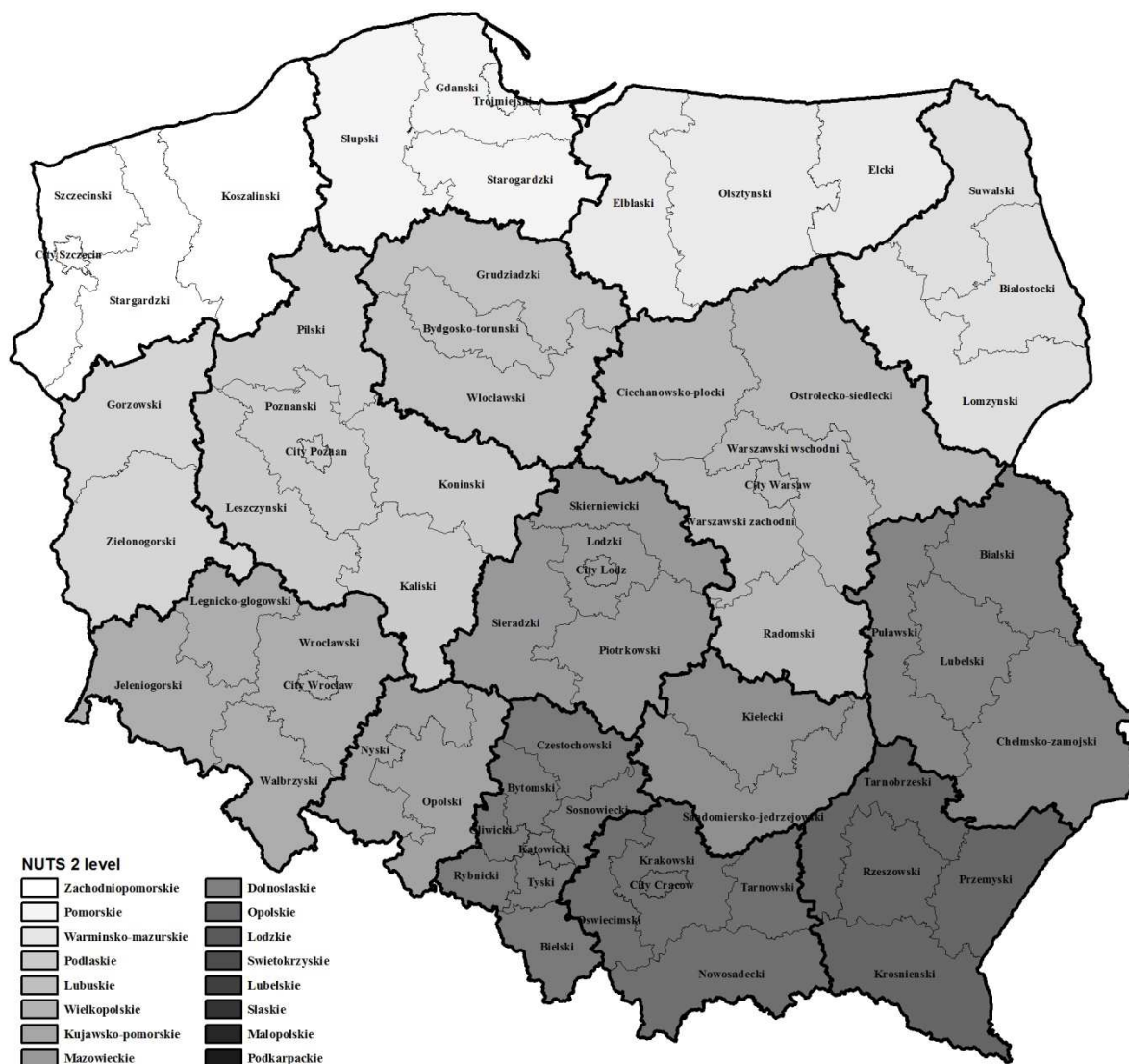
$$\mathbf{V}(\mathbf{W}) = \begin{bmatrix} v_{11} & \cdot & \cdot & v_{1n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ v_{n1} & \cdot & \cdot & v_{nn} \end{bmatrix}, \quad (4)$$

$$\frac{\partial E(Y_i)}{\partial x_j} = v_{ij} \beta_1, \quad (5)$$

where Y is the vector of the dependent variable, X is the vector of the explanatory variable, ρ is the coefficient of the spatially lagged dependent variable, \mathbf{W} is the standardized spatial weight matrix, β_1 is a structural parameter of the model, and ε is the disturbance term, $\mathbf{V}(\mathbf{W})$ is the matrix of potential impact.

On the basis of formula 2, the value of potential impact can be calculated for all pairs of regions. The elements v_{ij} of the matrix $\mathbf{V}(\mathbf{W})$ express the potential impact of the region j on the region i . Matrix of potential impact $\mathbf{V}(\mathbf{W})$ is not symmetric. This means that the potential impact of region i on region j and of region j on region i may vary. The key issue here is the choice of the spatial weight matrix, because the potential impact is determined based on the elements of this matrix. It should be noted, however, that in contrast to the classical linear regression model the impact of determinants varies depending on the considered regions (see: Ward and Kristian, 2008; LeSage and Pace, 2009; Pietrzak, 2013). The average impact resulting from the change of the explanatory variable in the region j into the dependent variable in the region i can be defined by formula (5). According to this formula, depending on the choice of regions i and j , the value of the structural parameter β_1 is weighted by the value of the potential impact v_{ij} .

Figure 1. The Polish administrative division - territorial composition of voivodships NUTS 2 and territorial composition of subregions NUTS 3

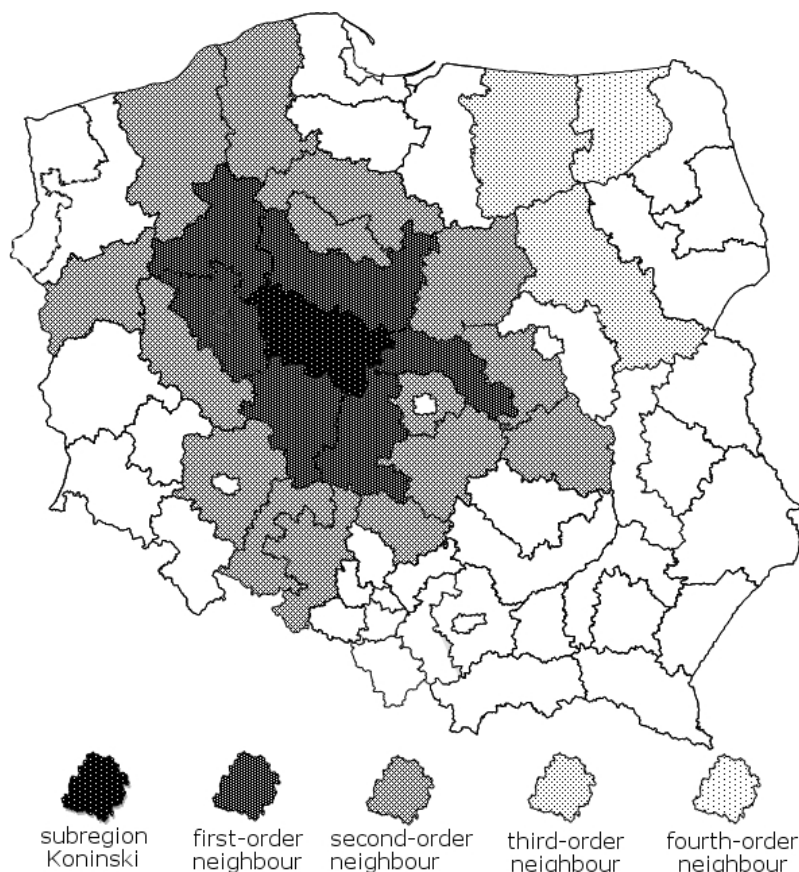


Source: Authors' compilation

The assumption of the existence of spatial dependence means that in the evaluation of the phenomenon in the selected region, in addition to the values of the variables in this region, one should also take into account the value of variables from other regions. In the case of the identification of spatial dependence, variables must be converted in such a way as to take into account possible interactions between regions. The resulting new variables will carry additional information about the trends in the spatial formation of the studied phenomenon. It is assumed that spatial dependence should not change significantly over time, since it results from the impact of a set of spatial mechanisms. A combination of these mechanisms creates a spatial structure, in which one can observe the impact of economic, historical, cultural or sociological factors. Given the persistence of spatial dependence, it is likely that over a few years the established trend will prove to be facts. It should be emphasized that the situation in the regions is also dependent on regional policy. Spatial mechanisms can have either a positive impact - supporting regional policies pursued, or negative - weakening them significantly. The situation in

regions is eventually influenced by the starting position of the regions, regional policies pursued as well as the spatial mechanisms. The taxonomic measure of development may be helpful in assessing the initial situation of regions, and the spatial taxonomic measure of development in identifying trend in the level of development of the phenomenon studied. This trend results from both the current situation of the phenomenon and from the spatial dependence.

Figure 2. Neighbourhoods for the Koninski subregion (NUTS 3 level)



Source: Authors' compilation

In order to demonstrate the formation of the potential impact v_{ij} , the matrix $\mathbf{V}(\mathbf{W})$ was considered for the territorial composition of 66 subregions in Poland (NUTS 3 level). This territorial composition with NUTS 2 voivodships composition is presented in Figure 1. Then, the Koninski subregion was selected arbitrarily based on which the potential impact from the other regions was discussed. Therefore, Figure 2 highlights the Koninski subregion for which additional first-order and second-order neighbourhood subregions are marked. The standardized spatial contiguity matrix \mathbf{W} was assumed in the research. The matrix of potential impact $\mathbf{V}(\mathbf{W})$ was determined according to formula 2, assuming arbitrarily the coefficient of the spatially lagged dependent variable ρ of 0.7, which means that there is strong spatial dependence. The results are shown in Table 1, where the second column presents the value of the potential impact v of selected subregions j (column 1) on the Koninski subregion i . The third column, in turn, shows the order of the neighbourhood for selected subregions to the Koninski subregion. Values v_{ij} presented in Table 1 are only a part of the matrix of potential impact $\mathbf{V}(\mathbf{W})$. From the matrix $\mathbf{V}(\mathbf{W})$ only one row corresponding to the Koninski subregion was selected. Then subregions were selected due to the order of neighbourhood. All first-order and second-order neighbours of the

Koninski subregion were selected. In addition, two adjacent third-order and one fourth-order order neighbours of the Koninski subregion were selected in order to demonstrate the formation of the potential impact of v_{ij} in this case. Presenting all of the elements of the matrix $V(W)$ would require the presentation of a table with 66x66 in dimension.

The highest value of the potential impact v_{ij} is obtained in the situation where both the region i and the region j constitute the Koninski subregion. The resulting value is greater than one (1.130). That means that if there is spatial dependence, the impact resulting from the change of a explanatory variable into the dependent variable within the same region is greater than the value of structural parameter β_1 (formula 5). Then, along with the increase in the order of the neighbourhood the level of potential impact decreases. In the case subregions that are first order neighbours of the Koninski subregion, the average value of the potential impact v_{ij} is 0.201. For the second order neighbours of the Koninski subregion the average value of the potential impact decreases significantly to the level of 0.044. However, in the case of the third order (or higher) neighbours of Koninski subregion, potential impact is close to zero.

Table 1: Potential impact between subregions

Subregion j	Potential impact v_{ij}	Order of the neighbourhood
Koninski	1.130	0
Wloclawski	0.192	1
Sieradzki	0.203	1
Skierniewicki	0.195	1
Kaliski	0.200	1
Pilski	0.214	1
Poznanski	0.203	1
Ciechanowsko-plocki	0.060	2
Radomski	0.014	2
Warszawski zachodni	0.035	2
Nyski	0.054	2
Opolski	0.032	2
Czestochowski	0.030	2
City Poznan	0.028	2
Koszalinski	0.032	2
Slupski	0.036	2
Leszczynski	0.089	2
Gorzowski	0.039	2
Lodzki	0.048	2
Piotrkowski	0.058	2
Wroclawski	0.054	2
Bydgosko-torunski	0.029	2
Grudziadzki	0.067	2
Ostrolecko-siedlecki	0.010	3
Olsztynski	0.009	3
Elcki	0.003	4

Source: Authors' compilation

Finding statistically significant spatial dependence for selected variable X_k indicates the need for its transformation to reflect this dependence in the values of the spatial taxonomic measure of development. To transform the variable X_k the matrix of potential impact $\mathbf{V}(\mathbf{W})$ can be used. To this end, for each variable X_k the matrix $\mathbf{V}(\mathbf{W})$ needs to be calculated (Formula 2) based on a SAR model described by the following formula:

$$\mathbf{X}_k = \rho \mathbf{W} \mathbf{X}_k + \boldsymbol{\varepsilon} \quad (6)$$

The finding of the matrix $\mathbf{V}(\mathbf{W})$ will allow the consideration of the potential spatial interaction between regions for the variable X_k using the following transformation:

$$\mathbf{Z}_k = (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X}_k = \mathbf{V}(\mathbf{W}) \mathbf{X}_k \quad (7)$$

The transformation of the variable X_k consists in adding the value of a variable from other regions to the value of this variable representing a selected region i . However, the values of the variable X_k from other regions will be weighed according to the rule that the greater the potential impact of the region j on the region i , the greater the extent will be considered.

In the case of spatial dependence statistically insignificant, the value of the variable should remain unchanged. Consequently, the value of the spatial taxonomic measure of development in the chosen region i will be impacted only by the value of the variable corresponding to this region.

The procedure for calculating the spatial taxonomic measure of development using the TOPSIS method can be summarized in the following steps:

1. Determining the research problem and the selection of a phenomenon for analysis in the context of the problem undertaken.
2. Establishing a set of objects O_1, O_2, \dots, O_m and the choice of the variables X_1, X_2, \dots, X_n describing the phenomenon analysed.
3. Testing spatial autocorrelation for each variable X_k using Moran test.
4. Taking into account the spatial dependence by transforming the variables.
 - a) In the case of finding statistically significant spatial dependence for selected variable X_k , the first step to be carried out is estimating the coefficient of the spatially lagged dependent variable ρ of the SAR model defined by formula (6). Then we should calculate the matrix of the potential impact $\mathbf{V}(\mathbf{W})$, which will allow the transformation of the variable X_k according to formula (7).
 - b) In the case of finding a statistically insignificant spatial dependence for the selected variable X_k , it is not transformed.
5. Determining the nature of the transformed variables Z_k (stimulant Z_k^s , destimulant Z_k^d). 'Stimulants' are variables that stimulate the development of the phenomenon under study. 'Destimulants' are those variables that inhibit that development (see: Hellwig 1972). Next converting destimulants to stimulants following the formula

$$Z_k^s = 1 / Z_k^d \quad (8)$$

6. Normalization of all variables (stimulants) Z_k^s according to the formula

$$Z_k^N = \frac{Z_k^s - m_k}{s_k} \quad (9)$$

where m_k, s_k are mean and standard deviation of the variable Z_k^s .

7. Establishing for each variable Z_k^N the pattern of development W_k as the maximum value of the variable Z_k^N for a set of objects O_1, O_2, \dots, O_m and the anti-pattern of development AW_k as the minimum value. In point 6 a different normalization method may be used and in point 7 a metric other than the Euclidean one.

8. Determining for each object O_i the Euclidean distance d_i^W from the pattern and the Euclidean distance d_i^{AW} from the anti-pattern.

9. The values of the spatial taxonomic measure of development sTMD for each object O_i are obtained through the calculation of the measure considering the proximity to the pattern and the distance from the anti-pattern, which can be described by the following equation

$$sTMD_i = \frac{d_i^{AW}}{d_i^W + d_i^{AW}}. \quad (10)$$

The values of the spatial taxonomic measure of development sTMD determine the trend in the level of development of the phenomenon under study and in most cases are contained in the range from zero to unity. Large values of the measure sTMD indicate an expected high level of the development of the phenomenon.

2. THE USE OF THE SPATIAL TAXONOMIC MEASURE OF DEVELOPMENT IN THE ANALYSIS OF THE LABOUR MARKET IN POLAND

The article presents an analysis of the situation on the labour market in Poland in 2013. The reason for choosing the year 2013 was the availability of all data. The analysis conducted was used to demonstrate the suitability of the use of the spatial taxonomic measure of development in economic research. For the purpose of assessing the development of the labour market in Poland both the taxonomic measure of development and the spatial taxonomic measure of development were applied. The results obtained made it possible to show the differences between the measures and to justify the simultaneous use of the two measures in the analyses performed. These measures are in fact complementary through mutual complementarity while evaluating economic phenomena.

The study used the territorial composition of 66 subregions (NUTS 3 level). The territorial composition taken for the study are shown in Figure 1. The description of the situation on the labour market employed the variables used in Table 2. All of the selected variables are the determinants with a significant impact on the situation of the labour market. The variables X_1 , X_2 , X_3 , X_4 function as stimulants, and the variable X_5 is a destimulant. The data on the selected variables were obtained from the Local Data Bank of the Main Statistical Office in Poland (www.stat.gov.pl). Subregions are administrative units where economic development and the situation on the labour market are relatively homogeneous. This means that the analysis carried out for the subregions should allow drawing correct conclusions, and the analysis of the voivodships can only serve to formulate initial and very general conclusions.

Table 2. Variables selected for the purpose of the description the situation on the labour market

Variable	Description
X_1	Average monthly gross wages and salaries
X_2	Entities of the national economy in the REGON register per capita (working age population)
X_3	Investment outlays per capita (working age population)
X_4	Labour productivity (GDP/employed persons)
X_5	Registered unemployment rate

Source: Authors' compilation

Therefore, the values of the taxonomic measure of development were calculated for subregions with the use of the TOPSIS method. The values of the measure TMD allowed the preparation of the ranking of subregions. Then, on the basis of the values of the measure the subregions were divided into five classes using the method of natural breaks. Relative to the situation on the labour market, these classes were subsequently named as follows: class 1 as 'very weak', class 2 as 'weak', class 3 as 'average', class 4 as 'good' and class 5 as 'very good'. The results are shown in Table 3. The second and the ninth columns contain the values of the TMR measure, the third and the tenth columns present the number of the ranking for each subregion, while the fourth and the eleventh columns show to which of the five classes a given subregion belongs.

Figure 3 shows the spatial variability of situation in the labour market as measured by means of TMD. The results presented in Table 3 and Figure 3a allow the assessment of the situation on the labour market in Poland. The assessment carried out in the article was reduced only to the selection of subregions (NUTS 3) with the best situation for each voivodship (NUTS 2). There is the possibility of dividing the Polish territory by the situation on the labour market and the voivodships were considered in the order of belonging to the three areas shown in Figure 3b. Area 1 'the eastern part' was created from the voivodships characterized by a weak situation on the labour market. Area 2 'the western part' of the country characterized by a well-developed labour market. The last one - area 3 'capital city' was created exclusively out of the Mazowieckie voivodship, which has a best situation on the labour market. Accordingly, the three areas with a similar labour market situation were separated arbitrarily.

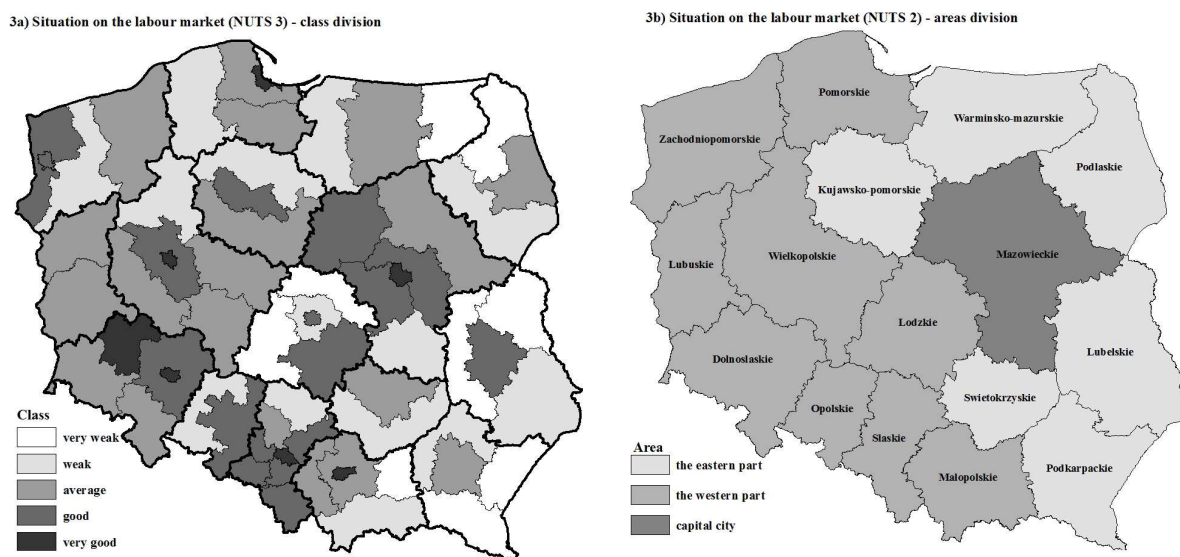
Area 2 referred to as 'the western part' is composed of nine voivodships. In the Zachodniopomorskie voivodship the situation on the labour market of the city of Szczecin and the Szczeciński subregion is good. Within the Pomorskie voivodship, the Trojmiejski subregion can boast of a very good situation on the labour market. Both subregions of the Lubuskie voivodship, the Gorzowski and Zielonogorski subregions, have the average situation on the labour market. In the Wielkopolskie voivodship a key centre generating a growth area is the city of Poznań, with a very good situation on the labour market. In addition, the Poznański subregion surrounding the city of Poznań has a good situation on its labour market. It should be emphasized, however, that the area of growth in this region is moving towards the Dolnośląskie voivodship. Within the Dolnośląskie voivodship, the city of Wrocław and the Legnicko-głogowski subregion can boast of a very good situation on their labour markets. Moreover, a good situation on the labour market occurs in the Wrocławski subregion. Within the Łódź voivodship, a good situation on the labour market can be seen in the city of Łódź and the Piotrkowski subregion. Within the Opolskie voivodship, the only subregion with a good situation in the labour market is the Opolski subregion. The Śląskie voivodship is characterized by the best-developed labour market compared to other voivodships. In the case of the Katowicki subregion, the situation on the labour market is very good. The following five subregions: the Rybnicki, Gliwicki, Sosnowiecki, Tyski, and Bielski subregions have a good situation on the labour market. In the Małopolskie voivodship, which is adjacent to the Śląskie voivodship, a very good situation on the labour market can be seen in the city of Cracow, which constitutes the main urban centre of the voivodship.

Table 3. The situation on the labour market in Poland at the level of subregions (NUTS 3)

Subregion	TMD index			sTMD index			Subregion	TMD index			sTMD index		
	M	R	K	M	R	K		M	R	K	M	R	K
City Warsaw	0.972	1	5	0.899	1	5	Olsztynski	0.166	34	3	0.189	43	2
City Poznan	0.490	2	5	0.509	2	5	Gdanski	0.166	35	3	0.211	35	3
Trojmiejski	0.477	3	5	0.450	4	5	Gorzowski	0.165	36	3	0.213	34	3
Legnicko-glogowski	0.427	4	5	0.414	6	5	Bytomski	0.164	37	3	0.246	27	3
Katowicki	0.419	5	5	0.476	3	5	Koszalinski	0.163	38	3	0.193	41	2
City Wroclaw	0.411	6	5	0.433	5	5	Bialostocki	0.162	39	3	0.155	56	2
City Cracow	0.406	7	5	0.407	8	5	Starogardzki	0.160	40	3	0.186	45	2
Warszawski zachodni	0.349	8	4	0.393	9	5	Oswiecimski	0.154	41	3	0.229	30	3
Tyski	0.349	9	4	0.409	7	5	Wloclawski	0.152	42	3	0.194	39	2
Rybnicki	0.315	10	4	0.391	11	5	Koninski	0.151	43	3	0.198	38	2
Gliwicki	0.310	11	4	0.392	10	5	Czestochowski	0.149	44	2	0.204	36	2
Poznanski	0.292	12	4	0.331	13	4	Lodzki	0.148	45	2	0.183	46	2
City Szczecin	0.289	13	4	0.305	15	4	Radomski	0.147	46	2	0.192	42	2
Piotrkowski	0.285	14	4	0.305	14	4	Tarnobrzesci	0.143	47	2	0.155	57	2
Ciechanowsko-plocki	0.280	15	4	0.284	21	4	Sandomiersko-jedrzejowski	0.142	48	2	0.187	44	2
City Lodz	0.277	16	4	0.284	20	4	Stargardzki	0.138	49	2	0.204	37	2
Sosnowiecki	0.259	17	4	0.298	18	4	Nowosadecki	0.131	50	2	0.167	54	2
Bydgosko-torunski	0.254	18	4	0.258	24	3	Grudziadzki	0.127	51	2	0.177	50	2
Bielski	0.243	19	4	0.341	12	4	Slupski	0.127	52	2	0.170	53	2
Lubelski	0.242	20	4	0.237	29	3	Pilski	0.126	53	2	0.177	49	2
Warszawski wschodni	0.241	21	4	0.299	17	4	Elblaski	0.124	54	22	0.176	51	2
Wroclawski	0.240	22	4	0.301	16	4	Nyski	0.120	55	2	0.175	52	2
Szczecinski	0.232	23	4	0.271	22	4	Chelmsko-zamojski	0.118	56	2	0.140	60	1
Opolski	0.226	24	4	0.286	19	4	Lomzynski	0.115	57	2	0.129	61	1
Leszczynski	0.182	25	3	0.259	23	3	Skierniewicki	0.110	58	1	0.182	47	2
Walbrzyski	0.182	26	3	0.227	31	3	Pulawski	0.110	59	1	0.148	58	1
Rzeszowski	0.181	27	3	0.179	48	2	Tarnowski	0.109	60	1	0.141	59	1
Kielecki	0.178	28	3	0.213	33	3	Sieradzki	0.095	61	1	0.166	55	2
Kaliski	0.176	29	3	0.239	28	3	Elcki	0.095	62	1	0.105	64	1
Zielonogorski	0.175	30	3	0.250	26	3	Suwalski	0.094	63	1	0.108	63	1
Jeleniogorski	0.174	31	3	0.251	25	3	Krosnienski	0.088	64	1	0.111	62	1
Ostrolecko-siedlecki	0.167	32	3	0.193	40	2	Bialski	0.067	65	1	0.097	65	1
Krakowski	0.167	33	3	0.223	32	3	Przemyski	0.064	66	1	0.083	66	1

M - value of the TMD or sTMD measure, R - position in the ranking, K - the class to which each of the regions was assigned. Source: Authors' compilation.

Figure 3: The situation on the labour market in Poland at the level of subregions (NUTS 3)



Source: Authors' compilation

Area 1 – ‘the eastern part’ consists of six voivodships. All voivodships in Area 1 have a low economic potential. The impact of the most developed cities is local, which translates into a weak situation on the labour market of the other subregions. Within the Kujawsko-pomorskie voivodship only the Bydgosko-torunski subregion has a good situation on the labour market. Further five voivodships - the Świętokrzyskie, Warmińsko-mazurskie, Podlaskie, Lubelskie and Podkarpackie voivodships are characterized by a significantly weaker labour market situation, if compared with the other eleven provinces. In this case, a good or average situation in the labour market is limited to the Kielecki, Olsztyński, Białostocki, Lubelski and Rzeszowski subregions, where the capital cities of these voivodships are situated. The remaining subregions of the five voivodships have a weak or very weak situation on the labour market.

In the case of Area 3 – ‘the capital city’ consisting solely of the Mazowieckie voivodship, the city of Warsaw has a very good situation on its labour market. Warsaw with the status of the Polish capital city can boast of the highest value of the TMD measure, therefore, it ranks first in the subregions ranking. The impact of the city of Warsaw reaches out far beyond the boundaries of the city. In consequence, the growth area of Warsaw has been extended to the Warszawski wschodni subregion, the Warszawski zachodni subregion and the Ciechanowski-płocki subregions. These subregions have a good situation on the labour market.

The next step was to calculate the spatial taxonomic measure of development in accordance with the procedure proposed in the article. For this purpose, first, Moran I statistic for each of the variables taken from Table 2 was calculated, where a standardized spatial contiguity matrix W was assumed. Moran I statistic is most commonly used in testing spatial autocorrelation of selected variables (see: Moran, 1950; Cliff and Ord, 1973,1981). The received test statistics are shown in Table 4 and p-values are given in parentheses. The results indicated statistically significant spatial dependence for all variables. Therefore, estimation of the parameters of the spatial autoregressive model SAR was made following the formula (6) (see Table 4). The obtained estimates of parameter ρ allowed the transformation of the variables using the formula (7). During the transformation of the variables the same spatial weight matrix was applied as in the case of the Moran test. In the last step, using the transformed variables, the values of the spatial taxonomic measure of development sTMD were calculated in accordance with the formula (10). The results allowed us to re-rank the subregions and to divide them again into 5 classes using the

methods of natural breaks. In this way subregions were ascribed to the following classes with different levels of the expected situation on the labour market : class 1 - 'very weak', class 2 - 'weak', class 3 - 'average', class 4 - 'good' and class 5 - 'very good'. 'The expected situation' is understood as a trend in the level of development of the labour market, resulting from the current situation and existing spatial mechanisms. The sTMD values, as in the case of TMD, are shown in Table 3. The fifth and the twelfth columns show the value of the measure, the sixth and the thirteenth columns contain the ranking number for each of the subregions, and the seventh and the fourteenth columns show the belonging of the subregions to the classes.

Table 4. Results of Moran test and of the estimation of the parameters of the SAR model

Variable	Moran I statistic	Estimate of the parameter ρ
X ₁	0.156 (0.019)	0.355 (0.009)
X ₂	0.086 (0.042)	0.236 (0.048)
X ₃	0.122 (0.039)	0.294 (0.023)
X ₄	0.458 (~0.00)	0.713 (~0.00)
X ₅	0.253 (~0.00)	0.606 (~0.00)

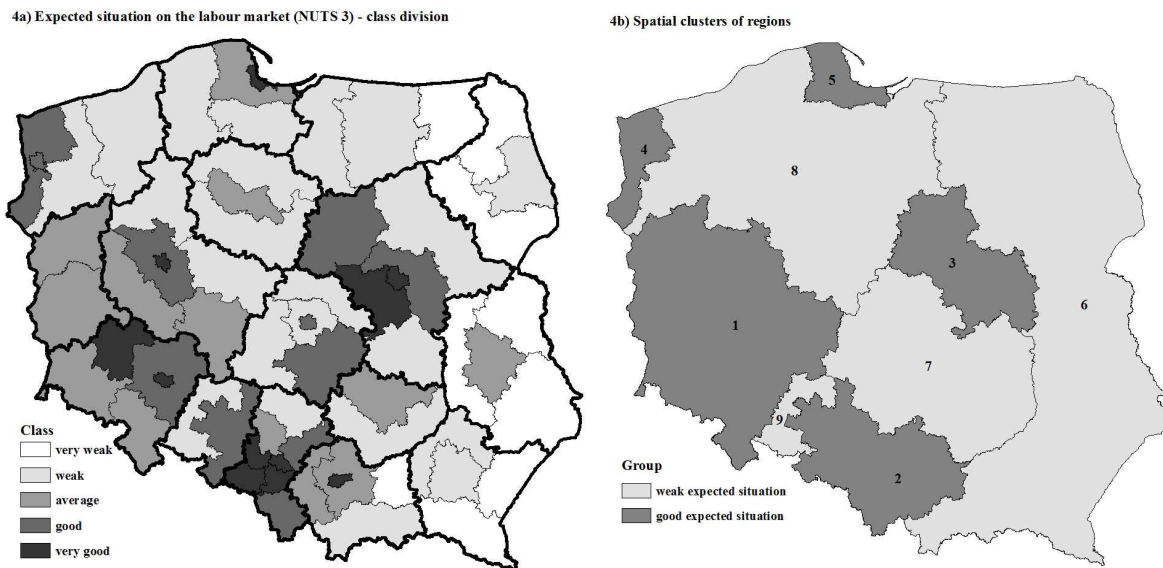
Source: Authors' compilation

The values of the spatial taxonomic measure of development, in turn, helped to establish the expected situation on the labour market in Poland. Figure 4a shows the spatial variability of the sTMD values (see Table 3). In accordance with the adopted division of subregions into classes, the following were included to the class with a very good expected situation on the labour market: the city of Warsaw, the city of Poznan, the city of Wroclaw, the city of Cracow and also the following subregions: the Trojmiejski, Legnicko-glogowski, Katowicki, Tyski, Gliwicki and Rybnicki. Together with the Polish capital city of Warsaw they constitute the most economically developed subregions. The good expected situation on the labour market contained the city of Lodz, the city of Szczecin and subregions: the Szczecinski, Warszawski wschodni, Ciechanowsko-plocki, Piotrkowski, Poznanski, Wroclawski, Opolski and Sosnowiecki. The average expected situation on the labour market included the following subregions: Gdanski, Gorzowski, Zielonogorski, Bydgosko-torunski, Leszczynski, Kaliski, Jeleniogorski, Walbrzyski, Kielecki, Lubelski, Bytomski, Oswiecimski, and city of Cracow. The remaining subregions form spatial clusters of a weak or very weak expected situation on the labour market. The results indicate the dominant role of the Mazowieckie, Slaskie, Wielkopolskie, and Dolnoslaskie voivodships.

The variability of the spatial taxonomic measure of development presented in Figure 4a allows the identification of spatial clusters of subregions with the similar expected situation on the labour market. Accordingly, the spatial clusters were divided into the following two groups: group 1 'good expected situation' and group 2 'weak expected situation' (see Figure 4b). Group 1 was assigned five clusters of the good expected situation on the labour market, and group 2 three clusters, where the expected situation is reverse. Among the first group of clusters, the largest spatial cluster proved to be the cluster composed of the following subregions belonging to the Lubuskie, Wielkopolskie and Dolnoslaskie voivodships (cluster no. 1). The major growth centres in the cluster are the cities of Poznan and of Wroclaw. The next largest cluster is composed of the Opolskie subregion and subregions of Slaskie, Malopolskie voivodships (cluster no. 2). The major growth centres here are the biggest cities in the Slaskie voivodship (Katowice, Sosnowiec, Gliwice, Tychy) and the cities of Cracow and of Opole. The third cluster is formed of the subregions of the Mazowieckie voivodship (cluster no. 3). This cluster is focused around

Warsaw and is the most dynamically developing area because of Warsaw's status of the capital city. Additionally, two areas in the Zachodniopomorskie and Pomorskie voivodships were depicted where the good expected situation on the labour market is limited to the city of Szczecin and the Szczecinski subregion (cluster no. 4), to the Trojmiasto subregion (the cities of Gdansk, Gdynia, Sopot), and to the Gdanski subregion (cluster no. 5). In the case of these two areas, the good expected situation on the labour market results from the fact that the cities of Szczecin, Gdansk, Gdynia are port cities with access to the Baltic Sea.

Figure 4: The expected situation on the labour market in Poland at the level of subregions (NUTS3)



Source: Authors' compilation

Figure 4b highlights significantly larger spatial clusters with the weak expected situation on the labour market. The largest spatial cluster of group 2 is formed by the subregions situated in eastern Poland (cluster no. 6). The cluster comprises the subregions of the Warminko-mazurskie, Mazowieckie, Podlaskie, Lubelskie, Podkarpackie and Malopolskie voivodships. In this cluster there are four major urban centres, the cities of Olsztyn, Bialystok, Lublin and of Rzeszow. Another cluster no. 7 is composed of the subregions belonging to the Mazowieckie, Lodzkie, Swietokrzyskie and Slaskie voivodships. In this cluster there are four major urban centres Kielce, Radom, Lodz, Czestochowa. The last cluster no. 8 consists of the subregions situated within the Zachodniopomorskie, Wielkopolskie, Pomorskie and Kujawsko-pomorskie voivodships. The major urban centres of the cluster are the cities of Bydgoszcz, Torun, Pila, Koszalin, and of Slupsk.

Despite the fact that in the selected clusters of group 2 are situated large urban centres, they do not have sufficient potential to create strong growth areas and their impact is purely local. In addition, these centres are developing more slowly than the urban centres of the clusters belonging to group 1. In the future, it may contribute to the perpetuation of the situation in the clusters of group 2 and to the extension of their territory by another subregions.

The difficult situation of the Nysa subregion (no. 10) needs to be emphasised. The situation of this subregion is unique because it is situated between two clusters having the good expected situation on the labour market (clusters no. 1, no. 2). In the case of the Nysa subregion, the close proximity of the city of Wroclaw has a negative impact on the development of that

subregion, where the most valuable resources are being relocated to the city of Wrocław and to the Wrocław subregion.

The taxonomic measure of development and the spatial taxonomic measure of development are mutually complementary measures in terms of information. The measure TMD allows the assessment of the current situation of the studied phenomenon and the measure sTMD allows determining the expected level of its development. The measure sTMD has a tendency to form spatial clusters of regions with a high or low expected level of development of the examined phenomenon. This is due to the fact that in order to determine the composite index, the initial values of the variables X_k for the selected region get converted. The values of selected variable X_k for neighbouring regions are added to the value of this variable of the region i (formula 7). This change causes the convergence of the value of the variable X_k for neighbouring regions. If a majority of assumed variables is characterized by significant spatial dependence, there will be a tendency to converge the spatial taxonomic measure of development value to the neighbouring regions.

In the case of the occurrence of spatial dependence for most variables, rankings of regions made on the basis of the measures TMD and sTMD may differ from each other. Any region can take a lower position in the ranking based on the measure TMD compared with the ranking based on the measure sTMD, if there is a strong spatial impact of their neighbours characterized by high values of the measure TMD. An opposite situation may occur in the case of the neighbourhood of regions with low values of the measure TMD. Since the assessment of the expected situation on the labour market may increase or decrease due to the spatial impact of neighbouring subregions, then subregions will naturally tend to cluster spatially into groups with similar values of the measure sTMD.

The differences in the rankings based on the measures TMD and sTMD will be discussed using the example of the Mazowieckie voivodship and the Lubelskie voivodship (see Table 3). The area of the Mazowieckie voivodship comprises six subregions - the city of Warsaw, the Warszawski wschodni subregion, the Warszawski zachodni subregion, the Ciechanowsko-plocki subregion, the Radomski subregion, and the Ostrołęcko-siedlecki subregion. When calculating the value of the measure sTMD for any subregion, the weighted values of the variables X_k from the remaining 65 subregions were added to the initial values of the same variables. Since the city of Warsaw has the highest values of the variables X_k , then the transformed values of the variables Z_k increase to a greater extent for the subregions neighbouring with Warsaw than for Warsaw itself. Therefore, as a result of the transformation of the variables, the value of the measure sTMD for Warsaw and its neighbouring subregions (the Warszawski wschodni and the Warszawski zachodni subregions) converged and the positions held in the ranking could be improved. In the case of the other subregions of the Mazowieckie voivodship, the situation is as follows: the Ciechanowsko-plocki and the Radomski subregions have achieved positions closer to Warsaw - the Ciechanowsko-plocki subregion due to the proximity of the Warszawski zachodni subregion and the Bydgosko-torunski subregion (those are subregions with a good situation on the labour market), and the Radomski subregion due to the proximity of the Warszawski zachodni subregion, Kielecki and Piotrkowski subregions (they have a good situation the labour market). The Ostrołęcko-siedlecki subregion, however, departed from Warsaw, due to the proximity of subregions with a much weaker labour market situation - the Lomżyński, Puławski, and Elcki subregions. Finally, the Warszawski wschodni subregion, the Warszawski zachodni subregion, the Ciechanowsko-plocki subregion and the Radomski subregion neared Warsaw's value of the measure sTMD, and the Ostrołęcko-siedlecki subregion departed from the value. This resulted in the identification of a spatial cluster around Warsaw classified as group 1 and the assignment of the Ostrołęcko-siedlecki subregion to a group 2 cluster. The differences in the values of the TMD and sTMD affect the results of the ranking. In the ranking based on the measure sTMD Warsaw remained on the first position. The Warszawski zachodni subregion and the Ciechanowsko-plocki subregion also remained unchanged in the ranking. The

Warszawski wschodni subregion and the Radomski subregion moved up in the ranking (the Warszawski wschodni one moved up from the 24th position to the 19th, and Radomski from the 51st to 44th position). The Ostrolecko-siedlecki dropped in the ranking from the 46th position to 51st, due to the proximity of the subregions with a very weak situation on the labour market in the northern and eastern parts of Poland.

The situation looks quite different in the case of the Lubelskie voivodship. The voivodship consists of the following four subregions: the Lubelski, Pulawski, Bielski and Chelmsko-zamojski subregions. In the voivodship only the Lubelski subregion, which includes the city of Lublin, has the potential for growth and has a good situation on the labour market. However, this potential is so weak that the Lubelski subregion does not significantly affect the neighbouring subregions with a much weaker labour market situation. The result is that the values of selected variables (Table 2), and the measure of TMD for the Lubelski subregion far outweigh the values of the variables for the neighbouring subregions: the Pulawski, Bialski and Chelmsko-zamojski subregions. The neighbourhood of the Lubelski subregion with the subregions with a very poor labour market situation resulted in a fall in the position of this subregion in the ranking based on sTMD, if compared with the ranking based on the TMD. The negative impact of the subregions adjacent to the Lubelski subregion resulted in a fall of this subregion from the 22nd to 31st position. The ranking positions of the Bielski subregion and the Chelmsko-zamojski subregion did not change. However, the ranking position of the Pulawski subregion rose from 59th to 57th. This is due to the spatial location of the subregion, which is adjacent to the Lubelski subregion, the Warszawski wschodni subregion and the Ostrolecko-siedlecki subregion with a significantly better situation on the labour market.

CONCLUSIONS

The realisation of the objective set in the present article allowed the development of the procedure for constructing the spatial taxonomic measure of development sTMD using the TOPSIS method. The need for the construction of the measure results from the problem of the presence of spatial dependence observed for most economic phenomena. Spatial dependence was taken into account while constructing the measure by using the matrix of potential impact $V(W)$. Therefore, the spatial taxonomic measure of development allows determining trend in the level of development of the analysed phenomena, assuming the impact of the existing spatial mechanisms.

The proposed construction of the sTMD measure has already been applied for the purpose of analysis of the situation on the labour market in Poland. The study allowed us to assess the situation in 2013 and to identify the trend in the development of the labour market. As a result of the study, two groups of spatial clusters were also identified. Clusters of group 1 consisted largely of subregions with the good expected situation on the labour market. Clusters of group 2 were dominated by subregions with the weak expected situation on the labour market. That means that with the existing spatial mechanisms the labour market tends to regional divergence. A high level of development of the labour market is predicted only for the areas around Poland's most developed urban centres. However, in the case of other areas the situation on the labour market is expected to worsen. This is due to the drain of valuable resources from subregions belonging to group 2 clusters, which naturally flow into clusters from group 1. The results received point to the fact that without intense, deliberate state policy, the current differences in the level of the labour market will continue to increase and the differences in the level of economic development will deepen.

The analysis appointed to the usefulness of the proposed measure sTMD, which is complementary to the use of the taxonomic measure of development in the process of explaining the economic reality. The use of the spatial taxonomic measure of development allows expanding the results obtained on the basis of TMD by conclusions concerning the trend in development of

the analysed phenomenon. This means that by taking into account spatial dependence the use of the measure sTMD in economic research gives us a fuller explanation of the variability of phenomena.

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