

Quaas, Georg

Working Paper

The reproduction number in the classical epidemiological model

Working Paper, No. 167

Provided in Cooperation with:

University of Leipzig, Faculty of Economics and Management Science

Suggested Citation: Quaas, Georg (2020) : The reproduction number in the classical epidemiological model, Working Paper, No. 167, Universität Leipzig, Wirtschaftswissenschaftliche Fakultät, Leipzig

This Version is available at:

<https://hdl.handle.net/10419/216820>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



UNIVERSITÄT
LEIPZIG

Wirtschaftswissenschaftliche
Fakultät
Faculty of Economics and
Management Science

Working Paper, No. 167

Georg Quaas

**The reproduction number in the
classical epidemiological model**

April 2020

ISSN 1437-9384

The reproduction number in the classical epidemiological model

Georg Quaas
Universität Leipzig
Grimmaische Straße 12
D-04109 Leipzig
quaas@uni-leipzig.de

April 28, 2020

Abstract:

The German Robert Koch Institute aims to “protect the population from disease and improve their state of health” (RKI 2017). To this end, it develops concrete, research-based recommendations for policymakers and makes data available to the expert public. Since March 4, 2020, it has been publishing the numbers of coronavirus infections reported by health authorities daily; since March 9, these data have included the numbers of people who have died of COVID-19; and since March 25, the RKI has reported the estimated numbers of those who have recovered. The important reproduction number, reported daily since April 7, has now largely replaced all other criteria used for decision-making. This paper aims to show that the calculation of this figure is neither theory-based nor particularly reliable. Nevertheless, there is a simple way to determine its change more or less conservatively and precisely.

Keywords:

Classic epidemic model, reproduction number, contact rate, COVID-19, mathematics of highly infectious diseases, public health

JEL-Classification:

C32, C61, I12, I18, J11

1. The theoretical framework

The classical epidemiological model (CEM), compared to econometric models of moderate size used for forecasting and simulating economic policy measures, is quite clear as well as plausible in its assumptions. Moreover, some parts of the model have a practical use. On the basis of daily data, it is characterized by the following equations (Hethcote 2000: 604):

$$(1) \quad \Delta S = -\beta IS/N$$

$$(2) \quad \Delta I = \beta IS/N - \Delta R$$

$$(3) \quad \Delta R = \gamma I$$

Here, β and γ are parameters to be determined empirically; N is the population size; S is the number of people susceptible to infection (at the beginning of the disease and without information about the number of immune people in the population, it is set as equal to N); I is the number of infected and infectious people (with an initial value close to zero); and R is the number of people who recover and are assumed to be immune (without information, this number is also set at zero at the beginning). The exact conceptual characteristics of these three population groups are not entirely unimportant because the “susceptible” need not be “healthy” and the “infected” need not be “sick” (Donsimoni et al. 2020). Whether this model can be applied to COVID-19 depends on whether those who have recovered are immune, at least for a while.

The logic behind this model becomes clear when defining proportions of the two population groups with and without active pathogens in the total population: $i = I/N$ und $s = S/N$. These proportions can be interpreted as the probability of accidentally encountering an infectious or a susceptible person in the population. The probability that these persons meet has the value $i \cdot s$ (Hamer 1906). The frequency of infections is assumed in the model to increase with the size of the population, resulting – with the exception of β – in the mean term of equation (2), which we abbreviate as H :

$$(4) \quad i s N = \frac{I S N}{N^2} = \frac{I S}{N} = H$$

From a practical perspective, H can be interpreted as the number of possibilities to become infected in a population of the extent N , so to speak, the “abstract risk situation.”

The parameter β reflects the actual infection process with regard to how many people are infected by an infectious agent per time unit (here: per day) on average. This depends on a variety of factors that are not explicitly included in the model, such as population density, the number of daily interactions, and common behaviors (hygiene, shaking hands, etc.). Some of these factors can be influenced pragmatically so that policymakers have access to the ongoing process. However, the effect on the parameter β is recorded with a time delay. In the case of the novel coronavirus, the media initially assumed that each infected person infects three more people during his or her infectious phase. The RKI reports a basic reproduction figure R_0 between 2 and 2.5 in its “coronavirus profile.” Initial experience suggests that the infectious phase lasts for about 10 days, so that an average can be set as $\beta = 0,3$, that is, as long as more accurate estimates are unavailable.

There is a fourth variable that does not play an explicit role in the above-formulated classical model, and that is the number of deaths, D . The model has been interpreted (an der Heiden & Buchholz 2020: p.1) in such a way that the number of deaths is included in the number of recoveries. This sounds a bit cynical, but from a statistical perspective, another aspect is important: If the number of deaths is empirically available, the model should be supplemented with it. Both the number of deaths and the number of recoveries reduce the number of infectious cases; therefore, equation (2) can be made more precise as:

$$(2a) \Delta I = \beta IS/N - \Delta R - \Delta D$$

The number of recoveries is determined by equation (3) if not previously estimated by the data producer. For the number of new reported deaths, the above assumption applies:

$$(4) \Delta D = \delta I$$

For this parameter, δ measures the current number of newly reported deaths based on the number of infected persons at an earlier time of infection. Deaths reduce the population number. Therefore, in the case of a short-term event with potentially high numbers of victims, it makes sense to supplement the population figure set constant in the model with the number of deaths and treat all other variables as time-dependent:

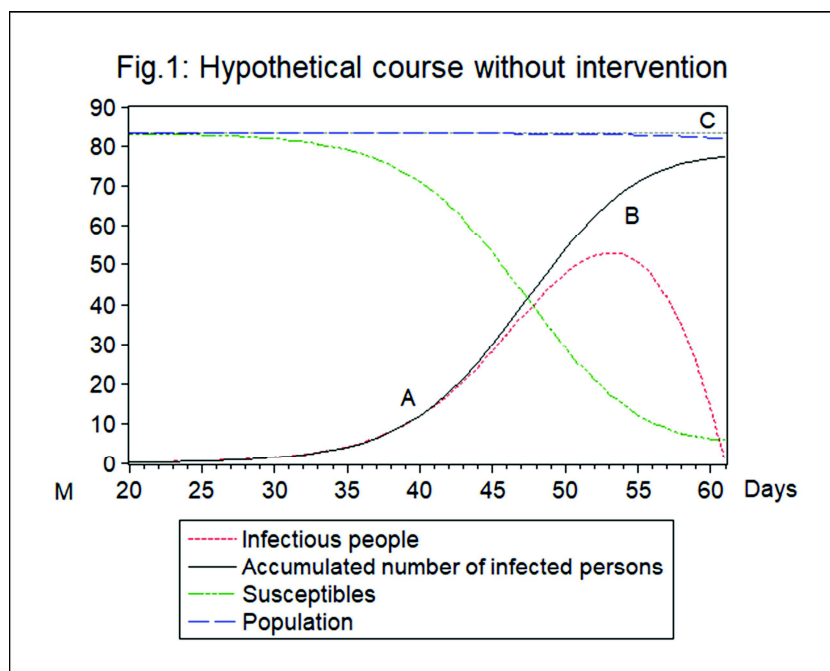
$$(5) S(t) + I(t) + R(t) = N - D(t) = N'(t)$$

In order to adapt the model to empirical evidence, it is essential to add time lags to individual variables according to the characteristics of the underlying disease. This requires

medical information (Schilling et al. 2020). Not all of the characteristics of COVID-19 are known; for the unknown parameters, assumptions must be made to “feed” the model. Particularly problematic is the average duration of infectivity of an infected person, which is, as it seems, difficult to determine at this time. It should also be noted that the accumulated number of infected persons reported by the RKI (or the number of persons reported as infected by public health authorities) does not correspond to the number of infectious persons $I(t)$ in the classical model.

2. Measurement of contact intensity

With the above equations, exponential growth is modeled in the first phase of the epidemic: The numbers increase slowly at first, but then they behave like an explosion. But it would be a mistake to assume exponential growth throughout and thus explain new infections. The more the probabilities change and political measures influence the contact rate β , the more the development deviates from exponential growth. Ultimately, the decisive factor for reversing development is a reduction in the number of people in the population susceptible to the pathogen either through recovery, death, or vaccination. Fig. 1 shows the course that would have been expected according to the CEM if no measures had been taken:



Parameters of the model: $\beta = 0.26$; $\delta = 0.026$; $N = 83.1$ million

The number of COVID-19 victims would be about one million; this can be seen in the figure by the slight decrease of the population curve (point C).

In the absence of a vaccine, the contact rate β should be the focus of pragmatic considerations, reflecting what can be influenced by policy. It is, by the way, relatively easy to determine β based on the daily number of new infections using equation (1) or (2) – preferably as an average to compensate for statistical variations. Since the average is always calculated over the last X days, it is a conservative estimate, which, when it comes to relaxing the restrictions, certainly does not encourage premature action. If the aim is to remain at a low level of new cases and detect changes as early as possible, a 3-day average is recommended, which also provides fairly stable measurement results.

The “replacement number” $R(t)$, which is sometimes called the “reproduction number” in literature as well as by the RKI, is more descriptive and, thus, easier to communicate to the public. It represents the number of secondary infections that a typical infected person produces during the period T of infectivity (Hethcote 2000: 603 f.). Due to the definitions, the following applies:

$$(6) \quad R(t) = \beta(t)T$$

In principle, the problem with this figure is that, for COVID-19, the duration T of being infectious is not yet known exactly and must be replaced by an assumption. However, since the main factor in the ongoing process is the change in the contact rate, this is not an obstacle to using either $R(t)$ or β as a basis for decision-making.

In Germany, policymaking was initially purely pragmatic and based on the available data, using the number of days in which the number of infected persons doubles. For reasons that are difficult to understand, the number 14, which was noted one day before the first relaxing of restrictions, that is, on April 15, was favored. Although criticism of this highly error-prone number is justified (Kaßmann 2020), it cannot be replaced by purely mathematical methods from an epidemiological perspective.

3. The reproduction number of the RKI

The RKI began reporting the value for the following variable in its daily situation reports on April 7, 2020:

The reproduction number is the number of persons in average infected by a case. This number can only be estimated and not directly extracted from the notification system. The current estimation is $R = 1.3$ (1.0-1.6). This is based on the number of cases with disease onset between 31/03/2020-03/04/2020 and 27/03/2020-30/03/2020 and an average generation time of 4 days. Cases with more recent disease onset are not included because their low number would lead to an unstable estimation. (Daily Situation Report, RKI 4/7/2020)

The RKI's estimate reflects the state of affairs dating back at least three days. Due to late notifications by health authorities, the actual number of new infections is correct only after at least three days. These figures are reported on the "dashboard" recommended by the RKI. After aggregation, they should correspond to the current figures in the daily situation reports, but this is not the case.

Reference is made to the methodological principles in the management report dated April 13. The preparation of the data for analysis is described in detail by Heiden and Hamouda (2020: 10 ff.): With the help of the average delay in the reporting system, the time of the event of infection is first determined (in some cases completely new). Based on this, the time-dependent reproduction rate according to the RKI is determined by assuming that it takes an average of four days for one infected person to infect the next ("generation time"). In another publication, the presumed duration of infectivity is given in days $T = 10$ (an der Heiden & Buchholz 2020). According to the RKI profile, the virus could still be detected in some infected persons eight days after the outbreak of the disease. Together with the average of two days of an infectious prelude during the incubation period, one must, therefore, expect an average of ten days of infectivity. Based on these facts, the RKI's method has to be critically evaluated for theoretical and empirical reasons:

With a constant generation time of 4 days, R is the quotient of the number of new cases in two consecutive periods of 4 days each. If the number of new cases has increased in the second time period, R is above 1. If the number of new cases is the same in both time periods, the re-production number is 1. This then corresponds to a linear increase in the number of cases. If, on the other hand, only every second case

infects another person, i.e. $R = 0.5$, then the number of new infections is halved within the generational period. (an der Heiden & Hamouda 2020: 13)

The use of generation time in the absence of the much-longer infectivity period in the theoretical justification of the method used raises doubts. Confrontation of this method with the CEM shows that the doubts are justified.

The basis for the calculation of the reproduction figure is the number of newly infected (or reported as infected) people ΔI , which is part of equation (2). If the numbers of those recovered and deceased are disregarded and two successive periods are added from 1 to 4 and 5 to 8, respectively, and we divide the second sum by the first, this provides the reproduction number of the RKI:

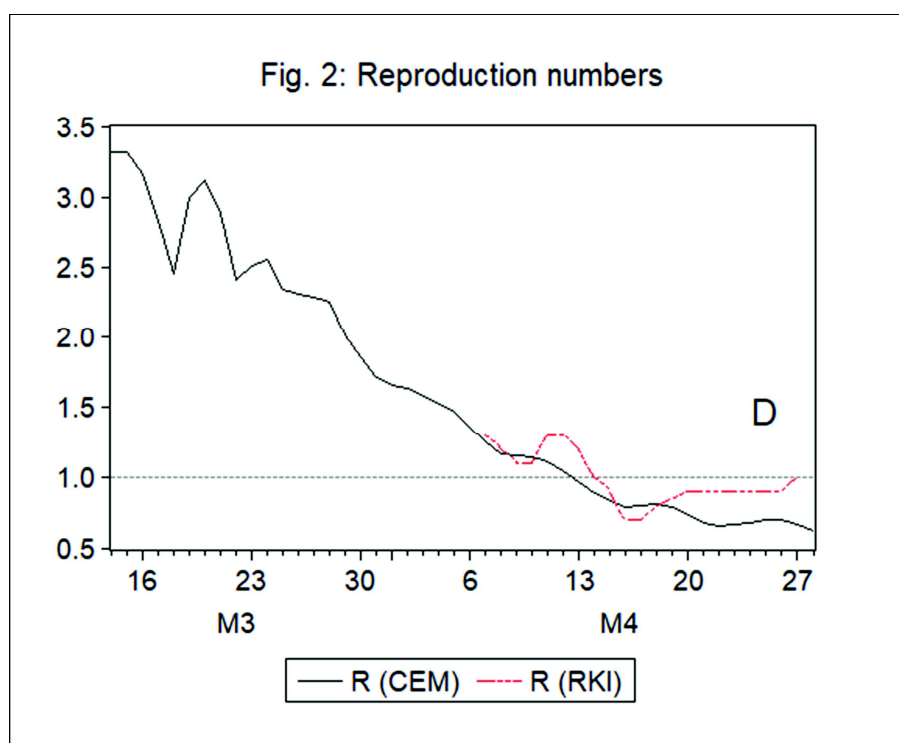
$$(7) R_{RKI} = \frac{\sum_{i=5}^8 \Delta I_i}{\sum_{i=1}^4 \Delta I_i} = \frac{\sum_{i=5}^8 (\beta IS/N)_i}{\sum_{i=1}^4 (\beta IS/N)_i} = \frac{\sum_{i=5}^8 \beta_i H_i}{\sum_{i=1}^4 \beta_i H_i} \approx \frac{\bar{\beta}_{5-8}}{\bar{\beta}_{1-4}} \neq \bar{\beta}_{1-8} T = \bar{R}(t_1, \dots, t_8)$$

Under the simplifying assumption that the term for the frequencies in the period in question does not change, the fraction can be shortened and one can see that the authors do not calculate the reproduction number $R(t)$ defined by the epidemiological model according to equation (6), but the (multiplicatively expressed) change in the four-day contact rates. However, since the frequencies H change over time to a similar extent as the contact rate, this effect overlays the change in the contact rate, so that, strictly speaking, we learn nothing about it – and, correspondingly, nothing about the change in the reproduction number, which expresses exactly the change in the contact rate (since T is constant).

Mathematically speaking, R_{RKI} is a rough measure of the curvature of the curve formed by the accumulated infection numbers (see Fig. 1). The curve is convex in the initial phase of the epidemic (“convex” in economic terminology), i.e., it provides $R_{RKI} > 1$ because the subsequent differences increase (the area around point A in Fig. 1). At a certain point, which mathematicians call the “turning point,” the curve becomes concave because the subsequent differences are smaller than the previous ones: Now it is $R_{RKI} < 1$ (point B in Fig. 1).

R_{RKI} provides a purely phenomenological characteristic of the epidemic's course, while the reproduction number is closer to the mechanism that determines the course. In the RKI method, the four-day "generation time" merely determines the length of the measurement interval for averaging errors in the data. If there were no errors, one would have to say that the shorter the measuring interval, the more precisely the curvature is recorded; the longer the measuring interval, the more inaccurate.

The curvature parameter R_{RKI} was specified for the first time on April 16, 2020, with a value below 1. At that time, the number of reproductions $R(t)$, calculated conservatively (10-day average) and multiplied by a very conservative duration $T = 12$ of being infectious, was already at 0.8. Figure 2 compares the two "reproduction numbers" (possible only at point D):



Data source: Daily Situation Reports of the RKI as well as own calculations from CEM.

As the dot-dash line in Fig. 2 shows, the RKI's measurements have, thus far, been quite volatile and are since April 18 for four days surprisingly constant. It is not very likely that the reproductive rate $R(t)$ will fluctuate as sharply under the regime of reduced contacts, and it should not remain constant under the condition of relaxing the restrictions. The increase in the RKI's reproduction figures for April 27 reflects a slight fluctuation in infection figures in

the 17th week of the year, i.e. one week before the first nationwide loosening of restrictions. No value was reported for April 19 due to “technical changes”; the missing value was inserted in the graph by the mean value between values reported on April 18 and 20.

Contrary to the preference for the true reproduction number $R(t)$ (acc. CEM) represented in this paper, an objection could be that the RKI controls the actual course with its method more closely. However, if one considers that the infection figures are, to a large extent, modified by the RKI prior to analysis, this is also doubtful. If, from an epidemiological perspective, it is important to keep an eye on the number of new infections, then the contact rate β and also the number of possibilities H to become infected in a population of 83 million people (the abstract risk situation) should be targeted, even if only the contact rate can be influenced politically. The theoretically expected number H of possibilities to become infected can help to explain the number of new infections and provide a realistic picture of the risk situation.

4. Conclusion

Under the authoritative expert advice of the RKI, German policymakers have succeeded in decisively reducing the contact rate in a relatively short time and, thereby, containing the spread of the first wave of the coronavirus epidemic. Furthermore, it will be important to prevent this rate from rising again, if possible, despite the easing of restrictions. In view of the high and, likely, increasing economic costs of each additional day of contact restriction, it would have been advisable to use a theoretically well-thought-out model to calculate the contact rate and the corresponding reproduction number, the parameters of which reflect actual, effective circumstances. Moreover, the classical epidemiological model allows more up-to-date and stable estimates.

References:

an der Heiden, M., U. Buchholz (2020): Modellierung von Beispielszenarien der SARS-CoV-2-Epidemie 2020 in Deutschland, DOI 10.25646/6571.2

an der Heiden, M., O. Hamouda (2020): Schätzung der aktuellen Entwicklung der SARS-CoV-2-Epidemie in Deutschland – Nowcasting, *Epid. Bull.* 2020, Nr. 17, S. 10 – 15. DOI 10.25646/6692

Donsimoni, J. R., R. Glawion, B. Plachter, K. Wälde (2020): Projektion der COVID-19-Epidemie in Deutschland, *Wirtschaftsdienst*, 100. Jahrgang, Heft 4, S. 272-276

Hamer, W. H. (1906): Epidemic Disease in England, *Lancet*, 1, pp. 733-739

Hethcote, H. W. (2000): The Mathematics of Infectious Diseases, *SIAM Review*, Vol. 42, No. 4. (Dec., 2000), pp. 599-653

Kaßmann, M. (2020): Die Fallstricke der Verdoppelung, URL: <https://www.math.uni-bielefeld.de/kassmann/index.php>

Robert-Koch-Institut (2017): Leitbild, URL: https://www.rki.de/DE/Content/Institut/Leitbild/Leitbild_node.html

RKI: SARS-CoV-2 Steckbrief zur Coronavirus-Krankheit-2019 (COVID-19), URL: www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Steckbrief.html

RKI: Tägliche Situationsberichte ab 4. März 2020, URL: https://www.rki.de/DE/Content/InfAZ/N/Neuartiges_Coronavirus/Situationsberichte/Archiv.html

Schilling, J., M. Diercke, D. Altmann, W. Haas, S. Buda (2020): Vorläufige Bewertung der Krankheitsschwere von COVID-19 in Deutschland basierend auf übermittelten Fällen gemäß Infektionsschutzgesetz, *Epid. Bull.* 2020, Nr. 17, S. 3 – 9, DOI 10.25646/6670

Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

Nr. 1	Wolfgang Bernhardt	Stock Options wegen oder gegen Shareholder Value? Vergütungsmodelle für Vorstände und Führungskräfte 04/1998
Nr. 2	Thomas Lenk / Volkmar Teichmann	Bei der Reform der Finanzverfassung die neuen Bundesländer nicht vergessen! 10/1998
Nr. 3	Wolfgang Bernhardt	Gedanken über Führen – Dienen – Verantworten 11/1998
Nr. 4	Kristin Wellner	Möglichkeiten und Grenzen kooperativer Standortgestaltung zur Revitalisierung von Innenstädten 12/1998
Nr. 5	Gerhardt Wolff	Brauchen wir eine weitere Internationalisierung der Betriebswirtschaftslehre? 01/1999
Nr. 6	Thomas Lenk / Friedrich Schneider	Zurück zu mehr Föderalismus: Ein Vorschlag zur Neugestaltung des Finanzausgleichs in der Bundesrepublik Deutschland unter besonderer Berücksichtigung der neuen Bundesländer 12/1998
Nr. 7	Thomas Lenk	Kooperativer Föderalismus – Wettbewerbsorientierter Föderalismus 03/1999
Nr. 8	Thomas Lenk / Andreas Mathes	EU – Osterweiterung – Finanzierbar? 03/1999
Nr. 9	Thomas Lenk / Volkmar Teichmann	Die fiskalischen Wirkungen verschiedener Forderungen zur Neugestaltung des Länderfinanzausgleichs in der Bundesrepublik Deutschland: Eine empirische Analyse unter Einbeziehung der Normenkontrollanträge der Länder Baden-Württemberg, Bayern und Hessen sowie der Stellungnahmen verschiedener Bundesländer 09/1999
Nr. 10	Kai-Uwe Graw	Gedanken zur Entwicklung der Strukturen im Bereich der Wasserversorgung unter besonderer Berücksichtigung kleiner und mittlerer Unternehmen 10/1999
Nr. 11	Adolf Wagner	Materialien zur Konjunkturforschung 12/1999
Nr. 12	Anja Birke	Die Übertragung westdeutscher Institutionen auf die ostdeutsche Wirklichkeit – ein erfolg-versprechendes Zusammenspiel oder Aufdeckung systematischer Mängel? Ein empirischer Bericht für den kommunalen Finanzausgleich am Beispiel Sachsen 02/2000
Nr. 13	Rolf H. Hasse	Internationaler Kapitalverkehr in den letzten 40 Jahren – Wohlstandsmotor oder Krisenursache? 03/2000
Nr. 14	Wolfgang Bernhardt	Unternehmensführung (Corporate Governance) und Hauptversammlung 04/2000
Nr. 15	Adolf Wagner	Materialien zur Wachstumsforschung 03/2000
Nr. 16	Thomas Lenk / Anja Birke	Determinanten des kommunalen Gebührenaufkommens unter besonderer Berücksichtigung der neuen Bundesländer 04/2000
Nr. 17	Thomas Lenk	Finanzwirtschaftliche Auswirkungen des Bundesverfassungsgerichtsurteils zum Länderfinanzausgleich vom 11.11.1999 04/2000
Nr. 18	Dirk Bütel	Continuous linear utility for preferences on convex sets in normal real vector spaces 05/2000
Nr. 19	Stefan Dierkes / Stephanie Hanrath	Steuerung dezentraler Investitionsentscheidungen bei nutzungsabhängigem und nutzungsunabhängigem Verschleiß des Anlagenvermögens 06/2000
Nr. 20	Thomas Lenk / Andreas Mathes / Olaf Hirschfeld	Zur Trennung von Bundes- und Landeskompetenzen in der Finanzverfassung Deutschlands 07/2000
Nr. 21	Stefan Dierkes	Marktwerte, Kapitalkosten und Betafaktoren bei wertabhängiger Finanzierung 10/2000
Nr. 22	Thomas Lenk	Intergovernmental Fiscal Relationships in Germany: Requirement for New Regulations? 03/2001
Nr. 23	Wolfgang Bernhardt	Stock Options – Aktuelle Fragen Besteuerung, Bewertung, Offenlegung 03/2001
Nr. 24	Thomas Lenk	Die „kleine Reform“ des Länderfinanzausgleichs als Nukleus für die „große Finanzverfassungs-reform“? 10/2001

Nr. 25	Wolfgang Bernhardt	Biotechnologie im Spannungsfeld von Menschenwürde, Forschung, Markt und Moral Wirtschaftsethik zwischen Beredsamkeit und Schweigen 11/2001
Nr. 26	Thomas Lenk	Finanzwirtschaftliche Bedeutung der Neuregelung des bundestaatlichen Finanzausgleichs – Eine allokoative und distributive Wirkungsanalyse für das Jahr 2005 11/2001
Nr. 27	Sören Bär	Grundzüge eines Tourismusmarketing, untersucht für den Südraum Leipzig 05/2002
Nr. 28	Wolfgang Bernhardt	Der Deutsche Corporate Governance Kodex: Zuwahl (comply) oder Abwahl (explain)? 06/2002
Nr. 29	Adolf Wagner	Konjunkturtheorie, Globalisierung und Evolutionsökonomik 08/2002
Nr. 30	Adolf Wagner	Zur Profilbildung der Universitäten 08/2002
Nr. 31	Sabine Klinger / Jens Ulrich / Hans-Joachim Rudolph	Konjunktur als Determinante des Erdgasverbrauchs in der ostdeutschen Industrie? 10/2002
Nr. 32	Thomas Lenk / Anja Birke	The Measurement of Expenditure Needs in the Fiscal Equalization at the Local Level Empirical Evidence from German Municipalities 10/2002
Nr. 33	Wolfgang Bernhardt	Die Lust am Fliegen Eine Parabel auf viel Corporate Governance und wenig Unternehmensführung 11/2002
Nr. 34	Udo Hielscher	Wie reich waren die reichsten Amerikaner wirklich? (US-Vermögensbewertungsindex 1800 – 2000) 12/2002
Nr. 35	Uwe Haubold / Michael Nowak	Risikoanalyse für Langfrist-Investments Eine simulationsbasierte Studie 12/2002
Nr. 36	Thomas Lenk	Die Neuregelung des bundesstaatlichen Finanzausgleichs auf Basis der Steuerschätzung Mai 2002 und einer aktualisierten Bevölkerungsstatistik 12/2002
Nr. 37	Uwe Haubold / Michael Nowak	Auswirkungen der Renditeverteilungsannahme auf Anlageentscheidungen Eine simulationsbasierte Studie 02/2003
Nr. 38	Wolfgang Bernhard	Corporate Governance Kodex für den Mittel-Stand? 06/2003
Nr. 39	Hermut Kormann	Familienunternehmen: Grundfragen mit finanzwirtschaftlichen Bezug 10/2003
Nr. 40	Matthias Folk	Launhardt'sche Trichter 11/2003
Nr. 41	Wolfgang Bernhardt	Corporate Governance statt Unternehmensführung 11/2003
Nr. 42	Thomas Lenk / Karolina Kaiser	Das Prämienmodell im Länderfinanzausgleich – Anreiz- und Verteilungsmittelnwirkungen 11/2003
Nr. 43	Sabine Klinger	Die Volkswirtschaftliche Gesamtrechnung des Haushaltssektors in einer Matrix 03/2004
Nr. 44	Thomas Lenk / Heide Köpping	Strategien zur Armutsbekämpfung und –vermeidung in Ostdeutschland: 05/2004
Nr. 45	Wolfgang Bernhardt	Sommernachtsfantasien Corporate Governance im Land der Träume. 07/2004
Nr. 46	Thomas Lenk / Karolina Kaiser	The Premium Model in the German Fiscal Equalization System 12/2004
Nr. 47	Thomas Lenk / Christine Falken	Komparative Analyse ausgewählter Indikatoren des Kommunalwirtschaftlichen Gesamt-ergebnisses 05/2005
Nr. 48	Michael Nowak / Stephan Barth	Immobilienanlagen im Portfolio institutioneller Investoren am Beispiel von Versicherungsunternehmen Auswirkungen auf die Risikosituation 08/2005
Nr. 49	Wolfgang Bernhardt	Familiengesellschaften – Quo Vadis? Vorsicht vor zu viel „Professionalisierung“ und Ver-Fremdung 11/2005
Nr. 50	Christian Milow	Der Griff des Staates nach dem Währungsgold 12/2005

Nr. 51	Anja Eichhorst / Karolina Kaiser	The Institutional Design of Bailouts and Its Role in Hardening Budget Constraints in Federations 03/2006
Nr. 52	Ullrich Heilemann / Nancy Beck	Die Mühen der Ebene – Regionale Wirtschaftsförderung in Leipzig 1991 bis 2004 08/2006
Nr. 53	Gunther Schnabl	Die Grenzen der monetären Integration in Europa 08/2006
Nr. 54	Hermut Kormann	Gibt es so etwas wie typisch mittelständige Strategien? 11/2006
Nr. 55	Wolfgang Bernhardt	(Miss-)Stimmung, Bestimmung und Mitbestimmung Zwischen Juristentag und Biedenkopf-Kommission 11/2006
Nr. 56	Ullrich Heilemann / Annika Blaschzik	Indicators and the German Business Cycle A Multivariate Perspective on Indicators of Ifo, OECD, and ZEW 01/2007
Nr. 57	Ullrich Heilemann	“The Soul of a new Machine” zu den Anfängen des RWI-Konjunkturmodells 12/2006
Nr. 58	Ullrich Heilemann / Roland Schuhr / Annika Blaschzik	Zur Evolution des deutschen Konjunkturzyklus 1958 bis 2004 Ergebnisse einer dynamischen Diskriminanzanalyse 01/2007
Nr. 59	Christine Falken / Mario Schmidt	Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil I: Einführende und Erläuternde Betrachtungen zum Systemwechsel im kommunalen Rechnungswesen 01/2007
Nr. 60	Christine Falken / Mario Schmidt	Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil II Bewertung der Informationsfunktion im Vergleich 01/2007
Nr. 61	Udo Hielscher	Monti della città di firenze Innovative Finanzierungen im Zeitalter Der Medici. Wurzeln der modernen Finanzmärkte 03/2007
Nr. 62	Ullrich Heilemann / Stefan Wappler	Sachsen wächst anders Konjunkturelle, sektorale und regionale Bestimmungsgründe der Entwicklung der Bruttowertschöpfung 1992 bis 2006 07/2007
Nr. 63	Adolf Wagner	Regionalökonomik: Konvergierende oder divergierende Regionalentwicklungen 08/2007
Nr. 64	Ullrich Heilemann / Jens Ulrich	Good bye, Professir Phillips? Zum Wandel der Tariflohdeterminanten in der Bundesrepublik 1952 – 2004 08/2007
Nr. 65	Gunther Schnabl / Franziska Schobert	Monetary Policy Operations of Debtor Central Banks in MENA Countries 10/2007
Nr. 66	Andreas Schäfer / Simone Valente	Habit Formation, Dynastic Altruism, and Population Dynamics 11/2007
Nr. 67	Wolfgang Bernhardt	5 Jahre Deutscher Corporate Governance Kodex Eine Erfolgsgeschichte? 01/2008
Nr. 68	Ullrich Heilemann / Jens Ulrich	Viel Lärm um wenig? Zur Empirie von Lohnformeln in der Bundesrepublik 01/2008
Nr. 69	Christian Groth / Karl-Josef Koch / Thomas M. Steger	When economic growth is less than exponential 02/2008
Nr. 70	Andreas Bohne / Linda Kochmann	Ökonomische Umweltbewertung und endogene Entwicklung peripherer Regionen Synthese einer Methodik und einer Theorie 02/2008
Nr. 71	Andreas Bohne / Linda Kochmann / Jan Slavik / Lenka Slaviková	Deutsch-tschechische Bibliographie Studien der kontingenten Bewertung in Mittel- und Osteuropa 06/2008
Nr. 72	Paul Lehmann / Christoph Schröter-Schlaack	Regulating Land Development with Tradable Permits: What Can We Learn from Air Pollution Control? 08/2008
Nr. 73	Ronald McKinnon / Gunther Schnabl	China's Exchange Rate Impasse and the Weak U.S. Dollar 10/2008
Nr. 74	Wolfgang Bernhardt	Managervergütungen in der Finanz- und Wirtschaftskrise Rückkehr zu (guter) Ordnung, (klugem) Maß und (vernünftigem) Ziel? 12/2008

Nr. 75	Moritz Schularick / Thomas M. Steger	Financial Integration, Investment, and Economic Growth: Evidence From Two Eras of Financial Globalization 12/2008
Nr. 76	Gunther Schnabl / Stephan Freitag	An Asymmetry Matrix in Global Current Accounts 01/2009
Nr. 77	Christina Ziegler	Testing Predictive Ability of Business Cycle Indicators for the Euro Area 01/2009
Nr. 78	Thomas Lenk / Oliver Rottmann / Florian F. Woitek	Public Corporate Governance in Public Enterprises Transparency in the Face of Divergent Positions of Interest 02/2009
Nr. 79	Thomas Steger / Lucas Bretschger	Globalization, the Volatility of Intermediate Goods Prices, and Economic Growth 02/2009
Nr. 80	Marcela Munoz Escobar / Robert Holländer	Institutional Sustainability of Payment for Watershed Ecosystem Services. Enabling conditions of institutional arrangement in watersheds 04/2009
Nr. 81	Robert Holländer / WU Chunyou / DUAN Ning	Sustainable Development of Industrial Parks 07/2009
Nr. 82	Georg Quaas	Realgrößen und Preisindizes im alten und im neuen VGR-System 10/2009
Nr. 83	Ullrich Heilemann / Hagen Findeis	Empirical Determination of Aggregate Demand and Supply Curves: The Example of the RWI Business Cycle Model 12/2009
Nr. 84	Gunther Schnabl / Andreas Hoffmann	The Theory of Optimum Currency Areas and Growth in Emerging Markets 03/2010
Nr. 85	Georg Quaas	Does the macroeconomic policy of the global economy's leader cause the worldwide asymmetry in current accounts? 03/2010
Nr. 86	Volker Grossmann / Thomas M. Steger / Timo Trimborn	Quantifying Optimal Growth Policy 06/2010
Nr. 87	Wolfgang Bernhardt	Corporate Governance Kodex für Familienunternehmen? Eine Widerrede 06/2010
Nr. 88	Philipp Mandel / Bernd Süsmuth	A Re-Examination of the Role of Gender in Determining Digital Piracy Behavior 07/2010
Nr. 89	Philipp Mandel / Bernd Süsmuth	Size Matters. The Relevance and Hicksian Surplus of Agreeable College Class Size 07/2010
Nr. 90	Thomas Kohstall / Bernd Süsmuth	Cyclic Dynamics of Prevention Spending and Occupational Injuries in Germany: 1886-2009 07/2010
Nr. 91	Martina Padmanabhan	Gender and Institutional Analysis. A Feminist Approach to Economic and Social Norms 08/2010
Nr. 92	Gunther Schnabl / Ansgar Belke	Finanzkrise, globale Liquidität und makroökonomischer Exit 09/2010
Nr. 93	Ullrich Heilemann / Roland Schuhr / Heinz Josef Münch	A "perfect storm"? The present crisis and German crisis patterns 12/2010
Nr. 94	Gunther Schnabl / Holger Zemanek	Die Deutsche Wiedervereinigung und die europäische Schuldenkrise im Lichte der Theorie optimaler Währungsräume 06/2011
Nr. 95	Andreas Hoffmann / Gunther Schnabl	Symmetrische Regeln und asymmetrisches Handeln in der Geld- und Finanzpolitik 07/2011
Nr. 96	Andreas Schäfer / Maik T. Schneider	Endogenous Enforcement of Intellectual Property, North-South Trade, and Growth 08/2011
Nr. 97	Volker Grossmann / Thomas M. Steger / Timo Trimborn	Dynamically Optimal R&D Subsidization 08/2011
Nr. 98	Erik Gawel	Political drivers of and barriers to Public-Private Partnerships: The role of political involvement 09/2011
Nr. 99	André Casajus	Collusion, symmetry, and the Banzhaf value 09/2011
Nr. 100	Frank Hüttner / Marco Sunder	Decomposing R^2 with the Owen value 10/2011
Nr. 101	Volker Grossmann / Thomas M. Steger / Timo Trimborn	The Macroeconomics of TANSTAAFL 11/2011

Nr. 102	Andreas Hoffmann	Determinants of Carry Trades in Central and Eastern Europe 11/2011
Nr. 103	Andreas Hoffmann	Did the Fed and ECB react asymmetrically with respect to asset market developments? 01/2012
Nr. 104	Christina Ziegler	Monetary Policy under Alternative Exchange Rate Regimes in Central and Eastern Europe 02/2012
Nr. 105	José Abad / Axel Löffler / Gunther Schnabl / Holger Zemanek	Fiscal Divergence, Current Account and TARGET2 Imbalances in the EMU 03/2012
Nr. 106	Georg Quaas / Robert Köster	Ein Modell für die Wirtschaftszweige der deutschen Volkswirtschaft: Das "MOGBOT" (Model of Germany's Branches of Trade)
Nr. 107	Andreas Schäfer / Thomas Steger	Journey into the Unknown? Economic Consequences of Factor Market Integration under Increasing Returns to Scale 04/2012
Nr. 108	Andreas Hoffmann / Björn Urbansky	Order, Displacements and Recurring Financial Crises 06/2012
Nr. 109	Finn Marten Körner / Holger Zemanek	On the Brink? Intra-euro area imbalances and the sustainability of foreign debt 07/2012
Nr. 110	André Casajus / Frank Hüttner	Nullifying vs. dummifying players or nullified vs. dummified players: The difference between the equal division value and the equal surplus division value 07/2012
Nr. 111	André Casajus	Solidarity and fair taxation in TU games 07/2012
Nr. 112	Georg Quaas	Ein Nelson-Winter-Modell der deutschen Volkswirtschaft 08/2012
Nr. 113	André Casajus / Frank Hüttner	Null players, solidarity, and the egalitarian Shapley values 08/2012
Nr. 114	André Casajus	The Shapley value without efficiency and additivity 11/2012
Nr. 115	Erik Gawel	Neuordnung der W-Besoldung: Ausgestaltung und verfassungsrechtliche Probleme der Konsumtionsregeln zur Anrechnung von Leistungsbezügen 02/2013
Nr. 116	Volker Grossmann / Andreas Schäfer / Thomas M. Steger	Migration, Capital Formation, and House Prices 02/2013
Nr. 117	Volker Grossmann / Thomas M. Steger	Optimal Growth Policy: the Role of Skill Heterogeneity 03/2013
Nr. 118	Guido Heineck / Bernd Süßmuth	A Different Look at Lenin's Legacy: Social Capital and Risk Taking in the Two Germanies 03/2013
Nr. 119	Andreas Hoffmann	The Euro as a Proxy for the Classical Gold Standard? Government Debt Financing and Political Commitment in Historical Perspective 05/2013
Nr. 120	Andreas Hoffmann / Axel Loeffler	Low Interest Rate Policy and the Use of Reserve Requirements in Emerging Markets 05/2013
Nr. 121	Gunther Schnabl	The Global Move into the Zero Interest Rate and High Debt Trap 07/2013
Nr. 122	Axel Loeffler / Gunther Schnabl / Franziska Schobert	Limits of Monetary Policy Autonomy and Exchange Rate Flexibility by East Asian Central Banks 08/2013
Nr. 123	Burkhard Heer / Bernd Süßmuth	Tax Bracket Creep and its Effects on Income Distribution 08/2013
Nr. 124	Hans Fricke / Bernd Süßmuth	Growth and Volatility of Tax Revenues in Latin America 08/2013
Nr. 125	Ulrich Volz	RMB Internationalisation and Currency Co-operation in East Asia 09/2013
Nr. 126	André Casajus / Helfried Labrenz	A property rights based consolidation approach 02/2014
Nr. 127	Pablo Duarte	The Relationship between GDP and the Size of the Informal Economy: Empirical Evidence for Spain 02/2014
Nr. 128	Erik Gawel	Neuordnung der Professorenbesoldung in Sachsen 03/2014
Nr. 129	Friedrun Quaas	Orthodoxer Mainstream und Heterodoxe Alternativen Eine Analyse der ökonomischen Wissenschaftslandschaft 04/2014
Nr. 130	Gene Callahan / Andreas Hoffmann	The Idea of a Social Cycle 05/2014

Nr. 131	Karl Trela	Klimaanpassung als wirtschaftspolitisches Handlungsfeld 06/2014
Nr. 132	Erik Gawel / Miquel Aguado	Neuregelungen der W-Besoldung auf dem verfassungsrechtlichen Prüfstand 08/2014
Nr. 133	Ulf Papenfuß / Matthias Redlich / Lars Steinhauer	Forschend und engagiert lernen im Public Management: Befunde und Gestaltungsanregungen eines Service Learning Lehrforschungsprojektes 10/2014
Nr. 134	Karl Trela	Political climate adaptation decisions in Germany - shortcomings and applications for decision support systems 11/2014
Nr. 135	Ulf Papenfuß / Lars Steinhauer / Benjamin Friedländer	Beteiligungsberichterstattung der öffentlichen Hand im 13-Länder-Vergleich: Erfordernisse für mehr Transparenz über die Governance und Performance öffentlicher Unternehmen 02/2015
Nr. 136	Gunther Schnabl	Japans Lehren für das Schweizer Wechselkursdilemma 02/2015
Nr. 137	Ulf Papenfuß / Christian Schmidt	Determinants of Manager Pay in German State-Owned Enterprises and International Public Policy Implications: 3-Year Study for Sectors, Performance and Gender 02/2015
Nr. 138	Philipp Mandel / Bernd Süßmuth	Public education, accountability, and yardstick competition in a federal system 05/2015
Nr. 139	Gunther Schnabl	Wege zu einer stabilitäts- und wachstumsorientierten Geldpolitik aus österreichischer Perspektive 06/2015
Nr. 140	Ulf Papenfuß / Matthias Redlich / Lars Steinhauer / Benjamin Friedländer	Forschend und engagiert lernen im Public Management: Befunde und Gestaltungsanregungen eines Service Learning Lehrforschungsprojektes – 2. aktualisierte Auflage 08/2015
Nr. 141	Friedrun Quaas / Georg Quaas	Hayeks Überinvestitionstheorie 10/2015
Nr. 142	Bastian Gawellek / Marco Sunder	The German Excellence Initiative and Efficiency Change among Universities, 2001-2011 01/2016
Nr. 143	Benjamin Larin	Bubble-Driven Business Cycles 02/2016
Nr. 144	Friedrun Quaas / Georg Quaas	Effekte des Kapitalmarktzins auf die Preis- und Produktivitätsentwicklung Eine Analyse der deutschen Volkswirtschaft 1970-2014 02/2016
Nr. 145	Thomas Lenk / Matthias Redlich / Philipp Glinka	Nachhaltige Stadtfinanzen - Akzeptanzsteigerung der bürgerschaftlichen Beteiligung an der Haushaltsplanung 02/2016
Nr. 146	Michael von Prollius / Gunther Schnabl	Geldpolitik, Arabellion, Flüchtlingskrise 10/2016
Nr. 147	David Leuwer / Bernd Süßmuth	The Exchange Rate Susceptibility of European Core Industries, 1995-2010 05/2017
Nr. 148	Gunther Schnabl	Monetary Policy and Wandering Overinvestment Cycles in East Asia and Europe 05/2017
Nr. 149	Ullrich Heilemann / Karsten Müller	Wenig Unterschiede – Zur Treffsicherheit internationaler Prognosen und Prognostiker 07/2017
Nr. 150	Gunther Schnabl / Sebastian Müller	Zur Zukunft der Europäischen Union aus ordnungspolitischer Perspektive 10/2017
Nr. 151	Gunther Schnabl	Ultra-lockere Geldpolitiken, Finanzmarktblasen und marktwirtschaftliche Ordnung 10/2017
Nr. 152	Pablo Duarte / Bernd Süßmuth	Implementing an approximate dynamic factor model to nowcast GDP using sensitivity analysis 02/2018
Nr. 153	Sophia Latsos	Real Wage Effects of Japan's Monetary Policy 03/2018
Nr. 154	Gunther Schnabl / Klaus Siemon	Die EU-Insolvenzrichtlinie zu vorinsolvenzlichen Verfahren aus ordnungspolitischer Perspektive The EU Directive on Preventive Restructuring Frameworks from a Ordoliberal Perspective 07/2018
Nr. 155	Marika Behnert / Thomas Bruckner	Cost effects of energy system stability and flexibility options – an integrated optimal power flow modeling approach 09/2018
Nr. 156	Gunther Schnabl	70 Years after the German Currency and Economic Reform: The Monetary, Economic and Political Order in Europe is Disturbed 10/2018
Nr. 157	Wolfgang Bernhardt	Corporate Governance und Compliance Bunte Streiflichter 2018 11/2018

Nr. 158	Friedrun Quaas	Der spezifische Liberalismus von Hayek im Spektrum des Neoliberalismus 01/2019
Nr. 159	Sophia Latsos	The Low Interest Policy and the Household Saving Behavior in Japan 03/2019
Nr. 160	Gunther Schnabl	Die Verteilungseffekte der Geldpolitik der Europäischen Zentralbank und deren Einfluss auf die politische Stabilität 06/2019
Nr. 161	Wolfgang Bernhardt	30 Jahre nach dem Fall der Mauer Einheit in Zweierheit? 07/2019
Nr. 162	Gunther Schnabl / Tim Sepp	30 Jahre nach dem Mauerfall Ursachen für Konvergenz und Divergenz zwischen Ost- und Westdeutschland 09/2019
Nr. 163	Karl-Friedrich Israel / Sophia Latsos	The Impact of (Un)Conventional Expansionary Monetary Policy on Income Inequality – Lessons from Japan 11/2019
Nr. 164	Wolfgang Bernhardt	30 Jahre nach dem Fall der Mauer am 9. November 1989 Einheit in Zweierheit – Teil II 11/2019
Nr. 165	Friedrun Quaas	Pluralismus in der Ökonomik – verpasste Chance, überfälliges Programm oder normalwissenschaftliche Realität? 03/2020
Nr. 166	Gunther Schnabl / Nils Sonnenberg	Monetary Policy, Financial Regulation and Financial Stability: A Comparison between the Fed and the ECB 04/2020
Nr. 167	Georg Quaas	The reproduction number in the classical epidemiological model 04/2020