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**Article**

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# Never-buyers of consumer non-durables

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In this article the concept of never-buyers of consumer non-durables is discussed. The traditional Negative Binomial Distribution approach of Ehrenberg to the question is presented. Previously unpublished work carried out at the Graduate School of Business Administration, University of the Witwatersrand, is reviewed and hypotheses are put forward that the observed large zero cell in the purchase frequency distributions may be caused by the existence of a group of never-buyers of the product, or by the superimposition of at least two distinct buying populations, previously identified as brand-loyal and multibrand/brand-switching households. The results of the research aimed at testing the first hypothesis are presented here. Two carefully monitored data sets were modelled using zero-augmented Negative Binomial and Sichel distributions. The data were previously shown to exhibit the necessary mean households purchase/consumption stationarity. Individual brands in one data set (purchases of toilet soap) were shown to follow the predictions of the traditional theory — the proportion of non-buyers decreasing with time. In the second data set (consumption of packaged soup) the proportion of non-consumers of the brands fell towards zero as the length of the time period studied was increased, but at a rate faster than that predicted by the theory. The hypothesis of the existence of never-buyers/users of individual brands in these two product classes was therefore rejected.

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In die artikel word die konsep van nooit-kopers van nie-duursame verbruikersgoedere bespreek. Ehrenbergse tradisionele Negatief-binomiaaldistribusie-benadering in verband met die vraag is voorgestel. Vorige ongepubliseerde werk uitgevoer by die Nagraadsebestuurskool, Universiteit van die Witwatersrand is bespreek, en hipoteses word voorgestel dat die waargenome groot nul-sel in die verkoops-frekwensiedistribusies miskien 'n gevolg mag wees van die bestaan van 'n groep nooit-kopers van die produk, of as 'n gevolg van die opeenplasing van ten minste twee afsonderlike koperpopulasies voorheen geïdentifiseer as handelsnaam-lojaal en handelsnaam-wisselende gesinne. Die resultate van die navorsing wat gedoen is om die eerste hipotese te toets, word gegee. Twee sorgvuldig gekontroleerde stelsels data is gemodelleer met gebruik van die nulvermeerderde Negatief-binomiaal- en Sichel-distribusies. Individuele handelsname in een datatesel (aankope van toiletseep) volg die voorspelling van die tradisionele teorie — die proporsie van nie-verbruikers neem mettertyd af. In die tweede datatesel (verbruik van verpakte sop) het die proporsie van nie-verbruikers van die handelsname na nul gedaal met verlenging van die tydperk wat bestudeer is, maar teen 'n snelheid vinniger as wat voorspel was deur die teorie. Die hipotese dat daar nooit-kopers/gebruikers van individuele handelsname in die twee produkklasse bestaan het, is dus verwerp.

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## Introduction

In the field of consumer non-durables, an interesting segment of the market, known as 'never-buyers', has caused some discussion in the literature.

Ehrenberg (1972) claimed that, given a sufficiently long period of time, every consumer will eventually purchase a product at least once, and so the never-buyer does not exist. If a study shows a number of non-buyers in a period of time, Ehrenberg believes that, had the time of the study been longer, these non-buyers would not have been found. In fact, his Negative Binomial Distribution (NBD) theory which proposes that, provided the market is stationary frequently purchased consumer non-durables can be modelled using the NBD, (Equation A1) predicts this outcome. As time increases, the proportion of buyers of the product as predicted by the NBD tends to unity (Equation A2). (A brief description of the statistical distributions mentioned in the text is given in the Appendix.)

Purely from a logical point of view one may seek to raise issue with Ehrenberg on the question of never-buyers. Non-smokers probably never buy cigarettes, non-drivers who do not own vehicles are unlikely to purchase petrol. Product classes can therefore be identified in which, by logical considerations alone, one would expect to find some never-buyers.

Nevertheless, one should concede to Ehrenberg that such examples probably represent a small proportion of the consumer non-durables market.

Ehrenberg also claimed that his model is as applicable to an individual brand as it is to the product class as a whole. However, with respect to individual brands the situation may be quite different. Ehrenberg and other overseas writers have, with few published exceptions, only tested the NBD model in its univariate form. Where they found departures from the model, they ascribed them to effects which they labelled 'the shelving phenomenon' or 'the variance discrepancy'. They believed this effect — an observed discontinuity in the frequency of heavier purchases at or close to the number of units, equal to the number of weeks or period in the time period of the study — to be caused by a small sector of the market purchasing the product regularly.

Sichel (1975) investigated this phenomenon using observations on packaged-soup consumption. By their very nature, being consumption-based, such data, according to Ehrenberg's theory, ought not to exhibit the effect described above. However, 'shelving' was observed in the data.

She also found that other statistical tests revealed significant departures from Ehrenberg's NBD model. As was subse-

quently demonstrated by Zachos (1977), although univariate distributions predicted by NBD theory may be accepted by the *chi-square* test, predictions in the bivariate form of the NBD often provide very unsatisfactory models.

As a result, both of her own work and an earlier study by Tucker (1973), Sichel (1975) suggested that the true phenomenon of repeat-consumption/buying may be represented by two distinctly different populations:

- (a) A hard-core of stable, brand-loyal households which consume regularly in accordance with Ehrenberg's original model. These therefore would be expected to follow the correlated NBD model (Equation A3).
- (b) 'A fair number of brand-switching, multibrand consuming households.' This group is statistically characterized by the uncorrelated bivariate NBD (Equation A5) in successive time periods.

She put forward the hypothesis that the observed bivariate distribution arose from the superimposition of these two populations. This hypothesis was supported by her work on the variation of market penetration, number of new and repeat consumers and correlation coefficients with increasing lengths of the observational period. She found that the observations usually lay between the predictions of the correlated and uncorrelated models.

Firer & Barnett (1979) found similar results in the toilet soap market.

Joannides (1976) formulated a binary model by combining the correlated and uncorrelated bivariate NBDs. He showed that his model could produce many of the observed shapes, patterns (including a shelf), and values of parameters which the original NBD theory could not.

In practice, purchase/consumption frequency distributions can be observed in which the size of the zero cell appears excessively large when compared to the distribution as a whole. This suggests that the distribution is not drawn from a single homogeneous population, but that at least two distinct populations are involved.

Joannides (1976) identified these populations as brand-loyal and multibrand, brand-switching households. However, the same distribution shape could arise from the superimposition of a buying population (which may, for a given time period contain non-buyers who will buy in a future period) and a group of never-buyers (whose distribution would consist of a certain frequency *in the zero cell only*).

The objective of the research reported in this article was to test these hypotheses by investigating the proportion of never-buyers in two frequently purchased South African consumer non-durables product classes.

## Data

The two data sets analysed were:

### Soap

The data referred to as 'soap' originate from the toilet soap product class. In South Africa this product class comprised more than thirty-five different brands, but was dominated by the two leading brands, Lux and Palmolive.

The data base used consisted of a sample of 614 White households' bimonthly purchases of toilet soap, over the period April to November 1978, details of which were collected by Market Research Africa using the consumer panel method.

Goodman (1979) tested the data and concluded that purchasing rates over the eight-month period were stationary. The observations had one peculiarity worthy of mention. When

they were plotted in the form of frequency distributions, even-numbered purchase frequencies appeared high relative to the odd-numbered purchase frequencies, giving a zig-zag shaped distribution (Figure 1). This was attributed by Goodman to a consumer preference for purchasing the product in even-numbered lots.

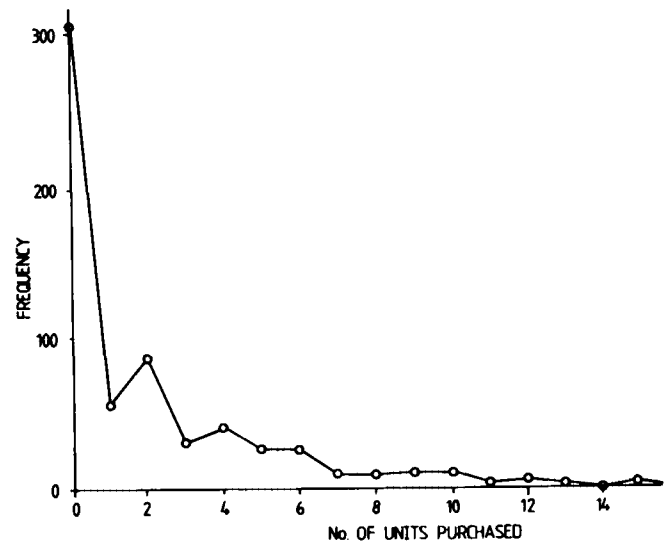


Figure 1 Observed frequency distribution for Palmolive soap — eight months (N = 614)

For the purpose of testing a distribution fit to this data it was therefore necessary to group the cells. The zero cell was allowed to stand alone, the one-plus-two purchase frequency cells, the three-plus-four cells, etc., were grouped together.

### Soup

In the early 1970s the packaged-soup market in South Africa was dominated by the leading brand (here designated brand A) followed at some distance by the only other brand with a reasonable market share (designated brand B). Nine other brands made up the rest of the market. All brands were presented in uniform packagings of four to six servings each.

The data referred to as 'soup' in this paper were collected by Sichel (1975) as part of a South African National Dustbin Panel Survey. Housewives were provided with a bin into which they discarded empty packets of soup, the contents of which had been consumed by their families. Every twenty-eight days (i.e. at standard monthly intervals) the housewives were visited by interviewers who recorded the contents of their bins.

Information relating to 512 households' consumption for sixteen twenty-eight day periods was collated.

Soup consumption displayed a seasonal pattern. It dropped during spring and rose in autumn. Three separate stationary periods could thus be identified:

- Periods one through four (the first stationary winter period);
- periods seven through ten (the stationary summer period);
- periods thirteen through sixteen (the second stationary winter period).

Similar mean consumption levels were found for the two winter periods.

This set of data differs fundamentally from the toilet soap observations in that it contains consumption and not purchase data. Thus idiosyncrasies of individual households such as the

frequency of shopping and the number of units purchased on each shopping occasion are removed. Consumption should therefore produce a more uniformly distributed set of data.

### Never-Buyers

#### Never-buyers of the product field

In the toilet soap product field, all of the 614 households in the sample had purchased some soap during the eight-month period of the survey. Thus no never-buyers of toilet soap existed in the sample.

However, in the packaged-soup market, after two winter periods, 20 of the 512 households had not yet consumed. Sichel (1975) concluded that, because there were so few non-consumers, these 20 households did not provide sufficient evidence of the existence of never-consumers. This conclusion is accepted by the author and the analysis below is thus devoted to individual brands.

#### Never-buyers of individual brands:

A study of the soap and soup data reveals that even after eight and sixteen months respectively, substantial proportions of the households in the samples had not purchased/consumed the leading brands (Table 1).

**Table 1** Non-buyers of leading brands

Brand	Proportions of non-buyers over period studied
Lux	0,450
Palmolive	0,492
Sunlight Mild	0,590
Breeze	0,651
Shield	0,694
Soup Brand A	0,203
Soup Brand B	0,309

A visual inspection of the brand purchase frequency distributions revealed certain interesting phenomena. First, in almost all cases a kink or a flattening out, and sometimes even a slight maximum, occurred at about the second or third cell. In the case of the toilet soap distributions, the problem of the consumers' favouring of even-numbered purchases (discussed above) may have been contributing towards this effect. Secondly, in visually sweeping a smooth curve upwards through the data points, one would have expected such a curve to intercept the frequency axis at a much lower point than that actually recorded. An example from each data set is shown in Figures 1 and 2.

The hypothesis put forward is that these two effects are caused by an 'excess' of zeros in the first cell of the distributions, and that this excess in fact comprises never-buyers/consumers of the brand.

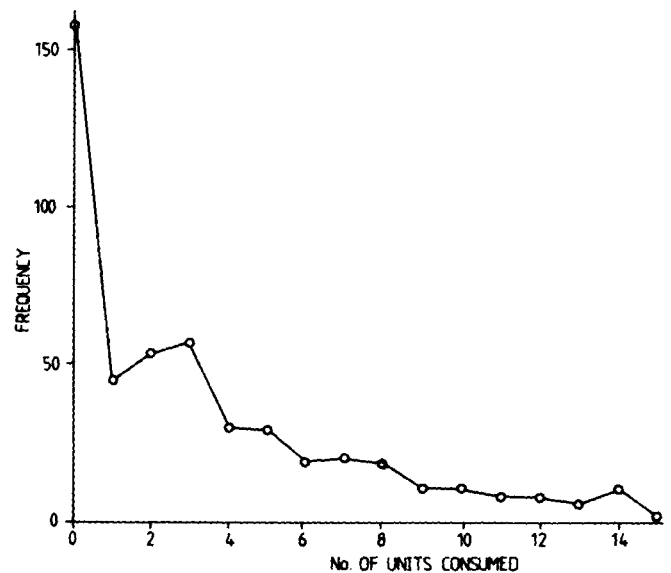
It was anticipated that the observations could best be modelled using a zero-augmented distribution (See (c) in Appendix).

### Results

Analysis of each of the data sets is discussed separately.

#### Analysis of the Soap Data

Goodman (1979) found that the fit of the Sichel distribution (Sichel, 1971) with  $\gamma = -2,5$  ((b) in Appendix) was generally superior to the NBD with respect to the toilet soap product class (all brands included). As a result of this finding, she ex-



**Figure 2** Observed frequency distribution for combined brands A and B of soup — first four months (N = 614)

amined only the Sichel distribution when analysing the individual brands within the product field.

According to Sichel (1980), individual brands need not necessarily follow the same distribution as that postulated for a total product field, and therefore Goodman's (1979) omission in not studying the zero-augmented NBD as a potential model for the individual brands is open to some criticism.

This study therefore began by attempting to fit a zero-augmented NBD model to the individual brand data for Lux and Palmolive soaps. Earlier research (Firer & Barnett, 1979) had shown that for all time periods studied, the *chi*-square test rejected the hypothesis of an ordinary NBD model for the data grouped as suggested above.

Parameter estimates for the model were made on ungrouped data. As can be seen from Table 2, the ungrouped data 'failed' the *chi*-square test. A detailed study of the tests revealed that the 'odd-even' purchase phenomenon was responsible for a substantial proportion of the total *chi*-square. The tests were thus repeated using grouped data. The zero-augmented Sichel distribution ( $\gamma = -0,5$ ) was then fitted to the same grouped data using the same tail-end grouping of cells of expected frequency less than 5,0.

From Table 2 it can be seen that, contrary to Goodman's expectations, the better fit was provided by the zero-augmented NBD model. An example of the fits obtained is shown in Figure 3.

Systematic deviations of the two models were found. The Sichel distribution was first too low, then too high, then too low again, relative to the NBD. According to Sichel (1980) the nature of the deviations led one to the conclusion that decreasing the value of  $\gamma$  from  $-0,5$  to  $-1,5$  or  $-2,5$  would not significantly improve the fit.

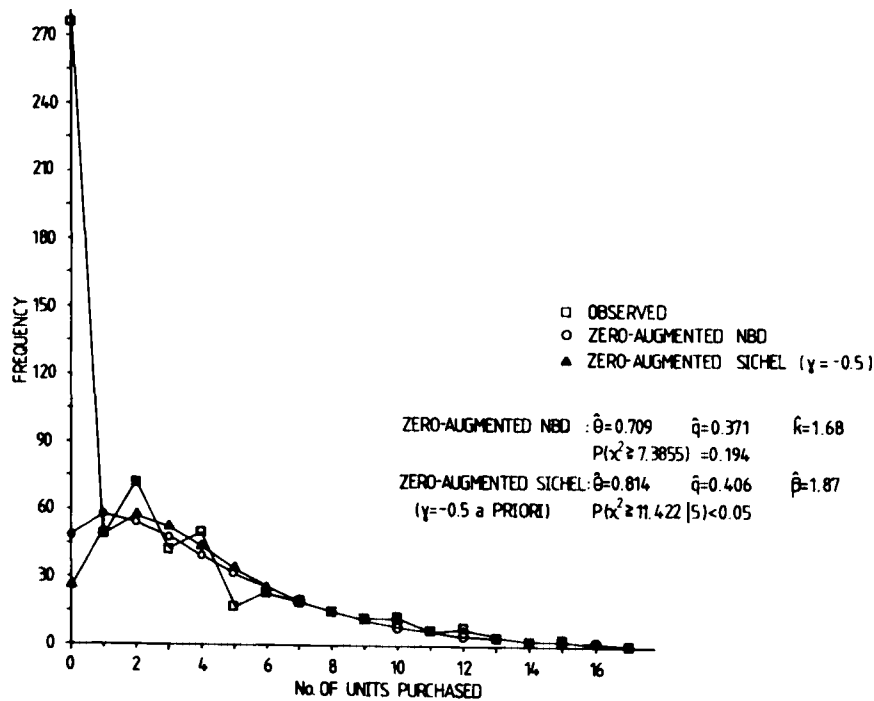
It was therefore concluded that, in the toilet soap field, individual brand purchase frequencies are best modelled by the univariate zero-augmented NBD.

One of the implications of the zero-augmented model is that estimates obtained for  $\hat{q}$ , the proportion of never-buyers in the sample, should be independent of time. Table 3 shows the results obtained from the fitting of both the NBD and Sichel zero-augmented models. These estimates are certainly not constant.

**Table 2 Soap: Results of chi-square tests for zero-augmented distributions**

	NBD Ungrouped	NBD Grouped	Sichel ( $\gamma = -0,5$ ) Grouped
<b>Lux</b>			
1st 2 months	$P(\chi^2 \geq 1,531   2) < = 0,465$	*	*
1st 4 months	$P(\chi^2 \geq 16,379   5) < = 0,050$	$P(\chi^2 \geq 2,023   2) = 0,364$	$P(\chi^2 \geq 2,548   2) = 0,275$
6 months	$P(\chi^2 \geq 29,982   8) < = 0,050$	$P(\chi^2 \geq 5,747   3) = 0,124$	$P(\chi^2 \geq 7,791   3) = 0,051$
8 months	$P(\chi^2 \geq 19,566   11) = 0,052$	$P(\chi^2 \geq 7,385   5) = 0,194$	$P(\chi^2 \geq 11,422   5) < 0,050$
<b>Palmolive</b>			
1st 2 months	$P(\chi^2 \geq 1,482   2) = 0,477$	*	*
1st 4 months	$P(\chi^2 \geq 25,474   5) < 0,050$	$P(\chi^2 \geq 2,536   1) = 0,111$	$P(\chi^2 \geq 2,916   1) = 0,088$
6 months	$P(\chi^2 \geq 28,420   7) < 0,050$	$P(\chi^2 \geq 5,991   3) = 0,112$	$P(\chi^2 \geq 8,398   3) < 0,050$
8 months	$P(\chi^2 \geq 27,037   9) < 0,050$	$P(\chi^2 \geq 9,181   4) = 0,057$	$P(\chi^2 \geq 12,931   4) < 0,050$

\*No degrees of freedom left for chi-square tests



**Figure 3** Observed and fitted zero-augmented distributions for Lux soap — eight months (N = 614)

**Table 3 Soap: Estimates of proportion of never-buyers  $\hat{q}$  derived from zero-augmented models**

	Proportion of never-buyers	
	NBD	Sichel
<b>Lux</b>		
1st 2 months	0,634	0,635
1st 4 months	0,498	0,515
6 months	0,410	0,440
8 months	0,371	0,407
<b>Palmolive</b>		
1st 2 months	0,686	0,686
1st 4 months	0,474	0,503
6 months	0,450	0,485
8 months	0,376	0,430

the proportion of never-buyers in the sample, whereas the proportion of non-buyers as predicted by Ehrenberg's theory would tend to zero with lengthening time periods (Ehrenberg, 1972).

Figure 4 is an example of the observed and expected proportions of non-buyers of Lux soap. The expected proportions were derived using Equation A4. The parameter estimates were made by the method of maximum likelihood using the observations of the first two-month period.

In no case did the observed proportion lie above that predicted by Ehrenberg's theory. As a result of these findings, the hypothesis of the existence of never-buyers for individual brands in the toilet soap market was rejected.

**Analysis of the Soap Data**

In this analysis the non-consumer over the entire sixteen-month period of the survey were considered, despite the fact that the data did not exhibit stationarity over the period studied.

Although the use of the sixteen-month period is theoretically not correct, it does at least give some guidance as to the proportion of non-consumers over a lengthy period. As will be seen from Figure 5, the observed point plotted at sixteen

If never-buyers do exist, one would expect the observed proportion of non-buyers of a product to eventually level out at

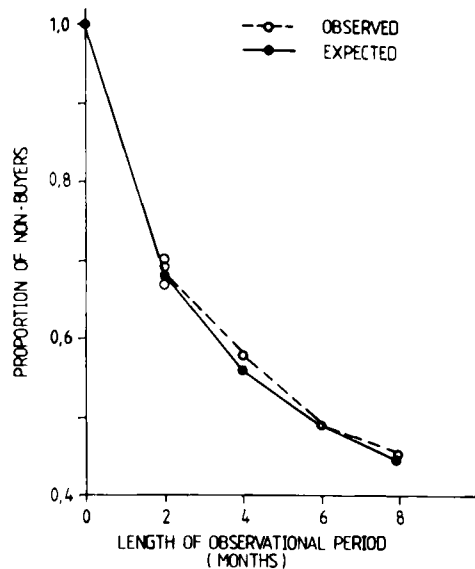


Figure 4 Proportion of non-buyers of Lux soap as a function of length of observational period (N = 614)

months is well within the general sweep of the curve.

The consumption frequency distributions of the individual brands were not modelled except for the case of the combined brands A and B over the eight stationary winter months. The results of the *chi-square* tests on the two zero-augmented models were:

$$\begin{array}{ll} \text{NBD} & P(\chi^2 \geq 25,732 \mid 27) = 0,531 \\ \text{Sichel} & P(\chi^2 \geq 33,304 \mid 26) = 0,166 \end{array}$$

The NBD model, which provided the better fit, gave an estimate of zero for the proportion of never-consumers. Because of this result, the fitting of models to the individual brands was omitted, and the graphs of observed and expected (via Ehrenberg's NBD theory) proportions of non-consumers of brand A, brand B and brands A and B combined were immediately studied.

The parameter estimates for brands A and B were based on averages of each of the first four months' estimates, calculated using the method of moments. For the combined brands the maximum likelihood parameter estimates for the first month were used. It was found that the observed proportions of non-consumers approached zero more rapidly than the expected proportions as predicted by the Ehrenberg theory. An example of the observed and expected proportions of non-consumers is shown in Figure 5.

On the basis of this evidence, one must therefore also reject the hypothesis of the existence of never-consumers of individual brands in the two data sets studied.

### Conclusions

For the two data samples available, no evidence could be found to support the hypothesis that the apparent excessive size of the zero cell in the purchase/consumption frequency distribution was due to the superimposition of a buying/consuming population and a group of never-buyers/consumers.

The  $\hat{q}$  parameters of the zero-augmented NBD (which it was hoped would provide an estimate of the proportion of never-buyers of toilet soap) were not constant over the various time periods studied. In addition, with lengthening time periods, the observed proportion of non-buyers decreased at almost exactly the same rate as that predicted by the Ehrenberg NBD

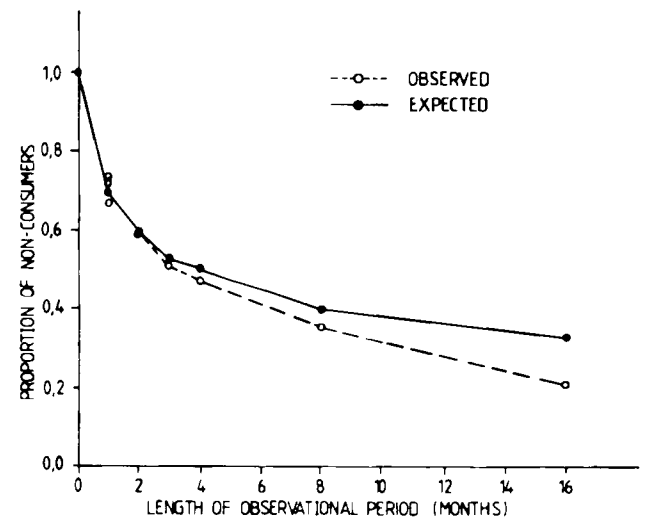


Figure 5 Proportion of non-consumers of brand A soup as a function of length of observational period (N = 512)

theory. This theory does not provide for the existence of never-buyers. A similar calculation was made for the individual brands in the soup product field.

Therefore, on the basis of the available evidence in two product fields, the never-buyer/consumer hypothesis was rejected.

The Binary Model of Joannides (1976) should thus be considered as the explanation for the observed distribution shapes. He showed that the marginal distributions obtained by summing a correlated NBD, representing loyal repeat-buyers, and an uncorrelated NBD, representing brand-switching/multi-brand buyers, displayed high zero cell frequencies as well as points of inflection or secondary modes close to the zero cell.

Thus the results obtained, together with a consideration of the Joannides model, lead one to the conclusion that perhaps brand-disloyal buyers/consumers are better represented by a distribution of frequencies rather than the subset thereof (zero cell frequencies only) implied by the never-buyer hypothesis.

These results have important implications for the marketers of consumer non-durables. Instead of devoting time and effort in persuading never-buyers to use their products (in a generic sense), they would be strongly advised to undertake advertising campaigns aimed at wooing existing users of other brands of the product into trying their particular brand.

Finally, this study highlighted the need to model the field of consumer non-durable purchase patterns from a multibrand perspective, rather than by looking at individual brands in isolation. Research in this area has been undertaken and will be reported on in the future.

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**Appendix**

A brief description of the statistical distributions mentioned in the text is given here.

**(a) The NBD Repeat-buying Model**

*(i) Univariate Case*

The model assumes that an individual household's purchases are characterized by a Poisson distribution.

In order to study market behaviour as a whole, the individual household's distributions are compounded using a  $\gamma$  distribution for the individual average purchase frequencies per unit time (Sichel, 1971).

The result is the Negative Binomial Distribution (NBD):

$$f(x|t) = \frac{(k/(k+tm))^k \Gamma(x+k) \{tm/(k+tm)\}^x}{\Gamma(k) \Gamma(x+1)} \quad (A1)$$

where  $x$  is the number of purchases in time  $t$ ,

$0 < m < \infty, 0 < k < \infty$ , and  $x = 0, 1, 2, \dots$

The proportion of buyers of the product in time  $t$  is given by

$$b_t = 1 - (1 + tm/k)^{-k} \quad (A2)$$

where a buyer is defined as one who buys at least one unit during time period  $t$ .

*(ii) The Bivariate Correlated NBD*

The bivariate model is obtained by mixing the product of two univariate Poisson distributions with the  $\gamma$  distribution, to form the bivariate correlated NBD model.

In its symmetric form (where successive purchase periods are of equal length) it can be written

$$f(x,y|t) = \frac{(1 + 2tm/k)^{-k} \Gamma(x+y+k) \{tm/(k+2tm)\}^{x+y}}{\Gamma(k) \Gamma(x+1) \Gamma(y+1)} \quad (A3)$$

where  $k > 0, m > 0, k(t) = k, m(t) = tm, x = 0, 1, 2, \dots, y = 0, 1, 2, \dots$

The market penetration (proportion of buyers of the product in time  $t$ ) is given by

$$b_t^2 = 1 - (1 + tm/k)^{-k} \quad (A4)$$

*(iii) The Uncorrelated Bivariate NBD*

Sichel (1980) derived the uncorrelated bivariate NBD in an attempt to model the behaviour of product-disloyal consumers (brand-switchers and multibrand buyers) whose consumptions are uncorrelated between two time periods. The uncorrelated NBD is obtained from the product of two univariate NBDs. It can be written in the form

$$f(x,y|t) = (1 + m/k)^{-2k} (1 + k/m)^{x+y} \binom{x+k-1}{x} \binom{y+k-1}{y} \quad (A5)$$

$$k > 0, m > 0, k(t) = tk, m(t) = tm,$$

$$x = 0, 1, 2, \dots, y = 0, 1, 2, \dots$$

In the case of this distribution, the proportion of buyers is

$$b_t^2 = 1 - (1 + m/k)^{-tk} \quad (A6)$$

**(b) The Sichel distribution**

This distribution was also derived by Sichel in 1971. It can be written in the form

$$\phi(r) = \frac{(\sqrt{1-\Theta})^\gamma (\alpha\Theta/2)^\gamma}{K_\gamma(\alpha\sqrt{1-\Theta})r!} \cdot K_{r+\gamma}^{(\alpha)} \quad (A7)$$

$$r = 0, 1, 2, \dots$$

It is derived by mixing the Poisson distribution with the flexible mixing distribution

$$f(\lambda) = \frac{[2\sqrt{(1-\Theta)}/\alpha\Theta]^\gamma}{2K_\gamma(\alpha\sqrt{1-\Theta})} \cdot \lambda^{\gamma-1} \cdot \exp\{-\frac{1}{\Theta}\lambda - \frac{\alpha^2\Theta}{4\lambda}\} \quad (A8)$$

In this distribution  $-\infty < \gamma < \infty, 0 < \Theta < 1$  and  $\alpha > 0$  are the three parameters and  $K_\gamma(\cdot)$  is the modified Bessel function of the second kind or order  $(\cdot)^\gamma$ .

$\phi(r)$  represents a family of discrete distributions. Many of the better known discrete distributions such as the NBD, Poisson, geometric, etc., are special or limiting forms of  $\phi(r)$  (Sichel, 1974).

Making gamma negative allows one to generate an entirely new set of discrete distributions. The distributions are mathematically difficult to handle unless  $\gamma$  is a half-integer.  $\gamma$  could therefore be set, *a priori*, to  $-0,5$  and values of  $-1,5$  or  $-2,5$  would be used in order to try and improve the fit if  $\gamma = -0,5$  is not satisfactory.

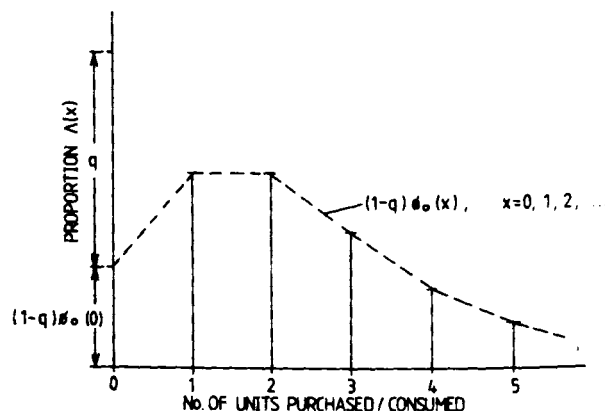
Two parameters,  $\alpha$  and  $\Theta$ , have therefore to be estimated. If the proportion of observed frequencies in the zero cell is large (generally above 50%), the method of equating mean and zero cell proportions is fairly efficient (Sichel, 1973). If this is not the case, a maximum likelihood method is available (Sichel, 1971). However, this method is very complicated, so when the zero cell is small, the method of moments is generally used (Sichel, 1974).

**(c) Zero-augmented distributions**

A large number of observed purchase/consumption frequency distributions have an (what appears to the eye), excessively large zero cell. Irrespective of the true cause of this effect, one requires to be able to model such distributions.

In order to do so, it is assumed that this cell is made up of two separate groups. The situation is illustrated graphically in Figure A1.

An implication of this model is that, if a hard core of never-buyers does exist, the estimate of  $q$  should be independent of time. The values of  $q$  for different brands within a product field need not, of course, be similar.



**Figure A1** A graphical representation of never-buyers

Morrison (1969) showed that the existence of never-buyers would introduce a systematic bias into the NBD model. He therefore proposed a model of the form given above. In this model three parameters require estimation: the  $k$  and  $m$  parameters of the NBD and  $\hat{q}$ , the proportion of never-buyers.

Sichel (1980) proposed a similar model based on the Sichel distribution. Here, too, three parameters require estimation:  $\alpha$  and  $\Theta$  of the Sichel, and  $\hat{q}$  the proportion of never-buyers.

The techniques of estimation of the parameters for both distributions are discussed by Firer (1980).