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## **Article**

Dating the business cycle: Evidence from Mongolia

Central Bank Review (CBR)

# **Provided in Cooperation with:**

Central Bank of The Republic of Turkey, Ankara

Suggested Citation: Luvsannyam, Davaajargal; Batmunkh, Khuslen; Buyankhishig, Khulan (2019): Dating the business cycle: Evidence from Mongolia, Central Bank Review (CBR), ISSN 1303-0701, Elsevier, Amsterdam, Vol. 19, Iss. 2, pp. 59-66, https://doi.org/10.1016/j.cbrev.2019.06.001

This Version is available at: https://hdl.handle.net/10419/217331

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Contents lists available at ScienceDirect

# Central Bank Review

journal homepage: http://www.journals.elsevier.com/central-bank-review/



# Dating the business cycle: Evidence from Mongolia

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b IMF, Mongolia



#### ARTICLE INFO

Article history: Received 20 February 2019 Received in revised form 25 April 2019 Accepted 12 June 2019 Available online 27 June 2019

JEL classification: C18 E17 E32

Keywords:
Business cycle
BBQ
l<sub>1</sub> trend filter
HP filter
Expansion
Contraction
Hamilton filter
BN filter

#### ABSTRACT

Business cycle is an important indicator for making policy and management decisions. This paper compares the business cycle estimates for Mongolia based on a graphical and parametric methods. We find that Bry Boschan Quarterly (BBQ) algorithm accurately dates the business cycle which is consistent with the economic expectations. When we indicate the result of Bry Boschan Quarterly algorithm as the benchmark for the business cycle,  $\ell_1$  trend filter provides a more precise estimate of the output gap for Mongolia.

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### 1. Introduction

Dating of business cycle is very crucial for policy makers and businesses. Business cycle is the upward and downward trend of the production or business. Especially macro business cycle, which represents the general economic prospects, plays an important role in policy and management decisions. For instance, when the economy is in downtrend, companies tend to act more conservative. In contrast, when the economy is in uptrend, companies tend to act more aggressive with the purpose of enhancing their market share. Also, one of the biggest challenges for countries with rich natural resources is to implement a countercyclical policy. A countercyclical policy aims to save the windfall in times of high commidy prices and stimulate the economy in times of commodity price collapse by spending the windfall. Keynesian business cycle theory suggests that the business cycle is an important indicator for

monetary policy to stabilize the fluctuations of the economy. Therefore, an accurate dating and forecasting of the business cycle can be fundamental to efficient and practical policy decisions.

Defining the business cycle continues to remain one of the important topics among economists and researchers. Bersch and Sinclair (2011) compare the output gap estimates for Mongolia based on a number of different methods. They find that a Blanchard and Quah-type joint model of output and inflation provides a more robust estimate of the output gap for Mongolia, Canova (1994), (1998examines the sensitivity of turning points classification to different detrending methods and the ability of each method to replicate NBER (National Bureau of Economic Research) dating. The output series detrended with the Hodrick and Prescott (HP) filter reproduce all NBER turning points with at most two quarters lead or lag, regardless of the dating rule used. Yamada and Jin (2013) estimates Japan's output gap using the recently developed  $\ell_1$ trend filter, which is an alternative to the popular HP filter. As they suggest this new filter provides a piece wise linear trend line, which means it possibly provides better output gap estimates than the HP filter does for an economy such as Japan that had experienced some structural breaks. Hamilton (2017) gives three reasons why one should not use the HP filter: (1) the HP filter produces series with

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Peer review under responsibility of the Central Bank of the Republic of Turkey.

spurious dynamic relations that have no basis in the underlying data-generating process, (2) filtered values at the end of the sample are very different from those in the middle, (3) there is a better alternative. Furthermore, the unobserved component model introduced by Watson (1986) and Clark (1987) is popular alternative method to detrend output in which both model the trend as random walk and the cycle as AR process.

In the academic study, the dating process of the business cycle has been evolved from a graphical orientation to quantitative measures extracted from parametric models. For instance, Burns and Mitchell (1946) explained the main concepts of the business cycle and introduced a graphical (classical) model that aims to calculate the peak and trough of the cycle. While Cooley and Prescott (1995) started to calculate the cycle by using the variable moments of the parametric (detrend) models. In this paper, we will calculate the cycle using both types of model and compare each parametric models with their ability to replicate the result of a graphical model. In other words, the secondary purpose of this paper is to find the most suitable parametric model for estimating the business cycle of Mongolia. The paper is organized as follows: the next section explains the basic concepts of business cycle such as peak, trough, duration and amplitude. In section 3, we review the Bry Boschan Quarterly algorithm, Beveridge-Nelson, Hodrick Prescott and other filters. In section 4 and 5, we present the empirical results and conclude this paper, respectively.

#### 2. Definition and illustration of the business cycle

Burns and Mitchell (1946) define that business cycle is a pattern seen in any series  $Y_t$  taken to represent aggregate economic activity. In the process of defining a cycle, we usually use the logarithm of any series  $Y_t$  ( $y_t = \ln(Y_t)$ ). Business cycles are identified as having four distinct phases: trough, expansion, peak, contraction (Fig. 1).

Characteristics of a cycle are defined as follows: Peak (A) is the turning point when the expansion transitions into the contraction phase. Trough (C) is the turning point when the contraction transitions into the expansion phase. Duration (AB length) is the number of quarters between peak and trough. Duration differs through the time measurements such as year, quarter or month. For example, if duration is equal to 4 with quarterly basis, contraction phase lasts around 4 quarter. Amplitude (BC length) is the height of differences between peak and trough. Amplitude measures the deepness of contraction. These characteristics are illustrated in Fig. 2.



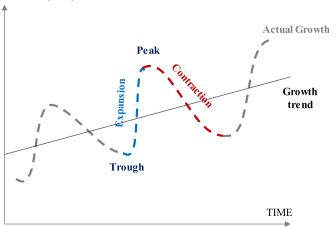


Fig. 1. Business cycle.

#### 3. Methodology

#### 3.1. Bry and Boschan quarterly (BBQ) algorithm

Recognising the turning points in the series is the very first step of detection and description of any cycle. After that, mark off periods of expansions and contractions use those dates. Sometimes we can visually detect the location of turning points. While performing the dating with the eye is also possible at filtering out "false turning points" i.e. movements which are either short-lived or of insufficient amplitude. Algorithm which translates the ocular judgments needs to perform at least three steps.

Step 1. Determination of possible turning points i.e. the peaks and troughs in a series.

Step 2. A procedure for alternating peaks and troughs

Step 3. A set of rules that re-combine the turning points established after steps one and two in order to satisfy predetermined criteria concerning the duration and amplitudes of phases and complete cycles.

Bry and Boschan (1971) introduced BB algorithm which performs these tasks associated with the NBER for monthly observations on a series. Determination of a local peak or trough as happening at time t is the core step of the algorithm.

$${a_{t-n} < a_t > a_{t+n}}, n = 1, ..., N$$
 (1)

$$\{b_{t-n} > b_t < b_{t+n}\}, n = 1, ..., N$$

In equation (1),  $a_t$  is the peak,  $b_t$  is the trough and n is generally set to five. The main criteria relating to the third step are that a phase must last at least six months and a complete cycle should have a minimum duration of fifteen months. When the data is measured at the quarterly frequency an analog to the first step of the BB algorithm would be to put n=2,  $a_t$  is a local maximum relative to two quarters.

$$\{\Delta_2 a_t > 0, \ \Delta a_t > 0, \ \Delta a_{t+1} < 0, \ \Delta_2 a_{t+2} < 0\}$$
 (2)

According to Harding and Pagan (2002a), BBQ is described as the quarterly version of the BB algorithm, combined with some important rules.

#### 3.2. Beveridge-Nelson (BN) filter

Beveridge and Nelson (1981) define the trend of a time series as its long-horizon conditional expectation minus any a priori has

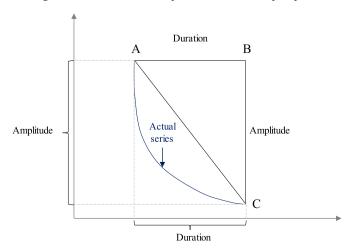
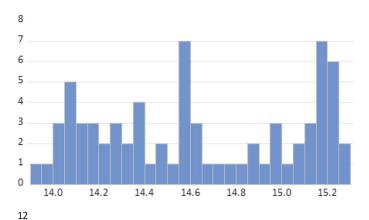


Fig. 2. Illustration of the contraction phase.

**Table 1** Descriptive statistics.



Variable	LRGDP_SA
Observation	72
Mean	14.63
Median	14.58
Maximum	15.29
Minimum	13.91
Standard.dev	0.43

10							_	
8								
6								
Ü								
4								
_								
2								
0								
-	14.0	14.2	14.4	14.6	14.8	15.0	15.2	
10								

Variable	LRGDP_T
Observation	72
Mean	14.63
Median	14.58
Maximum	15.28
Minimum	13.97
Standard.dev	0.43

8									
6									
4									
2									
0									
13.6	13.8	14.0	14.2	14.4	14.6	14.8	15.0	15.2	15.4

Variable	LRGDP
Observation	72
Mean	14.62
Median	14.62
Maximum	15.39
Minimum	13.61
Standard.dev	0.47

known future movements in the time series. In particular, letting  $\{y_t\}$  denote a time series process with a trend component that follows a random walk with constant drift, the BN trend at time t,  $\tau_t^{BN}$ , is

$$\tau_t^{BN} = \lim_{j \to \infty} E_t \left[ y_{t+j} - j * E[\Delta y_t] \right]$$
 (3)

By removing the deterministic drift,  $E[\Delta y_t]$ , the conditional expectation in (3) remains finite and becomes an optimal (minimum mean squared) estimate of the current trend component. To implement the BN decomposition, it is typical to specify a stationary forecasting model for the first differences  $\{\Delta y_t\}$  of the time series. Based on sample autocorrelation and partial autocorrelation functions for much macroeconomic time series, including the first differences of quarterly log real GDP, it is natural when implementing the BN decomposition to consider an AR(p) forecasting model.

$$\Delta y_t = c + \sum_{i=1}^p \varphi_i \Delta y_{t-i} + e_t \tag{4}$$

Kamber et al. (2018) imposed a lower signal-to-noise ratio, the resulting Beveridge-Nelson filter produces a more reliable and intuitive estimate of the output gap.

#### 3.3. Hodrick-Prescott (HP) filter

Hodrick and Prescott (1997) proposed a very popular method, which is commonly interpreted as decomposing an observed variable into trend and cycle. They proposed interpreting the trend component  $\mu_t$  as a very smooth series that does not differ too much from the observed  $y_t$ . It is calculated as

**Table 2**Result of BBO algorithm.

BBQ dates		Duration in Quarter					
Peak (P)	Trough (T)	Contraction	Expansion	Cycle			
		P to T	T to P	P to P	T to T		
2001Q4	2003Q1	5	5	10	8		
2004Q2	2005Q1	3	15	18	20		
2008Q4	2010Q1	5	6	11	8		
2011Q3	2012Q1	2	14	16	16		
2015Q3	2016Q1	2	3	5	_		
2016Q4	_	_	_	_	_		
	Mean	3.4	8.6	12	13		

$$\min \frac{1}{T} \sum_{t=1}^{T} (y_t - \mu_t)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} [(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})]^2$$
 (5)

When the smoothness penalty  $\lambda \to 0$ ,  $\mu_t$  would just be the series  $y_t$  itself, whereas when  $\lambda \to \infty$  the procedure amounts to a regression on a linear time trend (that is, produces a series whose second difference is exactly 0). The common practice is to use a value of  $\lambda = 1,600^1$  quarterly time series. Also, one sided HP filter is estimated by solving the following minimization problem:

$$\min \frac{1}{T} \sum_{t=1}^{T} (y_t - \mu_t)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} [(\mu_t - \mu_{t-1}) - (\mu_{t-1} - \mu_{t-2})]^2$$
 (6)

#### 3.4. l<sub>1</sub> trend Filter

 $\ensuremath{\mathfrak{l}}_1$  trend filtering is one of following the variation on HP filtering. The  $\ensuremath{\mathfrak{l}}_1$  trend filtering method produces trend estimates that are piecewise linear, and therefore it is well suited to analyzing time series with an underlying piecewise linear trend. The kinks, knots, or changes in slope of the estimated trend can be interpreted as abrupt changes or events in the underlying dynamics of the time series. By replacing square meaning of equation (5) to absolute meaning,  $\ensuremath{\mathfrak{l}}_1$  trend filtering can be estimated by minimization of the following equation:

$$\min \frac{1}{T} \sum_{t=1}^{T} (y_t - \mu_t)^2 + \frac{\lambda}{T} \sum_{t=2}^{T-1} \left| (\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1}) \right|$$
 (7)

#### 3.5. Markov regime switching filter

Hamilton (1989) explained a model by the typical historical behavior could be described with a first-order auto regression,

$$y_t = c_{s_t} + \Phi_{s_t} y_{t-1} + \varepsilon_t \tag{8}$$

with  $\varepsilon \sim i.i.d. \ N(0, \sigma^2)$ ,  $s_t$  is the realization of N-state Markov chain. Hamilton filter can be described by two equations in (9).

$$\xi_{t|t} = \frac{(\xi_{t|t-1} \odot \eta_t)}{1'(\xi_{t|t-1} \odot \eta_t)}$$
(9)

$$\xi_{t+1|t} = P^* \xi_{t|t}$$

Where  $\xi_{t|t}$  denotes a  $(2 \times 1)$  vector with conditional probability relative to  $s_t$ ,  $\xi_{t+1|t}$  denotes forecast of  $s_t$ , 1 denotes an  $(N \times 1)$  vector all of whose elements are unity,  $\odot$  denotes element-by-element multiplication and P denotes the matrix of transition probability.

#### 4. Empirical results

#### 4.1. Data

Our analysis uses quarterly real GDP<sup>2</sup> ( $RGDP_t$ ) for 2000Q1 until 2017Q4.  $LRGDP_t$  is the abbreviation of the logarithm of real GDP. Let's denote the seasonally adjusted real GDP with X-12 as  $RGDP\_SA_t$  and Tramo as  $RGDP\_T_t$ , respectively. We use the logarithmic series of  $LRGDP\_SA_t$ ,  $LRGDP_t$  series to calculate the cycle by BBQ algorithm and  $LRGDP\_SA_t$ ,  $LRGDP\_T_t$  series to calculate the cycle by filters (see Table 1).

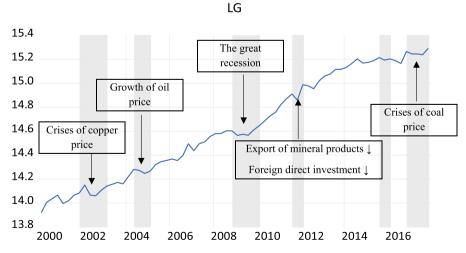


Fig. 3. Business cycle identified by BBQ algorithm.

<sup>1</sup> Hodrick and Prescott (1997), Farmer (1993).

<sup>&</sup>lt;sup>2</sup> Quarterly GDP data is only available from 2000Q1 in Mongolia.

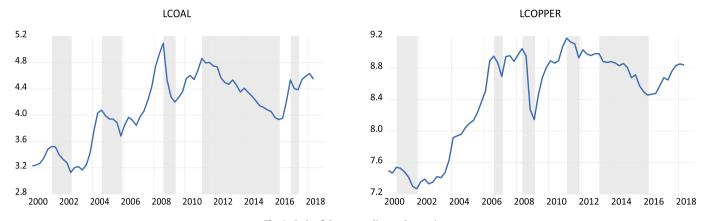
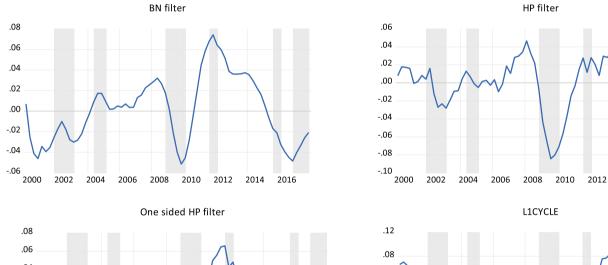


Fig. 4. Cycle of the commodity products price.

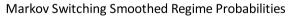


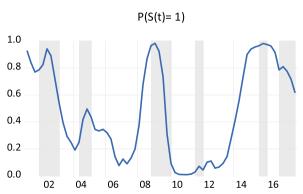




2014

2016





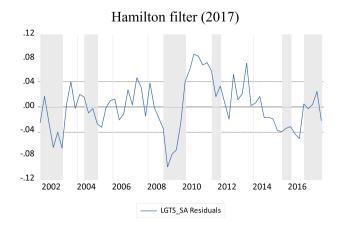


Fig. 5. Cycles estimated by filters (seasonally adjusted with Tramo).

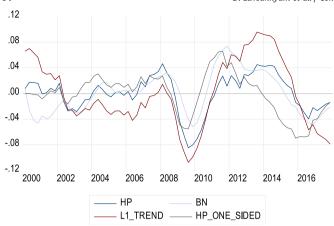


Fig. 6. A comparison of the business cycles.

#### 4.2. Business cycle identified by BBQ algorithm

We implemented BBQ algorithm<sup>3</sup> to calculate the business cycle of Mongolia. When estimating cycle with BBQ algorithm some paper use the seasonally adjusted data, while others don't. For the seasonally adjusted real GDP, the average duration of expansion and contraction are approximately 13 and 2 quarters, respectively. In contrast, just for real GDP, the average duration of expansion and contraction are approximately 9 and 4 quarters, respectively (Table 2). Average duration of expansion and contraction are more presice for the not seasonally adjusted real GDP. Because seasonal adjustment may change the dynamics of GDP and conflicts with the conditions of BBQ algorithm. So we used unadjusted real GDP for the BBQ algorithm only when detecting the turning points while we estimated output gap with seasonally adjusted data by using other filters.

Moreover, contraction phases of real GDP without seasonal adjustment can be explained by specific events that happened in the Mongolian economy. For example, we observed a collapse of the copper price during 2001Q4-2003Q1, a surge in oil prices during 2004Q2-2005Q1, recession from 2008Q4 through 2010Q1, contraction in minerals export and foreign direct investment from 2011Q3 through 2012Q1 and coal price collapse during 2016Q4 -2017Q4 (Fig. 3). In further analysis, we used seasonally adjusted real GDP series.

Khuslen and Davaajargal (2017) show that the economic export income cycles are very dependent on the world commodity market prices. We also implemented a BBQ algorithm to the prices of main mineral commodities such as copper, coal, gold, and oil. Fig. 4 shows the result of BBQ algorithm on commodity prices.

The cycle of mineral commodity price is a core defining factor of the Mongolian business cycle. Especially contraction phases of copper and coal prices occurred one to two quarters before the contraction phases of the business cycle of Mongolia. So, the BBQ algorithm defines the business cycle very efficiently.

#### 4.3. Business cycle estimated by filters

A result of Markov regime switch filter will show the probability of being in the contraction phase. When we estimate the cycle of real GDP (seasonally adjusted with X-12) with 6 types of filter, BN, HP, one sided HP and  $\ell_1$  trend filters are giving relatively similar results. Constant  $\lambda$  of HP, one sided HP and  $\ell_1$  trend filters are taken the value of 1600. Markov regime switch filter accurately identified all contraction phases. However, the probability of identification is too low for periods of 2004Q2 to 2005Q1 and 2011Q3 to 2012Q1. Other filters identified all contraction phases without mistake. But results show too noisy estimates of the cycle because of X-12 seasonal adjustment/Appendix 1/. Thus, we used TRAMO procedure to adjust seasonality of real GDP to get a smoother estimation of the cycle. Because when defining the business cycle, noisy estimation is not efficient.

Fig. 6 shows the correlation between cycles estimated by different filters. From Figs. 5 and 6, we can infer that BN, one sided HP and Hamilton filters estimate the cycle similarly. Also, HP and  $\ell_1$  trend filters give similar results.

In Fig. 5, all filters correctly identified first five contraction phases, but only  $\ell_1$  trend filter accurately identified last contraction phase which is the period of 2016Q4 to 2017Q4. We took first order differentiation from the estimated cycles and calculated the percentage of cycle identification by their match between signs ( $\pm$ ) and phases (expansion/contraction).

Fig. 7 shows that the percentage of cycle identification is high for  $\ell_1$  trend and HP filters as 59.15%, 57.75%, respectively. When the BBQ algorithm is a benchmark<sup>4</sup> for cycle identification,  $\ell_1$  trend

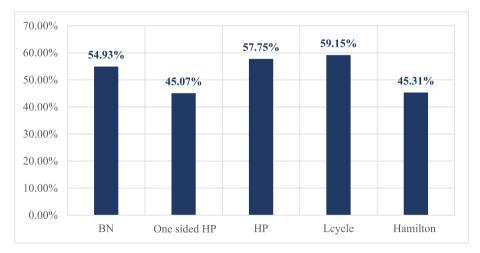


Fig. 7. Percentage of cycle identification (%, based on the sign).

<sup>&</sup>lt;sup>3</sup> Davaajargal and Khuslen (2017) have written the BBQ add-in of Eviews software.

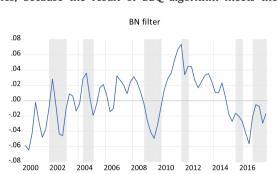
<sup>&</sup>lt;sup>4</sup> Canova (1994), Harding and Pagan (2002b).

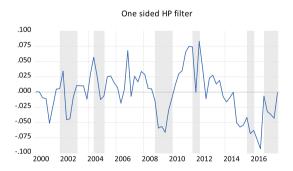
filter estimates the cycle with the lowest error. For the countries which has experienced several structural breaks  $\ell_1$  trend filter is quite promising and possibly estimates better cycle component. Mongolian economy has experienced some structural breaks during the estimation period. For the HP filter, filtered values at the end of the sample are very different from those in the middle. So, we implemented pseudo-real time analysis on HP and  $\ell_1$  trend filter. When we change the end period of sample,  $\ell_1$  trend filter is more consistent than HP filter/Appendix 2/.

#### 5. Conclusion

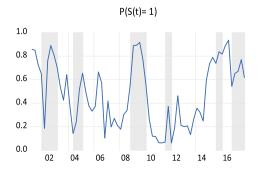
Defining and forecasting the country's business cycle accurately helps the policy makers both in the private and public sectors to make right decisions. To promote research-based policy making, many countries have established a separate institution to define the country's business cycles for policy making purposes. It is important for a country like Mongolia to estimate the business cycle on a continuous basis and apply it in economic decision-making. Therefore, establishing a separate institution for this purpose could be an option to pursue.

Mongolian business cycles which are identified by the BBQ algorithm matches the price cycles of coal, copper, gold, and oil. We claim that BBQ algorithm can be a good benchmark for identification of cycles, because the result of BBQ algorithm meets the





Markov Switching Filtered Regime Probabilities



economic expectations.

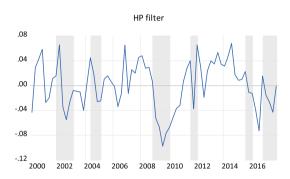
With the identification of BBQ algorithm, the collapse of copper price during 2001Q4 - 2003Q1, a surge in oil price during 2004Q2-2005Q1, economic recession during 2008Q4 -2010Q1, contraction in mineral exports during 2011Q3-2012Q1 and coal price shock during 2016Q4-2017Q4 drove all contraction phases in the Mongolian economy. This result suggests that the Mongolian economy is currently in the contraction phase. The average duration of the contraction phase is approximately 4 quarters, while the average duration of the expansion phase is approximately 9 quarters.

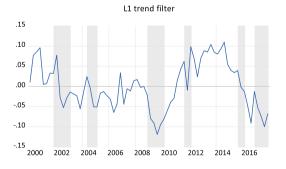
In this paper, we also compared several filters' estimation results of the cycles. When implementing filters for the cycle, the real GDP seasonally adjusted with Tramo gave smoother estimates than the real GDP seasonally adjusted with X-12.  $\ell_1$  trend filter most closely resembles the result of BBQ algorithm.

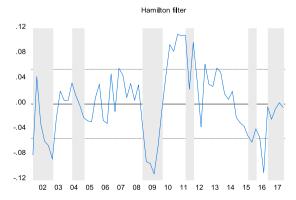
#### Acknowledgments

We would like to thank Tayyar Büyükbaşaran, the associate editor, and two anonymous referees. All remaining errors are entirely our own.

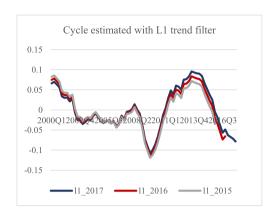
Appendix 1. Cycles estimated by filters (seasonally adjusted with X-12)

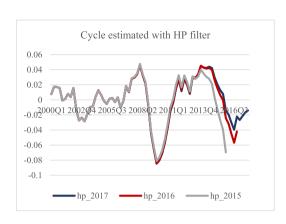






# Appendix 2. Cycles estimated by filters (seasonally adjusted with X-12)





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