

Tangian, Andranik S.

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German parliamentary elections 2005 in the mirror of party manifestos

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German parliamentary elections 2005 in the mirror of party manifestos

Andranik S. Tangian

Discussion paper Nr. 139E

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Privatdozent Dr. Dr. Andranik Tangian
WSI in the Hans-Böckler-Foundation
Hans-Böckler-Straße 39
D-40476 Düsseldorf
Tel: +49 211 7778-259
Fax: +49 211 7778-190
Andranik-Tangian@Boeckler.De

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Abstract

Five leading German political parties and their coalitions are evaluated with regard to party manifestos and results of the 2005 parliamentary elections. For this purpose, the party manifestos are converted into Yes/No answers to 95 topical questions (Relax the protection against dismissals? Close nuclear power plants? etc.). On each question, every party represents its adherents as well as those of the parties with the same position. Therefore, a party usually represents a larger group than its direct adherents.

The popularity of a party is understood to be the percentage of the electorate represented, averaged on all the 95 questions. The universality of a party is the frequency of representing a majority of electors. The questions are considered either unweighted, or weighted by an expert, or weighted by the number of GOOGLE-results for given keywords (the more important the question, the more documents in the Internet). The weighting however plays a negligible role because the party answers are backed up by the party “ideology” which determines a high intra-question correlations.

The SPD (Social-Democratic Party) did not receive the highest percentage of votes, remains nevertheless the most popular and the most universal German party. A comparison of the election results with the position of German Trade Union Federation (DGB) reveals its high representativeness as well.

Finally, all coalitions with two and three parties are also evaluated. The coalition CDU/SPD (which is currently in power) is the most popular, and the coalition SPD/Green/Left-Party (which failed due to personal conflicts) is the most universal.

Keywords: Parliamentary election, fractions, coalitions, theory of voting, mathematical theory of democracy, indices of popularity and universality, German Trade Union Federation (DGB).

JEL Classification: D71

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1 Introduction

The result of German parliamentary elections 2005 (BUNDESTAG 2005, BUNDESWAHLLEITER 2005, SUEDEUTSCHE 2005; see the “second votes” devoted to parties)

	SPD	CDU/CSU	Green	FDP	Left-Party	19 minor parties
Percentage of votes	34.2	35.2	8.1	9.8	8.7	4.0

is difficult to judge, because no party got an absolute majority of votes¹. It is known that the bottle-necks of simple majority voting systems are so drastic that the legitimacy of election results can be put in question (HELD 1996, SAMONS 2004).

Already in 1770, the member of the Royal Academy of Sciences in Paris J.-Ch. de Borda (1733–1799) warned against “wrong results” of simple majority vote for more than two parties (BLACK 1958). Borda has illustrated it with an example of three candidates A, B, C and 21 voters, see Figure 1. The winner is the candidate A who receives 8 votes. On the other hand, A is the most undesired for an absolute majority of 13 of 21 voters.

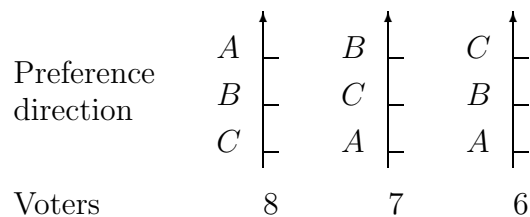


Figure 1: Example of Borda (1770, see BLACK 1958, p. 157)

In similar cases, more information than just the first choices should be considered: candidate rankings (preferences of electors with second and third priorities), preference grades, quantitative estimations, etc. Such problems are studied in the theory of voting and social choice since the 18 century, however, with no unambiguous solution (MUELLER 1989). Therefore, to attain a social consensus, each particular situation should be analyzed from several sides.

The given paper evaluates the five leading political parties in the German parliamentary elections on September 18 and in Dresden on October 2, 2005, following the mathematical model of democracy (TANGIAN 1991–2005). The data required for the application are similar to that used by the Internet program WAHL-O-MAT (2005): a

¹SPD is the Social Democratic Party, CDU is the Christian Democratic Union together with Bavaria’s Christian Social Union (conservators), FDP is the Free Democratic Party (neoliberals), and Left-Party is a fusion of the PDS (Party of Democratic Socialism—former East German communists) with the WASG (Voting Alternative for Employment and Social Justice—the separated left wing of the SPD).

tabular representation of party positions and individual opinions in the form of Yes/No answers to topical questions (Relax the protection against dismissals? Yes/No; Close nuclear power plants? Yes/No, etc.). The task of the mathematical model of democracy is different, however. The program WAHL-O-MAT helps the *individual user* to find the party to vote for (the program was originally aimed at involving young people into political participation). The mathematical model of democracy estimates how good the parties represent the *whole of electorate*. In other words, the WAHL-O-MAT tests the goodness of fit of single voters to parties, and the mathematical model of democracy tests the goodness of fit of parties to the electorate.

The parties are evaluated with indicators of *popularity* and *universality*. The indicators can be regarded as two global *social utility functions* which reflect the ability of parties and their coalitions to represent the electorate. The indicators are derived from the size of groups resulting from *crosscutting cleavages* (Pitkin 1967, Miller 1964, Wright 1978, Miller 1983, and Brams et al. 1998). In a sense, both indicators suggest a kind of correlation measure for estimating the proximity between party positions and voters' opinions introduced into political science by Achen (1977, 1978).

The crosscutting cleavages are determined by 95 dichotomous questions² (with Yes/No answers), each dividing the society into two groups, one with positive, and one with negative opinion on the question. The parties, answering these questions, represent some Yes-groups and some No-groups. The popularity of a party is measured by the size of the group represented, averaged on the 95 questions selected. The universality of a party is the frequency of representing a majority. Therefore, the popularity reflects the spatial aspect of representativeness, and the universality reflects its temporal aspect.

The questions are considered either unweighted, or weighted by an expert, or weighted by the number of GOOGLE-results for given keywords (the more important the question, the more documents in the Internet). It turns out that the weighting plays a negligible role. The party answers backed up by the party "ideology" are highly correlated, making the overall evaluation little sensitive to question weights.

The quantitative analysis shows that the SPD is the most popular and the most universal German party, although it did not receive the highest percentage of votes. A comparison of the election results with the position of German Trade Union Federation (DGB) reveals its high representativeness as well. Finally, all coalitions with two and three parties are also evaluated. The coalition CDU/SPD (which is actually in power) is

²The same number as of the *Theses* by Martin LUTHER (1517).

the most popular, and the coalition SPD/Green/Left-Party (which failed due to personal conflicts) is the most universal.

In Section 2, “Model”, initial data, basic assumptions, and indicators of popularity and universality of parties are introduced. In particular, all the 95 source questions with party answers and weightings are listed.

In Section 3, “Evaluation of parties”, the indices of popularity and universality of parties and of DGB are discussed, both for the whole of Germany and for federal states (Länder). The main conclusion is that the SPD has better indicators, although the CDU got the highest percentage of votes.

In Section 4, “Evaluation of coalitions”, a kind of coalition formation analysis is performed; for recent references see van Deemen (1997) and de Vries (1999). For this purpose, the indices of popularity and universality are extended to coalitions with two and three parties. The main conclusion is that the coalition of three left parties could be a better alternative to the centrist coalition currently in power.

In Section 5, “Summary”, the main statements of the paper are recapitulated.

In Section 6, “Computational issues”, the mathematical model is rigorously described. The computation formulas are derived and linked to tables and figures of the paper.

Section 7, “Proofs”, contains demonstrations to two theorems of the paper.

2 Model

Journalists Anne Graef (*Einblick*, DGB, Berlin) and Michael Schultheiss (Hannover) have analyzed comparative tables of party manifestos downloadable from the Internet like (GEWERKSCHAFT DER POLIZEI BREMEN 2005, WERNER ET AL. 2005) and published in popular journals. The distinctions in the form of Yes/No answers to 95 questions are collected in Table 1. The position of the German Federation of Trade Unions (DGB) has been specified by Anne Graef and Michael Schultheiss.

The unequal importance of the questions can be reflected by weight coefficients. Such a weighting (with grades 1–5) performed by Anne Graef as a DGB-expert is shown in the next to last column of Table 1. The last column of Table 1 contains the number of GOOGLE-results for given (German) keywords on September 25, 2005. The idea of using GOOGLE as a weighting instrument is as follows: the more important the question, the more frequently it is discussed, and the more documents in the Internet contain the related keywords.

It is assumed that the opinions of voters on all the 95 questions are represented by their

Table 1: Structuralized representation of party manifestos

	Opinions of parties and unions						Weighting	
	SPD	CDU	Green	FDP	Left-Party	Unions	Expert	Google results
	34.2%	35.2%	8.1%	9.8%	8.7%		18.09.05	25.09.05
Labour market								
Relax the protection against dismissals	No	Yes	No	Yes	No	No	5	25300
Sector-dependent minimal wages	Yes	No	Yes	No	No	Yes	4	367
Statutory minimal wage	Yes	No	Yes	No	Yes	Yes	3	32500
Prolong Unemployment Benefits I (ALG I) for old employees	Yes	No	Yes	No	Yes	Yes	5	32700
Equalize Unemployment Benefits II (ALG II) in East and West	Yes	No	Yes	No	Yes	Yes	5	65900
Increase Unemployment Benefits II	No	No	Yes	No	Yes	Yes	5	23500
Unemployment Benefits II: higher allowance for old-age provisions	No	No	Yes	No	Yes	Yes	5	14900
Cancel Hartz IV	No	No	No	No	Yes	No	4	25700
Increase the limit for mini-jobs up to 600 EUR	No	No	No	Yes	No	No	5	41
Adopt combined wages	No	Yes	No	No	No	No	3	54000
Statutory subsidies to low-paid	No	No	Yes	No	No	Yes	3	604
Abolish the Federal Employment Office	No	No	No	Yes	No	No	5	32000
Further subsidize self-employment (Ich-AGs)	Yes	No	No	No	No	No	4	24600
Narrow rights of employees	No	Yes	No	Yes	No	No	5	20100
Broaden laws for foreign workers	Yes	No	Yes	No	Yes	Yes	5	407
Efficient protection against dismissals also in small enterprizes	Yes	No	Yes	No	Yes	Yes	5	30000
Unemployment Benefits II: reconsideration of partner's income	No	No	Yes	No	Yes	Yes	5	548
Simplifying temporary employment	No	Yes	No	Yes	No	No	5	9180
More actively subsidize the labour market	Yes	No	Yes	No	Yes	Yes	5	40900
Co-determination								
Restrict the establishment of works councils	No	No	No	Yes	No	No	5	35900
Cancel the parity co-determination	No	No	No	Yes	No	No	5	27300
Pacts for employment and competitiveness	No	Yes	No	Yes	No	No	5	120000
Reduce the autonomy of collective bargaining	No	Yes	No	Yes	No	No	5	12400
Reduce the influence of trade unions	No	Yes	No	Yes	No	No	5	34100
Abolish generally binding collective agreements	No	Yes	No	Yes	No	No	5	980
Retain area collective agreements	Yes	No	Yes	No	Yes	Yes	5	34600
Trade union representatives in supervisory boards	Yes	Yes	Yes	No	Yes	Yes	5	712
Economy								
Reduce the corporation tax	Yes	Yes	No	Yes	No	No	3	359000
Deregulate employment, health, and environment protection	No	Yes	No	Yes	No	No	5	770000
More favorable credits for medium-sized business	Yes	Yes	No	Yes	No	No	3	472000
More generous financing of research	Yes	Yes	Yes	Yes	No	Yes	3	289000
Better control of hedge funds	Yes	Yes	Yes	No	Yes	Yes	4	57400
Disclose the income of firms' directors	Yes	Yes	Yes	No	Yes	Yes	4	156000
Increase public investments	Yes	No	No	No	Yes	Yes	5	454000
No further reduction of statutory social responsibility	Yes	No	No	Yes	No	Yes	5	60100
Privatization of state investments	No	No	No	Yes	No	No	4	56300

Table 1: (Continued) Structuralized representation of party manifestos

	Opinions of parties and unions						Weighting	
	SPD	CDU	Green	FDP	Left-Party	Unions	Expert	Google results
	34.2%	35.2%	8.1%	9.8%	8.7%		18.09.05	25.09.05
Taxes								
Increase the value added tax (VAT)	No	Yes	No	No	No	No	5	37400
Abolish trade income tax	No	No	No	Yes	No	No	5	29800
Reduce the highest tax rate	No	Yes	No	Yes	No	No	4	57800
Increase the highest tax rate	No	No	Yes	No	Yes	Yes	4	44300
Tax-free surcharges for work on weekends and at night	No	Yes	No	Yes	No	No	5	10200
Cut flat-rate benefits for commuter travels	No	Yes	Yes	Yes	No	No	5	32400
Tax on assets	No	No	Yes	No	Yes	Yes	5	220000
Increase the inheritance tax for large heritages	No	No	Yes	No	Yes	Yes	4	610
"Richness-tax" for high income	Yes	No	Yes	No	Yes	Yes	5	54000
Tax on realization of shares in joint-stock companies	No	Yes	No	Yes	Yes	Yes	4	561
Health and pensions								
Account the income from capital	Yes	No	Yes	No	Yes	Yes	5	519
Compulsory health insurance for all employed	Yes	No	Yes	No	Yes	Yes	5	743000
Equal contribution for all insured	No	Yes	No	No	No	No	5	226000
Abolish the compulsory health insurance	No	No	No	Yes	No	No	5	968000
Obligatory basic insurance at private health insurance	No	No	No	Yes	No	No	5	203
Strengthen private old-age provisions	Yes	Yes	Yes	Yes	No	No	4	94600
Prolong the duration of working life	Yes	Yes	Yes	Yes	No	Yes	3	37500
Obligatory insurance for pensions	No	No	Yes	No	Yes	Yes	4	1
Basic income	No	No	No	Yes	No	No	5	652
Gender								
Anti-discrimination law	Yes	No	Yes	No	No	Yes	5	316000
Equal-opportunity law for private firms	No	No	Yes	No	Yes	Yes	5	13300
Reform the splitting of income between spouses	No	No	Yes	No	Yes	Yes	3	646
Family								
More general right to part-time work	Yes	No	Yes	No	Yes	Yes	5	526
One-year parental benefits as wage substitute	Yes	No	Yes	No	Yes	Yes	5	152
Right to the parental leave for children under three years	Yes	No	Yes	No	Yes	Yes	4	12800
Child-bonus 50 EUR to the pension insurance	No	Yes	No	No	No	No	3	12600
Education								
Tuition fees for the first course of study	No	Yes	No	Yes	No	No	4	102000
Abolish the school system with three school types	No	No	Yes	No	Yes	No	2	608
Statutory apportionment of trainee positions	No	No	Yes	No	Yes	Yes	5	1740
Strengthen / subsidize professional training	Yes	No	Yes	Yes	Yes	Yes	4	198000
Establishing the all-day school	Yes	No	Yes	No	Yes	Yes	5	91500

Table 1: (Continued) Structuralized representation of party manifestos

	Opinions of parties and unions						Weighting	
	SPD	CDU	Green	FDP	Left-Party	Unions	Expert	Google results
	34.2%	35.2%	8.1%	9.8%	8.7%		18.09.05	25.09.05
East Germany								
Some exceptions from federal laws	No	Yes	No	Yes	No	No	5	21000
Subsidize households	Yes	Yes	No	Yes	Yes	No	3	313
Energy and environment								
Longer terms for nuclear power plants	No	Yes	No	Yes	No	No	4	9910
Subsidize renewable energy	Yes	No	Yes	No	Yes	Yes	3	212000
Subsidize black coal industry	Yes	No	No	No	Yes	Yes	4	46800
Continue the agriculture reform	Yes	No	Yes	No	Yes	Yes	3	568
Restrict genetic technologies	Yes	No	Yes	No	Yes	Yes	3	133000
Introduce the environmental code	Yes	No	Yes	No	No	Yes	3	18700
Abolish the can pledge	No	No	No	Yes	No	No	1	12300
Speed limit on motor ways	No	No	Yes	No	Yes	No	1	50500
Domestic policy								
Use army forces in domestic problems	No	Yes	No	Yes	No	No	3	46800
Abolish large bugging operations	No	No	Yes	Yes	Yes	Yes	2	606
Abolish the access of public authorities to bank accounts	No	No	No	Yes	No	No	3	215
Sharpen the immigration restrictions	No	Yes	No	No	No	No	4	62
Allow double nationality	No	No	Yes	No	Yes	Yes	4	107000
People's participation at the national level	No	No	Yes	Yes	Yes	Yes	3	17000
Improve labelling of foodstuffs	Yes	Yes	Yes	No	Yes	Yes	1	39000
Culture as the national objective in the Basic Law	Yes	No	Yes	Yes	No	Yes	1	19700
Foreign affairs								
Abolish the compulsory military service	No	No	Yes	Yes	Yes	No	2	38100
NATO is the most important security pact	No	Yes	No	Yes	No	No	3	37
Withdraw nuclear weapons from Germany	No	No	No	No	Yes	No	3	29200
Increase the development aid for other countries	Yes	No	Yes	No	Yes	Yes	3	104000
European politics								
Accept Turkey as the EU member state	Yes	No	Yes	Yes	Yes	Yes	4	51400
Strictly respect the stability pact	No	Yes	No	Yes	No	No	5	853
Reconsider the EU-constitution	No	No	No	Yes	Yes	No	5	548
Develop the European joint defense policy	Yes	Yes	Yes	Yes	No	Yes	3	28200
Liberalize the Single European Market for services	No	Yes	No	Yes	No	No	5	35000
Strengthen the social dimension of the EU	Yes	No	Yes	No	Yes	Yes	5	144000

favorite parties. It would be certainly better to use individual answers to each question but such data are not available.

Each party, as well as the DGB has its own Yes/No answer to each question. By voting for a Yes-party or for a No-Party (for a given question), the voters build two groups: Yes-group and No-Group. The *representativeness* of a Yes-Party for a given question is the size of the whole Yes-Group. Respectively, the *representativeness* of a No-Party for a given question is the size of the whole No-Group; see Figure 2.

For example, consider the first question “Relax the protection against dismissals?”. The SPD, Green, and Left-Party are against this measure, having the representativeness $34.2 + 8.1 + 8.7 = 51\%$, that is, representing 51% of voters. This No-group is shown in Figure 2 by red, green, and dark-red rectangles³ (with the lengths being the percentage of votes received by each party). These rectangles lie in the No-Domain to the left from the vertical axis 0%. CDU and FDP will relax the protection against dismissals and represent thereby $35.2 + 9.8 = 45.0\%$ of the electorate. This Yes-group is shown by black and yellow rectangles. It lies in the Yes-Domain to the right from the vertical axis. The sum of both groups is always 96%. The more the No-Group overbalances, the more the total 96%-rectangle is shifted to the left. The more the Yes-Group overbalances, the more it is shifted to the right.

On each question, every party represents its adherents as well as those of the parties with the same position. For example, the SPD’s *representativeness* is 51% on the first question, 44.2% on the second question, etc., although it obtained only 34.2% votes. This means that a party usually represents a larger group than its direct adherents.

The average percentage of the voters represented is understood to be the *popularity* of the party. For example, the computed popularity of the SPD is 67.5%, almost twice the percentage of its direct adherents.

The frequency of representing a majority ($\geq 50\%$) is called the *universality* of the party. For instance, the SPD represents a majority on 86 of 95 questions. This provides $\frac{86}{95} \cdot 100\% = 90.5\%$ universality.

The indices of popularity and universality can be computed directly (unweighted), or with weight coefficients of the questions. Then the popularity is defined to be the weighted average representativeness, and the universality is defined to be the weighted frequency of representing a majority; for details see Section 6.

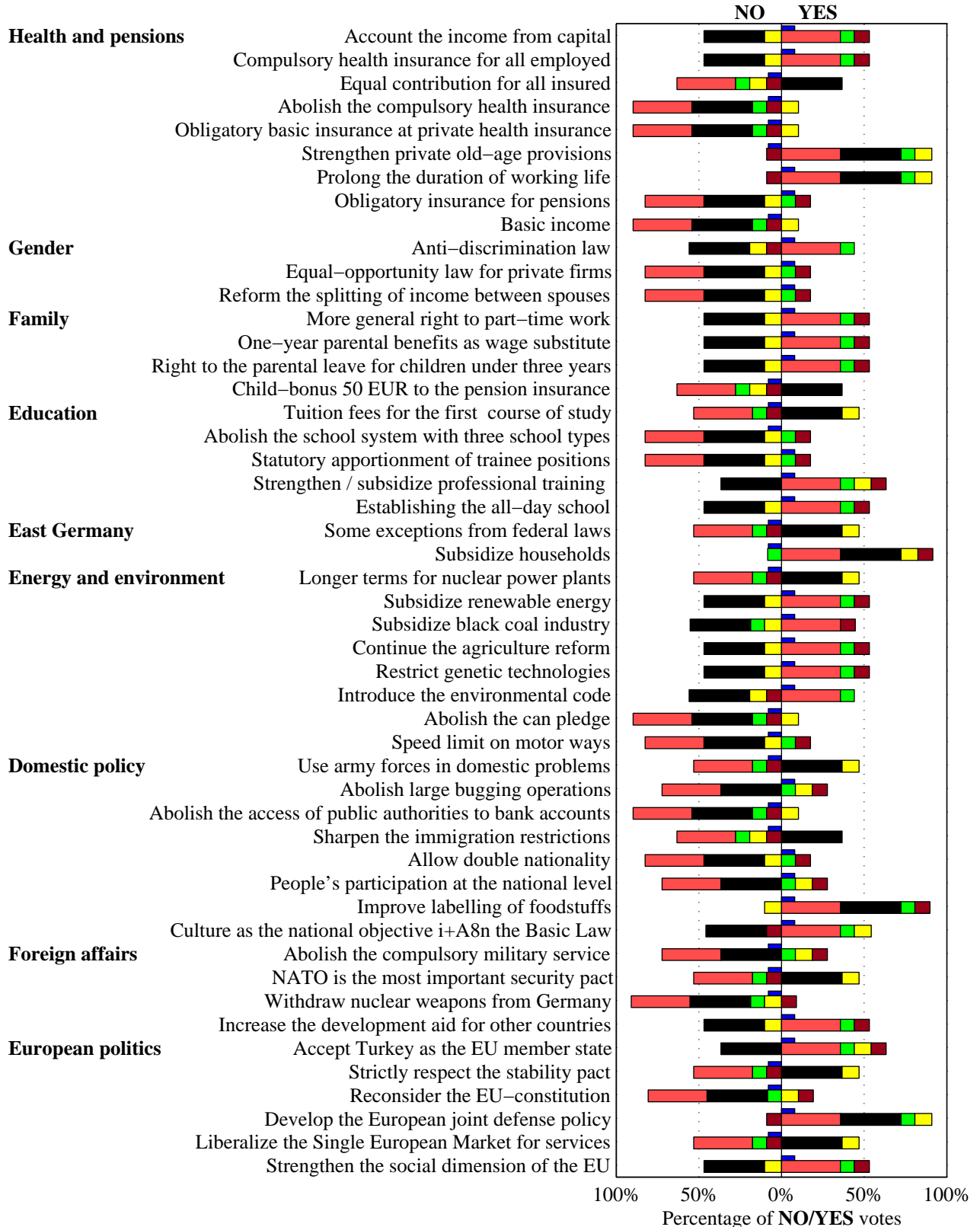
³Throughout the paper, the official colors of the five parties are used. The DGB’s unofficial color is red, but we use blue to distinct the DGB from the “red” parties.

Figure 2: What do the voters give their votes for?



Figure 2: (Continued) What do the voters give their votes for?

Percentage of voters, who with YES/NO to the questions of party manifestos answer: by voting for the
 SPD ■, CDU ■, Green ■, FDP ■, Left-Party ■. Position of unions (DGB) ■



3 Evaluation of parties

The popularity and universality of parties and of DGB are displayed in Figure 3. Each indicators is given in three versions:

- for *Unweighted* (u) questions,
- for by Anne Graef as *Expert* (e) weighted questions as well as
- by *GOOGLE*-results (g) weighted questions.

Which conclusions do follow from Figure 3?

1. In spite of shortage of votes, the SPD remains to be the most popular and most universal German party

The 67.5% unweighted popularity of the SPD means that on the average it represents the opinion of 67.5% voters. The unweighted 90.5% universality means that it represents a majority on 86 of 95 questions.

Note that both indices of the SPD are higher than that of the election winner CDU. This means that, despite the unfavorable vote ratio, the SPD represents the electorate better. The shortage of votes is rather due to a disappointment of citizens by the economic recession and governmental policy than due to the electoral program.

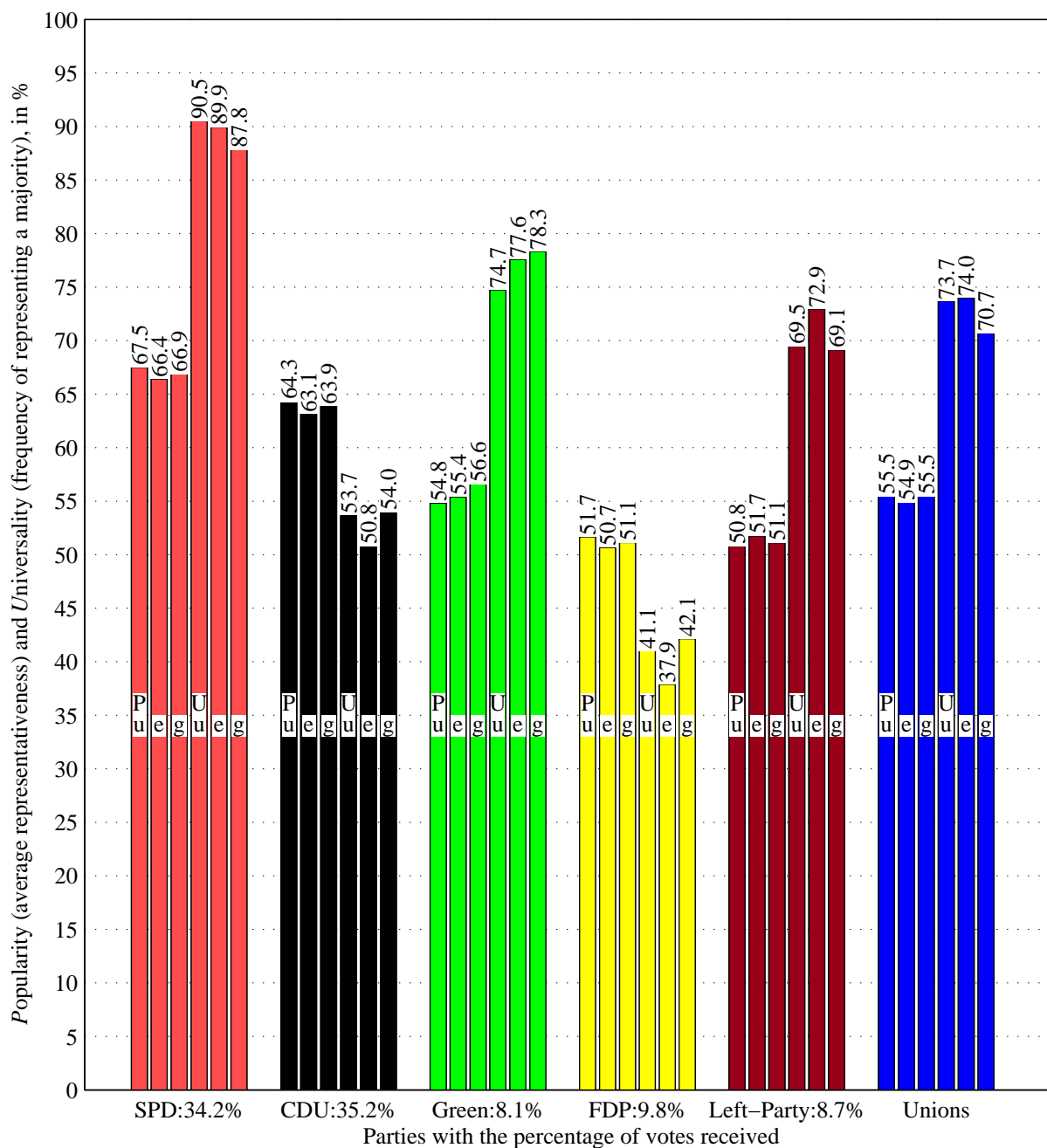
2. High representativeness of the German Trade Union Federation (DGB)

The high indices of the DGB mean that it well represents the public opinion and finds a significant support in the society. Note that this conclusion is obtained with no interrogation of public opinion but indirectly, by comparing the position of the DGB with the election results.

3. Weighting plays a negligible role in the evaluation

As one can see in Figure 3, the three weighting types of the questions (unweighted, expert-weighting, and GOOGLE-weighting) has a little influence on the indicators' values. Indeed, the party answers are backed up by the party "ideology" which determines a high intra-question correlations. Therefore, under-weighting and even omitting some questions plays a negligible role, because other questions carry superfluous information on the parties (over-weighting some questions is equivalent to under-weighting other questions).

Figure 3: Indicators of popularity and universality of parties and of DGB for unweighted (u) questions, for by an expert (e) weighted questions and for by GOOGLE-results (g) weighted questions



Henceforth, only unweighted indicators will be considered.

Table 2 displays the percentage of votes and unweighted indices of popularity and universality of the parties and of the DGB for all the 16 German federal states and for the whole of Germany. Besides, each column of the table is provided with a ranking R of the federal states with respect to the corresponding indicator.

For example, the SPD is the most popular party in Bremen (74.5%, rank 1), where its universality 98.9% is also maximal (the SPD represents a majority in 94 of 95 questions). However, its highest percentage of votes is attained in Niedersachsen with somewhat lower indices (5th and 3rd rank, respectively).

Both unweighted indicators are depicted in Figures 4 and 5. The CDU surpasses the SPD in Bavaria, Baden-Württemberg and Rheinland-Pfalz. At the party landscape, the CDU is seen behind the SPD in these three openings only.

The DGB attains its highest popularity in Brandenburg (65.8%) and the highest universality in Bremen (80%), whereas the CDU has a quite low standing in these federal states. The DGB is least popular and universal in the conservative Bavaria, where the CSU (the Bavarian partner of CDU) has the largest percentage of votes with rank 1 and represents a majority in all the 95 of 95 questions.

4 Evaluation of coalitions

Table 3 displays all possible coalitions with two and three parties with their indices of popularity and universality for unweighted questions. The first column contains the names of parties which constitute the coalition. The second column shows the coalition size in the total percentage of votes (= the percentage of parliament seats) with its ranking. For example, the first two-party coalition SPD/CDU has the percentage of votes $34.2 + 35.2 = 69.4\%$.

The third column shows the degree of unanimity of the coalition, expressed in % of questions on which all the member parties agree, also with a ranking. The following coalitions have the highest unanimity: Green/Left-Party (rank 1, $81.1\% = 78/95$ questions), SPD/Green (rank 2, $73.7\% = 70/95$ questions), CDU/FDP (rank 3, $70.5\% = 67/95$ questions), and SPD/Left-Party (rank 4, $66.3\% = 63/95$ questions). Then go the triplet-coalition SPD/Green/Left-Party with rank 5 and unanimity $61.1\% = 58/95$ questions. The much discussed “Jamaica-coalition” CDU/Green/FDP (called so by its black-green-yellow colors) agrees in $10.5\% = 10/95$ questions only and has rank 17.

Table 2: Unweighted popularity and universality of parties and of DGB with ranks R of the German federal states (Länder)

	SPD			CDU			Green			FDP			Left-Party			Unions	
	Votes	Popu- larity	Univer- sality	Votes	Popu- larity	Univer- sality	Votes	Popu- larity	Univer- sality	Votes	Popu- larity	Univer- sality	Votes	Popu- larity	Univer- sality	Popu- larity	Univer- sality
	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R	%/R
Baden-Württemberg	30.1/13	64.6/15	52.6/9	39.2/2	67.8/2	91.6/2	10.7/4	52.0/15	36.8/8	11.9/1	55.1/2	78.9/1	3.8/14	46.7/15	31.6/8	52.1/15	35.8/7
Bayern	25.5/15	61.3/16	44.2/10	49.3/1	73.6/1	100.0/1	7.9/7	47.7/16	28.4/9	9.5/6	57.8/1	70.5/2	3.4/15	42.9/16	25.3/9	47.7/16	27.4/8
Berlin	34.4/9	71.7/3	97.9/2	22.0/15	53.9/15	46.3/9	13.7/3	64.0/1	75.8/3	8.2/9	43.8/15	33.7/8	16.4/6	60.8/4	68.4/4	64.4/2	78.9/2
Brandenburg	35.8/6	72.2/2	93.7/5	20.6/16	52.1/16	50.5/6	5.1/12	63.9/2	71.6/6	6.9/13	41.7/16	37.9/5	26.6/1	64.3/1	72.6/2	65.4/1	76.8/3
Bremen	43.0/2	74.5/1	98.9/1	22.8/14	55.6/12	45.3/10	14.3/2	63.3/3	74.7/4	8.1/10	45.0/12	34.7/7	8.3/7	57.9/6	67.4/5	64.0/3	80.0/1
Hamburg	38.7/4	71.2/4	94.7/4	28.9/11	59.5/9	49.5/7	14.9/1	60.1/6	78.9/1	9.0/7	48.0/9	36.8/6	6.3/8	54.4/9	65.3/6	60.4/7	75.8/4
Hessen	35.7/7	67.9/12	90.5/8	33.7/6	64.2/6	53.7/3	10.1/5	54.8/12	74.7/4	11.7/2	52.6/5	41.1/4	5.3/10	49.7/12	69.5/3	55.4/12	73.7/6
Mecklenburg-Vorpommern	31.7/12	68.8/9	92.6/6	29.6/10	57.9/11	51.6/5	4.0/15	59.4/7	70.5/7	6.3/14	45.4/11	36.8/6	23.7/3	59.5/5	73.7/1	60.6/6	75.8/4
Niedersachsen	43.2/1	71.2/5	95.8/3	33.6/7	63.8/7	48.4/8	7.4/9	55.7/10	77.9/2	8.9/8	50.9/7	37.9/5	4.3/13	50.5/10	64.2/7	56.9/10	76.8/3
Nordrhein-Westfalen	40.0/3	69.6/7	91.6/7	34.4/5	64.4/5	52.6/4	7.6/8	54.9/11	73.7/5	10.0/5	51.9/6	42.1/3	5.2/11	50.0/11	68.4/4	55.9/11	74.7/5
Rheinland-Pfalz	34.6/8	66.5/13	52.6/9	36.9/3	66.5/3	91.6/2	7.3/10	52.5/14	36.8/8	11.7/2	54.2/3	78.9/1	5.6/9	47.9/14	31.6/8	53.2/14	35.8/7
Saarland	33.3/10	69.0/8	92.6/6	30.2/8	59.3/10	51.6/5	5.9/11	58.6/8	70.5/7	7.4/12	46.9/10	36.8/6	18.5/5	57.1/7	73.7/1	59.6/8	75.8/4
Sachsen	24.5/16	64.8/14	92.6/6	30.0/9	59.8/8	51.6/5	4.8/13	56.7/9	70.5/7	10.2/3	48.7/8	36.8/6	22.8/4	56.9/8	73.7/1	57.4/9	75.8/4
Sachsen-Anhalt	32.7/11	69.7/6	92.6/6	24.7/13	54.8/14	51.6/5	4.1/14	61.2/4	70.5/7	8.1/10	44.2/14	36.8/6	26.6/1	61.8/2	73.7/1	62.5/4	75.8/4
Schleswig-Holstein	38.2/5	68.5/11	90.5/8	36.4/4	65.5/4	53.7/3	8.4/6	54.1/13	74.7/4	10.1/4	52.6/4	41.1/4	4.6/12	49.1/13	69.5/3	54.9/13	73.7/6
Thüringen	29.8/14	68.6/10	92.6/6	25.7/12	55.6/13	51.6/5	4.8/13	60.7/5	70.5/7	7.9/11	44.6/13	36.8/6	26.1/2	61.3/3	73.7/1	61.8/5	75.8/4
Whole of Germany	34.2	67.5	90.5	35.2	64.3	53.7	8.1	54.8	74.7	9.8	51.7	41.1	8.7	50.8	69.5	55.5	73.7

Figure 4: Unweighted popularity of parties and of DGB in German federal states (Länder)

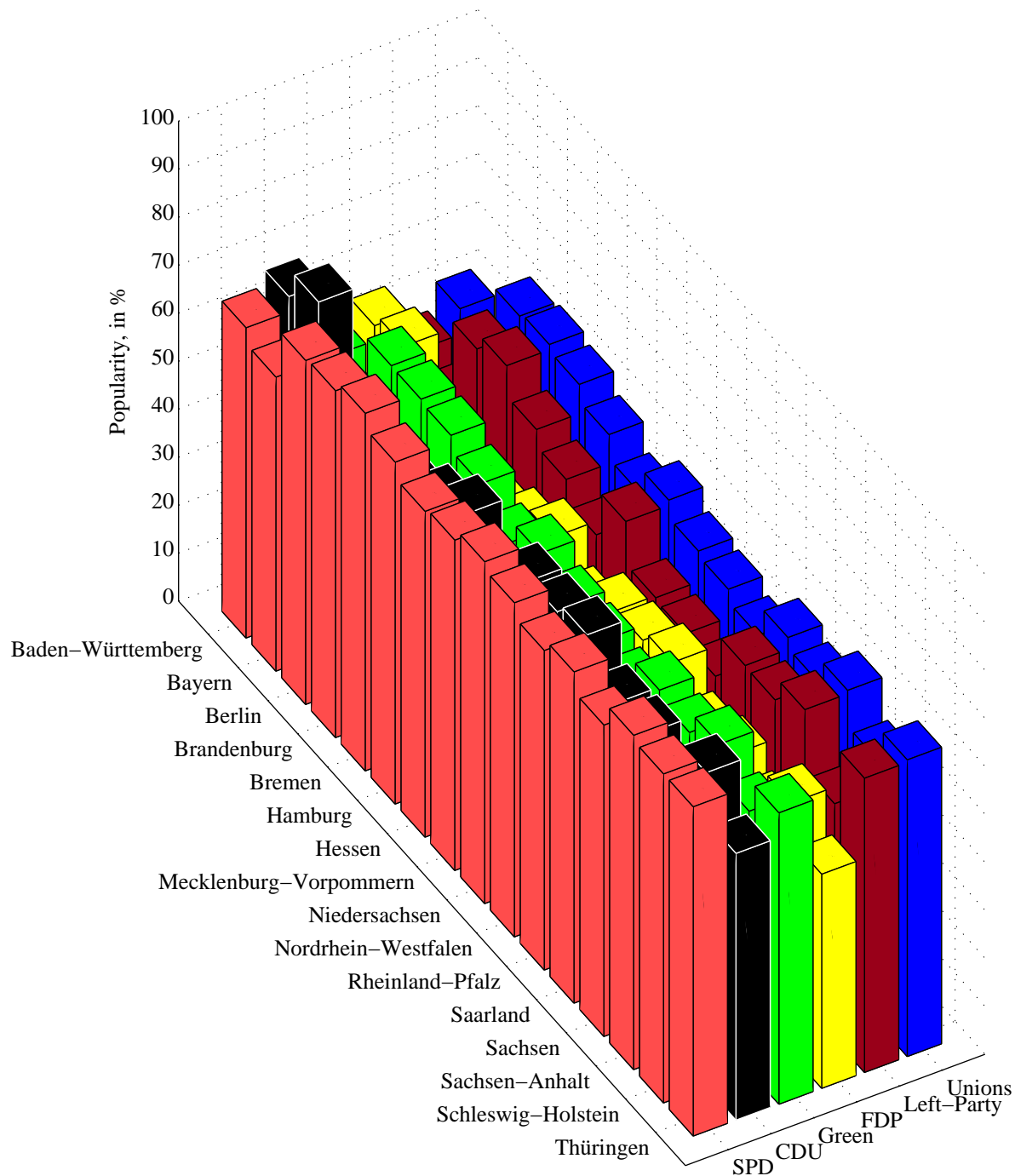
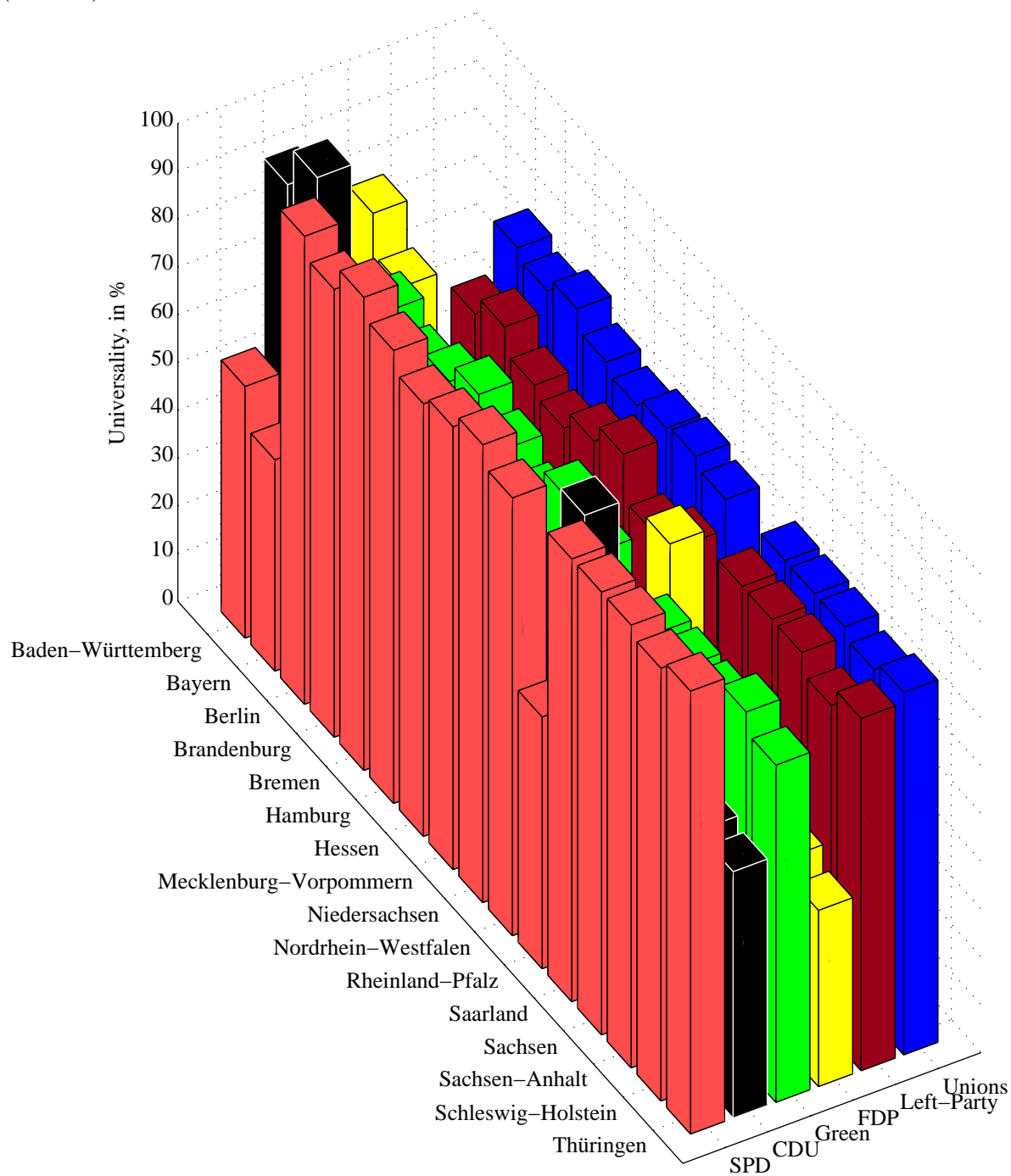


Figure 5: Unweighted universality of parties and of DGB in German federal states (Länder)



On every question, the coalition members can either agree or disagree. In the first case the coalition is unanimous and unambiguously represents the voters with the same opinion (either Yes-Group, or No-Group). In the second case the coalition is not unanimous and can come to any of alternative opinions. Usually, the impact of coalition members on final coalition opinions is proportional to their weights (number of votes received). However, such a proportionality holds only approximately.

In our model, the *proportionality of impact to weights* is described by a special parameter p . For example, let the weight ratio of parties within a coalition be 3 : 1. The maximal respect to weight $p = 1$ means that the larger party determines the coalition opinions with the proportional probability $\frac{3}{4}$, and the smaller party with probability $\frac{1}{4}$. No respect to weight $p = 0$ means equal impact, so that each of alternative opinions can be accepted by the coalition with probability $\frac{1}{2}$, regardless of member weights. The intermediate case $p = \frac{1}{2}$ corresponds to the impact probabilities $\frac{3}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{8}$ and $\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$. In the paper $p = \frac{1}{2}$ is accepted and applied to all coalitions considered.

Under the assumption, both indices of popularity and universality turn out to be random variables. The coalition's popularity is understood as the expected size of the voter group represented. It is shown in the fourth column of Table 3, *Expectation*. The prediction accuracy is specified in the fifth column of Table 3, *Standard deviation* (= square root of the variance) of the size of the group represented. The highest popularity 65.9% and the highest prediction accuracy (= lowest standard deviation) $\pm 0.5\%$ are inherent in the coalition SPD/CDU, which is currently in power.

The next to last column of Table 3 contains the coalition universality understood as the expected frequency of representing a majority. The last column shows the standard deviation of the underlying random variable, characterizing the prediction accuracy. The most universal coalition is SPD/Green with rank 1 with the expected universality 85.1% and the second best prediction accuracy $\pm 2.5\%$ (rank 2).

The location of coalitions in the space Popularity–Universality–Unanimity is depicted in Figure 6. As one can see, the coalitions differ in universality much more than in popularity. The results of principle component analysis are shown in Table 4. Recall that this type of analysis allows to approximate a cloud of observations with an ellipsoid, which first diameter is the direction of the maximal variance, the second diameter is the second maximal variance, etc. (JACKSON 1988, KRZANOWSKI 1988, and SEBER 1984). The contribution of universality absolutely predominates in the first two components. Consequently, the universality can be regarded as a more decisive indicator than popularity.

Table 3: Evaluation of coalitions for unweighted questions; the proportionality of impact to weights $p = \frac{1}{2}$

Nr.	Coalition size	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/R	%/R	%/R	%/R	%/R	%/R
1 SPD, CDU	69.4/4	44.2/6	65.9/1	$\pm 0.5/1$	72.0/6	$\pm 3.8/6$
2 SPD, Green	42.3/16	73.7/2	63.1/2	$\pm 1.5/3$	85.1/1	$\pm 2.5/2$
3 SPD, FDP	44.0/12	33.7/7	61.8/4	$\pm 1.7/7$	72.6/4	$\pm 4.0/7$
4 SPD, Left-Party	42.9/15	66.3/4	61.6/5	$\pm 1.7/10$	83.1/2	$\pm 2.8/4$
5 CDU, Green	43.3/14	28.4/8	61.0/7	$\pm 1.5/5$	60.9/12	$\pm 4.1/8$
6 CDU, FDP	45.0/11	70.5/3	59.8/10	$\pm 1.7/8$	49.2/20	$\pm 2.7/3$
7 CDU, Left-Party	43.9/13	25.3/9	59.6/11	$\pm 1.7/11$	59.2/14	$\pm 4.2/9$
8 Green, FDP	17.9/19	22.1/12	53.2/17	$\pm 2.2/16$	57.1/15	$\pm 4.5/13$
9 Green, Left-Party	16.8/20	82.1/1	52.8/18	$\pm 1.3/2$	72.1/5	$\pm 2.2/1$
10 FDP, Left-Party	18.5/18	18.9/13	51.2/20	$\pm 2.4/19$	54.8/18	$\pm 4.6/17$
11 SPD, CDU, Green	77.5/3	23.2/11	62.0/3	$\pm 1.5/4$	66.9/8	$\pm 4.4/11$
12 SPD, CDU, FDP	79.2/1	24.2/10	61.4/6	$\pm 1.6/6$	65.1/11	$\pm 4.4/10$
13 SPD, CDU, Left-Party	78.1/2	17.9/14	60.7/8	$\pm 1.7/9$	65.3/10	$\pm 4.5/14$
14 SPD, Green, FDP	52.1/8	14.7/15	58.1/13	$\pm 2.1/14$	68.1/7	$\pm 4.5/12$
15 SPD, Green, Left-Party	51.0/10	61.1/5	60.5/9	$\pm 1.8/12$	82.5/3	$\pm 3.1/5$
16 SPD, FDP, Left-Party	52.7/7	9.5/18	56.8/14	$\pm 2.3/17$	66.3/9	$\pm 4.6/18$
17 CDU, Green, FDP	53.1/6	10.5/17	56.7/15	$\pm 2.2/15$	54.9/17	$\pm 4.6/16$
18 CDU, Green, Left-Party	52.0/9	17.9/14	58.5/12	$\pm 1.9/13$	59.3/13	$\pm 4.5/15$
19 CDU, FDP, Left-Party	53.7/5	7.4/19	55.5/16	$\pm 2.3/18$	53.8/19	$\pm 4.7/19$
20 Green, FDP, Left-Party	26.6/17	11.6/16	51.4/19	$\pm 2.4/20$	56.6/16	$\pm 4.8/20$

Table 4: Principal component analysis of the “cloud of observations” of the 20 coalitions in the space Popularity–Universality–Unanimity

	Principle components		
	1st component	2nd component	3rd component (residual)
Popularity	0.0568	-0.2327	-0.9709
Universality	0.2677	-0.9333	0.2394
Unanimity	0.9618	0.2735	-0.0093
Standard deviation w.r.t. new axes	24.9417	8.3166	3.3827

Some more details can be found in Figures 7–8 which show the coalitions in planes Unanimity–Popularity and Unanimity–Universality. The prediction accuracy (standard deviation) of the indicators is depicted by vertical grey segments. Note the location of coalitions along the ascending diagonal, meaning that the higher the degree of unanimity, the higher the indices of popularity and universality.

The coalition SPD/CDU (now in power) has a high popularity but a low degree of unanimity and a mediocre universality. The coalition SPD/Green/Left-Party (much discussed but not realized) has a higher degree of unanimity, somewhat lower popularity but a much higher universality. The coalition CDU/FDP (which held before the elections) has a higher degree of unanimity but relatively low indices of popularity and universality.

All of these show that the best would be likely the coalition SPD/Green/Left-Party which failed due to personal conflicts between party leaders. One can only regret that the politicians could not overcome personal prejudgements and did not think first about their statesman duty to represent citizens.

5 Summary

1. **(Initial data)** The indices of popularity and universality of parties and their coalitions are derived from the results of the German parliament elections 2005 and party manifestos.
2. **(Indicators)** These allow us to evaluate the representativeness of parties and of the German Trade Union Federation (DGB), the degree of unanimity of coalitions as well as to predict their performance.
3. **(Most representative party)** According to the quantitative analysis, the SPD was the most representative German party at the time of the elections, although it was not the election winner.
4. **(Most representative coalition)** The actual coalition in power, CDU/SPD, might have a better alternative SPD/Green/Left-Party if the politicians could surmount personal conflicts.
5. **(Computational formulas)** The vector-matrix computing formulas derived in the next sections are easy to implement and provide a clear geometric interpretation of interactions between the model elements.

Figure 6: Location of coalitions in the space Popularity–Universality–Unanimity for un-weighted questions; the proportionality of impact to weights $p = \frac{1}{2}$

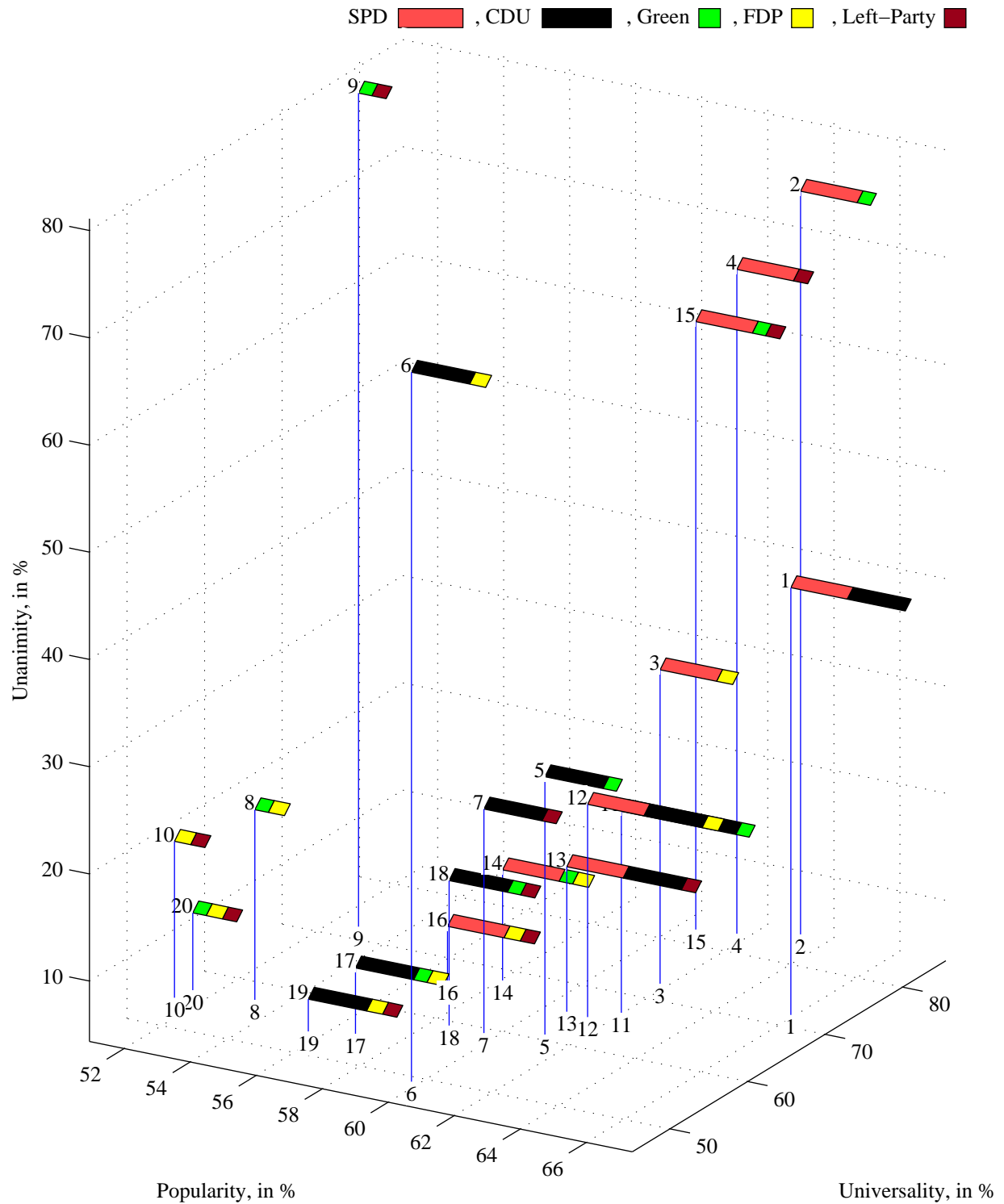


Figure 7: Degree of unanimity versus popularity of coalitions for unweighted questions; the proportionality of impact to weights $p = \frac{1}{2}$

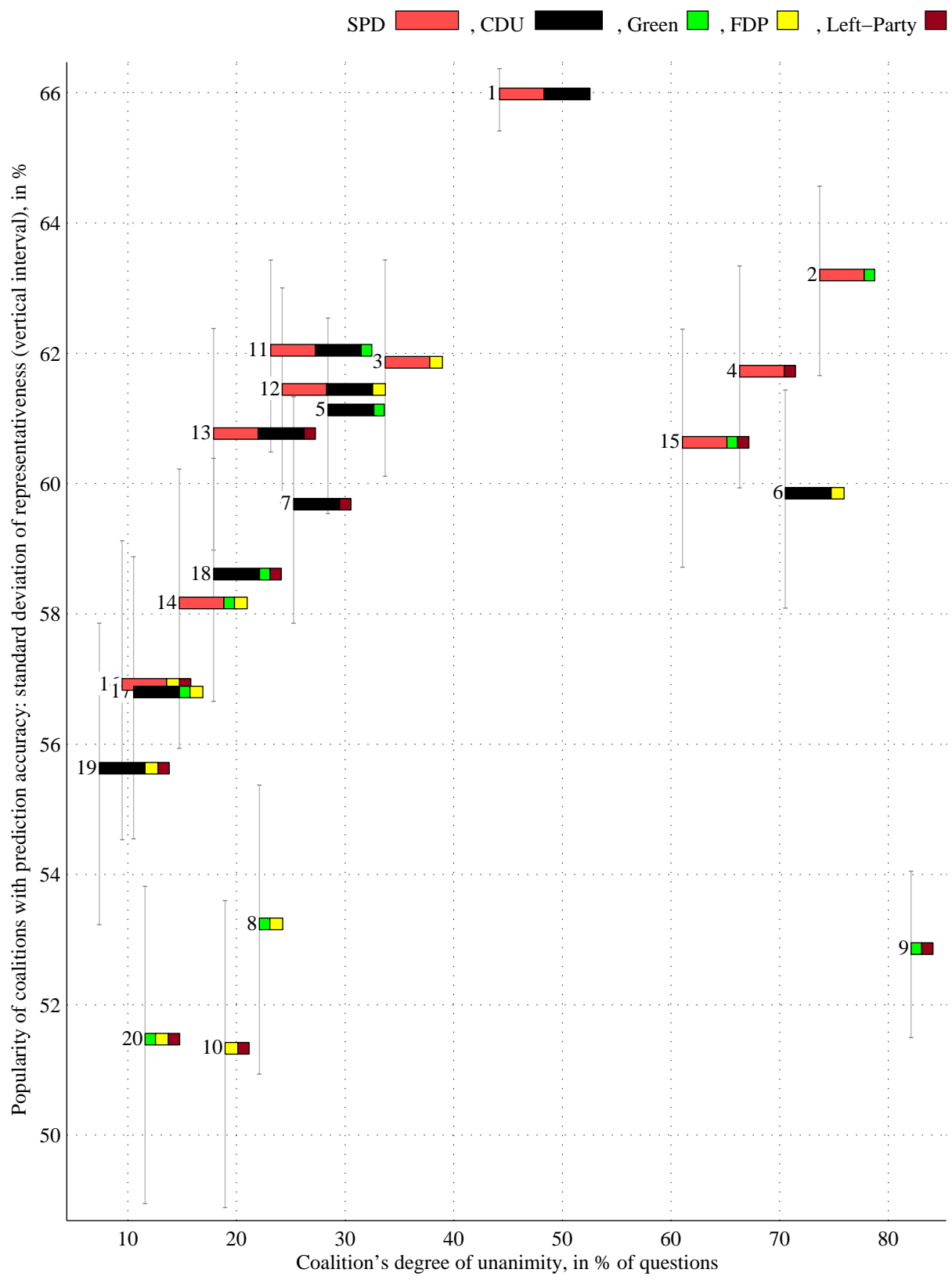
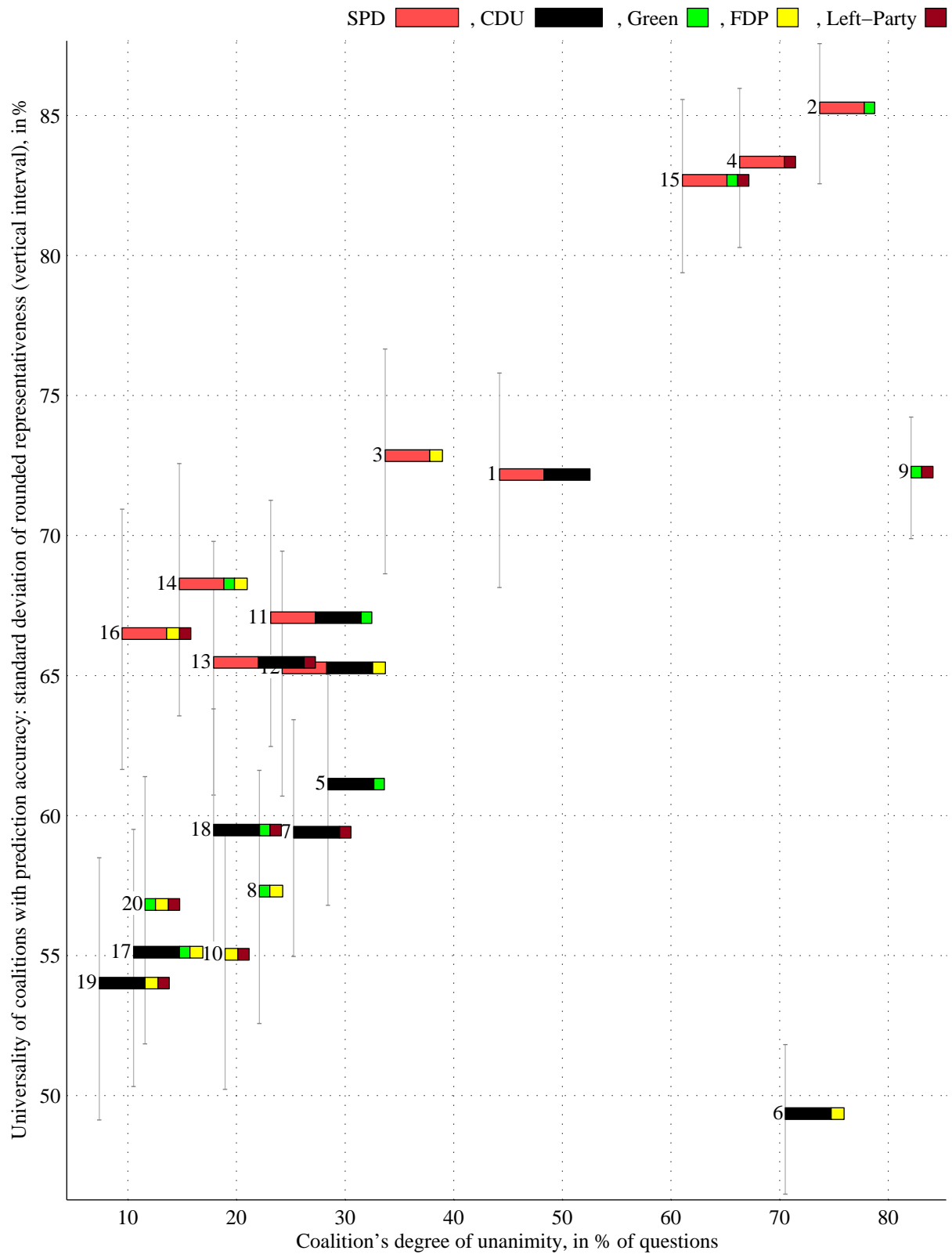


Figure 8: Degree of unanimity versus universality of coalitions for unweighted questions; the proportionality of impact to weights $p = \frac{1}{2}$



6 Computational issues

This section introduces notation and formulas for evaluating parties and their coalitions. Similar tasks of coalition analysis have been considered with the use of relational algebras (Schmidt and Ströchlein 1993, Brink et al. 1997, de Swart et al. 2003, Berghammer et al. 2005, and Rusinowska et al. 2005). The vector-matrix formulas adduced below are much simpler and have a clear geometric interpretation.

Questions/Agenda By Q denote the agenda with dichotomous questions q , that is, which evoke either positive or negative *opinions* (Yes/No answers) coded by ± 1 . In our applications, the list of $m = 95$ questions is given in the first column of Table 1.

The importance of questions is reflected by weights μ_q which constitute a *probability measure* μ on Q (the reference to “probability” can be misleading; in fact, we just need a normalized additive measure). It assumes

non-negativity

$$\mu_q \geq 0 \quad \text{for every } q \in Q ,$$

additivity

$$\mu_X = \sum_{q \in X} \mu_q \quad \text{for every subset } X \subset Q ,$$

and *normality*

$$\sum_q \mu_q = 1 \quad (\text{the totality is 100\%}) . \tag{1}$$

The question weights are collected into the column m -vector

$$\boldsymbol{\mu} = \{\mu_q\} .$$

In our application, “unweighted” means equal weights $\mu_q = 1/95$. The non-normalized expert weights and GOOGLE-weights are given in the last two columns of Table 1. To fulfill the normalizing condition (1), divide each weight by the total of the 95 weights in the given column.

Candidates Consider N candidates c for election; in our application the candidates are $N = 5$ parties. Their positive or negative opinions $b_{qc} = \pm 1$ on questions q are collected into the $(m \times N)$ -matrix of candidate opinions derived from columns 2–6 of Table 1⁴

$$\mathbf{B} = \{b_{qc}\}, \quad b_{qc} = \pm 1 .$$

⁴Comparing with (Tangian 2005), the matrix \mathbf{B} is transposed.

Balance of opinions in the society Consider the *society* of 45,430,368 voters for the five leading parties, that is, 96% of all the 47,287,988 voters with the valid second vote (for parties); see BUNDESWAHLLEITER (2005). The 4% voters for other 19 minor parties are not considered.

On each question q , the society falls into *protagonists* who answer the question positively, and *antagonists* who answer the question negatively. These groups, redefined for each question, are shown in Figure 2, assuming that voters' opinions are represented by the parties.

On every question q , the *balance of opinions* a_q is the predominance of protagonists over antagonists, expressed in fraction (%) of the society. In Figure 2, it corresponds to the difference between the length of the right-hand and left-hand rectangles. For example, for question $q = 1$ (Relax the protection against dismissals?) we obtain

$$a_1 = \frac{\overbrace{35.2}^{\text{Votes for CDU}} + \overbrace{9.8}^{\text{Votes for FDP}} - \overbrace{34.3}^{\text{Votes for SPD}} - \overbrace{8.1}^{\text{Votes for Green}} - \overbrace{8.7}^{\text{Votes for Left-Party}}}{\underbrace{35.2 + 9.8 + 34.3 + 8.1 + 8.7}_{\text{Total votes for the five leading parties}}} = -0.0635 = -6.35\% .$$

To facilitate computing, define the normalized *vector of candidate weights*, the coordinates of which are proportional to percentages of votes for the five parties⁵

$$\boldsymbol{\xi} = \{\xi_c\} = \frac{(34.2, 35.2, 8.1, 9.8, 8.7)'}{35.2 + 9.8 + 34.3 + 8.1 + 8.7} . \quad (2)$$

The column 95-vector *balance of opinions* in the society is the matrix-vector product⁶

$$\mathbf{a} = \{a_q\} = \mathbf{B}\boldsymbol{\xi} . \quad (3)$$

Popularity and universality of the candidates (parties) The *representativeness* r_{qc} of candidate c on question q is the size of the social group represented, measured in fraction (percentage) of the society

$$r_{qc} = \begin{cases} \text{total weight of protagonists in the society} & \text{if } b_{qc} = 1 \\ \text{total weight of antagonists in the society} & \text{if } b_{qc} = -1 \end{cases} .$$

The *popularity* of candidate c is the weighted average of his representativeness (= expected representativeness)

$$P_c = \sum_q \mu_q r_{qc} \quad (4)$$

⁵The figures in (2) are for the whole of Germany. For federal states use the voting results from Table 2.

⁶The balance of opinions in the society should be derived from individual opinions, as in (Tangian 2005). Here, the balance of opinions in the society is indirectly derived from the votes for candidates and their opinions, because direct data on individual opinions are not available.

The *universality* of candidate c is the weighted frequency with which he represents a non-strict majority (= expected rounded representativeness):

$$U_c = \sum_{q:r_{qc} \geq \frac{1}{2}} \mu_q = \sum_q \mu_q \text{round}[r_{qc}] \quad (5)$$

In a sense, the popularity reflects the spatial aspect of representativeness, and the universality reflects its temporal aspect.

Computing the indicators and their geometrical interpretation Introduce the following notation (all vectors are column vectors!):

' the operation of vector/matrix transpose

. the operation of element-by-element product of vectors and matrices of the same size, for example, $(1, 2) \cdot (3, 4) = (3, 8)$

.² the operation of element-by-element square of vectors and matrices, for example, $(2, 3)^2 = (4, 9)$

+ the addition of scalars to matrices or vectors by applying it to all matrix elements, for example, $0.5 + (1, 2) = (1.5, 2.5)$

diag \mathbf{a} the diagonal $(m \times m)$ -matrix with elements of vector \mathbf{a} on its main diagonal

sign \mathbf{a} the m -vector of *majority opinions* derived from the vector \mathbf{a} by applying the sign function to its coordinates

$$\text{sign}a_q = \begin{cases} +1 & \text{if } a_q > 0, & \text{i.e. the majority opinion on question } q \text{ is positive} \\ 0 & \text{if } a_q = 0, & \text{i.e. tie opinion on question } q \\ -1 & \text{if } a_q < 0, & \text{i.e. the majority opinion on question } q \text{ is negative} \end{cases}$$

$\delta_{\mathbf{a}} = 1 - \text{abs}(\text{sign}\mathbf{a})$ the m -vector of *indicators of tie opinion*, with the q th coordinate being 1 if the opinion on question q is tied, and 0 otherwise; we use this vector to express the total weight of questions with a tie opinion

$$\boldsymbol{\mu}'\delta_{\mathbf{a}} = \sum_{q:a_q=0} \mu_q \quad (6)$$

Theorem 1 (Computing the indicators and their geometric interpretation)

$$\underbrace{\mathbf{R}}_{\substack{(m \times N)\text{-matrix of} \\ \text{representativeness} \\ \text{of candidates } c \\ \text{on questions } q}} = \frac{1}{2} + \frac{1}{2} \underbrace{\text{diag} \mathbf{a}}_{\substack{\text{diagonal} \\ (m \times m)\text{-matrix of} \\ \text{balance of opinions} \\ \text{in the society}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}} \quad (7)$$

$$\underbrace{\{\mathbf{P}_c\}}_{\substack{\text{row } N\text{-vector of} \\ \text{popularity of} \\ \text{all candidates } c}} = \boldsymbol{\mu}' \mathbf{R} \quad (8)$$

$$= \frac{1}{2} + \frac{1}{2} \underbrace{(\boldsymbol{\mu} \cdot \mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of balance} \\ \text{of opinions}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}} \quad (9)$$

$$\underbrace{\{\mathbf{U}_c\}}_{\substack{\text{row } N\text{-vector of} \\ \text{universality of} \\ \text{all candidates } c}} = \boldsymbol{\mu}' \text{round}[\mathbf{R}] \quad (10)$$

$$= \frac{1}{2} + \frac{1}{2} \underbrace{\boldsymbol{\mu}' \delta_a}_{\substack{\text{total weight of} \\ \text{questions with} \\ \text{tie opinions} \\ \text{(constant scalar} \\ \text{independent of } c)}} + \frac{1}{2} \underbrace{(\boldsymbol{\mu} \cdot \text{sign} \mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of majority} \\ \text{opinion}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}}, \quad (11)$$

where the vector \mathbf{a} is computed from (3).

Thus, the most popular (universal) candidate c has the largest projection of his opinion vector \mathbf{b}_c (= the c th column of matrix \mathbf{B}) on the $\boldsymbol{\mu}$ -weighted social vector of balance of opinions $\boldsymbol{\mu} \cdot \mathbf{a}$ (respectively, on the $\boldsymbol{\mu}$ -weighted vector of majority opinion $\boldsymbol{\mu} \cdot \text{sign} \mathbf{a}$)⁷.

The formulas of the theorem are used to compute the indicators in Table 2 and for Figures 3, 4, and 5.

Remark 1 (Analogy with force vectors in physics)

Recall that in mechanics a work is produced by displacements. Accordingly, the only productive constituent of a force vector is its projection on the direction of motion. In Theorem 1, the “work for the society” of a candidate is measured by the projection of his opinion vector on the “main stream”, the social vector of balance of opinions, or social vector of majority opinion. Thus the variety of representatives and representative bodies with numerous opinions on the agenda is projected onto a single line axis, exactly like in the case of physical forces.

⁷The popularity of candidate c is a bilinear form of social opinions \mathbf{a} and candidate opinions \mathbf{b}_c (c th column of matrix \mathbf{B}). In our application, vectors \mathbf{a} and matrix \mathbf{B} are derived from candidate opinions, reducing the bilinear form to quadratic.

Evaluation of coalitions By definition, a *coalition* C is a subset of the set of candidates. The *coalition size* is the total percentage of voters for all the candidates of the coalition. For instance, the size of coalition $C = \text{SPD}/\text{CDU}$ is $34.2 + 35.2 = 69.4\%$.

The (relative) weights of members of coalition C are collected in the normalized vector

$$\overset{C}{\boldsymbol{\xi}} = \left\{ \xi_c = \frac{\xi_c}{\sum_{c \in C} \xi_c}, \quad c \in C \right\} .$$

The matrix of opinions of coalition members is the restriction of \mathbf{B} to columns $c \in C$:

$$\overset{C}{\mathbf{B}} = \{b_{qc}, \quad c \in C\} . \quad (12)$$

The *balance of opinions within coalition* C is the vector

$$\overset{C}{\mathbf{b}} = \left\{ b_q \right\} = \overset{C}{\mathbf{B}} \overset{C}{\boldsymbol{\xi}} . \quad (13)$$

The (degree of) *unanimity* of coalition C is the total weight of the questions on which the coalition members have equal positions. For instance, SPD and CDU agree in 42 of 95 questions, so that its unanimity is 44.2%.

If a coalition C is unanimous on question q its representativeness r_{qC} is equal to that of its every member. If the coalition is not unanimous, it can come to either positive, or negative opinion on question q . We assume that on question q a coalition C represents protagonists in the society with a probability ranging from the relative weight $\overset{C}{\xi}_q^+$ of coalition protagonists to the absolute uncertainty $\frac{1}{2}$:

$$\overset{C}{\xi}_q^+ p + \frac{1}{2}(1 - p), \quad 0 \leq p \leq 1 ,$$

where p denotes the *proportionality of impact to weights* of coalition members. If $p = 1$ the impact of coalition members is proportional to their weights. If $p = 0$ the coalition comes to ± 1 opinions with equal chances, so that both protagonists and antagonists in the society are represented with equal probabilities $\frac{1}{2}$.

Under these provisions, the representativeness and the indicators of popularity and universality of a coalition are random variables which behavior for non-unanimous questions depends on the parameter p . The *popularity* \mathbf{P}_C and *universality* \mathbf{U}_C of coalition C are understood as its expected representativeness and expected rounded representativeness. Besides, we compute the variance of representativeness and of rounded representativeness as a measure of accuracy of the coalition indicators.

Theorem 2 (Evaluation of coalitions)

$$\text{Unanimity of } C = 1 - \boldsymbol{\mu}' \boldsymbol{s}^C \quad (14)$$

$$\begin{aligned} P_C = \mathbb{E} r_C &= \underbrace{\overline{P}_C}_{= \sum_{c \in C} \xi_c^C P_c} - \frac{1}{2} \underbrace{(1-p)}_{\text{impact of uncertainty}} \underbrace{(\boldsymbol{\mu} \cdot \boldsymbol{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of balance} \\ \text{of opinions}}} \underbrace{\begin{pmatrix} C \\ \boldsymbol{s} \cdot \boldsymbol{b} \end{pmatrix}}_{\substack{m\text{-vector} \\ \text{of balance} \\ \text{of opinions} \\ \text{within the} \\ \text{coalition for} \\ \text{non-unanimous} \\ \text{questions}}} \end{aligned} \quad (15)$$

$$\begin{aligned} U_C = \mathbb{E} \text{round}[r_C] &= \underbrace{\overline{U}_C}_{= \sum_{c \in C} \xi_c^C U_c} - \frac{1}{2} \underbrace{(1-p)}_{\text{impact of uncertainty}} \underbrace{(\boldsymbol{\mu} \cdot \text{sign} \boldsymbol{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of majority} \\ \text{opinion}}} \underbrace{\begin{pmatrix} C \\ \boldsymbol{s} \cdot \boldsymbol{b} \end{pmatrix}}_{\substack{m\text{-vector} \\ \text{of balance} \\ \text{of opinions} \\ \text{within the} \\ \text{coalition for} \\ \text{non-unanimous} \\ \text{questions}}}, \end{aligned} \quad (16)$$

where

$$\boldsymbol{s}^C = \left\{ s_q^C = \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) \right\}, \quad \text{are indicators of the coalition non-unanimity on questions } q, \text{ with } n \text{ being the number of coalition members,}$$

$\overline{P}_C = \sum_{c \in C} \xi_c^C P_c$ is the weighted average popularity of coalition members, and

$\overline{U}_C = \sum_{c \in C} \xi_c^C U_c$ is the weighted average universality of coalition members.

Besides, if the coalition opinions on non-unanimous questions are independent (= independent negotiations on every question) then

$$\mathbb{V} r_C = \frac{1}{4} [(\boldsymbol{\mu} \cdot \boldsymbol{a}) \cdot]^2 \left[\boldsymbol{s}^C \cdot \left(1 - p^2 \boldsymbol{b} \cdot \right) \right] \quad (17)$$

$$\mathbb{V} \text{round}[r_C] = \frac{1}{4} [(\boldsymbol{\mu} \cdot \text{sign} \boldsymbol{a}) \cdot]^2 \left[\boldsymbol{s}^C \cdot \left(1 - p^2 \boldsymbol{b} \cdot \right) \right] . \quad (18)$$

The formulas of the theorem with $p = \frac{1}{2}$ are used to compute indicators in Table 3 and for Figures 6, 7, and 8.

Remark 2 (Coalition indicators in the simplest case)

If $p = 1$ (the impact of coalition members is proportional to their weights) then by (15) and (16) the popularity and universality of a coalition are equal to the weighted average of corresponding indicators of its members: $P_C = \overline{P}_C$ and $U_C = \overline{U}_C$.

7 Proofs

Proof of Theorem 1

On every question q , obviously

$$\text{The weight of non-strict majority/minority} = \underbrace{\frac{1}{2}}_{\substack{\text{half} \\ \text{the} \\ \text{society}}} \pm \frac{1}{2} \underbrace{|a_q|}_{\substack{\text{predominance} \\ \text{of protagonists} \\ \text{over antagonists}}} .$$

Consequently, the representativeness of candidate c on question q is determined by the sign of his opinion and by the sign of a_q

$$r_{qc} = \underbrace{\frac{1}{2}}_{\substack{\text{half} \\ \text{the} \\ \text{society}}} + \frac{1}{2} \underbrace{a_q}_{\substack{\text{predominance} \\ \text{of protagonists} \\ \text{over antagonists} \\ \text{in the society}}} \underbrace{b_{qc}}_{\substack{= \pm 1 \\ \text{opinion of} \\ \text{candidate } c}} , \quad (19)$$

which matrix form is

$$\mathbf{R} = \frac{1}{2} \mathbf{1} + \frac{1}{2} \text{diag} \mathbf{a} \mathbf{B} , \quad (20)$$

as required in (7). Multiplying $\boldsymbol{\mu}'$ by (20), as required in the definition (4), obtain (8) and (9):

$$\begin{aligned} \{\mathbf{P}_c\} &= \boldsymbol{\mu}' \mathbf{R} \\ &= \boldsymbol{\mu}' \left[\frac{1}{2} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \frac{1}{2} \text{diag} \mathbf{a} \mathbf{B} \right] \\ &= \frac{1}{2} \cdot \underbrace{\sum_q \mu_q}_{=1} + \frac{1}{2} \boldsymbol{\mu}' \text{diag} \mathbf{a} \mathbf{B} \\ &= \frac{1}{2} + \frac{1}{2} (\boldsymbol{\mu} \cdot \mathbf{a})' \mathbf{B} . \end{aligned} \quad (21)$$

To obtain (10), multiply $\boldsymbol{\mu}'$ by $\text{round}[\mathbf{R}]$, as required by definition (5). To obtain (11), express the rounded representativeness of candidate c on question q by analogy with (19):

$$\text{round}[r_{qc}] = \frac{1}{2} + \frac{1}{2} \text{sign} a_q b_{qc} + \frac{1}{2} \underbrace{\delta_{a_q}}_{\substack{= 1 \text{ if } a_q = 0 \\ = 0 \text{ if } a_q \neq 0}} \quad (22)$$

and proceed similarly to (21). ■

Proof of Theorem 2

The unanimity of coalition members $c \in C$ on question q means that either all $b_{qc} = 1$, or all $b_{qc} = -1$, implying $|\sum_{c \in C} b_{cq}| = n$, where n is the number of coalition members. Consequently,

$$s_q^C = \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) = \begin{cases} 0 & \text{if } c \in C \text{ are unanimous on question } q \\ 1 & \text{otherwise} \end{cases} \quad (23)$$

$$1 - s_q^C = 1 - \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) = \begin{cases} 1 & \text{if } c \in C \text{ are unanimous on question } q \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

Hence, the total weight of the questions on which the coalition is unanimous

$$\begin{aligned} \text{Unanimity of } C &= \sum_q \mu_q \left[1 - \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) \right] \\ &= \underbrace{\sum_q \mu_q}_{=1} - \boldsymbol{\mu}' \boldsymbol{s}^C, \end{aligned}$$

as required in (14).

Compute the expectation and variance of representativeness r_{qC} of a coalition C for a given question q . Consider two cases.

- **Coalition members are unanimous on question q .** Express the coalition's representativeness by analogy with (19) and note that r_{qC} is constant, implying

$$\mathbb{E} r_{qC} = \frac{1}{2} + \frac{1}{2} a_q \underbrace{b_q^C}_{=\pm 1 \text{ in case of unanimity}} \quad (25)$$

$$\mathbb{V} r_{qC} = 0. \quad (26)$$

- **Coalition members are not unanimous on question q .** The representativeness r_{qC} is a Bernoulli random variable, taking two values $\frac{1}{2} \pm \frac{1}{2} a_q$ with range $|a_q|$. The coalition can accept the opinion of majority in the coalition, or of its minority, representing respectively the social groups with the size:

$$\frac{1}{2} + \frac{1}{2} a_q \text{sign } b_q^C \quad \text{with probability} \quad \underbrace{\left(\frac{1}{2} + \frac{1}{2} \left| \frac{C}{b_q} \right| \right)}_{\text{majority in the coalition}} \quad p + \frac{1}{2}(1-p) \quad (27)$$

$$\frac{1}{2} - \frac{1}{2} a_q \text{sign } b_q^C \quad \text{with probability} \quad \frac{1}{2} - \frac{1}{2} \left| \frac{C}{b_q} \right| p. \quad (28)$$

By definition of expectation

$$\begin{aligned}
\mathbf{E} r_{qC} &= \frac{1}{4} \left[\left(1 + a_q \text{sign } b_q^C \right) \left(1 + \left| b_q^C \right| p \right) + \left(1 - a_q \text{sign } b_q^C \right) \left(1 - \left| b_q^C \right| p \right) \right] \\
&= \frac{1}{2} + \frac{1}{2} a_q \text{sign } b_q^C \left| b_q^C \right| p \\
&= \frac{1}{2} + \frac{1}{2} a_q b_q^C p .
\end{aligned} \tag{29}$$

By the known formula for the variance of Bernoulli random variables (Korn and Korn 1968, Table 18.8.3, case $n = 1$, Abramowitz and Stegun 1972, 26.1.20, case $n = 1$) obtain

$$\begin{aligned}
\mathbf{V} r_{qC} &= a_q^2 \left(\frac{1}{2} + \frac{1}{2} \left| b_q^C \right| p \right) \left(\frac{1}{2} - \frac{1}{2} \left| b_q^C \right| p \right) \\
&= \frac{1}{4} a_q^2 \left[1 - \left(b_q^C \right)^2 p^2 \right] .
\end{aligned} \tag{30}$$

Compute the popularity \mathbf{P}_C of a coalition C . Using (25) and (29) and applying (23)–(24) as indicators of (non-) unanimity, obtain

$$\begin{aligned}
\mathbf{P}_C = \mathbf{E} r_C &= \sum_{q \text{ unanimous}} \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q b_q^C \right) + \sum_{q \text{ non-unanimous}} \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q b_q^C p \right) \\
&= \sum_q \mu_q \left(1 - s_q^C \right) \left(\frac{1}{2} + \frac{1}{2} a_q b_q^C \right) + \sum_q \mu_q s_q^C \left(\frac{1}{2} + \frac{1}{2} a_q b_q^C p \right) \xrightarrow{\text{Identity}} \\
&= \sum_q \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q b_q^C \right) - \frac{1}{2} (1-p) \sum_q \mu_q a_q s_q^C b_q^C \xrightarrow{\text{by (12)}} \\
&= \underbrace{\sum_q \mu_q \left(\frac{1}{2} \sum_{c \in C} \xi_c^C + \frac{1}{2} a_q \sum_{c \in C} \xi_c^C b_{qc}^C \right)}_{=1} - \frac{1}{2} (1-p) \sum_q \mu_q a_q s_q^C b_q^C \\
&= \underbrace{\sum_{c \in C} \xi_c^C \sum_q \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q b_{qc}^C \right)}_{\text{by (4) and (19)}} \stackrel{=}{=} \sum_{c \in C} \xi_c^C \mathbf{P}_c \\
&= \underbrace{\sum_{c \in C} \xi_c^C \mathbf{P}_c}_{\overline{\mathbf{P}}_C} - \frac{1}{2} (1-p) (\boldsymbol{\mu} \cdot \mathbf{a})' \left(\begin{smallmatrix} C \\ \mathbf{s} \cdot \mathbf{b} \end{smallmatrix} \right) ,
\end{aligned} \tag{31}$$

as required in (15).

Compute the universality \mathbf{U}_C of a coalition C . If $a_q = 0$ (tie opinion on question q in the society) the rounded representativeness $\text{round}[r_C] = \text{round}[\frac{1}{2}] = 1$. If $a_q \neq 0$, by analogy with (27)–(28), the rounded representativeness of coalition C takes values

$$\begin{aligned}
\frac{1}{2} + \frac{1}{2} \text{sign } a_q \text{sign } b_q^C & \quad \text{with probability} \quad \frac{1}{2} + \frac{1}{2} \left| b_q^C \right| p , \\
\frac{1}{2} - \frac{1}{2} \text{sign } a_q \text{sign } b_q^C & \quad \text{with probability} \quad \frac{1}{2} - \frac{1}{2} \left| b_q^C \right| p .
\end{aligned}$$

Applying the indicator of tie opinion (6) and proceeding like in (29) and (31) obtain

$$\begin{aligned}
U_C &= \mathbf{E} \text{round}[r_C] \\
&= \underbrace{\sum_{q:a_q=0} \mu_q \cdot 1 + \sum_{q:a_q \neq 0} \mu_q \left(\frac{1}{2} + \frac{1}{2} \text{sign} a_q \frac{C}{b_q} \right)}_{\sum_{c \in C} \xi_c^C U_c} - \frac{1}{2}(1-p) \underbrace{\sum_{q:a_q \neq 0} \mu_q \text{sign} a_q \frac{C}{s_q} \frac{C}{b_q}}_{\sum_q \mu_q \text{sign} a_q \frac{C}{s_q} \frac{C}{b_q}} \\
&= \overline{U}_C - \frac{1}{2}(1-p)(\boldsymbol{\mu} \cdot \text{sign} \mathbf{a})' \left(\frac{C}{\mathbf{s}} \cdot \frac{C}{\mathbf{b}} \right),
\end{aligned}$$

as required in (16).

Compute the variance of representativeness r_{qC} of coalition C . Using (26) and (30), applying (23)–(24) as indicators of (non-)unanimity, and taking into account that the variance of a sum of independent random variables is the sum of their variances, obtain

$$\begin{aligned}
V r_C &= \sum_q \mu_q^2 \left(1 - \frac{C}{s_q}\right) \cdot 0 + \sum_q \mu_q^2 \frac{C}{s_q} \frac{1}{4} a_q^2 \left[1 - p^2 \left(\frac{C}{b_q}\right)^2\right] \stackrel{\text{Identity}}{\implies} \quad (32) \\
&= \frac{1}{4} [(\boldsymbol{\mu} \cdot \mathbf{a}) \cdot^2]' \left[\frac{C}{\mathbf{s}} \cdot \left(1 - p^2 \frac{C}{\mathbf{b}} \cdot^2\right) \right],
\end{aligned}$$

as required in (17).

The derivation of the variance for the rounded representativeness $\text{round}[r_{qC}]$ of coalition C is similar to (32), but there are two changes to be made:

- The range of the Bernoulli random variable $\text{round}[r_{qC}]$ is 1 instead of $|a_q|$, consequently, a_q^2 in (32) should be replaced by 1.
- If $a_q = 0$ in (32), that is, tie opinion in the society on question q , then $r_{qC} = \frac{1}{2}$, implying $\text{round}[r_{qC}] = \text{round}[\frac{1}{2}] = 1$. Hence, $V \text{round}[r_{qC}] = 0$. The variances for such questions q should be nullified by the multiplier $(\text{sign} a_q)^2$ which retains all other terms intact.

Thus,

$$V \text{round}[r_C] = \frac{1}{4} [\boldsymbol{\mu} \cdot^2 \cdot (\text{sign} \mathbf{a}) \cdot^2]' \left[\frac{C}{\mathbf{s}} \cdot \left(1 - p^2 \frac{C}{\mathbf{b}} \cdot^2\right) \right],$$

as required in (18). ■

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