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Article

# Indirect Evolution and Aggregate-Taking Behavior in a Football League: Utility Maximization, Profit Maximization, and Success

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**Abstract:** An evolutionary model of European football was applied to analyze a two-stage indirect evolution game in which teams choose their utility function in the first stage, and their optimal talent investments in the second stage. Given the second-stage optimal aggregate-taking strategy (ATS) of talent investment, it was shown that teams may choose a mix of profit or win maximization as their objective, where the former is of considerably higher relevance with linear weights for profits, and is more successful in the utility function. With linear weights for profit and win maximization, maximizing profits is the only evolutionarily stable strategy (ESS) of teams. The results change if quadratic weights for profits and wins are employed. With increasing talent productivity, win maximization dominates in the static and in the dynamic versions of the model. As a consequence, it is an open question whether the commercialization of football (and other sports) leagues will lead to more profit or win maximization.

**Keywords:** indirect evolution; football leagues; utility maximization; profit maximization; evolutionary stability

## 1. Introduction

The aim of this paper and its contribution to the literature is the analysis of whether utility (or win) maximization or profit maximization is an evolutionarily stable strategy (ESS) in European football (also known as soccer) if teams *choose* whether to maximize utility or profit. This question is of relevance for all kinds of business firms, as is documented by the long-standing debate on Scitovszky's [1] paper (see, for instance, References [2]; [3], pp. 152 ff.; [4]). In order to analyze the profit–win maximization conundrum in sports economics, the so-called “indirect evolution” concept was applied to this topic, in combination with an “optimal aggregate-taking strategy” (ATS). To the best of this author's knowledge, this is the first application of the combined concepts of “indirect evolution” and “optimal aggregate-taking strategy” to football leagues.

It seems to be a generally accepted view in sports economics that U.S. American sports leagues are characterized by profit maximization, whereas utility maximization, defined as maximization of the teams' winning percentages (or short: wins), is the main objective of teams in European football leagues [5–9]. Several years ago, the ‘Americanization’ of European football was forecasted [10]. Although professionalization and commercialization of European football advanced (see, for example, the Europe League and the Champions League, as well as the increase in the prices for broadcasting rights of football matches), it is not completely clear what the football clubs' objectives are. Nonetheless, Garcia-del-Barrio and Szymanski [11] found empirical evidence for win maximization in Spanish and English leagues. Alexander [12] presented evidence for profit-maximizing ticket prices in Major League Baseball (see also Reference [13], concerning play selection in American Football).

Recently, further aspects related to the consequences of profit or win maximization have been investigated. Késenne [14] included efficiency wages in a win maximization model with a flexible talent supply. Dietl, Franck, and Lang [15] shed light on overinvestment in talent in sports leagues and Dietl, Lang, and Werner [16] showed the social welfare effects of profit versus win maximization of sports clubs. Dietl, Grossmann, and Lang [17] analyzed the effects of revenue sharing in a model in which teams adopted a mix of profit and win maximization. Terrien et al. [18] demonstrated with data from the first French football league (Ligue 1) that teams may switch between profit and win maximization strategies over time, because of an uncertain industry environment that may provide respective opportunities at different points in time. In the paper, a profitability measure was employed to differentiate between profit maximization, win maximization with a hard budget constraint, and win maximization with a soft budget constraint. Osokin [19] used the approach of Reference [18] to investigate win and profit maximization strategies in the first Russian football league.

In the following formal analysis, the evolutionary game theoretical model of Grossmann [20], as extended by Langen and Prinz [21], is employed to investigate the above question in a two-stage game.

Furthermore, the concept of indirect evolution [22,23] was applied to analyze the choice of the clubs' goals in the first stage of the game. This concept is not part of ordinary evolutionary game theory, as the latter assesses strategies according to their success with respect to fitness, and does not directly take account of the motivation of agents (Reference [24], p. 183). In this sense, evolutionary game theory is not motivation-bound, but driven by success. In the indirect evolution concept, evolutionary success and motivations (utility functions) are connected with each other by an evolutionary feedback loop. Although evolutionary success remains the key driver of the process, preferences co-evolve with success. The crucial implication is that preferences and utility functions are neither fixed nor stable over time. However, at each point in time, they are seemingly given and stable (Reference [24], p. 183).

Preference changes are not such an exception in an evolutionary setting as is suggested by static microeconomic theory [25,26]. The indirect evolution approach [22,23] proposes the following dynamics: Although preferences motivate behavior, and behavior leads to success, success itself may be the key driver of the development and change of preferences via an evolutionary feedback process (References [27,28]; Hanson and Stuart [29] studied the so-called "Malthusian selection of preferences" in a model of population development with Malthusian features; for a survey on preference evolution, see Reference [30]). Individually, preferences may be fixed. In an evolutionary process, certain individuals will be "fitter," i.e., will receive higher payoffs, than others. If there is a certain type of preferences that supports these higher payoffs, those preferences will also co-evolve, whereas the other preferences will decline or even vanish. For instance, the dynamic evolution of preferences was analyzed by Norman [31] with a replicator dynamics approach. Therefore, even preferences, and, hence, utility functions, are not necessarily fixed over time for the population as a whole; they are, rather, endogenously determined by success and failure. In a sense, this is in accordance with the view of Alchian [32].

The connection between the concept of evolution and "aggregate-taking behavior" (i.e., there is no strategic reaction of the aggregate of players to the actions of a single player) was discussed by Possajennikov [33]. He found that this kind of behavior may be evolutionarily stable in symmetrical games with a finite number of players.

The rest of the paper is structured as follows. The indirect evolution of preferences concerning profit or utility maximization in a football league is analyzed in Section 2. The replicator dynamics of the game are investigated in Section 3. The model and its solution are discussed in Section 4; moreover, a variant of the model is studied there. Section 5 concludes.

## 2. Preference Choice in a Football League

### 2.1. Talent Investment

In a dynamic economic context, profit-maximizing firms do not necessarily perform better than firms with other objectives. As a consequence, different firm objectives may coexist in a competitive environment ([34–37]). In a game theoretical model that is not based on aggregate-taking behavior, Heifetz et al. [38] showed that there will be no convergence to payoff (or profit) maximization in an evolutionary equilibrium. However, aggregate-taking behavior may make the crucial difference here, as the interdependent behavioral effects between the players are much more restricted by aggregate-taking behavior. In accordance with this reasoning, Possajennikov [39] demonstrated that only so-called zero conjectures are evolutionarily stable; the latter means that players do not believe that the other players react to their actions.

The survival of payoff maximization was analyzed by indirect evolution in Güth and Peleg [40], with the result that its survival depends, with commonly known stimuli, on whether the success of a player is independent of the behavior of other players. For preferences in so-called Tullock [41] contests, Leininger [42] demonstrated that these (indirect) evolutionary preferences are more aggressive than those in a Nash equilibrium (see also Reference [43]).

However, it is not clear whether or to what extent these results hold true for all kinds of economic environments. In the following, a professional and commercial football league with a Tullock [41] contest success function is considered. In such a league, the success of one team automatically implies the failure of the other one, since the teams are matched up against each other (for the sake of simplicity, draws are excluded). As shown by Grossmann [20], applying Schaffer's [44] evolutionary game theory for finite populations, optimal talent investments of teams consist of an equilibrium strategy that is evolutionarily stable, but not a Nash equilibrium. Since all teams are identical, talent investment in the symmetrical evolutionarily stable equilibrium depends, among other things, on the relative weight of profits and wins in the teams' utility functions. This was the starting point for the following analysis.

As already stated above, the intention of this paper is to apply the indirect evolution approach [22, 23] to a sports league. Teams were assumed to choose their (identical) utility functions in the first stage of a two-stage game and their talent investments in the second stage, in such a way that the resulting equilibrium is evolutionarily stable according to the concept of optimal aggregate-taking strategies [45,46]. Solving this game by backwards induction, the question is: Which goal in the utility function will clubs select, given the evolutionarily stable talent investments of the second stage of the game? This procedure mimics "indirect evolution," as defined above: The optimal levels of talent investments in the second game stage are determined first; depending on these success-adjusted investment levels, clubs "adjust," so to speak, their objective function, anticipating the second stage result when deciding in the first stage of the game.

Consider a football league in which  $i = 1, \dots, n$  teams have the following utility function  $u_i$  (see also Reference [20]):

$$u_i(x_i) = \delta\pi_i + (1 - \delta)p_i \quad (1)$$

with:

- $x_i$ : investment in talent;
- $\delta$ : weight of profits,  $0 < \delta \leq 1$ ;
- $1 - \delta$ : weight of winning probability (wins);
- $\pi_i$ : profit of team  $i$ ;
- $p_i$ : winning probability of team  $i$ .

Note that  $\delta > 0$  is assumed, because otherwise teams would not be restricted when investing in talent (Reference [20], p. 119). Furthermore, the utility function in Equation (1) contains profits, usually measured in monetary units, and winning probabilities. As can be seen in the following, profits and

wins are measured in units of “talent.” Therefore, the measurement of utility is also in units of talent. Profits are defined by the difference of revenues and costs:

$$\pi_i = R_i - C_i = mp_i - \frac{b}{2}p_i^2 - cx_i \quad (2)$$

with:

$R_i$ : revenues of team  $i$ ;

$C_i$ : costs of team  $i$ ;

$m$ : market size (equal for all teams) with  $m > 1$ ;

$b > 1$ : parameter of the revenue function;

$c$ : constant marginal costs of talent.

Equation (2) is a standard formalization of the revenues (and costs) of a football club (see References [47,48] for a more general formalization). Revenues are supposed to depend linearly on the winning probability of the team, multiplied by the size of the team’s market. However, if a team wins too often, revenues may decline as the outcome uncertainty of the match declines. This is captured in the term  $-\frac{b}{2}p_i^2$ ;  $b$  is, therefore, called the competitive balance parameter. As a consequence, the revenue function has a maximum where the marginal revenue of the winning probability becomes zero:  $\frac{\partial R_i}{\partial p_i} = m - bp_i = 0 \Rightarrow p_i = \frac{m}{b}$ .

The marginal cost of talent,  $c > 0$ , is assumed to be constant. Hence, investment in more talented players does not increase marginal costs.

The winning probability is determined by a Tullock [41] contest success function:

$$p_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \quad (3)$$

with  $r > 0$  as talent productivity with respect to the winning probability (for further interpretations of this parameter see, for example, [48–51]).

To solve this game, the so-called “optimal aggregate-taking strategy” (ATS), introduced by Alós-Ferrer and Ania [45], was used. ATS is generally applicable in games where the payoff of one player depends on its own strategy choice, as well as on an *aggregate* of all players’ choices. The formal definition of a symmetric aggregative game reads as follows (Reference [45], p. 500, Definition 1):

“A (generalized) symmetric aggregative game with aggregate  $g$  is a Tuple  $\Gamma \equiv (N, S, \pi)$  where  $N$  is the number of players, the strategy set  $S$ , common to all players, is a subset of a totally ordered space  $X$ ,  $\pi : S \times X \rightarrow R$  is a real-valued function, and  $g : SN \rightarrow X$  is a symmetric and monotone increasing function, such that individual payoff functions are given by  $\pi_i(s) \equiv \pi(s_i, g(s))$  for all  $s = (s_1, \dots, s_N) \in SN$  and  $i = 1, \dots, N$ .”

The crucial aspect of a symmetrical aggregative game is that a player’s payoff is determined by its own strategy and the sum of all other players’ strategies (Reference [45], p. 500). The latter is replaced by the function  $g$ , which is symmetrical and monotone increasing.

The main reasons to apply this concept are firstly that an ATS is, in certain cases, more comprehensive than an evolutionarily stable strategy (ESS) (see below and Reference [45], p. 508, Proposition 2), and secondly that an ATS is much easier to understand intuitively than an ESS. Note that a strategy is evolutionarily stable if it is not possible that a small share of individuals deviating from the mixed strategy played in a population can successfully invade the population with the deviating strategy. The formal definition is as follows (Reference [52], p. 63, Theorem 6.4.1):

“The strategy  $\hat{p} \in S_N$  is an ESS if and only if  $\hat{p} \cdot Uq > q \cdot Uq$  for all  $q \neq \hat{p}$  in some neighbourhood of  $\hat{p}$  in  $S_N$ .” In this definition,  $U$  is the symmetric payoff matrix,  $\hat{p}$  and  $q$  are strategies and  $S_N$  is given by: ([52], p. 61):”

$$S_N = \{p = (p_1, \dots, p_N) \in \mathbb{R}^N : p_i \geq 0 \text{ and } \sum_{i=1}^N p_i = 1\}.$$

The issue of finiteness of the number of players in the ESS was solved by Schaffer [44]. Nevertheless, a pure strategy Nash equilibrium is evolutionarily stable with a finite number of players. Hence, it seems (but it is not correct) that the ESS equilibrium itself might be beaten by another strategy [46]. This confusion can be avoided by the ATS concept.

An ATS is formally defined as follows (Reference [45], p. 507):

“Let  $\Gamma \equiv (N, S, \pi)$  be a symmetric aggregative game. We say that  $s^* \in S$  is an optimal aggregate-taking strategy (ATS) if  $s^* \in \operatorname{argmax}_s \pi(s, g(s^*, \dots, s^*))$ . A strict ATS is an ATS which is a strict maximizer of this problem.”

Finally, ESS and ATS are connected with each other as follows ([45], p. 508, Proposition 2):

“Let  $\Gamma \equiv (N, S, \pi)$  be a symmetric aggregative game. Suppose  $\Gamma$  is quasisupermodular in individual strategy and the aggregate. If  $s^* \in S$  is an ESS, then  $s^*$  is also an ATS. If  $s^*$  is a strict ESS, then  $s^*$  is also a strict ATS.”

Applying ATS in the context of this paper requires, accordingly (note that here  $\sum_{j=1}^n x_j^{*r} = n \cdot (x^*)^r$ ):

$$\begin{aligned} \max_x u_i(x, g(x^*, \dots, x^*)) &= \delta(R_i - cx) + (1 - \delta)p_i = \\ \delta \left[ m \left( \frac{x^r}{n \cdot (x^*)^r} \right) - \frac{b}{2} \left( \frac{x^r}{n \cdot (x^*)^r} \right)^2 \right] - \delta cx_i + (1 - \delta) \left( \frac{x^r}{n \cdot (x^*)^r} \right) \end{aligned} \tag{4}$$

Equation (4) transforms the strategic choice of team  $i$  into a game where the team chooses its talent investment by supposing that all other teams will not change their choices due to the choice of team  $i$ 's talent investment. This means that team  $i$  is playing against all other teams at once with respect to talent investment, instead of playing against each other on a team-by-team basis. This can most easily be recognized by the winning probability  $p_i$  (see Equation (3) above):

$$p_i = \left( \frac{x_i^r}{\sum_{j=1}^n x_j^{*r}} \right).$$

The numerator represents team  $i$ 's choice of talent investment, whereas the denominator represents the sum of all teams' optimal talent investments, i.e., the aggregate of the game. Note that the latter is *not* influenced by the talent investment of team  $i$ ,  $x_i$ .

The supermodularity in the individual strategy and the aggregate requires:  $\frac{\partial u_i}{\partial x_i \partial x_j} > 0$  (see Reference [45], p. 501, Definition 3). The latter means that the strategies are complementary to each other. Put differently, if team  $i$  increases its talent investment, the reaction of team  $j$  is to increase its talent demand, too. The respective condition reads here:

$$\frac{\partial u_i}{\partial x_i \partial x_j} = r x_i^{r-1} (x_i + x_j)^{-2(1+r)} \{ -b \delta (x_i - 2r x_j) + [1 + \delta(m - 1)] (x_i + x_j)^r (x_i - r x_j) \}.$$

Given that the game between the teams is symmetrical and all teams play with the same parameters (i.e., they employ the same level of talent), the condition is fulfilled for  $\frac{1}{2} < r < 1$  (see below for the derivation of this relation).

The first-order condition for the maximization program in Equation (4) reads:

$$\frac{\partial u_i(\dots)}{\partial x} \Big|_{x=x^*} = \delta \left[ \frac{r}{nx} \left( m - \frac{b}{n} \right) \right] - \delta c + (1 - \delta) \frac{r}{nx} = 0. \tag{5}$$

Therefore,

$$x^* = x = \frac{r}{\delta cn} \left[ \delta \left( m - \frac{b}{n} \right) + (1 - \delta) \right], n \cdot x^* = \frac{r}{\delta c} \left[ \delta \left( m - \frac{b}{n} \right) + (1 - \delta) \right]. \tag{6}$$

However, note that this value of  $x^*$  is a globally stable ESS for *strict* ATS. ATS is strict if the  $u_i(\dots)$  function in Equation (12) is strictly concave in  $x$  (Reference [45], p. 508, Proposition 2), i.e., if

$$\frac{\partial^2 u_i(x, g(x^*, \dots, x^*))}{\partial x^2} < 0.$$

Since

$$\frac{\partial^2 u_i(x, g(x^*, \dots, x^*))}{\partial x^2} = \frac{x_i^{r-2}}{(n(x^*)^r)^2} \left\{ n(x^*)^r r(r-1)[1 + \delta(m-1)] - b\delta r(2r-1)x^r \right\}, \tag{7}$$

the sign of the second derivative depends on the term in the  $\{ \dots \}$  brackets:

$$\text{sign} \frac{\partial^2 u_i(x, g(x^*, \dots, x^*))}{\partial x^2} = \text{sign} \left\{ n(x^*)^r r(r-1)[1 + \delta(m-1)] - b\delta r(2r-1)x^r \right\}. \tag{8}$$

Hence,

$$\frac{\partial^2 u_i(x, g(x^*, \dots, x^*))}{\partial x^2} < 0 \text{ if } \frac{1}{2} < r < 1, \tag{9}$$

because (for  $m > 1$ )

$$n(x^*)^r r(r-1)[1 + \delta(m-1)] < 0 \text{ if } r < 1 \tag{10}$$

and

$$-b\delta r(2r-1)x^r < 0 \text{ if } r > \frac{1}{2}. \tag{11}$$

Note that the local extremum at  $x^*$  is a maximum for  $nm > b$ , since

$$\frac{\partial^2 u_i}{\partial x^2} \Big|_{x_i=x^*} = -\frac{r}{nx^2} \left[ \left( m - \frac{b}{n} \right) \delta + (1 - \delta) \right] < 0 \text{ if } nm > b. \tag{12}$$

Note that the condition  $nm > b$  means that the sum of market sizes of all teams together is larger than the competitive balance parameter,  $b$ .

These results are summarized in the following proposition.

**Proposition 1.** *The optimal aggregate-taking strategy (ATS) given by  $x^* = x = \frac{r}{\delta cn} \left[ \delta \left( m - \frac{b}{n} \right) + (1 - \delta) \right]$  is a globally stable ESS if  $m > 1$ ,  $nm > b$  and  $\frac{1}{2} < r < 1$ .*

**Proof.** See above. □

### 2.2. Choosing Goals

The question to be answered next is: Which value  $\delta$  maximizes utility in the (symmetrical) evolutionarily stable talent investment equilibrium of Proposition 1? The answer is shown in Proposition 2.

**Proposition 2.** Given that  $\frac{1}{2} < r < 1$ , profit maximization is optimal, i.e.,  $\delta^* = 1$ .

**Proof.** Inserting  $x^*$  from Equation (6) into the profit function (2), incorporating the result in the utility function in Equation (1) and rearranging terms yields:

$$U[x^*(\delta)] = \frac{\delta}{n} \left[ m(1-r) - \frac{b}{n} \left( \frac{1}{2} - r \right) \right] - \frac{(1-\delta)}{n} (r-1). \quad (13)$$

The utility function  $U[x^*(\delta)]$  is linear in  $\delta$ . Therefore, only corner solutions, i.e.,  $\delta = 0$  or  $\delta = 1$ , are possible. Utility at  $\delta = 0$  ( $\delta = 1$ ) is given by:

$$U[x^*(\delta = 0)] = -\frac{1}{n}(r-1), \quad (14a)$$

$$U[x^*(\delta = 1)] = \frac{1}{n} \left[ m(1-r) - \frac{b}{n} \left( \frac{1}{2} - r \right) \right]. \quad (14b)$$

Note that both utility levels are larger than zero if  $r < 1$ .

$$U[x^*(\delta = 1)] > U[x^*(\delta = 0)] \text{ if:} \\ \frac{1}{n} \left[ m(1-r) - \frac{b}{n} \left( \frac{1}{2} - r \right) \right] > -\frac{1}{n}(r-1) \text{ for } \frac{1}{2} < r < 1 \text{ and } m > 1. \quad (15)$$

Since  $\frac{1}{2} < r < 1$  and  $m > 1$  hold true by assumption,  $U[x^*(\delta = 1)] > U[x^*(\delta = 0)]$ .

Proposition 2 contains the result of the indirect evolution in a football league. The weights for the goals profit maximization on the one hand and winning probability maximization (wins) on the other hand are chosen in such a way that the teams' utility is maximized, given their maximum fitness, measured by their talent investments. In the context of a football league, only profit maximization maximizes fitness.  $\square$

### 3. Preference Dynamics in a Football League

Having analyzed the evolutionary stability of win versus profit maximization, the remaining question is whether the results of this analysis are stable if the weights for wins and profits themselves change over time. Such dynamic questions are studied via so-called replicator dynamics. In plain words, it works as follows: Assume that the teams in a football league choose either profit or win maximization. The share of teams that choose profit maximization is  $\alpha$ , the remaining share of teams that choose wins is  $\beta = 1 - \alpha$ . The respective average level of utility that is attained by all teams, given the resulting mix of profit and win maximization, is called (average) "fitness,"  $\bar{U}$ . The next question is how these shares of win or profit maximizing teams evolve over time in relation to this average level of "fitness" (Reference [52], p. 67):  $\frac{\dot{\alpha}}{\alpha} = \text{fitness } \alpha - \text{average fitness}$ . "Fitness," in this sense, means "success," measured by utility levels. The share of win (profit) maximizing teams with a utility level above the average will increase, and the share of teams with utility levels below the average will decrease.

To formalize the replicator dynamics for the choice of  $\alpha$ , the average payoff or "fitness" level is determined by

$$\bar{U} = \alpha U(\pi) + \beta U(p), \quad \beta(1 - \alpha). \quad (16)$$

Note that  $\alpha$  and  $\beta$  are the shares of teams in a league that have as their goal profit or win maximization, respectively.

The corresponding replicator equation for profit maximization reads

$$\frac{\dot{\alpha}}{\alpha} = U(\pi) - [\alpha U(\pi) + \beta U(p)] = \beta[U(\pi) - U(p)], \quad (17)$$



and for win maximization

$$\dot{\beta} = U(p) - [\alpha U(\pi) + \beta U(p)] = \alpha[U(p) - U(\pi)]. \tag{18}$$

Equation (17) says that the change of the profit objective in the utility function relative to its starting level is defined by the utility level attained by profit maximization minus the average “fitness” level. Equation (18) states the same for win maximization. The dynamic processes defined by Equations (17) and (18) come to an end if the respective fixed points are reached.

The fixed points of (17) and (18) are

$$\dot{\alpha} = 0 \Leftrightarrow U(\pi) = U(p); \dot{\beta} = 0 \Leftrightarrow U(p) = U(\pi). \tag{19}$$

**Proposition 3.** For  $r < \frac{2+b}{2(1+b)}$ , the fixed points for profit and win shares, respectively, of the replicator dynamics are

$$\alpha^* = \frac{2(r-1) - b(1-2r)}{2(m+1)(r-1)}, \tag{20}$$

$$\beta^* = 1 - \alpha^* = \frac{b(1-2r) + 2m(r-1)}{2(m+1)(r-1)}. \tag{21}$$

**Proof.** From Equation (13),  $U(\pi(x^*)) = U(p(x^*)) \Leftrightarrow \frac{\alpha}{n} [m(1-r) - \frac{b}{n} (\frac{1}{2} - r)] = \frac{(1-\alpha)}{n} (1-r)$ . Solving this equation for  $\alpha$  gives

$$\alpha = \frac{2(r-1) - b(1-2r)}{2(m+1)(r-1)}. \tag{22}$$

The denominator on the right-hand side of Equation (22) is smaller than zero for  $r < 1$ ; the nominator is smaller than zero for  $(\frac{1}{2} <) r < \frac{2+b}{2(1+b)} (< 1)$ ; therefore  $\alpha, \beta > 0$ .  $\square$

**Remark 1.** Note that  $\frac{\partial \alpha^*}{\partial r} = -\frac{b}{2(m+1)(r-1)^2} < 0$ ,  $\frac{\partial \beta^*}{\partial r} = \frac{b}{2(m+1)(r-1)^2} > 0$ . This implies that for increasing talent productivity  $r$ , the share of profit-maximizing teams decreases, and that of winning-probability-maximizing teams increases.

**Remark 2.** The fixed point determined in Proposition 3 is unstable:

$$\frac{\partial [U(\pi(x^*)) - U(p(x^*))]}{\partial \alpha} = \frac{1}{n} \left[ (m+1)(1-r) - \frac{b}{n} \left( \frac{1}{2} - r \right) \right] > 0 \text{ for } \frac{1}{2} < r < 1. \tag{23}$$

Hence, the fixed point defined by Equation (20) is a repeller.

**Proposition 4.** The stable fixed point of the replicator dynamics of the football league is  $\alpha = 1$ , i.e., pure profit maximization.

**Proof.** The utility difference  $U(\pi) - U(p)$  for optimal values of  $x$  increases monotonously in  $\alpha$ , as shown by Equation (23). Therefore, the largest value of admissible values of  $\alpha$ , i.e.,  $\alpha = 1$ , is the only stable fixed point of the replicator dynamics.  $\square$

The dynamic analysis emphasizes the results of the static investigation. Although there exists an unstable fixed point at  $\alpha^* = \frac{2(r-1) - b(1-2r)}{2(m+1)(r-1)}$ , where  $\alpha$  decreases with an increase of talent productivity,  $r$ , the only stable fixed point is at  $\alpha = 1$ . Therefore, pure maximization of profits results, although profit

maximization and win maximization may coexist for a certain restrictive configuration of values, as shown by Equations (23) and (24). If the talent productivity index  $r$  increases in this case, teams give more weight to the goal of win maximization than to maximizing profits.

#### 4. Discussion

In the above models, the reason for the zero–one decision with respect to profit or win maximization is the linearity of the weights of profits and wins in the utility function. In the following, the question of whether the results reached so far will change with a quadratic weighting of profits and wins is analyzed. Of course, this is only one particular version of a very large number of possible non-linear models.

Consider a football league in which  $i = 1, \dots, n$  teams have the following utility function  $u_i$ :

$$u_i(x_i) = \mu^2 \pi_i + (1 - \mu)^2 p_i, \tag{24}$$

$\mu$ : weight of profits,  $(1 - \mu)$ : weight of wins.

With the methods described in Section 2, the optimal symmetric investment in talent reads

$$x^* = x = \frac{r}{\mu^2 c n} \left[ \mu^2 \left( m - \frac{b}{n} \right) + (1 - \mu)^2 \right], \quad n \cdot x^* = \frac{r}{\mu^2 c} \left[ \mu^2 \left( m - \frac{b}{n} \right) + (1 - \mu)^2 \right]. \tag{25}$$

Accordingly, Proposition 5 is the new version of Proposition 1 above.

**Proposition 5.** *The optimal aggregate-taking strategy (ATS), given by  $x^* = x = \frac{r}{\mu^2 c n} \left[ \mu^2 \left( m - \frac{b}{n} \right) + (1 - \mu)^2 \right]$ , is a globally stable ESS if  $nm > b$  and  $\frac{1}{2} < r < 1$ .*

Similarly, Proposition 6 is the adjusted version of Proposition 2 above:

**Proposition 6.** *Given that  $\frac{1}{2} < r < 1$ , the optimal weight of profit maximization in the utility function is given by  $\mu^* = \frac{1-r}{(1-r)(m+1) - \frac{b}{n}(\frac{1}{2}-r)}$ ; the optimal weight of maximizing wins reads*

$$1 - \mu^* = \frac{(1 - r)m - \frac{b}{n}(\frac{1}{2} - r)}{(1 - r)(m + 1) - \frac{b}{n}(\frac{1}{2} - r)}.$$

**Remark 3.** *For  $r \rightarrow 1 \Rightarrow \mu^* \rightarrow 0$  and  $(1 - \mu^*) \rightarrow 1$ . i.e., higher talent productivity values render pure win maximization more likely. The same holds true for  $b \rightarrow 0$ , since  $\mu^* \rightarrow \frac{1}{m+1}$  and  $(1 - \mu^*) \rightarrow \frac{m}{m+1}$ .*

Moreover, the replicator dynamics (see Section 3 above) with the same methods as in Section 3 read

$$\frac{\dot{\alpha}}{\alpha} = 0 \Leftrightarrow U(\pi) = U(p); \quad \frac{\dot{\beta}}{\beta} = 0 \Leftrightarrow U(p) = U(\pi). \tag{26}$$

**Proposition 7.** *The replicator dynamic fixed points for profit and winning probability shares, respectively, for  $\frac{1}{2} < r < 1$  are*

$$\alpha = \frac{(r - 1) + \sqrt{m(1 - r^2)}}{m(1 + r) + (r - 1)}, \tag{27}$$

$$\beta = 1 - \alpha = \frac{m(1 + r) - \sqrt{m(1 - r^2)}}{m(1 + r) + (r - 1)}. \tag{28}$$

**Proof.** From Equation (13),  $U(\pi(x^*)) = U(p(x^*)) \Leftrightarrow \frac{\alpha}{n} \left[ m(1-r) - \frac{b}{n} \left( \frac{1}{2} - r \right) \right] = \frac{(1-\alpha)^2}{n} (1-r)$ . Solving this equation for  $\alpha$  gives

$$\alpha_{1,2} = \frac{(r-1) \pm \sqrt{m(1-r^2)}}{m(1+r) + (r-1)}. \quad (29)$$

Since the denominator on the right-hand side of Equation (29) is larger than zero, the nominator must also be larger than zero for  $\alpha \geq 0$ . Hence,  $\alpha = \frac{(r-1) + \sqrt{m(1-r^2)}}{m(1+r) + (r-1)}$ . Accordingly, the share of win maximization is given by  $1 - \alpha = \frac{m(1+r) - \sqrt{m(1-r^2)}}{m(1+r) + (r-1)}$ .  $\square$

**Remark 4.** For  $r \rightarrow 1 \Rightarrow \alpha^* \rightarrow 0$  and  $(1 - \alpha^*) \rightarrow 1$ . This implies that for increasing talent productivity it becomes more likely that the maximization of winning probabilities is the only goal of teams. The dynamic analysis also strengthens the results of the static investigation in this case.

Comparing the two versions of the football league model, the crucial question is what kind of utility function the football clubs may have. Moreover, the question of whether football clubs use linear or non-linear weights for profits and wins is also of importance. In addition, there exists a certain asymmetry between profit and win maximization. Pure win maximization is not allowed in both model versions, since a pure win orientation would violate the budget constraint (which is incorporated into the profit part of the utility function in these models). This is the reason why the productivity measure,  $r$ , must be smaller than unity. In contrast, pure profit maximization is always possible, since it encompasses the budget constraint.

The latter is also highly relevant for real football leagues. Pure win maximization will not be sustainable over time if all teams attempt it, because this will (sooner or later) violate the budget constraint and lead to team or even league bankruptcy. A certain degree of profit orientation must always exist to prevent disastrous results.

The empirical evidence on profit or win maximization is inconclusive. This could be an indication that a certain combination of both would be optimal.

## 5. Conclusions

Although preferences are generally seen as behavioral assumptions that are not subject to choice, they may nevertheless be chosen evolutionarily as a consequence of success and failure. This was analyzed for a football league.

It turns out that the productivity of talent investment with respect to the winning probability,  $r$ , plays a key role. In general, there will be a mix of profit and win maximization. Despite the fact that, with increasing talent productivity, win maximization increases, the only evolutionary goal in a model with linear weights for profits and wins is profit maximization. This was also the result of the replicator dynamics with respect to the shares of teams that choose either profit or win maximization. Nevertheless, the outcomes were different with a quadratic weighting scheme for profits and wins. In the latter model version, win maximization became more important than profit maximization.

The theoretical results of this paper emphasize that the evolution of European football leagues and of the major North American sport leagues depends on the weights for profits and wins in the clubs' utility functions. Nevertheless, the question of in which direction the commercialization of the major sport leagues may drive the orientations concerning profits and wins remains open. Although this question cannot be answered theoretically, the model versions discussed in this paper demonstrate that the answer will neither be simple nor self-evident.

Note, however, that the analysis presented in this paper faces several limitations:

- (a) The teams of the league are assumed to have the same market size.
- (b) The game the teams play is symmetrical.
- (c) The productivity of talent employed by the teams is deterministic.

- (d) The marginal cost of talent is constant; this implies a completely elastic supply of talent.
- (e) A ratio contest success function is applied.

The relaxation of these limitations remains a task for future research.

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