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A new estimation technique of sovereign default risk☆

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ABSTRACT

Using the fixed-point theorem, sovereign default models are solved by *numerical value function iteration* and *calibration* methods, which due to their computational constraints, greatly limits the models' quantitative performance and foregoes its country-specific quantitative projection ability. By applying the Hotz-Miller estimation technique (Hotz and Miller, 1993)- often used in applied microeconometrics literature- to dynamic general equilibrium models of sovereign default, one can estimate the *ex-ante* default probability of economies, given the structural parameter values obtained from country-specific business-cycle statistics and relevant literature. Thus, with this technique we offer an alternative solution method to dynamic general equilibrium models of sovereign default to improve upon their quantitative inference ability.

1. Introduction

In developing countries, economic crises are often followed by sovereign debt crises, which results in sudden drop or halt in foreign credit to real sector, a surge in cost of borrowing in the international financial markets, a sharp decline in foreign trade, a slowdown in economic growth, and as a consequence a decline in consumption and investment.[1] To avoid the aforementioned large negative impacts of sovereign debt crises, it is crucial for policy-makers to identify the optimal external borrowing need, assess debt sustainability, and understand the (speculative) pricing strategies of foreign investors. In addition, considering the high stakes, it also becomes crucial for foreign investors to identify default tendencies of countries during high global/local economic volatilities. With this motivation, in order to make the aforementioned quantitative inferences and to estimate the ex-ante default probabilities of countries, we develop the framework to apply the Hotz-Miller estimation technique on dynamic general equilibrium models of sovereign default.

The Hotz-Miller estimation technique (Hotz and Miller, 1993) is popularly used in microeconomics literature as an alternative to *Nested Fixed Point* (NFXP) estimation technique. This technique estimates the structural parameter values of a recursive competitive partial equilibrium (general equilibrium under some conditions) model with discrete choice, given the actual probability of the (discrete) endogenous choice extracted from real-time data. By applying this technique to the dynamic general equilibrium models of sovereign default, one can estimate the ex-ante default probability of economies for a given set of structural parameter values that are obtained from real business-cycle statistics and relevant literature. Thus, with this technique we offer an alternative solution method to dynamic general equilibrium models of sovereign default to improve upon their quantitative inference ability.

Macroeconomic studies on sovereign default literature became popular in the 1980s, with the seminal work of Eaton and Gersovitz, (1981) which studies sovereign default through an exchange economy general equilibrium model. However, during the period of 1990–2000 during which the financial problems of developing countries seemed to be predominantly concerned with privately issued debt and liquidity crises, this area of research lost attention. However, after Argentina's default in 2001, the macroeconomic literature on sovereign default revived with (Arellano, 2008) that reused the general equilibrium model of sovereign default introduced by the seminal paper of Eaton and Gersovitz, (1981).

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[1] Arteta and Hale (2008), Borensztein and Panizza (2008), Dias and Richmond (2009), Fuentes and Saravia (2010), Furceri and Zdzienicka (2011), Gelos et al. (2011), Rose (2005) and Yeyati and Panizza (2011) study the impacts of debt crises on economic activity.

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Following Arellano (2008), there is a still growing literature that focuses on identifying the factors that induce emerging economies to default despite the sanctions and economic costs[2] that followed[3]. The most common feature of the this literature is that they are all based on Eaton-Gersovitz model which endogenizes default. Almost all of these studies incorporate new features into the Eaton-Gersovitz model to refine its ability in matching the two most important empirical stylized facts of default: high debt-to-income ratio observed in developing countries, accompanied with high spread. Even though these studies achieve some improvements, their models' ability to predict default probabilities and optimal debt strategies is highly limited given their solution technique that is *numerical value function iteration* with *calibration*. For example, we often observe that the numerical value function iteration generates discontinuous policy functions. Moreover, calibratio[4] does not allow us to use the model to quantitatively study those countries that did not default in their past and binds the model to a fixed set of parameter values that sometimes are not economically intuitive.

By applying Hotz-Miller estimation technique to general equilibrium sovereign default models, this paper contributes to the literature in two aspects. First, by using this technique and avoiding calibration, we can quantify the debt-default strategies and most importantly estimate the ex-ante default probabilities of all countries, even those that have never defaulted. Second, through this technique, we avoid using the fixed-point theorem which together with calibration bind the model to a fixed set of parameter values that sometimes are not economically intuitive. As opposed to the numerical iteration function and calibration, this will give us room to study countries' debt-default strategies at any given set of parameter values that are economically intuitive and representative of their real business cycle facts.

The paper is organized as follows. In section 2, we define the sovereign default problem and characterize its recursive formulation. In section 3, we show the application of the Hotz-Miller estimation technique to our sovereign default problem. Section 4 will outline the econometric estimation. Finally, in section 5 we conclude.

2. The sovereign default model

The sovereign default model we introduce is adopted from Hatchondo and Martinez (2009). It is a dynamic general equilibrium (endowment) model, specifically with long-term debt and endogenous default[5]. The economy we characterize below is an emerging economy inhabited by a representative agent, in other words a sovereign.

2.1. The setup

The sovereign derives utility from consumption c_t. The sovereign's expected life-time utility is given by

$$U(c) = E\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \tag{1}$$

where the discount factor $\beta \in (0,1)$, the relative risk aversion parameter $\gamma \geq 1$.[6]

At period $t \geq 0$, the representative agent receives a stochastic endowment stream of a single tradable good, y_t, drawn from a compact set $Y = [\underline{y}, \overline{y}] \subset R_{++}$ with probability $\pi(y_t|y_{t-1})$ conditional on previous period realization of y_{t-1}. In this model, we can assume the income to follow an AR (1) process given as

$$\log y_t = (1-\rho)m + \rho \log y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma^2\right) \tag{2}$$

After the income realization, the sovereign is then required to repay its debt obligations. Given that the sovereign's debt stock is λ_t, its debt obligations is $[\delta + \kappa(1-\delta)]b_t$ where the first term is the portion of the debt that matures and the second term is the coupon payments of those that are still outstanding. However, the sovereign may opt to default, that is repudiate on its debt obligations, which in turn would incur an output loss of $\phi(y_t)$[7] and face autarky at the period of default. After, the sovereign does not incur a contraction[8] and with probability $(1-\mu)$ regains access to the capital markets. These assumptions are supported by empirical evidence.[9] On the other hand, if the sovereign opts to repay its debt, it maintains its access to the international capital markets to issue new debt in the form of long-term bonds, denoted by λ_{t+1}.

With the new bond issuance the economy's outstanding debt stock becomes

$$b_{t+1} = (1-\delta)b_t + \lambda_{t+1} \tag{3}$$

We assume $\delta \in (0,1)$ which implies that on average the bonds mature in $1/\delta$ periods. Note that if $\delta = 1$ the debt would become a one-period discount bond or if $\delta = 0$ and $\kappa > 0$ the debt would become a consol bond promising to pay κ units infinitely. In the above equation, $b_{t+1} < (1-\delta)b_t$ means that the sovereign is borrowing from the international financial markets by selling bonds, i.e. $\lambda_{t+1} < 0$ at a price of $q(b_{t+1}, y_t)$ set by risk-neutral investors. Additionally, $b_{t+1} > (1-\delta)b_t$ means that the sovereign is saving by purchasing bonds from the international financial markets amounting $\lambda_{t+1} > 0$ at the risk-free interest rate r. Finally,

[2] Default costs are identified as financial sanctions imposed by international creditors, decline in international trade, temporary output loss, and decline in output growth rate. Some of the papers that study default costs are Arteta and Hale (2008), Borensztein and Panizza (2008), Fuentes and Saravia (2010), Rose (2005) and Yeyati and Panizza (2011).

[3] For example, Aguiar and Gopinath (2006) proposes that it is more the permanent income shocks that induces emerging economies to accumulate debt. Additionally, Yue (2010) highlights the enforcing role of debt renegotiation and its recovery in an emerging economy's ex-ante incentive to default. More differently, Cuadra and Sapriza (2008) shows the direct effect of political uncertainty on the frequency of default by emerging economies (Lizarazo, 2013). characterizes foreign investors as risk averse agents (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009; Volkan, 2008). contribute mainly by increasing the term-structure of sovereign bonds. Finally, Bai and Zhang (2012), Mendoza and Yue, (2012), Park (2012) study impacts of sovereign default on production and investment.

[4] This is an estimation technique often used by macroeconomists to calibrate the values of the structural parameters of a general equilibrium model so that the model's simulated data statistics match to the target statistics obtained from the actual data.

[5] For another dynamic general equilibrium (endowment) model, specifically with long-term debt and endogenous default please see (Chatterjee and Eyigungor, 2012).

[6] When $\gamma = 1$ the period utility becomes logarithmic, i.e. $\ln(c_t)$.

[7] Empirical evidence shows that there are several mechanisms through which default affects output. Firstly, sovereign defaults are associated with direct output loss. It is evident that the direct loss increases when the economy suffers a banking crisis in addition to default. Second, sovereign defaults cause output loss through its restrains on trade, foreign direct investments and foreign as well as domestic credit to the private sector.

[8] Using quarterly data on defaults that occurred during 1970–2005 period (Yeyati and Panizza, 2011), finds that growth rates in post-default period are never significantly lower than in normal times. On the contrary, output reaches its minimum at the time of default and starts recovering after. Using yearly data on defaults that occurred during 1980–2000 period (Sandleris, 2012), finds similar results.

[9] Gelos et al. (2011) and Alessandro (2011) show that the duration of the exclusion is short-lived. Accordingly, in the aftermath of defaults, average period of autarky was 4.5 years during 1980–2000 and 2.9 years during 1990–2000.

$b_{t+1} = (1-\delta)b_t$ means that the sovereign is neither borrowing nor saving.

The risk-neutral investors price the bond, i.e. $q(b_{t+1}, y_t)$, so that they make zero-profit in expectations. Note that the investors can borrow at the risk-free interest rate and have perfect information regarding the emerging economy's endowment. Let $d(b_{t+1}, y_t)$ be an indicator function that represents the default decision of the sovereign at its outstanding debt stock and income. Accordingly, the function takes on value 1 for default and 0 for no default.

$$q(b_{t+1}, y_t) = \sum_{y_{t+1} \in Y} \left\{ \begin{matrix} [\delta + (1-\delta)[\kappa + q(\mathbf{b}(d(b_{t+1}, y_{t+1}), b_{t+1}, y_{t+1}), y_{t+1})]] \\ [1 - d(b_{t+1}, y_{t+1})]\pi(y_{t+1}|y_t) \end{matrix} \right\} \frac{1}{1+r}, \tag{4}$$

where $\mathbf{b}(d(b_{t+1}, y_{t+1}), b_{t+1}, y_{t+1})$ is the optimal debt policy of the sovereign at its outstanding debt stock and income given its default decision. As can be seen in the above equation, the price internalizes the information that in the event of default, they will not receive any repayment, otherwise, they will receive redemption of the debt that matures and the coupon payment due from the rest of the outstanding debt. Finally, this price is both the bid price of the new issues as well as the repurchase price of the outstanding bonds.

2.2. Recursive formulation

Let $\mathcal{V}(b, y)$ be the value function of the sovereign at the beginning of period t after the default or no default decision is made.

$$\mathcal{V}(b, y) = \max_{d(b,y) \in \{0,1\}} \{ dV_0(b, y) + (1 - d(b, y))V_1(y) \} \tag{5}$$

where d represents the optimal default policy of the sovereign for all pairs of outstanding debt and income. Thus, the state space $(b, y) \in \mathbb{B} \times Y \times \{0, 1\}$ consists of the outstanding debt stock, income and default decision at period t. We assume the debt stock level is from a compact set of $\mathbb{B} = [\underline{b}, 0] \subset \mathbb{R}_-$.

In the above equation, $V_0(b, y)$ is the value of no default defined as

$$V_0(b, y) = \max_{b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{y' \in Y} \mathcal{V}(b\prime, y\prime)\pi(y\prime|y)dy\prime \right\}, \tag{6}$$

with $c = y - [\delta + (1-\delta)\kappa]b - q(b\prime, y)[b\prime - (1-\delta)b] \geq 0$ and $\lambda\prime = b\prime - (1-\delta)b \leq 0$.

On the other hand, $V_1(y)$ is the value of default defined as

$$V_1(y) = \left\{ \frac{(y - \phi y)^{1-\gamma}}{1-\gamma} + \beta \sum_{y\prime \in Y} [\mu V_1(y\prime) + (1-\mu)\mathcal{V}(0, y\prime)]\pi(y\prime|y)dy\prime \right\}. \tag{7}$$

As introduced before with probability μ, the economy will stay in the default state, while with probability $(1-\mu)$ it regains access to the capital markets.

2.3. Definition of equilibrium

The equilibrium of the above recursive problem is a Markov Perfect Equilibrium which is characterized by a set of value functions $\{V_0(b, y), V_1(y), \mathcal{V}(b, y)\}$, a set of decision rules $\{d(b, y), \mathbf{b}(d, b, y)\}$, and a bond price

$$q(b\prime, y) = \sum_{y\prime \in Y} \left\{ \begin{matrix} [\delta + (1-\delta)[\kappa + q(\mathbf{b}(d(b\prime, y\prime), b\prime, y\prime), y\prime)]] \\ [1 - d(b\prime, y\prime)]\pi(y\prime|y) \end{matrix} \right\} \frac{1}{1+r},$$

such that given the bond price $q(b\prime, y)$, the set of decision rules $\{d(b, y), \mathbf{b}(d, b, y)\}$ solve the recursive problem defined by Equations (5)–(7).

3. Applying Hotz-Miller estimation technique

The application of the Hotz-Miller estimation technique will be illustrated using a stationary environment. Suppose the income in any period can take one of the values from $Y = \{y_1, y_2, y_3, \ldots, y_n, \ldots y_N\}$. Then transition of income can be given by a transition matrix $\Pi(y\prime|y)$. Note that AR (1) income process mentioned in Section 2 can be transformed into a Markov Process using the methodology introduced by Hussey and Tauchen (1991). Additionally, assume that in a given period, the country can choose among the following debt alternatives $b\prime \in (b_{-1}, b_0, b_1, b_2, b_3, \ldots, b_j, \ldots b_J)$ where b_{-1} or $j = -1$, represents the default decision and the following choices are the ascending debt level choices of the sovereign, respectively. Note also that b_0 or $j = 0$ represents the country's 0 debt level choice in the current period.

Let the state variable relevant for period utility be $s = (y, b, \varepsilon)$, such that $s \in \mathcal{S} = \mathcal{Y} \times \mathcal{B} \times \mathcal{F}$. First two state variables are already introduced. Then, following the functional form given in Section 2 the period utility can be written as

$$u(s, j) = \begin{cases} \dfrac{(y - \phi y)^{1-\gamma}}{1-\gamma} + \varepsilon_j & j = -1 \\ \dfrac{\left(y + [\delta + (1-\delta)\kappa]b - q(y, b_j)\left[b_j - (1-\delta)b\right]\right)^{1-\gamma}}{1-\gamma} + \varepsilon_j & j = 0, 1, 2, 3, \ldots J \end{cases},$$

where the additive unobserved components in the utility function (ε_j) have a continuous joint distribution function $G(\varepsilon)$ where $\varepsilon = (\varepsilon_{-1}, \varepsilon_0, .., \varepsilon_J)$.

Given the characterization above, the recursive formulation of the sovereign default model then becomes

$$V(s) = \max_{j\in J}\left[u(s,j) + \beta\sum_{s\prime\in s}\pi(s\prime|s,j)V(s\prime)\right], \tag{8}$$

where

$$\pi(s\prime|s,j) = \pi(y\prime,b\prime|y,b,j)g(\varepsilon\prime|y\prime,b\prime). \tag{9}$$

In the state transition in Equation (9), we assume that conditional on the state (y,b),[10] the unobserved component is conditionally independent over periods. This is a founding assumption in most microeconometric applications using the Hotz-Miller or Nested Fixed point environment. Violation of this assumption would bring more computational complexity, since then one should include all the relevant past states in the state transition equation for ε. Moreover presumably some of those past states will include the unobserved $\varepsilon\prime$ s, we will need to integrate out those components from the value functions. Therefore conditional independence assumption will be assumed throughout the representation of the estimation. However it should be clear that the technique can still be applied in a more general dependency structure in the transition of ε in the expense of computational flexibility.

For now, we will assume no autarky period, in other words, an instant access to the international capital markets after default. This assumption can be removed. However, for now, assuming no autarky provides us an applicative ease and removing this assumption presumably can only improve the quantitative performance of the estimation technique.

Given the setup presented above, the value equation (8) can be rewritten as the integrated valuation function

$$V(y,b) = \int\left(\max_{j\in J}\left[u_j(y,b) + \varepsilon(j) + \beta\sum_{y\prime\in Y}\pi(y\prime|y,j)V(y\prime,b\prime)\right]\times\right)g(d\varepsilon|y,b). \tag{10}$$

Now let the conditional choice probability (CCP) be defined as

$$P(b\prime|y,b) = \int I\left\{j = \arg\max_{j\in J}\left[V_j(y,b) + \varepsilon(j)\right]\right\}g(d\varepsilon|y,b), \tag{11}$$

where $V_j(y,b) = u_j(y,b) + \beta\sum_{y\prime\in Y}\pi(y\prime|y,j)V(y\prime,b\prime)$.

The right hand side of Equations (6), (7), and the bond price equation can be expressed in closed form given the distributional assumption about $G(\varepsilon)$. If ε are distributed extreme value type I, we have the following form for the conditional valuation functions.

$$V_{-1}(y,b) = \left\{\begin{array}{c} y - \phi y + \beta\mu\sum_{y\prime\in Y}\pi(y\prime|y)V_{-1}(y\prime,0) \\ +\beta(1-\mu)\sum_{y\prime\in Y}\pi(y\prime|y)\left(\lambda + \sum_{k=-1}^{J} e^{V_k(y\prime,0)}\right)\end{array}\right\} j = -1 \tag{12}$$

$$V_j(y,b) = \left\{\begin{array}{c} y + [\delta + (1-\delta)\kappa]b - q(y,b_j)\left[b_j - (1-\delta)b\right] \\ +\beta\sum_{y'\in Y}\pi(y\prime|y)\left(\lambda + \sum_{k=-1}^{J} e^{V_k(y\prime,b_j)}\right)\end{array}\right\} j = 0,1,..,J \tag{13}$$

where $\lambda = 0.577215665$ is the Euler's constant.

In Hotz and Miller (1993) representation, we know there is a one to one mapping between the conditional choice probabilities (CCPs) and the normalized valuation functions.

$$\ln\left(p_j(y,b) - p_{-1}(y,b)\right) = V_j(y,b) - V_{-1}(y,b) \quad where \quad j = 0,1,..,J$$

It remains how one can obtain the value of $q(y,b\prime)$ as a function of model parameters and fundamentals. It is given in the bond price equation that the value of $q(y,b\prime)$ depends on the solution to a fixed point. The natural way to accommodate this object in the Hotz-Miller framework is to make use of the CCPs. One can write the value of $q(y,b\prime)$ at a particular choice of debt level as:

$$q(y,b') = E_{((d'',b''),y'|y,b')}\left[[1-d'']\frac{[\delta + (1-\delta)[\kappa + q(y',b'')]]}{1+r}\right],$$

where $d'' = d(y\prime,b\prime)$ and $b'' = b(y\prime,b\prime)$ denotes the default and the debt level choices in the next period. The choice on the next period depends on the valuation function comparisons, therefore we can characterize the above equation in terms of CCPs and the finite number of $q(y,b)$'s given the discrete nature of the state space. Obviously existence of the unobserved shock in the utility function requires integrating this component.

$$q(y,b\prime) = \sum_{y'\in Y}\left\{\sum_{j\in J-\{0\}}^{J}\left[\frac{P\left(V_j(y\prime,b\prime) + \varepsilon_j - V_k(y\prime,b\prime) + \varepsilon_k\right)}{1+r}\right]\times\right\}\pi(y\prime|y)$$

The first term in the square brackets is the CCP of choosing action j, therefore we can write the simplified version of the above equation for q as

$$q(y,b') = \sum_{y\prime\in Y}\left\{\sum_{j=0}^{J}p_j(y\prime,b\prime)\frac{[\delta + (1-\delta)[\kappa + q(y\prime,b_j)]]}{1+r}\right\}\pi(y\prime|y),$$

and

$$q(y,b\prime) = \sum_{y\prime\in Y}\left\{\sum_{j=0}^{J}p_j(y\prime,b\prime)\frac{\delta + (1-\delta)[\kappa + q(y\prime,b_j)]}{1+r}\right\}\pi(y\prime|y)$$

[10] Transition from b to $b\prime$ is determined entirely by the choice j, therefore, in the following representations we will denote the function $\pi(y\prime,b\prime|y,b,j)$ by $\pi(y\prime|y,j)$.

for the possible values of $y\prime \in (y_1, y_2, \ldots, y_n, \ldots y_N)$ and $b'' \in \{b_0, b_1, \ldots, b_j, \ldots, b_J\}$, $q(y, b\prime) = H[(q(y_1, b_0), q(y_1, b_1), q(y_1, b_2), q(y_1, b_3), \ldots., q(y_3, b_3), \ldots, q(y_N, b_J))'']$.

Given the solutions to these functions, we may obtain the following representation of the problem using the stationarity of the problem. Let $\omega_j = E[\varepsilon_j | y, b\prime = b_j]$. In the case of Extreme Value Type I,

$$\omega_j = \lambda - \ln\Big(p_j(y, b)\Big)$$

Using the conditional valuation functions, the valuation function can be expressed as

$$V(y, b) = \sum_{j=-1}^{J} p_j(y, b)\Big\{u_j(y, b) + E\Big[\varepsilon_j\Big| y, b\prime = b_j\Big] + \beta \sum_{y\prime \in Y} \pi(y\prime|y) V(y\prime, b_j)\Big\}. \quad (14)$$

Let us substitute the conditional expectation of the errors and stack the M equations for each possible value of the state vector $(y, b) \in \mathcal{M} \equiv (\mathcal{Y} \times \mathcal{B})$. In compact matrix notation we get

$$V = \sum_{j \in J} P_j * [u_j + \omega_j + \beta F_j V], \quad (15)$$

where $*$ is the Hadaramad (element by element) product, V is the $M \times 1$ vector of value functions; P_j, u_j, and ω_j are $M \times 1$ vectors that stack the corresponding elements at all states for alternative j; and F_j is the $M \times M$ matrix of conditional transition probabilities $F_j = F(s\prime|s) = \pi(y\prime|y)P(b\prime|b_j)$. This system of fixed point equations can be solved for the value function V as a function of P where P is the $M(J+1) \times 1$[11] vector of CCPs

$$V = \Big(I_M - \beta F^U(P)\Big)^{-1}\left\{\sum_{j \in J} P_j * [u_j + \omega_j]\right\}, \quad (16)$$

where $F^U(P)$ is the $M \times M$ matrix of unconditional transition probabilities induced by P. Now, we can define and calculate the vector of expected utilities.

4. Estimation strategy

Standard estimates of dynamic discrete choice models involve forming the likelihood functions from the CCPs derived in Equation (11). This involves solving the value function for each iteration of the likelihood function. The method used to solve the value function depends on the nature of the optimization problems and falls into one of two cases; finite-horizon problems: in that case the solution will involve a backward induction starting from the last period of the model T $(t = 0, 1, \ldots, T)$; stationary infinite-horizon problem: the valuation is obtained by a contraction mapping as described in the model section. We will describe the estimation relying on the stationary infinite horizon environment, however model can be generalized to a finite life-cycle setting without loss of generality.

The estimation can be done in two ways, the first is a PML (as used in Aguirregabiria & Mira (2002) and the second is a GMM (as used in the original Hotz and Miller (1993). With M possible state variables, the PML needs to estimate the $\{P_j\}_{j=-1}^{J}$ probabilities which can be constructed as parametric or non-parametric functions of state variables. Assuming a parametric functional form for the P_j and denoting its dependence on the parameter vector θ as $p_j(y_m, b_m; \theta)$, the PML function can be obtained via maximizing

$$\widehat{\theta}_{PML} = \arg\max_{\theta}\left(\sum_{m=1}^{M}\sum_{j=-1}^{J} \ln\Big(p_j(y_m, b_m; \theta)\Big)\right), \quad (17)$$

Or a GMM estimator can be constructed using the inversion found in Hotz & Miller (1993). As already introduced, under the assumption that ε is distributed independently and identically as type I extreme values, then the Hotz and Miller inversion implies that

$$\ln\Big(p_j(y_m, b_m; \theta)/p_{-1}(y_m, b_m; \theta)\Big) = V_j(y_m, b_m; \theta) - V_{-1}(y_m, b_m; \theta) \quad (18)$$

for any normalized choice, but we set this choice to the default alternative conveniently. We can use the set of structural parameter values obtained from real business-cycle statistics and relevant literature to construct the valuation functions (specifically let $\Gamma = (\beta, \gamma, \rho, m, \sigma^2, \lambda, \delta, \kappa, \phi)$ denote those parameters used in the model introduced) up to the parameters θ of the CCPs. Then we can proceed to form empirical counterparts of equation (18) and estimate the parameters of the model. The moment conditions can be obtained from the difference in the conditional valuation functions calculated for choice j and the base choice -1. The following moment conditions are produced for a particular state variable (y_m, b_m):

$$\xi_{jm}(\theta) \equiv V_j(y_m, b_m; \theta) - V_{-1}(y_m, b_m; \theta) - \ln\Big(p_j(y_m, b_m; \theta) \times /p_{-1}(y_m, b_m; \theta)\Big). \quad (19)$$

Therefore, there are $J+1$ orthogonality conditions and thus $j = 0, \ldots, J$. Letting $\xi_m(\theta)$ be the vector of moment conditions at state m, these vectors are defined as $\xi_m(\theta) = (\xi_{0m}(\theta), \xi_{1m}(\theta), \ldots \xi_{Jm}(\theta))'$. Therefore, $E[\xi_m(\theta^o)|y_m, b_m; \Gamma]$ converges to 0 for every consistent estimator of true CCPs, $p_j(y_m, b_m; \theta)$, for $m \in \{1, \ldots, M\}$, and where θ^o is the true parameter of the model. Then the GMM estimate of θ is obtained via

$$\widehat{\theta}_{GMM} = \arg\min_{\theta}\left[1/M \sum_{m=1}^{M} \xi_m(\theta)\right]\prime\left[1/M \sum_{m=1}^{M} \xi_m(\theta)\right]. \quad (20)$$

4.1. Model moment functions and GMM estimator

Returning back to the model introduced. in Section 2, the moment conditions can be obtained by the differences in the choices considering the default and debt level choice. We have the following value function differences and moment conditions obtained from a particular state (y_m, b_m):

[11] $J+2$ is the number of choice alternative including the default $(b_{-1}, b_0, b_1, b_2, .., b_J)$. Knowing $J+1$ probabilities automatically gives the last one.

$$V_j(y_m,b_m)-V_{-1}(y_m,b_m)=\left\{\begin{array}{l}\dfrac{\left(y_m+\left[\delta+(1-\delta)\kappa\right]b_m-q(y_m,b_j)\left[b_j-(1-\delta)b_m\right]\right)^{1-\gamma}}{1-\gamma}\\ -\dfrac{(y_m-\phi y_m)^{1-\gamma}}{1-\gamma}\\ +\left[f\left(y\prime,b\prime|y_m,b\prime=b_j\right)\prime-\left(y\prime,b\prime|y_m,b\prime=b_{-1}\right)\prime\right]\\ \times\left(I_M-\beta F^U(P)\right)^{-1}\left\{\sum_{k\in J}P_k*[u_k+\omega_k]\right\}\end{array}\right\} \tag{21}$$

For $j=0,1,2,..,J$ and $m=1,..M$. $\xi_m(\theta)$ is the vector of moment conditions at state m as introduced before. This vector is defined as before:

$$\xi_m(\theta)=\begin{pmatrix}V_0(y_m,b_m)-V_{-1}(y_m,b_m)-\ln\left(\dfrac{p_0(y_m,b_m;\theta)}{p_{-1}(y_m,b_m;\theta)}\right)\\ \vdots\\ V_j(y_m,b_m)-V_{-1}(y_m,b_m)-\ln\left(\dfrac{p_j(y_m,b_m;\theta)}{p_{-1}(y_m,b_m;\theta)}\right)\\ \vdots\\ V_J(y_m,b_m)-V_{-1}(y_m,b_m)-\ln\left(\dfrac{p_0(y_m,b_m;\theta)}{p_{-1}(y_m,b_m;\theta)}\right)\end{pmatrix} \tag{22}$$

The optimal GMM estimator for, θ satisfies equation (20).

4.2. Estimation of the conditional choice probabilities

Let's denote the probability at the mth row of P corresponding to the jth choice as:

$$p_{mj}=f_j(y_m,b_m),$$

and also let p_m collect the choice alternatives for the state (y_m,b_m).

$$p_m=\left(p_{m,-1},p_{m0},\ldots,p_{mJ}\right)$$
$$=(f_{-1}(y_m,b_m),f_0(y_m,b_m),..,f_3(y_m,b_m))$$

The estimation methodology proposes a functional form for $f_j(y_m,b_m)$ and then the PML or the GMM estimates the function $f(.)$. For instance if we use logistic function for the probabilities and linear dependency to (y,b)[12] and using the fact that:

$$f_{-1}(y_m,b_m)+f_0(y_m,b_m)+..+f_J(y_m,b_m)=1$$

we obtain the following equations:

$$f_{-1}(y_m,b_m)=\frac{1}{1+\sum_{j=0}^{J}e^{a_j+b_jy_m+c_jb_m}}$$

$$f_0(y_m,b_m)=\frac{e^{a_0+b_0y_m+c_0b_m}}{1+\sum_{j=0}^{J}e^{a_j+b_jy_m+c_jb_m}}$$

$$\vdots$$

$$f_k(y_m,b_m)=\frac{e^{a_k+b_ky_m+c_kb_m}}{1+\sum_{j=0}^{J}e^{a_j+b_jy_m+c_jb_m}}$$

$$\vdots$$

$$f_J(y_m,b_m)=\frac{e^{a_J+b_Jy_m+c_Jb_m}}{1+\sum_{j=0}^{J}e^{a_j+b_jy_m+c_jb_m}}$$

Replacing the above functions to form the $F^U(P)$ and P_k terms in the value function differences in equation (21), and similarly obtaining $p_j(y_m,b_m;\theta)$ terms in equation (22) in terms of the functions defined for the CCPs, we obtain a natural framework for estimating the coefficients (a_0b_0,$c_0a_1b_1c_1$,…,a_j,b_j,c_j,…,a_J,b_J,c_J). The estimation can be done either by GMM or ML as introduced in Section 4. Once the coefficients are estimated, we can construct the CCPs associated with each state pair (y,b) using the logistic functions.[13]

4.3. Beyond the solution of the original model

The estimation technique we introduced for the sovereign default model in fact provides a solution to the model valuation functions by optimally estimating the CCPs consistent with those valuation functions. Therefore any policy simulation including generation of future paths from the equilibrium follows as usual as when the model is estimated traditionally. However, one should observe that other than finding the optimal solution to the model (which is essential and important for further analysis), the estimated CCP functions can serve for other purposes outside the model. For instance assuming a logistic form for the choice probabilities, once the coefficients are estimated, the flexible logit functional form gives us a tool for predicting default probabilities for a variety of possible states. Of those states, we are not restricted to the ones that are actually visited in the solution of the model in (y_m,b_m). This actually let the researcher to predict the default

[12] The p_{mj} functions can be constructed parametrically as the logit example introduced or non-parametrically. Only requirement for consistently estimating those functions would be that $\sum_{j=-1}^{J}p_{mj}=1$, and each probability $p_{mj}\geq 0$ for $j=-1,0,..,J$ and $m=1,..,M$.

[13] Logistic functions can be generalized to include interactions and higher order terms of the state variables (y_m,b_m). In this case the approximation will be uniformly better to the true CCPs.

probability associated with a particular scenario. We can further generalize this idea to the estimation of the sovereign default model itself. To estimate the probabilities consistent with a particular model (with the other model parameters are either calibrated, observed, estimated elsewhere as the current sovereign default models generally do), theoretically we are not required to have the country defaulted. Most of the literature use Argentina or countries with at least one default in their histories. However given the parametrization of the model with the relevant parameters from the literature, there exists a set of CCPs consistent with those parameters that solve the model.[14] In this association between the model and the CCPs, there is no reference to the default probability other than its influence on the calibrated, estimated, observed parameters used from the literature. This particular property makes the CCP framework a potential tool for further exploring the sovereign default models.

5. Conclusion

In this paper we present a new estimation technique, namely Hotz-Miller estimation technique (Hotz and Miller, 1993), to solve dynamic general equilibrium models of sovereign default. By applying Hotz-Miller estimation technique to general equilibrium sovereign default models, this paper contributes to the literature in two aspects. First, by using this technique and avoiding calibration, we can quantify the debt-default strategies and most importantly estimate the ex-ante default probabilities, even those that have never defaulted. Second, through this technique, we avoid using the fixed-point theorem which together with calibration bind the model to a fixed set of parameter values that sometimes are not economically intuitive. As opposed to the numerical iteration function and calibration, this gives us room to study countries' debt-default strategies at any given set of parameter values that are economically intuitive and representative of their real business cycle facts. Therefore the methodology is promising a substantial improvement in quantitative performance of dynamic general equilibrium models of sovereign default. Using the proposed technique, how well the results can replicate the main behaviors of the emerging economies is an empirical question, and is a topic of another paper currently in progress.

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[14] Certainly, one potential problem becomes the uniqueness of the solution to the CCP functions. Since the estimation framework is new, such topics are let for future research. However if one uses the information of past defaults to set a target (this is another way of saying defining a moment condition as traditionally the literature does), the uniqueness can be guaranteed with many functional forms.