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CESifo Working Paper, No. 7977

**Provided in Cooperation with:** Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Gersbach, Hans; Haller, Hans (2019) : Gainers and Losers from Market Integration, CESifo Working Paper, No. 7977, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at: https://hdl.handle.net/10419/214979

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### Impressum:

CESifo Working Papers ISSN 2364-1428 (electronic version) Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute Poschingerstr. 5, 81679 Munich, Germany Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de Editor: Clemens Fuest www.cesifo-group.org/wp

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## Gainers and Losers from Market Integration

## Abstract

We compare integration of economic, matching and networking markets. There can be losers from integration in all three cases, but their relative numbers depend on the type of market. There can be many losers from integration of pure exchange economies. There are relatively few losers from integration of networking markets. In the matching case, the relative numbers tend to lie between those of the other two cases.

JEL-Codes: C780, D020, D850.

Keywords: competitive exchange, matching theory, networks, market integration.

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This Version: November 2019

We would like to thank David Adjiashvili, Eric Bahel, Mario Gersbach, Bettina Klaus, Jeff Kline, Vincent Lohmann, Jonathan Lorand, Jan Christoph Schlegel and Simon Skok for helpful comments.

## 1 Introduction

When two hitherto separate economic or social systems are merged to become one, all previous outcomes are still feasible. Therefore, the integrated system can, in principle, achieve better or equally good outcomes. This optimistic outlook is correct from a social planner's perspective. Yet, some individuals can be worse off after integration when the outcome is determined in a competitive or strategic equilibrium.

Both theory and casual empiricism suggest that economic integration can have beneficial effects on some economic agents and detrimental effects on others. In this paper, we make some simple comparisons how integration in three well understood equilibrium models produces gainers and losers: competitive pure exchange economies, two-sided pairwise matching and strategic network formation.

We first consider pure exchange economies in the sense of Arrow-Debreu, that is economies as described in Debreu (1959). Second, we consider two-sided pairwise matching studied by Gale and Shapley (1962) and Roth and Sotomayor (1990), with stable matching as the solution concept. Finally, we consider strategic network formation. A number of recent contributions have treated social and economic networks as the outcome of a network formation game. The players of the game constitute the nodes of the network to be formed. In the purely noncooperative approach of Bala and Goyal (2000) adopted here, addition and deletion of links are unilateral decisions of the player from whom the respective links originate.<sup>1</sup> The player's strategy is a specification of the set of agents with whom he forms links. The costs of link formation are incurred only by the player who initiates the link. The formed links define the network.

Our investigation is focused on the gainers and losers from integration and, more precisely, their relative numbers. In the pure exchange context, almost all but not all consumers can lose. In the matching context, less than half of the members of each group can be losers. In the strategic network formation setting,

<sup>&</sup>lt;sup>1</sup>Pairwise stability à la Jackson and Wolinsky (1996) treats addition of a link in a network as a bilateral decision by the two players involved, whereas severance of a link constitutes a unilateral decision.

integration of two groups causes at most half the members plus one in one group to be losers and nobody in the other group. We shall comment on these results in Section 5 and relate them to enduring debates about gainers and losers from market integration in the literature.

The next three sections are devoted to the three different integration scenarios.

## 2 Pure Exchange Economies

Pure exchange economies belong to the canon of contemporary microeconomic theory. Therefore, we confine their formal description to the bare minimum.

#### 2.1 Brief Outline of the Model

A finite pure exchange economy is specified by a tuple  $\mathcal{E} = (X_i, \succeq_i, \omega_i)_{i \in I}$ . The economy consists of a finite set I of consumers. There exist a finite number of commodities  $l = 1, \ldots, \ell$ . Each consumer has consumption set  $X_i = \mathbb{R}^{\ell}_+$ . Superscripts denote commodities. Hence  $x_i^l$  stands for the quantity of commodity l consumed by  $i \in I$ . Consumer i's consumption bundles assume the form  $x_i = (x_i^1, \ldots, x_i^{\ell})$ . In our general theory,  $\ell$  can be any finite number. In the numerical examples,  $\ell = 2$ . Consumer i has complete and transitive preferences on  $X_i$ , represented by the binary relation  $\succeq_i$ . Finally, each consumer is endowed with a commodity bundle  $\omega_i \in X_i$ .

The following definitions apply to the economy  $\mathcal{E}$  at large:

Feasible Allocations:	A feasible allocation is a vector
	$x = (x_i)_{i \in I} \in \mathcal{X} \equiv \prod_{i \in I} X_i$ such that
	$\sum_{i \in I} x_i = \sum_{i \in I} \omega_i.$
Pareto Optima:	A feasible allocation $x = (x_i)_{i \in I}$ is
	Pareto optimal if there is no feasible allocation
	$(x'_i)_{i \in I}$ such that $x'_i \succ_i x_i$ for some $i \in I$ and
	$x'_i \succeq_i x_i \text{ for all } i \in I.$
Weak Pareto Optima:	A feasible allocation $x = (x_i)_{i \in I}$ is weakly
	Pareto optimal if there is no feasible allocation
	$(x'_i)_{i \in I}$ such that $x'_i \succ_i x_i$ for all $i \in I$ .
Competitive Equilibrium:	A competitive equilibrium is a pair $(x^*, p^*)$
	where $x^* = (x_i^*)_{i \in I}$ is a feasible allocation,
	$p^* \in \mathbb{R}_+^\ell$ is a price system, and for each consumer
	$i \in I$ :
	$p^* x_i^* \leq p^* \omega_i$ and $x_i \in X_i, x_i \succ_i x_i^*$ implies $p^* x_i > p^* \omega_i$ .
	$x^*$ is called an equilibrium allocation. $p^*$ is called
	an equilibrium or market clearing price system.

We shall invoke the following version of the first welfare theorem.

Proposition 1 (First Welfare Theorem).

- (a) Equilibrium allocations are weakly Pareto optimal.
- (b) If preferences are locally non-satiated, then equilibrium allocations are Pareto optimal.

Notice that (a) holds without any assumption on preferences whereas (b) holds for locally non-satiated and transitive but not necessarily complete preferences.

By Proposition 11.C.1 of Mas-Colell, Whinston and Green (1995), a finite pure exchange economy  $\mathcal{E} = (X_i, \succeq_i, \omega_i)_{i \in I}$  has a competitive equilibrium if  $\sum_{i \in I} \omega_i \gg 0$ and every consumer has continuous, strictly convex and strongly monotone preferences. These sufficient conditions are not necessary. In particular, a competitive equilibrium exists if all consumers have Cobb-Douglas preferences. However, nonexistence may occur if the sufficient conditions are violated. Of special interest to us is the case where each of two separate pure exchange economies has a competitive equilibrium, but there is no competitive equilibrium of the integrated economy. This means that market integration can have a destabilizing effect. Such a case will be presented in 2.4. We first consider a simple example to illustrate how integration generates gainers and losers.

#### 2.2 A First Example

We consider the separate economies of Aland and Eland and then the economy after integration of the two. The economy of Aland consists of three consumers, Anna, Bart, and Carl, labeled a, b and c, respectively. Let  $I_A = \{a, b, c\}$  denote the set of these consumers. Each  $i \in I_A$  has an endowment  $\omega_i = (1, 1)$  and consumption set  $X_i = \mathbb{R}^2_+$ . Preferences are given by the utility functions

$$U_{a}(x_{a}^{1}, x_{a}^{2}) = x_{a}^{1}(x_{a}^{2})^{3} \text{ for Anna;} U_{b}(x_{b}^{1}, x_{b}^{2}) = x_{b}^{1}x_{b}^{2} \text{ for Bart;} U_{c}(x_{c}^{1}, x_{c}^{2}) = \min\{x_{c}^{1}, x_{c}^{2}\} \text{ for Carl.}$$

Up to price normalization, this economy has a unique competitive equilibrium  $(x_A^*, p_A^*)$  given as  $p_A^* = (1, 5/3), x_A^* = (x_a^*, x_b^*, x_c^*), x_a^* = (2/3, 6/5), x_b^* = (4/3, 4/5), x_c^* = (1, 1), U_a(x_a^*) = 144/125, U_b(x_b^*) = 16/15, U_c(x_c^*) = 1.$ 

The economy of Eland consists of two consumers, Dennis and Esther, labeled d, and e, respectively. Let  $I_E = \{d, e\}$  denote the set of these consumers. Each  $i \in I_E$  has an endowment  $\omega_i = (1, 1)$  and consumption set  $X_i = \mathbb{R}^2_+$ . Preferences are given by the utility functions

$$U_d(x_d^1, x_d^2) = (x_d^1)^3 x_d^2 \text{ for Dennis;}$$
  
$$U_e(x_e^1, x_e^2) = x_e^1 x_e^2 \text{ for Esther.}$$

Up to price normalization, this economy has a unique competitive equilibrium

 $(x_E^*, p_E^*)$  given as  $p_E^* = (1, 3/5), x_E^* = (x_d^*, x_e^*), x_d^* = (6/5, 2/3), x_e^* = (4/5, 4/3), U_d(x_d^*) = 144/125, U_e(x_e^*) = 16/15.$ 

In the integrated economy with consumers in  $I = I_A \cup I_E$ , the unique equilibrium (up to price normalization) is  $(\hat{x}, \hat{p})$  with  $\hat{p} = (1, 1)$ ,  $\hat{x} = (\hat{x}_a, \hat{x}_b, \hat{x}_c, \hat{x}_d, \hat{x}_e)$ ,  $\hat{x}_a = (1/2, 3/2), \hat{x}_b = (1, 1), \hat{x}_c = (1, 1), \hat{x}_d = (3/2, 1/2), \hat{x}_e = (1, 1), U_a(\hat{x}_a) = 27/16, U_b(\hat{x}_b) = 1, U_c(\hat{x}_c) = 1, U_d(\hat{x}_d) = 27/16, U_e(\hat{x}_e) = 1$ . It follows that Anna and Dennis are gainers from integration, Bart and Esther are losers, and Carl is unaffected by integration. In this particular example, Bart and Esther actively trade in the respective equilibria before integration and do not trade after integration. More commonly, a consumer might trade before and after market integration, but nevertheless lose from integration.

Further notice that the allocation  $x^* = (x_i^*)_{i \in I}$  is of course feasible in the integrated economic, but not weakly Pareto optimal. It is strictly dominated by some feasible allocation x'. However, it is not dominated by  $\hat{x}$ .

#### 2.3 More General Analysis

We consider two countries whose economies are initially separated and compare original equilibrium outcomes with equilibrium allocations after market integration. As a consequence of the first welfare theorem, we obtain

#### Proposition 2.

- (a) Not all consumers can be losers from market integration.
- (b) In case preferences are locally non-satiated, if there exists a loser, then there also exists a gainer from market integration.

PROOF. Let the two countries A and B have respective sets of consumers  $I_A$  and  $I_B$ . Set  $I = I_A \cup I_B$ . Before integration, let  $(p_A^*, x_A^*)$  be a competitive equilibrium for A's economy, with  $x_A^* = (x_i^*)_{i \in I_A}$ , and  $(p_B^*, x_B^*)$  be a competitive equilibrium for B's economy, with  $x_B^* = (x_i^*)_{i \in I_B}$ . Furthermore, consider a competitive equilibrium

 $(\hat{x}, \hat{p})$  of the integrated economy, with  $\hat{x} = (\hat{x}_i)_{i \in I}$ . Finally, define  $x^* = (x_i^*)_{i \in I}$ , a feasible allocation of the integrated economy.

(a) Suppose all consumers are losers from market integration, that is,  $x_i^* \succ_i \hat{x}_i$  for all  $i \in I$ . Then  $\hat{x}$  is not weakly Pareto optimal, contradicting part (a) of Proposition 1. Hence to the contrary, not all consumers can be losers.

(b) Suppose that there are some losers from market integration and no gainers. That means  $x_i^* \succ_i \hat{x}_i$  for some  $i \in I$  and  $x_i^* \succeq_i \hat{x}_i$  for all  $i \in I$ . Therefore,  $\hat{x}$  is not Pareto optimal. But if preferences are locally non-satiated, then by part (b) of Proposition 1, the equilibrium allocation  $\hat{x}$  is Pareto optimal. Thus a contradiction results. Hence to the contrary, if there are some losers, there has to exist a gainer as well.

The proposition does not rule out the possibility that almost all consumers lose. Consider, for example, consumer populations  $I_A = \{1, \ldots, n\}$  and  $I_B = \{n + 1\}$  with  $n \ge 2$ . Let the commodity space be  $\mathbb{R}^2$  and each consumer *i*'s consumption set be  $X_i = \mathbb{R}^2_+$ . All consumers have Cobb-Douglas preferences represented by  $u_i(x_i^1, x_i^2) = x_i^1 x_i^2$  for  $(x_i^1, x_i^2) \in \mathbb{R}^2_+$ . Consumer 1 has endowment bundle  $\omega_1 = (1, 1)$ whereas consumers  $i = 2, \ldots, n + 1$  have endowment bundle  $\omega_i = (1, 2)$ . Then consumers 1 and n+1 gain from economic integration and all other consumers lose. At the other extreme, assume that all consumers have the endowment bundle (1, 1), all consumers within  $I_A$  have identical Cobb-Douglas preferences, all consumers within  $I_B$  have identical Cobb-Douglas preferences in  $I_A$  differ from those in  $I_B$ . Then all consumers benefit from market integration.

It is also possible that the welfare of consumers in country B, say, is unaffected by economic integration, but merely their presence in the integrated economy affects the consumers in country A. For example, let A's economy consist of consumers 1 and 2, both with endowment bundle (1, 1). Consumer 1 has lexicographic preferences:  $(x_1^1, x_1^2) \succ_1 (y_1^1, y_1^2)$  if  $x_1^1 > y_1^1$  or  $x_1^1 = y_1^1 \& x_1^2 > y_1^2$ . Consumer 2 only cares for commodity 2, hence has utility representation  $U_2(x_2^1, x_2^2) = x_2^2$ . Up to price normalization, A's economy has a unique competitive equilibrium  $(x_A^*, p_A^*)$ with  $p_A^* = (1, 1), x_A^* = (x_1^*, x_2^*) = ((2, 0), (0, 2))$ . Country B consists of consumer 3 with endowment bundle  $\omega_3 = (1, 1)$  and utility function  $U_3(x_3^1, x_3^2) = x_3^1 + 2x_3^2$ . Up to price normalization, this economy has a unique competitive equilibrium  $(x_B^*, p_B^*)$  with  $p_B^* = (1, 2)$  and  $x_B^* = x_3^* = \omega_3 = (1, 1)$ . Up to price normalization, the integrated economy has a unique competitive equilibrium  $(\hat{x}, \hat{p})$  with  $\hat{p} = (1, 2)$  and  $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3) = ((3, 0), (0, 1.5), (0, 1.5)).$ 

#### 2.4 Destabilizing Effect of Integration

This example relies on non-convex preferences of the consumers in country A. Let country A consist of two consumers 1 and 2, each with endowment bundle  $\omega_i =$ (1,1) and utility function  $U_i(x_i^1, x_i^2) = (x_i^1)^2 + (x_i^2)^2$ . Up to price normalization, this economy has two equilibria with price system  $p_A^* = (1,1)$  where one consumer chooses consumption bundle (2,0) and the other consumer chooses consumption bundle (0,2).

Let country B consist of consumer 3 with endowment bundle  $\omega_3 = (1, 1)$  and utility function  $U_3(x_3^1, x_3^2) = x_3^1 + 2x_3^2$ . Up to price normalization, this economy has a unique competitive equilibrium  $(p_B^*, x_B^*)$  with  $p_B^* = (1, 2)$  and  $x_B^* = x_3^* = \omega_3 = (1, 1)$ .

Now consider the integrated economy. Because of monotonicity of preferences, an equilibrium price system p has to satisfy  $p = (p_1, p_2) \gg 0$ . Let us take commodity 1 as numéraire so that  $p_1 = 1$ . We can distinguish several cases depending on the size of  $p_2$ . If  $p_2 < 1$ , then consumers 1 and 2 each demand more than 2 units of good 2; hence there is positive excess demand for good 2. If  $p_2 = 1$  then there are three subcases: In case both consumers in A demand two units of good 2, there is positive excess demand for good 2. In case one of the consumers in A demands 2 units of good 1 and the other demands 2 units of good 2, there is still positive excess demand for good 2, since consumer 3 will demand 2 units of that good. In case both consumers in A demand two units of good 1, then there is excess demand for good 1. If  $p_2 > 1$ , then consumers 1 and 2 each demand more than 2 units of good 1 and there is positive excess demand for good 1. Thus there are no relative prices at which the market in the integrated economy is cleared. This shows that market integration can have a destabilizing effect.

## 3 Pairwise Two-sided Matching

Our focus lies on two-sided pairwise matching as in the seminal contribution of Gale and Shapley (1962). In the marriage market interpretation, the population consists of men and women. A matching selects heterosexual couples such that each individual is matched with exactly one partner of the other sex or remains unmatched. Stability requires that no matched person prefers to be single and no pair consisting of a man and a woman prefers being a couple to the status quo. This presupposes that individuals have preferences over partners, including having no partner. In the standard setting of two-sided matching, in principle, a man can be matched with any woman and vice versa – if one disregards preferences.

We observe that a stable matching exists in any matching market where all individuals have complete and transitive preferences (Gale and Shapley 1962). We will establish as a general result that with strict preferences, the number of losers from integration cannot exceed the number of gainers. More precisely, if two or more separate groups are merged into one, then within each group, at most half of the members are losers from integration.

#### 3.1 Basic Model

Some of the notation and terminology is adopted from Roth and Sotomayor (1990). There is a finite population I that is partitioned into a non-empty male subpopulation  $M = \{m_1, \ldots, m_k\}$  and a non-empty female subpopulation  $W = \{w_1, \ldots, w_\ell\}$ . A bipartite graph or bigraph based on the given partition is an undirected graph whose edges are of the form  $\{m, w\}$  for some  $m \in M$  and some  $w \in W$ .

Matching. A matching is a bigraph  $\mathcal{M}$  based on the given partition such that no two of its edges share an element:  $\{m, w\} \cap \{m', w'\} = \emptyset$  for  $\{m, w\}, \{m', w'\} \in \mathcal{M}, \{m, w\} \neq \{m', w'\}$ . Elements of  $\mathcal{M}$  are called matches. A matching  $\mathcal{M}$  can be identified with a bijection  $\mu : I \to I$  with the properties that (a)  $\mu^{-1} = \mu$ ; (b)  $\mu(m) \notin W$  implies  $\mu(m) = m$  for  $m \in M$ ; (c)  $\mu(w) \notin M$  implies  $\mu(w) = w$  for  $w \in W$ . **Preferences.** Individuals have **preferences for partners**, including the possibility of not having a partner. Each  $m \in M$  has complete and transitive preferences on  $W \cup \{m\}$ , represented by an ordered list P(m). For example

 $P(m) = w_1, w_2, m, w_3, \ldots, w_\ell$ 

means that m's first choice is to be married to woman  $w_1$ , his second choice is to be married to woman  $w_2$ , and his third choice is to remain single. Women  $w_3$  to  $w_\ell$  are not acceptable to m. He would rather be single than be married to one of them. An abbreviated list contains only the acceptable women:

$$P(m) = w_1, w_2$$

Similarly, each  $w \in W$  has complete and transitive preferences on  $M \cup \{w\}$ , represented by an ordered list P(w). If a person is indifferent between several possible mates, then those are put in square brackets in the preference list:

 $P(w) = m_2, [m_3, m_4, w], m_1$ 

means that  $m_2$  is w's first choice; to be married to  $m_3$ , to be married to  $m_4$  and being single tie for second choice while  $m_1$  is her third choice. The abbreviated list is  $P(w) = m_2, [m_3, m_4, w]$ .

 $\mathbf{P} = (P(m_1), \ldots, P(m_k), P(w_1), \ldots, P(w_\ell))$  denotes a preference profile, that is a collection of lists, one for each individual. Then a particular marriage market is specified by a triple  $(M, W; \mathbf{P})$ . The notation  $w >_m w'$  means that m prefers woman w to woman w' and  $w \ge_m w'$  means that m likes w at least as much as w'.  $m >_w m'$  and  $m \ge_w m'$  are defined in an analogous way. Woman w is **acceptable** to man m if he likes her at least as much as remaining single, i.e.,  $w \ge_m m$ . Analogously, m is acceptable to w if  $m \ge_w w$ . An individual is said to have **strict preferences** if he or she is not indifferent between any two acceptable alternatives.

**Stable Matching.** Formally, we consider matchings that are stable according to the following

**Definition 1.** A matching  $\mathcal{M}$  (or, equivalently,  $\mu$ ) in the marriage market  $(M, W; \mathbf{P})$  is stable if:

- 1. For any  $m \in M$ ,  $w \in W$  who are matched in  $\mathcal{M}$ , neither partner wants to go single. That is,  $\{m, w\} \in \mathcal{M}$  (or, equivalently  $m = \mu(w)$ ) implies  $w \ge_m m$  and  $m \ge_w w$ .
- 2. There is no pair  $(m, w) \in M \times W$  who can get married to each other and who prefer this marriage to the status quo. That is,  $\mu(m) \ge_m w$  or  $\mu(w) \ge_w m$ .

In particular, a stable matching  $\mu$  is **individually rational**, that is  $\mu(i) \geq_i i$ for all  $i \in I$ . If a pair  $\{m, w\}$  renders a potential matching unstable, we say that  $\{m, w\}$  blocks or destabilizes the particular matching. That is,  $\{m, w\}$  blocks the matching  $\mu$  if  $w >_m \mu(m)$  and  $m >_w \mu(w)$ . This is the **strong version of** blocking. In the weak version of blocking, the pair  $\{m, w\}$  blocks the matching  $\mu$  if  $w >_m \mu(m)$ ,  $m \geq_w \mu(w)$  or if  $w \geq_m \mu(m)$ ,  $m >_w \mu(w)$ . The two notions of blocking coincide in the case of strict preferences.

#### **3.2** Desegregation of Marriage Markets

Segregation of various sorts, be it racial, religious or political, restricts the marriage market. As an actual example for the latter, consider the case of South and North Korea or former West and East Germany. How would desegregation affect the marriage market? We are going to study this question first in a simple example. More specifically, we consider the case where the population I is segregated into two sub-populations  $I_1 = M_1 \cup W_1$  and  $I_2 = M_2 \cup W_2$ , with  $M_i = M \cap I_i$  and  $W_i = W \cap I_i$ . Marriages are only possible within  $I_1$  and within  $I_2$ .

**Example 1** (Effect of desegregation). Let  $k = \ell = 8$  and

 $P(m) = w_1, w_2, \ldots, w_k$  for all  $m \in M$ ;

 $P(w) = m_1, m_2, \ldots, m_k$  for all  $w \in W$ .

Suppose the population  $I = M \cup W$  is segregated into two parts  $I_1 = M_1 \cup W_1$ and  $I_2 = M_2 \cup W_2$  where  $M_1 = \{m_1, m_2, m_3, m_4\}, M_2 = \{m_5, m_6, m_7, m_8\}, W_1 = \{w_2, w_3, w_4, w_5\}, W_2 = \{w_1, w_6, w_7, w_8\}$ . Marriages are only possible within  $I_1$  and within  $I_2$ . Let  $\mathbf{P_1}$  denote the restriction of the preference profile  $\mathbf{P}$  to members of  $I_1$  and  $\mathbf{P_2}$  denote the restriction of the preference profile  $\mathbf{P}$  to members of  $I_2$ . Then:

Under segregation, there exists a unique stable matching  $\mu$  given by

$$\mu(m_1) = w_2, \mu(m_2) = w_3, \mu(m_3) = w_4, \mu(m_4) = w_5$$
 for members of  $I_1$ ,

$$\mu(m_5) = w_1, \mu(m_6) = w_6, \mu(m_7) = w_7, \mu(m_8) = w_8$$
 for members of  $I_2$ .

After desegregation, there exists a unique stable matching  $\mu'$  given by

 $\mu'(m_i) = w_i \text{ for } i = 1, \dots, 8.$ 

Men  $m_1, m_2, m_3, m_4$  and woman  $w_1$  gain from desegregation whereas male  $m_5$  and females  $w_2, w_3, w_4, w_5$  lose.

In the example, there are equal numbers of losers and gainers in I, and also in each of the subpopulations  $I_1$  and  $I_2$ . This observation generalizes as follows:

**Proposition 3.** Suppose the population is segregated into  $K \ge 2$  non-empty subpopulations  $I_1, \ldots, I_K$ . Let  $\mu$  be any matching under segregation and  $\mu'$  be a stable matching after desegregation. Then

$$|\{i \in I_k : \mu'(i) <_i \mu(i)\}| \le |I_k|/2 \text{ for } k = 1, \dots, K.$$

PROOF. Suppose that  $|\{i \in I_k : \mu'(i) <_i \mu(i)\}| > |I_k|/2$  for some  $k \in \{1, \ldots, K\}$ .  $\mu'(i) <_i \mu(i)$  implies  $\mu(i) \neq i$  for  $i \in I_k$ . That is, all members of  $J_k \equiv \{i \in I_k : \mu'(i) <_i \mu(i)\}$  were matched under  $\mu$ . Since  $|J_k| > |I_k|/2$ , there exist two members of  $J_k$  who were matched with each other under  $\mu$ . Therefore, they can block  $\mu'$ , contradicting the assumed stability of  $\mu'$ . Hence to the contrary, the assertion of the proposition has to hold.

The proposition says that at most half of the people in each group are losers. It is possible that, indeed, half of the people in each group are losers. It is also possible that there are no losers at all. The result has been reported as Proposition 8 in Gersbach and Haller (2015). Proposition 2 of Ortega (2018) asserts that with strict preferences, at most half of the population are losers, which is a corollary of our result. He obtains additional results on the integration of matching markets.

## 4 Strategic Network Formation

Our analysis of strategic network formation games is going to be two-fold. First we will demonstrate, by means of an elaborate example with a heterogeneous player population, that integration can cause isolation of a previously central player. Second, we study the effects of integrating two groups where all individuals are homogeneous and linkage costs are small. We show that from an ex ante point of view, integration always proves desirable with a homogeneous player population (and small linkage costs). While ex post losers cannot be ruled out, we find striking restrictions on their numbers. As a rule, the upper bound on the number of losers from integration of two networking groups is less than the corresponding upper bound when two matching markets are integrated.

#### 4.1 Basic Model

Our basic model is the two-way flow connections model à la Galeotti, Goyal, and Kamphorst (2006) that incorporates cost and value heterogeneity. We adopt the notation of Haller, Kamphorst and Sarangi (2007) for the case of perfectly reliable links.

Let  $n \geq 3$ .  $N = \{1, \ldots, n\}$  denotes the set of players with generic elements i, j, k. N also constitutes the set of nodes of the network to be formed. For ordered pairs  $(i, j) \in N \times N$ , the shorthand notation ij or i, j is used and for non-ordered pairs  $\{i, j\} \subset N$  the shorthand [ij] is used. The symbol  $\subset$  for set inclusion permits equality. The model is specified by two families of parameters, indexed by ij, with  $i \neq j$ :

- Cost parameters  $c_{ij} > 0$ .
- Value parameters  $V_{ij} > 0$ .

In case  $c_{ij} \neq c_{kl}$  ( $V_{ij} \neq V_{kl}$ ) for some  $ij \neq kl$ , the model exhibits **cost** (value) heterogeneity; otherwise, it exhibits **cost** (value) homogeneity.

We only consider pure strategies. A pure strategy for player i is a vector  $g_i = (g_{i1}, \ldots, g_{ii-1}, g_{ii+1}, \ldots, g_{in}) \in \{0, 1\}^{N \setminus \{i\}}$ . The set of all pure strategies of agent i is denoted by  $\mathcal{G}_i$ . It consists of  $2^{n-1}$  elements. The joint strategy space is given by  $\mathcal{G} = \mathcal{G}_1 \times \cdots \times \mathcal{G}_n$ .

There is a one-to-one correspondence between the set of joint strategies  $\mathcal{G}$  and the set of all directed graphs or networks with vertex set N. Namely, to a strategy profile  $g = (g_1, \ldots, g_n) \in \mathcal{G}$  corresponds the graph (N, E(g)) with edge or node set  $E(g) = \{(i, j) \in N \times N | i \neq j, g_{ij} = 1\}$ . In the sequel, we shall identify a joint strategy g and the corresponding graph and use the terminology **directed graph** or **directed network** g. Since our aim is to model network formation,  $g_{ij} = 1$ is interpreted as a direct link between i and j is initiated by player i (edge ij is formed by i) whereas  $g_{ij} = 0$  means that i does not initiate the link (ij is not formed). Regardless of what player i does, player j can set  $g_{ji} = 1$ , i.e., initiate a link with i, or set  $g_{ji} = 0$ , i.e., not initiate a link with i.

**Benefits.** A link between agents i and j potentially allows for two-way (symmetric) flow of information. Accordingly, the benefits from network g are derived from its closure  $\overline{g} \in \mathcal{G}$ , defined by  $\overline{g}_{ij} := \max \{g_{ij}, g_{ji}\}$  for  $i \neq j$ . Moreover, a player receives information from others not only through direct links, but also via indirect links. To be precise, information flows from player j to player i, if i and j are linked by means of a path in  $\overline{g}$  from i to j. A path of length m in  $f \in \mathcal{G}$  from player i to player  $j \neq i$ , is a finite sequence  $i_0, i_1, \ldots, i_m$  of pairwise distinct players such that  $i_0 = i$ ,  $i_m = j$ , and  $f_{i_k i_{k+1}} = 1$  for  $k = 0, \ldots, m-1$ . Let us denote

 $N_i(f) = \{j \in N | j \neq i, \text{ there exists a path in } f \text{ from } i \text{ to } j\},\$ 

the set of other players whom player i can access or "observe" in the network f. Information received from player j is worth  $V_{ij}$  to player i. Therefore, player i's benefit from a network g with perfectly reliable links and two-way flow of

information is (as in Galeotti, Goyal, and Kamphorst (2006)):

$$B_i(g) = B_i(\overline{g}) = \sum_{j \in N_i(\overline{g})} V_{ij}.$$

Notice that  $\overline{g}$  belongs to the set  $\mathcal{H} = \{h \in \mathcal{G} | h_{ij} = h_{ji} \text{ for } i \neq j\}$ . In turn, there is a one-to-one correspondence between the elements of  $\mathcal{H}$  and the non-directed networks (graphs) with node set N. Namely, for  $h \in \mathcal{H}$  and  $i \neq j$ , [ij] is an edge of the corresponding non-directed network if and only if  $h_{ij} = h_{ji} = 1$ .

**Costs.** Player *i* incurs the cost  $c_{ij}$  when she initiates the direct link ij, i.e., if  $g_{ij} = 1$ . Hence when the network *g* is formed, *i* incurs the total costs

$$C_i(g) = \sum_{j \neq i} g_{ij} c_{ij}$$

**Payoffs.** Player *i*'s payoff from the strategy profile g is the net benefit

$$\Pi_i(g) = B_i(g) - C_i(g). \tag{1}$$

**Nash Networks.** Given a network  $g \in \mathcal{G}$ , let  $g_{-i}$  denote the network that remains when all of agent *i*'s links have been removed so that  $g_i \in \mathcal{G} \equiv \prod_{j \neq i} \mathcal{G}_i$ . Clearly  $g = g_i \oplus g_{-i}$  where the symbol  $\oplus$  indicates that g is formed by the union of links in  $g_i$  and  $g_{-i}$ . A strategy  $g_i$  is a **best response** of agent *i* to  $g_{-i}$  if

$$\Pi_i(g_i \oplus g_{-i}) \ge \Pi_i(g'_i \oplus g_{-i})$$
 for all  $g'_i \in \mathcal{G}_i$ .

Let  $BR_i(g_{-i})$  denote the set of agent *i*'s best responses to  $g_{-i}$ . A network  $g = (g_1, \ldots, g_n)$  is said to be a **Nash network** if  $g_i \in BR_i(g_{-i})$  for each *i*, that is if *g* is a Nash equilibrium of the strategic game with normal form  $(N, (\mathcal{G}_i)_{i \in N}, (\Pi_i)_{i \in N})$ . A strict Nash network is one where agents are playing strict best responses.

Efficient Networks. Let  $W_0 : \mathcal{G} \to \mathbb{R}$  be defined as  $W(g) = \sum_{i=1}^n \prod_i(g)$ . A network  $\widehat{g}$  is efficient (in the narrow traditional sense) if  $W(\widehat{g}) \ge W(g)$  for

all  $g \in \mathcal{G}$ . Efficiency is a major performance criterion for network designers or planners and plays a prominent role in the traditional network literature. It is most attractive for cost-benefit analysis when payoffs are monetary and side-payments between players are feasible. Efficiency constitutes an important benchmark for network performance even when network formation is decentralized and structured as a strategic game. In economics, the term "efficiency" is often used in a broader sense, as a synonym for Pareto optimality.

Some Graph-theoretic Concepts. We now introduce some definitions of a more graph-theoretic nature. The network with no links is called the **empty network** and will be denoted e. A network g is said to be **connected** if there is a path in  $\overline{g}$  between any two agents i and j. A connected network g is **minimally connected**, if it is no longer connected after the deletion of *any* link.

#### 4.2 Isolation Caused by Integration

Let  $A = \{a0, a1, \ldots, am\}$  with  $m \ge 1$  and  $B = \{b0, b1, \ldots, bn\}$  with  $n \ge 2$ . We assume  $V_{ja0} = 0$  for all  $j \ne a0$  and  $V_{ij} = 1$  otherwise. We assume costs as follows:  $m - 2 < c_{ak,a0} = c_A < m - 1$  for  $k = 1, \ldots, m$ ;  $c_{ak,b0} = c_B < 1$  for  $k = 1, \ldots, m$ ;  $c_{b\ell,b0} = c_B$  for  $\ell = 1, \ldots, n$ ; and  $c_{ij} = m + n + 2$  otherwise. Thus in equilibrium, a0 will not form any links and value-wise is worthless by himself. Still, his serving as an intermediary can benefit the other players and himself. Indeed, when network formation is confined to group A, then the empty network is Nash and the only non-empty Nash network is the periphery-sponsored star  $S_A$ with a0 as center. Similarly, when network formation is confined to group B, the periphery-sponsored star  $S_B$  with b0 at the center is the only Nash network while the empty network is not Nash.

After the two groups are integrated, the star  $S_B$  persists as an equilibrium subnetwork. Moreover, in equilibrium at least one link ak,b0 is formed, say a1,b0. First of all, there cannot be a Nash equilibrium where all links in  $S_A$  and  $S_B$ plus the link a1,b0 exist. Namely, if link a2,b0 does not exist, then player a2 is better off severing link a2,a0 and forming link a2,b0 and if link a2,b0 does exist, then player a2 is better off severing link a2,a0 and keeping link a2,b0. Hence in Nash equilibrium, some link ak,b0, say a1,b0, and all links in  $S_B$ , but not all links in  $S_A$  are formed. Let link  $a\ell,a0$  be one of the links in  $S_A$  that are no longer formed. Then the link  $a\ell,b0$  exists in equilibrium. Now let  $h \neq 0, \ell$ . If both ah,a0and ah,b0 exist, then ah's payoff is at most  $m + n - 1 - (c_A + c_B)$ . When ahsevers the link ah,a0, then the payoff is at least  $n + 1 - c_B = n - 1 - c_B + 2 =$  $m + n - 1 - c_B + 2 - m = m + n - 1 - c_B - (m - 2) > m + n - 1 - (c_B + c_A)$ . Hence ah prefers severing that link. If only ah,a0 exists, then ah is better off severing that link and forming ah,b0. If only ah,b0 exists, then ah does not want to change his strategy. It follows that in Nash equilibrium, all links in  $S_B$  and all links ak,b0exist and none of the links in  $S_A$ . Consequently, a0 is isolated now.

There are two possible scenarios. First, the empty network A may be group A's equilibrium network. Then all players benefit from integration except a0 who remains isolated. Second, the star  $S_A$  may be group A's equilibrium network. Then again all players benefit from integration except a0 who loses the privileged intermediary status and becomes isolated. The case m = n = 3 is depicted in the following diagram.



#### 4.3 Effects of Integration in Homogeneous Populations

Next let us consider the homogeneous model with  $V_{ij} = V$  and  $c_{ij} = c$  for all ij. We focus on the low cost case where c < V. Without loss of generality, we may proceed with V = 1 and c < 1. For that case, Bala and Goyal (2000, p. 1202) show that the Nash networks are all the minimally connected networks.<sup>2</sup> Starting with two separate groups A and B with |A| = m > 1 and  $|B| = n \ge 1$ , take any Nash network  $g_A$  in A and any Nash network  $g_B$  in B. Then the aggregate welfare in group A is m(m-1) - (m-1)c and the per capita welfare is m-1 - (1-1/m)c. The latter is also a player's ex ante expected payoff if all Nash networks have an equal chance of being formed. As a consequence, we obtain the first assertion of the next proposition.

**Proposition 4.** In the homogeneous model of network formation with V = 1 and c < 1, the following assertions hold:

- (a) Ex ante, all players benefit from integration.
- (b) Ex post, a player may lose.
- (c) Ex post, at least one player gains.
- (d) Ex post, all players may gain.

**PROOF.** (a) The ex ante expected payoff is increasing in the number of players.

(b) Suppose  $B = \{b\}$ . The periphery-sponsored star  $S_A$  with center  $a \in A$  is a Nash network in A. The center-sponsored star  $S_{A\cup B}$  with center a is a Nash network after integration. Thus a remains the intermediary, but instead of getting a free ride, he shoulders all the costs in the larger network. Player a obtains payoffs  $\Pi_a(S_A) = m - 1$  and  $\Pi_a(S_{A\cup B}) = m - mc$ . Hence in case c > 1/m, he is worse off after integration.

(c) A minimally connected network of m+n players has m+n-1 links. Hence ex post, at least one player does not form a link and gains from integration.

 $<sup>^2\</sup>mathrm{Haller},$  Kamphorst and Sarangi (2007) show that Nash networks do not exist in some heterogeneous models.

(d) If one labels i = 1, ..., m the players in A and i = m + 1, ..., m + n the players in B, then the network consisting of the links i, i+1 for i = 1, ..., m+n-1 is minimally connected and hence a Nash network after integration. In that network, each player forms at most one link and, therefore, gains from integration.

**Proposition 5.** In every Nash equilibrium after integration of the homogeneous model of network formation with V = 1 and c < 1, there are either no losers in group A and at most 1 + (n-1)/m losers in group B or no losers in group B and at most 1 + (m-1)/n losers in group A.

PROOF. A member of A enjoys the added benefit n from integration. In order to be a loser, the player has to form more than n/c links. Similarly, a loser in B has to form more than m/c links. Let  $L_A$  denote the number of losers in A and  $L_B$  denote the number of losers in B. Then all losers combined form more than  $L_A \cdot \frac{n}{c} + L_B \cdot \frac{m}{c}$  links. Since the total number of links is m + n - 1 in equilibrium,

$$L_A \cdot \frac{n}{c} + L_B \cdot \frac{m}{c} < m + n - 1$$

has to hold. This implies

$$L_A \cdot n + L_B \cdot m < m + n - 1$$

because of c < 1. If there is at least one user in each group, then  $L_A \cdot n + L_B \cdot m \ge m + n$ , a contradiction. Thus there is no loser in one of the groups, say group A. In the latter case,  $L_B \cdot m < m + n - 1$  or  $L_B < 1 + (n - 1)/m$ . In case there are no losers in B, then  $L_A < 1 + (m - 1)/n$ .

The proposition implies that if the two groups have equal even numbers of members, then at most 25% of all players lose from integration and all belong to one group. For instance,  $L_A < 1+(m-1)/n$  and  $n \ge 2$  imply  $L_A < 1+(m-1)/2 = (m+1)/2$  and  $L_A \le m/2 = |A|/2$ .

Inspection of the proof shows that the assertion of the proposition can be slightly sharpened by considering non-gainers instead of losers. Hence if the two groups have equal even numbers of members, then at least 75% of all players gain from integration and one of the groups consists only of gainers. Further inspection shows that a member of A is either a gainer or a loser if n/c is not a natural number — and a member of B is either a gainer or loser if m/c is not a natural number. Therefore,

**Corollary 1.** For generic cost parameters, including irrational c, ex post every player is either a gainer or a loser from integration.

We also obtain

**Corollary 2.** If  $c < k \cdot \min\{m, n\}/(n + m - 1)$ , then there are less than k losers.

## 5 Concluding Remarks

We have seen that the integration of commodity markets, matching or marriage markets, or networking markets can have both gainers and losers. Losers can be prevalent when commodity markets are integrated. The relative numbers of losers tend to be smallest when networking markets are merged. In the case of competitive pure exchange economies, consumers interact with the market, not one-on-one. Consequently, a change in the composition of the consumer population can alter the terms of trade to the disadvantage of most consumers. In the case of networking markets, one would expect occurrence of some losers from integration when network externalities are negative, like in the coauthor model. In contrast, the benchmark model underlying our analysis exhibits positive network externalities: A player benefits or at least is not harmed when someone else forms a link. Therefore, to have losers from market integration is not that plausible. Still, there can be some losers, though not very many. To be a loser, a player has to create many links. Then others get away with few links. In turn, the loser has an incentive to form the large number of links. In a sense, a loser gets boxed in at equilibrium.

We have limited ourselves to some simple comparisons regarding gainers and losers of integration of economic, matching and networking markets.

There are entire branches in economics that deal with gainers and losers in the

context of production. The famous 1941 Stolper-Samuelson result showed that for a two-goods, two-markets economy, market integration with a labor-abundant economy can yield lower real wages and thus hurt workers.<sup>3</sup>. Since 1941, the literature has repeatedly dealt with potential and actual negative consequences of market integration or "globalization" on subgroups and small regions within the economy, coexisting with positive aggregate effects (see Autor et al. (2013, 2015), Feenstra and Sasahara (2018), Krugman (2019)). Similarly, reducing obstacles to labor mobility and thus achieving more integration in the labor market has been the topic of much political debate and empirical analysis. Finally, there is an extended literature on how firms (and managers) are affected by market integration—with conclusions depending on market structures and the nature of competition.<sup>4</sup> Our study is complementary to the literature, as we focus on comparisons of relative gainers and losers across three different types of markets.

Let us conclude with two more observations. Our analysis also applies to secession or market segregation if one reads the results in reverse. And there may be other reasons than purely economic ones why a country wants to join or leave a group like the European Union. Gainers and loosers from politically motivated integration are an important area for thorough analysis.

<sup>&</sup>lt;sup>3</sup>Stolper and Samuelson (1941)

 $<sup>{}^{4}</sup>$ See, e.g., Gersbach and Schmutzler (2014) for the integration of Cournot markets, Eaton et al. (2011) for empirical evidence of a differential impact of international trade on French firms.

## References

- Autor, D. H., Dorn, D. and Hanson, G. H., 2013: "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," American Economic Review, 103, 2121-2168.
- Autor, D. H., Dorn, D. and Hanson, G. H., 2015: "Untangling Trade and Technology: Evidence from Local Labor Markets," *Economic Journal*, 125, 621-646.
- Bala, V. and Goyal, S., 2000: "A Non-Cooperative Model of Network Formation," *Econometrica*, 68, 1181-1229.
- Debreu, G., 1959: The Theory of Value: An Axiomatic Analysis of Economic Equilibrium, Wiley: New York.
- Eaton, J., Kortum, S. and Kramarz, F., 2011: "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 79, 1453-1498.
- Feenstra, R. C. and Sasahara, A., 2018: "The 'China Shock,' Exports and U.S. Employment: A Global Input-output Analysis," *Review of International Economics*, 26, 1053-1083.
- Gale, D. and Shapley, L. S., 1962: "College Admissions and the Stability of Marriage", American Mathematical Monthly, 69, 91-14.
- Galeotti, A., Goyal, S. and Kamphorst, J., 2006: "Network Formation with Heterogeneous Players," *Games and Economic Behavior*, 54, 353-372.
- Gersbach, H. and Haller, H., 2015: "Matching on Bipartite Graphs," CES Ifo Working Paper No. 5575.
- Gersbach, H. and Schmutzler, A., 2014: "Does Globalization Create Superstars? A Simple Theory of Managerial Wages", *European Economic Review*, 71, 34-51.
- Haller, H., Kamphorst, J. and Sarangi, S., 2007: "(Non-)Existence and Scope of Nash Networks," *Economic Theory*, 31, 597-604.

- Jackson, M. O., Wolinsky, A., 1996: "A Strategic Model of Economic and Social Networks," Journal of Economic Theory, 71, 44-74.
- Krugman, P., 2019: "What Did We Miss about Globalization?", Corden Public Lecture 2019, University of Melbourne. https://www.youtube.com/watch?v=rWQ3jCURzy0 (accessed November 4, 2019).
- Mas-Colell, A., Whinston, M. D. and Green, J. R., 1995: *Microeconomic Theory*. Oxford University Press: New York and Oxford.
- Ortega, J., 2018: "Social Integration in Two-sided Matching Markets," *Journal* of Mathematical Economics, 78, 119-126.
- Roth, A. E. and Sotomayor, M. A. O., 1990: Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis. Cambridge University Press: Cambridge, UK.
- Stolper, W. F. and Samuelson, P. A., 1941: "Protection and Real Wages," Review of Economic Studies, 9, 58-73.