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Investment in Quality Upgrade and Regulation of the Internet

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Abstract

This paper studies the investment decision by a monopolistic internet service provider (ISP) in different regulatory environments. We consider that the ISP could technically provide separate quality upgrades to two vertically differentiated content providers (CPs); therefore, it could potentially extract the CPs' marginal profits through an offer to provide the quality upgrades. Our results show that if unregulated, the ISP optimally provides asymmetric quality upgrades, in favor of the high-quality CP. This subsequently increases the degree of content differentiation, softening competition between the CPs. Imposing a nondiscrimination regulation that forces the ISP to provide an equal quality upgrade to both CPs, however, can reduce the ISP's investment incentive and social welfare. Furthermore, the investment level is higher if the regulated ISP is allowed to charge the CPs. Finally, a socially optimal investment can be opposite to the ISP's choice when the contents are enough substitutes.

JEL-Codes: L130, L510, L960.

Keywords: complementary, differentiation, investment, internet, regulation.

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1 Introduction

The internet is an essential platform that changed how business activities are conducted and home entertainment with the so-called end-to-end principle. In the past, individuals had to watch television and videos through broadcast cable at home or in a cinema, and at that time, internet surfing was a substitute entertainment source. Emerging data-intensive businesses, such as Netflix, YouTube, and other streamed media services, have increasingly become a substitute for television and other traditional media¹, requiring substantial improvement to bandwidth access. In addition, the ISP has the capability to determine average transmission speed or network quality at which each content is delivered to end users by physically providing prioritized data packets. Notably, Netflix has recently agreed to pay Comcast—the largest internet provider in the United States—an access fee for direct interconnection to ensure their content is transmitted to clients more rapidly with significantly improved quality². This could be considered a violation of the widely adopted net neutrality regulation that requires equal and open access to any online content without favoring certain traffic. Although net neutrality rules have been adopted by the European Parliament and other advanced economies, for example, South Korea and Singapore, they were successfully repealed by the US Federal Communications Commission that took effect on June 11, 2018. To this end, an expectation might be that imposing net neutrality regulations would considerably affect the investment incentives that are the key factor in promoting very fast broadband connections³.

Thus, our paper provides an economic model to analyze the impacts of regulatory intervention in the following context: an ISP with limited available bandwidth invests in upgrading infrastructure to fulfill broadband capacity demands. In our setting, the ISP provides an internet platform for a continuum of end users and two vertically differentiated CPs. Content and internet are complementary services because end users enjoy

¹According to Ofcom’s first annual Media Nations report in 2018, the time spent using the TV set overall remains constant, and viewing of nonbroadcast content (including Netflix, and YouTube viewing) increases to approximately 42 mins (17%) per day (source: https://www.ofcom.org.uk/___data/assets/pdf_file/0014/116006/media-nations-2018-uk.pdf). Furthermore, Netflix has approximately 60.6 million paying subscribers in the United States and 97.7 million overseas by September 2019 (source: <https://www.netflixinvestor.com/financials/quarterly-earnings/default.aspx>)

²The deal was announced on 23 Feb 2014, see full text at: <https://media.netflix.com/en/press-releases/comcast-and-netflix-team-up-to-provide-customers-excellent-user-experience-migration-1>

³For instance, the EU Digital Agenda clearly sets ambitious objectives for broadband infrastructure development (source: <https://ec.europa.eu/digital-single-market/en/broadband-strategy-policy>).

having both. Without investments, the ISP only provides the minimum quality of internet service. The ISP could invest to offer advanced services, including higher speed internet, by upgrading the bandwidth of existing connections, providing a higher utility that an end user derives from consuming content.

In the absence of regulation, the ISP is allowed to provide different quality upgrades above the minimum level and can charge CPs differently. Accordingly, there are three reasons for the ISP to invest. First, because the internet and content are complementary services, the ISP could invest to increase end users' utility, resulting in an outward shift of the internet demand curve (complementary effect). Second, the ISP can technically improve the speed of low-quality content delivery, relatively increasing the attractiveness of the low-quality CP. This action would intensify price competition in the content market, and subsequently increasing demand for internet service (competition effect). Third, in contrast with the previous reason, the monopolistic ISP can strategically invest to increase content differentiation to extract higher marginal profits from the CPs (differentiation effect⁴).

Related literature. Our research contributes to the well-established literature on the net neutrality debate whereby proponents and opponents' opinions differ on the specific rules and regulatory impacts (see Schuett (2010), Krämer *et al.* (2012) Greenstein *et al.* (2016) for excellent overviews). A common economic approach to net neutrality is related to traffic growth or network congestion, namely, traffic generation can be modeled based on the M/M/1 queuing process (Choi and Kim (2010), Bourreau *et al.* (2015) and Reggiani and Valletti (2016)⁵), or simply correlated to the number of CPs in the same platform (Njoroge *et al.* (2012)). Peitz and Schuett (2016) introduce congestion control techniques, whereby a CP sends additional traffic to reduce individual delay or uses compression to reduce traffic volumes; in turn, the quality of content is negatively affected by internet traffic inflation for a given network capacity. This motivates a network operator to provide a prioritized lane to avoid congestion, especially of time-sensitive traffic.

In this context, net neutrality regulations oblige an ISP to deliver a data package on a nondiscriminatory basis, a so-called best-effort delivery, affecting investment incentives, competition, and social welfare. In particular, Choi and Kim (2010) demonstrate that

⁴We refer to the classical analysis of vertical differentiation in Shaked and Sutton (1982) and Gabzewicz and Thisse (1979).

⁵Others include Cheng *et al.* (2011), Krämer and Wiewiorra (2012), and Choi *et al.* (2018).

the discriminatory regime may not provide an investment incentive, because less congestion could lower a CP's willingness to pay for the prioritized delivery service. However, Bourreau *et al.* (2015) introduce competition in an internet market with two-sidedness and find that the ISPs have more incentive to invest in broadband capacity because of the increased revenue in the discriminatory regime. Additionally, content innovation is lower because some highly congestion-sensitive content is excluded from the market under net neutrality rules. In Njoroge *et al.* (2012), the investment is determined by trade-offs between softening price competition on the consumer side and profits extracted from the CPs.

In practice, actual traffic generation and delivery speed are complex, or they could appear very different in real computer networks. Therefore, we develop a model that shares certain aspects with the aforementioned literature to focus on the costly provision of internet service. More precisely, our research is similar to Njoroge *et al.* (2012) because we also consider that a CP can access the basic quality of an internet service if the ISP does not invest in upgrading the infrastructure⁶. We introduce an additional assumption that the ISP can provide different quality upgrades to two CPs⁷. Regarding the CPs' heterogeneity, we consider that the quality differs between the two sets of content, whereas Reggiani and Valletti (2016) assume there is one large CP and a number of small CPs that constitute a fringe. Our main finding is that the unregulated ISP would find it attractive to provide asymmetric quality upgrades in favor of the high-quality CP, which is contrary to Choi and Kim (2010). The second result of this paper complements the findings of Bourreau *et al.* (2015), that is, imposing nondiscrimination regulations reduces the ISP's investment incentive but could enhance competition in the content market.

Our model is built upon the non-traffic based approach, which focuses on firm business models and strategic relationships in digital markets. More precisely, we observe in practice that an ISP could offer two types of preferential treatment. First, the ISP could allow CPs to pay an extra price to have higher internet speed (paid prioritization), or second, it could exclude some traffic from overall data caps (zero rating). In our setting, the quality upgrade could be interpreted as a form of paid prioritization (e.g., Gautier and Somogyi (2018)). Additionally, Musacchio *et al.* (2009) compare two-sided and one-sided pricing

⁶Haucap and Klein (2012) also consider that consumers could derive higher utility because of quality upgrades to the internet and content provided by firms in the markets.

⁷Our setting is related to the classical study on the incentive of a monopolist selling a broad range of qualities (Mussa and Rosen, 1978).

whereby an ISP can charge only end users. We borrow from Musacchio *et al.* (2009) the idea that firms adopt usage-based pricing and that internet quality is determined by the level of investment, which then affects demand for content. Although they show that the level of social welfare is determined by factors such as advertising rate, end-user price sensitivity, and competition in the internet market, we could explicitly compare social welfare under various regulatory regimes. Economides and Tag (2012) also provide a two-sided model in which CPs generate advertising revenue. They find that implementation of net neutrality regulation requiring zero fees to CPs does not generally increase social welfare, even in a duopoly model. In this article, we go further into the investment decision, the key issue for internet development. This issue is however not addressed in Economides and Tag (2012) (and Peitz and Schuett (2016) either).

With an unregulated internet, we consider that the ISP's investment is determined by the complementarity between content and internet service and the ability to extend monopoly power beyond the internet market. The former effect has been extensively studied in Haucap and Klein (2012), Broos and Gautier (2017), and Baranes and Vuong (2019)⁸ who have considered that a higher content value leads to higher internet demand and *vice versa*⁹. To account for the latter effect, such as in Carroni *et al.* (2018) and Montes *et al.* (2019), we follow the contractual framework initiated by Jehiel and Moldovanu (2000) to explore the ISP's market position to extract rents through lump-sum payments: The ISP could vary the quality upgrades; hence, it could capture all the CPs' marginal profits through a take-it-or-leave-it offer. In contrast to the results of the literature whereby the seller might obtain a higher profit from selling to only one buyer, we find that the ISP with market power can always extract profits from both CPs.

We are interested in analyzing two forms of nondiscrimination regulation in both quality and price and propose a setting that accommodates the effects of regulatory restrictions on the ISP's investment and social welfare. First, the ISP receives zero payment from the CPs, which is similar to the definition of a strong net neutrality regulation in Gans (2015) and Gans and Katz (2016). Second, the ISP can charge an equal price at which it fully extracts the marginal profit of the low-quality CP. Although we could extend our

⁸Indeed, our companion paper is built upon the same framework, but the focus is on the vertical integrated ISP's incentive to provide a higher quality upgrade to the competing CP.

⁹In the personal computer industry, a monopolist can raise extra profit when products are complementary (see for examples Whinston (1990), Carlton and Waldman (2002), Casadesus-Masanell *et al.* (2007), Yalcin *et al.* (2013)).

analysis in other directions, we aim to show that if this less-restricted regulation is imposed, the ISP must consider a CP's profitability when it makes an investment decision, affecting social welfare. In this setup, we could also perform an analysis of the socially optimal investment and then compare that with the ISP's choice. A similar comparison is conducted in Choi *et al.* (2018), but they focus on the CPs' incentives to invest in the quality of service by using alternative technological solutions, namely, content distribution networks and advanced compression technology.

The research question that we analyzed has been explored in the economic literature on effective regulatory instruments that promote competition and investment in network industries (e.g., Cambini and Jiang (2009), Bourreau *et al.* (2012), Briglauer *et al.* (2016)). Baranes (2014) studies the interplay between the investment decision of network operators who supply both old and new technologies. The author shows that a discriminatory regime, whereby the ISP could set a higher price for a CP when consumers of the CP are connected to the new technology, can improve the social welfare and investment incentive. Our article takes a similar perspective by studying the optimal investment to upgrade the existing infrastructure in the context that the unregulated ISP can invest in offering different quality upgrades and charges the CPs differently.

We attempt to evaluate the regulatory impacts by comparing the investment, content prices, and social welfare when the ISP invests under regulated and unregulated regimes. With an unregulated internet, the ISP could offer to provide quality upgrades to capture all of the CPs' marginal profits, namely, the ISP's optimal investment provides asymmetric quality upgrades to both CPs and in favor of the high-quality CP. Thus, content would become more differentiated from a consumer's viewpoint, and the high-quality CP can take advantage of a relatively stronger market position because of the ISP's investment. If a nondiscriminatory regulation is imposed, the ISP's investment becomes relatively lower. In this case, the ISP makes a higher investment if it is allowed to collect equal lump-sum payments from the CPs. Last, we characterize the socially optimal quality upgrades that can be opposite to the ISP's optimal levels for sufficient substitute content. The results imply a regulatory trade-off between competition in the content market, the ISP's investment, and social welfare.

The structure of our paper is organized as follows. In section two, we provide the model setup. In section three, we investigate the ISP's optimal investment in the absence

of regulation. In section four, we analyze the investment decision under different regulatory restrictions. In section five, we provide a brief conclusion and policy implications. Throughout this paper, we report only the main results; the Appendix provides detailed proofs.

2 Model setup

In this paper, a monopolistic ISP provides an internet platform for duopolistic CPs to connect with end users. The ISP can invest in providing costly quality upgrades above the minimum quality level for both CPs. We study the ISP's investment decision, with and without regulatory restrictions, and analyze subsequent impacts on competition and social welfare. The stylized structure of the model is represented below.

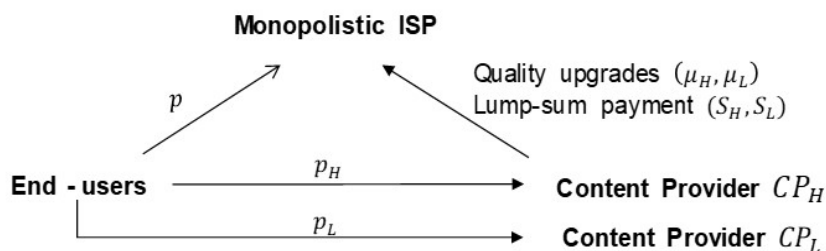


Figure 1 - Market players and model structure

The ISP. The ISP adopts usage-based pricing for end users¹⁰, denoted as p . We normalize the ISP's fixed and marginal costs of serving end users to 0. Without investment, internet service is offered at the minimum quality¹¹, normalized to 0. Both CPs pay zero to access the internet in this case.

In practice, the ISP can technically control the quality of internet service including the average speed of delivering content to end users. For this reason, we assume that the

¹⁰Usage-based pricing has been commonly adopted in practice, particularly by wireless broadband providers in the United States, Canada, the United Kingdom, and Australia, including Comcast's XFINITY plan, and the Cisco and Telus data plans. Additionally, Google Project Fi prices are based on per-megabyte consumed. Economic studies and discussions on usage-based pricing for internet service are available in Nevo *et al.* (2016), Musacchio *et al.* (2009), Lyons (2014)).

¹¹This assumption has also been employed in Haucap and Klein (2012) and Njoroge *et al.* (2013) and can simplify significantly our analysis. An interpretation of this assumption is that the ISP is required by the regulator to provide this minimum level to both CPs. In practice, the EC has proposed the introduction of minimum service levels in article 22.3 of the Universal Service Directive (see https://berec.europa.eu/eng/document_register/subject_matter/berec/download/0/1101-berec-guidelines-for-quality-of-service-_0.pdf).

ISP could invest separately and provides different quality upgrades to the CPs, denoted as $\mu_i > 0$ where $i = H, L$. Thus, the ISP cannot degrade the internet quality below the minimum level. Furthermore, when $\mu_H \neq \mu_L$, the ISP is said to offer asymmetric quality upgrades, in favor of content H if $\mu_H > \mu_L$, or conversely, content L if $\mu_H < \mu_L$. In turn, the unit cost associated with μ_i is $c(\mu_i) = \frac{\mu_i^2}{2}$. To this end, we assume that the ISP's total cost of providing both quality upgrades (μ_H and μ_L) is additive¹², that is:

$$C(\mu_H, \mu_L) = c(\mu_H) + c(\mu_L) = \frac{\mu_H^2}{2} + \frac{\mu_L^2}{2}$$

This critical assumption substantially reduces complications from interactions among the cost of investment at different levels of quality upgrades provided to both CPs and allows us to focus on the roles of content substitution and differentiation¹³. Additionally, this assumption is appropriate because we are interested in analyzing the ISP's investment in upgrading the network's infrastructure in this model. In summary, because we do not consider the cost associated with the deployment of the whole network, we ignore indivisibilities or the fixed cost of investment. We can observe that $C(\mu_H, \mu_L)$ is increasing and convex in μ_H and μ_L , that is, $C'() > 0$ and $C''() > 0$.

Furthermore, the ISP could collect lump-sum payments from CP_H and CP_L , denoted as S_H and S_L , respectively. Last, the quality upgrades and lump-sum payments could be subject to regulatory obligations.

CPs. Duopolistic CPs provide vertically differentiated contents. Quality of content L , normalized to 1, is lower than quality of content H , denoted as λ where $\lambda > 1$. Parameter λ measures the degree of content differentiation¹⁴.

We also assume that both CPs have zero fixed and marginal cost of production for analytical convenience. Moreover, we consider that end users incur no fixed cost of consumption, and the CPs' pricing is linear. We denote p_H and p_L for unit prices of content H and L , respectively¹⁵.

¹²We can also employ a more general cost function, e.g., $C(\mu_H, \mu_L) = k[c(\mu_H) + c(\mu_L)]$ where $k < 1$ accounts for a subadditivity property of investment in broadband infrastructure (see Beresteanu (2015) for empirical evidence of this in the US telecom industry).

¹³The classical monopoly problem with different varieties is analyzed in Mussa and Rosen (1978). In the context of the long-distance telephony, Economides (1999) also assumes a separable additive cost function for the monopolist in the provision of qualities of both long distance lines and local lines.

¹⁴In this paper, λ is exogenous to the investment decision because we do not consider the case that a CP could invest in improving the quality of content (see Choi *et al.* (2018) for an analysis).

¹⁵In practice, content services could be free, but we consider that consumers dislike most advertising displays (banner ads, pop-up, videos...) or sales' links when they access to content. For that reason,

Demand. One unit mass of end users consume services provided by both CPs through the internet platform. Because content and internet services are perfectly complementary, an end user must pay a composite price (p_i+p) for one unit of demand for composite service i , including internet and content i , where $i = H$ or L . We let α_i denote the perceived quality of composite service i which is the sum of content quality and internet quality¹⁶. More precisely, if the ISP provides the quality upgrades to both CPs, $\alpha_H = \lambda + \mu_H$ and $\alpha_L = 1 + \mu_L$. If only CP_H (or only CP_L) receives the quality upgrade, $\alpha_H = \lambda + \mu_H$ and $\alpha_L = 1$ (or, $\alpha_H = \lambda$ and $\alpha_L = 1 + \mu_L$).

Next, we follow Singh and Vives (1984) and assume the following quadratic utility function:

$$U(q_H, q_L) = \alpha_H q_H + \alpha_L q_L - \frac{1}{2} (q_H^2 + 2\beta q_H q_L + q_L^2) \quad (1)$$

where α_H and α_L are the perceived qualities of composite service H and L , respectively, and β represents the degree of content substitutability. We assume that $0 < \beta < \frac{1}{2}$ for technical intention.

Thus, the consumer's utility maximization problem is provided by

$$\max_{q_H, q_L} \{U(q_H, q_L) - (p_H + p)q_H - (p_L + p)q_L\} \quad (2)$$

Based on the consumer maximization problem in (2), we derive linear demand for content i that is decreasing in its composite price but increasing in content j 's composite price, as follows:

$$q_H = \frac{(\lambda - \beta) + \beta(p_L + p) - (p_H + p)}{(1 - \beta^2)} \quad (3)$$

$$q_L = \frac{(1 - \lambda\beta) + \beta(p_H + p) - (p_L + p)}{(1 - \beta^2)} \quad (4)$$

The demand q addressed to the ISP is simply the sum of content demands.

$$q = q_H + q_L = \frac{(1 + \lambda) - (p_L + p) - (p_H + p)}{(1 + \beta)}$$

We observe that internet demand is negatively affected by composite prices because the internet and content are perfectly complementary services.

this assumption can be interpreted as the cost for consumers to remove unwanted adverts on free-access content, or the price of ad-free access to content.

¹⁶This assumption of linearity is considerably more simple than the multiplicative form introduced in Valletti and Cambini (2005) to show that both quantity and utility are increasing in the network's quality.

3 The ISP's investment under an unregulated regime

In this section, we examine the ISP's optimal investment in an unregulated internet essentially based on the mechanism called "*auction with negative externalities*" initiated by Jehiel and Moldovanu (2000). More precisely, the ISP has all the bargaining power in determining the network quality of delivering each content to end users. Consequently, the ISP can extract maximal rents by offering the quality upgrades to both CPs. If a CP declines the offer, the ISP provides the quality upgrade to only the competing CP. Hence, the ISP can ask for the lump-sum payment S_i , which makes CP_i indifferent between accepting or declining the offer. Subsequently, the ISP determines the optimal quality upgrade provided to each CP.

We formalize the time structure of the game below:

Stage 1. The ISP makes a take-it-or-leave-it offer to provide the quality upgrades to both CPs. The ISP requires a lump-sum payment S_i from CP_i where S_i is equal to the difference between the CP_i 's profits when CP_i receives the quality upgrade and when only CP_j receives the quality upgrade.

Stage 2. The ISP invests in providing the quality upgrades to both CPs.

Stage 3. The ISP and the CPs set profit-maximizing prices to end users, who consume as a function of their perceived utility.

We solve the game backward by deriving a subgame perfect Nash equilibrium. In the first subsection, we compute the CPs' lowest profits when only the competitor receives the quality upgrade, and in the second subsection, we examine the ISP's optimal quality upgrades provided to both CPs.

3.1 One CP receives the quality upgrade

In this subsection, we study the ISP's optimal quality upgrade and the profit level of a CP when the ISP provides the quality upgrade to only the competing CP. Equilibrium values are subscripted with " A_i " where $i = H, L$, corresponding to the case in which the quality upgrade is provided to only CP_i .

We solve this game using backward induction, that is, we start by considering stage 3 and require that a quality upgrade be provided. When only content i is upgraded, the

ISP's profit function is

$$\pi_{ISP}(p, \mu_i) = pq - \frac{\mu_i^2}{2} + S_i$$

where S_i is the lump-sum payment from CP_i to the ISP. Indeed, S_i is endogenously determined by the ISP's investment, or we have

$$\pi_{ISP}(p, \mu_i) = pq + (p_i q_i - \pi_i^{Aj}) - \frac{\mu_i^2}{2} \quad (5)$$

where π_i^{Aj} denotes the CP_i 's profit when the ISP provides the quality upgrade to only CP_j . We can consider π_i^{Aj} an exogenous parameter in relation to the ISP's optimal investment decision in this case. As a consequence, the ISP's optimal quality upgrade is set to maximize the joint profits from selling the internet and content i . Additionally, the profit of CP_j who declines the offer is simply $\pi_j = p_j q_j$.

Solving simultaneously the firms' profit maximizations to derive the firms' prices based on the quality upgrade in stage 2, we replace these prices into (3), (4) and (5) to obtain the firms' profit functions in stage 2. Thus, we solve the first-order condition to find the ISP's optimal quality upgrade provided to only CP_i (denoted as μ_i^A) and the CP_j 's profit.

In the Appendix, we explicitly analyze case AH (and case AL) in which the ISP provides the quality upgrade to only CP_H (and CP_L , respectively). The results of the analysis are summarized in the following lemma.

Lemma 1 *When the ISP provides the quality upgrade to only the competitor, the profits of CP_H and CP_L are denoted by π_H^{AL} and π_L^{AH} , respectively, where:*

$$\pi_H^{AL} = \frac{(1 - \beta^2)[2\lambda\beta^4 - 3(\lambda - 1)\beta^3 - 2(6\lambda + 1)\beta^2 + (7\lambda - 19)\beta + 16\lambda - 6]^2}{(3 - \beta)^2(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)^2}$$

$$\pi_L^{AH} = \frac{(1 - \beta^2)[2\beta^4 + 3(\lambda - 1)\beta^3 - 2(\lambda + 6)\beta^2 - (19\lambda - 7)\beta - 6\lambda + 16]^2}{(3 - \beta)^2(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)^2}$$

In lemma 1, π_L^{AH} and π_H^{AL} represent the outside options that CP_L and CP_H , respectively, could obtain when the ISP provides the quality upgrade to only the competitor. We observe that both the profits are positive, and CP_H gains a higher profit than CP_L , that is, $\pi_H^{AL} > \pi_L^{AH} > 0$. Furthermore, the profit difference (defined as $\Delta\pi = \pi_H^{AL} - \pi_L^{AH}$) is increasing in the degree of vertical differentiation (λ) but decreasing in the degree of content substitutability (β).

In addition, the equilibrium quantities of both contents are always positive in case AL , namely, $q_H^{AL} > 0$ and $q_L^{AL} > 0$. Additionally, in a case where AH , the quantity of content H is always positive, whereas the quantity of content L is only positive, that is, $q_L^{AH} > 0$ if $\lambda < \lambda_1$ where:

$$\lambda_1 = \frac{2\beta^4 - 3\beta^3 - 12\beta^2 + 7\beta + 16}{(3\beta + 1)(3 - \beta)(\beta + 2)}$$

This condition is observed because the contents are substitutable and vertically differentiated, and the CPs compete in price¹⁷. In the remainder of this paper, we only focus on the range of the sufficiently low degree of content differentiation, that is, $1 < \lambda < \lambda_1$.

3.2 Both CPs receive the quality upgrades

In this subsection, the ISP provides separate quality upgrades (i.e., μ_H and μ_L) and collects lump-sum payments (i.e., S_H and S_L) from CP_H and CP_L , respectively. These payments lead to the indifferent CPs accepting or declining the ISP's offer. We compute the equilibrium values subscripted with "UU".

When the ISP provides the quality upgrades to both CPs, the content perceived qualities are $\alpha_H = \lambda + \mu_H$ and $\alpha_L = 1 + \mu_L$. The ISP's profit function is

$$\pi_{ISP}(p, \mu_H, \mu_L) = pq - \frac{\mu_H^2 + \mu_L^2}{2} + S_H + S_L \quad (6)$$

Then, equation (6) can be written as follows:

$$\pi_{ISP}(p, \mu_H, \mu_L) = [p(q_H + q_L) + p_H q_H + p_L q_L] - \frac{\mu_H^2 + \mu_L^2}{2} - (\pi_H^{AL} + \pi_L^{AH}) \quad (7)$$

We observe from equation (7) that the ISP is interested in maximizing the total revenues derived from both the internet and content markets less the investment. The last bracket in equation (7) essentially presents the CPs' profits, which are fixed and given in lemma 1.

Two conflicting effects on the competition between the CPs emerge in the aforementioned ISP's profit function. For clarity, we intentionally consider that at the interior equilibrium, one quality upgrade (i.e., μ_j) is fixed at zero; then, a slight departure of from zero has different effects on the ISP's profit as follows:

$$\frac{d\pi_{ISP}}{d\mu_i} = \frac{\partial p(q_H + q_L)}{\partial \mu_i} + \frac{\partial \pi_{ISP}}{\partial p} \frac{\partial p}{\partial \mu_i} + \frac{\partial \pi_{ISP}}{\partial p_H} \frac{\partial p_H}{\partial \mu_i} + \frac{\partial \pi_{ISP}}{\partial p_L} \frac{\partial p_L}{\partial \mu_i} \quad (8)$$

¹⁷We verify that $\lambda_1 > 1$ when $\beta < \frac{1}{2}$.

From (3) and (4), the first term of (8) can be rewritten as $\frac{\partial \pi_{ISP}}{\partial \mu_i} = p(\frac{\partial q_H}{\partial p_H} \frac{\partial p_H}{\partial \mu_i} + \frac{\partial q_L}{\partial p_L} \frac{\partial p_L}{\partial \mu_i}) = \frac{p}{\beta^2 - 1}(\frac{\partial p_H}{\partial \mu_i} + \frac{\partial p_L}{\partial \mu_i})$, describing the impact on the internet revenue of the ISP. The sign of $\frac{\partial \pi_{ISP}}{\partial \mu_i}$ is generally ambiguous because it depends on the degree of price competition between the CPs. In particular, there might be cases where the net effect is positive because the ISP provides a relative higher quality upgrade to CP_L , lowering the content's perceived qualities, and resulting in a significant decrease in p_H but a modest increase in p_L . This effect, which we refer to as the competition effect, intensifies competition in prices between the CPs, increasing the content or internet consumption. Notably, the competition effect is stronger for a higher value of β .

Furthermore, the second term of equation (8) is zero at equilibrium because of the envelope theorem, that is, $\frac{\partial \pi_{ISP}}{\partial p} = 0$. The impact on the ISP's profit derived from the content market is exemplified by the sum of the third and the last terms, that is, $q_H \frac{\partial p_H}{\partial \mu_i} + q_L \frac{\partial p_L}{\partial \mu_i}$. The net impact is positive when content H receives a relatively higher quality upgrade and *vice versa* (Shaked and Sutton, 1983). This eventually presents the differentiation effect. The intuition is that the ISP could extract a higher profit from the CPs by increasing the difference between the content's perceived qualities, and effectively softening competition between the CPs. Thus, the ISP's optimal investment in an unregulated internet environment would involve not only the absolute but also the relative levels of the quality upgrades. The results of this subsection confirm whether the unregulated ISP has an interest in creating the differentiation or competition effect.

We now directly compute the ISP's optimal quality upgrades. In stage 3, the firms set the profit-maximizing prices (i.e., p, p_H, p_L) depending on the quality upgrades (i.e., μ_H and μ_L). By replacing these prices into the ISP's profit function and solving for the first-order conditions simultaneously in stage 2, we obtain the optimal quality upgrades in an unregulated internet environment (i.e., μ_H^{UU} and μ_L^{UU}) as follows:

$$\begin{aligned}\mu_H^{UU} &= \frac{2\beta^4\lambda - (3\lambda - 5)\beta^3 - 4(\lambda + 1)\beta^2 + 4(\lambda - 4)\beta + 9\lambda - 1}{(2 - \beta^3 - 3\beta^2 - 2\beta)(\beta^3 - 5\beta^2 + 5\beta + 5)} \\ \mu_L^{UU} &= \frac{2\beta^4 + (5\lambda - 3)\beta^3 - 4(\lambda + 1)\beta^2 - 4(4\lambda - 1)\beta - \lambda + 9}{(2 - \beta^3 - 3\beta^2 - 2\beta)(\beta^3 - 5\beta^2 + 5\beta + 5)}\end{aligned}$$

In the range of the parameter values we discussed, the ISP's optimal quality upgrades are positive. Notably, the quality upgrade difference is defined as $\Delta\mu = \mu_H^{UU} - \mu_L^{UU} > 0$. This result indicates that if unregulated, the ISP's optimal strategy is to provide the positive quality upgrades to both CPs, and with a relatively higher level to CP_H . Furthermore,

$\Delta\mu$ tends toward 0 when λ is near 1, that is, the unregulated ISP provides equal quality upgrades in cases of symmetric content. Additionally, the higher the degree of content substitutability, the higher the quality upgrade asymmetry. Thus, the ISP would want to increase the content differentiation if the contents become more substitutable¹⁸.

Thus, we establish the following proposition.

Proposition 1 *For an unregulated internet, the ISP provides asymmetric quality upgrades to both CPs, in favor of the high-quality CP.*

The main implications of proposition 1 are as follows. First, if unregulated, the ISP would want to provide the different quality upgrades to both CPs. This result differs from that in Carroni *et al.* (2018) and Montes *et al.* (2019), who have proven that the seller could have a higher profit by providing an exclusive contract to only one buyer. The reason this occurs is that the case in which only one CP receives the quality upgrade is a special situation when the ISP offers two quality upgrades, but one of them must be constrained to zero. In our model, the unregulated ISP with market power could pose a credible threat of providing the quality upgrade to only the competing CP, inducing both CPs to accept the offer with the fixed positive profits. The second implication simply means that the strategy of the ISP with market power is to increase the difference in the content's perceived qualities to obtain maximal rent extraction. In that case, the optimal quality upgrade asymmetry is very large when both the degrees of vertical differentiation and substitutability are high. Put differently, we show that the differentiation effect is more important than the competition effect for maximizing the profit of the unregulated ISP.

Now, we easily find the equilibrium prices, denoted as $p^{UU}, p_H^{UU}, p_L^{UU}$ which are the higher degree polynomial functions of β and λ . We detect that the lower the level of β , causing end users to view the content that is less substitutable, the greater the prices for content L , and content H if $\lambda < \hat{\lambda}$. These relationships basically demonstrate the increased market power of the CPs for the lower values of content substitutability. When content H has very high quality compared with content L (i.e., $\lambda > \hat{\lambda}$), an increase in β would lead to an increase in demand for content H , inducing CP_H to increase content H 's

¹⁸Referring to footnote 12 on the property of subadditive costs, we prove that in the neighborhood of the additive cost (i.e., k is near 1), the ISP also provides a positive quality upgrade to CP_H , and a lower quality upgrade to CP_L .

price. Furthermore, the change in the internet price by deviating slightly β is dependent on the range of parameter values as follows. If the degree of content substitutability is very low (i.e., $\beta \lesssim 0.27$), the CPs do not strongly compete. Thus, when β is lower, the ISP can increase the internet price because of the complementarity between the content and internet services. By contrast, when $\beta \gtrsim 0.27$, an increase in β would trigger intense price competition between the CPs, leading to higher demand for the internet, and thereby the internet price.

Regarding the impact of an increase in the degree of vertical differentiation on the prices, we observe that given a fixed level of β , an increase in λ would lead to a higher price of content H , but a lower price of content L . This corresponds to the relative market positioning or the ability of each CP to extract rents. Moreover, a higher level of λ means that the contents become increasingly differentiated; hence, the ISP's market power increases because of the complementarity. In other words, the internet price would be set at a higher level. In the extreme, when the contents are independent (i.e., β goes to 0), we would have the highest price of content H if λ tends toward λ_1 , or of content L if λ tends toward 1.

Finally, we could easily compute the total investment (denoted as $TC^{UU} = \frac{(\mu_H^{UU})^2 + (\mu_L^{UU})^2}{2}$) and the profit (denoted as π_{ISP}^{UU}) of the ISP, and the social welfare (denoted as W^{UU}). We use these values to compare with those under nondiscrimination regulation.

4 Investment under regulatory restrictions

In this section, we analyze the optimal investment under different regulatory restrictions. Under a nondiscrimination regulatory regime, the ISP is obliged to provide identical quality upgrades to the CPs (nondiscriminatory quality), and it collects equal lump-sum payments (nondiscriminatory price). Accordingly, the regulated price is set at zero or a positive level. We then characterize the socially optimal investment.

4.1 Nondiscrimination regulation

When nondiscrimination regulation is imposed, an identical quality upgrade, denoted as μ , is provided to both CPs, who in return pay an identical fixed fee to the ISP. We consider that the ISP receives zero from the CPs in case RZ and a positive equal lump-

sum payment in case *RP*.

Case RZ. The ISP receives a zero lump-sum payment; hence the ISP's profit is

$$\pi_{ISP}(p, \mu) = pq - 2\left(\frac{\mu^2}{2}\right) \quad (9)$$

In this case, the ISP invests to benefit only from the complementary effect, that is, the higher quality upgrade leads to a higher demand for content and hence the internet.

Case RP. When the ISP could collect equal lump-sum payments (S) from both CPs, the ISP's profit function becomes

$$\pi_{ISP}(p, \mu) = pq - 2\left(\frac{\mu^2}{2}\right) + 2S \quad (10)$$

We aim to investigate a regulatory regime that allows the ISP to extract profit from CPs. Particularly, we propose that the regulated ISP is able to set the lump-sum payment to fully extract the CP_L 's marginal profit¹⁹. In this case, the ISP's profit function indicated in (10) becomes

$$\pi_{ISP}(p, \mu) = pq - \mu^2 + 2(\pi_L - \pi_L^{AH}) \quad (11)$$

The timing of the game is as follows:

Stage 1: The ISP determines the quality upgrades.

Stage 2: The ISP and the CPs simultaneously set prices. End users decide to buy the amount of each content.

As previously analyzed, it is straightforward to compute the ISP's optimal investment in stage 1, denoted by μ^{RZ} and μ^{RP} in case *RZ* and case *RP*, respectively as follows:

$$\begin{aligned} \mu^{RZ} &= \frac{\lambda + 1}{\beta^3 - 5\beta^2 + 3\beta + 7} \\ \mu^{RP} &= \frac{2\beta(1 - \beta) + (1 - 2\beta)\lambda + 7}{(\beta + 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \end{aligned}$$

Our results show that the regulated ISP invests more if it can extract profit from the CPs, that is, $\mu^{RZ} < \mu^{RP}$. This result is complementary to the so-called "*waterbed effect*" which indicates that the ISP's pricing strategy increases in fees for the CPs, which leads to a decrease in internet price (Greenstein *et al.* (2016)).

¹⁹In the previous analysis, we established that in the absence of regulation, CP_L is ready to pay for the quality upgrade at this level. We also show in the Appendix that CP_H always accepts the offer because CP_H 's profit is higher than the outside option indicated in lemma 1.

We now examine the impacts of an increase in the market parameters on the quality upgrades under a nondiscrimination regulation. First, we easily observe that the greater the value of λ , the higher the quality upgrades. Second, an increase in β always has a negative impact on μ^{RP} , but a positive impact on μ^{RZ} when $\beta \gtrsim 0.32$. The latter is possible because in case RZ , the ISP gains purely from selling the internet. Thus, if the degree of substitutability increases from a high level, the ISP would be encouraged to increase the quality upgrade to benefit from the increased internet demand because of the competition and the complementarity effects.

To explore the regulatory impacts, we compare the equilibrium levels of the ISP's investment, the content prices, and the social welfare in the unregulated and regulated regimes as follows.

Investment. Compared with ISPs under an unregulated environment, the regulated ISP provides the positive quality upgrades at a lower level to CP_H , but at a higher level to CP_L , that is, $\mu_H^{UU} > \mu^{RP} > \mu^{RZ} > \mu_L^{UU}$. The ISP's total investment in case RZ and case RP are $TC^{RZ} = (\mu^{RZ})^2$ and $TC^{RP} = (\mu^{RP})^2$, respectively. Obviously, $TC^{RZ} < TC^{RP}$ because $\mu^{RZ} < \mu^{RP}$. Then, we find that the ISP's total investment is higher in the absence of regulation, that is, $TC^{UU} > TC^{RP}$.

Content prices. The content prices in cases RP and RZ , are denoted by p_i^{RP} and p_i^{RZ} , respectively where $i = H, L$, are dependent on the quality upgrades. In particular, because content H receives a lower quality upgrade under a nondiscrimination regulation, the price of content H becomes relatively lower, that is, $p_H^{UU} > p_H^{RP} > p_H^{RZ}$. The result holds true in the following case: the content L 's price if the degree of vertical differentiation becomes substantial (i.e., $\lambda > \bar{\lambda}$). By contrast, we can show that the content L 's price becomes lower in case RZ , despite it receiving a relatively higher quality upgrade in this case, that is, $p_L^{UU} > p_L^{RZ}$ if $\lambda < \bar{\lambda}$. This thus suggests that a nondiscrimination regulation could be a useful instrument to enhance the content market's competitiveness.

Profits and social welfare. Denoting π_{ISP}^{RZ} and π_{ISP}^{RP} for the ISP's profits in case RZ and case RP , respectively, we have $\pi_{ISP}^{RZ} < \pi_{ISP}^{RP} < \pi_{ISP}^{UU}$. Similarly, we compute the social welfare under both regulated regimes, denoted as W^{RZ} and W^{RP} . We find that the social welfare is also lower if the regulation is imposed, that is, $W^{UU} > W^{RP} > W^{RZ}$. This implies that the regulated ISP's profit is higher if the CPs must pay for the quality upgrades. In turn, the ISP would increase investment, resulting in higher social welfare.

In summary, we state the investment and welfare impacts of a nondiscrimination regulation in proposition 2.

Proposition 2 *The implementation of a nondiscrimination regulation, which could enhance competition between the CPs, reduces the ISP's total investment and social welfare. The impacts are lower if the regulated ISP is allowed to charge the CPs for the quality upgrades.*

Intuitively, a public policy that forces the ISP to provide nondiscriminatory quality could support long-term competition between the CPs because the unregulated ISP would dampen competition by increasing content differentiation, such as in proposition 1. Our previous analysis of the content prices indicates that a nondiscriminatory regime could also trigger intense price competition between the CPs for the low degree of content differentiation. Nonetheless, proposition 2 clearly states that obtaining this benefit comes at the cost of lowering the investment level and social welfare. In other words, our comparisons indicate regulatory trade-offs between the ISP's investment incentive and competition in the content market, and social welfare.

4.2 Social welfare maximization

In this subsection, we consider that the benevolent social planner chooses the quality upgrades while expecting the ISP and the CPs to set profit-maximizing prices for end users by considering their investment. Therefore, a socially optimal investment can be simply analyzed in a two-stage game as follows.

Stage 1: The social planner chooses quality upgrades to maximize social welfare.

Stage 2: Price competition occurs. End users consume according to their perceived utility.

The analysis is similar to that of subsection 3.2. Technically, the main difference between the ISP's and the socially optimal investment is that the planner maximizes the joint profits, as the ISP does, plus consumer surplus.

In particular, price competition in stage 2 yields the same prices (p, p_H, p_L) based on the social quality upgrades chosen by the planner. In turn, we let μ_H^{SU} and μ_L^{SU} denote

the social quality upgrades provided to CP_H and CP_L , respectively in stage 1, where:

$$\begin{aligned}\mu_H^{SU} &= \frac{(4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 17)\lambda + 8\beta^3 - 6\beta^2 - 35\beta - 7}{2(-\beta^3 - 3\beta^2 - 2\beta + 1)(\beta^3 - 5\beta^2 + 5\beta + 4)} \\ \mu_L^{SU} &= \frac{4\beta^4 + (8\lambda - 6)\beta^3 - (6\lambda + 16)\beta^2 - (35\lambda - 11)\beta - 7\lambda + 17}{2(-\beta^3 - 3\beta^2 - 2\beta + 1)(\beta^3 - 5\beta^2 + 5\beta + 4)}\end{aligned}$$

We observe that $\mu_H^{SU} = \mu_L^{SU}$ only when $\lambda = 1$. This occurs when content is symmetric, which is similar to the ISP's choice. In addition, if $\beta \lesssim 0.32$, we have $\mu_H^{SU} > 0$, but $\mu_L^{SU} < 0$ if $\lambda > \lambda_2 = \frac{4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 7}{-8\beta^3 + 6\beta^2 + 35\beta + 7}$.

Hence, in the range $\lambda > \lambda_2$, we must compute the social investment subject to the minimum quality constraint, that is $\mu_L^{SU} \geq 0$. Because the constraint will become binding, the socially optimal quality upgrades would be $\mu_H^{SL} > \mu_L^{SL} = 0$ where

$$\mu_H^{SL} = \frac{3(\lambda + 1)\beta^3 + (\lambda + 17)\beta^2 - 20(\lambda - 1)\beta - 24\lambda}{8\beta^4 + 29\beta^3 + 23\beta^2 - 12\beta - 8}$$

When $\beta \gtrsim 0.32$, we find that $\mu_L^{SU} > 0$, but $\mu_H^{SU} > 0$ if $\lambda < \lambda_3 = \frac{1}{\lambda_2}$. Thus, when $\lambda > \lambda_3$, it is socially optimal to provide the quality upgrade to only CP_L , that is, $\mu_L^{SH} > \mu_H^{SH} = 0$ where

$$\mu_L^{SH} = \frac{3(\lambda + 1)\beta^3 + (17\lambda + 1)\beta^2 + 20(\lambda - 1)\beta - 24}{8\beta^4 + 29\beta^3 + 23\beta^2 - 12\beta - 8}$$

This allows us to formulate the proposition 3.

Proposition 3 *Provision of asymmetric quality upgrades, in favor of CP_H when $\beta \lesssim 0.32$, or CP_L when $\beta \gtrsim 0.32$, is socially optimal.*

We explore the social investment as a function of the market parameters, namely, the degree of substitutability (λ), and the content differentiation (β) (figure 2).

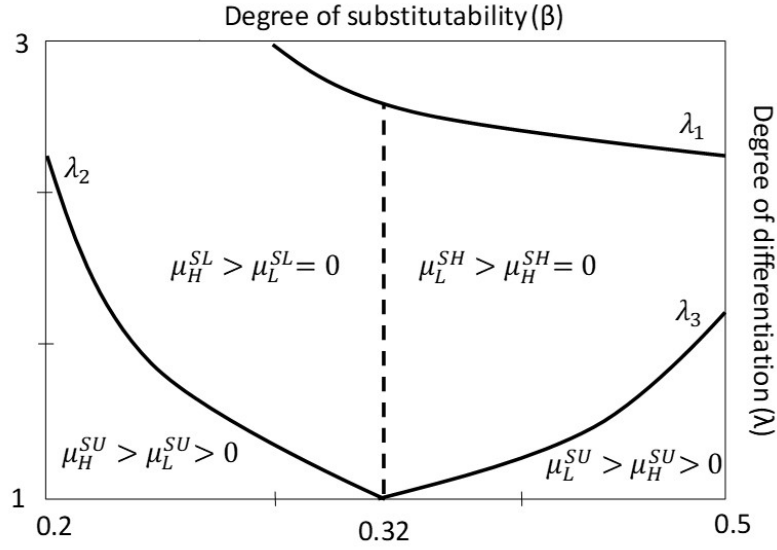


Figure 2 - The socially optimal quality upgrades

In figure 2, we only focus on the range $\lambda < \lambda_1$ in which the quantity of the content L is positive in case AH . When the content substitute is very low, that is, $\beta \lesssim 0.32$, the ISP's optimal investment is in line with the social choice but with a different level of asymmetry. Furthermore, the area below λ_2 (that is, $\lambda < \lambda_2$) shows the situation in which the ISP provides the positive quality upgrades to both CPs. In that case, our comparisons between the ISP's and the socially optimal quality upgrades show that $\mu_H^{SU} > \mu_H^{UU}$ and $\mu_L^{SU} < \mu_L^{UU}$. In addition, in the area above λ_2 which corresponds to the high values of λ (that is, $\lambda > \lambda_2$), only CP_L should be given the quality upgrade from the social perspective, that is, $\mu_H^{SL} > \mu_L^{SL} = 0$.

Notably, the opposite is chosen by the planner when $\beta \gtrsim 0.32$. More precisely, if $\lambda < \lambda_3$, it is socially desirable that CP_L receives a relative higher quality upgrade, which is obviously contrary to the ISP's optimal choice. Furthermore, only content L should be given the quality upgrade if $\lambda > \lambda_3$, that is, $\mu_L^{SH} > \mu_H^{SH} = 0$. Thus, if content H and content L are not too low substitutable, providing preferential treatment to content L can sharply force content H 's price down, increasing the consumer surplus²⁰. Subsequently, social welfare is higher because the increases in the consumer surplus and the ISP's profit derived from selling the internet service could outweigh the decrease in the CPs' profits. Intuitively, with an unregulated internet, the ISP's investment incentive is essentially determined by the differentiation effect. However, the competition effect becomes relatively

²⁰Price changes are provided in the Appendix.

more important to maximize social welfare when the degree of content substitutability is sufficiently high.

This result makes proposition 2 seem odd because it states that there is a negative impact of nondiscrimination rules on social welfare. The reason for this phenomenon is that the social quality upgrades do not consider the differentiation effect or the ISP's investment incentive. As previously analyzed, the ISP makes a relatively greater total investment under an unregulated environment; hence, imposing a nondiscrimination regulation could reduce consumer surplus and social welfare associated with the decreased investment.

5 Conclusion

The paper presents an economic model to analyze the investment decision in the digital economy while accounting for different regulatory restrictions and the ISP's ability to vary the quality of the internet service provided to two vertically differentiated CPs. Under the unregulated environment, the ISP obtains maximal rent extraction from CPs and invests in providing asymmetric quality upgrades, in favor of the low-quality CP. When a nondiscrimination regulation is implemented, investment and social welfare decrease. Moreover, the socially optimal investment might be in line with the ISP's choice if the content substitute is sufficiently low; if not, the low-quality content should be given a relatively higher quality upgrade than the high-quality content.

Our results indicate two notable policy implications regarding the investment and welfare effects of regulation. First, the opponents of net neutrality regulation often claim that the rules could hinder investment because the ISP essentially obtains a relatively lower profit from an investment. This remains true in this article because the ISP makes a relatively higher investment if unregulated. By contrast, we show that a nondiscriminatory quality regulation is a useful policy to promote competition in the content market. In this context, the investment incentive is higher if the regulated ISP is allowed to charge the CPs for the improved quality of service. Second, from the social welfare perspective, a nondiscriminatory regime is desirable only in the very strict condition (i.e., symmetric CPs). However, although the ISP always provides a higher quality upgrade to the high-quality CP, the opposite provision is chosen by the social planner if the content substitute

is sufficiently high. This finding suggests regulatory trade-offs between the ISP's investment incentive, competition in the content market, and social welfare. In other words, few regulatory instruments must be employed to promote investment in broadband infrastructure and competition in the content market.

Finally, the analysis could be extended by allowing a CP to take an active part in improving the content's perceived quality through applying congestion control techniques, potentially affecting the bargaining between the ISP and the CPs. In addition, in our model, the ISP cannot degrade the minimum quality level. However, this situation is very relevant for the implementation of net neutrality regulation in practice. Therefore, further research on investment incentives and regulation of the internet might consider that CPs can take costly actions to increase the quality of content or delivery speed, and manage the consequences of quality sabotage.

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6 Appendix

Proof of subsection 3.2.

Case AH. The content perceived qualities are $\alpha_H = \lambda + \mu_H$ and $\alpha_L = 1$.

In stage 3, we solve for the firms' maximization problems in taking the quality upgrades as given.

The optimal prices and quantities that are functions of the quality upgrades are as follows:

$$p = \frac{\mu_H + \lambda + 1}{2(3 - \beta)} \quad (12)$$

$$p_H = \frac{(2\beta^2 - \beta - 5)(\mu_H + \lambda) + 3\beta + 1}{2(\beta - 3)(2 + \beta)} \quad (13)$$

$$p_L = \frac{(3\beta + 1)(\mu_H + \lambda) + 2\beta^2 - \beta - 5}{2(\beta - 3)(2 + \beta)} \quad (14)$$

Thus, CP_L 's price is lower while CP_H 's price is higher if the quality upgrade is provided only to CP_H .

$$q_H = \frac{(2\beta^2 - \beta - 5)(\mu_H + \lambda) + 3\beta + 1}{2(2 + \beta)(\beta - 3)(1 - \beta^2)} \quad (15)$$

$$q_L = \frac{(3\beta + 1)(\mu_H + \lambda) + 2\beta^2 - \beta - 5}{2(2 + \beta)(\beta - 3)(1 - \beta^2)} \quad (16)$$

Using the firms' optimal prices in (12), (13), (14), and quantities in (15) and (16), we compute the ISP's profit from both the internet and content H as follows.

$$\pi_{ISP} = \frac{(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)\mu_H^2 + [8\beta^3\lambda + 4(3^2\lambda + 2)\beta^2 - 2(7\lambda - 5)\beta - 22\lambda - 2]\mu_H + [4\beta^3\lambda^2 + (6\lambda^2 + 8\lambda - 2)\beta^2 - (7\lambda^2 - 10\lambda + 3)\beta - 11\lambda^2 - 2\lambda - 3] - \pi_H^{AL}}{4(1 - \beta^2)(\beta - 3)(\beta + 2)^2} \quad (17)$$

Solving the FOC of (17), we obtain the ISP's optimal quality provided to CP_H in stage 2 as follows:

$$\mu_H^A = \frac{-4\beta^3\lambda - (6\lambda + 4)\beta^2 + (7\lambda - 5)\beta + 11\lambda + 1}{2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13} \quad (18)$$

The SOC is satisfied as $\partial_{\mu_H}'' \pi_{ISP} < 0$.

Substituting (18) into (16) and (14), we easily show the quantity and the profit of CP_L .

$$q_L^{AH} = \frac{2\beta^4 + 3(\lambda - 1)\beta^3 - 2(\lambda + 6)\beta^2 - (19\lambda - 7)\beta - 6\lambda + 16}{(3 - \beta)(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)} \quad (19)$$

The denominator of (19) is positive; thus, $q_L^{AH} > 0$ when $\lambda < \lambda_1 = \frac{2\beta^4 - 3\beta^3 - 12\beta^2 + 7\beta + 16}{(3\beta + 1)(3 - \beta)(\beta + 2)}$.

$$\pi_L^{AH} = \frac{(1 - \beta^2)[2\beta^4 + 3(\lambda - 1)\beta^3 - 2(\lambda + 6)\beta^2 - (19\lambda - 7)\beta - 6\lambda + 16]^2}{(3 - \beta)^2(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)^2} \quad (20)$$

Case AL. The perceived qualities are $\alpha_H = \lambda$ and $\alpha_L = 1 + \mu_L$.

In stage 3, firms' profit maximizations lead to the following prices:

$$p = \frac{\mu_L + \lambda + 1}{2(3 - \beta)} \quad (21)$$

$$p_L = \frac{(2\beta^2 - \beta - 5)(\mu_L + 1) + (3\beta + 1)\lambda}{2(3 - \beta)(\beta + 2)} \quad (22)$$

$$p_H = \frac{(3\beta + 1)(\mu_L + 1) + (2\beta^2 - \beta - 5)\lambda}{2(3 - \beta)(\beta + 2)} \quad (23)$$

$$q_H = \frac{(3\beta + 1)(\mu_L + 1) + (2\beta^2 - \beta - 5)\lambda}{2(1 - \beta^2)(\beta - 3)(\beta + 2)} \quad (24)$$

$$q_L = \frac{(2\beta^2 - \beta - 5)(\mu_L + 1) + (3\beta + 1)\lambda}{2(1 - \beta^2)(\beta - 3)(\beta + 2)} \quad (25)$$

As in the previous case, we compute the ISP's profits from the internet and content L as follows:

$$\pi_{ISP} = \frac{(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)\mu_L^2 + [8\beta^3 + 4(2\lambda + 3)\beta^2 + 2(5\lambda - 7)\beta - 2\lambda - 22]\mu_L + [4\beta^3 - 2(\lambda^2 - 4\lambda - 3)\beta^2 - (3\lambda^2 - 10\lambda + 7)\beta - 3\lambda^2 - 2\lambda - 11] - \pi_L^{AH}}{4(1 - \beta^2)(\beta - 3)(\beta + 2)^2} \quad (26)$$

We now solve the FOC of (26) to obtain the ISP's optimal quality upgrade provided to CP_L in stage 2.

$$\mu_L^A = \frac{-4\beta^3 - (4\lambda + 6)\beta^2 - (5\lambda - 7)\beta + \lambda + 11}{2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13} \quad (27)$$

The SOC is satisfied as $\partial_{\mu_L}'' \pi_{ISP} < 0$.

In this case, the profit of CP_H is given by:

$$\pi_H^{AL} = \frac{(1 - \beta^2)[2\lambda\beta^4 - 3(\lambda - 1)\beta^3 - 2(6\lambda + 1)\beta^2 + (7\lambda - 19)\beta + 16\lambda - 6]^2}{(3 - \beta)^2(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)^2} \quad (28)$$

The profit difference is $\Delta\pi = \pi_H^{AL} - \pi_L^{AH}$, and we have

$$\Delta\pi = \frac{4(1 - \beta^2)(\lambda^2 - 1)(\beta^4 - 7\beta^2 - 6\beta + 5)(\beta^4 - 3\beta^3 - 5\beta^2 + 13\beta + 11)}{(3 - \beta)^2(2\beta^5 + 2\beta^4 - 14\beta^3 - 20\beta^2 + 9\beta + 13)^2} \quad (29)$$

We verify that $\frac{\partial\Delta\pi}{\partial\lambda} > 0$ and $\frac{\partial\Delta\pi}{\partial\beta} < 0$.

Proof of subsection 3.3.

In stage 3, we simultaneously solve the firms' profit maximizations in (6) and derive the ISP's pricing as follows:

$$p = \frac{\mu_H + \mu_L + \lambda + 1}{2(3 - \beta)} \quad (30)$$

Likewise, the price of content i is increasing in its quality upgrade but decreasing in content j 's quality upgrade, as follows.

$$p_H = \frac{(-2\beta^2 + \beta + 5)\mu_H - (3\beta + 1)\mu_L - 2\beta^2\lambda + \beta\lambda - 3\beta + 5\lambda - 1}{2(3 - \beta)(\beta + 2)} \quad (31)$$

$$p_L = \frac{(-2\beta^2 + \beta + 5)\mu_L - (3\beta + 1)\mu_H - 2\beta^2 - 3\beta\lambda + \beta - \lambda + 5}{2(3 - \beta)(\beta + 2)} \quad (32)$$

Hence, content demand functions are

$$q_H = \frac{(-2\beta^2 + \beta + 5)\mu_H - (3\beta + 1)\mu_L - 2\beta^2\lambda + \beta\lambda - 3\beta + 5\lambda - 1}{2(3 - \beta)(\beta + 2)(1 - \beta^2)} \quad (33)$$

$$q_L = \frac{(-2\beta^2 + \beta + 5)\mu_L - (3\beta + 1)\mu_H - 2\beta^2 - 3\beta\lambda + \beta - \lambda + 5}{2(3 - \beta)(\beta + 2)(1 - \beta^2)} \quad (34)$$

Using (30), (31), (32), (33) and (34), we rewrite the ISP's profit given in (7) as $F(\mu_H, \mu_L)$ which is a quadratic polynomial function with two variables, namely, μ_H and μ_L . By solving the FOCs simultaneously, we obtain the ISP's optimal quality upgrades:

$$\mu_H^{UU} = \frac{2\beta^4\lambda - (3\lambda - 5)\beta^3 - 4(\lambda + 1)\beta^2 + 4(\lambda + 4)\beta + 9\lambda - 1}{-\beta^6 + 2\beta^5 + 8\beta^4 - 8\beta^3 - 35\beta^2 + 10} \quad (35)$$

$$\mu_L^{UU} = \frac{2\beta^4 + (5\lambda - 3)\beta^3 - 4(\lambda + 1)\beta^2 - 4(4\lambda - 1)\beta - \lambda + 9}{-(\beta^3 + 3\beta^2 + 2\beta - 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \quad (36)$$

We also verify that the Hessian matrix is negative definite at μ_H^{UU} and μ_L^{UU} , indicating that the quality upgrades correspond to a maximum.

We check that $\mu_H^{UU} > 0$ and $\mu_L^{UU} > 0$ when $0 < \beta < \frac{1}{2}$ and $1 < \lambda < \lambda_1$.

We define $\Delta\mu = \mu_H^{UU} - \mu_L^{UU} = \frac{2(\lambda-1)(\beta+1)}{-\beta^3-3\beta^2-2\beta+2} > 0$.

Finally, $\frac{\partial\Delta\mu}{\partial\beta} > 0$, and $\Delta\mu = 0$ if $\lambda = 1$.

The equilibrium prices are

$$p^{UU} = \frac{(\beta + 1)(3 - \beta)(\lambda + 1)}{2(\beta^3 - 5\beta^2 + 5\beta + 5)} \quad (37)$$

We check that $\frac{\partial p^{UU}}{\partial\lambda} > 0$, and $\frac{\partial p^{UU}}{\partial\beta} < 0$ if $\beta \lesssim 0.27$.

$$p_H^{UU} = \frac{(\beta^2 - 1)[2\lambda\beta^4 - 3(\lambda - 1)\beta^3 - 2(6\lambda + 1)\beta^2 + (7\lambda - 23)\beta + 16\lambda - 4]}{2(\beta^3 - 5\beta^2 + 5\beta + 5)(\beta^3 + 3\beta^2 + 2\beta - 2)} \quad (38)$$

We check that $\frac{\partial p_H^{UU}}{\partial\lambda} > 0$.

Additionally, $\frac{\partial p_H^{UU}}{\partial\beta} < 0$ if $\lambda < \hat{\lambda} = -\frac{3\beta^{10}-4\beta^9-50\beta^8+48\beta^7+36\beta^6+12\beta^5+452\beta^4+160\beta^3+121\beta^2+240\beta+230}{\beta^{10}+4\beta^9-14\beta^8-160\beta^7+144\beta^6+612\beta^5+52\beta^4-160\beta^3-329\beta^2-560\beta-70}$.

$$p_L^{UU} = \frac{(\beta^2 - 1)[2\beta^4 + 3(\lambda - 1)\beta^3 - 2(\lambda + 6)\beta^2 - (23\lambda - 7)\beta - 4\lambda + 16]}{2(\beta^3 - 5\beta^2 + 5\beta + 5)(\beta^3 + 3\beta^2 + 2\beta - 2)} \quad (39)$$

We check that $\frac{\partial p_L^{UU}}{\partial\beta} < 0$ and $\frac{\partial p_L^{UU}}{\partial\lambda} < 0$.

The ISP's total investment is $TC^{UU} = \frac{(\mu_H^{UU})^2 + (\mu_L^{UU})^2}{2}$, and we can show that

$$TC^{UU} = \frac{(2\beta^8 - 6\beta^7 + 9\beta^6 - 58\beta^4 + 16\beta^3 + 104\beta^2 + 52\beta + 41)(\lambda^2 + 1) + (20\beta^7 - 46\beta^6 - 50\beta^5 + 164\beta^4 + 192\beta^3 - 192\beta^2 - 296\beta - 18)\lambda}{(\beta^3 + 3\beta^2 + 2\beta - 2)^2(\beta^3 - 5\beta^2 + 5\beta + 5)^2} \quad (40)$$

Now, we substitute the prices and the quantities in (30), (31), (32), (33) and (34) and the optimal quality upgrades in (35), (36) into (1) and (2) to derive the equilibrium

consumer surplus (CS^{UU}) and the ISP's profit (π_{ISP}^{UU}), and consequently the social welfare $W^{UU} = \pi_{ISP}^{UU} + \pi_{ISP}^{AL} + \pi_L^{AH} + CS^{UU}$, which are the polynomials of a higher degree of the market parameters (i.e., λ and β).

Proof of subsection 4.1.

Price competition in stage 3 leads to the prices and the quantities as functions of investment as follows:

$$p = \frac{2\mu + \lambda + 1}{2(3 - \beta)} \quad (41)$$

$$p_H = \frac{2(\beta^2 + \beta - 2)\mu + 2\beta^2\lambda - \beta\lambda + 3\beta - 5\lambda + 1}{2(\beta - 3)(\beta + 2)} \quad (42)$$

$$p_L = \frac{2(\beta^2 + \beta - 2)\mu + 2\beta^2 + 3\beta\lambda - \beta + \lambda - 5}{2(\beta - 3)(\beta + 2)} \quad (43)$$

$$q_H = \frac{2(\beta^2 + \beta - 2)\mu + 2\beta^2\lambda - \beta\lambda + 3\beta - 5\lambda + 1}{2(1 - \beta^2)(\beta - 3)(\beta + 2)} \quad (44)$$

$$q_L = \frac{2(\beta^2 + \beta - 2)\mu + 2\beta^2 + 3\beta\lambda - \beta + \lambda - 5}{2(1 - \beta^2)(\beta - 3)(\beta + 2)} \quad (45)$$

Case RZ. The ISP is obliged to provide a non-negative equal quality upgrade free of charge.

From (41), (42), (43), (44) and (45), we easily derive the ISP's profit given in (9) as a function of investment:

$$\pi_{ISP} = \frac{-(2\beta^3 - 10\beta^2 + 6\beta + 14)\mu^2 + 4(\lambda + 1)\mu + (\lambda + 1)^2}{2(\beta + 1)(\beta - 3)^2} \quad (46)$$

Solving the FOC of (46), we obtain the ISP's optimal investment as follows:

$$\mu^{RZ} = \frac{\lambda + 1}{\beta^3 - 5\beta^2 + 3\beta + 7} \quad (47)$$

The SOC is satisfied as $\partial_\mu'' \pi_{ISP} < 0$.

We check that $\frac{\partial \mu^{RZ}}{\partial \lambda} > 0$, and $\frac{\partial \mu^{RZ}}{\partial \beta} > 0$ if $\beta \gtrsim 0.32$.

The ISP's total investment is

$$TC^{RZ} = (\mu^{RZ})^2 = \left(\frac{\lambda + 1}{\beta^3 - 5\beta^2 + 3\beta + 7} \right)^2 \quad (48)$$

The content prices are

$$p_H^{RZ} = \frac{(\beta + 1)[2\beta^3\lambda - (7\lambda - 3)\beta^2 - 2(\lambda + 4)\beta + 13\lambda - 1]}{2(\beta + 2)(\beta^3 - 5\beta^2 + 3\beta + 7)} \quad (49)$$

$$p_L^{RZ} = \frac{(\beta + 1)[2\beta^3 + (3\lambda - 7)\beta^2 - 2(4\lambda + 1)\beta - \lambda + 13]}{2(\beta + 2)(\beta^3 - 5\beta^2 + 3\beta + 7)} \quad (50)$$

As previously analyzed, after tedious calculus, we obtain the ISP's profits (π_{ISP}^{RZ}), the consumer surplus (CS^{RZ}) and the social welfare (W^{RZ}), which are the polynomials of higher degree of the market parameters (i.e., λ and β).

Case RP. The ISP must provide equal quality upgrades, and it can receive a positive equal lump-sum payment.

From (41), (42), (43), (44) and (45), we easily derive the ISP's profit given in (11) as a function of investment:

$$\pi_{ISP}^{RP} = \frac{(2\beta^4 - 6\beta^3 - 10\beta^2 + 30\beta + 20)\mu^2 + 4(2\beta^2 + 2\beta\lambda - 2\beta - \lambda - 7)\mu - \beta\lambda^2 - 2\beta\lambda - 2\lambda^2 - \beta - 4\lambda - 2}{(\beta + 2)(\beta + 1)(\beta - 3)^2} \quad (51)$$

Now, solving for the FOC of (51), we obtain the ISP's optimal investment in stage 1:

$$\mu^{RP} = \frac{-2\beta^2 - 2\beta\lambda + 2\beta + \lambda + 7}{(\beta + 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \quad (52)$$

The SOC is satisfied as $\partial_\mu''\pi_{ISP} < 0$.

We check that $\frac{\partial\mu^{RP}}{\partial\lambda} > 0$, and $\frac{\partial\mu^{RZ}}{\partial\beta} < 0$.

The content prices are

$$p_H^{RP} = \frac{(\beta + 1)[2\beta^3\lambda - (7\lambda - 3)\beta^2 + 2(\lambda - 6)\beta + 9\lambda + 3]}{2(\beta + 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \quad (53)$$

$$p_L^{RP} = \frac{(\beta + 1)[2\beta^3 + (3\lambda - 7)\beta^2 - 2(4\lambda + 1)\beta - \lambda + 13]}{2(\beta + 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \quad (54)$$

The ISP's total investment is

$$TC^{RP} = (\mu^{RP})^2 = \left(\frac{-2\beta^2 - 2\beta\lambda + 2\beta + \lambda + 7}{(\beta + 2)(\beta^3 - 5\beta^2 + 5\beta + 5)} \right)^2 \quad (55)$$

Thus, $TC^{RP} > TC^{RZ}$ because $\mu^{RP} > \mu^{RZ}$.

Now, we can straightforwardly calculate the ISP's profits (π_{ISP}^{RZ}), the consumer surplus (CS^{RZ}) and the social welfare (W^{RZ}) which are the polynomials of the higher degree of the market parameters (i.e., λ and β).

Now, we can conduct a few comparisons with the range of parameter values $\beta < \frac{1}{2}$ and $1 < \lambda < \lambda_1$ as follows:

First, from (35), (36), (47) and (52), we can show that $\mu_H^{UU} > \mu^{RP} > \mu^{RZ} > \mu_L^{UU}$.

Second, from (40), (48) and (55), we find that $TC^{UU} > TC^{RP} > TC^{RZ}$.

Third, from (38), (49) and (53), we find that $p_H^{UU} > p_H^{RP} > p_H^{RZ}$. Similarly, from (39), (50) and (54), we have $p_L^{RP} > p_L^{RZ} > p_L^{UU}$ if $\lambda > \bar{\lambda} = -\frac{2\beta^7 - 9\beta^6 + 13\beta^5 - \beta^4 - 46\beta^3 + 15\beta^2 + 51\beta + 47}{9\beta^6 - 33\beta^5 - 11\beta^4 + 100\beta^3 + 5\beta^2 - 119\beta - 23}$. By contrast, we can show that $p_L^{RP} > p_L^{UU} > p_L^{RZ}$ if $\lambda < \bar{\lambda}$.

In addition, we can show that $\pi_{ISP}^{UU} > \pi_{ISP}^{RP} > \pi_{ISP}^{RZ}$, and $W^{UU} > W^{RP} > W^{RZ}$.

Finally, we can compute the CP_H 's profit, and compare it with CP_H 's profit in case AL , as follows:

$$\pi_H^{RP} = \frac{(1 + \beta)[24\beta^9\lambda^2 - 12(4\lambda - 1)\beta^8\lambda + 3(67\lambda^2 - 50\lambda + 8)\beta^7 - (261\lambda^2 - 694\lambda + 253)\beta^6 - (373\lambda^2 + 1254\lambda - 847)\beta^5 + (1047\lambda^2 - 166\lambda - 701)\beta^4 + (259\lambda^2 + 3014\lambda - 1473)\beta^3 - (1799\lambda^2 + 1734\lambda - 2133)\beta^2 + (285\lambda^2 - 1626\lambda + 841)\beta + 1069\lambda^2 + 634\lambda - 1203]}{4(1 - \beta)(\beta^3 - 5\beta^2 + 3\beta + 8)^2(\beta^3 - 5\beta^2 + 5\beta + 5)(\beta + 2)^2} > \pi_H^{AL}$$

Proof of subsection 4.2.

Price competition in stage 2 leads to prices and quantities as given in (30), (31), (32), (33) and (34). Thus, it is straightforward for us to obtain the social welfare: the total sum of firms' profits and consumer surplus, denoted as $G(\mu_H, \mu_H)$, which is a function of the second degree of μ_H and μ_H .

Solving the FOCs simultaneously, and after tedious calculus, we obtain the socially optimal investment as follows:

$$\mu_H^{SU} = \frac{(4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 17)\lambda + 8\beta^3 - 6\beta^2 - 35\beta - 7}{(-\beta^3 - 3\beta^2 - 2\beta + 1)(\beta^3 - 5\beta^2 + 5\beta + 4)} \quad (56)$$

$$\mu_L^{SU} = \frac{(8\beta^3 - 6\beta^2 - 35\beta - 7)\lambda + 4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 17}{(4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 17)\lambda + 8\beta^3 - 6\beta^2 - 35\beta - 7} \quad (57)$$

Notably, the concavity is ensured by computing the Hessian matrix.

From (56), we check that when $\beta \lesssim 0.32$, we have $\mu_H^{SU} > 0$, but $\mu_L^{SU} > 0$ if $\lambda < \lambda_2$ where

$$\lambda_2 = \frac{4\beta^4 - 6\beta^3 - 16\beta^2 + 11\beta + 7}{-8\beta^3 + 6\beta^2 + 35\beta + 7}$$

Furthermore, in this range, we can compare that $\mu_H^{SU} > \mu_H^{UU}$ and $\mu_L^{SU} < \mu_L^{UU}$.

Hence when $\beta \lesssim 0.32$ and $\lambda > \lambda_2$, we have the socially optimal investment subject to the minimum quality condition, that is, $\mu_H \geq 0$. Then the Lagrangian is

$$\mathbf{L}_W = G(\mu_H, \mu_H) + \Phi \mu_H \quad (58)$$

where Φ is the Lagrange multiplier.

Solving simultaneously for the FOCs of (58) in stage 1, we can obtain the socially optimal quality upgrades under the minimum quality constraint as follows:

$$\mu_H^{SL} = \frac{3(\lambda + 1)\beta^3 + (\lambda + 17)\beta^2 - 20(\lambda - 1)\beta - 24\lambda}{8\beta^4 + 29\beta^3 + 23\beta^2 - 12\beta - 8} \quad (59)$$

$$\mu_L^{SL} = 0 \quad (60)$$

Notably, the concavity is ensured by computing the Hessian matrix.

Similarly, from (57), we check that when $\beta \gtrsim 0.32$, we have $\mu_L^{SU} > 0$ but $\mu_H^{SU} > 0$ if $\lambda > \lambda_3 = \frac{1}{\lambda_2}$.

Thus, when $\beta \gtrsim 0.32$ and $\lambda > \lambda_3$, we have the socially optimal investment subject to the minimum quality condition, that is, $\mu_L \geq 0$. Using Lagrange's equation and solving simultaneously for the FOCs in stage 1, we can obtain the socially optimal quality upgrades under the minimum quality constraint as follows:

$$\mu_L^{SH} = \frac{3(\lambda + 1)\beta^3 + (17\lambda + 1)\beta^2 + 20(\lambda - 1)\beta - 24}{8\beta^4 + 29\beta^3 + 23\beta^2 - 12\beta - 8} \quad (61)$$

$$\mu_H^{SH} = 0 \quad (62)$$

Notably, the concavity is ensured by computing the Hessian matrix.