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# Automation, stagnation, and the implications of a robot tax

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by

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# Automation, Stagnation, and the Implications of a Robot Tax

Emanuel Gasteiger[a,b] and Klaus Prettner[c]

February 2020

a) TU Wien,
Institute of Statistics and Mathematical Methods in Economics,
Wiedner Hauptstr. 8-10, 1040 Wien, Austria
email: emanuel.gasteiger@tuwien.ac.at

b) Instituto Universitário de Lisboa (ISCTE-IUL),
Business Research Unit (BRU-IUL),
Av. das Forças Armadas, 1649-026 Lisboa, Portugal

c) University of Hohenheim
Institute of Economics
Schloss, Osthof-West, 70593 Stuttgart, Germany
email: klaus.prettner@uni-hohenheim.de

**Abstract**

We assess the long-run growth effects of automation in the overlapping generations framework. Although automation implies constant returns to capital and, thus, an $AK$ production side of the economy, positive long-run growth does not emerge. The reason is that automation suppresses wage income, which is the only source of investment in the overlapping generations model. Our result stands in sharp contrast to the representative agent setting with automation, where sustained long-run growth is possible even without technological progress. Our analysis therefore provides a cautionary tale that the underlying modeling structure of saving/investment decisions matters for the derived economic impact of automation. In addition, we show that a robot tax has the potential to raise per capita output and welfare at the steady state. However, it cannot induce a takeoff toward positive long-run growth.

**JEL classification:** O33, O41, E60.
**Keywords:** Automation, robot taxes, stagnation, economic growth, fiscal policy.



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# 1 Introduction

Automation and its potential economic consequences have caught the attention of economists, policymakers, and the general public over the last few years. For the recent breathtaking development of automation technologies, see, for example, The Economist (2014), Ford (2015), Brynjolfsson and McAfee (2016), and Tegmark (2017) who provide many examples for advances in robotics and artificial intelligence (AI) that were considered impossible even ten years ago. The number of industrial robots that substitute for workers on assembly lines started to take off in the 1990s (International Federation of Robotics, 2015) and 3D printing is used to produce customized products like hearing aids and prostheses for which specialized labor was required in the past (Abeliansky et al., 2019). Currently, driverless cars and trucks that could soon revolutionize the employment-intensive transport business are being developed and tested.[1]

While there is widespread agreement that automation has a great potential to raise living standards, there are concerns that automation could (at least partly) be responsible for the stagnating wages of low-skilled workers, a phenomenon observed in the United States since the 1970s. Thus, automation might be a major driver of the rise in wage inequality and in the skill premium since the 1980s (Autor et al., 2003; Atkinson et al., 2011; Autor and Dorn, 2013; Piketty, 2014; Lankisch et al., 2019).[2]

On top of these concerns, Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) use an overlapping generations (OLG) setting to argue that automation could lead to lower wages and investment, overall economic stagnation, and even to decreasing welfare of future generations. This result is perhaps surprising because the literature on automation in the representative agent setting of Solow (1956), Cass (1965), and Koopmans (1965) implies that automation could lead to perpetual long-run growth even without (exogenous or endogenous) technological progress (Steigum, 2011; Prettner, 2019).

We aim to contribute to this strand of the literature along two lines. First, we explain the differences between the predictions of representative agent neoclassical growth models of automation and OLG models of automation from an analytical perspective. To this end, we propose a framework that differs in the following two ways from Sachs and Kotlikoff (2012): i) it allows to distinguish between traditional ma-

[1] Automation is not only confined to routine tasks: devices based on machine learning are starting to rival (and outcompete) doctors in the accuracy of diagnosing diseases, reporters in writing newsflashes, authors in writing books, and even scientists in formulating theories based on vast amounts of experimental data (see National Science Foundation, 2009; Schmidt and Lipson, 2009; Barrie, 2014; Ford, 2015). For an interesting recent contribution that discusses the differences between the economic effects of mechanization, automation, and artificial intelligence (AI), see Growiec (2019).

[2] See Autor (2010), Steigum (2011), Frey and Osborne (2013, 2017), Arntz et al. (2016, 2017), Hémous and Olsen (2016), Abeliansky and Prettner (2017), Acemoglu and Restrepo (2017, 2018b,c), Dauth et al. (2017), Autor and Salomons (2018), Graetz and Michaels (2018), Eden and Gaggl (2018), Prettner (2019), Cords and Prettner (2019), Guimarães and Mazeda Gil (2019), and Prettner and Strulik (2019) for different arguments in the debate.



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chines and robots and thereby to analyze the extent to which their effects on growth differ, and ii) it is analytically tractable. Using this framework, we show that the reason for the differential effects of automation between neoclassical growth models and OLG models is rooted in the implied life-cycle saving pattern rather than in the specifics of the production sector. The generation that builds up its assets for retirement in the OLG model can save *only* out of wage income. The resulting assets are in turn used to invest in standard physical capital and in automation capital. Since automation capital is a close substitute for labor, its accumulation suppresses wages and therefore diminishes the only source of investment in this model. As a result, automation is itself preventing the takeoff to long-run growth in the OLG economy. By contrast, households in the representative agent neoclassical growth model save out of their wage income *and* out of their asset income. Thus, they are able to benefit from the positive effects of automation on the rate of return on assets. This allows to sustain investment and to accumulate assets even though that wages stagnate.

This central result provides a cautionary tale that the underlying modeling structure of saving/investment decisions matters in assessing the effects of automation. This holds true although OLG models and the representative agent model usually lead to similar predictions, for example, on convergence patterns and on the effects of capital accumulation on growth in the medium run. Our result should not be misinterpreted in the sense that we believe that economic stagnation is the likely outcome of automation or that we think the OLG model is more appropriate than the representative agent setting. Instead, we merely aim to show that otherwise innocuous modeling choices on saving/investment turn out to have crucial effects when analyzing the economic consequences of automation.

Our insight is preserved when extending the analytically tractable baseline model to include i) three overlapping generations, ii) bequests, iii) a more general utility function, and iv) a more general production function that accounts for an imperfect substitutability between workers and robots. The extent to which the theoretical channel identified in our paper matters in reality depends on whether the process of asset accumulation is described better by the OLG setting or by the representative agent setting. This is ultimately an empirical question.

As a second contribution to the literature we analyze the effects of a robot tax coupled with a redistribution of the proceeds of the tax from robot income to labor income. We trace the effects of the tax-transfer scheme on the steady-state capital stock and thus on steady-state per capita output. We show that such a tax-transfer scheme cannot overcome the stagnation steady state and push the economy onto a path with positive long-run growth. However, the tax-transfer scheme has the potential of raising the steady-state levels of per capita capital, per capita output, and welfare for certain levels of the tax rate. The intuitive explanation for this result is that the robot tax distorts the optimal split between investments in traditional physical capital and in automation capital in favor of the former. This raises the wages of the workers in



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the next period and allows them to save more. Thus, the traditional capital stock and automation capital in the next period are higher, which, in turn, raises aggregate income.

A recent contribution by Guerreiro et al. (2018) analyzes the optimality of a robot tax in a static model of production. In contrast to the standard result on production efficiency that it is not optimal to tax production inputs (Diamond and Mirrlees, 1971a,b; Judd, 1985; Chamley, 1986), Guerreiro et al. (2018) show how a robot tax increases welfare as long as automation is not yet full. Since their paper is concerned with the static efficiency gains of taxes on robots, whereas our paper is concerned with the effects of a robot tax on the dynamic forces that determine long-run growth in the OLG economy with automation, we view these two papers and their results as highly complementary.

The paper is structured as follows. In Section 2, we present the basic formulation of the OLG model with automation. In Section 3, we analyze the equilibrium dynamics and show that such a model leads necessarily to long-run stagnation. In Section 4, we show that the baseline mechanism is robust with respect to the following extensions i) a three-period setting, ii) bequests, iii) the generalization of the utility function to allow for an intertemporal elasticity of substitution that is different from unity, and iv) the generalization of the production function to allow for imperfect substitutability between robots and human labor. In section 5, we analyze the effects of a robot tax on the steady-state capital stocks, on per capita income, and on welfare. In Section 6, we summarize and draw conclusions for policy makers.

## 2 Automation in the canonical OLG framework

Consider an economy in which time $t = 0, 1, 2 \ldots$ evolves discretely and households live for three time periods, youth, adulthood, and retirement. Children do not make any economic decisions and fulfill their needs via the consumption expenditures of their parents. Adults supply their available time on the labor market for the market clearing wage $w_t$ and save for retirement. Retirees do not work and finance their consumption expenditures at old age out of their savings carried over from adulthood. The population growth rate is exogenous and denoted by $n > -1$ such that the evolution of the population size is given by $N_{t+1} = (1+n)N_t$, where $N_t$ refers to the size of the adult cohort in period $t$.

Following Diamond (1965), households derive utility from consumption in adulthood, $c_{1,t}$, and from consumption in retirement, $c_{2,t+1}$. Assuming that households discount the future at rate $\rho > 0$, which implies a discount factor of $\beta = 1/(1+\rho)$, and a logarithmic utility function to guarantee analytical tractability, the household's lifetime utility is given by

$$U_t = \log(c_{1,t}) + \beta \log(c_{2,t+1}).$$



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Denoting the real interest rate on savings between time $t$ and time $t+1$ by $r_{t+1}$, the standard budget constraint of households is

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t, \tag{1}$$

where the left-hand side refers to discounted lifetime consumption expenditures and the right-hand side to lifetime labor income. Solving the household's intertemporal optimization problem yields the consumption Euler equation

$$\frac{c_{2,t+1}}{c_{1,t}} = \beta(1 + r_{t+1}) \tag{2}$$

describing the optimal consumption growth path for a given interest rate and a given discount factor. From this expression and the budget constraint, optimal consumption and savings of adults follow as

$$c_{1,t} = \frac{1}{1+\beta} w_t, \qquad s_t = \frac{\beta}{1+\beta} w_t. \tag{3}$$

Note that adults consume and save a fraction of their wage income in the first period, which allows them to build up assets for consumption when retired. However, young adults do not yet have any asset income that they could save, which stands in contrast to the models of Solow (1956), Cass (1965), and Koopmans (1965), where individuals start to accumulate assets at the first moment of their life.

While the consumption side is identical to the standard canonical OLG model, the production side changes in a fundamental way in the face of automation. There are now three production factors: labor, which is supplied by adults on the labor market, traditional physical capital in the form of machines, assembly lines, factory buildings, etc., which is an *imperfect* substitute for labor, and automation capital in the form of industrial robots, 3D printers, devices based on machine learning, etc., which is, according to its very definition, a *perfect* substitute for labor[3]. In Subsection 4.3, we relax this assumption and show that our results also hold in case of an imperfect substitutability between workers and automation capital. When investing their savings, households can choose to buy traditional physical capital or automation capital.

Given this setting, the representative firm has access to a production technology as described by Prettner (2019)

$$Y_t = K_t^{\alpha} (N_t + P_t)^{1-\alpha}, \tag{4}$$

where $Y_t$ denotes aggregate output (real GDP), $K_t$ denotes the stock of traditional physical capital, $P_t$ denotes the stock of automation capital[4], and $\alpha \in (0,1)$ is the elasticity of output with respect to traditional physical capital. This production func-

[3] See, for example, the definition of automation in Merriam-Webster (2017).

[4] The letter $P$ stands for "programmable labor" because $R$ for "robots" would lead to a confusion with the rate of return on capital.



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tion conceptualizes the distinctive feature of automation capital as a perfect substitute for labor.[5]

There is perfect competition in goods and factor markets such that all three production factors are paid their marginal value products. Using aggregate output as the numéraire, the profits of the representative firm are given by

$$\Pi_t = K_t^{\alpha}(N_t + P_t)^{1-\alpha} - w_t N_t - R_t^k K_t - R_t^p P_t,$$

where $R_t^k$ is the rate of return on traditional physical capital and $R_t^p$ is the rate of return on automation capital. The first term on the right-hand side is the revenue of the representative firm, whereas the last three terms are the costs of production in terms of the wage bill ($w_t N_t$), the expenses for traditional physical capital ($R_t^k K_t$), and the expenses for automation capital ($R_t^p P_t$). Profit maximization then implies the following factor rewards

$$w_t \stackrel{!}{=} R_t^p = (1-\alpha)\left(\frac{K_t}{N_t + P_t}\right)^{\alpha}, \tag{5}$$

$$R_t^k = \alpha\left(\frac{N_t + P_t}{K_t}\right)^{1-\alpha}. \tag{6}$$

Similar to the standard Diamond (1965) model, an increase in traditional physical capital raises the wage rate because it raises the machine intensity of the economy and therefore the productivity of workers. By contrast, an increase in automation capital has the opposite effect because automation capital competes closely with workers. Thus, an increase in the stock of automation capital does not raise worker's productivity as measured by their marginal value product but renders the workers more and more redundant. Notice, however, that labor productivity as measured by output per worker increases for both an increase in $K_t$ and an increase in $P_t$. The reason is that an increase in both types of capital implies more production for a given amount of labor input.

[5]Below we show that the use of a more general production function such as in Steigum (2011), where the elasticity of substitution between labor and automation capital is allowed to be lower than in case of Equation (4) does not alter our central insights. The reason is that – with a lower elasticity of substitution – the potential for positive long-run growth would be reduced and for low enough levels it would not even emerge in the representative agent setting. In this case, we would be back in the standard well-known stagnation steady state, irrespective of whether we consider the models of Solow (1956), Cass (1965), and Koopmans (1965), or the model of Diamond (1965) as the baseline framework.



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## 3 Equilibrium of the canonical model and main results

For low levels of the traditional capital stock and for low levels of automation capital, Equations (5) and (6) imply

$$\lim_{P_t \to 0} R_t^p = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} \qquad \text{and} \qquad \lim_{K_t \to 0} R_t^k = \infty.$$

Consequently, the Inada conditions are not fulfilled for automation capital such that the possibility of a corner solution emerges. If the traditional capital stock and the automation capital stock are close to zero, individuals would only want to invest in the accumulation of traditional physical capital because its return is higher. Only later, for a large enough traditional physical capital stock, an interior equilibrium on the capital market emerges. For certain parameters, investments in both types of capital then yield the same rate of return and individuals start to accumulate both, traditional physical capital and automation capital. Such an interior equilibrium of the capital market is characterized by a no-arbitrage relationship between both types of investment implying that $R_t^k = R_t^p$. From this condition, the following relationship between $P_t$ and $K_t$ emerges that holds in an interior capital market equilibrium

$$P_t = \left(\frac{1-\alpha}{\alpha}\right) K_t - N_t. \tag{7}$$

The intuition behind this relationship is best illustrated by referring to Equations (5) and (6): a higher stock of traditional physical capital ($K_t$) raises the rate of return on investment in automation capital ($P_t$) and reduces the rate of return on traditional physical capital. Hence, the stock of automation capital has to rise in response to re-establish the equality between the rates of return on traditional physical capital and on automation capital. By contrast, a larger cohort size of adults ($N_t$) implies that there are more workers available. In light of Equation (5), workers will then have lower wages as a result such that the incentives to invest in automation capital are reduced. This, in turn, leads to a decline in the equilibrium stock of automation capital (see also Abeliansky and Prettner, 2017; Acemoglu and Restrepo, 2018a, for theoretical considerations regarding this aspect and for supporting empirical evidence). Altogether, the behavior of the stock of automation capital is given by

$$P_t = \max\left\{0, \left(\frac{1-\alpha}{\alpha}\right) K_t - N_t\right\},$$

which takes into account that households do not invest in automation capital if Equation (7) is negative. In this case the production function collapses to the standard expression of the canonical OLG model as given by $Y_t = K_t^{\alpha} N_t^{1-\alpha}$ and, consequently, the steady-state per capita capital stock and per capita income are constant.

To solve for the steady state that is associated with an interior equilibrium of the



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capital market, we plug the no-arbitrage relationship (7) into the production function (4). This yields an $AK$-type of production technology in equilibrium

$$Y_t = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} K_t, \tag{8}$$

where $A \equiv [(1-\alpha)/\alpha]^{1-\alpha}$. As is well-known, such a production structure usually leads to perpetual growth because there are constant returns with respect to the accumulation of physical capital (see, for example, Romer, 1986; Rebelo, 1991). In neoclassical models of automation that admit a representative household along the lines of Solow (1956), Cass (1965), and Koopmans (1965), there is indeed the possibility of perpetual long-run growth for exactly this reason. For the theoretical derivation of these growth paths see Steigum (2011) and Prettner (2019). It is important to note that long-run growth is possible for a constant level of technology and it is *not* the result of knowledge spillovers due to a learning-by-doing mechanism. Instead, it follows directly from the feature of automation that it is a substitute for labor, which prevents the diminishing returns of capital accumulation from kicking in. Thus, the standard neoclassical convergence mechanism toward a steady state in which the economy stagnates is not operative in this setting.

However, as we show next, the fact that automation leads to an $AK$-type of production technology in case of an interior capital market equilibrium does not imply sustained growth in the OLG model. This stands in sharp contrast to the described findings of Steigum (2011) and Prettner (2019) for the representative agent neoclassical growth model with automation. Since the economy is closed and we follow the standard practice in OLG models by assuming that both types of capital fully depreciate over the course of one generation, the aggregate stock of assets at time $t+1$ is determined by investment in period $t$. This implies the following law of motion for the aggregate stock of assets

$$S_t = s_t N_t \stackrel{!}{=} K_{t+1} + P_{t+1} = \frac{\beta(1-\alpha)}{1+\beta}\left(\frac{K_t}{N_t + P_t}\right)^{\alpha} N_t. \tag{9}$$

We now define the competitive equilibrium of the economy with automation as follows.

**Definition 1.** *A competitive equilibrium is a sequence* $\{K_t, P_t, c_{1,t}, c_{2,t}, R_t, R_t^k, R_t^p, w_t\}_{t=0}^{\infty}$, *such that*

*i)* $\{R_t,\ R_t^k, R_t^p, w_t\}_{t=0}^{\infty}$ *satisfy* (5), (6), *and* $R_t = R_t^k = R_t^p$;

*ii)* $\{c_{1,t}, c_{2,t}\}_{t=0}^{\infty}$ *satisfy* (2) *and* (3);

*iii)* $\{K_t, P_t\}_{t=0}^{\infty}$ *satisfy* (7) *and* (9);

*iv)* $\{N_t\}_{t=0}^{\infty}$ *satisfies the population growth equation* $N_{t+1} = (1+n)N_t$.



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Dividing Equation (9) by the size of the adult cohort $N_{t+1}$ and plugging the no-arbitrage condition (7) into the result, we arrive at the capital accumulation equation

$$k_{t+1} = \alpha + \alpha \left(\frac{\beta}{1+\beta}\right)\left(\frac{1-\alpha}{1+n}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}. \tag{10}$$

It is immediately clear that there are no transitional dynamics and that the steady-state capital-labor ratio of the economy is constant at $k_{t+1} = k_t = k$. From inspecting Equation (8) it follows that GDP per capita stagnates and there is no potential for long-run economic growth. We summarize our main finding on the long-run growth effects of automation in the canonical OLG economy in the following proposition.

**Proposition 1.** *In the canonical overlapping generations model with automation and an interior capital market equilibrium in which both traditional physical capital and automation capital are accumulated:*

*i) the production structure resembles the properties of an AK type of growth model;*

*ii) the accumulation of automation capital reduces wages and therefore the savings/investments of households;*

*iii) the economy is trapped in a stagnation equilibrium because of the feedback effect between automation and wage income.*

This proposition implies that, in contrast to automation-augmented neoclassical growth models with a representative agent, the economy necessarily stagnates in the automation-augmented canonical OLG model even if agents invest in both types of capital. The reason is that investment is fully financed out of wage income as implied by (3). However, wage income itself is reduced by automation. In a sense, automation is therefore digging its own grave in the OLG model.

# 4 Extensions of the baseline model

Next, we clarify the effects of the following extensions of the model: i) a three-period setting, ii) bequests, iii) the generalization of the utility function to allow for an intertemporal elasticity of substitution that is different from unity, and iv) a generalization of the production function to allow for imperfect substitution between robots and human labor. We show that in these settings automation does not have the potential to lead to perpetual growth for the very same reasons as in the baseline model.

## 4.1 Three-period overlapping generations

One might surmise that a three-period structure would re-introduce the possibility of perpetual growth because in the second period, individuals would have asset income



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that they could save. If we allow for such a three-period OLG structure, the consumption side of the economy changes, while the production side does not. Households then live for three periods and maximize their utility as given by

$$U_t = \log(c_{1,t}) + \beta \log(c_{2,t+1}) + \beta^2 \log(c_{3,t+2})$$

subject to the budget constraints that take into account that individuals work in the first and in the second period of their lives but that they are retired in the third period:

$$\begin{aligned} c_{1,t} &= w_t - s_t, \\ c_{2,t+1} &= w_{t+1} + (1 + r_{t+1})s_t - s_{t+1}, \\ c_{3,t+2} &= (1 + r_{t+2})s_{t+1}. \end{aligned}$$

The resulting Euler equations are given by

$$\begin{aligned} \frac{c_{2,t+1}}{c_{1,t}} &= \beta(1 + r_{t+1}), \\ \frac{c_{3,t+2}}{c_{2,t+1}} &= \beta(1 + r_{t+2}). \end{aligned}$$

Using the Euler equations and the budget constraints, we can derive savings as

$$s_t = \left[\frac{\beta + \beta^2}{(1 + \beta + \beta^2)}\right] w_t - \left[\frac{1}{(1 + \beta + \beta^2)}\right] \frac{w_{t+1}}{(1 + r_{t+1})}.$$

From the no-arbitrage relation for investments in traditional physical capital and in automation capital (7), the equilibrium accumulation equation for the capital stock per worker follows as

$$\begin{aligned} \left(1 + \frac{1-\alpha}{\alpha}\right) k_{t+1} = 1 &+ \left[\frac{(\beta + \beta^2)(1-\alpha)}{(1 + \beta + \beta^2)(1+n)}\right] \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \\ &- \left[\frac{(1-\alpha)}{(1 + \beta + \beta^2)(1+n)}\right] \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}{\left[1 + \alpha\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1}\right]}. \end{aligned}$$

It is thus immediately clear that the stagnation result from the baseline model carries over to the three-period overlapping generations case.

### 4.2 Bequests

Another obvious conjecture is that allowing for bequests might change our central result because bequests lead to capital holdings of the young such that they have an additional source of income apart from their work.



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### 4.2.1 Warm glow motive of giving

One way of modeling bequests in our setting is by the warm glow motive of giving (Andreoni, 1989), according to which the members of the old generation derive utility from their bequests to the young. This implies that the preferences of households and the budget constraint change as compared to the baseline model. When turning old, the bequest that the individual has received when young is rented out to firms. In this period the individual begets an offspring, works, and dies. For simplicity, we assume that the young do not consume and work and that there is no population growth. Thus, the size of each cohort is normalized to unity (for the exposition of the problem without automation capital see Acemoglu, 2009, pp.342ff.).

Taking bequests into account, the income $y_{i,t}$ of individuals in generation $t$ consists of wage income and capital income from bequests

$$y_{i,t} \equiv w_t + (1 + r_t)b_{i,t-1}$$

where $b_{i,t-1}$ is the bequest that the offspring received. This income can then be spent on consumption and on bequests to the next generation such that

$$y_{i,t} = c_{i,t} + b_{i,t}.$$

Considering the normalization $N_t = 1$ for all $t$ and solving the household problem under the assumption that individuals live for two generations and have a logarithmic utility function as in the canonical model, the evolution of individual bequests and the evolution of aggregate bequests coincide and read

$$b_{i,t} = \frac{\beta}{1+\beta} y_{i,t} = \frac{\beta}{1+\beta} \left[ w_t + (1 + r_t)b_{i,t-1} \right],$$

where $\beta/(1 + \beta)$ can be interpreted as the bequest rate. Thus, the model has an endogenous wealth distribution that evolves over time and depends on the evolution of $w_t$ and $R_t$.

The production side is as in the canonical OLG model with automation that we described above. Capital market clearing requires that $K_{t+1} + P_{t+1} \stackrel{!}{=} B_t$. Dividing by the population size we get

$$k_{t+1} + p_{t+1} = \frac{\beta}{1+\beta} \left[ w_t + (1 + r_t)(k_t + p_t) \right].$$

Considering the no-arbitrage relationship between the rates of return on traditional physical capital and on automation capital, we derive the evolution of the traditional physical capital stock per worker as

$$k_{t+1} = \alpha + \alpha \frac{\beta}{1+\beta} \left\{ (1-\alpha) \left[ \frac{\alpha}{1-\alpha} \right]^{\alpha} - \alpha \left[ \frac{1-\alpha}{\alpha} \right]^{1-\alpha} \right\} + \alpha \frac{\beta}{1+\beta} \left[ \frac{1-\alpha}{\alpha} \right]^{1-\alpha} k_t,$$



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where we define the composite coefficients

$$\Lambda = \alpha \frac{\beta}{1+\beta} \left\{ (1-\alpha) \left[ \frac{\alpha}{1-\alpha} \right]^{\alpha} - \alpha \left[ \frac{1-\alpha}{\alpha} \right]^{1-\alpha} \right\},$$
$$\Gamma = \alpha \frac{\beta}{1+\beta} \left[ \frac{1-\alpha}{\alpha} \right]^{1-\alpha}.$$

The growth rate of $k$ is then given by

$$g_{k,t+1} := \frac{k_{t+1} - k_t}{k_t} = (\alpha + \Lambda) k_t^{-1} + (\Gamma - 1).$$

From this expression it follows that the long-run growth rate of $k_t$ converges to zero. At the steady state, we thus have $k_{t+1} = k_t = k$ such that the steady-state capital stock $k$ solves for

$$k = \left( \frac{\alpha + \Lambda}{1 - \Gamma} \right).$$

Again, as in the canonical model, stagnation prevails in the long run despite the presence of automation and bequests.

### 4.2.2 Altruistic linkages across generations

An alternative way of modeling bequests is to assume altruistic linkages across generations in line with Barro (1974). In doing so, we follow the exposition of Barro and Sala-i-Martin (2003, pp. 198-200) and assume the following dynastic utility function

$$U_t = \sum_{i=0}^{\infty} \left( \beta \frac{1+n}{1+\phi} \right)^i \left[ \log(c_{1,t+i}) + \beta \log(c_{2,t+1+i}) \right],$$

where $\phi \in (-1, \infty)$ measures the strength of intergenerational linkages, i.e., the degree of altruism. If $\phi = 0$, individuals attach the same weight to their offspring as to themselves such that there is perfect intergenerational altruism. If $\phi$ tends to infinity, by contrast, there is no intergenerational altruism and the utility function collapses to the one in the baseline model. $-1 < \phi < 0$ can be interpreted as a case where parents attach more weight to the utility of their offsprings relative to their own utility. As in Barro and Sala-i-Martin (2003), population growth $n$ can be positive.

The budget constraint and the production function are as in the baseline model and given by (1) and (4). Solving the household's optimization problem and plugging in the expressions for the interest rate and the no-arbitrage relation for investments in the two types of capital yields the following consumption Euler equation for aggregate consumption

$$\frac{c_{t+1}}{c_t} = \beta \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{1+\phi}.$$



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We observe that the right-hand side of the consumption Euler equation consists only of exogenous parameters such that, depending on the value of these parameters, three different cases can in principle emerge: i) if the right-hand side is greater than one, there is perpetual consumption growth over time such that perpetual long-run economic growth prevails; ii) if the right-hand side is equal to one, then consumption stagnates and the economy is at its steady state; iii) if the right-hand side is lower than one, consumption declines perpetually and converges to zero. These cases are qualitatively in line with the results of Steigum (2011) for the representative agent neoclassical growth model with automation. However, we will see next that the first case is impossible to occur for the routinely considered range of $\phi \in [0, \infty)$ and the generally agreed-upon values for $\alpha$ and $\beta$ in the literature.

To this end, we compute the value of intergenerational altruism ($\phi$) depending on $\alpha$ and $\beta$ at which consumption switches between growing and shrinking. This relation is given by

$$\phi = \beta\alpha^{\alpha}(1-\alpha)^{1-\alpha} - 1$$

and we have

$$\frac{c_{t+1}}{c_t} : \begin{cases} > 1, & \text{if} \quad \phi < \beta\alpha^{\alpha}(1-\alpha)^{1-\alpha}, \\ = 1, & \text{if} \quad \phi = \beta\alpha^{\alpha}(1-\alpha)^{1-\alpha}, \\ < 1, & \text{if} \quad \phi > \beta\alpha^{\alpha}(1-\alpha)^{1-\alpha}. \end{cases}$$

In Figure 1, we plot the values of $\phi$ and the associated consumption growth rates for $\alpha = 1/3$ (see, for example, Jones, 1995; Acemoglu, 2009) and $\beta = 0.6$, which corresponds to a yearly discount rate of 2%. We observe that positive long-run growth is only possible for negative values of $\phi$ such that individuals would attach more weight to their children than they do to themselves. Please note that the rather low value of the yearly discount rate makes it more likely that consumption growth could, in principle, be positive depending on the measure for intergenerational altruism, $\phi$. Raising the yearly discount rate would therefore imply that we needed even lower values of $\phi$ to generate perpetual long-run growth in the model.

To summarize, while intergenerational linkages by altruism would, in principle, allow for the emergence of a long-run balanced growth path with constant positive consumption growth in the automation-augmented OLG framework, the implied parameter value required for intergenerational altruism were negative and therefore outside of the routinely considered range.

### 4.3 The general model with isoelastic utility and CES production

In this extension, we investigate whether our result from the canonical model holds true in case of a more general utility function and a more general production structure.



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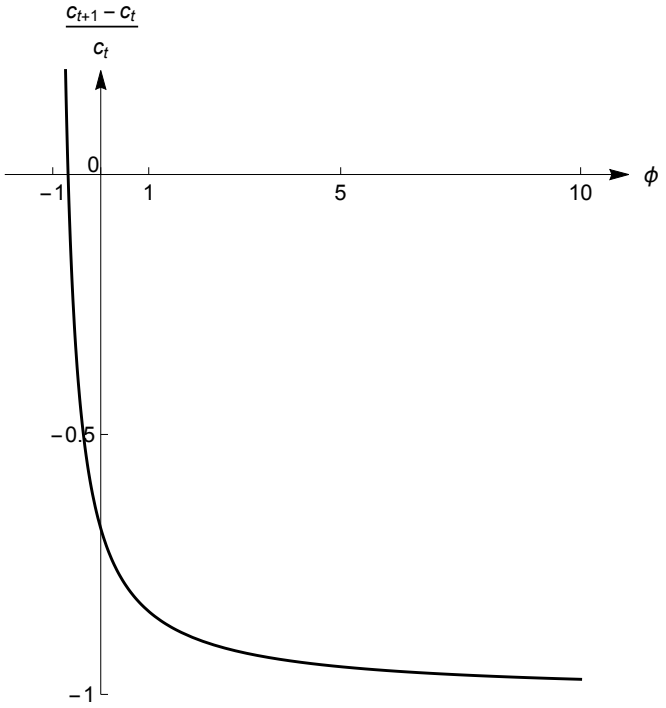
Figure 1: Consumption growth depending on intergenerational altruism as measured by $\phi$




Notes: Consumption growth depending on the parameter measuring intergenerational altruism ($\phi$). A smaller value of $\phi$ implies that parents attach a higher weight on the utility of their parents. In case of $\phi = 0$ there is perfect intergenerational altruism. The other parameters required for plotting consumption growth are given by $\alpha = 1/3$ (Jones, 1995; Acemoglu, 2009) and $\beta = 0.6$, which corresponds to a yearly discount rate of 2%. We observe that long.run consumption growth is only positive for negative values of $\phi$ implying that parents would attach more weight to the utility of their children than to themselves.

To this end, we assume that household's utility is derived from consumption according to the following iso-elastic utility function

$$U_t = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta},$$

where $\theta$ is the coefficient of relative risk aversion such that the elasticity of intertemporal substitution is given by $1/\theta$. Maximizing utility subject to the same budget constraint as in the canonical model yields the following modified Euler equation

$$\frac{c_{2,t+1}}{c_{1,t}} = [\beta(1 + r_{t+1})]^{\frac{1}{\theta}}.$$



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Together with the lifetime budget constraint, the Euler equation allows us to derive first period consumption and savings $s_t = w_t - c_{1,t}$ as

$$c_{1,t} = \frac{w_t}{\left[1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}\right]},$$
$$s_t = \left\{1 - \frac{1}{\left[1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}\right]}\right\} w_t.$$

For later reference, we follow Acemoglu (2009) and rewrite savings as

$$s_t = \frac{w_t}{\psi_{t+1}},$$

where

$$\psi_{t+1} \equiv 1 + \left[\beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}\right]^{-1}.$$

Aggregate savings are again given by $S_t = s_t N_t$ and the population grows at rate $n$.

On the production side, we allow for a more general CES specification along the lines of Steigum (2011) in which automation capital and labor are imperfect substitutes:

$$Y_t = K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}},$$

where $\mu \in (-\infty, 1]$ determines the substitutability between automation capital and workers and $\nu$ is the production share parameter of automation capital. For $\mu \to -\infty$, we would get a Leontief production function in which human labor and automation capital were perfect complements, whereas for $\mu \to 1$, we would get the perfect substitutability case. The factor prices are then given by

$$w_t = \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[(1-\nu)\mu N_t^{\mu-1}\right],$$
$$R_t^p = \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[\nu\mu P_t^{\mu-1}\right],$$
$$R_t^k = \alpha K_t^{\alpha-1}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}}.$$

From a qualitative perspective, similar interpretations obtain as in the canonical OLG model in which automation capital and human labor are perfect substitutes. Applying the no-arbitrage condition $R_t^k \stackrel{!}{=} R_t^p$ to the new setting implies the following relation between traditional physical capital and automation capital

$$K_t = \frac{\alpha\mu}{1-\alpha}\frac{\nu P_t^{\mu} + (1-\nu)N_t^{\mu}}{\nu\mu P_t^{\mu-1}}. \tag{11}$$

At the competitive equilibrium, we have $K_{t+1} + P_{t+1} = s_t N_t$ such that the evolution



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of assets is given by

$$K_{t+1} + P_{t+1} = \left[1 - \frac{1}{1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}}\right] \times \frac{1-\alpha}{\mu} K_t^{\alpha} [\nu P_t^{\mu} + (1-\nu) N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1} \left[(1-\nu)\mu N_t^{\mu-1}\right] N_t.$$

In terms of per capita variables, the following conditions apply

$$\begin{aligned} k_t &= \frac{\alpha}{\nu(1-\alpha)} \left[\nu p_t + (1-\nu) p_t^{1-\mu}\right], \\ R_t &= \alpha k_t^{\alpha-1} \left[\nu p_t^{\mu} + (1-\nu)\right]^{\frac{1-\alpha}{\mu}}, \\ w_t &= (1-\alpha)(1-\nu) k_t^{\alpha} \left[\nu p_t^{\mu} + (1-\nu)\right]^{\frac{1-\alpha-\mu}{\mu}}, \\ k_{t+1} + p_{t+1} &= \frac{w_t}{(1+n)\psi_{t+1}}. \end{aligned}$$

It is well-known that the general OLG model is not tractable analytically and this is also the case here. We therefore linearize the model and solve it numerically.[6] In doing so, we assume that a generation lasts for 25 years, which is a standard value in the OLG literature. As far as the parameter values are concerned, we set $\alpha = 1/3$ (Jones, 1995; Acemoglu, 2009) and $\beta = 0.6$ (corresponding to a yearly discount rate of 2%) as before, assume population growth at rate $n = 0.25$, which corresponds to a yearly population growth rate of 0.9% (World Bank, 2019), take $\mu = 0.5$ such that the elasticity of substitution between robots and workers is 2 (which is a plausible value given the substitutability between low-skilled and high-skilled workers reported by Acemoglu and Autor, 2012; DeCanio, 2016), and assume that $\nu = 0.1$, and $\theta = 2$ (Chetty, 2006; Guvenen, 2006).

The results for the stock of traditional physical capital per capita, automation capital per capita (multiplied by 100 to facilitate the comparison with the physical capital stock), the wage rate, and the capital rental rate are displayed in Figure 2. We observe that the economy converges to the steady-state solution with a stagnating income in the long run. Thus, the results of the canonical model carry over to the generalized case.

There are two additional extensions that do not invalidate our central result: i) the effects of labor-augmenting technological progress are as in the standard OLG model, which we demonstrate in Appendix A. Technological progress merely introduces exogenous growth to the setting presented here. It cannot lead to a situation in which automation would be an independent source of growth (in contrast to the representative agent setting); (ii) Lankisch (2017, pp. 46-51) extends our analysis to a setting in which two types of labor are available in the economy: low-skilled labor, which is a perfect substitute for robots, and high-skilled labor, which is an imperfect substitute

[6] The details for the calculations are provided in Appendix B.



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for robots. He demonstrates that the stagnation result remains valid in this setting.[7]

We summarize all the results of the extensions in the following remark.

**Remark 1.** *The result that automation does not have the potential to lead to perpetual growth in the OLG framework carries over from the canonical model to a setting with*

*i) three-period overlapping generations,*

*ii) bequests,*

*iii) a more general iso-elastic utility function and a more general CES production function,*

*iv) exogenous technological progress,*

*v) low-skilled and high-skilled labor in which only low-skilled labor is a perfect substitute for automation capital.*

At this stage we want to emphasize again that it is not our aim to argue that the OLG model is superior to the representative agent setting or that we believe that automation will lead to stagnation. By contrast, our aim is much more modest. We want to show that caution has to be applied when modeling the effects of automation because the result is not robust to the underlying model of the consumption/saving decision. An OLG framework can lead to very different predictions as compared with a representative agent neoclassical growth setting.

## 5 The effects of a robot tax

A natural question that emerges in our context is the extent to which redistribution policies can affect the impact of automation on the economy. In particular, a tax on robots is often suggested as a solution to mitigate some of the negative consequences of automation. For example, Bill Gates stated in an interview in 2017 that "[...]taxation is certainly a better way to handle it than just banning some elements of it." Gates also mentioned how such a tax could be designed: "Some of it can come on the profits that are generated by the labor-saving efficiency there. Some of it can come directly in some type of robot tax." (Delaney, 2017). Furthermore, some governments and even the European Parliament are considering ideas about a robot tax (see, for example, Prodhan, 2017). In the context of our model, it might be straightforward to conjecture that a tax on the income generated by robots and an associated redistribution of the proceeds of the tax toward workers who do not own assets could raise aggregate savings and enable the asset-poor parts of the population to participate in the gains of automation. In the following we show that, while such a scheme is not effective in

[7]For an early contribution analyzing the effects of computerization on workers with different skill levels, see Autor et al. (2003).



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overcoming long-run stagnation, it can raise the *levels* of per capita income and welfare at the steady state under certain conditions.

To conceptualize the tax-transfer scheme, we examine lump-sum transfers to the working age adults denoted by $\bar{\tau}_t$, which are financed by a tax on the use of automation capital by firms (the robot tax) at rate $\tau > 0$. The budget constraint of households in the model with taxes and redistribution has to be modified and is given by

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t + \bar{\tau}_t.$$

Please note that transfers generally depend on time $t$ because, as the stock of robots and their price change during transition, the tax base also changes (cf. Prettner and Strulik, 2019). Thus, even for a constant robot tax rate, the transfers to each individual might vary over time. Analogous to the previous section, the solution of individual savings is given by

$$s_t = \frac{w_t + \bar{\tau}_t}{\psi_{t+1}}.$$

The profit-maximization problem of the representative firm in case of the tax-subsidy scheme becomes to maximize

$$\Pi_t = K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}} - w_t N_t - (1+\tau)R_t^p P_t - R_t^k K_t$$

by choosing employment $N_t$, the use of traditional physical capital $K_t$, and the use of automation capital $P_t$. This profit function takes into account that a constant robot tax $\tau$ increases the cost of employing automation capital versus other types of machines. As a consequence, $\tau$ distorts the no-arbitrage condition between using traditional physical capital $K_t$ and automation capital $P_t$ in favor of using traditional capital $K_t$. Solving the no-arbitrage condition for the employment of traditional physical capital as above leads to the following relation between traditional physical capital and automation capital

$$K_t = (1+\tau)\left\{\frac{\alpha\mu}{1-\alpha}\frac{[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]}{\nu\mu P_t^{\mu-1}}\right\}.$$

When comparing this to Equation (11) of the model without the tax, more traditional physical capital is used in the production process when the robot tax rate is positive. This is intuitively clear because the robot tax increases the cost of producing with robots as compared to traditional machines such that firms would switch to the latter.

Next we calculate the lump-sum transfers to each adult recognizing that the aggregate proceeds of the robot tax are given by $\tau R_t^p P_t$. Dividing by the size of the adult



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cohort $N_t$, we get the amount that each individual receives as

$$\bar{\tau}_t = \tau R_t^p \left( \frac{P_t}{N_t} \right).$$

With these ingredients, we can derive the equilibrium capital accumulation equation by using again that $K_{t+1} + P_{t+1} = s_t N_t$ such that

$$K_{t+1} + P_{t+1} = \left\{ 1 - \frac{1}{\left[ 1 + \beta^{\frac{1}{\theta}} (1 + r_{t+1})^{\frac{1}{\theta}-1} \right]} \right\} \times \left\{ \frac{1-\alpha}{\mu} K_t^{\alpha} [\nu P_t^{\mu} + (1-\nu) N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1} \left[ (1-\nu)\mu N_t^{\mu-1} \right] + \tau_t R_t^p p_t \right\} N_t.$$

Using the conditions derived based on the no-arbitrage relationship, we again write the equations that fully describe our economy as

$$\begin{aligned} k_t &= (1+\tau) \left\{ \frac{\alpha}{\nu(1-\alpha)} \left[ \nu p_t + (1-\nu) p_t^{1-\mu} \right] \right\} \\ R_t &= \alpha k_t^{\alpha-1} \left[ \nu p_t^{\mu} + (1-\nu) \right]^{\frac{1-\alpha}{\mu}} \\ w_t &= (1-\alpha)(1-\nu) k_t^{\alpha} \left[ \nu p_t^{\mu} + (1-\nu) \right]^{\frac{1-\alpha-\mu}{\mu}} \\ k_{t+1} + p_{t+1} &= \frac{w_t + \tau R_t p_t}{(1+n)\psi_{t+1}}. \end{aligned}$$

Next, we linearize the model and solve it numerically. We display the results for the stock of traditional physical capital per capita, automation capital per capita (multiplied by 100 to facilitate the comparison with the physical capital stock), the wage rate, and the capital rental rate in Figure 3. As before, we observe that the economy converges to the steady-state solution with stagnating stocks of per capita traditional capital and per capita automation capital. Thus, also incomes stagnate in the long run and the result of the canonical model carries over to the economy with the robot tax.

While our analysis shows that the long-run growth result from the canonical OLG model is robust to the introduction of a robot tax that is redistributed to households, we now investigate what happens to the *levels* of the capital stocks and to income if such a robot tax is introduced. In doing so, we also trace the effects of different tax rates on wages, savings, consumption in the two periods of life, the rate of return on capital at the steady state, and the tax revenue. The results are shown in Figure 4. Overall, we observe "Laffer" effects in the sense that – for a low robot tax rate – the increase of the tax rate has positive effects on per capita income. However, these effects turn negative beyond a certain level of the tax rate.



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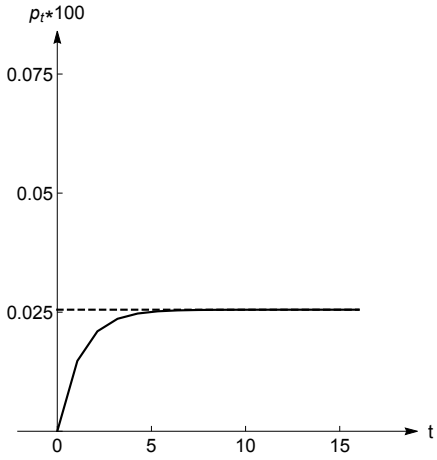
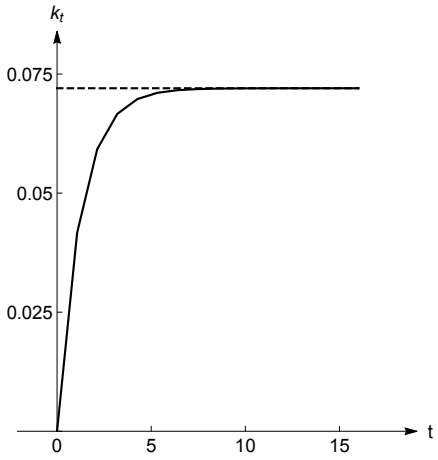
Figure 2: Simulation of the transitional dynamics of the general model

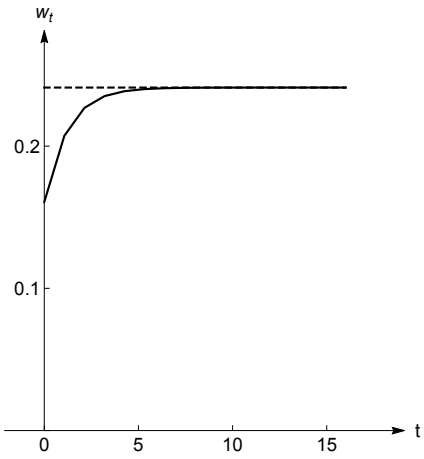
(a) Robots per worker scaled by 100



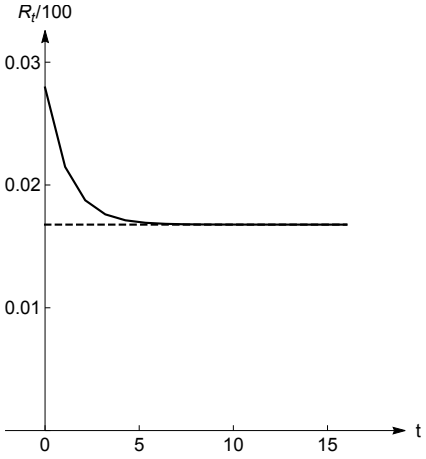

(b) Traditional capital stock per worker



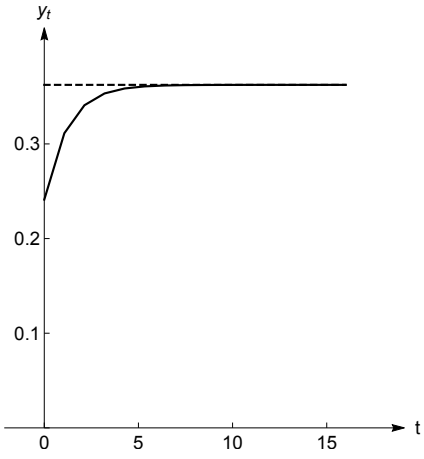

(c) Wage rate




(d) Capital rental rate




(e) Output per worker




Notes: Simulation of the general model with iso-elastic utility function and CES production function. Parameter values: $\alpha = 1/3$, $\beta = 0.6$, $n = 0.25$, $\mu = 0.5$, $\nu = 0.1$, and $\theta = 2$.



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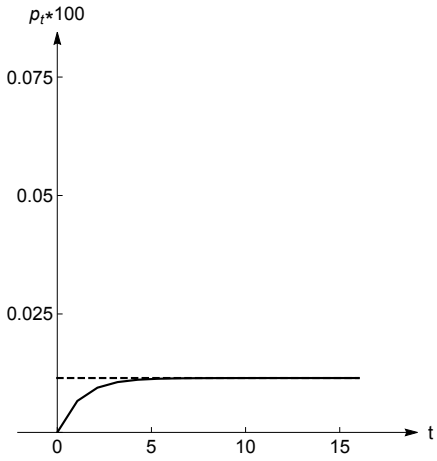
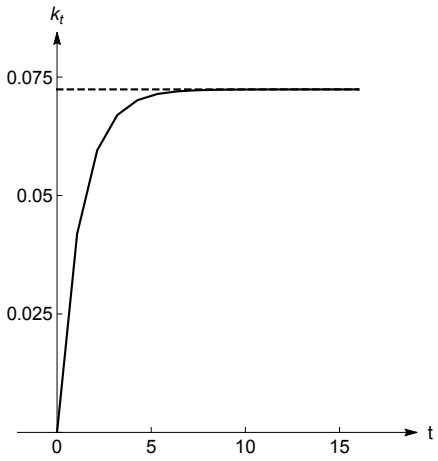
Figure 3: Simulation of the transitional dynamics of the general model with a positive robot tax rate $\tau = 0.5$

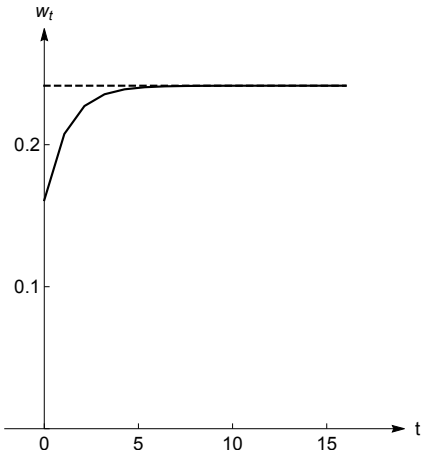
(a) Robots per worker scaled by 100



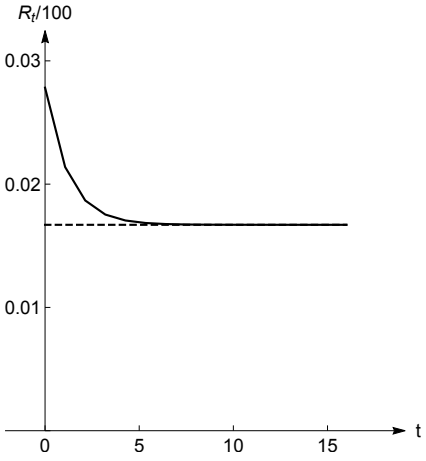

(b) Traditional capital stock per worker



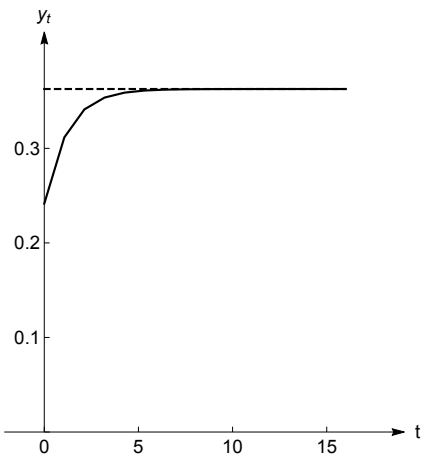

(c) Wage rate



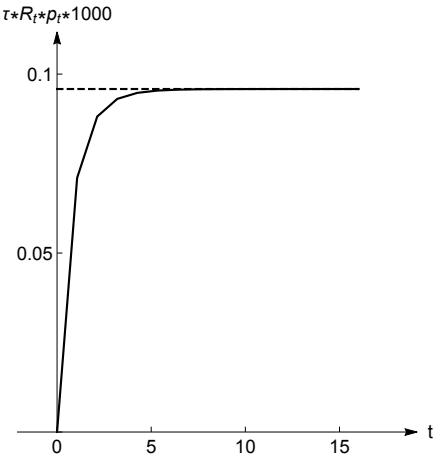

(d) Capital rental rate




(e) Output per worker




(f) Robot tax revenues scaled by 1000




Notes: Simulation of the general model with iso-elastic utility function and CES production function for the case of a robot tax at rate $\tau = 0.5$. Parameter values: $\alpha = 1/3$, $\beta = 0.6$, $n = 0.25$, $\mu = 0.5$, $\nu = 0.1$, and $\theta = 2$.



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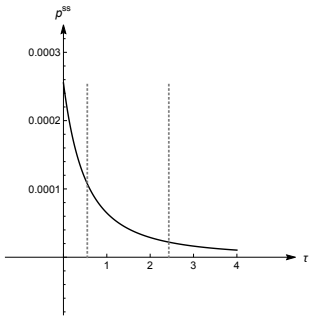
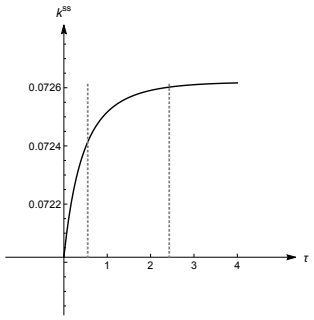
Figure 4: An analysis of the effects of robot taxes on the steady-state outcomes in the general model

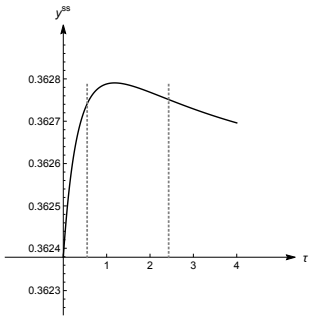
(a) Robots per worker scaled by 100



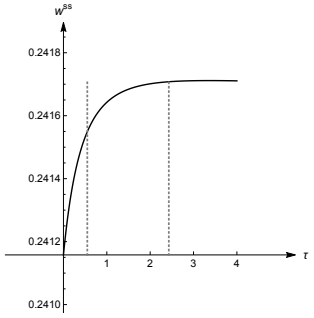

(b) Traditional capital stock per worker



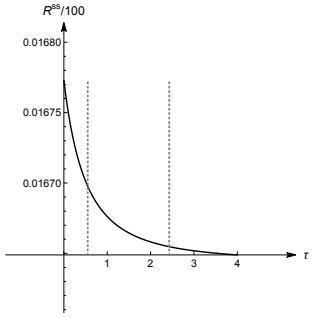

(c) Output per worker



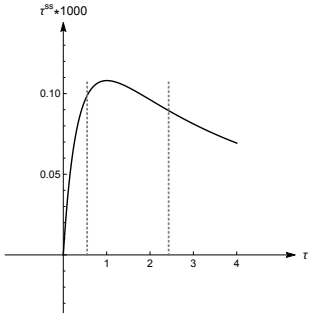

(d) Wage rate



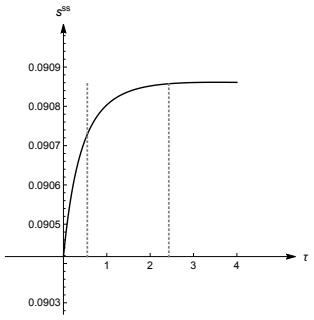

(e) Capital rental rate



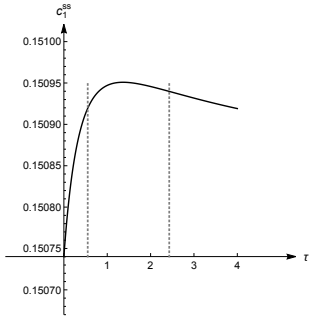

(f) Robot tax revenues scaled by 1000



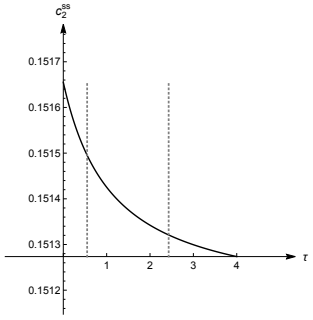

(g) Savings per worker




(h) Consumption per worker in the first period of live




(i) Consumption per worker in the second period of live




Notes: Steady-state outcomes of the general model with iso-elastic utility function and CES production function for a varying robot tax at rate $\tau \in [0, 4]$. Parameter values: $\alpha = 1/3$, $\beta = 0.6$, $n = 0.25$, $\mu = 0.5$, $\nu = 0.1$, and $\theta = 2$.



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The intuitive explanation for this result is that the robot tax distorts the optimal split between investments in traditional physical capital and in automation capital in favor of the former. Ceteris paribus, this raises the traditional physical capital stock and lowers the stock of automation capital as long as the robot tax is comparatively low. As is clear from the production function, this raises the wages of the workers in the next period, which, in turn, allows them to save and invest more. Thus, the traditional physical capital stock and the stock of automation capital increase in the next period, which leads to a higher aggregate income level. As the robot tax rate increases, however, the negative effect that the robot tax rate has on the rate of return of both types of capital becomes stronger. This discourages capital accumulation and eventually leads to a dampening of the wage rate. Eventually, with further rising robot taxes, output and consumption start to decline. Thus, there is an interior tax rate that maximizes per capita output at the steady state.

To calculate the welfare effects of the robot tax at the steady state, we follow Lucas (1990), Cooley and Hansen (1992) and others, and compute the consumption equivalent by which an individual needs to be compensated in case of the implementation of the robot tax such that she is equally well off in terms of utility compared with the situation before the implementation of the robot tax. Lifetime utility of an agent born in period $t$ at the steady state without the robot tax is

$$U_0 = \frac{c_{1,0}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{2,0}^{1-\theta} - 1}{1-\theta},$$

where $c_{1,0}$ and $c_{2,0}$ denote consumption when young and old at the steady state with $\tau = 0$. To get the consumption amount that is needed to compensate the agent for the effects of a robot tax, we solve

$$U_0 = \frac{[c_{1,\tau}(1+x_1)]^{1-\theta} - 1}{1-\theta} + \beta \frac{[c_{2,\tau}(1+x_2)]^{1-\theta} - 1}{1-\theta}$$

either for $x_1$ by setting $x_2 = 0$, or for $x_2$ by setting $x_1 = 0$. Here $c_{1,\tau}$ and $c_{2,\tau}$ are consumption when young and old at the steady state with $\tau > 0$. For $x_1$ and $x_2$ we get

$$x_1 = \left[ \frac{c_{1,0}^{1-\theta} + \beta \left( c_{2,0}^{1-\theta} - c_{2,\tau}^{1-\theta} \right)}{c_{1,\tau}^{1-\theta}} \right]^{\frac{1}{1-\theta}} - 1,$$

$$x_2 = \left[ \frac{\beta^{-1} \left( c_{1,0}^{1-\theta} - c_{1,\tau}^{1-\theta} \right) + c_{2,0}^{1-\theta}}{c_{2,\tau}^{1-\theta}} \right]^{\frac{1}{1-\theta}} - 1.$$

Then we can compute either $(x_1 \times c_{1,\tau})/y_\tau$, or $(x_2 \times c_{2,\tau})/y_\tau$, which is the compensating variation (CV) – leaving an agent equally well off after the implementation of the policy as before – either in units of consumption when young or in units of consumption



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when old, given as a fraction of output at the stead state with the robot tax. The consumption levels are given by

$$
\begin{aligned}
c_{1,0} &= \frac{w_0}{1+\beta^{\frac{1}{\theta}}(1+r_0)^{\frac{1}{\theta}-1}}, \\
c_{1,\tau} &= \frac{w_\tau+\bar{\tau}}{1+\beta^{\frac{1}{\theta}}(1+r_\tau)^{\frac{1}{\theta}-1}}, \\
c_{2,0} &= \frac{[\beta(1+r_0)]^{\frac{1}{\theta}}\, w_0}{1+\beta^{\frac{1}{\theta}}(1+r_0)^{\frac{1}{\theta}-1}}, \\
c_{2,\tau} &= \frac{[\beta(1+r_\tau)]^{\frac{1}{\theta}}\, (w_\tau+\bar{\tau})}{1+\beta^{\frac{1}{\theta}}(1+r_\tau)^{\frac{1}{\theta}-1}},
\end{aligned}
$$

where $c_{2,0}$ and $c_{2,\tau}$ follow from the corresponding Euler equations.

The result for a range of values $\tau \in [0, 4]$ is displayed in Figure 5. For our baseline parameter values, $\tau = 0.55$ maximizes welfare. If we reduce $\mu$ from 0.5 to 0.4 such that automation capital becomes a worse substitute for labor, the optimal tax rate rises to $\tau = 0.57$, whereas it decreases to $\tau = 0.54$ if we raise $\mu$ to 0.6 such that workers and robots become better substitutes. This implies that introducing a robot tax could be a valuable option for policymakers if an economy followed an OLG saving/investment structure.



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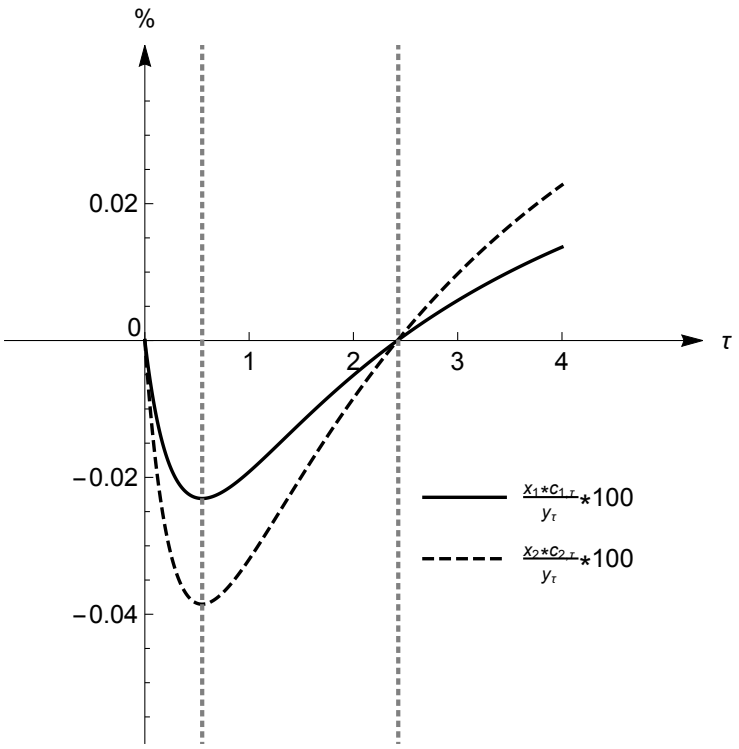
Figure 5: Welfare effects of the robot tax for a tax rate in the interval $[0, 4]$ in terms of the compensating variation (CV)




Notes: Steady-state compensating variation (CV) of the general model with iso-elastic utility function and CES production function for a varying robot tax at rate $\tau \in [0, 4]$. The solid line represents the CV in terms of consumption on the first period of the life-cycle, whereas the dashed line represents the CV in terms of consumption in the second period of the life-cycle. Parameter values: $\alpha = 1/3$, $\beta = 0.6$, $n = 0.25$, $\mu = 0.5$, $\nu = 0.1$, and $\theta = 2$.



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Altogether, the robot tax has the potential to raise per capita capital, per capita output, and welfare at the steady state. However, we have to emphasize that this result is only derived here for a closed economy, where capital in either form cannot move abroad. In an open economy setting, the robot tax faces the same difficulties as a tax on any mobile production factor: it is very easy to move a mobile production factor to a jurisdiction that does not impose such a tax and export the goods produced with robots in the foreign economy to the home market. A successful implementation of a robot tax then depends on whether or not it is implemented by all (or – at least – sufficiently many economically powerful) countries jointly. Thus, the results of our model with respect to the taxation of robots can only be interpreted to hold for a large entity such as all OECD countries taken together.

A second caveat applies if innovation and automation are endogenously determined as in Prettner and Strulik (2019). They show that the introduction of a robot tax reduces the incentives to invest in new technologies to the extent that technological progress and economic growth slow down after its introduction. Thus, the robot tax decreases income of future generations as compared to the baseline trajectories without the robot tax in such a setting.

# 6 Conclusions

Representative-agent neoclassical growth models and OLG models lead to different predictions regarding the consequences of automation for long-run economic growth. In representative agent neoclassical growth models with automation, the diminishing returns to capital accumulation are overcome by automation to the extent that the overall production structure resembles the properties of an $AK$ growth model and long-run economic growth becomes possible even without technological progress. By contrast, the canonical OLG model of Diamond (1965) always implies economic stagnation even if the diminishing returns to capital accumulation are overcome by automation. The reason for stagnation is that, in this framework, households save exclusively out of their labor income. By definition, however, automation competes with labor and depresses the wage rate. This reduces the saving/investment potential of households and prevents the economy from growing. We show that this baseline result holds true in various extensions of the model i) toward a three-period setting, ii) toward including bequests, iii) toward a generalization of the utility function with an elasticity intertemporal of substitution different from unity, and iv) toward a generalization of the production function with imperfect substitutability between robots and human labor.

We also analyze the effects of a robot tax in the more general setting with extensions iii) and iv) and show that it has the potential to raise per capita capital, per capita output, wages, and welfare at the steady state. However, it cannot overcome the stagnation equilibrium of the economy. This result with respect to robot taxation



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derived in a dynamic setting complements the result of Guerreiro et al. (2018) that a robot tax can be efficient in a static setting. A caveat to our result applies if innovation and automation are endogenously determined as in Prettner and Strulik (2019). In this setting a robot tax might reduce the incentives to invest in new technologies to the extent that technological progress and economic growth slow down. Thus, the robot tax would decrease the income of future generations as compared to the baseline situation without the robot tax. In addition, from a policy perspective, the implementation of a robot tax would require a coordinated move across countries because otherwise capital might move to jurisdictions without a robot tax.

Finally, we would like to stress once more that our main point is not that we believe that automation implies stagnation. We also do not think that the OLG model is a more realistic depiction of the saving/investment choices of households than the one in the representative agent neoclassical growth model. We merely provide a cautionary tale in the debate on automation by showing that the underlying modeling structure of saving/investment decisions matters in assessing the effects of automation, whereas it does not usually matter in the analysis of other phenomena related to economic growth. Thus, when analyzing the economic impact of automation, it is even more important to think carefully about the underlying framework's properties and to be transparent on the modeling choices and its potential implications. Otherwise, just by choosing a different underlying model formulation, very different conclusions could emerge. Ultimately, the relevance of the theoretical channel described in this paper needs to be assessed from an empirical perspective.

## Acknowledgements

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# Appendix

## A The effects of labor-augmenting technological progress

In this appendix we show that the general insights of the canonical model and the corresponding intuition do not change when we allow for labor-augmenting technological progress. In this case, the production side of the economy has to be modified, while the consumption side stays as in the baseline model. The modified production function of the canonical OLG model is then given by

$$Y_t = K_t^{\alpha}(A_t N_t + P_t)^{1-\alpha},$$

where technology evolves according to $A_{t+1} = (1+g)A_t$ with $A_0 = 1$ and $g > 0$. The effective capital-labor ratio $\tilde{k} \equiv K/AL$ is given by

$$\tilde{k} = \alpha + \alpha\left(\frac{\beta}{1+\beta}\right)\frac{(1-\alpha)}{(1+n)(1+g)}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \tag{A.1}$$

at the steady state. In this case, per capita variables grow along a balanced growth path at the rate of technological progress, $g$. For $g = 0$, Equation (A.1) collapses to Equation (10) and the economy is back in the stagnation steady state. Thus, with positive long-run growth, our result still holds true in the sense that automation does not represent an additional engine for long-run economic growth besides technological progress. Note that – in the representative agent setting in which individuals safe out of labor income and out of capital income – automation would act as an additional source of long-run growth apart from technological progress.

## B The general model with robot tax

In this Appendix, we provide the derivation of the model with an iso-elastic utility function and a CES production function. The equations for the case without the robot tax follow directly by setting $\tau = 0$.

### B.1 Consumption Side

Consider the following problem

$$\max_{c_{1,t}, c_{2,t+1}, s_t} U_t = u(c_{1,t}) + \beta u(c_{2,t+1}) \quad \text{s.t.}$$

$$c_{1,t} = w_t + \bar{\tau}_t - s_t, \tag{B.1}$$

$$c_{2,t+1} = (1 + r_{t+1})s_t, \tag{B.2}$$

where $r_{t+1} \equiv R_{t+1} - \delta$ is the real interest on a one-period loan from $t$ to $t+1$, $R_{t+1}$ is the rental rate of capital, and $\bar{\tau}_t$ is the transfer financed by the robot tax. The



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Lagrangian is

$$\begin{aligned}\mathcal{L}(\cdot) = &\left[u(c_{1,t}) + \lambda_t(w_t + \bar{\tau}_t - s_t - c_{1,t})\right] \\ &+ \beta\left[u(c_{2,t+1}) + \lambda_{t+1}((1 + r_{t+1})s_t - c_{2,t+1})\right].\end{aligned}$$

The FOCs are

$$\begin{aligned}\frac{\partial\mathcal{L}(\cdot)}{\partial c_{1,t}} &\stackrel{!}{=} 0 \quad \Leftrightarrow \quad \left[u'(c_{1,t}) - \lambda_t\right] \stackrel{!}{=} 0, \\ \frac{\partial\mathcal{L}(\cdot)}{\partial c_{2,t+1}} &\stackrel{!}{=} 0 \quad \Leftrightarrow \quad \beta\left[u'(c_{2,t+1}) - \lambda_{t+1}\right] \stackrel{!}{=} 0, \\ \frac{\partial\mathcal{L}(\cdot)}{\partial s_t} &\stackrel{!}{=} 0 \quad \Leftrightarrow \quad -\lambda_t + \beta\lambda_{t+1}(1 + r_{t+1}) \stackrel{!}{=} 0,\end{aligned}$$

with $\partial\mathcal{L}(\cdot)/\partial\lambda_t$ and $\partial\mathcal{L}(\cdot)/\partial\lambda_{t+1}$ establishing (B.1) to (B.2). Assuming iso-elastic utility

$$U_t = \frac{c_{1,t}^{1-\theta} - 1}{1 - \theta} + \beta\frac{c_{2,t+1}^{1-\theta} - 1}{1 - \theta},$$

the FOCs imply the following Euler equation

$$\frac{c_{2,t+1}}{c_{1,t}} = \left[\beta(1 + r_{t+1})\right]^{\frac{1}{\theta}}. \tag{B.3}$$

Next, we combine (B.1) and (B.2) by replacing $s_t$ to get

$$c_{2,t+1} = (1 + r_{t+1})(w_t + \bar{\tau}_t - c_{1,t}).$$

Using the Euler equation (B.3), we can obtain optimal consumption $c_{1,t}$ as

$$c_{1,t} = \frac{(w_t + \bar{\tau}_t)}{1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}}.$$

With this result, optimal savings $s_t$ can be derived by using (B.1) as

$$\begin{aligned}\frac{w_t}{1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}} &= w_t + \bar{\tau}_t - s_t \\ \Leftrightarrow s_t &= \left[1 - \frac{1}{1 + \beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}}\right](w_t + \tau_t).\end{aligned}$$

Rewriting along the lines of Acemoglu (2009) yields

$$s_t = \frac{w_t + \bar{\tau}_t}{\psi_{t+1}},$$

where $\psi_{t+1} \equiv 1 + \left[\beta^{\frac{1}{\theta}}(1 + r_{t+1})^{\frac{1}{\theta}-1}\right]^{-1}$.



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## B.2 Production Side

The representative firm rents the capital and labor available at period $t$ such that the problem of the representative firm is given by

$$\max_{K_t,P_t,N_t} \Pi_t = K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}} - w_t N_t - (1+\tau)R_t^p P_t - R_t^k K_t$$

with $\tau$ referring to the robot tax and the production function being as in Steigum (2011). The FOCs are

$$\begin{aligned}
\frac{\partial \Pi_t}{\partial N_t} \stackrel{!}{=} 0 \quad &\Leftrightarrow \quad \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[(1-\nu)\mu N_t^{\mu-1}\right] - w_t \stackrel{!}{=} 0, \\
\frac{\partial \Pi_t}{\partial P_t} \stackrel{!}{=} 0 \quad &\Leftrightarrow \quad \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[\nu\mu P_t^{\mu-1}\right] - (1+\tau_t)R_t^p \stackrel{!}{=} 0, \\
\frac{\partial \Pi_t}{\partial K_t} \stackrel{!}{=} 0 \quad &\Leftrightarrow \quad \alpha K_t^{\alpha-1}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}} - R_t^k \stackrel{!}{=} 0.
\end{aligned}$$

The factor prices are thus

$$\begin{aligned}
w_t &= \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[(1-\nu)\mu N_t^{\mu-1}\right], \\
(1+\tau_t)R_t^p &= \frac{1-\alpha}{\mu} K_t^{\alpha}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[\nu\mu P_t^{\mu-1}\right], \\
R_t^k &= \alpha K_t^{\alpha-1}[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}}
\end{aligned}$$

and the no-arbitrage condition for investments in the two types of capital pins down to

$$\begin{aligned}
R_t^k &\stackrel{!}{=} R_t^p \\
\Leftrightarrow (1+\tau_t)[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}] &= \frac{1-\alpha}{\alpha\mu} K_t \left[\nu\mu P_t^{\mu-1}\right] \\
\Leftrightarrow K_t &= (1+\tau_t)\left(\frac{\alpha\mu}{1-\alpha}\frac{[\nu P_t^{\mu} + (1-\nu)N_t^{\mu}]}{\nu\mu P_t^{\mu-1}}\right).
\end{aligned}$$

We assume that the total proceeds of the robot tax are redistributed to the households in a lump-sum manner such that

$$\bar{\tau}_t = \tau_t R_t^p \frac{P_t}{N_t} = \tau_t R_t^p p_t.$$

## B.3 Competitive Equilibrium

Capital markets clear such that the aggregate stock of capital consisting of traditional physical capital and of automation capital is equal to aggregate investment/saving because of full depreciation. Thus, we have $K_{t+1} + P_{t+1} = S_t = s_t N_t$ with $N_{t+1} =$



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$(1+n)N_t$. Plugging in the various results from above, we get

$$K_{t+1}+P_{t+1}=\left\{1-\frac{1}{\left[1+\beta^{\frac{1}{\theta}}(1+r_{t+1})^{\frac{1}{\theta}-1}\right]}\right\}$$
$$\times\left\{\frac{1-\alpha}{\mu}K_t^{\alpha}[\nu P_t^{\mu}+(1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[(1-\nu)\mu N_t^{\mu-1}\right]+\tau_t R_t^p p_t\right\}N_t.$$

In addition, we have the following conditions that have to hold in equilibrium:

$$K_t=(1+\tau_t)\left\{\frac{\alpha\mu}{1-\alpha}\frac{[\nu P_t^{\mu}+(1-\nu)N_t^{\mu}]}{\nu\mu P_t^{\mu-1}}\right\},$$
$$R_t=R_t^p=R_t^k=\left\{\frac{1-\alpha}{\mu}K_t^{\alpha}[\nu P_t^{\mu}+(1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}\left[\nu\mu P_t^{\mu-1}\right]\right\}(1+\tau_t)^{-1}$$
$$=\alpha K_t^{\alpha-1}[\nu P_t^{\mu}+(1-\nu)N_t^{\mu}]^{\frac{1-\alpha}{\mu}}.$$

Rewriting in terms of per capita variables, we get

$$k_t=(1+\tau_t)\left\{\frac{\alpha}{\nu(1-\alpha)}\left[\nu p_t+(1-\nu)p_t^{1-\mu}\right]\right\},$$
$$R_t=\alpha k_t^{\alpha-1}\left[\nu p_t^{\mu}+(1-\nu)\right]^{\frac{1-\alpha}{\mu}},$$
$$w_t=(1-\alpha)(1-\nu)k_t^{\alpha}\left[\nu p_t^{\mu}+(1-\nu)\right]^{\frac{1-\alpha-\mu}{\mu}},$$
$$k_{t+1}+p_{t+1}=\frac{w_t+\tau_t R_t p_t}{(1+n)\psi_{t+1}}.$$

We can linearize the above equations around the steady state $(k,p)$ as follows

$$k\widehat{k}_t=(1+\tau)\left\{\frac{\alpha}{\nu(1-\alpha)}\left[\nu p+(1-\nu)(1-\mu)p^{1-\mu}\right]\right\}\widehat{p}_t,$$
$$R\widehat{R}_t=\alpha k^{\alpha-1}\left[\nu p^{\mu}+(1-\nu)\right]^{\frac{1-\alpha}{\mu}}$$
$$\times\left\{(\alpha-1)\widehat{k}_t+\frac{1-\alpha}{\mu}\left[\nu p^{\mu}+(1-\nu)\right]^{-1}\nu\mu p^{\mu}\widehat{p}_t\right\},$$
$$w\widehat{w}_t=(1-\alpha)(1-\nu)k^{\alpha}\left[\nu p^{\mu}+(1-\nu)\right]^{\frac{1-\alpha-\mu}{\mu}}$$
$$\times\left\{\alpha\widehat{k}_t+\frac{1-\alpha-\mu}{\mu}\left[\nu p^{\mu}+(1-\nu)\right]^{-1}\nu\mu p^{\mu}\widehat{p}_t\right\},$$
$$k\widehat{k}_{t+1}+p\widehat{p}_{t+1}=\frac{w}{(1+n)\psi}\widehat{w}_t+\frac{\tau Rp}{(1+n)\psi}\widehat{p}_t$$
$$+\left[\frac{\tau p}{(1+n)\psi}+\frac{w+\tau Rp}{(1+n)\psi^2}\left(\frac{1}{\theta}-1\right)\beta^{-\frac{1}{\theta}}R^{-\frac{1}{\theta}}\right]R\widehat{R}_{t+1},$$

where $\widehat{k}_t\equiv(k_t-k)/k$, $\widehat{p}_t\equiv(p_t-p)/p$, $\widehat{w}_t\equiv(w_t-w)/w$, and $\widehat{R}_t\equiv(R_t-R)/R$. We are also interested in the evolution of output per capita and can express it as

$$y_t=k_t^{\alpha}\left[\nu p_t+(1-\nu)\right]^{\frac{1-\alpha}{\mu}}$$



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with the linearized version being

$$y\widehat{y}_t = k^{\alpha}\left[\nu + (1-\nu)\right]^{\frac{1-\alpha}{\mu}} \times \left[\alpha\widehat{k}_t^{\alpha} + \frac{(1-\alpha)}{\mu}\left[\nu p^{\mu} + (1-\nu)\right]^{-1}\nu\mu p^{\mu}\widehat{p}_t\right].$$



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**Contact**

Research Group Economics
Institute of Statistics and Mathematical Methods in Economics
Vienna University of Technology

Wiedner Hauptstraße 8-10
1040 Vienna
Austria

Phone: +43-1-58801-1053- 1
Fax: +43-1-58801-1053-99
E-mail: wps@econ.tuwien.ac.at

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