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# Insurance-markets Equilibrium with a Non-convex Labor Supply decision, Unobservable Effort, and Incentive ("Fair") Wages

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**Abstract.** *The purpose of this note is to describe the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and incentive ("fair") wages. The presence of indivisible labor creates a market incompleteness, which requires that an insurance market for employment be put in operation to "complete" the market.*

**Keywords:** Indivisible labor, Lotteries, Unobservable effort, Fair wages, Insurance.

**JEL Classification Codes:** E1, J22, J41.

## 1. Introduction and Motivation

The purpose of this note is to describe the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and incentive ("fair") wages a la Danthine and Kurmann ([1]). We show how lotteries as in Rogerson ([3]) can be used to convexify consumption sets. With a discrete labor supply decisions, the markets are incomplete. The particular focus in this paper is on the lottery- and insurance-market equilibrium in an economy with indivisible labor supply, unobservable effort and incentive ("fair") wages. The presence of non-convexity requires that an insurance market for employment be put in operation to achieve market completeness.

## 2. Model Setup

The theoretical setup follows to a great extent Vasilev ([4]). To simplify the analysis, the model economy here is static, without physical capital, and agents will face a non-convex labor supply decision. Effort exerted by workers is a productive input in the final goods sector, but unobservable, and thus not directly contractible. However, producers understand that while workers do not like exerting effort, they derive utility from returning the gift of a generous wage by supplying a higher effort level even in an environment of costly monitoring. This leads to the firm paying an incentive wage. Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by  $i$  and distributed uniformly on the  $[0; 1]$  interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index  $i$  to save on notation.

### 2.1 Description of the model

Each household maximizes the following utility function:

$$\ln c - hG(e) \tag{1}$$

where  $c$  denotes consumption of household  $i$ ,  $h$  is the fraction of time available to household  $i$  that is spent working, and  $e$  is the level of effort exerted. The total time endowment available to each household  $i$  is normalized to unity, thus leisure,  $l = 1 - h$  is implicitly expressed as time off-work. The novelty here is the  $G(e)$  utility term, which, as in Vasilev ([5]), is included to capture that workers may derive additional dis-utility from exerting effort.

As in Hansen ([2]) and Rogerson ([3]) household  $i$ 's labor supply is assumed to be indivisible, i.e.  $h \in \{0; 1\}$ . The problem faced by a household that decides to work full-time is then to set  $h = 1$  and enjoy

$$U^w = \ln c^w - G(e^w), \tag{2}$$

where  $c^w = w + \pi$  and  $e^w$  are the consumption and effort levels when working. Note that the effort level will be determined implicitly from its optimality condition  $G'(e) = 0$ , which does not depend on the other model variables. In contrast, a household that decides not to work chooses  $h = 0$  and enjoys

$$U^u = \ln c^u, \tag{3}$$

where  $c^u = \pi$  is the consumption level when the household is not working.

## 2.2 Effort function

As in Danthine and Kurmann ([1]), the effort function is modelled as follows:

$$G(e) = (e - (\phi_0 + \phi_1 \ln w + \phi_2 \ln N))^2, \quad (4)$$

where  $\phi_0, \phi_1 > 0$ ,  $\phi_2 < 0$ , and  $N$  denotes aggregate employment. Thus, in equilibrium,

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N. \quad (5)$$

In other words, if the worker receives an incentive ("fair") wage, s/he would supply a higher level of effort. On the other hand, the higher the employment, or the tighter the labor market, the lower the incentive to exert effort. Note that the optimal effort level is independent of consumption, and varies only with the wage rate and aggregate employment, which the household takes as given.

## 2.3 Stand-in firm

There is a representative firm in the model economy. It produces a homogeneous final product using a production function that requires labor  $H$  as the only input. For simplicity, output price will be normalized to unity. The production function  $f(H)$  features decreasing returns to scale (for any effort level):  $f'(eH) > 0$ ,  $f''(eH) < 0$ ,  $f'(0) = 1$ ,  $f'(e) = 0$ . The representative firm acts competitively by setting the wage rate  $w$  and choosing  $H$  to maximize profit by stimulating optimal effort:

$$\Pi = f(eH) - wH \quad s.t. \quad 0 \leq H \leq 1 \quad (6)$$

and

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N. \quad (7)$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

## 3. Insurance Market: Stand-in Insurance Company

An alternative way to represent the labor selection arrangement is to regard workers as participants in a lottery with the proportion employed equal to

the probability of being selected for work. Therefore, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received independent of the employment status. More specifically, the structure of the insurance industry is as follows: there is one representative insurance company, which services all households and maximizes profit. It receives revenue if a household is working in the market sector and makes payment if it is not. At the beginning of the period, the households decide if and how much insurance to buy against the probability of being chosen for work. Insurance costs  $q$  per unit, and provides one unit of income if the household is not employed. Thus, household will also choose the quantity of insurance to purchase  $b$ ; we can think of insurance as bonds that pay out only in case the household is not chosen for work.

The amount of insurance sold by the insurance company is a solution to the following problem: Taking  $q(i)$  as given,  $b(i)$  solves

$$\max_{b(i)} \lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i). \quad (8)$$

With free entry profits are zero, hence

$$\lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i) = 0, \quad (9)$$

hence the insurance market for each household clears.

## 4. Decentralized Competitive Equilibrium (DCE) with lotteries

### 4.1 Definition of the DCE with lotteries

A competitive Equilibrium with Lotteries for this economy is a list

$$(c(i)^w, c(i)^u, e(i)^w, \lambda(i), w, \pi) \quad (10)$$

such that the following conditions are fulfilled.

**1. Consumers maximization condition.** Taking prices  $w, \pi$  as given, for each  $i$ , the sequence

$$\sigma = (c(i)^w, c(i)^u, e(i)^w, \lambda(i)) \quad (11)$$

solves the maximization problem

$$\max_{\sigma \in \Sigma} \lambda(i) \ln c(i)^w + [1 - \lambda(i)] \ln c(i)^u - G(e(i)^w) \quad (12)$$

s.t

$$\lambda(i)c(i)^w + [1 - \lambda(i)]c(i)^u = \lambda(i)w + \pi, \quad (13)$$

with

$$c(i)^w \geq 0, c(i)^u \geq 0, 0 < \lambda(i) < 1, \forall i, \quad (14)$$

where  $\Sigma$  is the constraint defined by relations (13)-(14).

**2. Firm maximization condition.** Taking prices  $w, \pi$  as given,

$$\max_{0 \leq H \leq 1} f(eH) - wH. \quad (15)$$

and

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N. \quad (16)$$

**3. Market-clearing condition.** We have

$$\int_i \lambda(i) di = H, \quad (17)$$

$$\int_i \{\lambda(i)c(i)^w + [1 - \lambda(i)]c(i)^u\} di = f(eH), \quad (18)$$

where the first equation describes the clearing in the labor market, while the second equation captures the goods-market clearing.

## 4.2 Characterizing the DCE

The household's problem is as follows:

$$\begin{aligned} \mathcal{L} = \max_{\sigma \in \Sigma} & \lambda(i) \ln c(i)^w + [1 - \lambda(i)] \ln c(i)^u - G(e(i)^w) \\ & - \mu [\lambda(i)c(i)^w + (1 - \lambda(i))c(i)^u - \lambda(i)w - \pi], \end{aligned} \quad (19)$$

where  $\mu$  is the Lagrangian multiplier in front of the households' budget constraint. The first-order optimality conditions are as follows:

$$c(i)^w : \frac{1}{c(i)^w} = \mu, \forall i, \quad (20)$$

$$c(i)^u : \frac{1}{c(i)^u} = \mu, \forall i. \quad (21)$$

it follows that

$$c = c(i)^w = c(i)^u = 1/\mu, \forall i. \quad (22)$$

We simplify the Lagrangian by suppressing all consumption superscripts and  $i$  notation in the derivations to follow

$$\lambda(i) : cG(e(i)^w) = w \quad (23)$$

This condition states that the marginal rate of substitution between effort in the market sector and consumption equals the wage rate. This implicitly characterizes optimal market sector participation rate  $\lambda$ . Note that it is optimal from the benevolent planner/government point of view to choose randomly  $\lambda$  and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved. Now we extend the commodity space to include insurance markets explicitly.

## 5. Decentralized Competitive Equilibrium (DCE) with insurance markets

### 5.1 Definition of the DCE with insurance markets

A competitive Equilibrium with Lotteries and insurance markets for this economy is a list

$$(c(i)^w, c(i)^u, e(i)^w, \lambda(i), b(i), q(i), p, w, \pi) \quad (24)$$

such that the following conditions are fulfilled.

**1. Consumers maximization condition.** Taking prices  $p, w, \pi$  as given, for each  $i$ , the sequence

$$\sigma = (c(i)^w, c(i)^u, e(i)^w, \lambda(i), b(i), q(i)) \quad (25)$$

solves the maximization problem

$$\max_{\sigma \in \Sigma} \lambda(i)[\ln c(i)^w - G(e(i)^w)] + (1 - \lambda(i)) \ln c(i)^u \quad (26)$$

s.t.

$$pc(i)^w + b(i)q(i) = w + \pi \quad (27)$$

$$pc(i)^u = b(i) + \pi \quad (28)$$

$$c(i)^w \geq 0, c(i)^u \geq 0, 0 < \lambda(i) < 1, \forall i \quad (29)$$

or

$$pc(i)^w + pq(i)c(i)^u = w + (1 + \pi)q(i), \quad (30)$$

where  $\Sigma$  is the constraint defined by relations (27)-(29).

**2. Firm maximization condition.** Taking prices  $w, \pi$  as given,

$$\max_{0 \leq H \leq 1} f(eH) - wH. \quad (31)$$

and

$$e = \phi_0 + \phi_1 \ln w + \phi_2 \ln N. \quad (32)$$

**3. Insurance-company condition.** Taking  $q(i)$  as given,  $b(i)$  solves

$$\max_{b(i)} \lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i). \quad (33)$$

With free entry profits are zero, hence

$$\lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i) = 0, \quad (34)$$

hence the insurance market for each household clears.

**4. Market-clearing condition.** We have

$$\int_i \lambda(i)di = H, \quad (35)$$

$$\int_i \{\lambda(i)c(i)^w + [1 - \lambda(i)]c(i)^u\}di = f(eH), \quad (36)$$

where the first equation describes the clearing in the labor market, while the second equation captures the goods-market clearing.

## 5.2 Characterization of the DCE with insurance markets

$$\begin{aligned} \mathcal{L} = \max_{\sigma \in \Sigma} & \lambda(i)[\ln c(i)^w - G(e(i)^w)] + (1 - \lambda(i)) \ln c(i)^u \\ & - \mu[pc(i)^w + pq(i)c(i)^u - w - (1 + \pi)q(i)] \end{aligned} \quad (37)$$



Without loss of generality, normalize  $p = 1$ . The resulting first-order conditions are as follows:

$$c(i)^w : \frac{\lambda(i)}{c(i)^w} = p\mu, \forall i, \quad (38)$$

$$c(i)^u : \frac{1 - \lambda(i)}{c(i)^u} = pq(i)\mu, \forall i. \quad (39)$$

Optimal  $\lambda$  ( $\lambda(i) = \lambda, \forall i$ ) is implicitly characterized by the zero-profit condition from the insurance company:

$$\frac{\lambda}{1 - \lambda} = \frac{1}{q}, \quad (40)$$

which implies that the price of the insurance equals the ratio of probabilities of the two events (“the odds ratio”). Combining this with the other optimality condition, we obtain that households buy full insurance to equalize consumption,

$$c^w = c^u, \forall i. \quad (41)$$

That is, in the presence of uncertainty, we need insurance companies to achieve market completeness.

## 6. Conclusion

This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and incentive (“fair”) wages. The presence of indivisible labor supply created a market incompleteness, which in turn requires that an insurance market for employment be put in operation to “complete” the market.

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