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Optimal Redistributive Wealth Taxation When Wealth Is More Than Just Capital

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Optimal Redistributive Wealth Taxation When Wealth Is More Than Just Capital

Abstract

We show how normative standpoints determine optimal taxation of wealth. Since wealth is not equal to capital, we find very different welfare implications of land rent-, bequest- and capital taxation. It is mainly land rents that should be taxed. We develop an overlapping generations model with heterogeneous agents and calibrate it to OECD data. We compare three normative views. First, the Kaldor-Hicks criterion favors the laissez-faire equilibrium. Second, with prioritarian welfare functions based on money-metric utility, high land rent taxes are optimal due to a portfolio effect. Third, if society disapproves of bequeathing, bequest taxation becomes slightly more desirable.

JEL-Codes: D310, D630, E620, H210, H230, Q240.

Keywords: optimal taxation, social welfare, wealth inequality, land rent tax, Georgism.

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1. Introduction

Wealth inequality is on the rise again in many OECD countries (Alvaredo et al., 2018). The increase of wealth inequality in several OECD countries over the last decades coincided with political decisions to reduce taxation of wealth (Piketty, 2014; Drometer et al., 2018). To counter undesirably high levels of wealth inequality, several scholars have recommended to increase taxes based on individuals' wealth (for example Benhabib et al., 2011; Piketty, 2014; Stiglitz, 2016; and also the open letter by Saez and Zucman, 2019, to U.S. Senator Warren). This has been accompanied by recent scholarly discussion (Mattauch et al., 2018; Piketty and Saez, 2013; Stiglitz, 2018; Straub and Werning, 2020) that calls into doubt earlier results of optimal taxation theory suggesting that an efficient redistributive tax system has low or zero taxes on wealth (for example by Chamley, 1986 and Judd, 1985).

Much of this discussion about how to address wealth inequality, however, has neglected the fact that wealth is composed of different types of assets. Wealth is not just capital. The discussion sparked by Piketty (2014) and the data on wealth compiled by Piketty and his collaborators revealed the crucial role of housing and land for the distribution of wealth and its evolution over time (Maclennan and Miao, 2017; Stiglitz, 2016). Stiglitz, for example, shows that models that equate wealth with capital, and thus ignore land as a second component of wealth, cannot reproduce the growth rate of wealth that Piketty and Zucman (2014) have documented. Moreover, he argues that including land is crucial to understand the increase in wealth inequality.

However, to the best of our knowledge, there is no published work on the optimal redistributive taxation of wealth when there is a fixed factor of production such as land. Typically, the models used for the analysis of wealth taxation treat wealth as equal to capital.² The present paper aims at closing this gap in the literature. Therefore, we analyze distributional and welfare effects of taxes on income flows from wealth in general

¹Taxation of wealth (when it is modeled only as capital and fixed factors are not taken into account) may also be optimal if households face tight borrowing constraints or are subject to uninsurable idiosyncratic income risk (Hubbard et al., 1986; Aiyagari, 1995; İmrohoroğlu, 1998). A second situation in which the optimal capital tax is positive arises when allowing for the existence of households of different ages who make the same type of decisions (consumption/savings and labor supply) at a given point in time and age-dependent taxes are not available (Alvarez et al., 1992; Garriga, 2019). In this paper, however, we focus on a different issue. Therefore, we abstract from the latter two features for the purpose of analytical clarity.

²Examples include Chamley (1986), Judd (1985), Piketty and Saez (2013) and the literature on quantitative macroeconomic models surveyed in De Nardi and Fella (2017).

equilibrium and show explicitly how the choice of the normative standpoint determines the optimal policy. In particular, we consider taxes on land rents, capital income and bequests. To do so, we make two extensions to the standard overlapping generations (OLG) model introduced by Diamond (1965). First, households may invest in two factors of production – land and capital. Thereby, we ascommodate the fact that wealth is not just capital, but is instead composed of different types of assets. Second, we model agents that differ in preferences for leaving voluntary bequests (modeled as warm glow, cf. Andreoni, 1989).³

We calibrate the model to data on average wealth inequality in the OECD countries. The rich literature on optimal taxation deals extensively with tools to analyze ability heterogeneity among households. However, tools to analyze welfare when preferences are heterogeneous have only recently been developed. To make interpersonal welfare comparisons, we use money metric utility and the concept of equivalent income as discussed, for example, by Fleurbaey (2009) (for an axiomatic justification of the aggregation of money metric utility, see Bosmans et al., 2018).⁴

Our main contribution is to explicitly show how policy conclusions depend on the choice of the normative standpoint. We compare three different normative views. First, applying the Kaldor-Hicks criterion forbids any policy reform in most cases we analyze and, hence, reflects a status quo bias. This reflects the fact that an OLG economy with an (essential) fixed factor is dynamically efficient (Rhee, 1991) and taxes cannot achieve any Pareto improvement. However, application of the Kaldor-Hicks criterion means that one assumes that a dollar in the hand of a millionaire is the same as a dollar in a beggar's hat. Therefore, as

³Voluntary bequests are a key determinant of the distribution of wealth (Cagetti and De Nardi, 2008). De Nardi (2004) and De Nardi and Yang (2014) show that in particular the voluntary bequest motive helps to explain the observed wealth inequality within her model, while accidental bequests appear less important. Moreover, the alternative bequest motive where parents' utility depends on their offspring's utility would mean that the OLG model collapses again to an infinitely lived agent (ILA) model. The fact that we want to include land necessitates modeling overlapping generations such that there is a market for land in the steady state (old selling land to the young). In an infinitely lived agent (ILA) model, there would be no such market. Also, in the steady state of an ILA only the most patient households actually bequeathe, while all other agents end up behaving like pure life-cyclers, independent of their degree of altruism. Hence, our approach can be seen as complementary to the ILA-based model used by Michel and Pestieau (2005). Note, however, that the latter do not include land. Finally, since we want to focus on implications of different welfare criteria, a more detailed analysis of bequest- and inheritance taxation is beyond the scope of the present paper. See, for example, Kopczuk (2013) for a review of that strand of literature.

⁴Equivalent incomes can be used to compare wellbeing between different individuals when several non-income dimensions matter, e.g., related to personal health or the quality of the environment (see, e.g., Decancq et al., 2015). In our case, though, the only non-income dimension is the price vector (interest-, wage- and tax rates).

second normative standpoint, we consider giving priority to the worse-off using equivalent incomes and aggregating them using a weakly convex transformation, that is, we apply a prioritarian social welfare function (Adler, 2012). Choosing the latter ethical criterion suggests very high optimal land rent taxes and a potential role for bequest taxation but not for capital income taxation.⁵ Third, a society may see the practice of inheritances alltogether as morally undesirable, for example due to its interference with equality of opportunity (Mill, 1909, p. 808 f.; Haslett, 1986; Rawls, 1999, p. 245). To reflect this normative viewpoint, the social welfare function should take into account individuals' utility only if that individual derives it from his or her own consumption and not from the warm glow of leaving bequests. Since we model preference heterogeneity only in terms of the warm glow, individuals' different utility functions can be reduced to identical functions from a benevolent government's perspective. With identical utility functions that depend only on individuals' own consumption and life-cycle savings, a standard utilitarian welfare function is well defined. Then, land rent and bequest taxation becomes slightly more desirable.

Optimal rates for taxes on income streams from wealth in general become higher, the lower the intertemporal elasticity of substitution (IES) is. For values of the elasticity that have been determined empirically by Havranek et al. (2015) for those OECD countries to which we have calibrated our model, it is rather likely, that positive bequest tax rates enhance welfare relative to a laissez-faire equilibrium. Our model confirms findings of previous literature on the sensitivity of optimal bequest taxes (Piketty and Saez, 2013) and capital taxes (Straub and Werning, 2020) to the IES and extends the analysis to the case of land rent taxation. In our model, lowering the IES implies a lower elasticity of aggregate bequests to the net-of-tax bequest tax rate, which Piketty and Saez (2013) also have shown to increase the optimal bequest tax. Straub and Werning (2020) show that if the IES has a value lower than one, positive capital tax rates can become optimal.

Taxing the rent from the fixed factor land is welfare improving for almost all cases that

⁵In calculating the equivalent income a specific reference price is used to compare different outcomes. In Appendix C, we also discuss implications of an alternative social welfare measure that also gives priority to the poor, but is based on the compensating variation – and, hence, does not have one single reference price. Our main results turn out to be robust with respect to this alternative specification. Evaluation ex-post, as with the compensating variation, is especially important when there are potentially catastrophic outcomes, as they are analyzed, e.g., in the economics of climate change. The compensating variation (also referred to as willingness-to-accept an outcome) can then be infinitly high, while the equivalent variation or willingness-to-pay is always constrained by an individual's budget. In the present paper, however, we do not consider such catastrophic outcomes.

we consider. Thus in our model, taxing rents is not neutral. This stands in stark contrast to conventional economic wisdom about fixed factors (see Schwerhoff et al., 2019, for an extensive discussion of the taxation of economics rents). Absent spending requirements, for example for productive public investment, taxing fixed factors should result in no welfare change, no dead-weight loss and no change of allocations. It should be neutral. Our results contradict conventional wisdom because we have designed our model to be able to take portfolio effects into account: Taxing the land rent makes investments in land less attractive and spurs investments in productive capital.

Our research builds on the literature about portfolio effects. This strand of the literature sheds new light on an old debate about the taxation of fixed factors, which dates back to Henry George's call for a land tax (George, 1879/1894/2002). The first to break with conventional economic wisdom that taxing fixed factors is non-distortionary was Feldstein (1977). In his seminal contribution, he formalized the intuition that land rent taxes in general are not neutral. Instead, if land is used as an asset for investments, taxing it can shift investments towards productive capital and thereby increase production. Based on the insight that such portfolio effects can occur, Edenhofer et al. (2015) find that when capital is underaccumulated due to imperfect intergenerational altruism, land rent taxation can be socially optimal. However, the authors consider heterogeneity only between generations and not within, and do not consider capital income or bequest taxes. Franks et al. (2018) determine the impact of taxes on land rents, capital income and bequests on output and the intra-generational wealth distribution. The authors find that with combinations of land rent and bequest taxes, governments can reduce wealth inequality without sacrificing output. However, they do not specify a social welfare function and therefore cannot make statements about the optimal tax portfolio. Therefore, in this paper, we include intragenerational wealth inequality and an extensive discussion of how different specifications of social welfare and different normative standpoints determines optimal tax policy.

In the following, we describe our model (Section 2) and discuss the criterion on which we base our normative statements about social welfare (Section 3). Then, in Sections 4 and 5 we present our welfare analysis of wealth taxation and assess the robustness of our results with respect to different underlying assumptions. Section 6 concludes the paper.

2. Model description

To make normative statements about the desirability of different policies, we extend the purely descriptive general equilibrium model introduced in Franks et al. (2018). In that paper, the authors construct a simple overlapping generations model with discrete time steps $t \in \{1, ..., T\}$. There is one representative firm producing a generic final good using capital, labor and land as inputs and households that live for two periods. In the first time period, that is, when they are young, households use their income to finance consumption in young age c_t^y and savings s_t . In the second period of their life, households can use the savings to pay for consumption in old age c_{t+1}^o or to leave bequests to their offspring b_{t+1} . Households are heterogeneous with respect to their warm-glow preferences for leaving bequests (Andreoni, 1989) and are labelled $i \in \{1, ..., N\}$. We assume that the offspring of a household has the same preferences as its parents.⁶ Moreover, savings are assumed to be invested in productive accumulable capital and a fixed factor (land), according to a no-arbitrage condition.

2.1. Agents of the economy

The utility of households is given by a household specific isoelastic function with elasticity parameter η . The only source of heterogeneity is the parameter β_i .

$$u_i\left(c_{i,t}^y, c_{i,t+1}^o, b_{i,t+1}\right) = \frac{(c_{i,t}^y)^{1-\eta} + \mu(c_{i,t+1}^o)^{1-\eta} + \beta_i \left(b_{i,t+1}(1-\tau_B)\right)^{1-\eta}}{1-\eta}$$
(1)

Assuming heterogeneous preferences for bequests is in line with the empirical literature (Hurd, 1989; Kopczuk and Lupton, 2007; Ameriks et al., 2011). Similar to our approach, Farhi and Werning (2013) base their theoretical analysis of optimal estate taxation on the assumption of heterogeneity in preferences for leaving bequests. Allowing for additional heterogeneity in μ does not change our results and would not yield additional insights.

⁶Assuming perfect transmission of preferences can potentially exaggerate the importance of preference heterogeneity in the steady-state wealth-distribution. However, De La Croix and Michel (2002) and Black et al. (2015), for example, provide evidence suggesting that our simplifying assumption is justified as a first-order approximation.

We assume that $\mu, \beta_i \in (0,1)$. Households budget equations are.

$$c_{i,t}^{y} + s_{i,t} = w_t + b_{i,t}(1 - \tau_B) + g_t \tag{2}$$

$$s_{i,t} = k_{i,t+1}^s + p_t l_{i,t+1} \tag{3}$$

$$c_{i,t+1}^{o} + b_{i,t+1} = (1 - \delta_K + R_{t+1}(1 - \tau_K))k_{i,t+1}^{s} + l_{i,t+1}(p_{t+1} + q_{t+1}(1 - \tau_L))$$
(4)

A young household (i, t) earns wages w_t , receives bequests from the currently old generation $b_{i,t}$, pays taxes τ_B on the bequests and receives the governments tax revenues as a transfer g_t . Savings $s_{i,t}$ are invested in capital $k_{i,t+1}^s$ or land $l_{i,t+1}$, which are assumed to be productive in the next period and may be taxed at rates τ_K and τ_L , respectively. We assume fixed labor supply. An extension to include endogenous labor supply would easily be possible. However, further numerical experiments showed that the results we obtain are independent of whether labor supply is fixed or endogenous (not shown).⁷ Thus, we abstract from a labor-leisure choice here, to keep the analysis as tractable as possible.

Note that we do not explicitly model social security. All savings for retirement are captured in $s_{i,t}$. The government transfer g_t can be interpreted as public spending that benefits the young working generation. In Section 5 we show that our results do not change qualitatively when the government gives a certain fraction of transfers (or all) to the old generation.

Capital is the numeraire good and depreciates with rate δ_K . Land has the price p. When old, households receive the return on their investments according to the interest rate R_{t+1} , the price of land p_{t+1} , and the land rent q_{t+1} . We define household wealth $v_{i,t}$ as the sum of the values of the stocks of capital and land and the returns to investments in these stocks. Old households consume or leave bequests, which is expressed in (4).

The first-order conditions of the households' optimizations are given by the budget

⁷Consistent with our numerical observations, in a simplified analytical model version with labor l and heterogeneous skills, but without land, one can see that changes of bequest- and capital income taxation entail no interaction effects with changes in labor taxes τ_w . The model follows (Acemoglu, 2008, Ch. 9). Consider, for example, logarithmic utility $u_{i,t} = \log(c_{i,t}) + \beta_i \log\left(b_i(1-\tau_B)\right) + \gamma \log(1-l_{i,t})$ derived from consumption, leaving bequests and leisure (1-l). Income is $y_{i,t} = w_{i,t}l_{i,t}(1-\tau_w)+(1+R_t(1-\tau_K))\,b_{i,t-1}(1-\tau_B)$; all savings are bequeathed and invested in capital, $k_{t+1} = \frac{1}{N}\sum_i b_{i,t}$. Assuming an interior solution, the households' optimum is determined by $\frac{w_{i,t}(1-\tau_w)}{y_{i,t}} + \frac{\frac{dR}{dl_{i,t}}(1-\tau_K)(1-\tau_B)b_{i,t-1}}{y_{i,t}} = \frac{\gamma}{1+\beta_i}\frac{1}{1-l_{i,t}}$. Marginal utility of income and of bequeathing are balanced with marginal utility of leisure. The first term on the left captures effects of labor taxation, the second effects of bequest- and capital income taxation. There is no interaction term that would depend on both types of taxation.

equations (2) - (4) and

$$(c_{i,t+1}^o)^{\eta} = \mu(1 - \delta_K + R_{t+1}(1 - \tau_K))(c_{i,t}^y)^{\eta}$$
(5)

$$\beta_i (1 - \tau_B)^{1-\eta} (c_{i,t+1}^o)^{\eta} = \mu b_{i,t+1}^{\eta} \tag{6}$$

$$\frac{p_{t+1} + q_{t+1}(1 - \tau_L)}{p_t} = 1 - \delta_K + R_{t+1}(1 - \tau_K). \tag{7}$$

The conditions (5) and (6) relate marginal utility of consumption when young and old and leaving bequests quite intuitively. The no-arbitrage condition (7) ensures that households invest in capital and land in such a way that the returns are equalized across the two assets. The no-arbitrage condition is useful to understand how a portfolio effect can arise: An increase in the land rent tax, for example, shifts savings towards capital. Increasing τ_L decreases the value of the expression on the left-hand side. For the right-hand side to decrease as well, the return to capital R_{t+1} has to fall. Assuming decreasing marginal productivity, the capital stock, thus, has to increase.

The representative firm produces one type of final good using capital k, land l, and labor, where the latter two are assumed to be fixed factors. We assume that the production function has constant elasticity of substitution. In intensive form it is defined as

$$f(k_t) = A_0 \left[\alpha (A_k k_t)^{\sigma} + \gamma l^{\sigma} + 1 - \alpha - \gamma \right]^{\frac{1}{\sigma}},$$

where A_0 is total factor productivity, A_k is capital productivity, and $\sigma = \frac{\epsilon - 1}{\epsilon}$ is determined by the elasticity of substitution ϵ . The total stock of capital k_t that the firm uses in production in period t equals the aggregate of capital $k_{i,t}^s$ that is supplied by households in period t. Thus, clearing of the factor markets is given by

$$k_t = \frac{1}{N} \sum_{i=1}^{N} k_{i,t}^s$$
 and $l = \frac{1}{N} \sum_{i=1}^{N} l_{i,t}$.

In each period the firm maximizes its profit, which we assume to be zero due to perfect competition. Thus, the first-order conditions are

$$f_k(k_t) = R_t$$
 and $f_l(k_t) = q_t$,

and wages are given by $w_t = f(k_t) - R_t k_t - q_t l$.

The government levies taxes on capital income τ_K , land rents τ_L , or bequests τ_B . Public revenues $g_t = \tau_K R_t k_t + \tau_L q_t l + \frac{1}{N} \sum_i \tau_B b_{i,t}$ are recycled as transfers to the young generation on an equal per capita basis.

2.2. Calibration

The heterogeneity of household preferences and the introduction of land as an additional factor of production yield complex results, which go beyond that which is analytically tractable. Since we cannot obtain closed form solutions, we solve the model numerically using the optimization framework GAMS (Brooke et al., 2005).⁸ We calibrate the model such that in the steady state it reproduces empirically observed data as well as possible. We use average OECD data for (i) GDP and (ii) household wealth (OECD, 2015) and (iii) the average OECD level of aggregate capital and (iv) the ratio of values of capital and land (OECD, 2016, Dataset 9B).

More precisely, we formulate the calibration as non-linear optimization problem. The objective is to minimize \mathcal{Z} , the quadratic percentage error that the model makes relative to the OECD data in terms of the steady state values of GDP f^* , household wealth ν_i^* , i=1,...,5, aggregate capital k^* and the capital-land ratio $\frac{k^*}{p^*l^*}$. The control variables are the parameters of production technology $\mathcal{A}=(\alpha,\gamma,\epsilon,A_0,A_k)$, household behavior $\mathcal{B}=(\beta_i,\mu,\eta)$, and the initial endowments $\mathcal{C}=(k_0,l_0)$. Hence, the problem is

$$\min_{\mathcal{A},\mathcal{B},\mathcal{C}} \mathcal{Z} := \left(\frac{f^* - f^{\text{OECD}}}{f^{\text{OECD}}}\right)^2 + \sum_i \left(\frac{\nu_i^* - \nu_i^{\text{OECD}}}{\nu_i^{\text{OECD}}}\right)^2 + \left(\frac{k^* - k^{\text{OECD}}}{k^{\text{OECD}}}\right)^2 + \left(\frac{\frac{k^*}{p^*l^*} - \frac{k^{\text{OECD}}}{p^{\text{OECD}}l^{\text{OECD}}}}{\frac{k^{\text{OECD}}}{p^{\text{OECD}}l^{\text{OECD}}}}\right)^2$$

subject to the model equations specified in Section 2.1.

The values that we find for the parameters are summarized in Table A.1 in Appendix A. We present a comparison of the data with the model output in Table 1, which shows that our model matches the data quite well, albeit not perfectly.

⁸For the model code and data visualization scripts, see the electronic supplementary material.

GDP per capita	1,121,631 US\$ per generation	1,112,778 US\$ per generation
Gini coefficient	0.75	0.73
Capital	75,462 US\$	76,000 US\$
Capital-land ratio	2.53	2.53
Wealth holdings of the five quintiles		
Q1	$2,\!356$ US\$	15,082 US\$
Q2	48,790 US\$	49,006 US\$
Q3	136,132 US\$	136,048 US\$
Q4	262,057 US\$	262,180 US\$
Q5	922,703 US\$	925,231 US\$

Table 1: Comparison of average OECD data and model output. Data taken from OECD (2015) and OECD (2016), currency in 2005 US\$ per capita, one generation equals 30 years.

3. Social welfare criterion

Assuming households have the same preference, aggregation with an appropriately weighted utilitarian social welfare function is easily done. However, with heterogeneous preferences, it is not straight forward to define a social welfare criterion. To determine how two given policy scenarios differ in terms of social welfare, we use, amongst others, the method of equivalent incomes (see for example Fleurbaey, 2009) and we apply a prioritarian social welfare function (Adler, 2012).

Let utility function u_i descibe the preferences of household i. Let (u_0, y_0, z_0) be utility level, income level and prices that household i faces in some initial policy scenario, say a situation of laissez faire where all taxes are set to zero. This will be the reference scenario. In the reference scenario, the household chooses the bundle (c_i^y, c_i^o, b_i) . Hence, $u_0 = u_i(c_i^y, c_i^o, b_i)$. Let $(\tilde{u}, \tilde{y}, \tilde{z})$ be utility, income and prices after a policy reform. In the policy reform scenario, the household consumes the bundle $(\tilde{c}^y, \tilde{c}^o, \tilde{b})$. Then, we can define the equivalent income y_i^* as the amount of income household i needs to reach the utility level $\tilde{u} = u_i(\tilde{c}^y, \tilde{c}^o, \tilde{b})$ but when facing reference prices z_0 . More formally, let v be the indirect utility function that maps income and prices to the according utility level. Then, it holds that

$$v(y_0, z_0) = u_0 = u(c^y, c^o, b)$$
$$v(\tilde{y}, \tilde{z}) = \tilde{u} = u(\tilde{c}^y, \tilde{c}^o, \tilde{b})$$
$$v(y^*, z_0) = \tilde{u}$$

In fact, for the model described in Section 2, the indirect utility function v can be calculated easily (see Appendix D). When fixing prices to any vector of (positive) reference prices \bar{z} , then $v(.,\bar{z})$ is a strictly increasing function of income, thus it's invertible. Consider the map

$$(v(.,z_0))^{-1} =: \phi_{z_0} : \text{utility} \longmapsto \text{income}$$

A household's equivalent income with reference prices from the laissez faire scenario is then given by

$$y^* = \phi_{z_0}(\tilde{u}).$$

Construction of a welfare function

We now describe the construction of the social welfare function to be applied to the problem of finding optimal taxes in the model. Within one generation, the equivalent incomes of all households could easily be added up. If a policy reform causes the sum of equivalent incomes to increase, the winners of the reform could in principle compensate the losers to "buy them in". The reform would – within that generation – pass the Kaldor-Hickstest. Across generations, equivalent incomes are weighted according to the benevolent government's preferences, i.e. the pure rate of time preference. As benchmark case, we'll assume that it's zero. We relax this assumption in Section 5, where we show how our results depend on the government's pure rate of time preference.

Following a more general approach than the Kaldor-Hicks-test, we aggregate the equivalent incomes to result in social welfare \mathcal{W} that gives some form of priority to those households with lower equivalent incomes.

$$W = \sum_{i,t} g(y_{i,t}^*) \tag{8}$$

where g is a weakly concave function. As benchmark case, we will assume that $g(y) = (y/\overline{y})^{\frac{1}{2}}$, where \overline{y} is the income of the median household in the steady state of the reference scenario.⁹ In the reference scenario equivalent income is by definition actual income. Since the main objective of this paper is to show how the optimal tax depends on normative decisions like, for example, the specific form of the function g, we will vary its parametrization in the following. The function \mathcal{W} as defined here in (8) is referred to in the literature as prioritarian social welfare function (Adler, 2012). It prioritizes households that are worse-off in terms of their equivalent income. Giving a poor household one additional dollar increases welfare by more than giving the dollar to a rich household. Expressed mathematically, it holds that

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}y_1^*} \ge \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}y_2^*} \iff y_1^* \le y_2^*,$$

which is also illustrated in Figure 1.

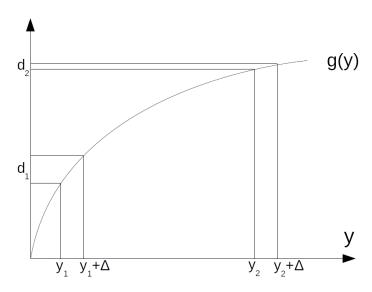


Figure 1: Illustration of the function g used in the definition of the prioritarian social welfare function (8). Increasing equivalent income of the poor household i = 1 by a small amount Δ yields a higher increase in social welfare d_1 than the same income increase for the richer household i = 2.

Social welfare functions based on equivalent incomes have been criticized for their de-

 $^{{}^{9}}$ We use \overline{y} to scale the equivalent income such that the order of magnitude of the possible arguments of the square root function are close to unity. Thereby, the social welfare function will exhibit more meaningful differences between the marginal contribution of individual households' equivalent income to welfare.

pendence on the reference price vector (Slesnick, 1991; Roberts, 1980). On a theoretical level, Fleurbaey (2009) defends the equivalent incomes approach in this regard and argues that reference prices need not be based on an arbitrary choice. On a more practical level, we have conducted a robustness analysis and compared reference prices derived from several policy scenarios. None of the prices taken from those scenarios changed our results based on the reference prices from the laissez-faire equilibrium. Moreover, in Appendix C, we also discuss implications of an alternative social welfare measure that also gives priority to the poor, but is based on the compensating variation – and, hence, does not have one single reference price. It turns out that our results are robust with respect to this alternative specification

4. Policy implications of different normative standpoints

In this section, we discuss how the optimal taxation of wealth varies with the underlying concept of social welfare and the normative standpoint of a benevolent government. When we define social welfare as aggregation of individuals' money metric utility, we want to be able to vary the degree to which the weakly concave function g prioritizes the worse-off. To meet this requirement, a conveniant functional form is, for example, that $g(y) = y^{\alpha}$ and that $\alpha \in (0,1)$. Then a lower value of the exponent α implies that the worse-off are given higher priority than the better off as described in the previous section. The limiting case in which $\alpha = 1$ would correspond to the well known Kaldor-Hicks criterion. Moving along the spectrum from the latter to the former gives increasing weight to the worse off. We discuss this variation in Section 4.1.

However, varying the curvature of g changes nothing about the implied moral standpoint that individuals' different preferences for leaving bequests should not be a reason for discrimination by the government. By contrast, bequeathing could be considered socially undesirable because it creates inequality among individuals who cannot change anything about it. We discuss this case in Section 4.2. Then, different preferences (which we model as the additive term in utility with coefficient β_i) should be a reason for discrimination. To express this moral standpoint, social welfare can be defined as the sum of all individuals' utility, but without the additive term for bequests. To illustrate this point, consider two individuals with the same income but different preferences for leaving bequests, $\beta_1 < \beta_2$. Both will divide their income differently between own consumption and leaving bequests. Due to aggregation via summation, the two individuals' contributions to social welfare are perfect substitutes. But since individual 1 is more "efficient" in transforming income to social welfare, the optimal policy will favor individual 1 over individual 2, expressing the standpoint that bequeathing is undesirable.

4.1. Variation of how much priority to give to the worse off

Our first result is shown in Figures 2 and 3. We compare the outcomes for different rates of land rent-, bequest- and capital income taxes. We implement only one tax at a time. When using the prioritarian social welfare function, increasing the land rent tax rate above zero has a positive effect. The optimal land rent tax rate at which social welfare peaks is $\tau_L =$ 0.8. Taxes on capital income and bequests, on the other hand, reduce welfare substantially. All three taxes reduce wealth inequality in the steady state, as measured by the Gini coefficient. Since the bequest tax targets the only source of heterogeneity in our model, it reduces the Gini coefficient much more than the other two taxes. Capital taxes reduce steady state per capita GDP substantially, while land rent taxes actually increase GDP. Bequest taxation has a negative effect on GDP but is less detrimental than capital taxes. The impact of capital income and land rent taxes are due to the portfolio effects they induce (on portfolio effects, see for example Feldstein, 1977; Edenhofer et al., 2015; Franks et al., 2018). Households react to taxes on one of the two assets by shifting their savings toward the other. Land is fixed in supply, capital is not. A tax on land thus increases the capital stock and therefore GDP, while a tax on capital income reduces the capital stock but does not increase the amount of available land. For a more detailed descriptive discussion of the macroeconomic impacts of capital income, land rent and bequest taxes, see Franks et al. (2018).

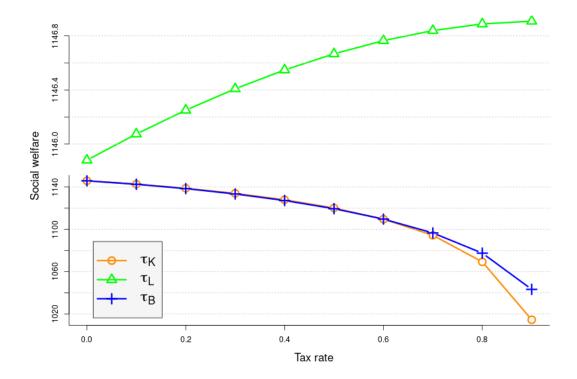


Figure 2: Social welfare based on equivalent incomes and a prioritarian social welfare function under variation of the tax rate for one single tax instrument. Compared to the no-tax case, the land rent tax improves welfare slightly. Taxes on capital income and bequests reduce social welfare substantially compared to the no-tax case. Note that for better exposition we split the y-axis and show different scales for positive and negative values.

Social welfare, steady state GDP and steady state wealth distribution

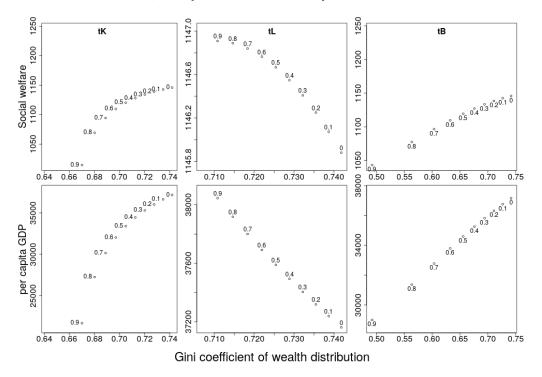


Figure 3: Impact of taxes on social welfare, steady state GDP and the distribution of wealth in the steady state. Tax rates are shown as labels next to each data point. Per capita GDP is measured in 2005 US\$ per annum.

The results presented in Figures 2 and 3 are based on the normative standpoint that some form of priority should be given to the poor. In particular, for the simulations we have chosen $g(y) = (y/\overline{y})^{\frac{1}{2}}$, where \overline{y} is the income of the median household in the steady state of the reference scenario. Choosing a different functional form for g yields substantially different results. In Figure 4 we illustrate this by varying the exponent of the concave function g.

If social welfare is simply defined as the sum of equivalent incomes, it represents the Kaldor-Hicks criterion. If a government applies the Kaldor-Hicks criterion, any increase of the land rent tax reduces welfare. The status quo without any taxation is the best outcome that can be achieved. When, instead, we give priority to worse off individuals by choosing a concave functional form for g, the picture changes. Now, the burden placed on the rich by an increase of the land rent tax is overcompensated by the benefits to the poorer wealth

groups and social welfare increases with the level of the land rent tax. Capital income and bequest taxes remain undesirable for all functions we have applied.

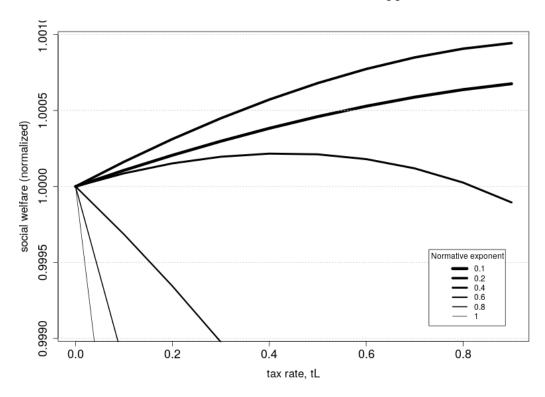


Figure 4: Welfare effects of land rent tax reforms under different degrees to which priority is given to the poor. The normative standpoint has strong implications for the optimal policy.

It is well known in the literature that OLGs with land are dynamically efficient (Rhee, 1991). Hence, the analysis of the Kaldor-Hicks criterion here confirms that the government cannot use taxes to achieve a Pareto improvement. Using the prioritarian social welfare function, hence, implies choosing a different point on the Pareto frontier.

4.2. When bequeathing is socially undesirable

In contrast to the moral standpoint on which the preceding section was based, there could be a social consensus that leaving bequests is undesirable in general. Such a consensus could be based on the conviction that the practice of bequeathing generates wealth inequality among individuals who have no responsibility for that inequality and no way of influencing it. Then, we can define social welfare as the sum of individuals' utility without the additive term representing the preference for leaving bequests:

$$W = \sum_{i,t} \frac{(c_{i,t}^y)^{1-\eta} + \mu(c_{i,t+1}^o)^{1-\eta}}{1-\eta}$$
(9)

For the case that no further welfare weights are applied, Figure 5 shows that both land rent and bequest taxation become slightly more desirable than in the case of non-discrimination between different preferences for leaving bequests. The optimal land rent tax increases from 80% to a confiscatory rate; the bequest tax becomes less harmful than the capital income tax.

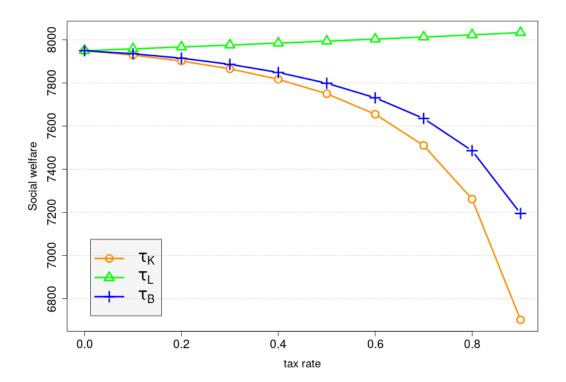


Figure 5: Welfare when discrimination based on preferences for leaving bequests is socially desired and the warm glow of bequests is disregarded.

We can introduce inequality aversion ψ to the utilitarian social welfare function (9) by writing

$$W = \frac{1}{1 - \psi} \sum_{i,t} \left[\frac{(c_{i,t}^y)^{1-\eta} + \mu(c_{i,t+1}^o)^{1-\eta}}{1 - \eta} \right]^{1-\psi}$$
(9')

When the inequality aversion parameter ψ is equal to zero, the standard utilitarian version of the function prevails. With increasing ψ the government becomes more averse to inequality between different households $\{(i,t)\}_{i\in\{1,\dots,N\},t\in\{1,\dots,T\}}$, that is, both within and across generations. While results change quantitatively, the ordering of taxes according to their effect on social welfare remains the same, as Figure 6 shows.

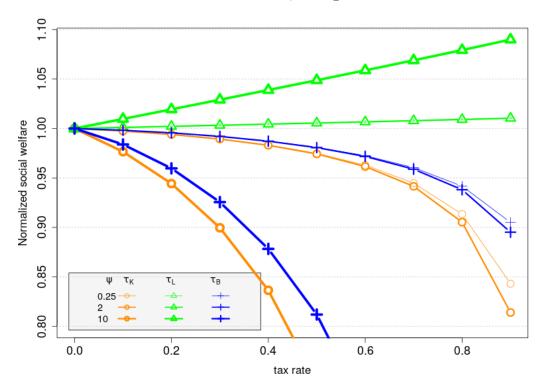


Figure 6: Welfare when discrimination based on preferences for leaving bequests is socially desired and the warm glow of bequests is disregarded. Variation of inequality aversion parameter $\psi \in \{0.25, 2, 10\}$. Note that normalized welfare under τ_L is almost identical for $\psi = 0.25$ and $\psi = 2$.

Let us conclude this section by summarizing what we have learned. Which policy is optimal strongly depends on the normative viewpoint on which the definition of social welfare is based. Based on the Kaldor-Hicks criterion, zero taxation is socially optimal – reflecting a status quo bias. If the poor are given higher priority, high land rent taxes are optimal. The beneficial impact of land rent taxation on GDP due to the portfolio effect is strong enough to outweigh the losses of the first old generation, which is always hurt by any form of taxation, and the rich households. Finally, if a strong preference for bequests is seen as unethical, social welfare is defined as the sum of individuals' utilities without the additive component of the warm glow. Then, the optimal policy is qualitatively similar to the case

of the approach using the prioritarian social welfare function. While the optimal land rent tax rate reaches confiscatory rates, optimal capital income and bequest taxes still remain at zero. Nevertheless, there are cases in which optimal capital income and bequest tax rates turn positive. We discuss these in the next section, in which we assess the robustness of our main results.

5. Robustness

Robustness analysis shows that plausible cases exist in which not only the land rent tax is socially optimal, but also positive rates of capital income and bequest taxes. In particular, the intertemporal elasticity of substitution $\frac{1}{\eta}$ is a crucial behavioral parameter that determines how households react to policy reforms. Moreover, we have performed an extensive one-at-a-time variation of all other descriptive model parameters, but find that our qualitative results are generally quite robust with respect these variations (see supplementary material). Normative parameters, that is, parameters that correspond to different moral viewpoints as to what is socially desirable, in contrast, have a stronger impact on optimal tax rates. Therefore, we also report how varying the government's pure rate of time preference changes our results (Section 5.2). Finally, in Section 5.3, we discuss alternative assumptions about tax revenue recycling.

5.1. Intertemporal elasticity of substitution

For all three taxes on wealth that we consider, there exists a threshold for the parameter η , the inverse of the intertemporal elasticity of substitution. Below the threshold it is socially optimal to remain in the laissez-faire equilibrium. Above the threshold, a certain positive tax rate is optimal. The reason is the dependence of households' savings behavior on η . Franks et al. (2018) showed that households react differently to taxation for different values of η . The higher (lower) the parameter is, the more households react to wealth taxation by increasing (reducing) their savings, which positively (negatively) affects capital accumulation and thus output.

Different threshold values are associated with different tax instruments and the thresholds also depend on which normative viewpoint is chosen. We give an overview over the thresholds in Table 2. We observe two patterns with respect to the conditions under which

taxation is socially desirable. First, comparing the rows in the table, applying the Kaldor-Hicks criterion is the most restrictive case and the welfare criterion based on the conviction that bequeathing is unethical is the most permissive. Second, the range of η for which positive tax rates are optimal is largest for the land rent tax, smallest for the capital income tax with the range for the bequest tax in the middle.

	$ au_K$	$ au_L$	$ au_B$
Kaldor-Hicks	N/A	1.1	N/A
Prioritarian	N/A	0.45	1.6
Bequeathing unethical			

Table 2: Threshold value of the elasticity parameter η , above which a positive tax rate is optimal and below which the optimal tax rate is zero. Note that feasible solutions for single tax rate experiments exist only up to $\eta = 2.5$. Hence, we cannot assign a definite value for capital income and bequest taxation when the Kaldor-Hicks criterion is applied, nor for capital income taxation when using a prioritarian social welfare function.

According to a meta analysis by Havranek et al. (2015), the median parameter value for the OECD countries to which our model is calibrated is 2.2. However, our model's best fit to the OECD data is achieved for a value of $\eta = 0.01$. The approximation error rises exponentially with increasing η values, where the average error stays below 10% as long as $\eta \leq 1.2$ (see appendix, Figure A.1). Therefore, as benchmark value, we have chosen $\eta = 0.6$ and show here, how the parameter choice affects outcomes.

Note further that the existence of thresholds is consistent with the properties of the optimal bequest tax rate determined by Piketty and Saez (2013), who find that the lower e_B (the elasticity of aggregate bequest flow with respect to the net-of-bequest-tax rate $1-\tau_B$) is, the higher the optimal bequest tax is. In our model, η has a strong impact on that elasticity. The higher η , the lower the elasticity and the higher the optimal bequest and capital income tax (see Table B.2 in the appendix).

5.2. Normative assumptions on discounting

A further normative parameter determines the shape of the social welfare function. In this section, we discuss how variations of an annual pure rate of time preference ζ change optimal policy.

$$\mathcal{W}_{\zeta} = \sum_{i,t} g(y_{i,t}^*)/(1+\zeta)^{\Delta} \tag{8'}$$

where Δ is the duration of one generation in years.

The above derived properties of bequest- and capital income taxation do not change qualitatively under variations of the government's pure rate of time preference. Thus, we concentrate on land rent taxation. In Figure 7 we show the impact of the latter two on social welfare (8') when we vary ζ . The figure shows that land rent taxation becomes more beneficial to social welfare, the more the government places equal weights on all generations. Intuitively, future generations are the main beneficiaries of the growth enhancing portfolio effect of the land rent tax.

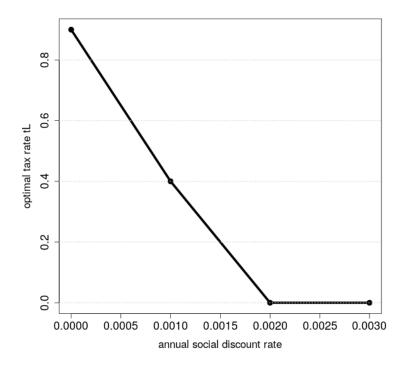


Figure 7: Optimal land rent tax rates as a function of the government's pure rate of time preference. Below a rate of 0.2%, positive land rent tax rates are desirable.

5.3. Revenue recycling

To derive our main results, we had assumed that the government recycles all tax revenue as transfer exclusively to the young generation – see equation (2). Here, we briefly show the impact giving part or all of the revenues to the old generation. In particular, we introduce a parameter $\delta \in [0,1]$ that determines the fraction of the revenues that are given instead to the old generation. Households now face budget equations

$$c_{i,t}^{y} + s_{i,t} = w_t + b_{i,t}(1 - \tau_B) + (1 - \delta)g_t$$

$$c_{i,t+1}^{o} + b_{i,t+1} = (1 - \delta_K + R_{t+1}(1 - \tau_K))k_{i,t+1}^{s} + l_{i,t+1}(p_{t+1} + q_{t+1}(1 - \tau_L)) + \delta g_t$$

If the Kaldor-Hicks criterion is used, a variation of δ , the fraction of tax revenue given to the old, does not change our results. No form of taxation can improve welfare over the laissez-faire equilibrium. However, it has some impact on the desirability of land rent taxation if we use a prioritarian social welfare function with based on equivalent incomes (Figure 8). Reducing the transfer to the young and instead giving more to the old reduces social welfare. Still, any non-negative land rent tax rate is desirable from a social welfare perspective, even if all tax revenue is transfered to the old generation, $\delta = 1$. Why does social welfare decrease when more tax revenues are given to the old generation? Two related effects cause households to save less in reaction to an increase of the land rent tax: First, giving more transfers to the old reduces the households' incentive to save for retirement – the "missing incentive" effect. This effect overcompensates the additional bequests that the old make due to their increase in income. Second, reducing transfers to the young reduces their income and hence the amount of savings – the "missing income" effect. ¹⁰

 $^{^{10}}$ For a detailed discussion, see also Franks et al. (2018).

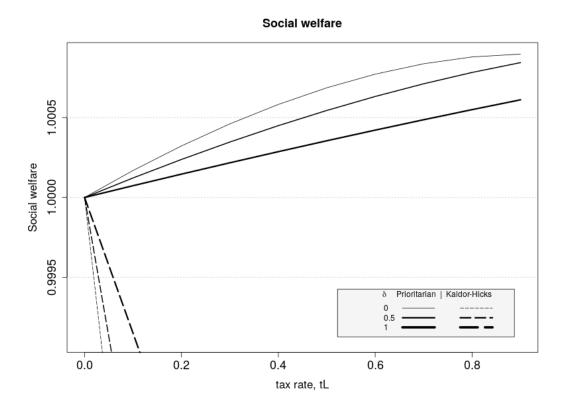


Figure 8: Social welfare as a function of the land rent tax rate. Different line widths are used to show a variation of the fraction of tax revenue recycled to the old generation. Solid lines show results for the prioritarian social welfare function, dashed lines for the Kaldor-Hicks criterion.

6. Conclusion

We have shown that taxation of land rents is not neutral in general due to the portfolio effect it induces and instead may enhance welfare. In particular, when social welfare is measured in terms of equivalent income and a prioritarian function, high levels of land rent taxation are socially optimal for a broad range of assumptions. Similarly, high land rent taxation is optimal if in the aggregation of individuals' utility the warm glow of leaving bequests is disregarded. This may, for example, be motivated by the conviction that the practice of inheritances is undesirable because it inhibits equality of opportunity for a generation that is neither responsible for, nor has the ability to change that inequality. Application of the Kaldor-Hicks criterion leads to a dictatorship of the status quo of market outcomes under most assumptions. Then, the optimal tax on any form of income from wealth is

zero. However, these results are sensitive to the choice of the intertemporal elasticity of substitution. For all three tax instruments, the capital income tax, the bequest tax and the land rent tax, it holds that the lower the elasticity, the more likely it is that a positive tax rate is optimal. Except for the case of bequest and capital income taxation under the Kaldor-Hicks criterion and capital income taxation for the prioritarian welfare function, the thresholds we find for the elasticity are well in the range of empirically plausible values.

Policy makers might object against taxation of land rents because it could disproportionately hurt middle class house owners. We leave such questions of horizontal equity for future research. Taking into account heterogeneity in asset portfolios across different households, for example, lies beyond the scope of the present paper. However, horizontal equity poses a fundamental political problem since the typical middle class household owns a house, which constitutes the largest part of that household's wealth. These households typically have incurred debt to purchase the house. Therefore, they would be hit overproportionately hard by a reform introducing higher land rent taxation, compared to households who hold the same level of wealth but have a different portfolio composition. Even though we have established a further argument in favor of land rent taxation beyond what was known so far, political problems remain. The political economy of compensation for the losers of a land rent tax still needs to be explored.

Nevertheless, we believe that by clarifying what role different normative viewpoints play for the optimal taxation of wealth, we can help to structure the debate about policy recommendations on the taxation of wealth.

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A. Model parameters and calibration error

Preferences	Elasticity parameter	η	0.6
	Preferences for consumption when old	μ_1	0.04
	Preferences for leaving bequests	β_1	0.0001
		β_2	0.06
		β_3	0.12
		β_4	0.17
		β_5	0.30
Production	Share parameter of capital	α	0.3
	Share parameter of land	γ	0.2
	Elasticity of substitution	ϵ	0.65
	Total factor productivity	A_0	543
	Capital productivity	A_K	0.04
	Depreciation rate	δ_k	1
Tax rates	Capital income tax	$ au_K$	0.2
	Land rent tax	$ au_L$	0
	Bequest tax	$ au_B$	0
Other	Initial capital	k_0	120,000 US\$ per capita
	Initial land	l_0	31 land units per capita

Table A.1: Benchmark parameters that reproduce observed data on the wealth distribution in OECD countries.

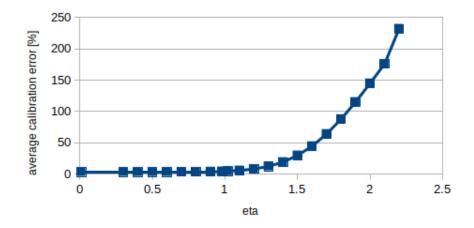


Figure A.1: Calibration error with varying η .

B. Additional material

τ_B	$\eta = 0.6$	$\eta = 1.6$	$\eta = 2.0$	$\eta = 2.5$
0.1	0.10744071	0.002585640	-0.015070162	-0.01898100
0.3	0.09896329	-0.017147047	-0.057077372	-0.13236813
0.5	0.10182980	-0.039703745	-0.103566255	-0.22521097
0.7	0.12389140	-0.071694641	-0.162940388	-0.29139123
0.9	0.21354733	-0.143245053	-0.261900275	-0.36193276

Table B.2: Elasticity of aggregate bequest flow with respect to the net-of-bequest-tax rate $1 - \tau_B$ for different values of η and τ_B . Numbers reported here are median values over the entire time horizon. The numbers do not change qualitatively when looking at individual time slices, maximum and minimum values.

C. Policy implications of a social welfare function based on the compensating variation

C.1. Definitions

Let (u, y, z) be utility, income and prices that a household faces in the initial policy scenario. Let $(\tilde{u}, \tilde{y}, \tilde{z})$ be utility, income and prices after a policy reform. Then, we define the willingness-to-accept that reform (WTA) as the transfer that this household has to be compensated with after the reform, such that he or she is indifferent between the initial

policy scenario and the proposed reform. In the literature, our concept of WTA is also referred to as compensating variation. If WTA is positive, the household is worse off after the reform. If WTA is negative, the reform is desirable for that household. More formally, let v be the indirect utility function that maps income and prices to the according utility level. Then, it holds that

$$v(y, z) = u$$
$$v(\tilde{y}, \tilde{z}) = \tilde{u}$$
$$v(WTA + \tilde{y}, \tilde{z}) = u$$

In fact, for the model described in Section 2, the indirect utility function v can be calculated easily (Sections D and E). When fixing prices to some value z_0 , then $v(., z_0)$ is a strictly increasing function of income, thus it's invertible. Consider the map

$$(v(.,z_0))^{-1} =: \phi_{z_0} : \text{utility} \longmapsto \text{income}$$

A household's willingness to accept the policy reform is then given by

$$WTA = \phi_{\tilde{z}}(u) - \tilde{y}.$$

Construction of a welfare function

We now describe the construction of the social welfare function to be applied to the problem of finding optimal taxes in the model. Within one generation, WTAs of all households could easily be added up. If a policy reform is such that that sum is negative the winners of the reform could in principle compensate the losers to "buy them in". The reform would – within that generation – pass the Kaldor-Hicks-test. Across generations, WTAs are weighted according to the benevolent government's preferences, i.e. the pure rate of time preference.

Following a more general approach than the Kaldor-Hicks-test, we aggregate the WTAs

to result in social welfare \mathcal{W} based on appropriate welfare weights:

$$W = \sum_{i,t} -WTA_{i,t}\rho_{i,t} \tag{10}$$

We choose the weights ρ such that they guarantee a welfare function that gives some form of priority to the poor. A common approach to implementing that property is to base the weights on a households marginal utility of income – when there is no preference heterogeneity – or even more simple, to base the weights on the inverse of income (proportional to marginal utility of income when households' utility is assumed to be the logarithm of income). In our situation, preference are heterogeneous. This leads to the question whether different preferences for leaving bequests β_i should be a reason for discrimination. If, for example, one chooses the moral standpoint that different preferences should *not* be a reason for discrimination, then the social welfare criterion should disregard differences between the β_i . This view gives rise to a representative utility function \bar{u} that determines to how much utility society thinks a household is entitled – given a certain income level and certain prices. Alternatively, one could also rationalize such weights as based on a democratic vote among citizens, who determine the \bar{u} .

$$\bar{u}(c_{i,t}^{y}, c_{i,t+1}^{o}, b_{i,t+1}) = \frac{(c_{i,t}^{y})^{1-\eta} + \mu(c_{i,t+1}^{o})^{1-\eta} + \hat{\beta}b_{i,t+1}^{1-\eta}}{1-\eta}.$$
(11)

Using the households first order conditions (2) - (7), we show in the appendix (Section E) that the representative indirect utility function is given by

$$\bar{v}(y_{i,t}, z_{i,t}) = y_{i,t}^{1-\eta} \frac{\alpha_{i,t}^{\eta}}{1-\eta},$$
where $\alpha_{i,t} = 1 + \mu^{\frac{1}{\eta}} r_{t+1}^{\frac{1-\eta}{\eta}} + \hat{\beta}^{\frac{1}{\eta}} [r_{t+1}(1-\tau_B)]^{\frac{1-\eta}{\eta}}$
and $r_{t+1} = 1 - \delta_K + R_{t+1}(1-\tau_K).$

In this case, we can follow the common approach and define weights to be the marginal representative utility of income – a measure of how valuable society thinks that income

should be to household (i, t):

$$\rho_{i,t} := \frac{\mathrm{d}\bar{v}}{\mathrm{d}y}|_{y = \widetilde{y_{i,t}}} \tag{12}$$

Willingness to accept vs. willingness to pay

Here, we use the concept of willingness to accept a policy change (WTA) instead of the equivalent variation or willingness to pay for a policy change (WTP). The most important difference in terms of the economic intuition is that prices of the economy after the policy reform determine WTA, while WTP is dermined by prices before the reform. However, a policy reform that might seem optimal from an ex-ante point of view can turn out to be sub-optimal after the reform has been implemented, requiring further reform to achieve optimality. Such a divergence ocurs, for instance, when checking whether our results are independent of the choice of the status quo, that is, the initial policy scenario before a policy reform. It turns out, that the WTA-based welfare criterion is indeed independent, while the WTP-based measure is not.¹¹

C.2. Policy implications

In Figures C.2 and C.3, we compare the outcomes for different rates of land rent-, capital income- and bequest taxes. We implement only one tax at a time. When social welfare is defined as the weighted sum of individuals' willingness to accept the policy change, as above, increasing the land rent tax rate above zero has a positive effect. The optimal land rent tax rate at which social welfare peaks is $\tau_L = 0.8$. Taxes on capital income and bequests, on the other hand, reduce welfare substantially. All three taxes reduce wealth inequality in the steady state, as measured by the Gini coefficient. Since the bequest tax targets the only source of heterogeneity in our model, it reduces the Gini coefficient much more than the other two taxes. Capital taxes reduce steady state per capita GDP substantially, while land rent taxes actually *increase* GDP. Bequest taxation has a negative effect on GDP but is less detrimental than capital taxes. The impact of capital income and land rent taxes

The assessment, we fix some initial policy scenario, that is a vector $\tau^0 = (\tau_K^0, \tau_L^0, \tau_B^0) \in [0, 0.9]^3$, and determine the optimal policy reforms $\tau^{*,\chi} = (\tau_K^{*,\chi}, \tau_L^{*,\chi}, \tau_B^{*,\chi}), \chi \in \{\text{WTP,WTA}\}$ for both the WTP- and the WTA-based welfare criterion. Then, we use that $\tau^{*,\chi}$ as new initial policy scenario and determine the new optimal policy reform $\tau^{**,\chi}$. In our numerical calculations, we found examples for $\tau^{0,\text{WTP}}$ such that $\tau^{*,\text{WTP}} \neq \tau^{**,\text{WTP}}$, while for all $\tau^{0,\text{WTA}} \in \{0,0.1,...,0.9\}^3$, the process converges immediately and $\tau^{*,\text{WTA}} = \tau^{**,\text{WTA}}$.

are due to the portfolio effects they induce (on portfolio effects, see for example Feldstein, 1977; Edenhofer et al., 2015; Franks et al., 2018). Households react to taxes on one of the two assets by shifting their savings toward the other. Land is fixed in supply, capital is not. A tax on land thus increases the capital stock and therefore GDP, while a tax on capital income reduces the capital stock but does not increase the amount of available land. For a more detailed descriptive discussion of the macroeconomic impacts of capital income, land rent and bequest taxes, see Franks et al. (2018).

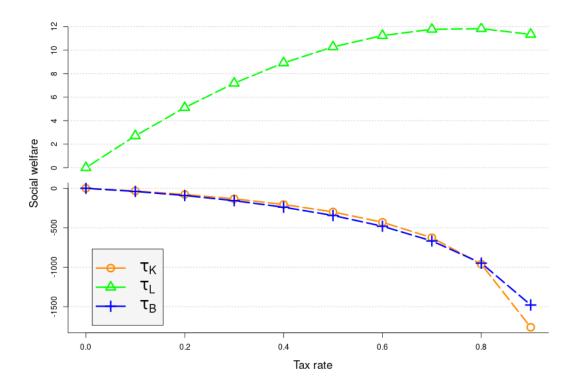


Figure C.2: Social welfare based on WTA under variation of the tax rate for one single tax instrument. Compared to the no-tax case, the land rent tax improves welfare slightly. Taxes on capital income and bequests reduce social welfare substantially compared to the no-tax case. Note that for better exposition we split the y-axis and show different scales for positive and negative values.

Social welfare, steady state GDP and steady state wealth distribution

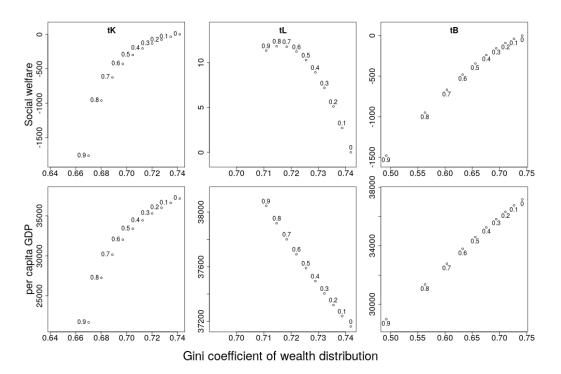


Figure C.3: Impact of taxes on social welfare, steady state GDP and the distribution of wealth in the steady state. Tax rates are shown as labels next to each data point. Per capita GDP is measured in 2005 US\$ per annum.

The results presented in Figures C.2 and C.3 are based on the normative standpoint that some form of priority should be given to the poor, i.e. on welfare weights $\rho_{i,t}$ as described by equation (12). Choosing different weights yields substantially different results. Figure C.4 illustrates a linear variation of the weights

$$\rho_{i,t} = (1 - \lambda) \frac{\sum_{i,t} \frac{d\overline{v}}{dy}|_{y = \widetilde{y_{i,t}}}}{NT} + \lambda \frac{d\overline{v}}{dy}|_{y = \widetilde{y_{i,t}}}.$$

When $\lambda \in [0, 1]$ setting $\lambda = 0$ represents the Kaldor-Hicks criterion and $\lambda = 1$ represents weights in accordance with marginal utility of income as defined in equation (12). If a government applies the Kaldor-Hicks criterion, any increase of the land rent tax reduces welfare. The status quo without any taxation is the best outcome that can be achieved. When, instead, we place different weights on the housholds according to their marginal

representative utility of income, the picture changes. Now, the burden placed on the rich by an increase of the land rent tax is overcompensated by the benefits to the poorer wealth groups and social welfare increases with the level of the land rent tax – up to the very high value of $\tau_L = 80\%$, when $\lambda = 1$. Capital income and bequest taxes remain undesirable for all λ .

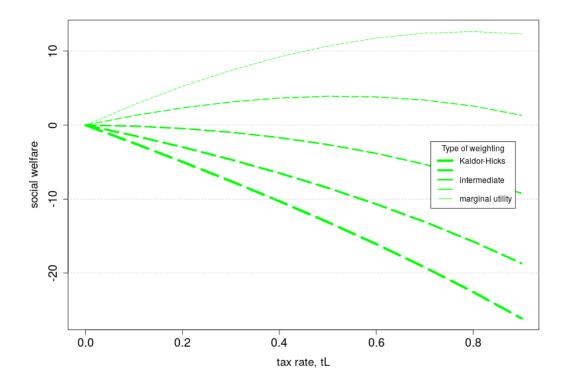


Figure C.4: Welfare effects of land rent tax reforms under different welfare weights. We vary the weights linearly, starting from the Kaldor-Hicks criterion and ending at weights based on marginal representative utility. The normative standpoint has strong implications for the optimal policy.

D. Mapping utility to income

We need a map ϕ_{scenario} : utility \longmapsto income, which depends on the policy scenario. The policy scenario determines the prices households face, e.g., the interest rate and tax rates. Note that we have to distinguish between regular generations, which live for two periods, and the first old generation, which only lives for the first period in the model.

D.1. first old generation

Budget equation

$$y^{\text{gross}} = b_{i,1} + c_{i,1}^{o}$$

Reformulation of FOC

$$\beta_{i}(c_{i,1}^{o})^{\eta}(1-\tau_{B})^{1-\eta} = \mu b_{i,1}^{\eta}$$

$$\implies c_{i,1}^{o} = b_{i,1} \left[\frac{\mu}{\beta_{i}} (1-\tau_{B})^{\eta-1} \right]$$

Reformulation of utility function to get map from u to b

$$u_{i}^{1\text{st old}} = \frac{\mu(c_{i,1}^{0})^{1-\eta} + \beta_{i} \left[(1 - \tau_{B})b_{i,1} \right]^{1-\eta}}{1 - \eta}$$

$$\Rightarrow u_{i}^{1\text{st old}}(1 - \eta) = \mu \left\{ b_{i,1} \left[\frac{\mu}{\beta_{i}} (1 - \tau_{B})^{\eta - 1} \right] \right\}^{1-\eta} + \beta_{i} \left[(1 - \tau_{B})b_{i,1} \right]^{1-\eta}$$

$$\Rightarrow u_{i}^{1\text{st old}}(1 - \eta) = b_{i,1}^{1-\eta} \left\{ \mu \left[\frac{\mu}{\beta_{i}} (1 - \tau_{B})^{\eta - 1} \right]^{1-\eta} + \beta_{i} (1 - \tau_{B})^{1-\eta} \right\}$$

$$\Rightarrow b_{i,1}^{1-\eta} = \frac{u_{i}^{1\text{st old}}(1 - \eta)}{\mu \left[\frac{\mu}{\beta_{i}} (1 - \tau_{B})^{\eta - 1} \right]^{1-\eta} + \beta_{i} (1 - \tau_{B})^{1-\eta}}$$

$$\Rightarrow b_{i,1} = \left\{ \frac{u_{i}^{1\text{st old}}(1 - \eta)}{\mu \left[\frac{\mu}{\beta_{i}} (1 - \tau_{B})^{\eta - 1} \right]^{1-\eta} + \beta_{i} (1 - \tau_{B})^{1-\eta}} \right\}$$

$$\Rightarrow b_{i,1} = \left[u_{i}^{1\text{st old}}(1 - \eta) \right]^{\frac{1}{1-\eta}} \left\{ \mu \left[\frac{\mu}{\beta_{i}} (1 - \tau_{B})^{\eta - 1} \right]^{1-\eta} + \beta_{i} (1 - \tau_{B})^{1-\eta} \right\}^{\frac{1}{1-\eta}}$$

Reformulation of budget equation to get map from u to y^{gross}

$$y^{\text{gross}} = b_{i,1} + c_{i,1}^{o} = b_{i,1} \left[1 + \left(\frac{\mu}{\beta_i} (1 - \tau_B)^{\eta - 1} \right)^{\frac{1}{\eta}} \right]$$

$$\implies y^{\text{gross}} = \left\{ \frac{u_i^{\text{1st old}} (1 - \eta)}{\mu \left[\frac{\mu}{\beta_i} (1 - \tau_B)^{\eta - 1} \right]^{1 - \eta} + \beta_i (1 - \tau_B)^{1 - \eta}} \right\}^{\frac{1}{1 - \eta}} \left[1 + \left(\frac{\mu}{\beta_i} (1 - \tau_B)^{\eta - 1} \right)^{\frac{1}{\eta}} \right]$$

D.2. regular generations

Budget equation

$$c_{i,t}^{y} + \underbrace{\frac{c_{i,t+1}^{o} + b_{i,t+1}}{1 + R_{t+1}(1 - \tau_{K}) - \delta_{K}}}_{=:r_{t+1}} = y_{i,t}$$

Reformulation of FOCs

$$c_{t+1}^{o} = c_{i,t}^{y} \left[\mu r_{t+1} \right]^{\frac{1}{\eta}}$$

$$b_{i,t+1} = c_{i,t+1}^{o} \left[\frac{\beta_{i}}{\mu} (1 - \tau_{B})^{1-\eta} \right]^{\frac{1}{\eta}}$$

$$= c_{i,t}^{y} \left[\mu r_{t+1} \right]^{\frac{1}{\eta}} \left[\frac{\beta_{i}}{\mu} (1 - \tau_{B})^{1-\eta} \right]^{\frac{1}{\eta}}$$

$$= c_{i,t}^{y} \left[\beta_{i} r_{t+1} (1 - \tau_{B})^{1-\eta} \right]^{\frac{1}{\eta}}$$

Reformulation of utility function to get map from c to u

$$u_{i,t} = \frac{(c_{i,t}^{y})^{1-\eta} + \mu(c_{i,t+1}^{o})^{1-\eta} + \beta_{i} \left(b_{i,t+1}(1-\tau_{B})\right)^{1-\eta}}{1-\eta}$$

$$u_{i,t} = u(c_{i,t}^{y}) = \frac{1}{1-\eta} \left\{ (c_{i,t}^{y})^{1-\eta} + \dots \\ \dots \mu \left[c_{i,t}^{y} (\mu r_{t+1})^{\frac{1}{\eta}} \right]^{1-\eta} + \dots \\ \dots \beta_{i} \left[c_{i,t}^{y} (\beta_{i} r_{t+1}(1-\tau_{B})^{1-\eta})^{\frac{1}{\eta}} (1-\tau_{B}) \right]^{1-\eta} \right\}$$

$$= \frac{(c_{i,t}^{y})^{1-\eta}}{1-\eta} \left\{ 1 + \mu(\mu r_{t+1})^{\frac{1-\eta}{\eta}} + \beta_{i} \left[\beta_{i} r_{t+1}(1-\tau_{B}) \right]^{\frac{1-\eta}{\eta}} \right\}$$

$$= (c_{i,t}^{y})^{1-\eta} \underbrace{\frac{1}{1-\eta}} \left\{ 1 + \mu^{\frac{1}{\eta}} r_{t+1}^{\frac{1-\eta}{\eta}} + \beta_{i}^{\frac{1}{\eta}} \left[r_{t+1}(1-\tau_{B}) \right]^{\frac{1-\eta}{\eta}} \right\}$$

$$= (c_{i,t}^{y})^{1-\eta} \xi_{i,t}$$

$$= (c_{i,t}^{y})^{1-\eta} \xi_{i,t}$$

$$\Rightarrow c_{i,t}^{y} = \left(\frac{u_{i,t}}{\xi_{i,t}} \right)^{\frac{1}{1-\eta}}$$

Reformulation of budget equation to get map from u to y

$$c_{i,t}^{y} + \overbrace{k_{i,t+1}^{s} + p_{t}l_{i,t+1}}^{s_{t,t+1}} = y_{i,t}$$

$$c_{i,t+1}^{o} + b_{i,t+1} = r_{t+1}k_{i,t+1}^{s} + l_{i,t+1} \underbrace{(p_{t+1} + q_{t+1}(1 - \tau_{L}))}_{=p_{t}r_{t+1}}$$

$$= r_{t+1}(k^{s} + pl)$$

$$= r_{t+1}(y - c^{y})$$

$$\frac{c_{i,t+1}^{o} + b_{i,t+1}}{r_{t+1}} = y - c^{y}$$

$$y_{i,t} = c_{i,t}^{y} + \frac{c_{i,t+1}^{o} + b_{i,t+1}}{r_{t+1}}$$

$$y_{i,t} = c_{i,t}^{y} + \frac{1}{r_{t+1}} \left[c_{i,t}^{y} (\mu r_{t+1})^{\frac{1}{\eta}} + c_{i,t}^{y} (\beta_{i} r_{t+1}(1 - \tau_{B})^{1-\eta})^{\frac{1}{\eta}} \right]$$

$$y_{i,t} = c_{i,t}^{y} + \frac{1}{r_{t+1}} \left[c_{i,t}^{y} (\mu r_{t+1})^{\frac{1}{\eta}} + c_{i,t}^{y} (\beta_{i} r_{t+1}(1 - \tau_{B})^{1-\eta})^{\frac{1}{\eta}} \right]$$

$$y_{i,t} = c_{i,t}^{y} + \frac{1}{r_{t+1}} \left[c_{i,t}^{y} (\mu r_{t+1})^{\frac{1}{\eta}} + c_{i,t}^{y} (\beta_{i} r_{t+1}(1 - \tau_{B})^{1-\eta})^{\frac{1}{\eta}} \right]$$

$$y_{i,t} = c_{i,t}^{y} + c$$

E. Welfare weights

E.1. Regular generations

From Section D.2, we have

$$y_{i,t} = u_{i,t}^{\frac{1}{1-\eta}} \left(\frac{1-\eta}{\alpha_{i,t}^{\eta}} \right)^{\frac{1}{1-\eta}},$$

and can thus also express utility as a function of income (indirect utility with given prices)

$$u_{i,t} = y_{i,t}^{1-\eta} \frac{\alpha_{i,t}^{\eta}}{1-\eta}.$$

Then, marginal utility of income is given by

$$\frac{\mathrm{d}u_{i,t}}{\mathrm{d}y_{i,t}} = y_{i,t}^{-\eta} \alpha_{i,t}^{\eta} = y_{i,t}^{-\eta} \left\{ 1 + \mu^{\frac{1}{\eta}} r_{t+1}^{\frac{1-\eta}{\eta}} + \beta_i^{\frac{1}{\eta}} [r_{t+1}(1-\tau_B)]^{\frac{1-\eta}{\eta}} \right\}^{\eta}$$

Note that for the actual welfare weights, we need to choose a representative utility function. Our benchmark case will be $\bar{\beta} = 0$.

E.2. 1st old generation

Here, we have from above

$$y_i^{\text{1st old}} = (u_i^{\text{1st old}})^{\frac{1}{1-\eta}} \left\{ \frac{(1-\eta)}{\mu \left[\frac{\mu}{\beta_i} (1-\tau_B)^{\eta-1} \right]^{1-\eta} + \beta_i (1-\tau_B)^{1-\eta}} \right\}^{\frac{1}{1-\eta}} \left[1 + \left(\frac{\mu}{\beta_i} (1-\tau_B)^{\eta-1} \right)^{\frac{1}{\eta}} \right],$$

which yields

$$\left[\frac{y_i^{\text{1st old}}}{1 + \left(\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right)^{\frac{1}{\eta}}}\right]^{1 - \eta} = \frac{u_i^{\text{1st old}}(1 - \eta)}{\mu \left[\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right]^{1 - \eta} + \beta_i(1 - \tau_B)^{1 - \eta}} \\
u_i^{\text{1st old}} = \frac{\mu \left[\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right]^{1 - \eta} + \beta_i(1 - \tau_B)^{1 - \eta}}{1 - \eta} \left[\frac{y_i^{\text{1st old}}}{1 + \left(\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right)^{\frac{1}{\eta}}}\right]^{1 - \eta}} \\
u_i^{\text{1st old}} = (y_i^{\text{1st old}})^{1 - \eta} \frac{\mu \left[\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right]^{1 - \eta} + \beta_i(1 - \tau_B)^{1 - \eta}}{(1 - \eta)\left[1 + \left(\frac{\mu}{\beta_i}(1 - \tau_B)^{\eta - 1}\right)^{\frac{1}{\eta}}\right]^{1 - \eta}}}\right]$$

Then, the marginal utility of income is

$$\frac{\mathrm{d}u_{i}^{1\text{st old}}}{\mathrm{d}y_{i}^{1\text{st old}}} = (y_{i}^{1\text{st old}})^{-\eta} \frac{\mu \left[\frac{\mu}{\beta_{i}}(1-\tau_{B})^{\eta-1}\right]^{1-\eta} + \beta_{i}(1-\tau_{B})^{1-\eta}}{\left[1+\left(\frac{\mu}{\beta_{i}}(1-\tau_{B})^{\eta-1}\right)^{\frac{1}{\eta}}\right]^{1-\eta}} \\
= (y_{i}^{1\text{st old}})^{1-\eta} \frac{\mu \left[\frac{\mu}{\beta_{i}}(1-\tau_{B})^{\eta-1}\right]^{1-\eta} + \beta_{i}(1-\tau_{B})^{1-\eta}}{(1-\eta)\left[1+\left(\frac{\mu}{\beta_{i}}(1-\tau_{B})^{\eta-1}\right)^{\frac{1}{\eta}}\right]^{1-\eta}} \frac{1-\eta}{y_{i}^{1\text{st old}}} \\
= u_{i}^{1\text{st old}} \frac{1-\eta}{y_{i}^{1\text{st old}}}$$