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## The Multivariate DCC-GARCH Model with Interdependence among Markets in Conditional Variances' Equations

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**The Multivariate DCC-GARCH Model with  
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# The Multivariate DCC-GARCH Model with Interdependence among Markets in Conditional Variances' Equations

**JEL Classification:** C32

**Keywords:** *DCC-GARCH model, interdependence, conditional variance*

**Abstract:** The article seeks to investigate the issue of interdependence that during crisis periods in the capital markets is of particular importance due to the likelihood of causing a crisis in the real economy. The research objective of the article is to identify this interdependence in volatility. Therefore, first we propose our own modification of the DCC-GARCH model which is so designed as to test for interdependence in conditional variance. Then, the DCC-GARCH-In model was used to study interdependence in volatility of selected stock market indices. The results of the research confirmed the presence of interdependence among the selected markets.

## Introduction

The development of the modern global world economy caused the occurrence of interdependence between individual economies, in fact, in every aspect of their economic activities. This interdependence is a natural

mechanism that ensures a proper functioning of capital markets. Another property of the global nature of capital markets is the transmission of shock phenomena that easily transfer from one market to another. The existence of interdependence between markets allows mitigating shocks of various kinds. The observed transmission of shocks causes disruption in the functioning of markets, but in the long run the situation regains its balance. In the global economy there are also situations where transmissions of shocks cannot be mitigated by the existing interdependence between markets. In a specific crisis period, subsequent transmissions of shocks, in addition to creating distortions in the functioning of markets, may also raise significantly the level of interdependence between markets. An increase in interdependence may, in turn, strengthen unpredictably a crisis situation and lead to upsetting the functioning of both financial markets and real economies. It means that considering interdependence between markets is significant for a better understanding of the functioning of stock markets (see: Bekaert, Wu, 2000; Pritsker, 2001; Forbes, Rigobon, 2002; Baur, 2003; Pericoli, Sbracia, 2003; Corsetti, Pericoli, Sbracia, 2005; Fiszeder, 2009; Billio, Caporin, 2010; Doman, Doman, 2014; Burzała, 2014; Burzała, 2015).

Under the process of on-going globalization one can observe a steady increase in interdependence between markets resulting from fundamental linkages. In addition, during a crisis period fundamental linkages are disrupted by shock transmissions. The implication is that there is a change in fundamental linkages. The result is that under crisis periods it is hardly possible to distinguish explicitly the impact of fundamental linkages on the increase in interdependence between markets from the impact of other factors. Under a tranquillity period we probably deal with the transmission of shocks stemming mostly from fundamental linkages. The existing interdependence between markets allows the elimination of shocks and regaining the state of balance by markets. Under a crisis period it is possible to generate shock transmissions caused mostly by factors not related to the existing fundamental linkages and evoke a change in fundamental linkages resulting from these factors. An unpredictable change in fundamental linkages in conjunction with the growth of interdependence between markets may contribute to the development of a crisis even to a larger degree than non-fundamental factors.

Research into the phenomenon of interdependence between markets is very important, since in today's economy a crisis situation on capital markets, or, more broadly, on financial markets, can shift very quickly to the real sphere and lead to an economic crisis. In addition, the 2008 world financial crisis pointed to the fact that collapses of large

financial entities with international connections or even of countries' financial systems are quite likely to happen. Moreover, an increase in the level of interdependence among markets resulting from the on-going globalization, on the one hand, allows a better neutralization of subsequent shock transmissions in a crisis situation; on the other hand, however, under contagious crisis periods it may contribute to the strengthening and extension of this situation. Therefore, identification of interdependence between markets seems to be an essential element in the evaluation of risk related to the functioning of capital markets, and, in consequence, in the determination of the tools and actions for such situations.

The objective of the present article is to identify interdependence between markets for volatility. The article proposes the DCC-GARCH-In (In for Interdependence) model whose design takes into account the impact of the volatility of other markets. This model is an extended specification of the DCC-GARCH model, and this extension can be regarded as the determination of the level of interdependence between markets in volatility. The DCC-GARCH model was applied in this paper due to its advantages, *i.e.*, the relatively easy parameters estimation and simple interpretation of results. The model specification proposed by the authors will be used for the purpose of analysis of selected capital markets regarding interdependence in volatility. The research performed allowed the identification of a significant increase in interdependence for conditional variance during selected periods. Furthermore, the estimation of the proposed DCC-GARCH-In model parameters showed that interdependence between markets largely determines the volatility of individual capital markets and should be taken into account when modelling conditional variance.

### **The DCC-GARCH-In model specification**

The introduction of the GARCH class models allowed the modelling of the conditional variance for individual assets or indices. However, it was pointed very quickly to the need to account for interdependence between the studied markets. In 2002 Engle proposed a DCC-GARCH<sup>1</sup> model whose construction made it possible to analyse

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<sup>1</sup> An alternative to the DCC-GARCH model could be the BEKK-GARCH model (see: Osińska, 2006; Doman, Doman, 2009; Fiszeder, 2009). An important goal in constructing multivariate GARCH models is to make them parsimonious enough, but still maintain flexibility. In most cases the BEKK-GARCH model

interdependence between markets by estimating the time-varying conditional correlation. The DCC-GARCH model can be written as (Engle, 2009):

$$\mathbf{Y}_t = \boldsymbol{\mu}_t + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t | F_{t-1} \sim N(0, \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t) \quad (1)$$

$$\mathbf{D}_t^2 = \text{diag}\{\mathbf{H}_t\}, \quad \mathbf{H}_t = \mathbf{V}_{t-1}(\boldsymbol{\eta}_t) \quad (2)$$

$$H_{i,t} = \omega_i + \alpha_i \eta_{i,t-1}^2 + \beta_i H_{i,t-1} \quad (3)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1} \boldsymbol{\eta}_t \quad (4)$$

$$\mathbf{R}_t = \text{diag}\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \text{diag}\{\mathbf{Q}_t\}^{-1/2} \quad (5)$$

$$\mathbf{Q}_t = \boldsymbol{\Omega} + \alpha \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1} + \beta \mathbf{Q}_{t-1}, \quad \boldsymbol{\Omega} = \bar{\mathbf{R}}(1 - \alpha - \beta) \quad (6)$$

where

$\mathbf{Y}_t$  - the multivariate process of returns,  $\boldsymbol{\mu}_t$  - the vector of conditional means of returns,

$H_{i,t}$  - the conditional variance equation for  $i$ -th returns, where  $i = 1, \dots, N$ ,

$\mathbf{V}_{t-1}(\boldsymbol{\eta}_t)$  - the covariance matrix of the residuals  $\boldsymbol{\eta}_t$ ,

$\mathbf{R}_t$  - the time-varying conditional correlation matrix,

$\mathbf{Q}_t$  - a positive-definite quasi correlation matrix for each time period,

$\omega_i, \alpha_i, \beta_i$  - the parameters of the conditional variance equation, where  $i = 1, \dots, N$ ,

$\alpha, \beta$  - the parameters of the conditional correlation equation,

$\bar{\mathbf{R}} = \frac{1}{T} \sum_{i=1}^T \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t$  - the unconditional covariance matrix.

In the DCC-GARCH model the conditional variance depends on the lagged conditional variance and on squared returns with the consideration of conditional means. However, in the case of the conditional correlation equation, the variables describing are the standardized residuals  $\boldsymbol{\varepsilon}_t$  and lagged conditional correlations  $\mathbf{Q}_{t-1}$ . In addition, to guarantee  $H_t$  to be positive definite the parameters  $\alpha, \beta$  must satisfy the conditions,  $\alpha \geq 0, \beta \geq 0$  and  $\alpha + \beta < 1$ .

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parametrization is too complex which leads to insignificance of some parameters. In addition, the model parameters do not have simple interpretation in contrast to the DCC-GARCH model and the BEKK-GARCH model estimation is highly demanded even for four variables.

The authors propose to extend the DCC-GARCH model by taking into account the volatility of other markets in the conditional variance equation. This procedure will allow one to capture the interdependence between markets in volatility. The DCC-GARCH-In model<sup>2</sup> specification is as follows:

$$\mathbf{Y}_t = \boldsymbol{\mu}_t + \boldsymbol{\eta}_t \quad \boldsymbol{\eta}_t | F_{t-1} \sim N(0, \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t) \quad (7)$$

$$\mathbf{D}_t^2 = \text{diag}\{\mathbf{H}_t\}, \quad \mathbf{H}_t = \mathbf{V}_{t-1}(\boldsymbol{\eta}_t) \quad (8)$$

$$H_{i,t} = \omega_i + \alpha_i \eta_{i,t-1}^2 + \beta_i H_{i,t-1} + \gamma_i \bar{H}_{i,t-1} \quad (9)$$

$$\bar{H}_{i,t-1} = \sum_{j=1}^N w_{i,j} H_{j,t-1} = w_{i,1} H_{1,t-1} + \dots + w_{i,N} H_{N,t-1}, \quad (10)$$

where  $w_{i,j} = 0$  for  $i = j$ .

$$\sum_{j=1}^N w_{i,j} = w_{i,1} + \dots + w_{i,N} = 1 \quad (10)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1} \boldsymbol{\eta}_t \quad (11)$$

$$\mathbf{R}_t = \text{diag}\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \text{diag}\{\mathbf{Q}_t\}^{-1/2} \quad (12)$$

$$\mathbf{Q}_t = \boldsymbol{\Omega} + \alpha \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1} + \beta \mathbf{Q}_{t-1}, \quad \boldsymbol{\Omega} = \bar{\mathbf{R}}(1 - \alpha - \beta) \quad (13)$$

$\bar{H}_{i,t-1}$  - the weighted mean of the conditional variances based on other  $H_{j,t-1}$  where  $j = 1, \dots, N$  and  $j \neq i$ ,

$\gamma_i$  - the parameter of the  $\bar{H}_{i,t-1}$ ,

$\mathbf{V}_{t-1}(\boldsymbol{\eta}_t)$  - the covariance matrix of the residuals  $\boldsymbol{\eta}_t$ ,

$H_{j,t-1}$  - the conditional variance estimated in the DCC-GARCH model for  $j = 1, \dots, N$ ,

$w_{i,j}$  - the weights specifying the share of conditional variance,

$\mathbf{R}_t$  - the time-varying conditional correlation matrix,

$\mathbf{Q}_t$  - a positive-definite quasi correlation matrix for each time period,

$\bar{\mathbf{R}} = \frac{1}{T} \sum_{i=1}^T \boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}'_i$  - the unconditional covariance matrix.

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<sup>2</sup> Eckner (2006) proposed the DCC-GARCH model which takes into account interdependence between markets in volatility as well. In that paper it is proposed that for every conditional variance equation one should add the squared returns with zero mean for the remaining assets.

In this paper the normal distribution is assumed as the error distribution. It is possible to assume the fat-tailed distribution, *e.g.*, *t*-distribution or skew *t*-distribution which may lead to a better fit of the model to the data.

The factor  $\bar{H}_{i,t-1}$  exhibits the weighted mean of conditional variances from remaining equations of the model. The introduction of the factor  $\bar{H}_{i,t-1}$  allows taking into account the  $N - 1$  conditional variances  $H_{j,t-1}$  in each equation.

A separate issue is the determination of the weights matrix. Due to the possibility of transferring capital between stock markets, it is assumed that all of the stock markets under study are considered to be neighbours to each other. Therefore, it is necessary to determine the values of the elements of the weights matrix that will reflect the share of *j*-th conditional variances  $H_{j,t-1}$  in formula **Błąd! Nie można odnaleźć źródła odwołania.**

. The weights matrix can be expressed by the following formula:

$$\mathbf{W} = \begin{bmatrix} w_{1,1} & \dots & w_{1,N} \\ \vdots & \ddots & \vdots \\ w_{N,1} & \dots & w_{N,N} \end{bmatrix} = \begin{bmatrix} 0 & w_{1,2} & \dots & w_{1,N} \\ w_{2,1} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & w_{1,N-1} \\ w_{N,1} & \dots & w_{N,N-1} & 0 \end{bmatrix} \quad (14)$$

It is assumed that the weight values  $w_{i,j}$  for  $i = j$  are equal to 0, since in the *i*-conditional variance equation (9) the variable  $H_{i,t-1}$  is already present. In this paper it is assumed that weights are given prior to the estimation. One may wish to estimate weights, but this leads to a high number of additional parameters to estimate in high dimensional cases, which could be not feasible. The method of determining the weights can be based on information describing individual markets or on the determination of arbitrary weights drawing on the researcher's knowledge and experience. The weights can be determined on the basis of stock market capitalization, GDP *per capita*, or on the basis of the share of stock market capitalization in GDP. In the article the authors took capitalization of stock markets as the criterion. Therefore, the values of the weight matrix can be determined in the following way:



$$w_{i,j} = \begin{cases} \frac{\text{capitalization}_j}{\sum_{\substack{j=1 \\ i \neq j}}^N \text{capitalization}_j} & j = 1, \dots, N \\ 0 & \text{for } i = j \end{cases} \quad (15)$$

where the sum of the weights for each row is 1.

The estimation of the DCC-GARCH-In model parameters can be carried out using the maximum likelihood method. Similar to the case of the DCC-GARCH model, one may use the two-stage estimation method described by Engle (2002, 2009), where the following likelihood function is used:

$$L(\theta) = L_1(\theta_1) + L_2(\theta_2 | \theta_1) \quad (16)$$

The two-stage method is a simplification consisting in estimating separately the parameters of the conditional variance equations and means in the first phase, and in estimating the parameters of the conditional correlation equations in the second phase. It is worth noting that in the first step all of the parameters of the conditional variance equations and means are estimated simultaneously utilizing the likelihood function  $L_1(\theta_1)$ .

$$L_1(\theta_1) = -0.5 \sum_t (n \ln(2\pi) + 2 \ln |\mathbf{D}_t| + \boldsymbol{\eta}_t' \mathbf{D}_t^{-2} \boldsymbol{\eta}_t) \quad (17)$$

The reason behind this is that the  $i$ -th conditional variance depends on all remaining conditional variances.

The second phase is about estimating the parameters of the conditional correlation equation. The logarithm of likelihood function  $L_2(\theta_2 | \theta_1)$  can be expressed using the following

$$\text{formula: } L_2(\theta_2 | \theta_1) = -0.5 \sum_t (-\boldsymbol{\varepsilon}_t' \boldsymbol{\varepsilon}_t + \ln |\mathbf{R}_t| + \boldsymbol{\varepsilon}_t' \mathbf{R}_t^{-1} \boldsymbol{\varepsilon}_t) \quad (18)$$

## Empirical research

The empirical research used time series of the selected stock market indices - WIG, BUX, DAX, S&P 500, FTSE 100, CAC 40, KOSPI, BOVESPA, SSE, HSI, RTS, SSMI, NIKKEI 225, ATG, MIBTEL<sup>3</sup> and adequate capitalization of stock markets<sup>4</sup>. For this purpose we took daily observations covering the period from 3 January 2000 to 3 January 2012, which gave us a total of  $T = 3000$  observations. The assumption of such a long time period allowed the consideration of two crisis periods (the 2000-2002 dot-com bubble and the 2007-2009 global financial crisis). The study used logarithmic returns  $r_t = 100\% * (\ln(P_t) - \ln(P_{t-1}))$ . For the estimation purpose, we applied the maximum likelihood method with a conditional normal distribution for both the DCC-GARCH and the DCC-GARCH-In models.

**Table 1.** The assumed weight matrix based on the capitalization of stock markets

Variable	$w_{i,1}$	$w_{i,2}$	$w_{i,3}$	$w_{i,4}$	$w_{i,5}$	$w_{i,6}$	$w_{i,7}$	$w_{i,8}$	$w_{i,9}$	$w_{i,10}$	$w_{i,11}$	$w_{i,12}$	$w_{i,13}$
WIG	0	0.001	0.04	0.492	0.087	0.053	0.021	0.023	0.077	0.026	0.019	0.029	0.11
BUX	0.003	0	0.04	0.491	0.087	0.053	0.021	0.023	0.076	0.026	0.019	0.029	0.11
DAX	0.003	0.001	0	0.511	0.09	0.055	0.022	0.023	0.08	0.027	0.019	0.03	0.115
SP500	0.006	0.001	0.079	0	0.17	0.104	0.041	0.044	0.15	0.051	0.036	0.057	0.216
FTSE100	0.004	0.001	0.044	0.537	0	0.058	0.023	0.025	0.084	0.028	0.02	0.032	0.121
CAC40	0.003	0.001	0.042	0.518	0.091	0	0.022	0.024	0.081	0.027	0.02	0.031	0.116
KOSPI	0.003	0.001	0.041	0.501	0.088	0.054	0	0.023	0.078	0.027	0.019	0.03	0.112
BOVESPA	0.003	0.001	0.041	0.501	0.089	0.054	0.021	0	0.078	0.027	0.019	0.03	0.113
SSE	0.004	0.001	0.043	0.531	0.094	0.058	0.022	0.024	0	0.028	0.02	0.031	0.119
HSI	0.003	0.001	0.041	0.503	0.089	0.055	0.021	0.023	0.078	0	0.019	0.03	0.113
RTS	0.003	0.001	0.041	0.499	0.088	0.054	0.021	0.023	0.078	0.026	0	0.029	0.112
SSMI	0.003	0.001	0.041	0.505	0.089	0.055	0.021	0.023	0.079	0.027	0.019	0	0.113
NIKKEI 225	0.004	0.001	0.045	0.551	0.097	0.06	0.023	0.025	0.086	0.029	0.021	0.033	0

Source: own calculations.

<sup>3</sup> The data was downloaded from <http://www.finance.yahoo.com>

<sup>4</sup> The data was downloaded from <http://www.data.worldbank.org>

**Table 2.** The results of the estimation of the multivariate DCC-GARCH model parameters

Parameter (Stock index)	Estimate	Std. Error	p-value	Parameter (Stock index)	Estimate	Std. Error	p-value
$\omega_1$ (WIG)	0.0184	0.0042	0.000	$\omega_9$ (SSE)	0.0327	0.0045	0.000
$\alpha_1$ (WIG)	0.0642	0.0057	0.000	$\alpha_9$ (SSE)	0.075	0.0053	0.000
$\beta_1$ (WIG)	0.9265	0.0064	0.000	$\beta_9$ (SSE)	0.915	0.0053	0.000
$\omega_2$ (BUX)	0.0624	0.0101	0.000	$\omega_{10}$ (HSI)	0.0144	0.0035	0.000
$\alpha_2$ (BUX)	0.0901	0.008	0.000	$\alpha_{10}$ (HSI)	0.0664	0.0057	0.000
$\beta_2$ (BUX)	0.8865	0.0098	0.000	$\beta_{10}$ (HSI)	0.9281	0.0061	0.000
$\omega_3$ (DAX)	0.0233	0.0046	0.000	$\omega_{11}$ (RTS)	0.1186	0.0115	0.000
$\alpha_3$ (DAX)	0.0963	0.0084	0.000	$\alpha_{11}$ (RTS)	0.1059	0.0064	0.000
$\beta_3$ (DAX)	0.8961	0.0088	0.000	$\beta_{11}$ (RTS)	0.8718	0.0077	0.000
$\omega_4$ (S&P 500)	0.0135	0.0019	0.000	$\omega_{12}$ (SSMI)	0.0236	0.0041	0.000
$\alpha_4$ (S&P 500)	0.0799	0.0067	0.000	$\alpha_{12}$ (SSMI)	0.1186	0.01	0.000
$\beta_4$ (S&P 500)	0.9123	0.0072	0.000	$\beta_{12}$ (SSMI)	0.8678	0.0111	0.000
$\omega_5$ (FTSE 100)	0.0126	0.0029	0.000	$\omega_{13}$ (NIKKEI 225)	0.0398	0.0077	0.000
$\alpha_5$ (FTSE 100)	0.1035	0.0086	0.000	$\alpha_{13}$ (NIKKEI 225)	0.1023	0.0076	0.000
$\beta_5$ (FTSE 100)	0.8916	0.0085	0.000	$\beta_{13}$ (NIKKEI 225)	0.884	0.0089	0.000
$\omega_6$ (CAC 40)	0.0192	0.0043	0.000	$\omega_{14}$ (ATG)	0.0228	0.0053	0.000
$\alpha_6$ (CAC 40)	0.0959	0.0082	0.000	$\alpha_{14}$ (ATG)	0.1026	0.0077	0.000
$\beta_6$ (CAC 40)	0.8984	0.0086	0.000	$\beta_{14}$ (ATG)	0.8943	0.0073	0.000
$\omega_7$ (KOSPI)	0.0246	0.0054	0.000	$\omega_{15}$ (MIBTEL)	0.0138	0.0028	0.000
$\alpha_7$ (KOSPI)	0.0793	0.0067	0.000	$\alpha_{15}$ (MIBTEL)	0.0956	0.0071	0.000
$\beta_7$ (KOSPI)	0.9151	0.007	0.000	$\beta_{15}$ (MIBTEL)	0.9013	0.0073	0.000
$\omega_8$ (BOVESPA)	0.0813	0.0133	0.000	Parameter	Estimate	Std. Error	p-value
$\alpha_8$	0.0722	0.0068	0.000	$\alpha$	0.0076	0.00047	0.000

(BOVESPA)							
$\beta_8$ (BOVESPA)	0.9045	0.0084	0.000	$\beta$	0.9861	0.00112	0.000

Source: own calculations.

Table 1 shows the weights matrix determined by the capitalization of the stock markets in accordance with formula (15). As can be seen, the diagonal elements of the weights matrix equals zero, which corresponds to the previous assumptions of the model. For instance, the conditional variance equation of the DCC-GARCH-In model for the WIG index take the following form:

$$\begin{aligned}
H_{WIG,t} = & \omega_1 + \alpha_1 r_{WIG,t-1}^2 + \beta_1 H_{WIG,t-1} + \gamma_1 (0.001H_{BUX,t-1} + 0.04H_{DAX,t-1} + \\
& + 0.492H_{S\&P500,t-1} + 0.087H_{FTSE100,t-1} + 0.053H_{CAC40,t-1} + 0.021H_{KOSPI,t-1} + \\
& + 0.023H_{BOVESPA,t-1} + 0.077H_{SSE,t-1} + 0.026H_{HSI,t-1} + 0.019H_{RTS,t-1} + \\
& + 0.029H_{SSMI,t-1} + 0.110H_{NIKKEI225,t-1} + 0.003H_{ATG,t-1} + 0.02H_{MIBTEL,t-1}) \quad (19)
\end{aligned}$$

Tables 2 and Table 3 contain the results of the estimation of the DCC-GARCH and DCC-GARCH-In models parameters. In the case of the stock market indices under study, the constant was not taken into account in the conditional mean equation, since for each equation the constant was found to be statistically insignificant.

The parameter was found to be statistically significant for the following indices: WIG, BUX, DAX, S&P 500, FTSE 100, CAC 40, BOVESPA, RTS, SSML, NIKKEI 225, MIBTEL, which indicates the presence of interdependence in volatility between the markets. In the case of the ATG, HSI, SSE and KOSPI indices, the parameter  $\gamma_i$  was found to be statistically insignificant at the significance level of 5%. The highest parameter estimate at the level of 0.0615 was obtained for the conditional variance equation of the BOVESPA. The lowest parameter estimate at the level of 0.0046 was obtained for the parameter  $\gamma_i$  in the equation for the KOSPI index. Statistically significant parameters  $\gamma_i$  indicate the presence of a stronger linkage in volatility between the selected index and remaining markets. Additionally, we compared the DCC-GARCH model with the DCC-GARCH-In model using standard Lagrange Ratio test, but applying it only to the conditional variance equations, *i.e.*, utilizing  $L_1(\hat{\theta}_1)$ . The obtained test statistic (LR=106.15 with  $p$ -value  $8.81e-16$ ) means that the

DCC-GARCH-In fits the data better than the standard DCC-GARCH model.

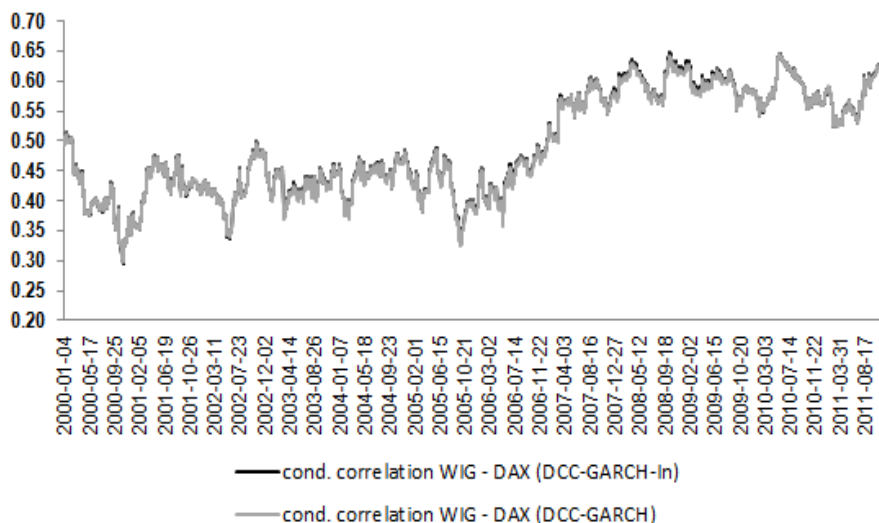
**Table 3.** The results of the estimation of the multivariate DCC-GARCH- In model parameters

Parameter (Stock index)	Estimate	Std. Error	p-value	Parameter (Stock index)	Estimate	Std. Error	p-value
$\omega_1$ (WIG)	0.0169	0.0046	0.0002	$\omega_9$ (SSE)	0.0375	0.0054	0.0000
$\alpha_1$ (WIG)	0.065	0.0061	0.0000	$\alpha_9$ (SSE)	0.0742	0.0052	0.0000
$\beta_1$ (WIG)	0.9199	0.0078	0.0000	$\beta_9$ (SSE)	0.9166	0.0052	0.0000
$\gamma_1$ (WIG)	0.0059	0.0029	0.0441	$\gamma_9$ (SSE)	-0.003	0.0017	0.0565
$\omega_2$ (BUX)	0.0575	0.0119	0.0000	$\omega_{10}$ (HSI)	0.0124	0.004	0.0022
$\alpha_2$ (BUX)	0.0919	0.0088	0.0000	$\alpha_{10}$ (HSI)	0.0664	0.0058	0.0000
$\beta_2$ (BUX)	0.8688	0.0134	0.0000	$\beta_{10}$ (HSI)	0.925	0.0069	0.0000
$\gamma_2$ (BUX)	0.022	0.0078	0.0047	$\gamma_{10}$ (HSI)	0.0044	0.0043	0.3053
$\omega_3$ (DAX)	0.0094	0.0067	0.161	$\omega_{11}$ (RTS)	0.1077	0.0126	0.0000
$\alpha_3$ (DAX)	0.0956	0.0095	0.0000	$\alpha_{11}$ (RTS)	0.1066	0.0066	0.0000
$\beta_3$ (DAX)	0.8758	0.0124	0.0000	$\beta_{11}$ (RTS)	0.8662	0.0082	0.0000
$\gamma_3$ (DAX)	0.0314	0.0092	0.0007	$\gamma_{11}$ (RTS)	0.0171	0.0085	0.0456
$\omega_4$ (S&P 500)	-0.002	0.0047	0.5797	$\omega_{12}$ (SSMI)	0.0185	0.0049	0.0002
$\alpha_4$ (S&P 500)	0.0795	0.0069	0.0000	$\alpha_{12}$ (SSMI)	0.1213	0.0112	0.0000
$\beta_4$ (S&P 500)	0.8919	0.0106	0.0000	$\beta_{12}$ (SSMI)	0.8504	0.0163	0.0000
$\gamma_4$ (S&P 500)	0.0204	0.0052	0.0001	$\gamma_{12}$ (SSMI)	0.0124	0.0063	0.0493
$\omega_5$ (FTSE 100)	-0.009	0.0082	0.2531	$\omega_{13}$ (NIKKEI 225)	0.0311	0.0099	0.0017
$\alpha_5$ (FTSE 100)	0.1151	0.0124	0.0000	$\alpha_{13}$ (NIKKEI 225)	0.096	0.0078	0.0000
$\beta_5$ (FTSE 100)	0.8197	0.0277	0.0000	$\beta_{13}$ (NIKKEI 225)	0.8538	0.0144	0.0000
$\gamma_5$ (FTSE 100)	0.0547	0.0183	0.0029	$\gamma_{13}$ (NIKKEI 225)	0.0437	0.0101	0.0000
$\omega_6$ (CAC 40)	-0.001	0.0093	0.899	$\omega_{14}$ (ATG)	0.0191	0.0063	0.0023

$\alpha_6$ (CAC 40)	0.0991	0.0099	0.0000	$\alpha_{14}$ (ATG)	0.1049	0.0081	0.0000
$\beta_6$ (CAC 40)	0.8541	0.0194	0.0000	$\beta_{14}$ (ATG)	0.8884	0.0086	0.0000
$\gamma_6$ (CAC 40)	0.0565	0.0194	0.0036	$\gamma_{14}$ (ATG)	0.0065	0.0051	0.2057
$\omega_7$ (KOSPI)	0.0232	0.0063	0.0002	$\omega_{15}$ (MIBTEL)	0.0061	0.005	0.2182
$\alpha_7$ (KOSPI)	0.0802	0.0069	0.0000	$\alpha_{15}$ (MIBTEL)	0.0966	0.0076	0.0000
$\beta_7$ (KOSPI)	0.9115	0.0078	0.0000	$\beta_{15}$ (MIBTEL)	0.8885	0.0097	0.0000
$\gamma_7$ (KOSPI)	0.0046	0.0056	0.405	$\gamma_{15}$ (MIBTEL)	0.0157	0.0076	0.0393
$\omega_8$ (BOVESPA)	0.1214	0.0255	0.0000	Parameter	Estimate	Std. Error	p- value
$\alpha_8$ (BOVESPA)	0.0796	0.0089	0.0000				
$\beta_8$ (BOVESPA)	0.8491	0.0211	0.0000	$\alpha$	0.0074	0.00047	0.000
$\gamma_8$ (BOVESPA)	0.0615	0.0193	0.0014	$\beta$	0.9864	0.00112	0.000

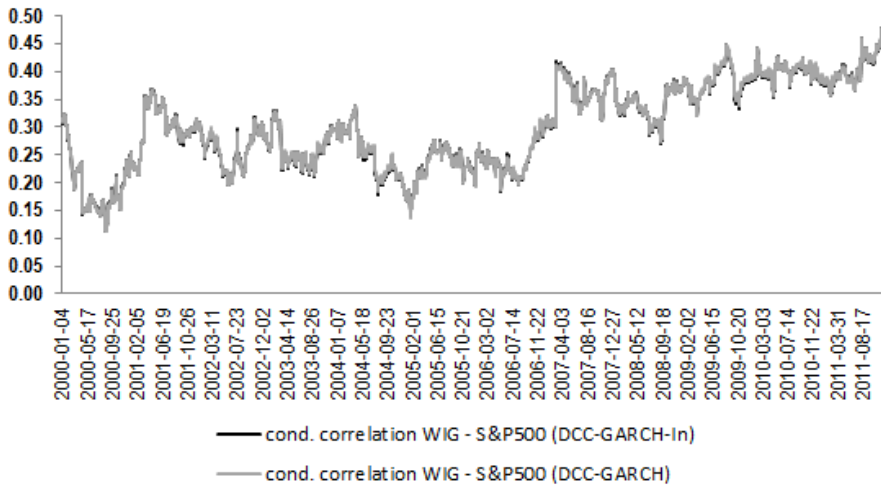
Source: own calculations.

**Figure 1.** The conditional correlation between the WIG and the DAX indices



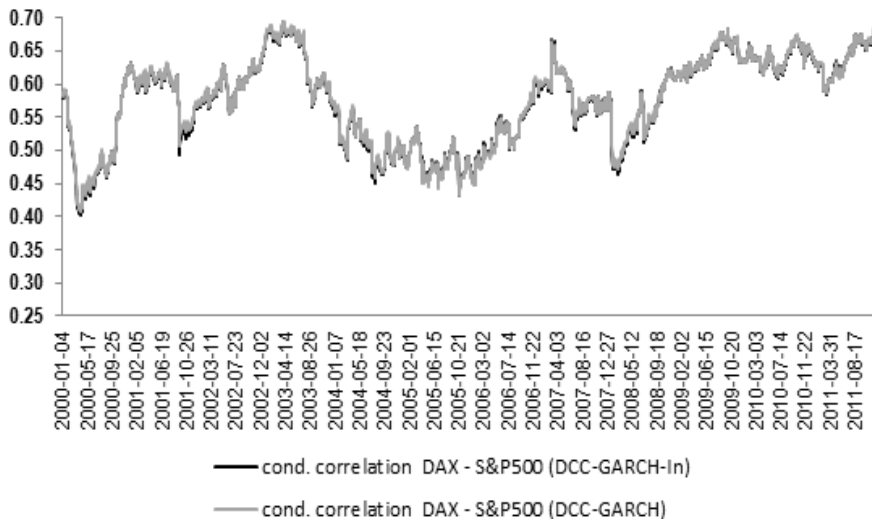
Source: own elaboration.

**Figure 2.** The conditional correlation between the WIG and the S&P500 indices



Source: own elaboration.

**Figure 3.** The conditional correlation between the DAX and S&P500 indices

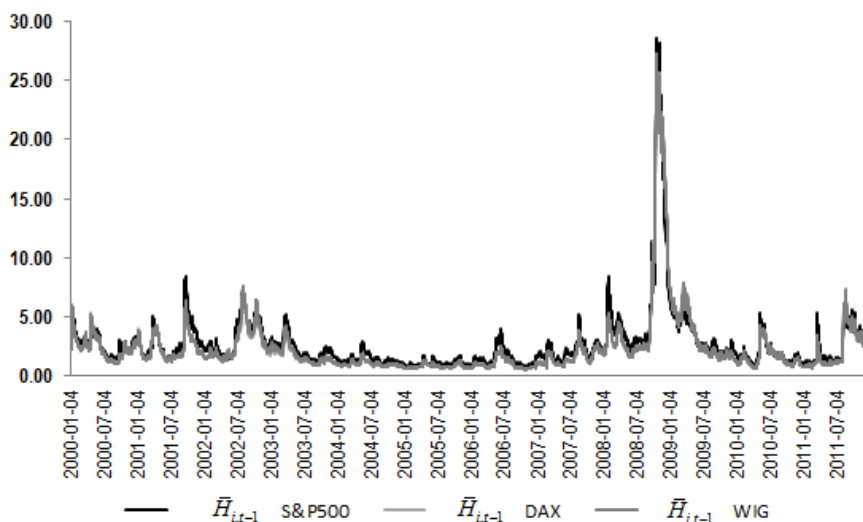


Source: own elaboration.

Conditional correlations for both models are usually very similar. This is due to the fact that essentially the conditional variance does not impact the conditional correlation. The only linkage that exists refers to the standardized residuals  $\varepsilon_t$ . The results obtained for three indices (WIG, DAX, S&P500) will be interpreted to provide an example. Figures 1 to 3 show the values of conditional correlations obtained from the DCC-GARCH and the DCC-GARCH-In for pairs of the WIG-DAX, WIG-S&P500, DAX-S&P500.

The values of the factor  $\bar{H}_{i,t-1}$  are similar for all markets, too. The factor  $\bar{H}_{i,t-1}$  allows taking into account the weighted mean of volatility of other markets. In the case of the WIG and the DAX the values of this factor are at a similar level. However, the values of the factor  $\bar{H}_{i,t-1}$  for the S&P500 index achieve a higher level (see Figure 4). We can see that the volatility of the factor  $\bar{H}_{i,t-1}$  is much higher under turmoil periods in the capital markets, than in periods of tranquillity, due to the design of this factor.

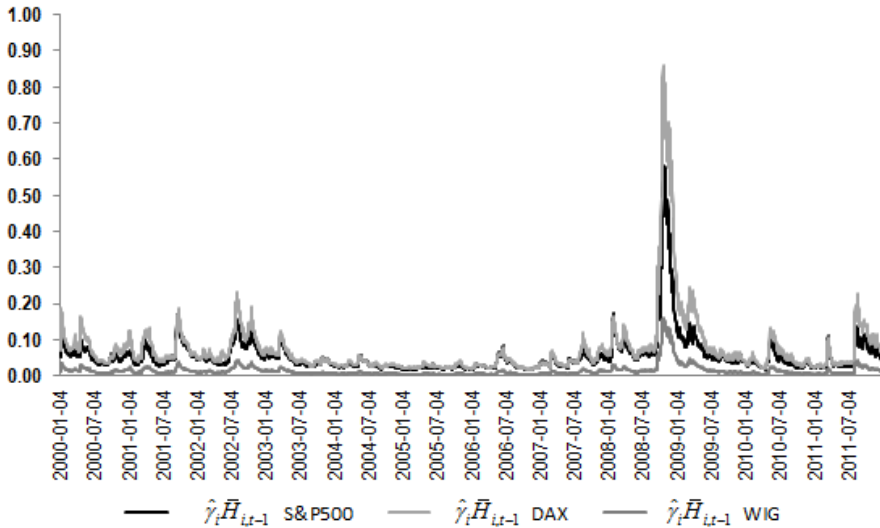
**Figure 4.** The variable  $\bar{H}_{i,t-1}$  for indices DAX, S&P500 and WIG



Source: own elaboration.



**Figure 5.** The values of  $\hat{\gamma}_i \bar{H}_{i,t-1}$  for indices DAX, S&P500 and WIG

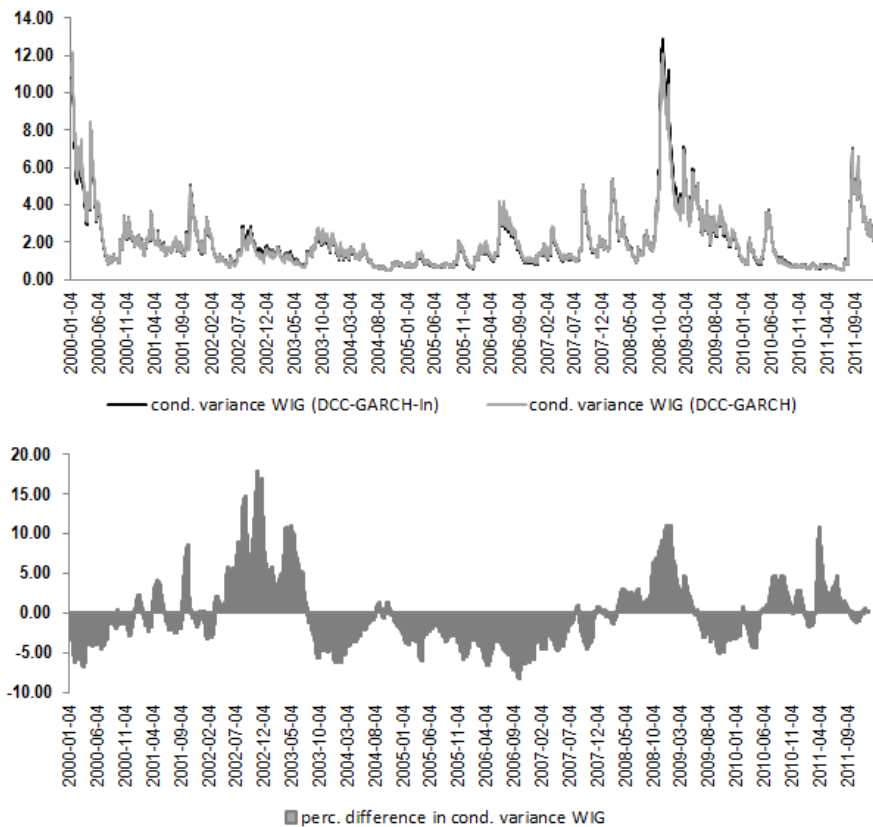


Source: own elaboration.

Next, Figure 5 shows the values of the factor  $\bar{H}_{i,t-1}$  multiplied by the parameter estimate of  $\gamma_i$  for the S&P500, WIG and DAX indices. For all of the three indices the parameter  $\gamma_i$  was found to be statistically significant. The greatest value of the parameter estimate  $\gamma_i$  was obtained for the DAX index, and the lowest for the WIG index. The parameter estimate  $\gamma_i$  for the DAX index is one and half times greater than the estimate for the S&P500 index, and five times higher than the estimate for the WIG index. Due to the expression  $\hat{\gamma}_i \bar{H}_{i,t-1}$ , the DCC-GARCH-In model specification allows for the consideration of the interdependence between selected markets in volatility. The entire interdependence is partially composed of the interdependence of fundamental linkages and partially of contagion stemming from crisis periods. It is not possible to clearly separate and determine the structures of these two parts. Based on the parameter estimates of the DCC-GARCH-In model, it can be asserted, however, that there occurs interdependence within the volatility of the studied markets. The entire study period was highly heterogeneous and contained as many as two crisis periods. In subsequent crisis periods, *i.e.*,

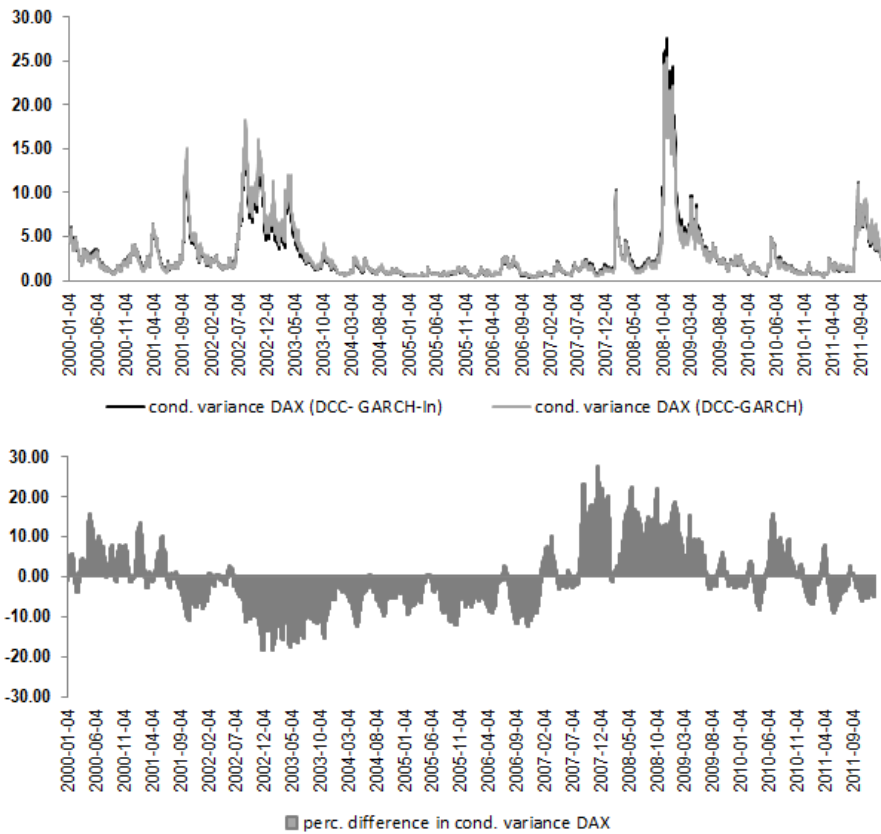
during the 2000-2002 dot-com bubble and the 2007-2009 global financial crisis, the interdependence of market volatility and its structure could change. It seems reasonable to attempt in further research to estimate parameters for the two crisis periods as well as for the tranquillity period. A considerable difference in the parameter estimates  $\gamma_i$  for the tranquillity period and for the crisis period would indicate a considerable change in the interdependence between markets in volatility.

**Figure 6.** The conditional variance obtained from the DCC-GARCH and DCC-GARCH-In models for the WIG index



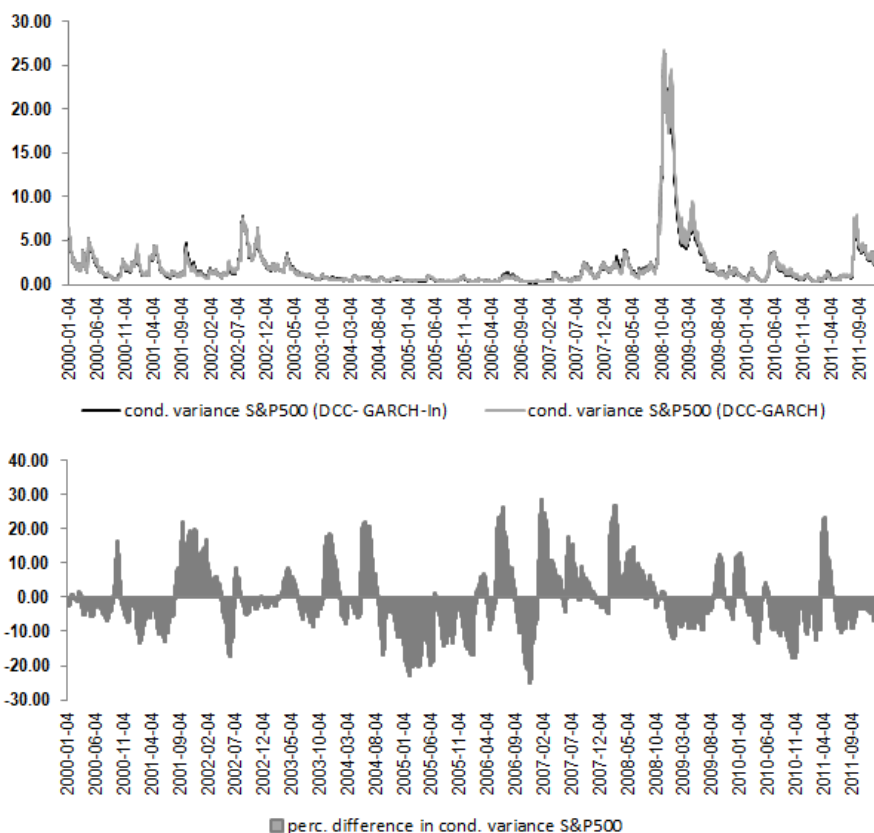
Source: own elaboration.

**Figure 7.** The conditional variance obtained from the DCC-GARCH and DCC-GARCH- In models for the DAX index



Source: own elaboration.

**Figure 8.** The conditional variance obtained from the DCC-GARCH and DCC-GARCH-In models for the S&P500 index



Source: own elaboration.

Figures 6 to 8 present the values of conditional variance obtained for the S&P500, WIG and DAX indices from the DCC-GARCH and DCC-GARCH-In models as well as the percentage difference between them. The percentage difference was determined as the difference between the level of conditional variance resulting from the DCC-GARCH-In model and the level of variance from the DCC-GARCH model. A shared feature of all of the three graphs is that the values of the conditional variance from the DCC-GARCH-In model are larger in some periods, and are smaller in the remaining ones, if compared to the variance from the DCC-GARCH model. This means that taking into account the interdependence between the

markets due to additional information (the factor  $\bar{H}_{i,t-1}$ ) results in either an increase or a decrease in the value of the conditional variance in relation to the value resulting from the DCC-GARCH model. It should be emphasized that the differences in the conditional variance for both models reach 40%, which indicates that the factor  $\bar{H}_{i,t-1}$  within the process of explaining the conditional variance is significant. As a result of the consideration of the information about the interdependence of the markets, we received different levels of volatility, compared with the levels of volatility obtained exclusively based on information on the domestic market (DCC-GARCH).

## Conclusions

The article is a study of interdependence between markets that in a crisis situation on the capital markets can lead to an economic crisis in one or more economies. The size and range of interdependence are constantly increasing along with increasing globalization. The identification and description of the mechanism of interdependence between markets appear to be an important research problem and its solution would allow determining a strategy to be implemented during a crisis period as well as developing system tools with a view to improving the functioning of financial systems. The research objective of the article was to identify interdependence for volatility. Within the realisation of the objective we proposed our own specification of the DCC-GARCH-In model, whose design takes into account the volatility of all other markets. This model is an extension of the specification of the DCC-GARCH model, which allows considering the volatility of all other markets.

The article analyses the interdependence of selected capital markets in conditional variance. The empirical part used time series of the WIG, DAX, WIG, BUX, DAX, S&P 500, FTSE 100, CAC 40, KOSPI, BOVESPA, SSE, HSI, RTS, SSMI, NIKKEI 225, ATG, MIBTEL stock indices in the time period from 3 January 2000 to 3 January 2012, and the corresponding capitalization of stock markets. The conducted study allowed us to present the application possibilities of the DCC-GARCH-In model in multidimensional modelling of the expected value and conditional variance. In addition, the use of the DCC-GARCH-In model allowed the determination of the presence of volatility interdependence on the markets surveyed. The results indicate the need to continue our research on the identification of changes in interdependence under the following crisis

periods: the 2000-2002 dot-com bubble and the 2007-2009 global financial crisis.

## References

- Baur D., (2003), Testing for contagion – mean and volatility contagion, *Journal of Multinational Financial Management*, 13, 405-422.
- Bekaert G., Wu G., (2000), Asymmetric Volatility and Risk in Equity Markets, *Review of Financial Studies*, 13, 1-42.
- Billio M., Caporin M., (2010), Market linkages, variance spillover and correlation stability: empirical evidences of financial contagion, *Computational Statistics & Data Analysis*, 54(11), 2443–2458.
- Burzała M., (2014), *Wybrane Metody Badania Efektów Zarażania na Rynkach Kapitałowych*, Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu, Poznań.
- Burzała M., (2015), Zmiana wartości oczekiwanej i wariancji stóp zwrotu z indeksów giełdowych w czasie kryzysu. Zarażanie czy wzajemne zależności rynków kapitałowych?, *Zeszyty Naukowe Uniwersytetu Szczecińskiego Finanse, Rynki Finansowe, Ubezpieczenia*, 75, 83-96.
- Corsetti G., Pericoli M., Sbracia M., (2005), Some Contagion, Some Interdependence: More Pitfalls in Testing for Contagion, *Journal of International Money and Finance*, 24, 1177-1199
- Doman M., Doman R., (2009), *Modelowanie zmienności i ryzyka : metody ekonometrii finansowej*, Seria Akademicka - Wolters Kluwer Polska, Kraków.
- Doman M., Doman R., (2014), *Dynamika zależności na globalnym rynku finansowym*, Difin, Warszawa.
- Eckner A. (2006), *A Multivariate Garch Model with Volatility Spill-Over and Time-Varying Correlations*, Available at SSRN: <http://ssrn.com/abstract=963937> or <http://dx.doi.org/10.2139/ssrn.963937>.
- Engle R.F., (2002), Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, *Journal of Business & Economic Statistics*, 20, 339-350.
- Engle R.F., (2009), *Anticipating Correlations A New Paradigm for Risk Management*, Princeton University Press, Princeton and Oxford.
- Fiszeder P., (2009), *Modele klasy GARCH w empirycznych badaniach finansowych*, Wydawnictwo Naukowe UMK, Toruń.
- Forbes K., Rigobon R., (2002), No Contagion, Only Interdependence: Measuring Stock Market Comovements, *The Journal of Finance*, 57/5, 2223-2261.

Osińska M., (2006), *Ekonometria finansowa*, Polskie Wydawnictwo Ekonomiczne, Warszawa.

Pericoli M., Sbracia M., (2003), A Primer on Financial Contagion, *Journal of Economic Surveys*, 17/4, 571-608.

Pritsker M., (2001), The Channel for Financial Contagion, in: *International Financial Contagion: How It Spreads and How It Can Stopped*, S. Claessens, K.J. Forbes (eds.), Kluwer Academic Publishers, Boston/Dordrecht/London.