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## Leaky bucket Paradoxes in income inequality perceptions: an experimental investigation

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# LEAKY-BUCKET PARADOXES IN INCOME INEQUALITY PERCEPTIONS: AN EXPERIMENTAL INVESTIGATION

by Eva Camacho-Cuena, Tibor Neugebauer, Christian Seidl



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# LEAKY-BUCKET PARADOXES IN INCOME INEQUALITY PERCEPTIONS: AN EXPERIMENTAL INVESTIGATION

by

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## Abstract

Leaky-bucket transactions can be regarded as a generalization of the transfer principle allowing for transaction costs. In its most rudimentary form, leaky-bucket transactions trace out the maximum leakage of transaction costs such that a transfer still pays at the margin. Yet “to pay at the margin” bears at least two different connotations: It could refer to the minimum transactions costs before a welfare loss is experienced, or before income inequality is exacerbated. These two aspects have not always been made explicit in earlier work. This notion suggests that a smaller, but positive amount of income has to reach the transferee in order to keep the degree of income inequality or the aggregate social welfare at the same level.

However, this conjecture is theoretically wrong for the degree of income inequality, and partly wrong for aggregate social welfare. Rather there exists a unique benchmark such that the above holds only for transfers among income recipients below the benchmark. When they are both above the benchmark, then the transferee has to be given more than the amount taken from the transferor, and when they are on opposite sides of the benchmark, both should experience an income loss. Notice that these three cases cover only progressive transfers. Three more cases apply to regressive transfers, and six more cases apply to income donations. Each of these twelve cases ordains different theoretical results.

Yet experimental research, calibrated against the Atkinson, generalized Gini, and entropy income inequality measures and their associated social welfare functions, shows that this generalized theory of the transfer principle is as poorly evidenced as is the plain transfer principle. This applies both to the income-inequality approach and to the social-welfare approach. At most one third of the subjects behave sometimes according to theory. The rest seems to follow some notion of compensating justice: If someone loses (gains) income, the other person involved should be negatively (positively) compensated to maintain the alleged degree of income inequality. This behavioral pattern is, however, at variance with theory.

JEL Classification: D31, D63, C91.

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# 1 Introduction

Virtually all income inequality measures satisfy the *transfer principle*, that is, if income is transferred from a richer income recipient to a poorer income recipient without changing the order of income recipients [this is called a *progressive transfer*], then all eligible income inequality measures should decrease because income becomes more equally distributed in this society. Moreover, a progressive transfer increases aggregate welfare when welfare is measured in terms of strictly S-concave social welfare functions. The opposite holds if the transfer goes from a poorer to a richer income recipient [this is called a *regressive transfer*].

However, when a transfer commands transaction costs, then the question arises as to the maximum leak of the bucket which transfers the income such that the degree of income inequality or aggregate welfare remains intact in this society. Preliminary reasoning would suggest that a smaller, but *positive* amount of income has to reach the transferee in order to keep the degree of income inequality or aggregate social welfare at the same level.<sup>1</sup>

Yet it can be shown that this conjecture is theoretically wrong, at least for the degree of income inequality [Seidl (2001); Lambert and Lanza (2003)]. Rather there exists a *unique benchmark* as a function of the income inequality measure and the income distribution applied, such that the above conjecture holds only when both parties to a progressive transfer have incomes *below* the benchmark. When both parties to a progressive transfer happen to have incomes *above* the benchmark, then the transferee of a progressive transfer has to be given even more than the amount of the transfer to maintain the degree of income inequality within the society. Finally, when the parties to a progressive transfer happen to lie on *opposite sides* of the benchmark, then the “transferee” should also suffer an income loss in order to keep the degree of income inequality in the society intact.

Note that all income inequality measures are associated with particular families of social welfare functions [Blackorby and Donaldson (1978)]. Preliminary reasoning may, thus, suggest that, although the above was theoretically evidenced for income inequality measures, it must not hold for aggregate social welfare. That is, it may be argued that a smaller, but positive, amount of income has to reach the transferee in order to keep the level of aggregate social welfare intact. Yet it can be theoretically shown that this conjecture fails to hold for some social welfare functions associated with respective income inequality measures. For instance, while it holds for the Atkinson social welfare function and the generalized Gini social welfare function, it may be *violated* for the entropy social welfare function. Indeed, benchmarks with similar properties can be identified for several entropy social welfare functions.

As to experimental investigation of the theoretical relationships, a comprehensive experimental design asks for four cases: The income of an income recipient may either be *increased* or *decreased* by a certain amount, and the other person, whose income is to be adapted, may either be *richer* or *poorer* than the considered person. For the analysis of the results, the data of each of these four experimental cases have then to be winnowed into three categories, according to whether the respective incomes are above, below, on opposite sides of the benchmark. This makes twelve cases for the analysis of the results.<sup>2</sup>

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<sup>1</sup>These beliefs were entertained, e.g., by Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).

<sup>2</sup>Even the seminal theoretical paper of Lambert and Lanza (2003), which treats leaky-buckets in terms of inequality measures exclusively, focuses on progressive transfers only, which covers just three out of

So far, experimental investigation of the leaky-bucket phenomenon has focused attention only on one of these twelve cases, viz. to the case of a progressive transfer where both parties are below the benchmark, or where no benchmark exists<sup>3</sup> (which is the case for some social welfare functions). This restriction is revealed through the chosen experimental design which allowed only responses of income adjustments which were restricted to be positive and smaller than the amount of the income reduction of the richer party.<sup>4</sup>

The experiment dealt with in this paper relies on a full-fledged experimental design of all four basic cases and, using all 84 transactions among seven income recipients, allows tracing all twelve response categories for three income inequality measures, viz. Atkinson<sup>5</sup>, generalized Gini<sup>6</sup>, and entropy<sup>7</sup>. Although, under our experimental design, subjects were instructed to adjust the other person's income such as to keep the degree of income inequality within the society constant, we investigated also whether the results conform to social-welfare considerations as well.

The paper is arranged as follows: Section 2 gives an appraisal on the theory of leaky-bucket transactions, Section 3 presents the experiment, Section 4 contains the results, and Section 5 concludes.

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twelve possible configurations.

<sup>3</sup>Obviously, the experimenters were not aware of the respective theory which establishes the existence of benchmarks. Yet these are the proper case which result from theory. This will be shown in Theorems 8, 11, and 14 below.

<sup>4</sup>Some experimenters, e.g., Amiel et al. (1999), asked their subjects to state the minimum amount that needs to be given to the transferee "to make the transfer worthwhile" without stating how "worthwhile" should be understood. It could refer to the minimum amount before a welfare loss is experienced, or before income inequality is exacerbated [Cf. Lambert and Lanza (2003, p. 17)]. Beckman et al. (2003b) employed an experimental design of leaky-bucket lotteries. Beckman et al. (2003a) investigated inefficiencies in income redistribution in a setting of a majoritarian democracy. They found "that voters are willing to support very large efficiency losses to transfer income if they do not have to pay and do not have any chance of obtaining the top positions. Support for redistribution, however, drops off markedly when voters are asked to contribute to the transfer of if they know their position before voting and they are at the top. Preferences for mandated redistribution thus are strongly conditioned by self interest and the opportunities which exist to occupy positions which receive the highest payoffs." [Beckman et al. (2003a, pp. 3-4.)] In contrast to that, our experimental design avoids both risk and redistributive connotations. Subjects were rather solicited to focus on the *maintenance of the degree of income inequality*. This did not prevent us from analyzing our results also under the fiction that our subjects had, in spite of our instruction, actually acted as if they had followed the social-welfare-approach.

<sup>5</sup>This income inequality measure is based on the concept of the *equally distributed equivalent income*, which was foreshadowed by Champernowne (1952), and first formulated by Kolm (1969). Atkinson (1970) re-established this concept and made it the centerpiece of his famous income inequality measure.

<sup>6</sup>The geometric interpretation of the Gini coefficient is that it is twice the area between the diagonal and the Lorenz curve in the unit square. The Lorenz curve was first proposed by Lorenz (1905). The Gini coefficient was introduced by Gini (1912; 1914). It can be expressed in several different ways. For a concise survey cf., e.g., Anand (1982, appendix). The Gini coefficient may, akin to the Atkinson income inequality measure, be extended to include an inequality-aversion parameter. Pioneering work was done by Donaldson and Weymark (1980), Weymark (1981), Yitzhaki (1983), and Chakravarty (1988).

<sup>7</sup>Entropy income inequality measures were developed by Bourguignon (1979), Cowell (1980), and Shorrocks (1980; 1984) to identify income inequality measures which are decomposable for different homogeneous subgroups of income recipients. Decomposition occurs with respect to a within-group component (a weighted sum of the inequality indices of the subgroups), and a between-group component (the inequality measure evaluated by assigning every member of a subgroup the mean income of the respective subgroup). Entropy income inequality measures are, too, driven by an inequality-aversion parameter. Note that they are generalizations of two non-parameterized income inequality measures originally proposed by Theil.

## 2 Theory

In this section we shall give an account on the theory of leaky-bucket transactions only in so far as to provide a basis to evaluate our experimental results vis-à-vis theoretical standards. This means that we avail ourselves of simplifying assumptions which facilitate the demonstration of the theory. For instance, we allow only for strictly positive incomes, which are different for different income recipients. The number of income recipients is considered finite. As we shall deal only with differentiable income inequality measures and social welfare functions, we assume differentiability throughout.

Thus, we will work only with income distributions  $y = \{y_i \mid i = 1, \dots, n; 1 < n < \infty\}$  such that  $0 < y_1 < y_2 < \dots < y_n < \infty$ . The set of such income distributions is denoted by  $Y$ . Income inequality measures are denoted by  $I : \mathbb{R}_{++}^n \rightarrow \mathbb{R}_+$ ,<sup>8</sup> and social welfare functions are denoted by  $W : \mathbb{R}_{++}^n \rightarrow \mathbb{R}$ . Both are assumed to be differentiable throughout.

Income inequality measures may either require that incomes are arranged in increasing (or decreasing) order, or are able to process them in any order. The former ones are called *positional* income inequality measures, the latter ones are called *nonpositional* income inequality measures. In this paper, we will use only incomes arranged in increasing order for the positional income inequality measures.

DEFINITION 1:  $I(\cdot)$  is scale invariant if  $I(y) = I(\lambda y)$  for all  $y \in \mathbb{R}_{++}$ ,  $\lambda > 0$ .

LEMMA 2:  $I(\cdot)$  is scale invariant if

$$\sum_{i=1}^n \frac{\partial I}{\partial y_i} y_i = 0.$$

PROOF: Consider a scale variation  $\lambda y$ ,  $\lambda > 0$  of  $y$ . Then  $I(\lambda y) \equiv I(y)$  for all  $\lambda > 0$  implies

$$\lim_{\lambda \rightarrow 1} \frac{\partial I(\lambda y)}{\partial \lambda} = \sum_{i=1}^n \frac{\partial I(y)}{\partial y_i} y_i \doteq 0.$$

Q.E.D.

DEFINITION 3:  $I(\cdot)$  is inequality averse if

$$\frac{\partial I}{\partial y_k} < \frac{\partial I}{\partial y_j} \text{ for } y_k < y_j.$$

DEFINITION 4:  $W(\cdot)$  is inequality averse if

$$\frac{\partial W}{\partial y_k} > \frac{\partial W}{\partial y_j} \text{ for } y_k < y_j.$$

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<sup>8</sup>It might sound strange to some readers that we do not confine the range of income inequality measures to the unit interval. Yet there are some recognized income inequality measures which do not satisfy this condition, most notably the entropy income inequality measure. When its inequality aversion parameter  $c$  converges to  $+\infty$  or  $-\infty$ , then the entropy income inequality measure converges to  $+\infty$ .

Let  $e_i$  denote an  $n$ -dimensional vector with a 1 on the  $i$ -th position and zeros everywhere, and let  $\delta > 0$  be such that  $\delta < \min_{i,\ell \in \{1,\dots,n\}} |y_i - y_\ell|$ .

DEFINITION 5:  $I(\cdot)$  satisfies the transfer principle if

$$I(y + \delta e_k - \delta e_j) < I(y) \text{ and } I(y - \delta e_k + \delta e_j) > I(y) \text{ for all } y_k < y_j.$$

DEFINITION 6:  $I(\cdot)$  is Lorenz-consistent if  $I(y) < I(y')$  when  $y$  Lorenz-dominates  $y'$ .

DEFINITION 7:  $I(\cdot)$  is strictly S-convex if  $I(yP) < I(y)$  for all bistochastic matrices except permutation matrices.  $W(\cdot)$  is strictly S-concave if  $W(yP) > W(y)$  for all bistochastic matrices except permutation matrices.

THEOREM 8: The following statements are equivalent for scale-invariant inequality measures:

- (i)  $I(\cdot)$  is inequality averse and there exists a benchmark  $y^*$ ,  $y_1 < y^* < y_n$  such that  $\frac{\partial I}{\partial y_i} < 0$  for all  $y_i < y^*$ , and  $\frac{\partial I}{\partial y_i} > 0$  for all  $y_i > y^*$ .
- (ii)  $I(\cdot)$  satisfies the transfer principle.
- (iii)  $I(\cdot)$  is S-convex.
- (iv)  $I(\cdot)$  is Lorenz-consistent.

PROOF:

- (i) Scale invariance implies

$$\sum_{i=1}^n \frac{\partial I}{\partial y_i} y_i = 0.$$

Provided that not all items of the summation equal zero, this requires the occurrence of negative and positive components. As  $y_i > 0$ ,  $i = 1, \dots, n$ , this extends to negative and positive partial derivatives. By inequality aversion, we have  $0 > \frac{\partial I}{\partial y_1} < \dots < \frac{\partial I}{\partial y_n} > 0$ . This implies the existence of a benchmark  $y^*$  such that  $\frac{\partial I}{\partial y_i} < 0$  for all  $y_i < y^*$  and  $\frac{\partial I}{\partial y_i} > 0$  for all  $y_i > y^*$ .

- (ii) A transfer of a small unit  $\delta > 0$ ,  $\delta < \min_{i,\ell \in \{1,\dots,n\}} |y_i - y_\ell|$  from  $j$  to  $k$ ,  $y_j > y_k$ , implies

$$0 > \Delta I = \left( \frac{\partial I}{\partial y_k} - \frac{\partial I}{\partial y_j} \right) \delta$$

, which can hold only if  $\frac{\partial I}{\partial y_k} < \frac{\partial I}{\partial y_j}$  for all  $y_k < y_j$ . This shows inequality aversion.

By scale invariance  $\frac{\partial I}{\partial y_1} < 0 < \frac{\partial I}{\partial y_n}$ . This demonstrates the existence of a benchmark  $y^*$  such that  $\frac{\partial I}{\partial y_i} < 0$  for all  $y_i < y^*$  and  $\frac{\partial I}{\partial y_i} > 0$  for all  $y_i > y^*$ .

(iii) The transfer principle is equivalent to S-convexity. By scale invariance  $\frac{\partial I}{\partial y_i} < 0$  for all  $y_i < y^*$  and  $\frac{\partial I}{\partial y_i} > 0$  for all  $y_i > y^*$ .

(iv) When  $y$  Lorenz-dominates  $y'$ , then  $\frac{y}{\mu}$  results from the multiplication of  $\frac{y'}{\mu'}$  with a bistochastic matrix  $P$ . By Lorenz-consistency we have

$$I(y) = I\left(\frac{y}{\mu}\right) = I\left(\frac{y'}{\mu'}P\right) = I(y'P) < I(y').$$

This demonstrates S-convexity which establishes  $\frac{\partial I}{\partial y_i} < 0$  for all  $y_i < y^*$  and  $\frac{\partial I}{\partial y_i} > 0$  for all  $y_i > y^*$ .

Q.E.D.

DEFINITION 9: A general income inequality measure defined on  $Y$ ,  $I : \mathbb{R}_{++}^n \rightarrow \mathbb{R}_+$ , is defined as

$$I(y) = \Psi[J(y)], \Psi' > 0, \text{ where } J(y) = \frac{1}{n} \sum_{i=1}^n w(i)u(z_i), \text{ and } z_i = \frac{y_i}{\mu}.$$

(i) For the nonpositional income inequality measures we have  $w(i) \equiv 1$ , and

- (a)  $u(z) = z^c$  and  $\Psi[J] = \frac{J-1}{c(c-1)}$  for  $c \neq 0, 1$   
 $u(z) = z \ln z$  and  $\Psi[J] = J$  for  $c = 1$   
 $u(z) = \ln \frac{1}{z}$  and  $\Psi[J] = J$  for  $c = 0$   
 for the entropy class of income inequality measures;
- (b)  $u(z) = z^{1-e}$  and  $\Psi[J] = 1 - J^{\frac{1}{1-e}}$ ,  $e \neq 1$ ,  $e > 0$   
 $u(z) = \ln z$  and  $\Psi[J] = 1 - e^J$ ,  $e = 1$   
 for the Atkinson inequality measure.

(ii) For positional income inequality measure we have  $u(z_i) = z_i$ ,  $\Psi[J] = J$ , and

$$w(i) = 1 + n \left[ f\left(\frac{n-i}{n}\right) - f\left(\frac{n-i+1}{n}\right) \right], f(0) = 0, f(1) = 1, f'(t) > 0 \forall t \in (0, 1)$$

for the Yaari (1988) inequality index, where this index simplifies for  $f(t) = t^\nu$ ,  $\nu > 1$ , to the extended Gini coefficient, and for  $\nu = 2$  to the Gini coefficient.

THEOREM 10: We have

(i) for nonpositional income inequality measures

$$\frac{\partial I}{\partial y_k} \geq 0 \Leftrightarrow \frac{y_k}{\mu} \geq z^* = (u')^{-1} \left[ \frac{1}{n} \sum_{i=1}^n z_i u'(z_i) \right].$$

In particular, we have



$$\begin{aligned}
z^* &= 1 && \text{for the entropy income inequality index where } c = 0, \\
&&& \text{and for the Atkinson income inequality index where } e = 1; \\
z^* &= \exp \left[ \frac{1}{n} \sum_{i=1}^n z_i \ln z_i \right] && \text{for the entropy income inequality index where } c = 1; \\
z^* &= \left[ 1 + \frac{1}{n} \sum_{i=1}^n (z_i^c - 1) \right]^{\frac{1}{c-1}} && \text{for the entropy income inequality index where } c \neq 0, 1; \\
z^* &= \left[ \frac{1}{n} \sum_{i=1}^n z_i^{1-e} \right]^{\frac{1}{e}} && \text{for the Atkinson income inequality index where } e \neq 1, e > 0;
\end{aligned}$$

(ii) for positional income inequality measures:

$$\frac{\partial I}{\partial y_k} \geq 0 \Leftrightarrow \frac{w(k)}{I} \geq 1,$$

where  $w(k)$  denotes the weight associated with  $y_k$ . In particular, we have

$$\frac{w(k)}{I} = \frac{1 + n \left[ f\left(\frac{n-k}{n}\right) - f\left(\frac{n-k+1}{n}\right) \right]}{\frac{1}{n} \sum_{i=1}^n \left\{ 1 + n \left[ f\left(\frac{n-i}{n}\right) - f\left(\frac{n-i+1}{n}\right) \right] z_i \right\}}$$

for the Yaari income inequality index, where this index simplifies for  $f(t) = t^\nu$ ,  $\nu > 1$ , to the extended Gini coefficient, and for  $\nu = 2$  to the Gini coefficient.

PROOF: Lambert and Lanza (2003).

**THEOREM 11 (LEAKY-BUCKET THEOREM):** Let  $I(\cdot)$  denote a differentiable, inequality averse and scale invariant income inequality measure with  $y^*$  as its benchmark. Consider two income recipients,  $j$  and  $k$ , and assume that  $j$ 's income is changed by  $\delta$ , where  $|\delta| < \min_{i, \ell \in \{1, \dots, n\}} |y_i - y_\ell|$ . Determine the change  $\gamma$  of  $k$ 's income such that  $\Delta I = 0$ . Then:

$$\frac{\gamma}{\delta} \doteq \alpha := -\frac{\partial I / \partial y_j}{\partial I / \partial y_k},$$

where we have for  $\delta > 0$ :

Benchmark	$j$ poorer than $k$	$j$ richer than $k$
$y^* > y_j, y_k$	$\alpha < -1$	$-1 < \alpha < 0$
$y^* < y_j, y_k$	$-1 < \alpha < 0$	$\alpha < -1$
$y^*$ between $y_j$ and $y_k$	$\alpha > 0$	$\alpha > 0$

For  $\delta < 0$  the boundaries in this table change their signs and the inequality signs are reversed.

PROOF: By differentiability we have

$$\frac{\partial I}{\partial y_j} \delta + \frac{\partial I}{\partial y_k} \gamma \doteq 0, \text{ which implies } \frac{\gamma}{\delta} = -\frac{\partial I / \partial y_j}{\partial I / \partial y_k}.$$

The cells in the above table result from Definition 3 and Theorem 8. For  $\delta < 0$  the sign of  $\alpha$  is changed.

Q.E.D.

Income inequality measures have associated social welfare functions. This is immediate for the Atkinson inequality index which is derived from a parent social welfare function. Other income inequality measures have to be doctored to be transformed into social welfare functions. We propose to follow Blackorby and Donaldson's (1978, pp. 69–70) suggestion to first multiply an income inequality measure by  $(-1)$  to change its welfare implication from a decreasing to an increasing scale, then add 1 to normalize for the value of 1 for income equality, and finally multiply by the mean income  $\mu$  to enter an efficiency component. Notice that this procedure does not give rise to unique social welfare functions, but defines only one representative of a whole family of social welfare functions compatible with the respective income inequality measure.<sup>9</sup>

DEFINITION 12: Associated with income inequality measures are the respective social welfare functions:

(i) Atkinson social welfare function:

$$W(y) = \begin{cases} \left[ \frac{1}{n} \sum_{i=1}^n y_i^{1-e} \right]^{\frac{1}{1-e}}, & \text{for } e \neq 1, e > 0; \\ \frac{1}{n} \sum_{i=1}^n \ln y_i, & \text{for } e = 1. \end{cases}$$

Note that the Atkinson social welfare function satisfies the population principle and becomes a variant of the Harsanyi social welfare function. If  $\frac{1}{n} \sum_{i=1}^n \ln y_i$  is exponentially transformed, it becomes a Cobb–Douglas social welfare function  $\prod_{i=1}^n y_i^{\frac{1}{n}}$ .

(ii) Yaari and Gini social welfare functions:

$$W(y) = \sum_{i=1}^n \left[ f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right) \right] y_i,$$

$$f(0) = 0, f(1) = 1, f'(t) > 0, \forall t \in (0, 1);$$

for  $f(t) = t^\nu$ ,  $\nu > 1$ , it becomes the extended Gini social welfare function, for  $\nu = 2$  the Gini social welfare function.

(iii) Entropy social welfare function

$$W(y) = \frac{\mu \left[ 1 + c(c-1) - \frac{1}{n} \sum_{i=1}^n \left( \frac{y_i}{\mu} \right)^c \right]}{c(c-1)}, \quad c \neq 0, 1;$$

---

<sup>9</sup>For further work on the relationship between income inequality measures and social welfare functions see Dagum (1990; 1993).

$$W(y) = \frac{1}{n \ln n} \left[ n\mu \ln n\mu - \sum_{i=1}^n y_i \ln y_i \right], \quad c = 1;$$

$$W(y) = \mu \left[ 1 - \frac{1}{n} \sum_{i=1}^n \ln \frac{\mu}{y_i} \right], \quad c = 0.$$

One can immediately check that all these social welfare functions are inequality averse. Benchmarks  $y^*$  equivalent to the benchmarks observed for income inequality measures would demand that  $\frac{\partial W}{\partial y_i} < 0$  when  $y^* < y_i \leq y_n$ . However, for the collection of social welfare functions in Definition 12,  $\frac{\partial W}{\partial y_i} < 0$  may arise only for the entropy social welfare function.

THEOREM 13: For the entropy social welfare function we have

$$\frac{\partial W}{\partial y_k} \geq 0 \Leftrightarrow y_k \leq y^*, \text{ where } y^* = \mu \left[ \frac{1 + c(c-1)}{c} - \frac{1}{cn}(c-1) \sum_{i=1}^n \left( \frac{y_i}{\mu} \right)^c \right]^{\frac{1}{c-1}}.$$

PROOF: Compute  $\frac{\partial W}{\partial y} \doteq 0$  for the entropy social welfare function and isolate the value  $y^*$  for which it holds.

Q.E.D.

This allows us to formulate a leaky-bucket theorem for social welfare functions.

THEOREM 14 (LEAKY-BUCKET THEOREM FOR SOCIAL WELFARE FUNCTIONS): Let  $W(\cdot)$  denote a differentiable, inequality averse social welfare function with  $y^*$  as its benchmark (if it exists). Consider two income recipients,  $j$  and  $k$ , and assume that  $j$ 's income is changed by  $\delta$ , where  $|\delta| < \min_{i, \ell \in \{1, \dots, n\}} |y_i - y_\ell|$ . Determine the change  $\gamma$  of  $k$ 's income such that  $\Delta W = 0$ . Then we have

$$\frac{\gamma}{\delta} \doteq \beta := -\frac{\partial W / \partial y_j}{\partial W / \partial y_k},$$

where we have for  $\delta > 0$ :

Benchmark	$j$ poorer than $k$	$j$ richer than $k$
no benchmark or $y_j, y_k < y^*$	$\beta < -1$	$-1 < \beta < 0$
$y^*$ between $y_j$ and $y_k$	$\beta > 0$	$\beta > 0$
$y_j, y_k > y^*$	$-1 < \beta < 0$	$\beta < -1$

For  $\delta < 0$  the boundaries in this table change their signs and the inequality signs are reversed.

PROOF: By differentiability we have

$$\frac{\partial W}{\partial y_j} \delta + \frac{\partial W}{\partial y_k} \gamma \doteq 0, \text{ which implies } \frac{\gamma}{\delta} = -\frac{\partial W / \partial y_j}{\partial W / \partial y_k}.$$

The cells in the above table result from Definition 4 and the possible existence of a benchmark. Q.E.D.

Recall that most leaky-bucket experiments not only rely exclusively on the case that  $j$ 's income exceeds  $k$ 's income and that  $\delta < 0$ , but also restrict subjects' responses to  $0 < \frac{\gamma}{\delta} < 1$ , that is, the (poorer) transferee receives a positive transfer which is smaller than the amount spared by the (richer) transferor. This holds only for the social-welfare-function approach with no benchmark, or for both income recipients below the benchmark. With respect to the inequality-measure approach it holds only if both income recipients are below the benchmark. However, this covers only a segment of the leaky-bucket universe.

## 3 The Experiment

### 3.1 The Experimental Design

Subjects were recruited from the students of Kiel University, mostly students of the economics and business departments. At the beginning, students received a training in handling income distributions. This was administered to them by means of a power point program. In this training, the concepts of income distributions, scale invariance, and the transfer principle were explained. Subjects were alerted that the evaluation of income distributions may follow different routes: They may, first, be judged according to a subject's perception of being *more equally distributed*. Second, they may be judged according to warranting a greater GDP for the economy, irrespective of the distribution of the incomes. Third, they may be judged according to securing higher aggregate social welfare, where it was stressed that social welfare had to be judged according to the values of the beholder. Fourth, they may approach the evaluation problem by relying on their preferences in which society they would rather live (under an ex-ante veil of ignorance).

Then the working of the experiment was explained to them. Its hub is an income distribution [500 €, 750 €, 1000 €, 1250 €, 1500 €, 1750 €, 2000 €], which contains monthly incomes of seven equally numerous groups of income recipients. These incomes were presented in the upper half of a computer screen in terms of numbers and in terms of proportional bars which were vertically arranged. The same picture appeared on the lower half of the screen. Upon touching any one key, 100 € would have been added or subtracted from some income in the lower part of the screen. This was shown by a black background of the respective number, and a broken part of the respective bar. At the same time, another income in the lower part of the screen would be set equal to zero with a white background, and the respective bar would have become empty. This pattern was administered according to a random device within our software program. In total, 84 combinations [ $7 \times 6$ , each for +100 € and -100 €] were presented to each subject in a random order.

The upper half of the screen continued to present the original picture save that the income changed by +100 € or -100 € was now highlighted in white, and the other income was highlighted in black. This allowed the subject to memorize, first, the original situation, and, second, which income was changed by the program and which other income should be adapted.

The subject was prompted to adapt the second income in the lower half of the screen such that the *degree of income inequality within this society should stay put*. Any attempt was shown to the subject and the subject was asked whether he or she would consider this move as definitive and wanted to enter it, or whether he or she wants to try out some other moves. The subject was allowed to play round as long as he or she wanted to do so before making his or her move definitive. Then the next round began until all 84 combinations were presented. The view of the screen is shown in Figure 1.

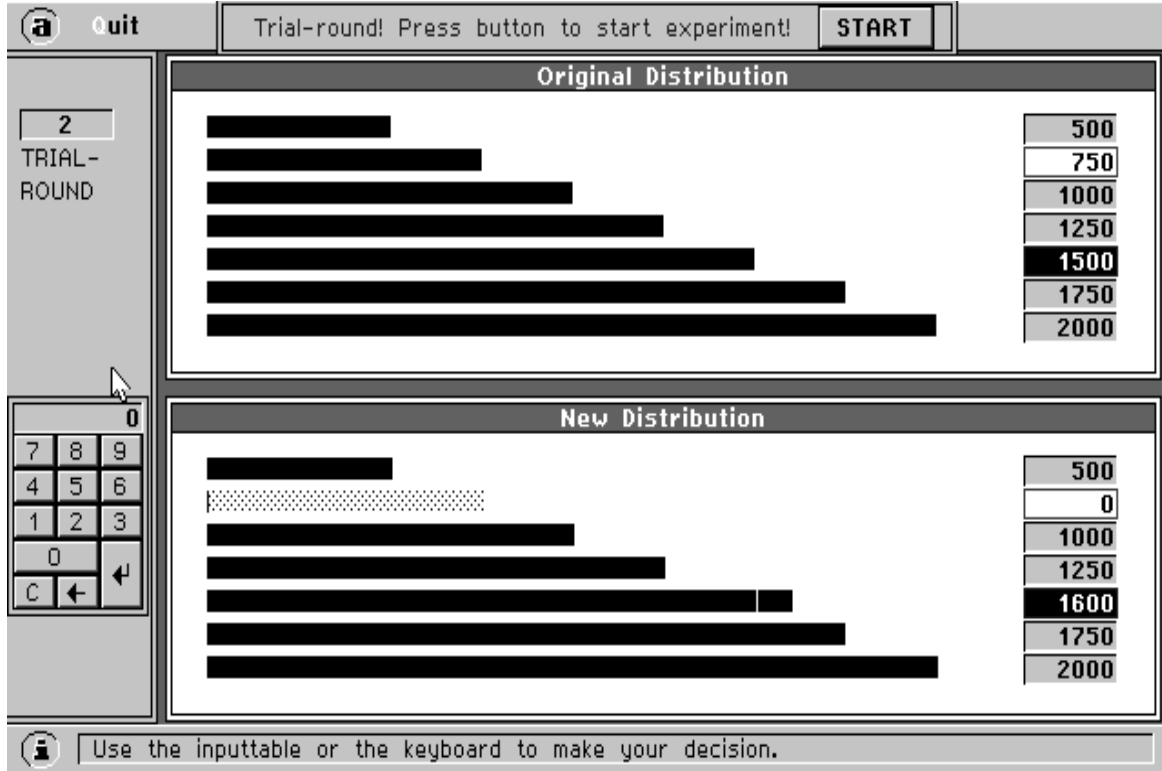


Figure 1: Screenshot

In order to eliminate emotions, subjects were told that the change in an income recipient's income by 100€ was only caused by some chance event beyond the control of any party.<sup>10</sup> The subject was asked to indicate the hypothetical change in some other income recipient's income which would restore the initial degree of income inequality within the society. The subject should not entertain the view that the other income recipient's income should *actually* be changed. The questions aimed at answering the problem of what hypothetical adaption would actually restore the initial degree of income inequality.

Before embarking on the phase of real data collection, subjects were invited to trial plays in order to become fully acquainted with the functioning of our program. Only when they pressed a "start"-button did the actual experiment begin. Moreover, an

<sup>10</sup>Other experimenters, e.g., Amiel et al. (1999, pp. 94-95), state explicitly in their instructions that some amount is *taken* from a person and ask for the minimum amount that needs to be *given* to some other person to make the transfer worthwhile. This addresses an act of redistribution and appeals to subjects' desire for transfers or feelings of social envy rather than to their perception of the degree of income inequality in this society. We were only interested in the latter aspect.

experimenter was present all the time who could be addressed in case of questions or problems.

This experiment required much attention and effort on the part of the subjects. Therefore, we decided to rely on subjects' interest in the topic to participate, rather than paying them a show-up fee which might have attracted subjects for pecuniary reasons only. This was reinforced, as we did not know of an incentive-compatible payment régime which would not have biased subjects' responses. In total, 44 subjects participated in our experiment. The data of three subjects had to be eliminated because these subjects had provided absurd responses.<sup>11</sup>

### 3.2 The Numerical Benchmarks

Note from Theorems 10 and 13 that the benchmark is implicitly defined by setting the partial derivatives equal to zero. Now, the partial derivatives are functions of both the income distributions and the parameters of the respective measures applied. As the income distribution involved is given by our experimental design, the partial derivatives are but functions of the income inequality parameters.

When plotting these functions (their graphs are given in the Appendix), we found that the function for the entropy inequality measure is S-shaped as a function of  $c$ , the function for the Atkinson inequality measure is ogival-shaped (counter-S-shaped) as a function of  $e$ ,<sup>12</sup> and that the extended Gini inequality measure is a decreasing convex function of  $\nu$ . For the entropy social welfare function we observe something similar to a function with two hyperbolic branches for negative and positive values of  $c$  [the right branch is slightly increasing after having reached a minimum, which is not strictly hyperbolic], with an undefined interval for values of  $c$  somewhat between -2 and 0.

Inserting the values of the seven incomes of our experimental design into the functions for the zero positions of the partial derivatives gives us the parameter values for the critical benchmarks. They are summarized in Table 1.

Measures	Income levels $y_i$						
	500	750	1000	1250	1500	1750	2000
Atkinson's $e$	$3.2 \times 10^7$	6.77	3.11	1	-1.68 <sup>a</sup>	-8.88 <sup>a</sup>	$-3.3 \times 10^6$ <sup>a</sup>
Gini's $\nu$ <sup>d</sup>	<sup>b</sup>	18.12	7.27	3.54	1.72	<sup>b</sup>	<sup>b</sup>
Entropy inc. inequ. $c$	$-3.26 \times 10^6$	-5.77	-2.11	0	2.68	9.88	$3.2 \times 10^6$
Entropy swf. $c$	$-3.27 \times 10^6$	-5.87	-3.82	-3.297	-3.08	-2.973	$\begin{matrix} +4.918 & ^c \\ -2.915 & ^c \end{matrix}$

<sup>a</sup> Domain of inequality sympathy.

<sup>b</sup> Not defined.

<sup>c</sup> Not uniquely defined.

<sup>d</sup> Rank positions converted into income levels.

Table 1: Zero Positions of Partial Derivatives Evaluated at  $y_i$ ,  $i = 1, \dots, 7$ .

<sup>11</sup>We conjecture that they were perhaps tempted to test the limits and confines of our software.

<sup>12</sup>This shape results from taking the real numbers as the domain of  $e$ . Notice, however, that  $e < 0$  indicates income inequality sympathy. Therefore, this part of the domain of  $e$  is usually excluded from the analysis.

Table 1 shows us that only benchmarks between 750€ and 1500€ are associated with reasonable values of the income inequality parameters. All other benchmarks require excessive values of the income inequality parameters, which can hardly be defended.

## 4 Results

### 4.1 Compensation Tests

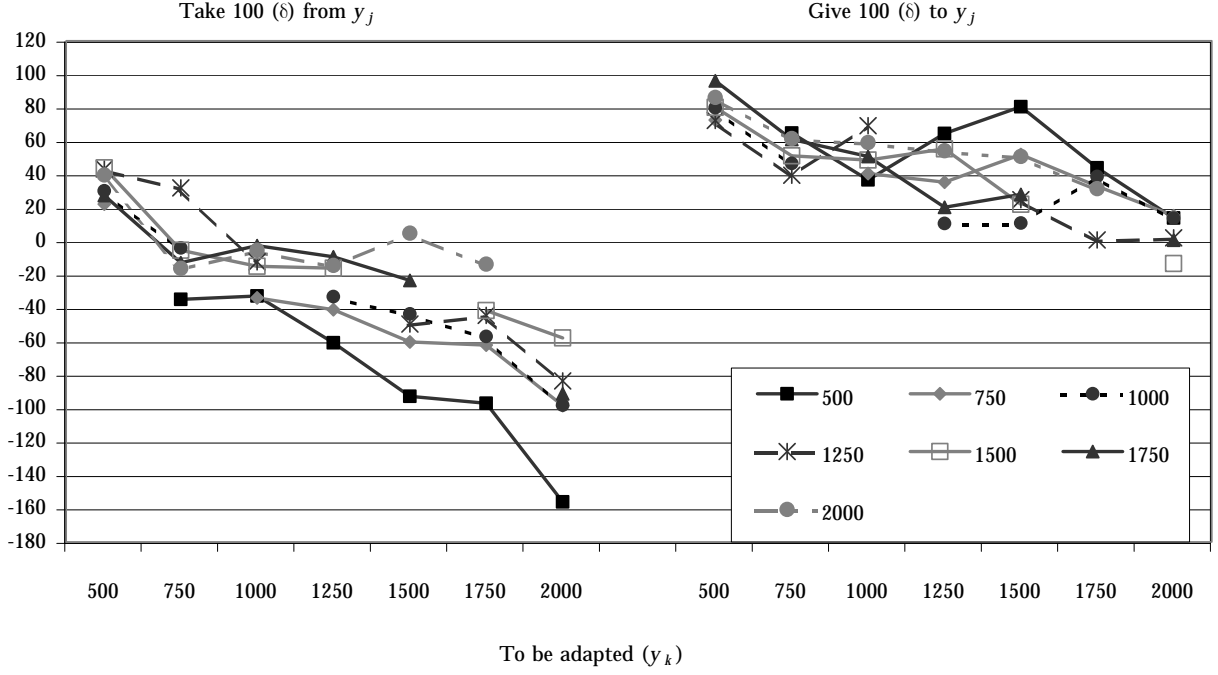
When we arrange subjects' mean responses in terms of effects on sensation on the one hand, and in terms of effects of stimuli on the other, we can check whether the empirical patterns comply with the theoretical results of the Leaky Bucket Theorem 11.

Effects on sensation can be demonstrated by increasing or decreasing a particular income  $y_j$  by 100€ and looking for the mean adaption of the other incomes  $y_k$ ,  $k = 1, \dots, 7$ ,  $k \neq j$ , to re-establish the former degree of income inequality. This gives seven curves for subtracting 100€, and seven curves for adding 100€. The two families of these curves are presented in Figure 2.

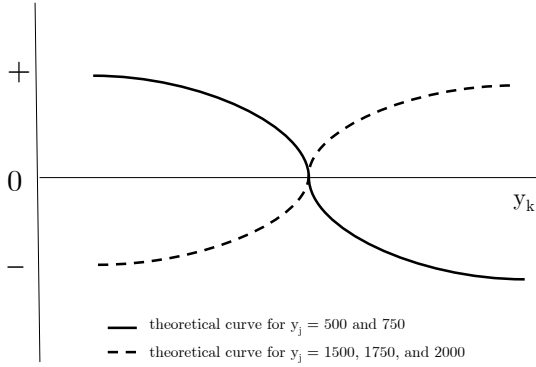
How should the theoretical curves look like? Let us assume that the benchmark lies somewhere between 750€ and 1500€, then the curves for  $y_j = 500$  and 750 on the left-hand side of Figure 2 should have positive values below the benchmark, and negative values above the benchmark. For  $y_j = 1500, 1750, 2000$ , the curves should have negative values below the benchmark, and positive values above the benchmark. The empirical curves violate this theoretical pattern. First, they are all decreasing for higher incomes to be adapted rather than being increasing for the curves for  $y_j = 1500, 1750, 2000$ . Second, they have positive values only at the very beginning, and, on top of that, for the wrong curves. While the curves for  $y_j = 1500, 1750, 2000$ , should all have negative values for 500€, the empirical data show positive values. Whereas the curve for  $y_j = 500$  should have a positive value greater than 100 for 750€, it has in fact a negative value. Out of seven curves, there is only one, viz. the curve for  $y_j = 750$ , which complies with the theory for the case that the benchmark lies in fact between 750 and 1000. These results largely invalidate the theoretical precepts. On the other hand, this pattern invalidates, too, the traditional experimental design, which allows subjects only positive responses [cf., e.g., Amiel et al. (1999)]. Out of the 21 cases in which the transferor has a higher income than the transferee, only 8 responses have positive values and 13 responses are negative. According to this theory, all 21 respective responses should be positive. Thus, the social-welfare escape to rescue the experimental treatment in terms of social welfare functions fails as well. Notice that positive responses should also hold for the remaining 21 cases (not considered by the traditional experimental design) under a social-welfare interpretation: If somebody loses income, then somebody else should receive income in order to maintain aggregate social welfare.<sup>13</sup>

Concerning the curves on the right-hand side of Figure 2, there is hardly a definite trend. Moreover, nearly all curves have positive values. Under the same assumptions about the benchmark, the theoretical curves for  $y_j = 500$  and 750 should have negative values below the benchmark, and positive values above the benchmark. For  $y_j = 1500, 1750, 2000$ , the theoretical curves should have positive values below the benchmark, and negative values above the benchmark. Yet there is only one correct result,

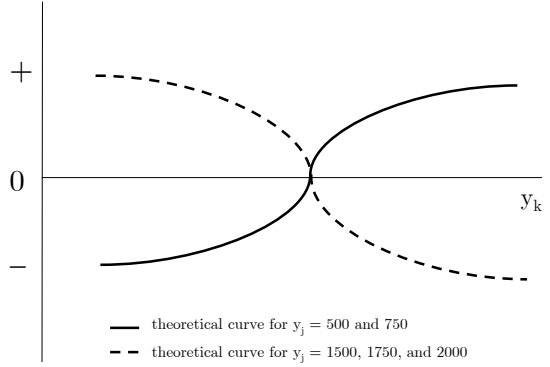
<sup>13</sup>The entropy social welfare function follows the pattern of the inequality measures.



(a) Mean responses



(b) Theoretical curves  $\delta < 0$



(c) Theoretical curves  $\delta > 0$

Figure 2: Effects on Sensation.

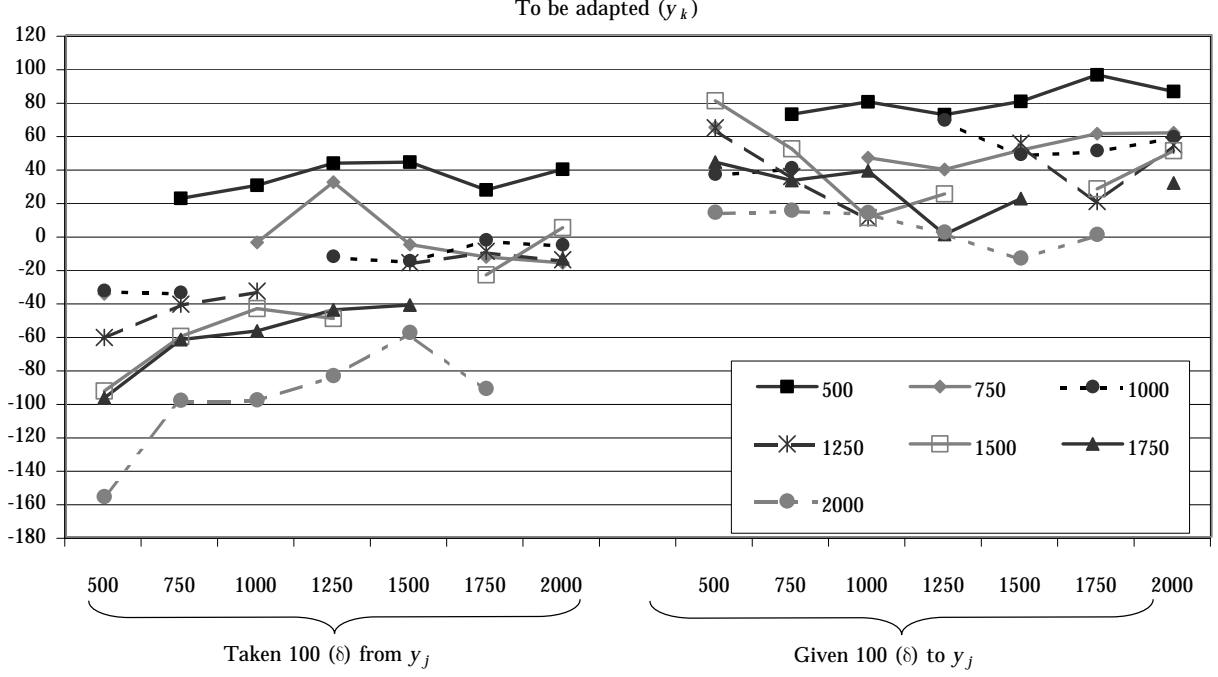
viz. a negative entry for the curve for  $y_j = 1500$  for 2000€. Thus, theory is also not evidenced for the family of the empirical curves on the right-hand side of Figure 2. Note that the social-welfare interpretation is also invalidated in this case.<sup>14</sup> If somebody receives additional income, somebody else should lose income in order to maintain aggregate social welfare.<sup>15</sup> Yet we hardly observe negative mean responses.

Let us now cast a look at the effects of changing the stimuli. Effects of stimuli can be demonstrated by increasing or decreasing all incomes  $y_j$  in succession by 100€ and looking for its effects on the adaptation of some income  $y_k$ ,  $k = 1, \dots, 7$ ,  $k \neq j$ . This produces again two times seven curves, which are presented in Figure 3, on the left-hand side for -100€ and on the right-hand side for +100€.

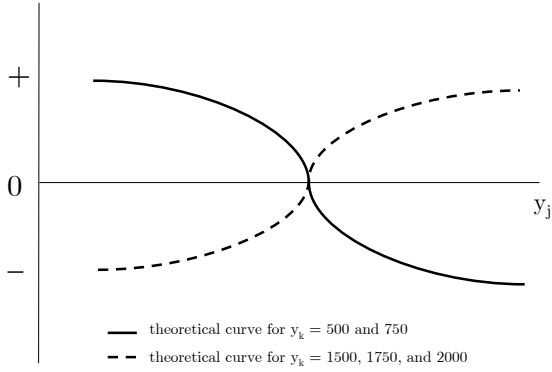
<sup>14</sup>This becomes also clear from the symmetry of Theorems 11 and 14.

<sup>15</sup>The entropy social welfare function follows the pattern of the inequality measures.

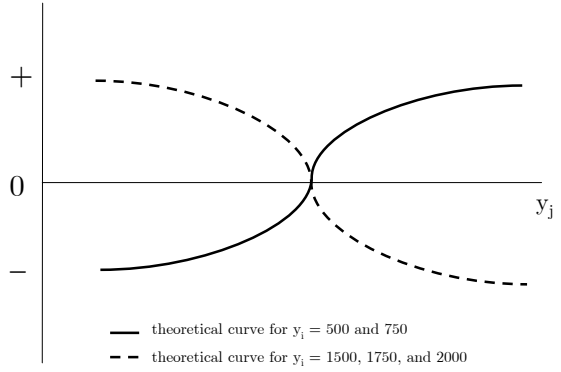




(a) Mean responses



(b) Theoretical curves  $\delta < 0$



(c) Theoretical curves  $\delta > 0$

Figure 3: Effects of Stimuli.

Assume again that the benchmark lies somewhere between 750€ and 1500€. Consider first  $\delta < 0$  [left-hand side of Figure 3]. According to Theorem 11, the curve for  $y_k = 500$ € should be decreasing, having a positive value for  $y_j = 750$  and negative values for  $y_j = 1500, 1750, 2000$ . Yet the empirical curve is rather flat and has only positive values. The curve for  $y_k = 750$ € should have a positive value for  $y_j = 500$ , and negative values for  $y_j = 1500, 1750, 2000$ . While the latter condition is moderately satisfied, the former is invalidated. The curves for  $y_k = 1500$ €,  $1750$ €, and  $2000$ € should be increasing, showing negative values for  $y_j = 500$  and  $750$ , and positive values for incomes  $y_j = 1500, 1750, 2000$ . Yet, while they exhibit an increasing pattern, they do not reach the positive range (let alone for  $y_j = 2000$  for the  $y_k = 1500$ €-curve). As to the traditional social-welfare approach, we again observe only 8 positive values among the 42 cases; however, all 42 values should be positive according to theory.<sup>16</sup>

<sup>16</sup>The entropy social welfare function follows the pattern of the inequality measures.

Consider now  $\delta > 0$  [right-hand side of Figure 3]. The curves for  $y_k = 500\text{€}$  and  $750\text{€}$  should have negative values for incomes below the benchmark, and positive values for incomes above the benchmark. For  $y_k = 1500\text{€}$ ,  $1750\text{€}$ , and  $2000\text{€}$ , the curves should have positive values for incomes below the benchmarks and negative values for incomes above the benchmarks. Yet with the exception of one value we have only positive values. Note that the social-welfare interpretation fails too.<sup>17</sup> If somebody receives additional income, then somebody else should lose income when aggregate social welfare should be restored.<sup>18</sup> Yet there is only one negative response among the 42 mean responses.

## 4.2 Categorical Tests

Categorical tests should answer the question as to the proper intervals of the effective benchmark. This could then allow inferences on the proper intervals of the effective parameter values of the income inequality measures applied, and, hopefully, on the goodness of the inequality measures (or, possibly, the social welfare functions) themselves. For this purpose we tested all twelve cases of Theorem 11 for six benchmark intervals lying between the seven incomes of our experimental design. Alas, with the exception of the interval between  $1750\text{€}$  and  $2000\text{€}$ , we received largely the same percentages for theoretically consistent behavior across the five remaining intervals for the benchmarks. Yet Table 1 shows us that the highest interval is not supported by the popular income inequality measures. Thus, our results by and large boil down to twelve percentages of behavior consistent with theory, which did, however, hardly differ for the first five benchmarks, while the last one was highly implausible.

This prompted us to look at the behavior of the individual subjects rather than at their aggregate behavior in the various cases. Thus, we screened all subjects for the four cases resulting from the combination of  $\delta \leq 0$  with  $y_j \leq y_k$ . Each of these cases covers  $(7 \times 6)/2 = 21$  combinations of incomes. For each combination we tried four benchmark intervals, viz.  $750 < y^* < 1000$ ,  $1000 < y^* < 1250$ ,  $1250 < y^* < 1500$ ,  $1500 < y^* < 1750$ , and registered for each subject the number of hits for each of this benchmark intervals according to Theorem 11. For each of this cases, 21 is the maximum number of hits which a subject could secure. A binomial test shows that 14 hits or more represent a 9.4% significance level such that they can be considered as different from pure chance.

Table 2 lists the instances of behavior consistent with theory (i.e., having 14 or more hits). For some subjects more than one benchmark interval shows 14 hits or more. These multiple instances of theoretically consistent behavior were included in Table 2. In total we found 15 out of 41 subjects with at least one instance of behavior consistent with theory. This amounts to 36.6%.

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<sup>17</sup>This becomes also clear from the symmetry of Theorems 11 and 14.

<sup>18</sup>The entropy social welfare function follows the pattern of the inequality measures.

Benchmark	$\delta < 0$		$\delta > 0$	
	$y_j > y_k$	$y_j < y_k$	$y_j > y_k$	$y_j < y_k$
$750 < y^* < 1000$	5	1	1	4
$1000 < y^* < 1250$	4	1	6	5
$1250 < y^* < 1750$		1	5	3
$1500 < y^* < 1750$			2	3

Number of subjects=41 in all entries

Table 2: Behavior Consistent with Theory.

Table 2 shows that the applicable benchmarks tend to be lower for  $\delta < 0$  than for  $\delta > 0$ . Moreover, this latter case can command more instances of consistent behavior than the former. Notice that only about one third of our subjects show sometimes behavior consistent with theory. By the symmetry of Theorems 11 and 14, this result carries over to the social-welfare interpretation of our results as well.

Is there any pattern which governs subjects' behavior in greater generality? A striking instance is the preponderance of negative income compensations for income losses and of positive income compensations for income gains, which became also evident from inspecting Figures 1 and 2. Table 3 offers another breakdown of the data. It shows that, although there are some 30% more negative responses than positive responses for the "traditional case" of progressive transfers [ $\delta < 0$ ,  $y_j > y_k$ ], their aggregate amount is somewhat less than that of the positive responses, which causes a mean positive response of 4.4€.<sup>19</sup> For regressive transfers negative responses abound in contrast to Theorem 11, which requires negative responses only if the benchmark lies between the transferor's and the transferee's incomes. For income gains [ $\delta > 0$ ], positive responses dominate.

Responses	$\delta < 0$		$\delta > 0$	
	$y_j > y_k$	$y_j < y_k$	$y_j > y_k$	$y_j < y_k$
Number of positive responses	341	194	662	517
Number of negative responses	452	632	156	278
Number of 0 responses	68	34	42	65
Total number of responses	861	860	860	860
Mean response in €	4.4	-65.2	57.8	28.2
Standard deviation	88.1	105.5	86.1	111.4

Table 3: Sign and Size of Responses

Broadly speaking, subjects' behavior seems to follow a simple precept: If someone loses income, the other person involved should be negatively compensated, and if someone gains income, the other person involved should be positively compensated. This expresses sort of compensating justice rather than restoration of the former degree of income inequality, or of the former amount of aggregate welfare. Except for a minority of less than one third of subjects, the precepts of the leaky-bucket theory are not supported by subjects' perceptions.

<sup>19</sup>At the 1% significance level this mean is not significantly different from zero according to both a t-test and a Wilcoxon test. At the 5% significance level it is not different from zero according to a t-test, while a Wilcoxon test indicates difference. All other means in Table 3 are significantly different from zero.

## 5 Conclusion

During the last 110 years, admirable advances have been made in the fields of theoretical and empirical research of income inequality measurement and the related field of concentration measurement.<sup>20</sup> Comprehensive information on this research can be gained from a great number of excellent surveys and textbooks.<sup>21</sup> Yet it is only a bit more than a decade since disillusion with the popular acceptance of central axioms of income inequality measurement began to undermine faith in the validity of inequality measurement. A number of questionnaire and experimental studies showed poor acceptance of central distributional axioms such as scale invariance, the income equalizing effects of income translations, the population principle, Pareto-dominance, Lorenz-dominance, and, most importantly of all, of the transfer principle.<sup>22</sup> Even for the simplest experimental designs in terms of numbers, the acceptance rates of these axioms hardly exceed some 40%. However, there seem to be response-mode effects. This means that, if the axioms are presented in verbal form, agreement rises to some 60%. Subjects seem to lack ability to transform verbal convictions into numbers.

The theoretical analysis of leaky-bucket transactions, which can be seen as a generalization of the transfer principle for transaction costs,<sup>23</sup> is of recent origin [Seidl (2001); Lambert and Lanza (2003)]. It has opened up new avenues of analysis. Rather than tracing out the maximum leakage of transaction costs such that a transfer still “pays at the margin”,<sup>24</sup> theory has shown a plethora of possible results. In Theorems 11 and 14 we have shown that leaky-bucket transactions encompass twelve cases each of which ordains different results. Only one of them covers the traditional case of transfers which allows the transferee to receive a positive fraction of the transfer taken from a richer transferor.

Yet experimental research shows that this new theory is poorly evidenced by the data. At most one third of the subjects behave sometimes according to theory. The rest seems to follow some notion of compensating justice: If someone loses income, the other person involved should be negatively compensated, and if someone gains income, the other person involved should be positively compensated. This behavioral pattern is, however, at variance with the theory. In this respect, empirical support for the generalized principle of transfer with transaction costs shares the fate of the axioms for income inequality measurement, to wit, it is rejected by the majority of subjects.

Our experiment was carried out in terms of numbers. It would not be worth while to ask respective questions in verbal form. While the transfer principle can be communicated to any simpleton, the theory of leaky-bucket transactions has proved to have been a sealed book even to first-rate experts who did not allow the respondents to their questionnaire the appropriate domain of responses [Amiel et al. (1999)].

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<sup>20</sup>Cf., e.g., Pareto (1895), Lorenz (1905), Gini (1912; 1914), Dalton (1920), Bonferroni (1930), Herfindahl (1950), Champernowne (1952; 1974), Amato (1968), David (1968), Kolm (1969), Atkinson (1970), Piesch (1975), Fishburn and Willig (1984), Ok (1995).

<sup>21</sup>Cf., e.g., Cowell (1977; 2000), Nygård and Sandström (1981), Kanbur (1984), Foster (1985), Lambert (1989), Chakravarty (1990), Jenkins (1991), Champernowne and Cowell (1998), Silber (1999).

<sup>22</sup>Cf. Amiel and Cowell (1992; 1994a,b; 1998; 1999a,b; 2000); Ballano and Ruiz-Castillo (1993); Harrison and Seidl (1994a,b); Bernasconi (2002); Traub et al. (2003).

<sup>23</sup>In our experiment, we avoided stories which could have evoked emotions, such as “it is unjust to take away income from somebody and transfer it to somebody else”. Therefore, we coached it in a neutral frame. Yet theory is immune to such connotations.

<sup>24</sup>Cf., e.g., Atkinson (1970, p. 5), Okun (1975, pp. 91–95), Jenkins (1991, pp. 28–29), and Amiel et al. (1999, pp. 87–89).

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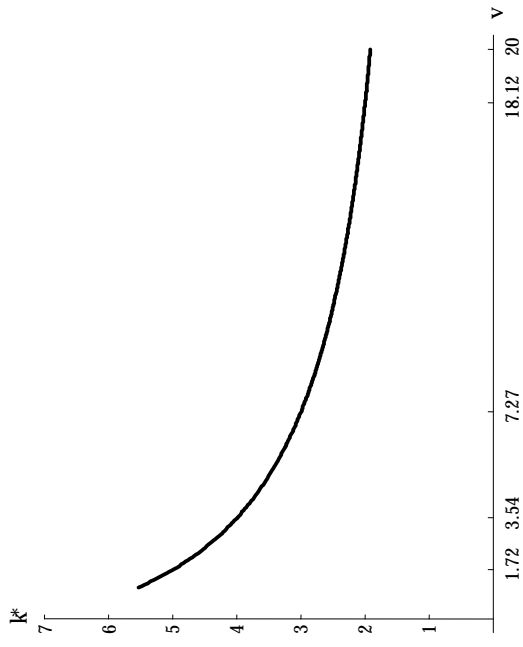
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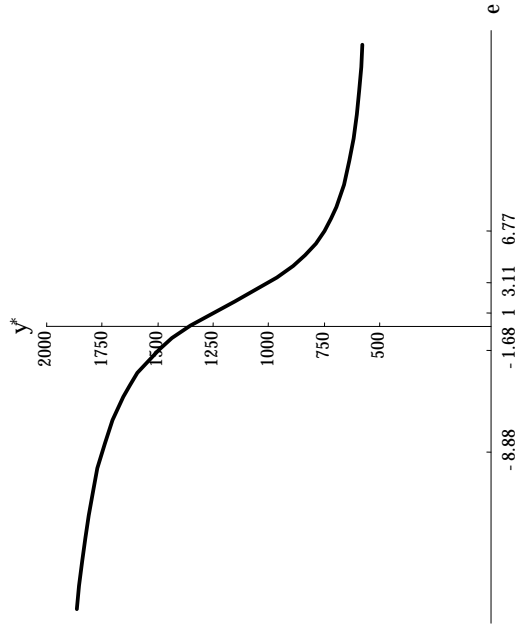
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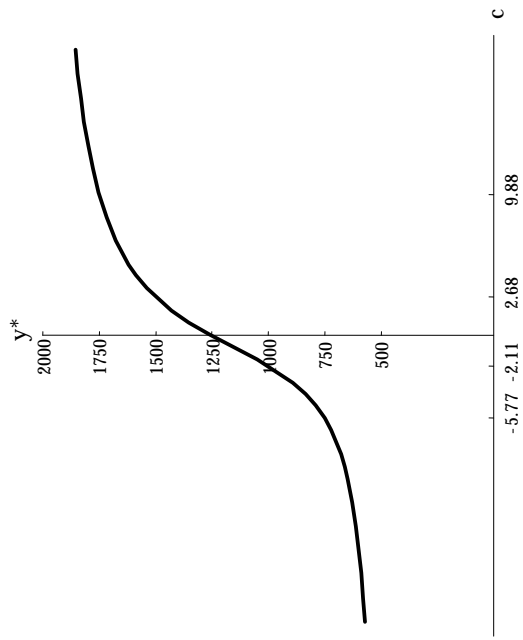




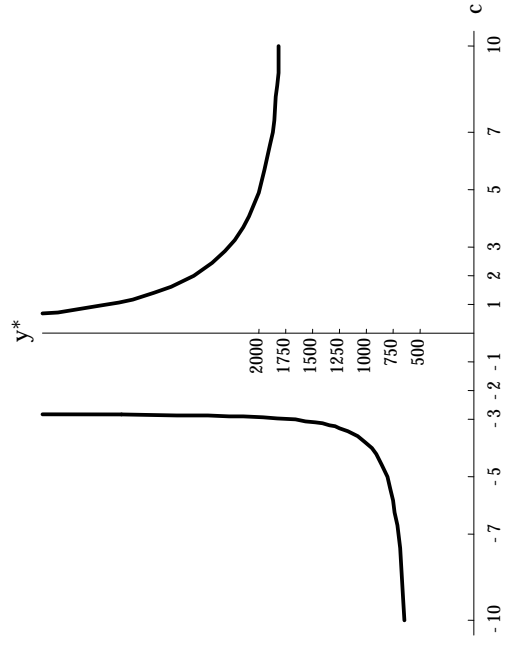
(a) Benchmarks of the Extended Gini Inequality Measure



(b) Benchmarks of the Atkinson Inequality Measure



(c) Benchmarks of the Entropy Inequality Measure



(d) Benchmarks of the Entropy Social Welfare Function

Figure 4: Benchmarks