

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Ahmed, Rashid; Tahir, M. H.; Rajab, Muhammad; Daniyal, Muhammad

### Article

# An overview on method of cyclic shifts for the construction of experimental designs useful in business and commerce

Pakistan Journal of Commerce and Social Sciences (PJCSS)

**Provided in Cooperation with:** Johar Education Society, Pakistan (JESPK)

*Suggested Citation:* Ahmed, Rashid; Tahir, M. H.; Rajab, Muhammad; Daniyal, Muhammad (2020) : An overview on method of cyclic shifts for the construction of experimental designs useful in business and commerce, Pakistan Journal of Commerce and Social Sciences (PJCSS), ISSN 2309-8619, Johar Education Society, Pakistan (JESPK), Lahore, Vol. 14, Iss. 1, pp. 49-62

This Version is available at: https://hdl.handle.net/10419/216864

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.



https://creativecommons.org/licenses/by-nc/4.0/

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.





Pakistan Journal of Commerce and Social Sciences 2020, Vol. 14 (1), 49-62 Pak J Commer Soc Sci

## An Overview on Method of Cyclic Shifts for the Construction of Experimental Designs Useful in Business and Commerce

Rashid Ahmed (Corresponding author) Department of Statistics, The Islamia University of Bahawalpur, Pakistan Email: rashid701@hotmail.com

M. H. Tahir Department of Statistics, The Islamia University of Bahawalpur, Pakistan Email: mtahir.stat@gmail.com

Muhammad Rajab Department of Statistics, The Islamia University of Bahawalpur, Pakistan Email: rajabmalik@yahoo.com

Muhammad Daniyal Department of Statistics, The Islamia University of Bahawalpur, Pakistan Email: muhammad.daniyal@iub.edu.pk

#### Abstract

The competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation (quality control). This goal can be achieved through the use of experimentation to identify and confirm the effects of the process variables. Therefore, design of experiments should be an integral part of process management. Experimental designs such as treatment balanced, neighbor balanced and balanced repeated measurements designs, have been extensively used in different fields, especially in business and commerce. Different methods have been used for their construction but method of cyclic shifts is the easiest one. This paper provides the overview, how this method has been used for the construction of the useful designs.

**Keywords:** cyclic shifts; experimental designs; competitive nature of business; polygonal designs; process management; quality control.

#### 1. Introduction

Method of cyclic shifts (MOCS) has been widely used to construct different designs. This method was developed by Iqbal (1991). In this method, we can study some standard characteristics and properties of the specific design only through the set(s) of shifts. This method is quite simple to construct several types of important designs. Some of these are balanced incomplete block designs (BIBDs), partially BIBDs, polygonal designs (PDs), neighbor balanced designs (NBDs) and repeated measurements designs (RMDs). In this

article, an overview is provided that how the MOCS is used to construct several types of important designs. This method consists of Rule I & Rule II.

According to Sanders et al. (2002), the competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation. The need of this knowledge (the continuous improvement of processes) must be quicker than ever before. This goal requires the ongoing use of experimentation to identify and confirm the effects of the process variables. Therefore, designed experiments should be viewed as an integral part of process management. Whereas the statistical design of experiments (DOE) is a valuable tool for rapidly accumulating this process knowledge. In industrial experimentation, blocks are frequently a period of time when extraneous variables (variables not explicitly manipulated in the experiment) can reasonably be expected to remain constant while an experiment takes place. Periods of time (blocks) for subsequent experiments are selected so that while variables not explicitly manipulated can reasonably be expected to remain constant during the execution of a DOE, some might have changed between experiments (blocks). Polygonal designs are useful for survey and marketing.

#### 2. Construction of BIBDs and PBIBDs

BIBDs and PBIBDs have the most significant role in experimental designs to compare each pair of treatments with equal or almost equal precision. Such experimental designs provide the assurance that treatments can be compared with same precision. Using method of cyclic shifts, Yasmin *et al.* (2015) and Jamil *et al.* (2017) presented BIBD and PBIBD respectively.

#### 2.1 How to Obtain a BIBD and PBIBD Using Rule I.

MOCS is described in this Section only for BIBDs and PBIBDs.

Let  $\underline{S}_{j} = [q_{j1}, q_{j2}, ..., q_{j(k-1)}], 1 \le q_{ji} \le v-1$ . A design will be BIBD if each of 1, 2, ..., v-1 appears  $\lambda$  times in  $S_{j}^{*}$ .  $\underline{S}_{j}^{*} = [q_{j1}, q_{j2}, ..., q_{j(k-1)}, (q_{j1}+q_{j2}), (q_{j2}+q_{j3}), ..., (q_{j(k-2)}+q_{j(k-1)}), (q_{j1}+q_{j2}+q_{j3}), (q_{j2}+q_{j3}+q_{j4}), ..., (q_{j(k-3)}+q_{j(k-2)}+q_{j(k-1)}), ..., (q_{j1}+q_{j2}+...+q_{j(k-1)}), and complement of each element], here <math>v$ - $q_{i}$  is complement of  $q_{i}$ . It will be PBIBD (two associate) if  $\lambda$  has two values  $\lambda_{2} = \lambda_{1}+1$ .

Example 2.1: S = [3, 1, 2, 2, 1, 1, 1] provide BIBD for v = 15, k = 8 in 15 blocks through MOCS (Rule I).

Here

$S^{*} = [3,1,2,2,1,1,1,4,3,4,3,2,2,6,5,5,4,3,8,6,6,5,9,7,7,10,8,11,12,14,13,13,14,14,14,11,1,14,11,11$
12,11,12,13,13,9,10,10,11,12,7,9,9,10,6,8,8,5,7,4]. Here each element from 1, 2, 3,, 14
appears 4 times. Hence it is BIBD with $\lambda = 4$ .

						]	Block	S						
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	0	1	2
4	5	6	7	8	9	10	11	12	13	14	0	1	2	3
6	7	8	9	10	11	12	13	14	0	1	2	3	4	5
8	9	10	11	12	13	14	0	1	2	3	4	5	6	7
9	10	11	12	13	14	0	1	2	3	4	5	6	7	8
10	11	12	13	14	0	1	2	3	4	5	6	7	8	9
11	12	13	14	0	1	2	3	4	5	6	7	8	9	10

2.2 How to Obtain a BIBD and PBIBD Using Rule II

Let  $\underline{S}_{j} = [q_{j1}, q_{j2}, ..., q_{j(k-2)}]t$ ,  $1 \le q_{ji} \le v-2$ . A design will be BIBD if each of 1, 2, ..., v-2 appears  $\lambda$  times in  $S_{j}^{*}$ . Where  $\underline{S}_{j}^{*} = [q_{j1}, q_{j2}, ..., q_{j(k-2)}, (q_{j1}+q_{j2}), (q_{j2}+q_{j3}), ..., (q_{j(k-3)}+q_{j(k-2)}), (q_{j1}+q_{j2}+q_{j3}), (q_{j2}+q_{j3}+q_{j4}), ..., (q_{j(k-4)}+q_{j(k-3)}+q_{j(k-2)}), ..., (q_{j1}+q_{j2}+...+q_{j(k-2)})]$ , here,  $v-1-q_i$  is complement of  $q_i$ . If  $\lambda$  has two values  $\lambda_2 = \lambda_1+1$  then it is PBIB design with 2-association scheme.

Example 2.2: BIBD is constructed from the sets of shifts [1,2,3] and [1,4]t for v = 8, k = 4 in 14 blocks through Rule II. Here S<sup>\*</sup> = [1,2,3,3,5,6,1,4,5,6,5,4,4,2,1,6,3,2] Here each element from 1, 2, 3, ..., 6 appears 3 times. Hence it is BIBD with  $\lambda = 3$ .

						B	lock	S					
1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	1	2	3	4	5	6	0	1	2	3	4	5	6
1	2	3	4	5	6	0	1	2	3	4	5	6	0
3	4	5	6	0	1	2	5	6	0	1	2	3	4
6	0	1	2	3	4	5	7	7	7	7	7	7	7

#### **3.** Construction of Polygonal Designs

PDs are important in survey sampling. Using MOCS, Tahir *et al.* (2011) constructed PDs for k = 3 with  $\lambda = 1, 2, 3, 4, 6, 12$  and  $\alpha = 2$ , Intizar *et al.* (2016) constructed PDs in blocks of two different sizes 4 and 2. Following is the procedure to obtain PDs.

3.1 How to Obtain Polygonal Designs Using Rule I

Let  $\underline{S}_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}]$ , where  $1 \le q_{ji} \le v-1$ .

- > If each of 2, ..., v-2 appears  $\lambda$  times in S\* except 1 and v-1 which do not appear then this design is CPD with  $\alpha = 1$ .
- > If each of 3, ..., v-3 appears  $\lambda$  times in S\* except 1, 2, v-1 and v-2 which do not appear then this design is CPD with  $\alpha = 2$ .
- > If each of  $\alpha$ +1, ...,  $\nu$ -( $\alpha$ +1) appears  $\lambda$  times in S\* except 1, 2,...,  $\alpha$ ,  $\nu$ -1,  $\nu$ -2,...,  $\nu$ - $\alpha$  which do not appear then this design is CPD with joint distance  $\alpha$ .

Here, S\* contains (i) each shift of S, (ii) sum (mod v) of each of two, three, ..., k-1 consecutive shifts, and (iii) complement of each element in (i) & (ii). Here,  $v-q_i$  is complement of  $q_i$ .

For a circular polygonal design with  $\alpha = 1$ , the concurrence matrix is:

$$NN' = \begin{pmatrix} r & 0 & \lambda & \lambda & \cdots & \lambda & 0 \\ 0 & r & 0 & \lambda & \cdots & \lambda & \lambda \\ \lambda & 0 & r & 0 & \cdots & \lambda & \lambda \\ \lambda & \lambda & 0 & r & \cdots & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \lambda & \cdots & r & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r' \end{pmatrix}$$

For a circular polygonal design with joint distance  $\alpha = 2$  the concurrence matrix is

$$NN' = \begin{pmatrix} r & 0 & 0 & \lambda & \cdots & \lambda & 0 & 0 \\ 0 & r & 0 & 0 & \cdots & \lambda & \lambda & 0 \\ 0 & 0 & r & 0 & \cdots & \lambda & \lambda & \lambda \\ \lambda & 0 & 0 & r & \cdots & \lambda & \lambda & \lambda \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \dots & \cdots & r & 0 & 0 \\ 0 & \lambda & \lambda & \lambda & \cdots & 0 & r & 0 \\ 0 & 0 & \lambda & \lambda & \cdots & 0 & 0 & r \end{pmatrix}$$

Example 3.1: A circular polygonal design for v = 11,  $k_1 = 3$ ,  $k_2 = 2$ ,  $\lambda = 1$  &  $\alpha = 1$  is constructed using [2, 3]+[4]. The required design is:

	Blocks														
1	2	3	4	5	6	7	8	9	10	) 11					
0	1	2	3	4	5	6	7	8	9	10					
2	3	4	5	6	7	8	9	10	0	1					
5	6	7	8	9	10	0	1	2	3	4					
	Blocks														
12	13	14	15	16	17	18	19	20	21	22					
0	1	2	3	4	5	6	7	8	9	10					
4	5	6	7	8	9	10	0	1	2	3					

3.2 How to Obtain Polygonal Designs Using Rule II

Let  $\underline{S}_{j} = [q_{j1}, q_{j2}, ..., q_{j(k-2)}]t$ , where  $1 \le q_{ji} \le v-1$ .

- > If each of 2, ..., v-3 appears  $\lambda$  times in S\* except 1 and v-2 which do not appear then this design is CPD with  $\alpha = 1$ .
- > If each of 3, ..., v-4 appears  $\lambda$  times in S\* except 1, 2, v-2 and v-3 which do not appear then this design is CPD with  $\alpha = 2$ .
- > If each of  $\alpha$ +1, ...,  $\nu$ -1-( $\alpha$ +1) appears  $\lambda$  times in S\* except 1, 2,...,  $\alpha$ ,  $\nu$ -2,...,  $\nu$ -1- $\alpha$  which do not appear then this design is CPD with joint distance  $\alpha$ .

Example 3.2: [2,3]+[4]t provide PD for v = 8, k = 3,  $\lambda = 2$  and  $\alpha = 1$ .

Here  $S^* = [2,3,5,4,5,4,2,3]$  Here each element from 2, 3, ..., 5 appears exactly twice. Hence it is polygonal design with  $\lambda = 2$  and  $\alpha = 1$ .

						B	Block	S					
1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	1	2	3	4	5	6	0	1	2	3	4	5	6
2	3	4	5	6	0	1	4	5	6	0	1	2	3
5	6	0	1	2	3	4	7	7	7	7	7	7	7

#### 4. Construction of Balanced and Strongly Balanced RMDs

RMDs have application in many branches such as business, commerce, agriculture, food science, animal husbandry, biology, education, psychology, and pharmacology. In the following literature, MOCS is used to construct RMDs.

Iqbal and Jones (1994), Iqbal and Tahir (2009), Iqbal et al. (2010), Bashir et al. (2018), Rajab et al. (2018), Rasheed et al. (2018), Ahmed et al. (2018), Khan et al. (2019), Daniyal et al. (2019), Rasheed et al. (2019), and Ahmed et al. (2019) constructed CBRMDs & CSBRMDs in periods of equal and unequal sizes. Jabeen et al. (2019), Nazeer et al. (2019), Jabeen et al. (2019) constructed minimal CSPBRMDs in periods of equal and two different sizes. Hussain et al. (2020) constructed CWBRMDs in two different periods sizes. Abdullah et al. (2019) constructed such designs in non-circular periods. Following is the to obtain the BRMDs.

4.1 How to Obtain BRMDs in Circular Periods using Rule I

Let  $S = [q_1, q_2, ..., q_{p-1}]$ , where  $1 \le q_i \le v-1$ . If each of 1, 2, ..., v-1 appears  $\lambda'$  in S\* then it be CBRMD in periods of size p, where  $S^* = [q_1, q_2, ..., q_{p-1}, v - (q_1+q_2+...+q_{p-1}) \mod v]$ . Design will be non-binary if sum of any two, three, ..., or (p-1) consecutive elements of S is 0 (mod v), otherwise binary.

Example 4.1: Set of shifts [1,2,8]+[3,4,6] produce non-binary CBRMD for v = 9 and p = 4.

Here  $S^* = [2,1,8,7,3,4,6,5]$  Here each element from 1,2,3,...,8 appears once. Therefore, it is minimal CBRMDs with  $\lambda' = 1$ .

				Subje	cts									
Periods	1	2	3	4	5	6	7	8	9					
1	02	13	24	35	46	57	68	70	81					
2	10	21	32	43	54	65	7 <sub>6</sub>	87	08					
3	31	42	<b>5</b> <sub>3</sub>	64	7 <sub>5</sub>	86	07	18	20					
4	23	34	45	56	67	7 <sub>8</sub>	80	01	12					
		Subjects												
Periods	10	11	12	13	14	15	16	17	18					
1	$0_4$	15	26	37	48	50	61	72	8 <sub>3</sub>					
2	30	41	<b>5</b> <sub>2</sub>	63	74	85	06	17	28					
3	7 <sub>3</sub>	84	05	16	27	38	40	51	62					
4	47	5 <sub>8</sub>	60	71	<b>8</b> <sub>2</sub>	03	14	25	36					

4.2 How to Obtained BRMDs in Circular Blocks Using Rule II

Let  $S_1 = [q_{11}, q_{12}, ..., q_{p_1-1}]$  and  $S_2 = [q_{21}, q_{22}, ..., q_{p_2-2}]t$ , where  $1 \le q_{ij} \le v-2$ . If each of 1, 2, ..., *v*-2 appears  $\lambda'$  in S\* then it is CBRMD in periods of size *p*, where S\* =  $[q_{11}, q_{12}, ..., q_{p_1-1}, (v-1)-\{(q_{11}+q_{12}+...+q_{p_1-1}) \mod(v-1)\}, q_{21}, q_{22}, ..., q_{p_2-2}].$ 

Example 4.2: Sets of shifts [2,1,6]+[3,4]t provide following non-binary CBRMD for v = 8, k = 4.

Here  $S^* = [1,2,6,5,3,4]$  and each element from 1, 2, 3, ..., 6 appears exactly once. Hence it is minimal CBRMDs with  $\lambda' = 1$ .

						Su	bject	S						
Periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	02	13	24	35	4 <sub>6</sub>	50	61	07	17	27	37	47	57	67
2	20	31	42	<b>5</b> <sub>3</sub>	64	05	16	30	41	52	63	$0_4$	15	26
3	32	43	54	65	06	10	21	03	14	25	36	40	51	62
4	23	34	45	56	60	01	12	70	71	72	7 <sub>3</sub>	74	7 <sub>5</sub>	7 <sub>6</sub>

4.3 How to Obtain a CSBRMD Using Rule I

Let  $S_1 = [q_{11}, q_{12}, ..., q_{1(p-1)}]$ , where  $0 \le q_{ij} \le v-1$ . If each of 0, 1, 2, ..., *v*-1 appears  $\lambda' S^*$  then it is CSBRMDs in period of size *p*, where  $S^* = [q_{11}, q_{12}, ..., q_{1(p-1)}, v-(q_{11}+q_{12}+...+q_{1(p-1)}) \mod v]$ . It will be minimal if  $\lambda' = 1$ .

Example 4.3: [1,3,2,4,6,5] provides following CSBRMDs for v = 7 and p = 7.

Here  $S^* = [1,3,2,4,6,5,0]$  Here each element from 0,1,2,3,...,6 appears exactly once. Hence it is minimal CSBRMDs with  $\lambda' = 1$  using rule I.

			Subje	ects			
Periods	1	2	3	4	5	6	7
1	00	11	22	3 <sub>3</sub>	44	<b>5</b> <sub>5</sub>	66
2	10	21	32	43	54	65	06
3	41	52	63	$0_4$	15	26	30
4	64	05	1 <sub>6</sub>	20	31	42	5 <sub>3</sub>
5	36	40	51	62	03	$1_{4}$	25
6	23	34	45	56	60	01	12
7	02	13	24	35	4 <sub>6</sub>	50	61

4.4 How to Obtain BRMDs in Linear Periods Using Rule I

Let S = [ $q_1, q_2, ..., q_{p-1}$ ], where  $1 \le q_i \le v-1$ . If each of 1, 2, ..., v-1 appears  $\lambda'$  in S\* then it is BRMD in linear periods of size p, where S\* = [ $q_1, q_2, ..., q_{p-1}$ ].

Example 4.4: Sets of shifts [2, 1, 3,4]+[5, 6, 8, 7] provide following CBRMDs in linear periods for v = 9 in p = 5.

Here  $S^* = [2,1,3,4,5,6,8,7]$  and each element from 1, 2, 3,..., 8 appears once, therefore, it is minimal BRMDs with  $\lambda' = 1$  in linear periods, using rule I.

Dowinda								Subj	jects									
rerious	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
2	20	31	42	5 <sub>3</sub>	64	7 <sub>5</sub>	86	07	18	50	61	72	83	$0_4$	15	26	37	48
3	32	43	54	65	7 <sub>6</sub>	87	08	10	21	25	36	47	5 <sub>8</sub>	60	71	82	03	14
4	63	74	05	16	27	38	40	51	62	12	23	34	45	56	67	7 <sub>8</sub>	80	01
5	26	37	40	51	62	73	$0_{4}$	15	06	81	02	13	24	35	46	57	68	70

4.5 How to Obtain BRMDs in Linear Periods Using Rule II

Let  $S_j = [q_{j1}, q_{j2}, ..., q_{j(p-1)}]$  and  $S_i = [q_{i1}, q_{i2}, ..., q_{i(p-2)}]t$ , where  $1 \le q_{ij} \le v-2$ . If each of 1, 2,..., v-2 appears  $\lambda'$  in S\* then it is non-linear BRMD in periods of size p, where S\*=  $[q_{j1}, q_{j2}, ..., q_{j(p-1)}, q_{i1}, q_{i2}, ..., q_{i(p-2)}] \mod(v-1)$ .

Example 4.5: Sets of shifts [1,2,3,4]+[5,6,7]t provide following CBRMDs for v = 9 & p = 5 in linear periods, using Rule II.

Here  $S^* = [1,2,3,4,5,6,7]$  and each element from 0, 1, 2, 3, ..., 7 appears exactly once. Hence it is minimal BRMDs in linear blocks with  $\lambda' = 1$ , using Rule II.

n · 1							S	bubje	cts									
Periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
2	$1_{0}$	21	32	43	54	65	76	87	08	50	61	72	8 <sub>3</sub>	$0_4$	15	26	37	48
3	31	42	5 <sub>3</sub>	64	7 <sub>5</sub>	86	07	18	20	25	36	47	5 <sub>8</sub>	60	71	82	03	$1_{4}$
4	63	74	85	06	17	28	30	41	<b>5</b> <sub>2</sub>	02	13	24	35	46	57	68	70	81
5	16	27	38	40	51	62	73	84	05	80	81	82	83	84	85	86	87	88

4.6 How to Obtain SBRMD in Linear Periods Using Rule I

Let S =  $[q_1, q_2, ..., q_{p-1}]$ , where  $0 \le q_i \le v-1$ . If each of 1, 2, ..., v-1 appears  $\lambda'$  in S\* then it is BRMD in linear periods of size p, where S\* =  $[q_1, q_2, ..., q_{p-1}]$ .

Example 4.6: Set of shifts [1,2,3,0]+[4,5,6,7] produce SBRMD for v = 8 & p = 5 in linear periods.

Dominda						Su	bject	ts								
renous	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	06	17	20	31	42	5 <sub>3</sub>	64	7 <sub>5</sub>	06	17	20	31	42	5 <sub>3</sub>	64	75
2	10	$2_1$	32	43	54	65	7 <sub>6</sub>	07	40	51	62	73	$0_4$	15	26	37
3	31	42	5 <sub>3</sub>	64	7 <sub>5</sub>	06	17	20	$1_4$	25	36	47	50	61	72	03
4	63	74	05	16	27	30	41	52	$7_{1}$	02	13	24	35	4 <sub>6</sub>	57	60
5	66	77	00	11	$2_{2}$	33	44	55	67	$\overline{7}_0$	01	12	23	34	45	56

Here  $S^* = [1,2,3,0,4,5,6,7]$  Here each element from 0,1,2,3,...,7 appears exactly once. Hence it is minimal SBRMDs in linear blocks with  $\lambda' = 1$ , using Rule I.

#### 5. Construction of Neighbor Balanced Design

NBDs are used to control neighbor effects. NBDs constructed through MOCS up to 2011 can be found in Ahmed et al. (2011). Ahmed and Akhtar (2011), Ahmed and Akhtar (2012 a, b) Yasmin et al. (2013), Ahmed et al. (2013), Ahmed and Akhtar (2013), Yasmin et al. (2014), Ahmed et al. (2014), Ahmed et al. (2015), Ahmed et al. (2016), Shahid et al. (2017), Ahmed et al. (2017), Khalid et al. (2018), and Shahid et al. (2019) constructed circular and non-circular NBDs. Following is the procedure to obtain NBDs.

5.1 How to Obtain NBDs in Linear Blocks Using Rule I

Let  $\underline{S}_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}]$ , where  $1 \le q_{ji} \le v-1$ . If each of 1, 2, ..., v-1 appears  $\lambda'$  times in S\* then design is NBD in linear blocks, where  $S^* = [q_{j1}, q_{j2}, ..., q_{j(k-1)}, v-q_{j1}, v-q_{j2}, ..., v-q_{j(k-1)}]$ .

Example 5.1: Sets of shifts [1,2,3]+[4,5,6] provide NBD for v = 13 in linear blocks of size 4 with  $\lambda' = 1$ .

Here  $S^* = [1,2,3,4,5,6,12,11,10,9,8,7]$  and each element from 1, 2, 3, ..., 12 appears exactly once. Hence it is minimal NBD in linear blocks using Rule I.

					]	Block	S								
1	2	3	4	5	6	7	8	9	10	11	12	13			
0	1	2	3	4	5	6	7	8	9	10	11	12			
1	2	3	4	5	6	7	8	9	10	11	12	0			
3	4	5	6	7	8	9	10	11	12	0	1	2			
6	7	8	9	10	11	12	0	1	2	3	4	5			
	Blocks														
14	15	16	17	18	19	20	21	22	23	24	25	26			
0	1	2	3	4	5	6	7	8	9	10	11	12			
4	5	6	7	8	9	10	11	12	0	1	2	3			
9	10	11	12	0	1	2	3	4	5	6	6	7			
2	3	4	5	6	7	8	9	10	11	12	0	1			

5.2 How to Obtain NBDs in Linear Blocks Using Rule II

Let  $\underline{S}_j = [q_{j1}, q_{j2}, ..., q_{j(k-2)}]t$ , where  $1 \le q_{ji} \le v-2$ . If each of 1, 2, ..., v-2 appears  $\lambda'$  times in S\*, design is NBD in linear blocks, where  $S^* = [q_{j1}, q_{j2}, ..., q_{j(k-2)}, v-1-q_{j1}, v-1-q_{j2}, ..., v-1-q_{j(k-2)}].$ 

Example 5.2: [1,2]+[3]t produce minimal NBD for v = 8 & k = 3.

Here  $S^* = [1,2,3,6,5,4]$  and each element from 1, 2, 3, ..., 6 appears exactly once. Hence it is minimal NBD in linear blocks using Rule II.

Blocks													
1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	1	2	3	4	5	6	0	1	2	3	4	5	6
1	2	3	4	5	6	0	3	4	5	6	0	1	2
3	4	5	6	0	1	2	7	7	7	7	7	7	7

5.3 How to Obtain Nearest Neighbor Balanced Design Using Rule I

Rule I: Let  $S = [q_1, q_2, ..., q_{k-1}]$ , where  $1 \le q_i \le v-1$ . If each of 1, 2, ..., v-1 appears  $\lambda'$  in S\* then it is CNBD, where  $S^* = [q_1, q_2, ..., q_{k-1}, (q_1+q_2+...+q_{k-1}) \mod v, v-(q_1), v-(q_2), ..., v-(q_{k-1}), v-(q_1+q_2+...+q_{k-1}) \mod v]$ .

Example 5.3: Set of shifts [1,2,3,9,5] provides minimal NBD for v = 13 and k = 6 from.

Blocks												
1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	0
3	4	5	6	7	8	9	10	11	12	0	1	2
6	7	8	9	10	11	12	0	1	2	3	4	5
2	3	4	5	6	7	8	9	10	11	12	0	1
7	8	9	10	11	12	0	1	2	3	4	5	6

Here  $S^* = [1,2,3,9,5,7,6,8,4,10,11,12]$  and each element from 1, 2, 3, ..., 12 appears once, therefore, it is minimal CNBDs, using Rule I.

5.4 How to Obtain Nearest Neighbor Balanced Design Using Rule II

Let  $S = [q_1, q_2, ..., q_{k-2}]t$ , where  $1 \le q_i \le v-2$ . If each of 1, 2, ..., v-2 appears  $\lambda'$  in S\* then it is CNBD, where  $S^* = [q_1, q_2, ..., q_{k-1}, v-1-q_1, v-1-q_2, ..., v-1-q_{k-2}]$ .

Example 5.4: Sets of shifts [1, 1, 1, 1, 1, 1, 1, 1, 1, 1](1/11)+[2, 3, 4, 5, 5, 4, 3, 2, 1]t provide minimal NBD for v = 12 and k = 11.

Blocks											
1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	0	1
2	5	6	7	8	9	10	0	1	2	3	4
3	9	10	0	1	2	3	4	5	6	7	8
4	3	4	5	6	7	8	9	10	0	1	2
5	8	9	10	0	1	2	3	4	5	6	7
6	1	2	3	4	5	6	7	8	9	10	0
7	4	5	6	7	8	9	10	0	1	2	3
8	6	7	8	9	10	0	1	2	3	4	5
9	7	8	9	10	0	1	2	3	4	5	6
10	11	11	11	11	11	11	11	11	11	11	11

59

#### REFERENCES

Abdullah, M., Ahmed, R., Tahir, M. H. and Daniyal, M. (2019). Minimal non-circular balanced and strongly balanced repeated measurements designs. *Communications in Statistics-Simulation and Computation*, [In Press]. Available Online: https://www.tandfonline.com/doi/abs/10.1080/03610918.2019.1614624

Ahmed, R. and Akhtar, M. (2011). Construction of Neighbor-Balanced Designs in Linear Blocks. *Communications in Statistics - Theory and Methods*, 40(17), 3198-3205.

Ahmed, R. and Akhtar, M. (2012a). Designs partially balanced for neighbor effects. *Aligarh Journal of Statistics*, *32*, 41-53.

Ahmed, R. and Akhtar, M. (2012b). Computer generated neighbor designs. *Communications in Statistics-Simulation and Computation*, *41*, 1834-1839.

Ahmed, R. and Akhtar, M. (2013). Neighbor designs in circular blocks of size 7. *Journal of Statistics*, 20, 69-87.

Ahmed, R. and Akhtar, M. (2015). Some classes of binary block neighbor designs. *Communications in Statistics-Simulation and Computation*, 44(5), 1253-1256.

Ahmed, R., Akhtar, M. and Yasmin, F. (2011). Brief review of one dimensional neighbor balanced designs since 1967. *Pakistan Journal of Commerce and Social Sciences*, 5 (1), 100-116.

Ahmed, R., Iqbal, I. and Akhtar, M. (2014). Universally optimal designs balanced for neighbor effects. *Communications in Statistics-Theory and Methods*, *43*, 3371-3379.

Ahmed, R., Iqbal, Z., Nawaz, M. and Akhtar, M. (2017). Construction of neighbor designs in binary blocks of some large sizes. *Aligarh Journal of Statistics*, *37*, 1-10.

Ahmed, R., Rajab, M., Iqbal, M. J. and Rasheed, H. M. K. (2018). A catalogue of minimal circular balanced repeated measurements designs in periods of two different sizes. *Aligarh Journal of Statistics*, *38*, 43-58.

Ahmed, R., Shehzad, F. and Akhtar, M. (2013). Minimal neighbor designs in linear blocks. *Communications in Statistics-Simulation and Computation*, 42(9), 2135-2139.

Ahmed, R., Shehzad, F. and Akhtar, M. (2016). Some infinite series of neighbor designs in circular blocks of small sizes. *Journal of Statistics*, 23, 12-16.

Ahmed, R., Shehzad, F., Rajab, M., Daniyal, M. and Tahir, M. H. (2019). Minimal circular balanced repeated measurements designs in periods of unequal sizes. *Communications in Statistics-Theory and Methods*, 48(21), 5223-5232.

Bashir, Z, Ahmed, R., Tahir, M. H., Ghazali, S. S. A. and Shehzad, F. (2018). Some extensions of circular balanced and circular strongly balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, 47(9), 2183-2194.

Daniyal, M., Ahmed, R., Shehzad, F., Tahir, M. H. and Iqbal, Z. (2019). Construction of repeated measurements designs strongly balanced for residual effects. *Communications in Statistics-Theory and Methods*. [In Press]. Available Online: https://www.tandfonline.com/doi/abs/10.1080/03610926.2019.1599019

Hussain, S., Ahmed, R., Aslam, M., Shah, A. and Rasheed, H. M. K. (2020). Some new construction of circular weakly balanced repeated measurements designs in periods of two different sizes. *Communications in Statistics-Theory and Methods*, 49(9), 2253-2263.

Intizar, M. N., Ahmed, R. and Shehzad, F. (2016). Catalogue of minimal cyclic polygonal designs in two different block sizes 4 and 2. *World Journal of Agricultural Science*, *12* (6), 414-420.

Iqbal, I. (1991), *Construction of experimental design using cyclic shifts*, Ph.D, Thesis, University of Kent at Canterbury, U.K.

Iqbal, I. and Jones, B. (1994). Efficient repeated measurements designs with equal and unequal period sizes. *Journal of Statistical Planning and Inference*, 42, 79-88.

Iqbal, I. and Tahir, M. H. (2009). Circular strongly balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, *38*, 3686-3696.

Iqbal, I., Tahir, M. H. and Ghazali, S. S. A. (2010). Circular first-and second-order balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, *39*, 228-240.

Jabeen, R., Ahmed, R., Sajjad, M., Rasheed, H. M. K. and Khan, A. (2019). Circular strongly partially-balanced repeated measurements designs in periods of two different sizes using method of cyclic shifts (Rule II). *Journal of King Saud University- Science*, *31*, 519-524.

Jabeen, R., Rasheed, H. M. K., Ahmed, R. and Shehzad, F. (2019). Construction of circular strongly partially-balanced repeated measurements designs. *Journal of King Saud University- Science*, *31*, 345-351.

Jamil, M., Shehzad, F., Ahmed, R. and Hussain, T. (2017). Construction of partially balanced incomplete block designs in blocks of size three using cyclic shifts. *Journal of Statistics*, *24*, 150-158.

Khalid, A., Shehzad, F., Ali, A. and Ahmed, R. (2018). Some important classes of neighbor balance designs in linear blocks of small sizes. *Journal of King Saud University- Science*, *30*, 311-315.

Khan, A., Ahmed, R., Shehzad, F., Tahir, M. H. and Ghazali, S. S. A. (2019). Construction of circular partially-balanced repeated measurement designs using cyclic shifts. *Communications in Statistics-Simulation and Computation*, 48(2), 506-515.

Nazeer, Y., Ahmed, R., Jabeen, R., Daniyal, M. and Tahir, M. H. (2019). Circular strongly partially-balanced repeated measurements designs in periods of two different sizes. *Journal of Probability and Statistical Science*, *17*(1), 85-95.

Rajab, M., Ahmed, R., Shehzad, F. and Tahir, M. H. (2018). Some new constructions of circular balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, 47, 4142-4151.

Rasheed, H. M. K., Rasul, M., Ahmed, R., Batool, M., Tahir, M. H. and Shehzad, F. (2019). Circular balanced repeated measurement designs in periods of three different sizes. *Communications in Statistics-Simulation and Computation*, 48(10), 3022-3030.

Rasheed, U., Rasheed, H. M. K., Rasheed, M. and Ahmed, R. (2018). Minimal circular strongly balanced repeated measurements designs in periods of three different sizes. *Communications in Statistics-Theory and Methods*, 47, 4088-4094.

Sanders, D., Leitnaker, M. G. and McLean, R. A. (2002). Randomized complete block designs in industrial studies. *Quality Engineering*, *14*(1), 1-8.

Shahid, M. R., Ahmed, R., Shehzad, F. and Muhammad, Y. S. (2019). Development of some useful generators to obtain partially neighbor balanced designs. *Journal of King Saud University-Science*, *31*, 24-26.

Shahid, M. R., Zakria, M., Shehzad, F. and Ahmed, R. (2017). Some important classes of generalized neighbor designs in linear blocks. *Communications in Statistics-Simulation and Computation*, 46(3), 1991-1997.

Tahir, M. H., Iqbal, I. and Aggarwal, M. L. (2011). Cyclic Polygonal Designs with Block Size 3 and Joint Distance  $\alpha = 2$ . *Communications in Statistics Theory and Methods*, 40(14), 2583-2590.

Yasmin, F., Ahmed, R. and Akhtar, M. (2013). A catalogue of neighbor balanced designs in circular blocks of size 4. *Pakistan Journal of Commerce and Social Sciences*, 7 (1), 166-173.

Yasmin, F., Ahmed, R. and Akhtar, M. (2014). Some more neighbor balanced designs in circular blocks of size five. *Pakistan Journal of Statistics*, *30*(2), 169-180.

Yasmin, F., Ahmed, R. and Akhtar, M. (2015). Construction of balanced incomplete block designs using cyclic shifts. *Communications in Statistics-Simulation and Computation*, 44(2), 525-532.