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Using genetic algorithm in dynamic model of speculative attack

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**Using genetic algorithm in dynamic model of
speculative attack**

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Wojciech Grabowski**

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Using genetic algorithm in dynamic model of speculative attack

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Abstract: Evolution of speculative attack models show certain progress in developing idea of the role of expectations in the crisis mechanism. Obstfeld (1996) defined expectations as fully exogenous. Morris and Shin (1998) endogenised the expectations with respect to noise leaving information significance away. Dynamic approach proposed by Angeletos, Hellwig and Pavan (2006) operates under more sophisticated assumption about learning process that tries to reflect time-variant and complex nature of information in the currency market much better. But this model ignores many important details like a Central Bank cost function. Genetic algorithm allows to avoid problems connected with incorporating information and expectations into agent decision making process to an extent. There are some similarities between the evolution in Nature and currency market performance. In our paper an assumption about rational agent behaviour in the efficient market is criticised and we present our version of the dynamic model of a speculative attack, in which we use a genetic algorithm to define decision-making process of the currency market agents. The results of our simulation seem to be in line with the theory and intuition. An advantage of our model is that it reflects reality in quite complex way, i.e. level of noise changes in time (decreasing), there are different states of fundamentals (with “more sensitive” upper part of the scale), number of inflowing agents can be low or high (due to different globalization phases, different capital flow phases, different uncertainty levels).

Introduction

Speculative attack models try to catch a complicated relation between information and expectations. Informed agents (provided by either private signals or common knowledge or both of these) formulate their expectations and due to these expectations make strategies either to attack or hold. This mechanism from information through expectations to attack has always been extremely difficult to cover in any theoretical framework. Evolution of speculative attack models show certain progress in developing idea of the role of expectations in the crisis mechanism. Obstfeld (1996) defined expectations as fully exogenous. Morris and Shin (1998) endogenised the expectations with respect to noise leaving information significance away. They proposed static model, including information but excluding any possibility of so-called common knowledge in the currency market. Dynamic approach proposed by Angeletos, Hellwig and Pavan (2006) operates under more sophisticated assumption about learning process that tries to reflect time-variant and complex nature of information in the currency market much better. But this model ignores many important details like a Central Bank cost function.

If we look at the speculative attack as at the optimisation problem, why not to use genetic algorithm to present the agent behaviour in the market? Genetic algorithm allows to avoid problems connected with incorporating information and expectations into agent decision making process to an extent. Evolution means that the species that are prepared to the environment worse, have smaller chances to survive, and as the time passes by, improved species appear. There are some similarities between the evolution in Nature and currency market performance. In the currency market the speculators make wrong decisions and are eliminated from the market by these speculators who generate high pay-offs. Therefore, we can assume that learning in the currency market may in fact be characterised like species adaptation process to the environment. That is why we believe that introducing genetic algorithm may be a right step towards finding some optimal solutions for the speculative attack model.

This paper is organized as follows. In the second section assumption about rational agent behaviour in the efficient market is criticised and we explain why we use genetic algorithm. In the third section dynamic model of a speculative attack is presented. In the fourth section optimal strategies for the Central Bank and for speculators are defined. In the fifth section genetic algorithm that reflects decision making process is described. In the sixth section our results are presented. We also show evolution of a learning process. The last section contains conclusions.

1. Critical approach towards rational agent behaviour in the efficient market

Foreign exchange market can not be characterised as a good example of strong efficiency paradigm by Fama (1970). Information is not equally available to all agents. The market is rather decentralised and trade transparency is low (see Lyons (2001)). It is well known, that this distinguishes the foreign exchange market from other financial markets. Moreover, results of the surveys (Sarno and Taylor (2002)), especially these based on the microstructural logic suggest that the static expectation hypothesis should be rejected. The results confirm heterogeneity of expectations.

Using behavioural finance perspective we can say that although an agent may store and process only a tiny part of the relevant information, the agent is not brainless. If we agree to abandon traditional rational expectation model that assumes perfect knowledge of the market participants, then it is possible to redefine an individual forecasting strategy, which is neither fully rational (in a sense of *homo oeconomicus*) nor fully irrational. It is in line with heuristics rules taken from the psychology. So-called *trial and error* strategy represents bounded rationality framework and means *ex post* checking how profitable certain rule is while comparing it with some others. If the rule does not prove to be the profitable one, then the agent switches to the better one. If the agent's strategy turns out to be successful, then she/he sticks to it. *Trial and error* strategy is rooted in Nature, it has got strongly evolutionary character.

In the behavioural model of exchange rate by De Grauwe and Grimaldi (2006) the mechanism of making forecasts by the agents is well described. The authors show that in the foreign exchange market the agents follow *trial and error* strategy, no matter if they are so-called "fundamentalists" or "chartists" (no matter if they analyse macroeconomic fundamentals or they rely on technical analysis to forecast the exchange rate). *Ex post* assessment of the forecasting strategies may transform "fundamentalist" into "chartist" or vice versa. It is worth mentioning that according to Tversky, Kahneman (1991) the agents need some time to adopt a new strategy, they are slightly conservative, therefore "status quo bias" must be considered in their decision making process even though it is true that the agents react to the relative profitability of the rules. *Trial and error* strategy is thus a dynamic process that requires further assumptions concerning "memory" of the agent. De Grauwe and Grimaldi (2006) use the short-run memory hypothesis that implies that the agent refer just to last period's squared forecast error to make their decision.

Frydman and Goldberg (2007) formulate some critical remarks towards rational expectation and efficient market hypothesis too. They are quite close to the behavioural economists' point of view. The authors pay attention to the fact that the individuals in the foreign exchange market must cope with imperfect knowledge. They stress importance of the revision of the agent forecasting strategies over time at the same time mentioning that even "social context" should be considered as important determinant of the strategy formulation process. They also describe the agents as conservative, defining this as follows: "*an individual's forecast of the future exchange rate is not too different from the forecast she would have had if she did not revise her forecasting strategy*" (Frydman and Goldberg, 2007, p. 184).

It seems that formulating a model that would reflect true agent behavior in the foreign exchange market in a proper way is more complicated task than the supporters of traditional efficient market hypothesis would like to present. Such a model should have an evolutionary, dynamic character, show making decision processes based on *trial and error* strategy which are treated as optimisation, however, under imperfect knowledge assumption. Genetic algorithm appears to be quite suitable to imitate agents' behavior in the foreign exchange market in the real world if we want to meet majority of these criteria.

Methodology of the research

2. Dynamic Model of Speculative Attack

Both models by Obstfeld (1986) and by Morris and Shin (1998) have some shortcomings and in this paper these models are extended (especially Morris and Shin's one) and made more applicable. Neither "multiple equilibria" approach nor "uniqueness" take into account time as important factor, they are both static. Therefore, in our paper dynamics of the model is introduced. We follow some elements of the model proposed by Angeletos, Hellwig and Pavan (2006). Their model offers rather general framework how to apply dynamic global games into a regime change mechanism. It can be applied for modeling speculation against a currency peg (which is of our priority interest), at the same time the model can be also used for some other purposes like explaining run against a bank or some other (not strictly economic) processes, for example a revolution against a dictator. There are two important features of the model. Firstly, it allows the agents to learn, therefore, the multiplicity is connected with information dynamics. And secondly, the fundamentals matter for the regime outcome prediction, although not for timing and number of attacks. However, the model presents only one side perspective, i.e. the speculator

one, and the payoff function of Central Bank is not analysed. Moreover, we are not quite sure if it is fully satisfying to accept:

“summarizing the private information by the agent about θ at any given period in a one dimensional sufficient statistic, and capturing the dynamics of the cross-sectional distribution of the static in a parsimonious way (Angeletos, Hellwig and Pavan ,2006, p. 1-2)”,and then to apply this algorithm to examine the effects of learning on equilibria in the model. Instead, we offer well defined genetic algorithm to simulate learning process, and as we think that the Central Bank can also learn, in fact the genetic algorithm is used to show how decisions of two categories of agents are changing as far as their knowledge on the proportion of attacking speculators is concerned.

In our model time is discrete and indexed by $t \in \{1, 2, \dots\}$. Agents are indexed by $n \in \{1, \dots, N_t, N_t + 1\}$, where agents $1, \dots, N_t$ are speculators and agent $N_t + 1$ is the Central Bank. Subscript t is used, since we assume that number of speculators considering attack evaluates in time. Therefore there is a sequence $\{N_t\}_{t \in \{1, 2, \dots\}}$, which is not observed. The Central Bank receives ex post information about the number of speculators attacking denoted by α . We assume that each speculator considering attack, attacks with the same probability, therefore we have relationship:

$$\alpha_t = N_t \kappa_t, \quad t = 1, 2, \dots \quad (1)$$

where κ_t denotes probability that a chosen speculators attacks. α is observed ex post, however N and κ are unobserved. Of course α and κ evaluate in time too, therefore we have sequences $\{\alpha_t\}_{t \in \{1, 2, \dots\}}$ and $\{\kappa_t\}_{t \in \{1, 2, \dots\}} \cdot \{er_t\}_{t \in \{1, 2, \dots\}}$ is a sequence of observed exchange rates and $\{\theta_t\}_{t \in \{1, 2, \dots\}}$ is a sequence of the true values of fundamentals. Similarly as in the model of Morris and Shin (1998) we assume that there are only 2 possible states of exchange rate. Exchange rate is pegged at a level e^* or depends on the fundamentals and is equal to $f(\theta)$. An action set for the Central Bank is binary, which means that the Central Bank can defend the exchange-rate peg or abandon it. Since speculators can attack the exchange-rate peg or refrain from doing so, their action set is binary too. We assume that $er_1 = e^*$. The game is continued until a state $er_t = f(\theta_t)$ is reached or if after a finite number of periods dominant strategy is not to attack.

According to the model of Angeletos, Hellwig, Pagan (2006) each player receives a private signal $x_t^n = \theta_t + \varepsilon_t^n$, where for $n=1, \dots, N_t$

$\varepsilon_t^n \sim N\left(0, \frac{1}{\beta_t}\right)$ is noise, independent identically distributed across agents.

In the case of the Central Bank we assume that a noise $\varepsilon_t^{N_t+1} \sim N\left(0, \frac{1}{\tilde{\beta}_t}\right)$

is independent of noises $\varepsilon_t^1, \dots, \varepsilon_t^{N_t}$ and we assume that for all t the inequality $\tilde{\beta}_t > \beta_t$ is valid because knowledge of the level of fundamentals is more precise in the case of the Central Bank than in the case of speculators. It is assumed in our paper that uncertainty concerning the level of fundamentals decreases and therefore $\forall_{s>t} \beta_s > \beta_t$. Morris and Shin

(1998) and Angeletos, Hellwig, Pagan (2006) assumed that the level of fundamentals was random too, however in our model we consider different nonrandom trajectories of $\{\theta_t\}_{t \in \{1, 2, \dots\}}$. In our paper $c(\cdot, \cdot, \cdot, \cdot, \cdot)$ denotes the Central Bank cost function. This cost depends similarly as in the paper of Morris and Shin (1998) the state of fundamentals θ . In our paper this function depends on the total number of speculators considering attack N and probability that a chosen speculator attacks κ . We assume that the total number of speculators considering attack evaluates according to the formula:

$$N_t = N_1 + (t-1)\tau. \quad t = 1, 2, \dots$$

(2)

Our extension of the paper of Morris and Shin (1998) is to make cost of intervention dependent of the level of reserves r too. We assume that

$c(N_1, \tau, \kappa, \theta, r)$ is a continuous function and $\frac{\partial c(N_1, \tau, \kappa, \theta, r)}{\partial N_1} > 0$,

$\frac{\partial c(N_1, \tau, \kappa, \theta, r)}{\partial \tau} > 0$, $\frac{\partial c(N_1, \tau, \kappa, \theta, r)}{\partial \kappa} > 0$, $\frac{\partial c(N_1, \tau, \kappa, \theta, r)}{\partial \theta} < 0$ and

$\frac{\partial c(N_1, \tau, \kappa, \theta, r)}{\partial r} < 0$. Total number of speculators in the beginning

period is not known but it has to be predicted by each agent. Therefore for each n , N_1^n denotes predicted by the n -th agent total number of speculators considering attack on the foreign exchange market. Similarly τ

and κ is not known and has to be predicted by all agents. τ is constant but predictions of this quantity change in time, therefore for each agent we have sequence $\{\tau_t^n\}_{t \in \{1,2,\dots\}}$. Analogously we have a sequence $\{\kappa_t^n\}_{t \in \{1,2,\dots\}}$.

Since our model is dynamic, we define time-dependent cost function $c_t(N_1, \tau, \kappa_{t-1}, \theta_{t-1}, r_{t-1})$, $t = 2, 3, \dots$. Lagged variables are included, because action is done in period $t-1$ and results of choice are observed in period t . Comparing to the paper of Morris and Shin (1998), one of extensions is based on the fact that the cost function is specified. We choose linear specification:

$$c_t = \gamma_1(N_1 + \tau(t-2))\kappa_{t-1} + \gamma_2\theta_{t-1} + \gamma_3r_{t-1}, \quad t = 2, 3, \dots, \quad (3)$$

where $\gamma_1 > 0, \gamma_2 < 0, \gamma_3 < 0$ are nonrandom and known constants.

Similarly as in the model of Morris and Shin (1998), if the Central Bank defends the peg, it receives value v but faces a cost c .

3. Optimal strategies for the Central Bank and for speculators

We suppose that all agents do action in the period $t-1$ and the result of this action is observed in period t . Let $\{ST_t^n\}_{t \in \{1,2,\dots\}}$ denotes a sequence of strategies chosen by the n -th agent. $pay_t^n(ST_{t-1}^n)$ denotes the payoff in period t for an agent number n , if this agent chooses action ST_{t-1}^n in period $t-1$. As we mentioned above $ST_t^n \in \{0,1\}$ for all n . In the case of speculators we suppose that ST_t^n is 1 if speculator decides to attack currency and is 0 otherwise. If $ST_t^{N_i+1} = 1$, the Central Bank decides to defend the exchange-rate peg and if $ST_t^{N_i+1} = 0$, then the Central Bank abandons the exchange-rate peg. The following table shows payoffs for speculators in period t in the period $t-1$ exchange rate is pegged at the level e^* :

Table 1. Payoffs for speculators in period t ($er_{t-1} = e^*$)

$ST_{t-1}^{N_{t-1}+1} \backslash ST_{t-1}^n$	0	1
0	0	0
1	$(e^* - f(\theta_t)) - tr$	$-tr$

Source: Own calculations

Central Bank's payoff depends on the proportion of speculators attacking, state of fundamentals and the level of reserves. The payoff is defined in the following way:

$$pay_t^{N_t+1}(ST_{t-1}^{N_{t-1}+1}) = \begin{cases} 0 & \text{if } ST_{t-1}^{N_{t-1}+1} = 0, \\ v - \gamma_1(N_1 + \tau(t-2))\kappa_{t-1} - \gamma_2\theta_{t-1} - \gamma_3r_{t-1} & \text{if } ST_{t-1}^{N_{t-1}+1} = 1. \end{cases} \quad (4)$$

Since neither the true proportion of speculators attacking nor state of fundamentals are known in the period of attack, expected payoff is calculated. This expected payoff is given by formula:

$$E[pay_t^{N_t+1}(ST_{t-1}^{N_{t-1}+1})] = \begin{cases} 0 & \text{if } ST_{t-1}^{N_{t-1}+1} = 0, \\ v - \gamma_1(N_1^{N_{t-1}+1} + \tau_{t-1}^{N_{t-1}+1}(t-2))\kappa_{t-1}^{N_{t-1}+1} - \gamma_2x_{t-1}^{N_{t-1}+1} - \gamma_3r_{t-1} & \text{if } ST_{t-1}^{N_{t-1}+1} = 1. \end{cases} \quad (5)$$

Firstly we consider the border cases. We define a binary variable bad_t which is 1 if the state of fundamentals and reserves is extremely bad and even in the case of “no attack” the exchange-rate peg is abandoned and we define a binary variable $good_t$ which is 1 if the state of fundamentals and reserves is extremely good. “Extremely good state of reserves and fundamentals” means that even in the case of all speculators attacking in period t , then $E[pay_{t+1}^{N_{t+1}+1}(1)] > 0$. Value of variable bad_t is defined below:

$$bad_t = 1 \left\{ (x_t^{N_t+1}, r_t) : v - \gamma_2x_t^{N_t+1} - \gamma_3r_t < 0 \right\}. \quad (6)$$

Analogously variable $good_t$ is defined by the following formula:

$$good_t = 1 \left\{ (x_t^{N_t+1}, r_t) : v - \gamma_1 - \gamma_2x_t^{N_t+1} - \gamma_3r_t > 0 \right\}. \quad (7)$$

If a variable bad_t is 1, then a dominant strategy for the Central Bank is to abandon the exchange-rate peg. Otherwise if $good_t$ is 1, then a dominant strategy is to defend the exchange rate peg. There is no reason to attack for the speculators, if payoff from attacking (even if the attack is successful) is smaller than a transaction cost, which means that:

$$e^* - f(\theta_t) < tr.$$

(8)

Then a dominant strategy for speculators is to refrain from attacking.

If conditions (6) and (7) are not satisfied, then there exists such $\tilde{\kappa}_{t-1}^{N_{t-1}+1}$ that solves the following equation against $\kappa_{t-1}^{N_{t-1}+1}$:

$$v = \gamma_1 \left(N_1^{N_{t-1}+1} + \tau_{t-1}^{N_{t-1}+1} (t-2) \right) \kappa_{t-1}^{N_{t-1}+1} + \gamma_2 x_{t-1}^{N_{t-1}+1} + \gamma_3 r_{t-1}.$$

(9)

Then an optimal strategy for the Central Bank is defined as follows:

$$ST_{t-1}^{N_{t-1}+1} = 1 \left\{ \kappa_{t-1}^{N_{t-1}+1} < \frac{v - \gamma_2 x_{t-1}^{N_{t-1}+1} - \gamma_3 r_{t-1}}{\gamma_1 \left(N_1^{N_{t-1}+1} + \tau_{t-1}^{N_{t-1}+1} (t-2) \right)} \right\}.$$

(10)

Similarly as in the case of the Central Bank, critical value $\tilde{\kappa}_{t-1}^n$ is defined for each speculator. This value is calculated analogously changing an index $N_{t-1}+1$ by n for $n=1, \dots, N_{t-1}$. Speculators do not have any information on a state of fundamentals observed by the Central Bank and predicted by the Central Bank values of parameters N_1 and predicted by the Central Bank probability of attacking by a chosen speculator κ . Therefore they have to rely on their own observations and predictions to formulate the payoff function of the Central Bank. There is a reason to attack for the speculators if the predicted probability of attacking exceeds the critical value. Therefore if inequality (8) is not satisfied, then an optimal strategy for speculators is given by the following formula:

$$ST_{t-1}^n = 1 \left\{ \kappa_{t-1}^n > \frac{v - \gamma_2 x_{t-1}^n - \gamma_3 r_{t-1}}{\gamma_1 \left(N_1^n + \tau_{t-1}^n (t-2) \right)} \right\}.$$

(11)

As we have already mentioned, parameters of the cost function of the Central Bank are known only to the CB but unknown to the speculators.

4. Genetic algorithm in the process of learning

In the first period for each n predicted total number of speculator considering attack is equal to N . Similarly, predictions of τ and κ are purely random for each agent. In the second period Central Bank knows value of $\alpha_1 = \kappa_1 N_1$. The Central Bank assumes that probability of attacking by individual speculator in a given period is the same as this probability in previous period and therefore predicts values of κ and τ in the next periods using the following recursive formula:

$$\kappa_{t+1}^{N_{t+1}+1} = \frac{\kappa_t^{N_t+1} N_t^{N_t+1}}{\alpha_t}.$$

(12)

Parameter τ is predicted according to the following formula:

$$\tau_{t+1}^{N_{t+1}+1} = \frac{\left(\frac{\alpha_t}{\kappa_t^{N_t+1}} - \frac{\alpha_1}{\kappa_1^{N_1+1}} \right)}{t-1}.$$

(13)

Since forex market is not fully transparent, we assume that speculators do not have any information concerning number of speculators attacking in the previous period. But they observe their own payoffs and if payoff for the first speculator is higher than payoff for the second one, then this first speculator has higher chance to “survive” than the second one. It is obvious that speculators with higher payoffs are satisfied with their decisions and they do not have any incentive to change the tactic. Speculators with negative payoffs decide to change their tactic. They learn tactic from the speculators with positive payoffs. New speculators enter the FX market. Dynamics on the FX market imitates nature, where only correctly fitted spiches survive. Spiches that are not able to adapt to the environment are replaced by the spiches that are better fitted. Crossover can be interpreted as a knowledge exchange. Considering all this FX market can be modeled as evolving system of the autonomous interacting agents and hence the genetic algorithm can be applied here. Analyzing formula for optimal strategy of n -th speculator in period t , we can notice that this strategy depends on parameters κ_{t-1}^n , γ_1 , γ_2 , γ_3 , τ_{t-1}^n , N_1^n . Speculators do not know these parameters and their knowledge concerning them is changing from first period to the second one. We assume that these parameters are different for different periods and different speculators. We denote γ_{it}^n as predicted value of parameter γ_i , $i = 1, 2, 3$ in period t by n -th speculator.

In order to use genetic algorithm we have to define range of possible values of parameters. It is obvious that:

$$\forall_{t \in \{2,3,\dots\}} \forall_n \kappa_{t-1}^n \in (0,1].$$

(14)

We have to choose minimum and maximum values of parameters $\gamma_1, \gamma_2, \gamma_3, \tau, N_1$, after choosing these values, we have:

$$\begin{aligned} \forall_{t,n} \gamma_{1t}^n \in \langle \gamma_1^{\min}, \gamma_1^{\max} \rangle, \forall_{t,n} \gamma_{2t}^n \in \langle \gamma_2^{\min}, \gamma_2^{\max} \rangle, \forall_{t,n} \gamma_{3t}^n \in \langle \gamma_3^{\min}, \gamma_3^{\max} \rangle, \\ \forall_{t,n} \tau_t^n \in \langle \tau^{\min}, \tau^{\max} \rangle, \forall_{t,n} N_1^n \in \langle N_1^{\min}, N_1^{\max} \rangle. \end{aligned}$$

If we choose precision ε , then number of gens in one chromosome is equal to:

$$\begin{aligned} GENS = \left\lceil \log_2 \frac{1}{\varepsilon} \right\rceil + \sum_{i=1}^3 \left\lceil \log_2 \frac{(\gamma_i^{\max} - \gamma_i^{\min})}{\varepsilon} \right\rceil + \log_2 \frac{(\tau^{\max} - \tau^{\min})}{\varepsilon} + \log_2 \frac{(N_1^{\max} - N_1^{\min})}{1} + \\ + 6 - 1 \left\{ \log_2 \frac{1}{\varepsilon} \in Z \right\} - \sum_{i=1}^3 1 \left\{ \log_2 \frac{(\gamma_i^{\max} - \gamma_i^{\min})}{\varepsilon} \in Z \right\} - 1 \left\{ \log_2 \frac{(\tau^{\max} - \tau^{\min})}{1} \in Z \right\} + \\ (15) \\ - 1 \left\{ \log_2 \frac{(N_1^{\max} - N_1^{\min})}{1} \in Z \right\}, \end{aligned}$$

where $[x]$ denotes integer value of x . First $\left\lceil \log_2 \frac{1}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{1}{\varepsilon} \in Z \right\}$

gens represent strategy concerning the expected probability of attacking by a chosen speculator, next

$$\left\lceil \log_2 \frac{(\gamma_1^{\max} - \gamma_1^{\min})}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{(\gamma_1^{\max} - \gamma_1^{\min})}{\varepsilon} \in Z \right\} \quad \text{gens represent}$$

strategy concerning the value of parameter γ_1 etc. In order to introduce crossover and mutation we define quantities $pc(\kappa), pc(\gamma_1), pc(\gamma_2), pc(\gamma_3), pc(\tau), pc(N_1)$, which denote probabilities of crossover for gens of two chromosomes for a given parameter. $pm(\kappa), pm(\gamma_1), pm(\gamma_2), pm(\gamma_3), pm(\tau), pm(N_1)$ denote probability of mutation of respective gens and $pr(\kappa), pr(\gamma_1), pr(\gamma_2), pr(\gamma_3), pr(\tau), pr(N_1)$ denote proportion of

gens in appropriate part of chromosome, which are crossed over. Before we calculate fitness function, for simplicity we define the following variables:

$$\begin{aligned}
 SS_t^n &= 1 \left\{ e^* - f(\theta_t + \varepsilon_t^n) - tr > 0 \right\}, \quad n=1, \dots, N_t, \quad t=1, 2, \dots, \\
 SB_t &= 1 \left\{ v - \gamma_1 \kappa_t (N_1^{N_t+1} + \tau_t^{N_t+1} (t-1)) - \gamma_2 (\theta_t + \varepsilon_t^{N_t+1}) - \gamma_3 r_t > 0 \right\}, \\
 t &= 1, 2, \dots, \quad (16) \\
 SBS_t^n &= 1 \left\{ v - \gamma_1^n \kappa_t^n (N_1^n + \tau_t^n (t-1)) - \gamma_2^n (\theta_t + \varepsilon_t^n) - \gamma_3^n r_t > 0 \right\}, \\
 n &= 1, \dots, N_t.
 \end{aligned}$$

If $SS_t^n = 1$, then attack is considered by the n -th speculator in period t . Otherwise transaction costs exceed payoff and attack is not taken into account. Value of variable SB_t informs us about the strategy of the Central Bank. Central Bank abandons the exchange-rate peg if $SB_t = 0$ and defends it otherwise. Variable SBS_t^n can be interpreted as expected (by the n -th speculator) strategy of the Central Bank. SBS_t^n is 1 if the n -th speculator predicts that the Central Bank will defend the exchange-rate peg and 0 otherwise. Since payoff may be negative, we do monotonic transformation in order to calculate the value of the fitness function:

$$F_t^n = \exp(\text{pay}_{t+1}^n). \quad (17)$$

The fitness function in our model is given by the following formula:

$$F_t^n = \begin{cases} \exp(-tr) & \text{if } SS_t^n SB_t = 1 \wedge SBS_t^n = 0, \\ \exp(0) = 1 & \text{if } SS_t^n SBS_t^n = 0, \\ \exp(e^* - f(\theta_t) - tr) & \text{if } SS_t^n SBS_t^n = 1 \wedge SB_t = 0. \end{cases} \quad (18)$$

We use roulette-wheel selection method in order to choose appropriate chromosomes in the next period.

In the first simulation experiment probability of abandoning the exchange-rate peg is calculated. If after 20 periods the Central Bank does not change its strategy and still defends exchange-rate peg or dominant

strategy for speculators is not to attack, then we assume that the exchange-rate peg is abandoned. Experiment is done for different states of fundamentals (weak fundamentals, medium fundamentals, strong fundamentals), different levels of reserves (low reserves, medium reserves, high reserves) and different values of parameter τ . We assume that

$$N_1 = 10, \quad N_1^{\min} = 7, \quad N_1^{\max} = 13. \quad \text{If } \tau = 2, \text{ then } \tau^{\min} = 1 \text{ and } \tau^{\max} = 3.$$

Else if $\tau = 5$, then $\tau^{\min} = 3$ and $\tau^{\max} = 7$. In our experiment predicted numbers of speculators in the first period are randomly selected in the following way:

$$P\left(N_1^n = k\right) = \begin{cases} \frac{1}{6} & \text{for } k = 7, \dots, 13, \\ 0 & \text{for } k \notin \{7, \dots, 13\} \end{cases} \quad \text{for } n = 1, 2, \dots, N_1$$

(19a)

and

$$P\left(N_1^{N_1+1} = k\right) = \begin{cases} \frac{1}{3} & \text{for } k = 9, \dots, 11, \\ 0 & \text{for } k \notin \{9, \dots, 11\} \end{cases}.$$

(19b)

κ_1^n is selected randomly from $U(0,1)$ for all agents. Similarly τ^n is selected randomly for all agents in the first period. We assume that if $\tau = 2$, then

$$P\left(\tau_1^n = k\right) = \frac{1}{3}, \quad \text{for } k = 1, 2, 3 \quad \text{and} \quad n = 1, \dots, N_1 + 1.$$

(20a)

If $\tau = 5$, then:

$$P\left(\tau_1^n = k\right) = \frac{1}{5}, \quad \text{for } k = 3, 4, 5, 6, 7 \quad \text{and} \quad n = 1, \dots, N_1$$

(20b)

and

$$P\left(\tau_1^{N+1} = k\right) = \frac{1}{3}, \quad \text{for } k = 4, 5, 6. \quad (20c)$$

In the case of the Central Bank, values of parameter τ in periods $t = 2, 3, \dots$ are obtained according to the formula (13) and values of parameter κ are obtained according to the formula (12). In the case of speculators, values of parameters τ and κ in periods $t = 2, 3, \dots$ result from the use of genetic algorithm. Speculators with higher payoffs have higher chances not to change their strategy. Speculators with negative payoffs have higher chances to use a different strategy in the next period. Constant parameters are as follows:

$$\begin{aligned} e^* = 2, v = 3, \quad \gamma_1 = 0,8, \quad \gamma_2 = -0,7, \quad \gamma_3 = -0,8, \quad \gamma_1^{\min} = 0, \quad \gamma_1^{\max} = 2, \\ \gamma_2^{\min} = -2, \quad \gamma_2^{\max} = 0, \quad \gamma_3^{\min} = -2, \quad \gamma_3^{\max} = 0, \quad \beta_t = 5t, \quad \tilde{\beta}_t = 10t, \\ f(\theta_t) = \theta_t, \quad tr = 0,35. \end{aligned}$$

In the case of weak fundamentals, θ_t is given by the following formula:

$$\theta_t = 1,2 + 0,01t,$$

in the case of medium fundamentals we have:

$$\theta_t = 1,4 + 0,01t,$$

in the case of strong fundamentals θ_t is given by formula:

$$\theta_t = 1,6 + 0,01t,$$

whereas in the case of very strong fundamentals we have:

$$\theta_t = 1,7 + 0,01t$$

In the case of low reserves, r_t is given by the following formula:

$$r_t = 1 + 0,01t,$$

in the case of medium reserves we have:

$$r_t = 3 + 0,01t$$

and in the case of high reserves we have:

$$r_t = 5 + 0,01t.$$

τ takes on values 2 and 5.

We conducted 10 000 replications and calculated probability of abandoning the exchange-rate peg. We received the following results:

Table 2a. Probability of Defending the Exchange-Rate Peg if number of speculators increases slowly

$\tau = 2$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	0,0770	0,1006	0,2807	
Medium	0,2826	0,3214	0,5763	0,8281
High	0,5479	0,5735	0,8291	0,9832

Source: Own calculations

Table 2b. Probability of Defending the Exchange-Rate Peg if number of speculators increases fast

$\tau = 5$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	0,0063	0,0120	0,0915	0,5573
Medium	0,0437	0,0679		0,8092
High	0,1367	0,1779		0,9827

Source: Own calculations

According to the results from the tables 2a and 2b, probability of defending the exchange-rate peg increases if state of fundamentals improves. The same relation concerns level of reserves. If the Central Bank keeps high level of reserves then the probability of abandoning the exchange-rate peg is higher than in the case of medium and weak reserves. Comparing values in the table 2a to the corresponding values in the table 2b, we can notice that probability of defending the exchange-rate peg is higher when number of speculators increases slower. It means that with the intensification of globalization and financial markets liberalization process, the Central Bank has lower chances to defend the exchange-rate peg.

In the second simulation experiment mean payoff for speculators is calculated. The same parameters are used as in the first experiment and the same variants are considered. Experiment is based on 10 000 replications. The results are as follows:

Table 3a. Mean payoff for speculators if number of speculators increases slowly

$\tau = 2$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	0,28	-0,96	-1,72	
Medium	-1,35	-2,25	-2,24	-0,56
High	-2,23	-2,89	-2,25	-0,46

Source: Own calculations

Table 3b. Mean payoff for speculators if number of speculators increases fast

$\tau = 5$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	1,56	0,02	-1,58	-0,84
Medium	0,56	-1,15		-0,78
High	-0,39	-2,31		-0,63

Source: Own calculations

According to the tables 3a and 3b, speculators reach positive payoffs for weak fundamentals and low reserves. For increasing level of reserves and for better fundamentals, mean payoff for speculators decreases, but this relation is non-linear. It can be seen that in the case of very strong fundamentals mean payoff for speculators is higher than in the case of strong or sometimes even medium fundamentals. This phenomenon results from the fact that if fundamentals are very strong then speculators know that probability of inefficient attack is higher. Therefore lower number of speculators decides to attack and even if they attack, they change tactic after first period of inefficient attack. Comparing values in the table 3a to

the corresponding values in the table 3b, we notice that if number of speculators increases faster, then mean payoff for speculators is higher.

In the third simulation experiment average time of collapse the exchange-rate peg is calculated. Though we assume that if the exchange-rate peg survives 20 periods, game is over, we put value 20 if the exchange-rate peg is defended. The same parameters are used as in the first experiment and the same variants are considered. Experiment is based on 10 000 replications. The results are as follows:

Table 4a. Mean time of duration of the exchange rate-peg, when number of speculators increases slowly

$\tau = 2$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	3,80	4,40	7,39	
Medium	8,54	9,19	12,90	16,73
High	13,59	13,98	17,40	19,68

Source: Own calculations

Table 4b. Mean time of duration of the exchange rate-peg, when number of speculators increases fast

$\tau = 5$				
Fundamentals Reserves	Weak	Medium	Strong	Very strong
Low	2,01	2,25	4,04	
Medium	3,80	4,32		16,41
High	6,60	4,59		19,67

Source: Own calculations

Comparing results in the tables 4a and 4b, we can notice positive relation between the state of fundamentals and duration of the exchange-rate peg and positive relation between the level of reserves and duration of

the exchange-rate peg. If the fundamentals are weak and the level of reserves is low, exchange-rate peg collapses very fast. If fundamentals are very strong and the level of reserves is high, then the mean duration of the exchange-rate peg is close to 20 periods, which means that in most situation exchange-rate peg is defended. This result agrees with the result from the first Monte Carlo experiment. Comparing values in the table 4a to the corresponding values in the table 4b, we can notice that average duration of the exchange-rate peg is higher in the case of slower pace of increasing of number of speculators.

Conclusions

1. The model seems to catch reality in more complex way: level of noise changes in time (decreasing), there are different states of fundamentals (with “more sensitive” upper part of the scale), number of inflowing agents can be low or high (due to different globalization phases, different capital flow phases, different uncertainty levels).
2. Dynamic nature of the model is also reflected in defining some kind of continuity of CB and agent behavior. Both sides must formulate their strategies in continuous way, and therefore simultaneously, which is a shift from a single-action approach (single nonreplicable attack, located in the short run and therefore sequential decision making process) to a longer perspective.
3. In fact, the results are in line with intuition, which may confirm that the usage of genetic algorithm was a right decision. Weaker level of fundamentals decreases probability of defending the exchange rate peg and increases pay-off for speculators. If dynamics of inflow of agents is higher, then probability of defending peg is lower, pay-off for speculators is higher and mean time of peg maintenance duration is lower. The higher the level of international reserves the higher probability of peg defending, the lower pay-off for speculators and the higher duration of peg maintenance.

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