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## Looking for the missing rich: tracing the top tail of the wealth distribution

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# Online Appendix: Looking for the missing rich: Tracing the top tail of the wealth distribution

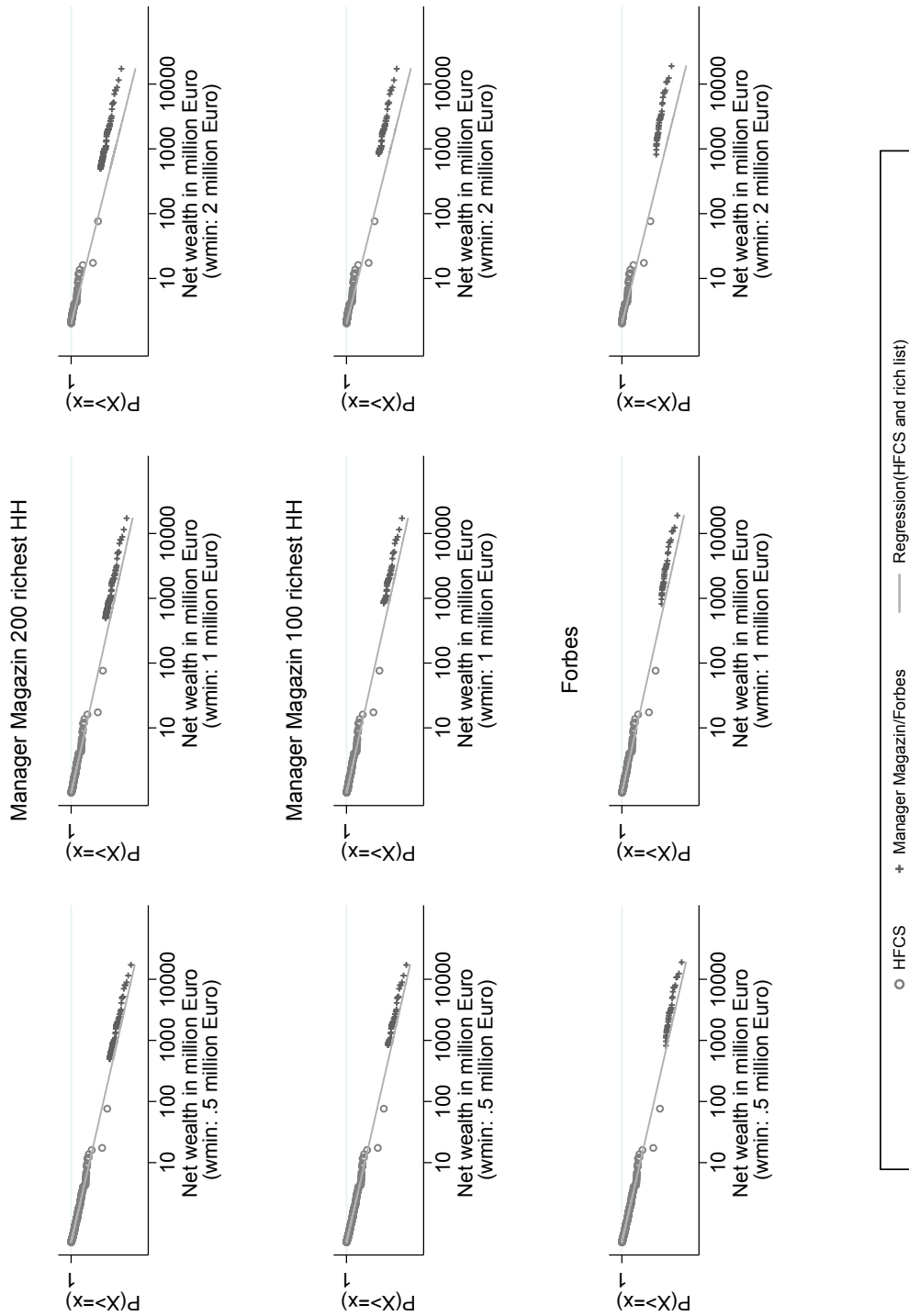
October, 2019

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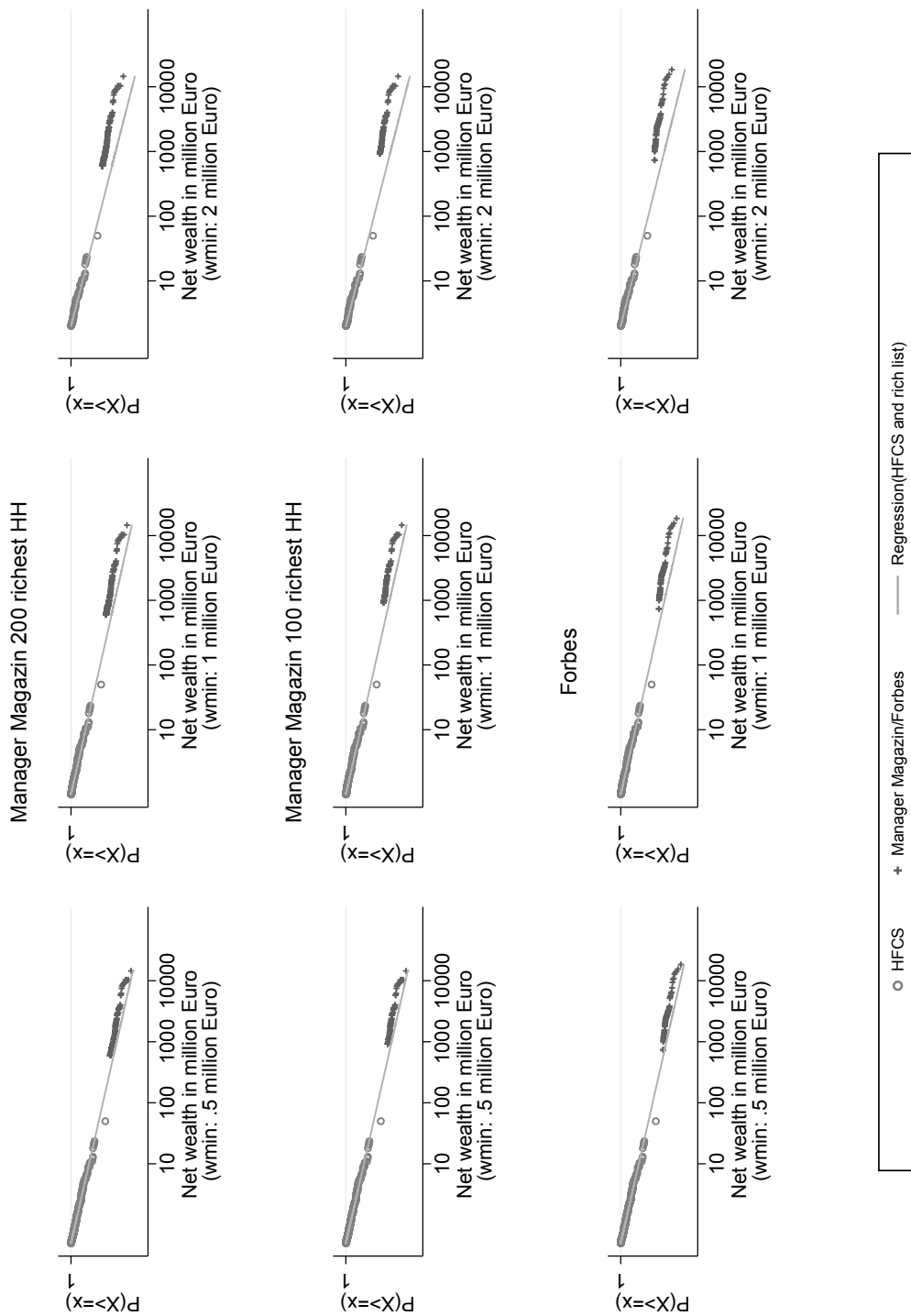
# 1 Tail wealth distributions by rich list and minimum wealth

Figure 1: Tail wealth distribution by rich list and minimum wealth, Germany, first wave of the HFCS



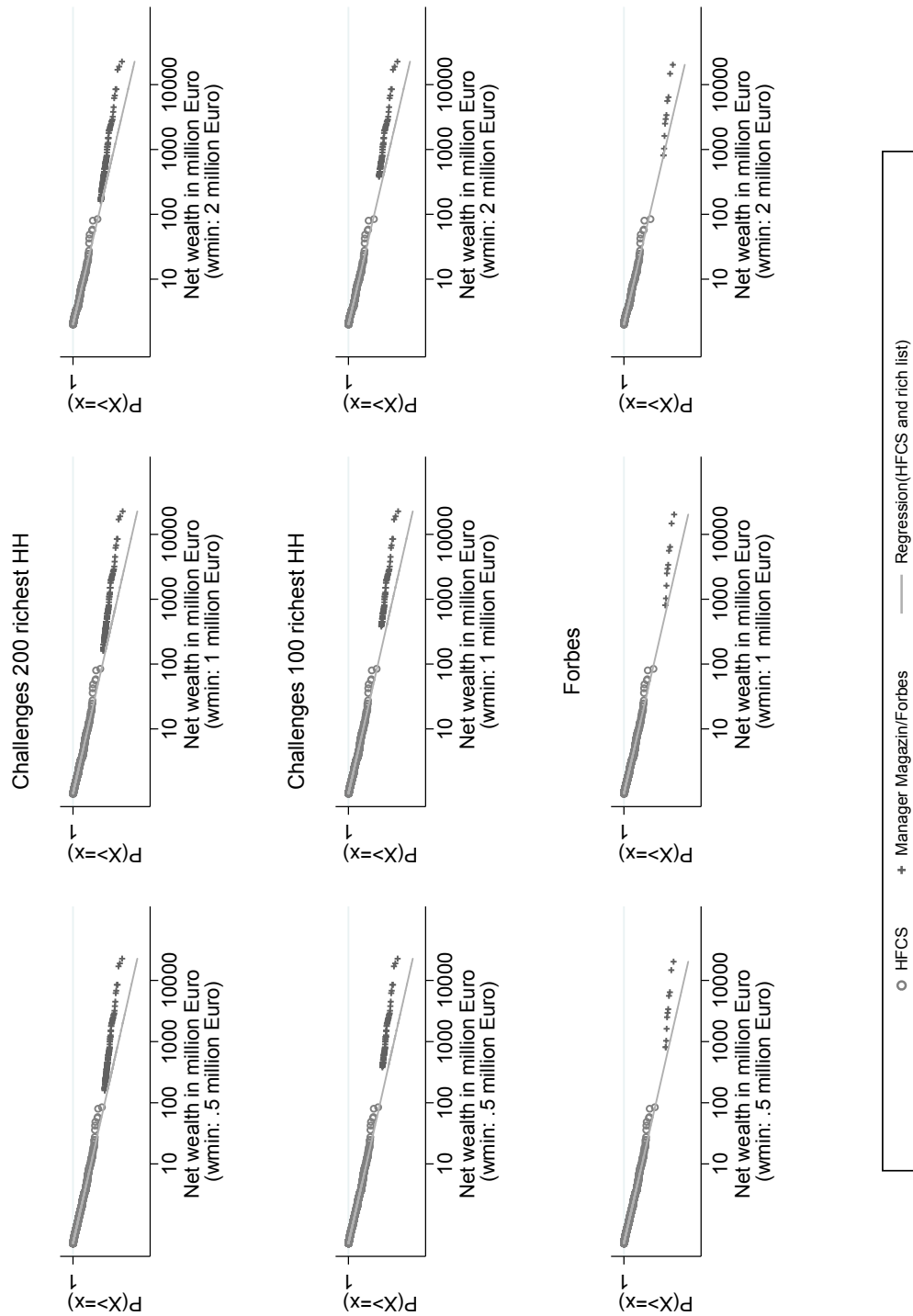
Source: HFCS (first wave), Manager magazin (2011) and Forbes (2011); own calculations.

Figure 2: Tail wealth distribution by rich list and minimum wealth, Germany, second wave of the HFCS



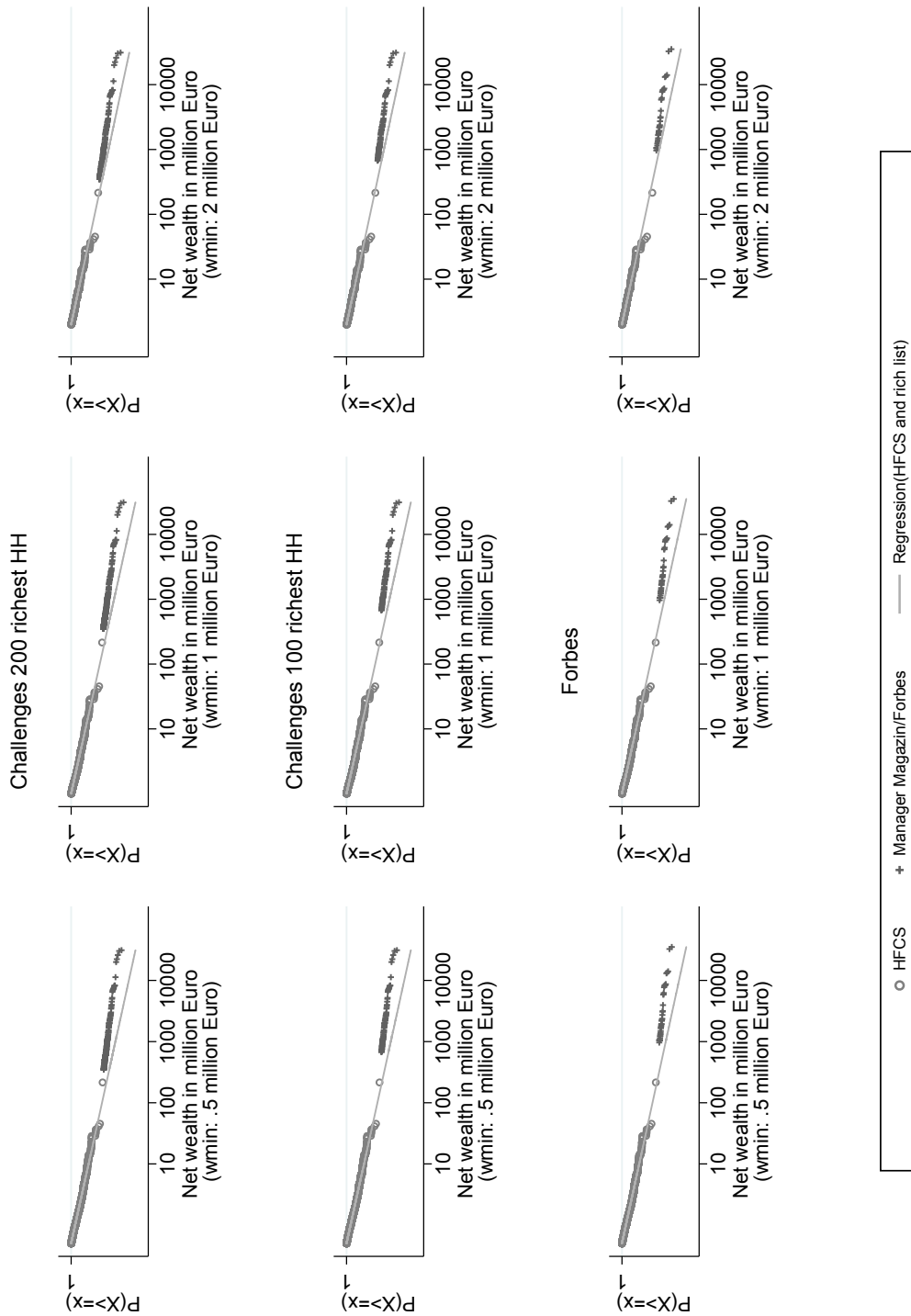
Source: HFCS (second wave), Manager magazin (2014) and Forbes (2014); own calculations.

Figure 3: Tail wealth distribution by rich list and minimum wealth, France, first wave of the HFCS



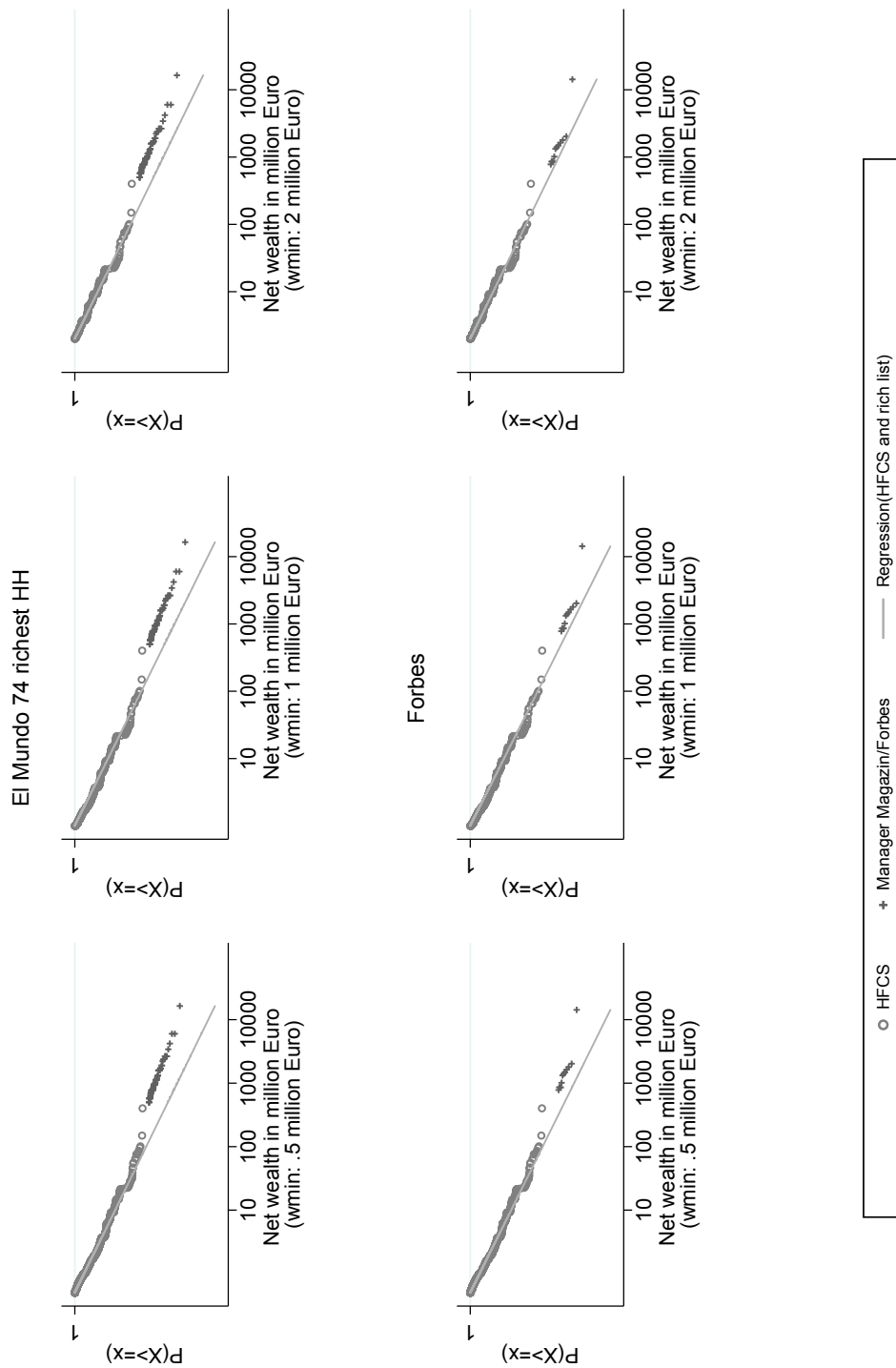
Source: HFCS (first wave), Challenges (2010) and Forbes (2010); own calculations.

Figure 4: Tail wealth distribution by rich list and minimum wealth, France, second wave of the HFCS



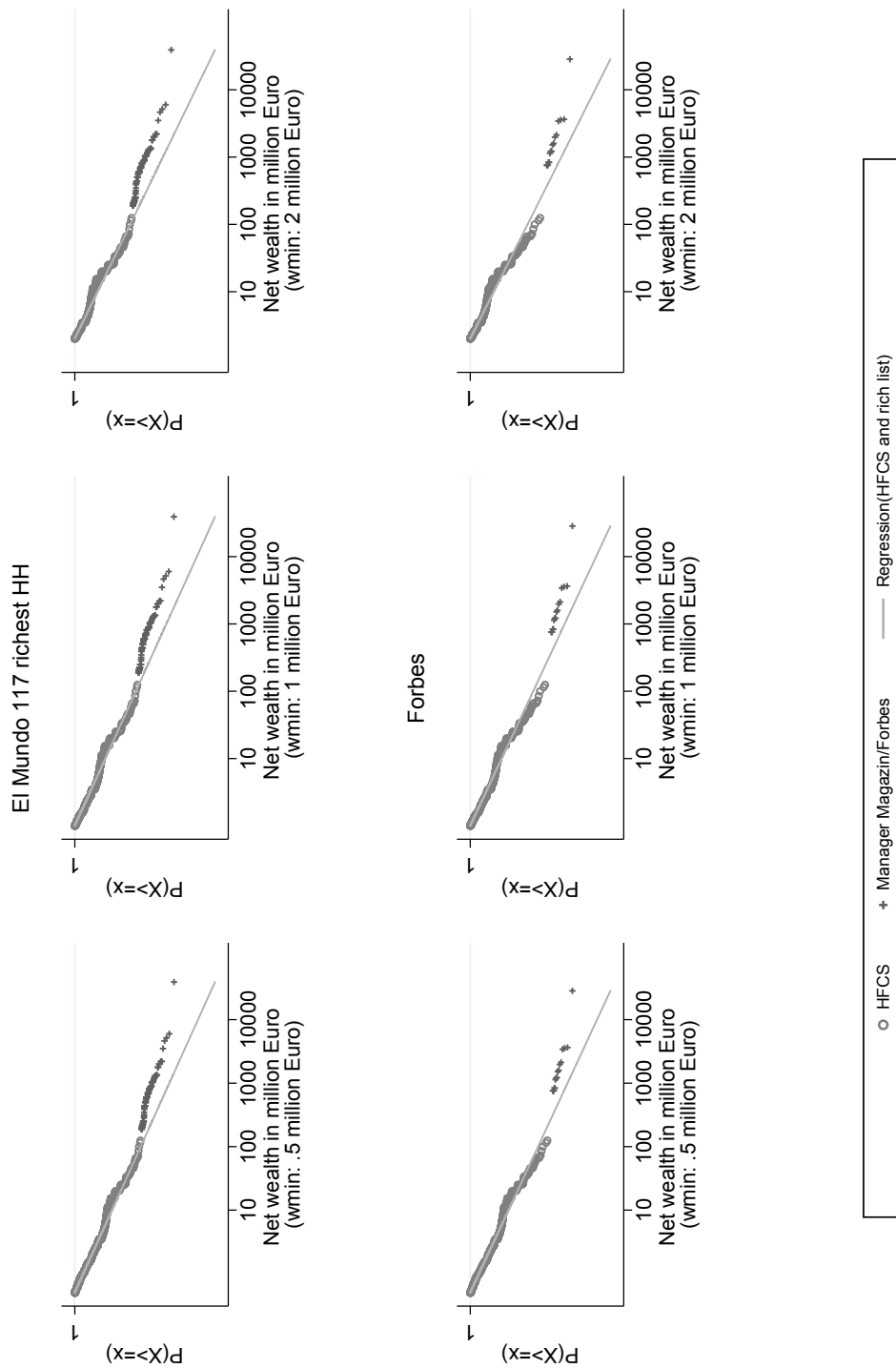
Source: HFCS (second wave), Challenges (2015) and Forbes (2015); own calculations.

Figure 5: Tail wealth distribution by rich list and minimum wealth, Spain, first wave of the HFCS



Source: HFCS (first wave), El Mundo (2009) and Forbes (2009); own calculations.

Figure 6: Tail wealth distribution by rich list and minimum wealth, Spain, second wave of the HFCS

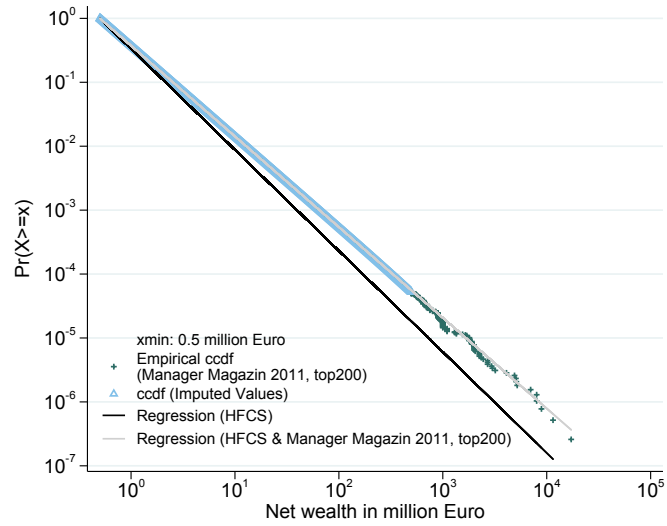


Source: HFCS (second wave), El Mundo (2012) and Forbes (2012); own calculations.



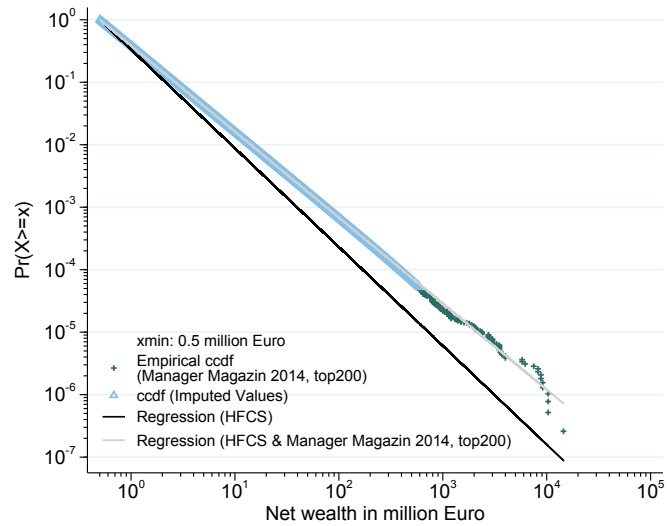
## 2 Adjusted tail wealth distributions

Figure 7: Adjusted tail wealth distribution, Germany - first wave of the HFCS



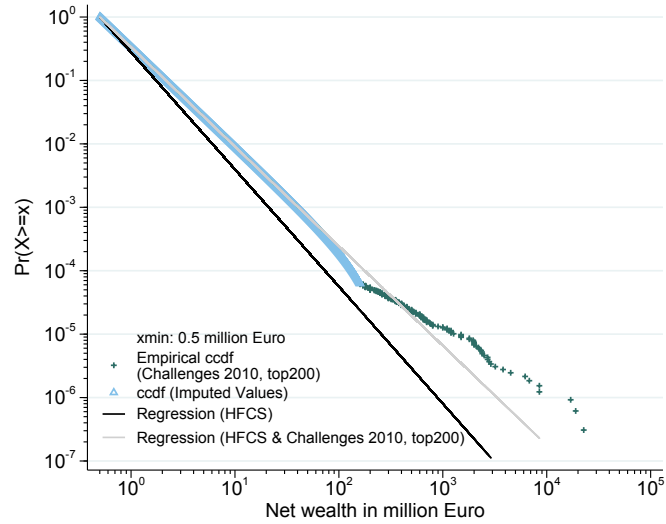
Source: HFCS (first wave), Manager magazin (2011) and Forbes (2011); own calculations.

Figure 8: Adjusted tail wealth distribution, Germany - second wave of the HFCS



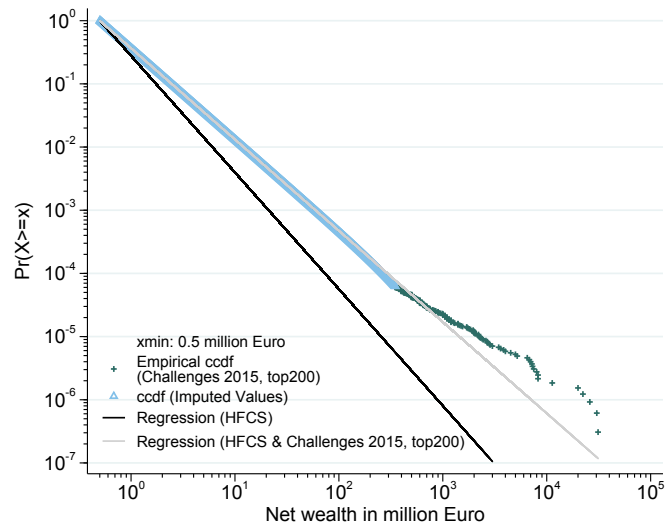
Source: HFCS (second wave), Manager magazin (2014) and Forbes (2014); own calculations.

Figure 9: Adjusted tail wealth distribution, France - first wave of the HFCS



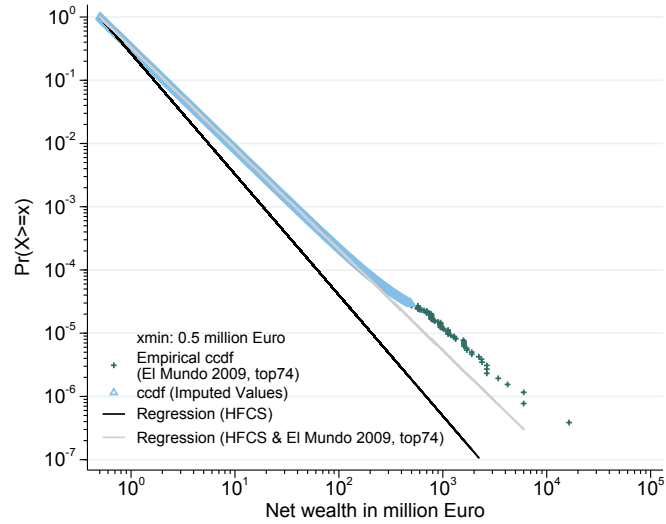
Source: HFCS (first wave), Challenges (2010) and Forbes (2010); own calculations.

Figure 10: Adjusted tail wealth distribution, France - second wave of the HFCS



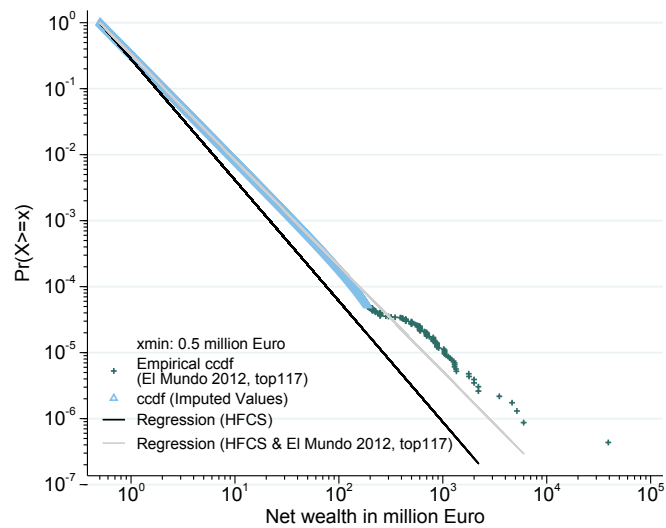
Source: HFCS (second wave), Challenges (2015) and Forbes (2015); own calculations.

Figure 11: Adjusted tail wealth distribution, Spain - first wave of the HFCS



Source: HFCS (first wave), El Mundo (2009) and Forbes (2009); own calculations.

Figure 12: Adjusted tail wealth distribution, Spain - second wave of the HFCS



Source: HFCS (second wave), El Mundo (2012) and Forbes (2012); own calculations.

### 3 Wealth distribution: Distributional tables

Table 1: The distribution of household net wealth in Germany, first wave of the HFCS (2010/2011)

Fractiles household net wealth	Database HFCS				Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total	%	Percentile 1000 Euro	bill. Euro	Total	%
1st - 5th decile	\	217		2.8	\	217		2.4
6th decile	51	290		3.7	51	290		3.3
7th decile	97	495		6.4	97	495		5.4
8th decile	163	837		10.8	163	837		9.2
9th decile	261	1,322		17.1	261	1,322		14.5
10th decile	442	4,582		59.2	442	5,973		65.4
Total	\	7,743		100.0	\	9,134		100.0
Top 5%	661	3,532		45.6	714	4,885		53.5
Top 1%	1,929	1,882		24.3	2,261	3,057		33.5
Top 0.1%	13,581	306		3.9	11,739	1,544		16.9
Gini coefficient		0.7444				0.7814		
Entropy meas. <sup>a)</sup>								
GE(1)		1.2867				1.8451		
GE(2)		5.5961				271.8905		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 200 richest entries of the *manager magazin* 2011 and assume a  $w_{min}$  of 0.5 million Euro.

a) GE(1) is the Theil index, and GE(2) is half of the square of the coefficient of variation.

Source: HFCS (First wave), Manager magazin (2011), own calculations.

Table 2: The distribution of household net wealth in Germany, second wave of the HFCS (2014)

Fractiles household net wealth	Database HFCS				Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total	%	Percentile 1000 Euro	bill. Euro	Total	%
1st - 5th decile	\	214		2.5	\	214		2.1
6th decile	61	335		3.9	61	335		3.3
7th decile	112	557		6.5	112	557		5.5
8th decile	175	893		10.5	175	893		8.8
9th decile	274	1,422		16.7	274	1,422		14.0
10th decile	469	5,080		59.8	469	6,718		66.3
Total	\	8,500		100.0	\	10,138		100.0
Top 5%	727	3,932		46.3	781	5,535		54.6
Top 1%	2,320	2,010		23.6	2,511	3,520		34.7
Top 0.1%	8,864	531		6.3	13,329	1,823		18.0
Gini coefficient		0.7477				0.7870		
Entropy meas. <sup>a)</sup>								
GE(1)		1.2865				1.9326		
GE(2)		5.2522				326.9958		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 200 richest entries of the *manager magazin* 2014 and assume a  $w_{min}$  of 0.5 million Euro.

a) GE(1) is the Theil index, and GE(2) is half of the square of the coefficient of variation.

Source: HFCS (Second wave), Manager magazin (2014), own calculations.

Table 3: The distribution of household net wealth in France, first wave of the HFCS (2009/2010)

Fractiles household net wealth	Database HFCS			Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total %	Percentile 1000 Euro	bill. Euro	Total %	%
1st - 5th decile	\	351	5.4	\	351	4.8	
6th decile	116	406	6.2	116	406	5.5	
7th decile	175	577	8.9	175	577	7.9	
8th decile	237	780	12.0	237	780	10.7	
9th decile	329	1,138	17.5	329	1,142	15.6	
10th decile	511	3,248	50.0	511	4 064	55.5	
Total	\	6 500	100.0	\	7,320	100.0	
Top 5%	775	2,375	36.5	800	3,189	43.6	
Top 1%	1,782	1,166	17.9	2,263	1 822	24.9	
Top 0.1%	6,959	458	7.0	9,970	832	11.4	
Gini coefficient		0.6751			0.7101		
Entropy meas. <sup>(a)</sup>							
GE(1)		1.0226			1.3919		
GE(2)		6.4762			417.1204		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 200 richest entries of the Challenges 2010 and assume a  $w_{min}$  of 0.5 million Euro.

a) GE(1) is the Theil index, and GE(2) is half of the square of the coefficient of variation.

Source: HFCS (First wave), Challenges (2010), own calculations.

Table 4: The distribution of household net wealth in France, second wave of the HFCS (2014/2015)

Fractiles household net wealth	Database HFCS			Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total %	Percentile 1000 Euro	bill. Euro	Total %	%
1st - 5th decile	\	439	6.3	\	455	5.0	
6th decile	113	408	5.9	116	420	4.6	
7th decile	170	582	8.3	173	599	6.6	
8th decile	235	799	11.5	241	830	9.2	
9th decile	331	1,200	17.2	342	1 267	14.0	
10th decile	536	3,545	50.8	565	5 490	60.6	
Total		6,973	100.0		9,061	100.0	
Top 5%	810	2,612	37.5	917	4,465	49.3	
Top 1%	1 814	1,307	18.7	2,821	2 769	30.6	
Top 0.1%	7,660	512	7.3	14,084	1 410	15.6	
Gini coefficient		0.6737			0.7365		
Entropy meas. <sup>(a)</sup>							
GE(1)		1.99			1.7037		
GE(2)		5.5264			794.9589		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 200 richest entries of the Challenges 2015 and assume a  $w_{min}$  of 0.5 million Euro.

a) GE(1) is the Theil index, and GE(2) is half of the square of the coefficient of variation.

Source: HFCS (Second wave), Challenges (2015), own calculations.

Table 5: The distribution of household net wealth in Spain, First wave (2008/2009)

Fractiles household net wealth	Database HFCS				Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total %		Percentile 1000 Euro	bill. Euro	Total %	
1st - 5th decile	\	643	13.0		\	643	11.3	
6th decile	183	346	7.0		183	346	6.1	
7th decile	228	437	8.8		228	437	7.7	
8th decile	289	568	11.5		289	568	10.0	
9th decile	387	813	16.4		387	816	14.3	
10th decile	608	2,152	43.4		606	2,887	50.7	
Total	\	4,958	100.0		\	5,695	100.0	
Top 5%	879	1,532	30.9		942	2,257	39.6	
Top 1%	1,857	737	14.9		2,629	1 280	22.5	
Top 0.1%	7,453	295	6.0		11,415	582	10.2	
Gini coefficient		0.5724				0.6254		
Entropy meas. <sup>a)</sup>								
GE(1)		0.7498				1.1085		
GE(2)		8.1610				132.1785		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 74 entries of *el mundo* 2009 and assume a  $w_{min}$  of 0.5 million Euro.

Note: a)  $GE(1)$  is the Theil index, and  $GE(2)$  is half of the square of the coefficient of variation.

Source: HFCS (First wave), El Mundo (2009), own calculations.

Table 6: The distribution of household net wealth in Spain, Second wave (2011/2012)

Fractiles household net wealth	Database HFCS				Database HFCS including imputed top wealth distribution			
	Percentile 1000 Euro	bill. Euro	Total %		Percentile 1000 Euro	bill. Euro	Total %	
1st - 5th decile	\	574	12.0		\	574	10.9	
6th decile	160	316	6.6		160	316	6.0	
7th decile	205	406	8.5		205	406	7.7	
8th decile	265	539	11.3		265	539	10.2	
9th decile	359	760	15.9		359	766	14.5	
10th decile	542	2,173	45.6		554	2 673	50.7	
Total	\	4,768	100.0		\	5,274	100.0	
Top 5%	864	1,586	33.3		860	2,084	39.5	
Top 1%	1,860	779	16.3		2,386	1,174	22.3	
Top 0.1%	9,808	307	6.4		10,254	528	10.0	
Gini coefficient		0.5925				0.6301		
Entropy meas. <sup>a)</sup>								
GE(1)		0.7965				1.1295		
GE(2)		3.7212				533.6497		

Note: The imputation of the top wealth relies on the approach described in section 3. We use the 117 entries of *el mundo* 2012 and assume a  $w_{min}$  of 0.5 million Euro.

Note: a)  $GE(1)$  is the Theil index, and  $GE(2)$  is half of the square of the coefficient of variation.

Source: HFCS (Second wave), El Mundo (2012), own calculations.

Table 7: Gross wealth share of the top 1 percent for different sample choices and  $w_{min}$

Country, wave	Excluding rich list			Including rich list									Forbes		
				Top 100			National rich list Top 200			Top 300					
	$w_{min}$ (in Euro)														
	$\geq 2$ M	$\geq 1$ M	$\geq .5$ M	$\geq 2$ M	$\geq 1$ M	$\geq .5$ M	$\geq 2$ M	$\geq 1$ M	$\geq .5$ M	$\geq 2$ M	$\geq 1$ M	$\geq .5$ M	$\geq 2$ M	$\geq 1$ M	$\geq .5$ M
GERMANY, 1ST	20.8	22.1	24.2	28.8	32.8	32.9	33.0	33.1	35.7	29.1	29.1	34.6	29.4	29.5	30.9
GERMANY, 2ND	23.8	24.8	27.0	32.6	32.5	32.7	32.5	32.4	32.6	31.9	31.7	31.8	29.8	34.3	32.7
FRANCE, 1ST	16.9	17.0	16.0	22.4	24.9	23.9	21.3	23.7	23.3	20.6	22.8	22.6	19.4	20.0	18.3
FRANCE, 2ND	23.5	19.7	17.6	32.2	30.5	27.7	30.9	29.9	28.0	29.9	29.1	27.7	30.0	27.3	24.1
SPAIN, 1ST	14.3	15.0	13.7	19.4	22.2	20.9	-	-	-	-	-	-	15.3	16.5	15.2
SPAIN, 2ND	14.9	15.7	15.1	19.2	21.1	20.6	-	-	-	-	-	-	17.1	18.0	17.2

*Note:* All underlying Pareto estimates rely on the regression method (OLS), as described in section 3.1, however, net wealth has been replaced by gross wealth. The HFCS top tail is replaced by synthetic households that follow the estimated Pareto distribution. The very end is replaced by the corresponding rich list (exempt for HFCS sample (excluding rich list)).

National rich list in Germany: manager magazin, in France: Challenges, and in Spain: El Mundo. The Top 100 in Spain contains 74 observations in the first, and 117 in the second wave.

*Source:* HFCS (both waves), Manager magazin (2011, 2014); Challenges (2010, 2015); El Mundo (2009, 2012) and Forbes (2011, 2014, 2010, 2015, 2009, 2012) own calculations.

## 4 Aggregate statistics on assets and liabilities

Table 8: Asset and liabilities of households in Germany according to national and financial accounts, 2010 (End-of-year level)

ESA 2010	Assets	billion Euro	%	ESA 2010	Liabilities	billion Euro	%
	Non-financial assets <sup>a)</sup>	6,040	57.8		Loans and other liabilities	1,520	14.5
AN.111	Dwellings	3,483	33.3	AF.41	Short-term loans	75	0.7
AN.112	Other buildings and structures	413	4.0	AF.42	Long-term loans	1,434	13.7
AN.113	Machinery and equipment	134	1.3	AF.8	Other liabilities	11	
AN.2111	Land underlying buildings and structures	1,775	17.0				
AN.2112-9	Land under cultivation, other land	212	2.0				
	Other non-financial assets <sup>b)</sup>	23	0.2				
	Financial assets	4,411	42.2				
AF.21	Currency	106	1.0				
AF.22	Transferable deposits	694	6.6				
AF.23	Other deposits	913	8.7				
AF.3	Debt securities	219	2.1				
AF.511	Listed shares	191	1.8				
AF.512	Unlisted shares	46	0.4				
AF.519	Other equity	184	1.8				
AF.52	Investment fund shares or units	396	3.8				
AF.61	Non-life insurance technical reserves	243	2.3		Net wealth	8,932	85.5
AF.62	Life insurance and annuity entitlements	765	7.3		Net wealth less currency, non-life insurance technical reserves, pension entitlements	7,969	76.2
AF.63	Pension entitlements	614	5.9				
AF.8	Other financial assets	39	0.4				
	Total	10,451	100.0		Total	10,451	100.0

Note: a) Including non-profit institutions serving households (NPIISH).

b) Cultivated assets and other natural resources, intellectual property products, inventories.

Source: Federal Statistical Office, national accounts; Deutsche Bundesbank, financial accounts.



Table 9: Asset and liabilities of households in Germany according to national and financial accounts, 2014 (End-of-year level)

ESA 2010	Assets	billion Euro	%	ESA 2010	Liabilities	billion Euro	%
	Non-financial assets <sup>a)</sup>	7,037	58.0		Loans and other liabilities	1,587	13.1
AN.111	Dwellings	4,047	33.4	AF.41	Short-term loans	65	0.5
AN.112	Other buildings and structures	436	3.6	AF.42	Long-term loans	1,506	12.4
AN.113	Machinery and equipment	141	1.2	AF.8	Other liabilities	17	
AN.2111	Land underlying buildings and structures	2,100	17.3				
AN.2112-9	Land under cultivation, other land	286	2.4				
	Other non-financial assets <sup>b)</sup>	27	0.2				
	Financial assets	5,093	42.0				
AF.21	Currency	128	1.1				
AF.22	Transferable deposits	981	8.1				
AF.23	Other deposits	889	7.3				
AF.3	Debt securities	162	1.3				
AF.511	Listed shares	234	1.9				
AF.512	Unlisted shares	69	0.6				
AF.519	Other equity	206	1.7				
AF.52	Investment fund shares or units	443	3.6				
AF.61	Non-life insurance technical reserves	307	2.5		Net wealth	10,542	86.9
AF.62	Life insurance and annuity entitlements	886	7.3		Net wealth less currency, non-life insurance technical reserves, and pension entitlements	9,355	77.1
AF.63	Pension entitlements	752	6.2				
AF.8	Other financial assets	36	0.3				
	Total	12,129	100.0		Total	12,129	100.0

Note: a) Including non-profit institutions serving households (NPISH).

b) Cultivated assets and other natural resources, intellectual property products, inventories.

Source: Federal Statistical Office, national accounts; Deutsche Bundesbank, financial accounts.

Table 10: Asset and liabilities of households in France according to national and financial accounts, 2010 (End-of-year level)

ESA2010	Assets	billion Euro	%	ESA2010	Liabilities	billion Euro	%
	Non-financial assets	7,042	63.5		Loans and other liabilities	1,323	11.9
AN.111	Dwellings	3,076	27.7	AF.4	Loans	1,057	9.5
AN.112	Other buildings and structures	150	1.4	AF.8	Other liabilities	266	2.4
AN.113	Machinery and equipment	43	0.4				
AN.2111	Land underlying buildings and structures	3,164	28.5				
AN.2112-9	Land under cultivation, other land	430	3.9				
	Other non-financial assets <sup>a)</sup>	179	1.6				
	Financial assets	4,043	36.5				
AF.21	Currency	49	0.4				
AF.22	Transferable deposits	288	2.6				
AF.29	Other deposits	773	7.0				
AF.3	Debt securities	77	0.7				
AF.4	Loans	25	0.2				
AF.511	Listed shares	160	1.4				
AF.512	Unlisted shares	330	3.0				
AF.519	Other equity	306	2.8				
AF.52	Investment fund shares or units	257	2.3				
AF.61	Non-life insurance technical reserves	87	0.8		Net wealth	9,763	88.1
AF.62	Life insurance and annuity entitlements	1,249	11.3		Net wealth, less currency, non-life insurance technical reserves, and pension entitlements	9,463	85.4
AF.63	Pension entitlements	164	1.5				
AF.8	Other financial assets	277	2.5				
	Total	11,085	100.0		Total	11,085	100.0

Note: a) Cultivated assets and other natural resources, intellectual property products, inventories.

Source: INSEE, national accounts; Banque de France and European Central Bank, financial accounts.

Table 11: Asset and liabilities of households in France according to national and financial accounts, 2014 (End-of-year level)

ESA2010	Assets	billion Euro	%	ESA2010	Liabilities	billion Euro	%
	Non-financial assets	7,141	61.1		Loans and other liabilities	1,354	11.6
AN.111	Dwellings	3,435	29.4	AF.4	Loans	1,178	10.1
AN.112	Other buildings and structures	147	1.3	AF.8	Other liabilities	176	1.5
AN.113	Machinery and equipment	37	0.3				
AN.2111	Land underlying buildings and structures	2,907	24.9				
AN.2112-9	Land under cultivation, other land	441	3.8				
	Other non-financial assets <sup>a)</sup>	175	1.5				
	Financial assets	4,538	38.9				
AF.21	Currency	65	0.6				
AF.22	Transferable deposits	319	2.7				
AF.29	Other deposits	888	7.6				
AF.3	Debt securities	77	0.7				
AF.4	Loans	31	0.3				
AF.511	Listed shares	189	1.6				
AF.512	Unlisted shares	373	3.2				
AF.519	Other equity	366	3.1				
AF.52	Investment fund shares or units	291	2.5				
AF.61	Non-life insurance technical reserves	109	0.9		Net wealth	10,326	88.4
AF.62	Life insurance and annuity entitlements	1,415	12.1		Net wealth less currency, non-life insurance technical reserves, and pension entitlements	9,964	85.3
AF.63	Pension entitlements	188	1.6				
AF.8	Other financial assets	228	2.0				
	Total	11,680	100.0		Total	11,680	100.0

Note: a) Cultivated assets and other natural resources, intellectual property products, inventories.  
Source: INSEE, national accounts; Banque de France and European Central Bank, financial accounts.

Table 12: Asset and liabilities of households in Spain according to national and financial accounts, 2009 (End-of-year level)

ESA 2010	Assets	billion Euro	%	ESA 2010	Liabilities	billion Euro	%
	Non-financial assets <sup>a)</sup>	5 884	77.5		Loans and other liabilities	942	12.4
	Financial assets	1 711	22.5	AF4	Loans	901	11.9
AF21	Currency	92	1.2	AF8	Other liabilities	41	0.5
AF22	Transferable deposits	300	3.9				
AF29	Other deposits	408	5.4				
AF3	Debt securities	39	0.5				
AF511	Listed shares	104	1.4				
AF512	Unlisted shares	253	3.3				
AF519	Other equity	45	0.6				
AF52	Investment fund shares or units	151	2.0				
AF61	Non-life insurance technical reserves	24	0.3		Net wealth	6 653	87.6
AF62	Life insurance and annuity entitlements	110	1.4		Net wealth less currency, non-life insurance technical reserves, and pension entitlements	6 394	84.2
AF63	Pension entitlements	143	1.9				
AF8	Other financial assets	42	0.6				
	Total	7 595	100.0		Total	7 595	100.0

Note: a) Only real estate assets. Including non-profit institutions serving households (NPISH).

Source: Banco de España and European Central Bank, financial accounts; estimation on real estate assets by Banco de España.

Table 13: Asset and liabilities of households in Spain according to national and financial accounts, 2011 (End-of-year level)

ESA 2010	Assets	billion Euro	%	ESA 2010	Liabilities	billion Euro	%
	Non-financial assets <sup>a)</sup>	5,208	74.7		Loans and other liabilities	918	13.2
	Financial assets	1,761	25.3	AF.4	Loans	868	12.5
AF.21	Currency	89	1.3	AF.8	Other liabilities	50	0.7
AF.22	Transferable deposits	299	4.3				
AF.29	Other deposits	434	6.2				
AF.3	Debt securities	82	1.2				
AF.511	Listed shares	90	1.3				
AF.512	Unlisted shares	237	3.4				
AF.519	Other equity	83	1.2				
AF.52	Investment fund shares or units	121	1.7				
AF.61	Non-life insurance technical reserves	19	0.3		Net wealth	6,051	86.8
AF.62	Life insurance and annuity entitlements	116	1.7		Net wealth less currency, non-life insurance technical reserves and pension entitlements	5,805	83.3
AF.63	Pension entitlements	139	2.0				
AF.8	Other financial assets	52	0.8				
	Total	6,969	100.0		Total	6,969	100.0

Note: a) Only real estate assets. Including non-profit institutions serving households (NPIH).

Source: Banco de España and European Central Bank, financial accounts; estimation on real estate assets by Banco de España.

## 5 Construction of an adjusted micro data base for the wealth top tail

In our paper, we replace the HFCS wealth top tail by imputed households that follow the Pareto distribution. The very end of the top tail we replace by observations from rich lists, given that these lists provide an empirical estimate for the very top. In this section, we describe the different steps of how we constructed this adjusted top tail. First of all, it is helpful to recall the characteristics of the Pareto distribution: The probability density function of the Pareto distribution is defined on the wealth interval  $[w_{min}; \infty[$  as

$$f(w) = \frac{\alpha w_{min}^\alpha}{w^{\alpha+1}}; \alpha > 0, \quad (1)$$

$\alpha$  being the slope parameter and  $w_{min}$  the minimum wealth which determines the lower bound of the top tail. When representing the top tail of the wealth distribution by the Pareto distribution, we can calculate total tail wealth as

$$\int_{w_{min}}^{\infty} f(w) w N_{tt} dw = \int_{w_{min}}^{\infty} \alpha N_{tt} \left(\frac{w_{min}}{w}\right)^\alpha dw \quad (2)$$

$N_{tt}$  being the top tail population, measured by the sum of weighted households in the HFCS with a net worth of  $w_{min}$  or higher.

In the following, we outline the different steps to derive the adjusted wealth top tail based on actual micro data.

1. Create a dataset with  $z$  observations (households) whose net wealth increases on a logarithm scale up to ' $\infty$ ', where the first household has net wealth higher than  $w_{min}$ . In this paper, we approximate  $\infty$  by 200 billion Euros which is equivalent to  $w_z$ , net wealth of observation  $z$ .<sup>1</sup>
2. For each interval between two observations in the wealth distribution  $[w_{i-1}, w_i]$ , calculate the number of households,  $IH_i$ , which falls into that interval as

$$IH_i = \int_{w_{i-1}}^{w_i} N_{tt} f(w) dw; i = 1, \dots, z \text{ and } w_0 = w_{min}. \quad (3)$$

We index  $IH$  by  $i$ , as it will serve as weight for household  $i$  in the adjusted top tail.

3. For each interval between two observations in the wealth distribution  $[w_{i-1}, w_i]$ , calculate total net wealth,  $IW_i$ , according to the Pareto distribution:

$$IW_i = \int_{w_{i-1}}^{w_i} f(w) w N_{tt} dw = \int_{w_{i-1}}^{w_i} \alpha N_{tt} \left(\frac{w_{min}}{w}\right)^\alpha dw; i = 1, \dots, z \text{ and } w_0 = w_{min}. \quad (4)$$

4. Calculate the final net wealth of each household,  $w^*_i$ , which falls in the wealth interval  $[w_{i-1}, w_i]$  such that

$$w^*_i IH_i = IW_i \quad (5)$$

and

$$w^*_i = \frac{IW_i}{IH_i} \quad (6)$$

where  $w^*_i > w_i$  and  $w^*_i < w_{i+1}$ . As a result, the weighted sum of top tail household net wealth,  $w^*_i$ , matches the sum calculated according to formula 2, given that ' $\infty$ ' is approximated by the same number, here 200 billion Euros.

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<sup>1</sup>The choice of the proxy for ' $\infty$ ', e.g. 200 billion Euros, has an impact on the estimate of total net wealth. Increasing the proxy for  $\infty$  for any given  $\alpha$ ,  $w_{min}$ , and  $N_{tt}$  shrinks the difference between total tail wealth - calculated according to equation 2 and replacing  $\infty$  by the proxy - and its theoretical expected value (calculated as  $N_{tt} w_{min} \frac{\alpha}{\alpha-1}$  (Vermeulen, 2016, p.15)). For instance, when setting  $\infty$  to 5 trillion Euros the two calculations yield almost the same result. However, given that in reality, there is no household with net wealth of infinity, we are confident that approximating  $\infty$  by a lower net wealth value, based on the rich list, is the appropriate approach.

5. Finally, replace the very end of this new discrete top tail Pareto distribution by the rich list, where each entry has a weight of one. Accordingly, we calculate total top tail wealth in our micro data as

$$Total\ tail\ wealth = \sum_i^m w *_{i} IH_i + \sum_{j=1}^R w_j; i = 1, \dots, m j = 1, \dots, R, \quad (7)$$

where  $m$  is the richest household in the imputed top tail which has net wealth below the net wealth of the poorest rich list entry and  $R$  is the number of rich list entries.

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