# ECONSTOR 

Make Your Publications Visible.

# Working Paper <br> Panel Bayesian VAR modeling for policy and forecasting when dealing with confounding and latent effects 

EERI Research Paper Series, No. 15/2018

## Provided in Cooperation with:

Economics and Econometrics Research Institute (EERI), Brussels


#### Abstract

Suggested Citation: Pacifico, Antonio (2018) : Panel Bayesian VAR modeling for policy and forecasting when dealing with confounding and latent effects, EERI Research Paper Series, No. 15/2018, Economics and Econometrics Research Institute (EERI), Brussels


This Version is available at:
https://hdl.handle.net/10419/213548

## Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

[^0]
## Panel Bayesian VAR Modeling for Policy and Forecasting when dealing with confounding and latent effects

Antonio Pacifico

## EERI Research Paper Series No 15/2018

ISSN: 2031-4892


EERI
Economics and Econometrics Research Institute
Avenue Louise
1050 Brussels
Belgium
Tel: +32 22719482
Fax: +32 22719480
www.eeri.eu

# Panel Bayesian VAR Modeling for Policy and Forecasting when dealing with confounding and latent effects 

Antonio Pacifico


#### Abstract

The paper develops empirical implementations of the standard time-varying Panel Bayesian VAR model to deal with confounding and latent effects. Bayesian computations and mixed hierarchical distributions are used to generate posteriors of conditional impulse responses and conditional forecasts. An empirical application to Eurozone countries illustrates the functioning of the model. A survey on policy recommendations and business cycles convergence are also conducted. The paper would enhance the more recent studies to evaluate idiosyncratic business cycles, policy-making, and structural spillovers forecasting. The analysis confirms the importance to separate common shocks from propagation of country- and variable-specific shocks.


JEL classification: 13/A2, 13/A4, 13/A5, 13/D1, 13/D2

Keywords: Hierarchical Mixture Distributions in Normal Linear Model; Bayesian Model Averaging; Panel VAR; Forecasting; Structural Spillovers; MCMC Implementations.

## 1 Introduction

The paper develops an approach to jointly model and quantify latent behaviors ${ }^{1}$ and confounding effects ${ }^{2}$. The underlying logic is very useful in econometrics in order to avoid "omitted biases" for the case of serial correlations and not (directly) observed linkages among economic-financial variables, mainly if the lagged endogenous variables tend to show some kind of irregular or common behaviour. I estimate a structural time-varying Extended Panel Bayesian VAR (EPBVAR) model by specifying hierarchical priors through a mixture of distributions. In this study, Bayesian methods and Maximum Likelihood Estimates (MLE) are used to reduce the dimensionality of the model, put structure on the time variations, and simultaneously evaluate omitted variable biases and issues of endogeneity. In the case of fully hierarchical priors, a MCMC implementation is employed to calculate posterior distributions of Conditional Generalized Impulse Response Fuctions (CGIRFs) and Conditional Forecasts (CFs) experiments $3^{3}$ to unexpected perturbations in the innovations of the factors of the system. A Normal Linear Regression (NLR) model is evaluated in order to work with smaller systems in which the regressors are observable, directly measured, and time-varying linear combinations of the right-hand side variables of the structural EPBVAR. The advantage of this approach is that it is easier to match endogenous variables with omitted factors and the framework is valid if and only if prior assumptions are satisfied and a hierarchical structure is provided. Thus, an analysis of joint and conditional densities and sequential factorizations are required.

An empirical application to a pool of the current members of the European Monetary Union (EMU) illustrates the functioning of the model, with particular attention on the recent recession and post-crisis consolidation. To be more precise, the paper focuses on three interesting macroeconomic-financial issues left even now unanswered. First, how dimension and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. A survey on policy recommendations and business cycle convergence are also assessed. This implies that idiosyncratic

[^1]business cycles among the G7 countries or international business cycles are not discussed, unless they contain interesting results from the viewpoint of the present paper. Likewise, the analysis does not review studies focusing on regional cycles in Europe. Overall, the analysis is consistent and robust with the more recent literature on business cycles, which recognizes the importance of accounting for both group-specifc and global factors in evaluating cross-country spillovers and to separate common shocks from propagation of country- and variable-specific shocks when studying economic-financial linkages.

Growth spillovers are evaluated through structural Bilateral Net Spillover Effects (BNSEs) and Systemic Contributions (SCs). The former incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of a potential shock. The SC index represents the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events. Finally, the Generalized Theil (GT) index is estimated in order to investigate on business cycle convergence and, hence, divergence across member states.

The outline of this paper is as follows. Section 2 reviews the existing literature on business cycles and cross-country spillovers. Section 3 presents the econometric model and empirical specifications. Section 4 provides the empirical analysis. The final section contains some concluding remarks.

## 2 Discussion and Relationship with the literature

In recent years, several theoretical and applied models have given a new impulse to the literature on business cycles and policy making. Nevertheless, these studies often reach very different or mixed conclusions due to the selection of variables used, diverging methodologies to identify spillover effects and alternative ways to investigate co-movement, heterogeneity, and interdependence across countries.

For example, Blanchard and Perotti (2002) estimated dynamic effects of shocks in government spending and taxes on U.S. activity in the postwar period, by using a mixed structural VAR model. The results consistently show positive government spending shocks as having a positive effect on output, and positive tax shocks as having a negative effect. However, the model developed is not consistent with large sample and the computations are costly.

Pesaran (2006) and Pesaran et al. (2007) developed a multicountry Global Vector Autoregres-
sion (GVAR) approach in which estimate spillover effects of a domestic budget balance shock on the members of the EA by combining all country-specific VAR models in one multicountry model and treating all variables as endogenous. They focused on identification of shocks to the US economy, particularly the monetary policy shocks, and considered the time profiles of their effects on the EA. From a policy perspective, they showed that: $(i)$ financial shocks (equity and bond prices) tend to be transmitted much faster than shocks to real output and/or inflation; (ii) the transmission of real shocks is rather slow, normally taking 2-3 years, or in some cases even more, before their full impacts are felt; (iii) the long run impacts of the real shocks are larger than what might be expected from a simple trade perspective; and (iv) the transmission of shocks does not take place only through trade, but also as importantly through the impact of real shocks on financial variables with subsequent spillover effects on real variables. However, they concluded that the GVAR presents a complicated spatio-temporal structure for the analysis of the world economy and, hence, it need to be modified and extended further.

Geweke (2001), Raftery and Lewis (1997) and Raftery et al. (1997) developed Bayesian econometrics analyses and forecasting via posterior simulations using MCMC integration. To be more precise, they described different methods to use posterior simulation output to estimate integrated likelihoods. They focused on the basic Laplace-Metropolis estimator for models without random effects. For models with random effects, they introduced a compound Laplace-Metropolis estimator. Then, they computed a Bayes factor as the ratio of two integrated likelihoods in order to compare the two models. In this way, they were able to obtain good approximations for integrated likelihoods in hierarchical models, favoring the model incorporating random effects. Nevertheless, these methods have been discussed in time since they refer to a broad class of conditionally independent hierarchical models and the integrals involved are of low dimension.

Canova and Ciccarelli (2009) estimated a Bayesian multicountry VAR model to assess economicfinancial linkages, to test specification hypotheses and to conduct policy exercises, with cross-unit interdependencies, unit-specific dynamics, and time variations in the coefficients. They conducted an empirical analysis on G7 countries by addressing Bayesian computations. Impulse responses and conditional forecasts are obtained with the output of an MCMC routine. The transmission of certain shocks across countries are analyzed. However, the model is devoid of a structural framework in order to be able to add time variant factors.

Giannone et al. (2009) and Koop (1996) evaluated large Bayesian VARs through hierarchical priors. They examined both forecasting accuracy and structural analysis of the effect of a mon-
etary policy shock adding additional macroeconomic variables and sectoral information on the results of De Mol and co-workers (2008). The shrinkage is obtained by using Minnesota priors that often is uninformative since it is based on an approximation which involves replacing the variance-covariance matrix with an estimate. Moreover, the framework is extended by using a set of dummy variables and, hence, difficult to model.

In a quite recent work, Ciccarelli et al. (2012) investigated heterogeneity and spillovers in macrofinancial linkages across developed economies, focusing on the most recent recession. They developed a time-varying panel Bayesian VAR model including real and financial variables and identified a statistically significant common component. The main findings they found were the need to allow for cross-country and cross-variable interdependencies when studying real-financial linkages and the presence of a heterogeneous pattern across members with a common component. Nevertheless, the empirical model used is non-structural and constrained due to time-invariant or exogenous factors in the system and, hence, unable in identifying structural and institutional differences across countries, different reactions behind a common unexpected shock, and the causality among real and financial variables.

According to the above-mentioned literature, my empirical specifications differ from previous studies in four ways. First, I represents a structural time-varying approach as a sum of three terms: (i) a vector of lagged endogenous variables; (ii) a vector of serially correlated factors to deal with confounding effects; and (iii) a vector of latent factors to minimize further omitted biases. All terms are treated as endogenous in order to capture and discriminate statistical relationships from structural and causal connections. Second, dynamic relationships between these terms hold and are allowed to be country- and variable-specifics minimizing heterogeneity biases. Third, interdependence, heterogeneity and commonality can be evaluated without overlooking the contribution of different channels and/or structural constraints through which these effects might have occurred. Fourth, synchronization and convergence of business cycles can be achieved and shocks can be directly linked to policy recommendations, without imposing particular short-run and long-run restrictions.

## 3 Econometric Model and Empirical Specifications

### 3.1 Model Estimation

The econometric model has the form:

$$
\begin{equation*}
Y_{i, t}=A_{i t, 0}+\sum_{\lambda=1}^{l}\left[A_{i t, j}(L) Y_{i, t-\lambda}+B_{i t, j}(L) W_{i, t-\lambda}+C_{i t, j}(L) Z_{i, t-\lambda}\right]+\varepsilon_{i t} \tag{1}
\end{equation*}
$$

where the subscripts $i, j=1,2, \ldots, N$ are country indeces, $t=1,2, \ldots, T$ denotes time, $L$ stands for the lag operator, $A_{i t, 0}$ is an $N \cdot 1$ vector of intercepts, $A_{i t, j}$ is an $N M \cdot N M$ matrix of coefficients for each $j, Y_{i, t-\lambda}$ is an $N M \cdot 1$ vector of variables lagged, $B_{i t, j}$ is an $N Q \cdot N Q$ matrix of coefficients for each $j, W_{i, t-\lambda}$ is a $N Q \cdot 1$ vector including serially correlated factors, $C_{i t, j}$ is an $N \Xi \cdot N \Xi$ matrix of coefficients for each $j, Z_{i, t-\lambda}$ is a $N \Xi \cdot 1$ vector of hidden factors, $\varepsilon_{i t} \sim i . i . d . N(0, \Sigma)$ is a $N \cdot 1$ vector of disturbance terms. The subscripts $\lambda=1,2, \ldots, l$ are lags for each of the $m=1, \ldots, M$ endogenous variables, $q=1, \ldots, Q$ serially correlated factors, and $\xi=1, \ldots, \Xi$ hidden factos.

Firstly, let $k=1+N[m+q+\xi] l$ be the number of coefficient in each equation of the EPBVAR, a $N k \cdot N k$ matrix $X_{t}=\left(1, Y_{t-1}^{\prime}, Y_{t-2}^{\prime}, \ldots, Y_{t-\lambda}^{\prime}, W_{t}^{\prime}, W_{t-1}^{\prime}, W_{t-2}^{\prime}, \ldots, W_{t-\lambda}^{\prime}, Z_{t}^{\prime}, Z_{t-1}^{\prime}, Z_{t-2}^{\prime}, \ldots, Z_{t-\lambda}^{\prime}\right)^{\prime}$ can be defined containing endogenous variables, serially correlated and hidden factors. Then, I define a $N k \cdot 1$ vector $\gamma_{t}=\operatorname{vec}\left(\Gamma_{i t}^{k}\right)^{4}$ which contains, stacked into a vector, the intercepts and all the $M, Q$ and $\Xi$ coefficients of the matrices $A_{i t, j}, B_{i t, j}$ and $C_{i t, j}$, respectively, with $\Gamma_{i t}^{k}=$ $\left(A_{i t, 0}^{\prime}, A_{i t}^{1^{\prime}}, A_{i t}^{2^{\prime}}, \ldots, A_{i t}^{m^{\prime}}, B_{i t}^{1^{\prime}}, B_{i t}^{2^{\prime}}, \ldots, B_{i t}^{q^{\prime}}, C_{i t}^{1^{\prime}}, C_{i t}^{2^{\prime}}, \ldots, C_{i t}^{\xi^{\prime}}\right)^{\prime}$. With all these definitions, I can write the EPBVAR in terms of multivariate normal distribution:

$$
\begin{equation*}
Y_{t}=\left(I_{N k} \otimes X_{t}\right) \gamma_{t}+E_{t} \tag{2}
\end{equation*}
$$

where $Y_{t}=\left(y_{1 t}^{\prime}, \ldots, y_{N t}^{\prime}\right)^{\prime}$ and $E_{t}=\left(\varepsilon_{1 t}^{\prime}, \ldots, \varepsilon_{N t}^{\prime}\right)^{\prime}$ are $N \cdot 1$ vectors containing the dependent variables and the random disturbances of the model, respectively. Here, $E_{t} \sim N\left(0, \Sigma \otimes I_{t}\right)$ and $Y_{i, t}$ is expressed in terms of $X_{t}$ and contains, stacked, all $T$ observations on the first dependent variable, then all $T$ observations on the second dependent variable, and so on. Moreover, there is no subscript $i$ since variables of all countries in the system are stacked in $X_{t}$.

Now, since the coefficient vectors in $\gamma_{t}$ vary in different time periods for each country-variable

[^2]pair and there are more coefficients than data points, following the framework in Koop (1996) and Ciccarelli et al. (2012), a flexible factorization for $\gamma_{t}$ can be assumed to estimate them without restrictions and loss of efficiency:
\[

$$
\begin{equation*}
\gamma_{t}=\sum_{k=1}^{K} D_{k} \cdot \beta_{k t}+u_{t}=D \cdot \beta_{t}+u_{t} \quad \text { with } \quad u_{t} \sim N(0, \Sigma \otimes V) \tag{3}
\end{equation*}
$$

\]

where $D_{k}=\left[D_{1}, D_{2}, \ldots, D_{K}\right]$ are conformable matrices of coefficients, with elements equal to zero and one, $u_{t}$ captures unmodelled variations present in $\gamma_{t}$ that are not captured from the system, $V=\sigma^{2} \cdot I_{k}$ is a $k \cdot k$ matrix with $\sigma^{2}$ known by assumption, and $\operatorname{dim}\left(\beta_{t}\right) \ll \operatorname{dim}\left(\gamma_{t}\right)$ by construction. The idea is to shrink $\gamma_{t}$ into $\beta_{k t}$ obtaining a much smaller dimensional vector.

In equation 3, all factors are permitted to be time-varying and, hence, time variant structures can be obtained via implementations of MCMC algorithms. Moreover, time variations in the variance of shocks $u_{t}$ to the factors $\beta_{k t}$ are also allowed in order to capture in $Y_{t}$ possible heterogeneity, interdependence, and commonality across countries and among variables. Running equations 2 and 3 for equation 1, the factorization is:

$$
\begin{equation*}
D_{k} \cdot \beta_{k t}=D_{1} \cdot \beta_{1 t}+D_{2} \cdot \beta_{2 t}+\ldots+D_{K} \cdot \beta_{K t} \tag{4}
\end{equation*}
$$

Finally, the factorization of $\gamma_{t}$ becomes exact as long as $\sigma^{2}$ converges to zero.
With these specifications, the reduced-form EPBVAR model in equation 2 can be trasformed into a Normal Linear Regression (NLR) model with an error covariance matrix of an Inverse Wishart distribution ${ }^{5}$ By equations 2 and 3, the NLR mode ${ }^{6}$ can be written as:

$$
\begin{equation*}
Y_{t}=\Theta_{t}\left(D \beta_{t}+u_{t}\right)+E_{t} \equiv \chi_{t} \beta_{t}+\eta_{t} \quad \text { with } \quad \Theta_{t}=\left(I_{N k} \otimes X_{t}\right) \tag{5}
\end{equation*}
$$

where $\chi_{t} \equiv \Theta_{t} D$ is a $N \cdot k$ matrix which stacks all the coefficients of the system, $\beta_{t}$ is a $k \cdot 1$ vector containing all the regression coefficients of the NLR model, $\eta_{t} \equiv \Theta_{t} u_{t}+E_{t}$ is a $N \cdot 1$ vector of random disturbances and distributed as a normal with zero-mean and covariance matrix equals to a $N \cdot N$ matrix $\Omega$, with $\Omega=\left(\Sigma \otimes I_{k T}\right)$ by assumption ${ }^{[7]}$

[^3]By construction, $\chi_{i t}$ are linear combinations of right-hand side variables of the system and correlated among each other. The correlation decreases as $k$ increases. The vectors of $M$ endogenous variables, $Q$ serially-correlated factors and $\Xi$ hidden factors depend on a small number of observable indices, $\chi_{i t}$, and the factors $\beta_{i t}$ load on the indices. They are time-varying vectors to be estimated with a gain in efficiency. In fact, they are smooth (observed) linear functions of the lagged variables.

### 3.2 Prior Information

In hierarchical models, many problems involve multiple parameters which can be regarded as related in some way by the structure of the problem. A joint probability model for those parameters should reflect their mutual dependence. Typically, the dependence can be summarized by viewing these parameters as a sample from a common population distribution. Hence, the problem can be modelled hierarchically, with observable outcomes $\left(Y_{i}\right)$ created conditionally on certain parameters $\left(\alpha_{t}\right)$, which themselves are assigned a distribution in terms of further (possibly common) parameters, hyperparameters $(\phi)$. This hierarchical thinking may help solve the trade-off between inaccurate fit and overfitting, and also plays an important role in developing computational strategies.

Given the NLR model described in equation 5, accounting for $j$ and stacking all $i$ country indices, it can be alternatively written as:

$$
\begin{equation*}
Y_{j}=\chi_{j} \beta+\eta_{j} \quad \text { with } \quad i=1,2, \ldots, N \quad \text { and } \quad j=1,2, \ldots, J \tag{6}
\end{equation*}
$$

where $Y_{j}=\left(Y_{1 j}, Y_{2 j}, \ldots, Y_{N j}\right)^{\prime}$ is a $N \cdot 1$ vector, $\chi_{j}=\operatorname{diag}\left(\chi_{1 j}^{\prime}, \chi_{2 j}^{\prime}, \ldots, \chi_{N j}^{\prime}\right)$ is a $N \cdot k$ matrix, $\beta=\left(\beta_{1}, \beta_{2}, \ldots, \beta_{N}\right)^{\prime}$ is a $k \cdot 1$ vector, and $\eta_{j}=\left(\eta_{1 j}, \eta_{2 j}, \ldots, \eta_{N j}\right)^{\prime}$ is a $N \cdot 1$ vector, with $\eta_{j} \sim N(0, \Sigma)$. Stacking further:

$$
\begin{equation*}
Y=\chi \beta+\eta \quad \text { with } \quad \eta \sim N(0, \Omega) \quad \text { and } \quad \Omega=(\Sigma \otimes I) \tag{7}
\end{equation*}
$$

where $Y=\left(Y_{1}, Y_{2}, \ldots, Y_{J}\right)^{\prime}, \chi=\left(\chi_{1}, \chi_{2}, \ldots, \chi_{J}\right)^{\prime}, \eta=\left(\eta_{1}, \eta_{2}, \ldots, \eta_{J}\right)^{\prime}$, and $\Omega=\operatorname{diag}\left(\Sigma^{\prime}, \Sigma^{\prime}, \ldots, \Sigma^{\prime}\right)$.
The specifications in equations 6 and 7 allow heteroskedasticity and assume zero correlation across countries in order to obtain no perfect collinearity. The non-zero covariances imply that
equation 7 is related and individual regressions are tied into a system of equations that can be analyzed together. However, variances can also differ across $j$, while $\eta_{j}$ are independent across $i$. Possible heterogeneity, interdependence, and commonality across countries and among variables are absorbed in the coefficient vectors $\beta_{t}$, assuming the following state-space structure:

$$
\begin{gather*}
Y_{t}=\chi_{t} \beta_{t}+\eta_{t}  \tag{8}\\
\beta_{t}=\beta_{t-1}+v_{t} \quad \text { with } \quad v_{t} \sim N(0, \mathcal{H}) \tag{9}
\end{gather*}
$$

where $v_{t} \sim N(0, \Sigma \otimes V), \mathcal{H}=\operatorname{diag}\left(\overline{\mathcal{H}}_{1}, \ldots, \overline{\mathcal{H}}_{F}\right)$ is a block diagonal matrix, and $\mathcal{H}_{f}=h_{f} \cdot I$, with $f=1, \ldots, F$ and $h_{f}$ controls the tightness of factor $i$ in the coefficients. In the model 8 , it is assumed a general AR structure and the block diagonality of $\mathcal{H}$ is needed to guarantee the orthogonality of the factors, which is preserved a-posterior and, hence, their identifiability. In this way, prior assumptions can be specified and, hence, Bayesian computations are feasibles.

### 3.3 Prior Assumptions

Let $\phi_{0}=\left(\Omega^{-1}, h_{f}, \beta_{0}\right)$ to be the prior densities, two tentative beliefs (assumptions) can be defined accounting for the model described in equation 7. (i) Conditional Normality: $p\left(\eta \mid \phi_{0}\right)=$ $N(0, \Omega)$. This is a hierarchical prior for $\eta$. (ii) Conditional Independence: $p\left(\eta \mid \phi_{0}\right)=p\left(\eta \mid \phi_{0}\right) p\left(\chi \mid \phi_{0}\right)$.

A hierarchical prior for $\eta$ has been already specified. Thus, in order to complete the model, a prior moments on $\left(\Omega^{-1}, h_{f}, \beta_{0}\right)$ need to be defined. The likelihood function can be derived from the sampling density $p\left(Y \mid \phi_{0}\right)$ and it can be shown to be of a form that breaks into a mixture of distributions. In words: (i) A Normal distribution for factors $\beta$ given $\Omega$; (ii) a Wishart distribution for $\Omega^{-1}$; (iii) an Inverse Gamma distribution for $h_{f}$, where $h_{f}=\operatorname{vec}(\mathcal{H})$. That is:

$$
\begin{equation*}
\beta \mid \Omega^{-1}, Y \sim N\left(\hat{\beta}, \Omega^{-1} \otimes\left(\chi^{\prime} \chi\right)^{-1}\right) \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
\Omega^{-1} \mid Y \sim W\left(S^{-1}, T-k-1\right) \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
h_{f} \left\lvert\, Y \sim I G\left\{\frac{\bar{\omega}}{2}, \frac{v S}{2}\right\}\right. \tag{12}
\end{equation*}
$$

where $\hat{\beta}=\left(\sum_{t} \chi_{t}^{\prime} \chi_{t}\right)^{-1}\left(\sum_{t} \chi_{t}^{\prime} Y_{t}\right)$ is the OLS estimate of $\beta, S=\left(Y_{t}-\hat{\beta} \chi_{t}\right)^{\prime}\left(Y_{t}-\hat{\beta} \chi_{t}\right)$ is the sum of squared errors, and $\hat{\Sigma}=S /(T-k)$ is the OLS estimate of $\Sigma$.

Furthermore, such prior assumptions will generally be influenced, for example, by common or subjective beliefs about marginal effects of economic variables. Hence, Independent Normal Wishart Prior is used in this analysis, since it assumes that tentative beliefs on $\phi_{0}=\left(\beta_{0}, \Omega^{-1}, h_{f}\right)$ derive from separate considerations.

According to equations 8 and 9, a MCMC methods and alternatives can be employed to conduct inference on the time-varying $\beta_{t}\left(\right.$ with $\left.\mathcal{H}_{i t} \neq 0\right)$ and construct their exact posterior distributions. The result would be a Bayesian Model Averaging (BMA). Let data run from $(-\tau, T)$, where $(-\tau, 0)$ is a training sample used to estimate features of the prior, when such a sample is unavailable, it is sufficient to modify the expressions for the prior moments in equations 10, 11, and 12 as:

$$
\begin{equation*}
p\left(\Omega^{-1}, h_{f}, \beta_{0}\right)=p\left(\Omega^{-1}\right) \cdot \Pi_{f} p\left(h_{f}\right) \cdot p\left(\beta_{0}\right) \tag{13}
\end{equation*}
$$

where, $p\left(\Omega^{-1}\right)=W\left(\beta_{1}, z_{1}\right), p\left(h_{f}\right)=I G\left(\frac{\bar{\omega}_{0}}{2}, \frac{S_{0}}{2}\right), p\left(\beta_{0} \mid \mathscr{F}_{-1}\right)=N\left(\bar{\beta}_{0}, \bar{R}_{0}\right)$, and $\mathscr{F}_{-1}$ denotes the information available at time -1. Here, $N()$ stands for Normal, $W()$ for Wishart, and $I G()$ for Inverse Gamma distributions. The prior for $\beta_{0}$ and the law of motion for the factors imply that $p\left(\beta_{t} \mid \mathscr{F}_{-1}\right)=N\left(\bar{\beta}_{t-1 \mid t-1}, \bar{R}_{t-1 \mid t-1}+\mathcal{H}_{t}\right)$, where $\bar{\beta}_{t-1 \mid t-1}$ and $\bar{R}_{t-1 \mid t-1}$ are, respectively, the mean and the variance-covariance matrix of the conditional distribution of $\bar{\beta}_{t \mid t}$. The hyperparameters are all known. To be more precise, collecting them in a vector $\delta$, where $\delta=\left(z_{1}, Q_{1}, \bar{\omega}_{0}, S_{0}, \bar{\beta}_{0}, \bar{R}_{0}\right)$, they are treated as fixed and are either obtained from the data to tune the prior to the specific applications (this is the case for $\bar{\beta}_{0}$ and $\beta_{1}$ ) or selected a-priori to produce relatively loose priors (this is the case for $z_{1}, S_{0}, \bar{R}_{0}$ ).

Whenever $\Omega$ is not replaced by an estimate $8^{8}$ the only fully Bayesian approach which leads to analytical results requires the use of a natural conjugate prior. Here, the prior, likelihood and posterior come from the same family of distribution. According to the equations 10, 11, and 12 , and letting time variant factors, the natural conjugate prior has the form:

[^4]\[

$$
\begin{gather*}
\beta_{t} \mid \Omega^{-1}, Y^{T} \sim N\left(\bar{\beta}_{t \mid t}, \Omega \propto \bar{R}_{t \mid t}\right) \quad \text { or } \quad p\left(\beta_{t} \mid \Omega^{-1}, Y^{T}\right)=N\left(\bar{\beta}_{t \mid t}, \Omega \propto \bar{R}_{t \mid t}\right)  \tag{14}\\
\Omega^{-1} \mid Y^{T} \sim W\left(\beta_{1}, z_{1}\right) \quad \text { or } \quad p\left(\Omega^{-1} \mid Y^{T}, \beta\right)=W\left(\beta_{1}, z_{1}\right) \tag{15}
\end{gather*}
$$
\]

where, $\bar{\beta}_{t \mid t}$ and $\bar{R}_{t \mid t}$, correspond to hyperparameters collected in the vector $\delta$, and GIRFs and CFs can be obtained with the same approach by Monte Carlo integration. That is, draws of $\mathcal{H}$ derive from equation $9, \Omega^{-1}$ can be obtained from equation 8 and, conditional on these, draws of $\beta$ can be taken from equation 8 . Then, draws of impulse responses can be computed using these drawn values of $\Omega^{-1}$ and $\beta$. If $\mathcal{H}=0$, allowing for time variant factors, draws of $h_{f}$ can be taken from a Normal-Inverse Gamma distribution.

According to the natural conjugate prior, $\beta_{t} \mid \Omega$ and $\Omega^{-1}$ have Normal and Wishart distributions, respectively. The fact that the prior for $\beta_{t}$ depends on $\Omega$ implies that $\beta_{t}$ and $\Omega$ are not independent of one another. To be more precise, the estimation works with a prior which has VAR coefficients and error covariance being independent of one another. To allow for different equations in the VAR to have different explanatory variables, previous specification have to be modified. Given the NLR model in equation 8, a general prior which does not involve the restrictions inherent in the natural conjugate prior is the independent Normal-Wishart prior:

$$
\begin{equation*}
p\left(\beta_{t}, \Omega^{-1} \mid Y^{T}\right)=p\left(\beta_{t} \mid Y^{T}\right) \cdot p\left(\Omega^{-1} \mid Y^{T}\right) \tag{16}
\end{equation*}
$$

where,

$$
\begin{align*}
\beta_{t} \mid Y^{T} \sim N\left(\bar{\beta}_{t \mid t}, \bar{R}_{t \mid t}\right) & \text { or } \tag{17}
\end{align*} \quad p\left(\beta_{t} \mid Y^{T}\right)=N\left(\bar{\beta}_{t \mid t}, \bar{R}_{t \mid t}\right)
$$

where $i W()$ stands for inverse Wishart distribution. Here, the prior allows for the prior covariance matrix, $\bar{R}_{t \mid t}$, to be anything the researcher chooses, rather the restrictive $\left(\Omega \mid Y^{T} \otimes \bar{R}_{t \mid t}\right)$ form of the natural conjugate prior.

[^5]
### 3.4 Posterior Distributions and MCMC Integration

The posterior distributions for $\phi=\left(\Omega^{-1}, h_{f},\left\{\beta_{t}\right\}_{t=1}^{T}\right)$ are calculated combining the prior with the (conditional) likelihood on initial conditions of the data, which is proportional to:

$$
\begin{equation*}
L\left(Y^{T} \mid \phi\right) \propto(\Omega)^{-\frac{T}{2}} \exp \left\{-\frac{1}{2}\left[\Sigma_{t}\left(Y_{t}-\left(\Theta_{t} D\right) \beta_{t}\right)^{\prime} \Omega^{-1} \Sigma_{t}\left(Y_{t}-\left(\Theta_{t} D\right) \beta_{t}\right)\right]\right\} \tag{19}
\end{equation*}
$$

where, $Y^{T}=\left(Y_{1}, \ldots, Y_{T}\right)$ denotes the data, $\phi=\left(\Omega^{-1}, h_{f},\left\{\beta_{t}\right\}\right)$ refers to the unknowns whose joint distribution needs to be found, with $\phi_{-k}$ standing the vector $\phi$ excluding the parameter $k$.

Despite the dramatic parameter reduction obtained with equation 8, analytical computation of posterior distributions $\left(\phi \mid Y^{T}\right)$ is unfeasible. Thus, through Monte Carlo techniques, a variant of Gibbs sampler approach can be used in this framework by making use of Kalman filter ${ }^{10}$ so it only requires knowledge of the conditional posterior distribution of $\phi$. The latter is extremely useful for investigating the issue of parameters constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period. To be more precise, the Kalman filter technique consists of two equations: the transition equation describing the evolution of the state variables and the measurement equation describing how the observed data are generated from the state variables. For the conditional posterior of $\left(\beta_{1}, \ldots, \beta_{T} \mid Y^{T}, \phi_{-\beta_{t}}\right)$, it gives the following recursions:

$$
\begin{gather*}
\beta_{t \mid t}=\tilde{\beta}_{t-1 \mid t-1}+\left[\tilde{R}_{t \mid t-1}\left(\Theta_{t} D\right) F_{t \mid t-1}^{-1}\right]\left[Y_{t}-\left(\Theta_{t} D\right) \theta_{t}\right]  \tag{20}\\
R_{t \mid t}=\left[I-\tilde{R}_{t \mid t-1}\left(\Theta_{t} D\right)^{\prime} F_{t \mid t-1}^{-1}\left(\Theta_{t} D\right)\right]\left(\tilde{R}_{t-1 \mid t-1}+\mathcal{H}\right)  \tag{21}\\
F_{t \mid t-1}=\left(\Theta_{t} D\right)^{\prime} \tilde{R}_{t \mid t-1}\left(\Theta_{t} D\right)+\Omega_{t} \tag{22}
\end{gather*}
$$

Hence, in order to obtain a sample $\left\{\beta_{t}\right\}$ from the joint posterior distribution $\left(\beta_{1}, \ldots, \beta_{T} \mid Y^{T}, \phi_{-\beta_{t}}\right)$, the output of the Kalman filter is used to simulate $\beta_{T}$ from $N\left(\beta_{T \mid T}, R_{T \mid T}\right)$, $\beta_{T-1}$ from $N\left(\beta_{T-1}, R_{T-1}\right)$, and $\beta_{1}$ from $N\left(\beta_{1}, R_{1}\right)$. The recursion can be started choosing $R_{0 \mid 0}$ to be diagonal with elements equal to small values, whereas $\beta_{0 \mid 0}$ can be estimated in the training sample or initialized using a constant coefficient version of the model. Convergence only requires the algorithm to be able to visit all partitions of the parameter-space in a finite number of iterations.

[^6]Thus, the marginal distributions of $\beta_{t}$ can be computed averaging over draws in the nuisance dimensions and the posterior distributions for $\phi$ are:

$$
\left.\begin{array}{rll}
\beta_{t} \mid Y^{T}, \phi_{-\beta_{t}} & \sim N\left(\tilde{\beta}_{t \mid T}, \bar{R}_{t \mid T}\right) & \text { or }
\end{array} \quad p\left(\beta_{t} \mid Y^{T}, \phi_{-\beta_{t}}\right)=N\left(\tilde{\beta}_{t \mid T}, \bar{R}_{t \mid T}\right) ~ 子 \begin{array}{rl} 
& \\
\Omega \mid Y^{T}, \phi_{-\Omega} \sim i W\left(\hat{z}_{1}, \hat{\beta}_{1}\right) & \text { or }
\end{array} \quad p\left(\Omega \mid Y^{T}, \phi_{-\Omega}\right)=i W\left(\hat{z}_{1}, \hat{\beta}_{1}\right)\right\}
$$

where,

$$
\begin{gather*}
\tilde{\beta}_{t \mid T}=\tilde{R}_{t \mid T}\left[\bar{R}_{t \mid T}^{-1} \bar{\beta}+\sum_{t=1}^{T}\left(\Theta_{t} D\right)^{\prime} \Omega^{-1}\left(\Theta_{t} D\right) \hat{\beta}\right]  \tag{26}\\
\tilde{R}_{t \mid T}=\left[\bar{R}_{t \mid T}^{-1}+\sum_{t=1}^{T}\left(\Theta_{t} D\right)^{\prime} \Omega^{-1}\left(\Theta_{t} D\right)\right]^{-1} \tag{27}
\end{gather*}
$$

The $\hat{\beta}$ is the GLS estimator, with $\hat{\beta}=\left[\left(\Theta_{t} D\right)^{\prime} \Omega^{-1}\left(\Theta_{t} D\right)\right]^{-1} \cdot\left(\Theta_{t} D\right)^{\prime} \Omega^{-1} Y_{t}$. Rearraging terms, the equation 26 can be rewritten as:

$$
\begin{equation*}
\tilde{\beta}_{t \mid T}=\tilde{R}_{t \mid T}\left[\bar{R}_{t \mid T}^{-1} \bar{\beta}+\sum_{t=1}^{T}\left(\Theta_{t} D\right)^{\prime} \Omega^{-1} Y_{t}\right] \tag{28}
\end{equation*}
$$

where $\tilde{\beta}_{t \mid T}$ and $\tilde{R}_{t \mid T}$ denote the smoothed one-period-ahead forecasts of $\beta_{t}$ and of the variancecovariance matrix of the forecast error, respectively, $\hat{z}_{1}=z_{1}+T, \hat{\beta}_{1}=\beta_{1}+\sum_{t}\left(Y_{t}-\left(\Theta_{t} D\right) \beta_{t}\right)$, $\bar{\omega}_{f}=K+\bar{\omega}_{0}, \bar{S}=S_{0}+\Sigma_{t}\left(\beta_{t}^{f}-\beta_{t-1}^{f}\right)^{\prime}\left(\beta_{t}^{f}-\beta_{t-1}^{f}\right)$, with $\beta_{t}^{f}$ denoting the $f^{t h}$ subvector of $\beta_{t}$ and $f$ refers to the factors described in equation 3 .

In this framework, it is common to burn some number of samples at the beginning and, hence, consider only any $n t h$ sample when averaging values to compute expectation. Moreover, the regressors of the NLR model in equation 8 are correlated, but the presence of correlation, even of extreme form, does not create problems in identifying the loadings as long as the priors are proper. In addition, the choice of making $E_{t}$ and $u_{t}$ correlated allows conjugation between the prior and the likelihood, avoids identification issues, and greatly simplifies the computation of the posterior. Here, the forecast error $\eta=Y_{t}-\left(X_{t} \Xi\right) \beta_{t}$ has the form $\left(\eta \mid \sigma^{2}\right) \sim N\left(0, \sigma_{t} \Omega\right)$ and, unconditionally,
$\eta_{t}$ has a multivariate $t$-distribution centered at 0 , with a scale matrix proportional to $\Omega$ and $\nu_{n}$ degrees of freedom. Hence, the innovations of the model described in equation 8 are endogenously allowed to have fat tails. Finally, since the fit improves when $\sigma^{2} \rightarrow 0$, the model in equation 8 presents an exact factorization of $\gamma_{t}$.

## 4 Application

### 4.1 Theoretical Background

The Euro Area is an unique form of a monetary union without historical precedence. The member states of the euro have assigned the framing of monetary policy to a common monetary authority, the European Central Bank (ECB), set up as a highly independent central bank to insure that it will be able to carry out a policy of price stability. The adoption of a common monetary policy in Europe has eliminated the possibility to use monetary policy for the stabilization of country-specific shocks. This is generally considered as the main cost of forming a monetary union. How large this cost actually is depends on what alternative mecchanisms are available to ensure economic adjustment to idiosyncratic shocks. As monetary policy can no longer address country-specific shocks and factor mobility does not solve the problem being notoriously low within and across countries, the only remaining instrument in the hands of national authorities and capable to stabilize local macroeconomic-financial conditions is fiscal policy. In the context of the European Monetary Union (EMU), fiscal flexibility is hampered by large public debts and formal institutional constraints (such as the Maastricht rules and the Stability and Growth Pact). Thus, policy coordinations became an important issue in order to assess whether decisions about a given policy instrument should be taken at the central level or be decentralized. Even though the interaction with the ECB is a key aspect to determine whether coordination is desirable, the debate often remain focused on the magnitude and the signs of the cross-country spillovers that could justify or not a more cooperative approach in the member states in response to bad shocks. Nevertheless, the theoretical literature does not provide a clear-cut analytical answer. For istance, in classic analyses of policy coordination ${ }^{111}$, ad-hoc fixed-price models generally assume direct and positive demand spillovers. By contrast, micro-founded models $\left[^{12}\right.$ of EMU tend to conclude in

[^7]favour of negative spillovers.
In the last decade, the recent financial crisis that started in mid-2007 and affected the whole world by September 2008 was one of the most challenge episodes for policy makers both at governments and central banks since the introduction of the euro. In a worldwide context, the effects of this disruption was not limited to the financial sector. Global real output and trade declined dramatically, and central banks took unprecedented coordinated action, in part, to alleviate the adverse impacts of the financial markets shocks on real activity. To be more precise, the theoretical literatur ${ }^{133}$ achieved three main findings. First, there are institutional and economic interdependencies across countries, specially between Eurozone countries having relinquished independent monetary and exchange rate policies. Second, there may still be a substantial degree of heterogeneity with some common behaviours in economic-financial linkages across countries and that those linkages may have changed over time due to different transmission channels. Third, the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. Nevertheless, when dealing with multicountry data, these studies have not yet achieved an empirical consensus or have reached very different conclusions. Most of these differences can be related to diverging methodologies to assume structural relationships or lagged interdependencies across factors and alternative ways to assess spillover effects (such as, time invariant factors, exogenous variables, restrictions on selected priors).

For the above reasons, it makes sense to try to estimate the structural EPBVAR model as described in Section 3. To be more precise, I focused on a pool of the current members of the EMU addressing three important macroeconomic-financial issues, with particular attention on the recent financial crisis and post-crisis consolidation. First, how dimension and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. A survey on policy recommendations and business cycle convergence are also assessed.

[^8]
### 4.2 Empirical Analysis

My baseline model consists of 8 EA countries: Italy (IT), Spain (ES), France (FR), Austria $(A T)$, Germany $(D E)$, Ireland (IE), Greece $(G R)$, and Portugal $(P T)$. The dataset contains the following collection of variables. (i) Six endogenous variables are involved to describe real (real ${ }_{i t, j}$ ) and financial $\left(f i n_{i t, j}\right)$ economy, so split: three real variables (general government spending, gross fixed capital formation, GDP growth rate) and three financial variables (general government debt, current account balance, interest rate). (ii) Bilateral flows of trade (rweights $s_{i, j}$ ) and capital ( fweights $_{i t, j}$ ) are used to capture inter-linkages between country- and variable- specific factors in real and financial dimension. (iii) Five (directly) observed variables are used as proxy for accounting for latent macroeconomic-institutional implications (structures $i_{i t, j}$ ), so shared: one indicator monitoring external positions (net international investments); one indicator capturing competitiveness developments and catching-up effects (nominal labour cost); and three indicators reflecting internal imbalances (general government consumption, private sector consumption, and change of unemployment rate). (iv) The productivity is used to evaluate the effects of structural spillovers on total economy given an unexpected shock and corresponds to the logarithm of the GDP per capita for each country. The weights ${ }_{i t,} \sqrt{[14}$ and the structures ${ }_{i t, j}$ components are treated endogenously and used to investigate confounding and latent effects, respectively.

The series are expressed in standard deviation with respect to the same quarter of the previous year $\left(q_{t} / q_{t-1}\right)$ and seasonally and calendar adjusted. All EA data points are originated from the Eurostat database. The estimation sample covers the period from March, 1999 to December, 2013. It amounts, without restrictions, to 3360 regression parameters. To be more precise, each equation of the structural EPBVAR has $k=[1+8(6+2+5) \cdot 1]=105$ coefficients and there are 60 equations in the system. Since this span of data includes a sufficient number of quarters describing the recent financial crisis and fiscal consolidation, the model is able to capture not only possible time variation around business cycle phases, but also time variation caused by possible structural and economic changes. Finally, according to the Schwartz-Bayesian Information Criterion, the model is estimated with only one lag of all variables and factors in the system.

Here, the structural $\operatorname{EPBVAR}(1)$ has the form:

$$
\begin{equation*}
Y_{i t}=A_{i t, 0}+A_{i t, j}(L) Y_{i, t-1}+B_{i t, j}(L) W_{i, t-1}+C_{i t, j}(L) Z_{i, t-1}+\varepsilon_{i t} \tag{29}
\end{equation*}
$$

[^9]where $i, j=1,2, \ldots, 8$ are the country indeces, $A_{i t, j}$ is a $[(8 \cdot 6) \cdot(8 \cdot 6)]$ matrix of $r e a l_{i t, j}$ and $f i_{i t, j}$ coefficients, $B_{i t, j}$ is a $[(8 \cdot 2) \cdot(8 \cdot 2)]$ matrix of weights ${ }_{i t, j}$ coefficients, $C_{i t, j}$ is a $[(8 \cdot 5) \cdot(8 \cdot 5)]$ matrix of structures $_{i t, j}$ coefficients, $Y_{i, t-1}$ is a $[(8 \cdot 6) \cdot 1]$ vector of the endogenous variables lagged containing real and financial variables, $W_{i, t-1}$ is a $[(8 \cdot 2) \cdot 1]$ vector including bilateral trade and financial flows, and $Z_{i, t-1}$ is a $[(8 \cdot 5) \cdot 1]$ vector containing (directly) observed variables used as proxy for economic-institutional implications. The analysis assumes that the coefficient vector $\gamma_{t}$ in equation 3 dependes on eight factors. Each of them is the result of estimates of interaction terms. Thus,
\[

$$
\begin{equation*}
D \beta_{t}=D_{1} \beta_{1 t}+D_{2} \beta_{2 t}+D_{3} \beta_{3 t}+D_{4} \beta_{4 t}+D_{5} \beta_{5 t}+D_{6} \beta_{6 t}++D_{7} \beta_{7 t}++D_{8} \beta_{8 t} \tag{30}
\end{equation*}
$$

\]

$D_{1, i t}=\sum_{m_{1}=1}^{3} \cdot \sum_{j} y_{i t m_{1} k_{1} l-j}$ and $D_{2, i t}=\sum_{m_{2}=4}^{6} \cdot \sum_{j} y_{i t m_{2} k_{1} l-j}$ are matrices of dimensions $N M_{1} k_{1} \cdot N$ and $N M_{2} k_{1} \cdot N$ respectively, with $k_{1}=[1+N M] l$ and $M_{1}, M_{2} \leq M$ denote real and financial variables, $\beta_{1 t}$ and $\beta_{2 t}$ are mutually orthogonal $N M_{1} k_{1} \cdot 1$ and $N M_{2} k_{1} \cdot 1$ vectors, respectively. They capture movements in the coefficient vector $\gamma_{t}$ which are country-specifics. They account for the only real $l_{i t, j}$ and fin $_{i t, j}$ components in order to evaluate restricted structural spillover effects and obtain a basis for matching additional effects.
$D_{3, i t}=\sum_{m_{1}=1}^{3} \cdot \sum_{j} y_{i t m_{1} k_{2} l-j}$ and $D_{4, i t}=\sum_{m_{2}=4}^{6} \cdot \sum_{j} y_{i t m_{2} k_{2} l-j}$ are matrices of dimensions $N M_{1} k_{2} \cdot N$ and $N M_{2} k_{2} \cdot N$ respectively, with $k_{2}=[1+N M+N q] l, \beta_{3 t}$ and $\beta_{4 t}$ are mutually orthogonal $N M_{1} k_{2} \cdot 1$ and $N M_{2} k_{2} \cdot 1$ vectors, respectively. They capture movements in the coefficient vector $\gamma_{t}$ which are country-specifics, accounting for two components: real $l_{i t, j}$ with $r_{\text {weights }}^{i t, j}$ and fin $_{i t, j}$ with fweights $_{i t, j}$. These factors are able to assess the role of transmission channels in driving the spreading of a shock in real and financial dimension, capturing possible homogeneous and/or heterogeneous patterns across countries.
$D_{5, i t}=\sum_{m_{1}=1}^{3} \cdot \sum_{j} y_{i t m_{1} k l-j}$ and $D_{6, i t}=\sum_{m_{2}=4}^{6} \cdot \sum_{j} y_{i t m_{2} k l-j}$ are matrices of dimensions $N M_{1} k$. $N$ and $N M_{2} k \cdot N$ respectively, with $k=[1+N M+N q+N \xi] l, \beta_{5 t}$ and $\beta_{6 t}$ are mutually orthogonal $N M_{1} k \cdot 1$ and $N M_{2} k \cdot 1$ vectors, respectively. They capture movements in the coefficient vector $\gamma_{t}$ which are country-specifics, accounting for two components: real ${ }_{i t, j}$ with rweights ${ }_{i t, j}$ and structures $_{i t, j}$ and fin $_{i t, j}$ with fweights $_{i t, j}$ and structures $_{i t, j}$. These factors are able to investigate the effective importance and impact of economic-institutional implications in driving the propagation and transmission of country-specific shocks in real and financial dimension, capturing possible hidden and/or not directly observed interdependencies across countries.
$D_{7, i t}=\sum_{i=1}^{8} \cdot \sum_{j} y_{i t m_{g} l-j}$ is a matrix of dimension $N M k \cdot M_{g}$ and $\beta_{7 t}$ is mututally orthogonal $N M_{g} \cdot 1$ vector capturing movements in $\gamma_{t}$ which are variable-specifics, where $M_{g}=1,2,3,4 \leq M$ denotes the number of variable groups. Thus, it corresponds to four groups of variables: (i) real ${ }_{i t, j}$ with rweights $_{i t, j}$; (ii) fin $_{i t, j}$ with fweights $_{i t, j}$; (iii) real ret,j with rweights $_{i t, j}$ and structures ${ }_{i t, j}$; and (iv) fin $_{i t, j}$ with fweights $_{i t, j}$ and structures $_{i t, j}$. The factor is able to capture possible commonality, heterogeneity and interdependence across variables accounting for both confounding and latent effects.
$D_{8, i t}=\sum_{i=1}^{8} \sum_{m_{c}=1}^{2} \cdot \sum_{j} y_{i t m_{c} l-j}$ is a matrix of dimension $N M_{c} k \cdot 1$ and $\beta_{8 t}$ is mutually orthogonal $N \cdot 1$ vector capturing movements in $\gamma_{t}$ which are common across all countries and variables, with $M_{c}=1,2 \leq M$ denotes the number of common variable groups. Thus, it accounts for two groups: real $_{i t, j}$ and fin $_{i t, j}$ with weights $_{i t, j}$ and real $_{i t, j}$ and fin $_{i t, j}$ with structures ${ }_{i t, j}$. It highlights possible reasons about different reactions or co-movements across countries and variables given a common unexpected shock.

Hence, $\beta_{t}=\left(\beta_{1 t}^{\prime}, \beta_{2 t}^{\prime}, \beta_{3 t}^{\prime}, \beta_{4 t}^{\prime}, \beta_{5 t}^{\prime}, \beta_{6 t}^{\prime}, \beta_{7 t}^{\prime}, \beta_{8 t}^{\prime}\right)^{\prime}$ is a (54•1) vector and the parsimonius NLR model having the form:

$$
\begin{align*}
Y_{t} & =\sum_{i=1}^{8} X_{t}^{i}\left(D_{f} \beta_{f t}+u_{t}\right)+E_{t} \\
& \equiv \chi_{f t} \beta_{f t}+\eta_{t} \equiv \\
& \equiv \chi_{1 t} \beta_{1 t}+\chi_{2 t} \beta_{2 t}+\ldots+\chi_{8 t} \beta_{8 t}+\eta_{t} \quad \text { with } \quad f=1,2, \ldots, 8 \tag{31}
\end{align*}
$$

where $\chi_{t} \equiv X_{t} \cdot D$, with $D=\left[D_{1}, D_{2}, D_{3}, D_{4}, D_{5}, D_{6}, D_{7}, D_{8}\right], \beta_{t}=\beta_{t-1}+v_{t}$, and $f$ stands for the factorization assumed for $\gamma_{t}$ in order to construct hierarchical priors and obtain posterior mixed distributions. In equation 31, $\chi_{1 t} \beta_{1 t}, \chi_{2 t} \beta_{2 t}, \chi_{3 t} \beta_{3 t}, \chi_{4 t} \beta_{4 t}, \chi_{5 t} \beta_{5 t}$, and $\chi_{6 t} \beta_{6 t}$ are observable country-specific indicators for $Y_{t}, \chi_{7 t} \beta_{7 t}=\chi_{7 t}^{1} \beta_{7 t}^{1}, \chi_{7 t}^{2} \beta_{7 t}^{2}, \chi_{7 t}^{3} \beta_{7 t}^{3}, \chi_{7 t}^{4} \beta_{7 t}^{4}$ is observable cross-country variable-specific groups indicator for $Y_{t}$, and $\chi_{8 t} \beta_{8 t}=\chi_{8 t}^{1} \beta_{8 t}^{1}, \chi_{8 t}^{2} \beta_{8 t}^{2}$ is observable common indicator for $Y_{t}$. The hyperparameters are all known ${ }^{15}$. To be more precise, the values used are: $z_{1}=N \cdot(M+4+2), Q_{1}=\hat{Q}_{1}, \bar{\omega}_{0}=10^{4+2}, S_{0}=1.0, \bar{\theta}_{0}=\hat{\theta}_{0}$, and $\bar{R}_{0}=I_{f}$. Here, $\hat{Q}_{1}$ is a block diagonal matrix, with $\hat{\theta}_{1}=\operatorname{diag}\left(Q_{11}, \ldots, Q_{1 N}\right)$ and $Q_{1 i}$ is the estimated covariance matrix for each country VAR, and $\hat{\beta}_{0}$ is obtained with the OLS version of equation 31 .

In order to show how dynamic analysis can be undertaken, accurate simulations are run. To

[^10]be more precise, the total number of draws is $5000+1000=6000$, which corresponds to the sum of final number of draws to discard and draws to save, respectively. The study checked convergence recursively calculating the first two moments of the posterior of the parameters using 1000, $2000,3000,4000,5000$ draws and found that convergence is obtained about 1000 draws. A total of 1000 draws were used to conduct posterior inference at each $t$. The CGIRFs are computed as the difference between conditional ${ }^{[16}$ and unconditional ${ }^{17}$ projection of output growth for each country in the period $1999 q 1: 2016 q 1$. Here, outcomes absorb conditional forecasts computed on the time frame of 9 quarters ( 2.25 years). The natural conjugate prior refers to two subsamples, 2006q4-2009q4 and 2010q1-2013q4, in order to highlight the impact of the recent financial crisis and fiscal consolidation, respectively.

Finally, Table 1 shows some main diagnostic tests in order to verify the robustness and consistency of the model. The estimates would be asymptotically consistent given the absence of serial correlations between residuals. According to time variant factors and the Schwarz approximation, the (conditional) ML is tested. The latter confirms the exact $\gamma_{t}$ 's factorization as the p-value of the test equals $1.41 e^{-4}$.

Table 1: Diagnostic Tests

| Test | Test Statistics | degree of freedoms | p-value |
| :---: | :---: | :---: | :---: |
| $L G B_{m}$ | 5762 | 2430 | 0.00 |
| $P_{m}$ | 1137.2 | 1215 | 0.9451 |
| $M L E_{f}$ | 38.22 | 12 | 0.000141 |

Here, $L G B_{m}$ stands for Multivariate Ljung-Box Test on the series, with lags $m=30 . P_{m}$ refers to Portmanteau (Asymptotic) Test on the residuals, with lags $m=30 . M L E_{f}$ is the Marginal (Conditional) Likelihood Estimation Test obtained through the Schwartz approximation, with $f=8$.

### 4.3 Spillover Effects and Shock Transmission

Given the benchmark model in equation 31, structural spillover effects given an unexpected shock in real and financial dimension, accounting for both counfounding and latent effects, can be assessed. The output deriving from the model is able to absorbe each single draw obtained from the posterior of regression coefficients. Firstly, I construct a spillover matrix (Table 2) in order to define (individual) Bilateral Spillover Effects (BSEs). The latter describe the dynamics of

[^11]impulse responses from a shock in real and financial variables within the EA as weighted average of responses of each variable. Since BSEs can either be negative or positive, two components can be defined: the average sum of the impulse responses to others defines Bilateral OUT Spillover Effects (equation 32) and the average sum of the impulse responses from others defines Bilateral $I N$ Spillover Effects (equation 33). They incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of a potential shock.

Table 2: Structural Spillover Matrix

| Shock/Response | $y_{1}$ | $y_{2}$ | $\ldots$ | $y_{n}$ | To Others |  |
| :---: | :---: | :---: | :---: | :---: | :--- | :---: |
| $y_{1}$ | $I R_{y_{1} \rightarrow y_{1}}$ | $I R_{y_{1} \rightarrow y_{2}}$ | $\cdots$ | $I R_{y_{1} \rightarrow y_{n}}$ | $\sum_{j=1}^{N} I R_{y_{1} \rightarrow y_{j}}$ | $j \neq 1$ |
| $y_{2}$ | $I R_{y_{2} \rightarrow y_{1}}$ | $I R_{y_{2} \rightarrow y_{2}}$ | $\ldots$ | $I R_{y_{2} \rightarrow y_{n}}$ | $\sum_{j=1}^{N} I R_{y_{2} \rightarrow y_{j}}$ | $j \neq 2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |  | $\vdots$ |
| $y_{n}$ | $I R_{y_{n} \rightarrow y_{1}}$ | $I R_{y_{n} \rightarrow y_{2}}$ | $\cdots$ | $I R_{y_{n} \rightarrow y_{n}}$ | $\sum_{j=1}^{N} I R_{y_{n} \rightarrow y_{j}}$ | $j \neq n$ |
| From Others | $\sum_{j=2}^{N} I R_{y_{j} \rightarrow y_{1}} \sum_{j=1}^{N} I R_{y_{j} \rightarrow y_{2}} \ldots \sum_{j=1}^{N-1} I R_{y_{j} \rightarrow y_{n}} \sum_{j=1}^{N}\left(I R_{y_{i} \rightarrow y_{j}}-I R_{y_{j} \rightarrow y_{i}}\right)$ |  |  |  |  |  |

Note: Row variables are the origin of the unexpected shock. Column variables are the respondents or spillover receivers.

$$
\begin{align*}
B S E_{\text {OUT }, y_{i} \rightarrow *} & =\sum_{j=1}^{N} I R_{y_{i} \rightarrow y_{j}} \text { with } \quad i=j=1, \ldots, 8  \tag{32}\\
B S E_{I N, * \rightarrow y_{i}} & =\sum_{j=1}^{N} I R_{y_{j} \rightarrow y_{i}} \quad \text { with } \quad i=j=1, \ldots, 8 \tag{33}
\end{align*}
$$

By the same token, Bilateral Net Spillover Effects (BNSEs) can be defined as the difference between the conditional impulse responses sent and received to/from another variable (equation (34). When the BNSE is positive/negative, the variable (country) is a net sender/net receiver of the system, respectively.

$$
\begin{equation*}
B N S E_{y_{i, j}}=B S E_{O U T, y_{i} \rightarrow *}-B S E_{I N, * \rightarrow y_{i}} \quad \text { with } \quad i=j=1, \ldots, 8 \tag{34}
\end{equation*}
$$

where $\sum_{j=1}^{N}\left(I R_{y_{i} \rightarrow y_{j}}-I R_{y_{j} \rightarrow y_{i}}\right)=0$.
Thus, I have suitable instruments to study dimension and intensification of spillover effects. To be more precise, I calculate the Systemic Contribution index, defined as the ratio between the BNSEs and the Total Net Positive Spillover (TNPS) of the system (equation 35). It represents the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events.

$$
\begin{equation*}
S C_{y_{i, j}}=\frac{B N S E_{y_{i, j}}}{T N P S_{y_{i, j}}} \tag{35}
\end{equation*}
$$

The Figure 1, in which I consider the first two country-specific indicators, confirms that the literature reached. Overall, there is a heterogenous pattern across countries in financial dimension $\left(\hat{S}_{f i n}=0.310\right)$ and even more in real dimension $\left(\hat{S}_{\text {real }}=0.342\right)$, with some common behaviors. These commonalities seem to be larger in financial dimension (Figure 1b). There are not significant findings about the signs of the spillovers, confirming the need to consider further empirical evidence. In real dimension (Figure 1a), most countries tend to be net receivers of the system and, hence, unexpected country-specific shocks directly affect own output growth in financial dimension and then in real economy because of consistent cross-country interdependencies.


Figure 1: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system. They account for real $_{i t, j}$ (plot $a$ ) and fin $_{i t, j}$ (plot $b$ ) components, corresponding to $\chi_{1 t} \hat{\beta}_{1 t}$ and $\chi_{2 t} \hat{\beta}_{2 t}$ cross-country indicators, respectively.

Accounting for the weights $_{i t, j}$ and structures $_{i t, j}$ components (Figure 22), important modelling and policy perspectives are found.

Firstly, focusing on the only weights ${ }_{i t, j}$ component (Figures 2a and 2b), a significant common pattern emerges in both real and financial dimension. In the latter, commonalities tend to largely occurr. From a modelling perspective, it highlights that the transmission is more intense across countries in their financial dimension and, from a policy perspective, that consolidations over time occurred simultaneously behind more coordinated fiscal actions across member states. Focusing on the spreading of spillovers, there is a larger intensification, while their signs do not show significant considerations. A deeper homogeneity is observed in financial dimension. Moreover, a consistent degree of heterogeneity persists across countries, mainly in real dimension $\left(\hat{S}_{\text {real }}=0.375\right)$ than in financial dimension $\left(\hat{S}_{f i n}=0.306\right)$.

Finally, considering both components (Figures 2c and 2d), I prove that economic and institu-
tional differences among national policy are implicitly involved when assessing growth spillovers and different transmission channels. To be more precise, most countries are net receivers in real economy and net senders in financial dimension, proving that shock transmissions are larger among capital flows than trade exposures. Cross-country homogeneity and co-movements in financial dimension are driven by large public debts and formal institutional constraints ${ }^{18}$ (e.g., the Maastricht rules and the Stability and Growth Pact, which forbids public deficits exceeding 3\% of GDP). Higher heterogeneity is observed across countries in their real dimension $\left(\hat{S}_{\text {real }}=0.342\right)$ and, hence, despite a common monetary policy, national policies of investments and structural reforms in labour and complementary markets remain heterogeneous across the EA. It might have contributed to the current emergence of different country-specific developments of competitiveness, consumption, investment, and production affecting national economy. From a policy perspective, despite its large size, Germany shows a limited role in generating growth spillovers. This result, in part, reflects Germany's own dependence on growth in the rest of the Eurozone. Furthermore, inward spillovers tend to be greater than outward spillovers, proving large trade exposures with other european countries. Finally, PIIGS countries ${ }^{[9]}$ show larger conditional responses due to their high dimension in cross-border trade accounts.


Figure 2: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system. They account for real $_{i t, j}$ with rweights $_{i t, j}$ component (plot $a$ ), fin $_{i t, j}$ with fweights $_{i t, j}$ component (plot b), real $_{i t, j}$ with $r$ weights $s_{i t, j}$ and structures $_{i t, j}$ components (plot $c$ ), and fin $_{i t, j}$ with fweights $s_{i t, j}$ and structures ${ }_{i t, j}$ components (plot d), corresponding to $\chi_{3 t} \hat{\beta}_{3 t}, \chi_{4 t} \hat{\beta}_{4 t}, \chi_{5 t} \hat{\beta}_{5 t}$, and $\chi_{6 t} \hat{\beta}_{6 t}$ cross-country indicators, respectively.

[^12]Different results have been found for the variable-specific factors (Figure 3). To be more precise, accounting for the only weights $s_{i t, j}$ component (Figure 3a), on average, countries tend to generate larger outward growth spillovers ( $\bar{\beta}_{7 t}^{2}=0.33$ ) in financial dimension than in real economy ( $\bar{\beta}_{7 t}^{1}=$ $0.25)$. Thus, I confirm the consistent role of capital income flows in absorbing the effects of variablespecific shocks in contrast of standard theoretical model.20 In addition, despite a substantial degree of heterogeneity, Figure 3a shows a certain homogeneity across countries and, hence, the analysis is in line with rapidly increasing cross-border trade and financial linkages.

Matching both weights ${ }_{i t, j}$ and structures $_{i t, j}$ components (Figure 3b), most countries tend to be net senders of the system in financial dimension, confirming that trade channels matter relatively less than financial channels. Moreover, outward and inward growth spillovers follow a homogeneous pattern across members, except Spain (possibly due to larger capital exposures). Thus, highly indebted countries were forced into equally taking wide-ranging austerity measures, having lost access to the financial markets. This has led to call for stronger cross-country differentiation and for temporary stimulus measures in countries not facing financial market pressure. Moreover, crossborder spillovers seem to have hampered the effects of consolidations, accounting for a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dynamic.


Figure 3: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system. They account for the four groups of variables: real $l_{i t, j}$ and fin $_{i t, j}$ with weights ${ }_{i t, j}$ component (plot $a$ ) and real ${ }_{i t, j}$ and fin $_{i t, j}$ with weights ${ }_{i t, j}$ and structures ${ }_{i t, j}$ components (plot $b$ ), corresponding to $\chi_{7 t} \hat{\beta}_{7 t}$ variable-specific indicator.

The last factor described in equation 31 is drawn in Figure 4 It shows that there is a significant common component across eurozone members in financial dimension and country-specific heterogeneities tend to matter more in real economy. In fact, in Figure 4a, in which the common indicator is matched with the corresponding transmission channels for real and financial dimension, I demonstrate that growth shocks appear to be predominantly transmitted via financial linkages and there is a more important consistent common component in financial dimension because of

[^13]stronger interdependencies among variables. The Figure 4b, in which the common indicator is matched with the only variables capturing economic-institutional factors in real and financial dimension, shows larger heterogeneities among growth spillovers observed across countries. The analysis confirms that, in the EA, structural reforms without coordinated national fiscal actions negatively affect the adjustment capacity of the currency union as a whole because of high degree of divergence.


Figure 4: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system. They account for the two common groups: real $i_{t, j}$ and $f i_{i t, j}$ with weights $_{i t, j}$ component (plot $a$ ) and real $_{i t, j}$ and fin $_{i t, j}$ with structres $s_{i t, j}$ component (plot b), corresponding to $\chi_{8 t} \hat{\beta}_{8 t}$ common indicator.

### 4.4 Crisis Period and Post-Crisis Consolidation

In recent years, the transmission of shocks is faster and deeper in financial dimension because of stronger economic and structural interdependencies (Figure 5). I found deeper co-movents and larger degree of homogeneity across countries in real dimension and even more in financial dimension during the recent recession (Figures 5a and 5c) compared to successive post-crisis consolidations (Figures 5 b and 5d). These results cast three main findings. First, interdependencies because of strongly common economic-institutional linkages matter more during trigger events. Second, coordinated fiscal actions do not necessarily yield better outcomes. Third, the sign of growth spillovers are not significant in determining whether coordination should lead to a more expansionary or more restrictive fiscal stance in the member states. Thus, although recent theoretical studies suggest that the imbalances have been reduced and the macroeconomic policy mix with a discretionary fiscal expansion and a neutral monetary policy were likely to mitigate output growth during recession and successive consolidations, without the appropriate adjustment of the private and public sector, Eurozone imbalances and different degrees of productivity growth would tend to persist in the future (Figure 6).


Figure 5: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system, focusing on the recent financial crisis (plots $a$ and $c$ ) and post-crisis consolidation (plots $b$ and $d$ ) periods. They account for real $_{i t, j}$ with rweights $_{i t, j}$ and structures $_{i t, j}$ components (plots $a$ and $b$ ) and fin $_{i t, j}$ with fweights ${ }_{i t, j}$ and structures ${ }_{i t, j}$ components (plots $c$ and $d$ ), corresponding to $\chi_{5 t} \hat{\beta}_{5 t}$ and $\chi_{6 t} \hat{\beta}_{6 t}$ cross-country indicators.


Figure 6: The Figure draws conditional forecasts of the productivity (prodea), general government debt (debtea), real GDP growth rate (gdpgea), current account balance (currea), general government spending (govea), and the generalized entropy index (theil) in the eurozone from 1999q1 to $2020 q$ 2. The latter corresponds to Theil's Entropy and is computed by weighing the GDP with the population in terms of proportions with respect to the total. It can be viewed as a measure of divergence and economic inequality. Here, forecasts from $2016 q 1$ to $2020 q 2$ correspond to conditional projections of each variable drawn in the $E P B V A R(1)$.

Accounting for the variable-specific indicator (Figure 7), I confirm that trade channels matter relatively less than capital linkages. In fact, most countries tend to be net senders in financial dimension and net receivers in real economy. During post-crisis consolidations (Figure 7b), inward growth spillovers are a lot more frequent and large because of tight institutional and economic interdependencies. Thus, cross-border spillovers have exacerbated the negative effects of consolidations due to (individual) domestic policies designed to counteract the events of the recession and that, when successive consolidations occured, proved to be ineffective and counter-productive
for the domestic economy.


Figure 7: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system, focusing on the recent financial crisis (plot $a$ ) and post-crisis consolidation period (plot $b$ ). They account for real $_{i t, j}$ and fin $_{i t, j}$ with weights $_{i t, j}$ and structures ${ }_{i t, j}$ components, corresponding to $\chi_{7 t} \hat{\beta}_{7 t}$ variable-specific indicator.

The common indicator (Figure 8) shows that economic-institutional interdependencies matter more than different transmission channels in driving the spreading of common unexpected shocks. In fact, I observe deeper commonalities in Figures 8d and 8d. Moreover, contrarily when assessing country- and variable-specific shocks, I demonstrate that common shocks tend to spill over in a heterogeneous way during the recent recession (Figures 8a and 8c), while a larger homogeneous pattern showed during successive consolidations (Figures 8b and 8d). Thus, it would seem to demonstrate that several countries actually started to put in practice a fiscal consolidation package and national fiscal actions have been adopted in some sort of coordinated way.


Figure 8: Systemic Contributions of the productivity given a $1 \%$ shock to real and financial dimension are drawn in standard deviation of the variables in the system, focusing on the recent financial crisis (plots $a$ and $c$ ) and post-crisis consolidation (plots $b$ and $d$ ) periods. They account for the two common groups: real $l_{i t, j}$ and $f i_{i t, j}$ with weights $s_{i t, j}$ component (plots $a$ and $b$ ) and real $_{i t, j}$ and fin $_{i t, j}$ with structres $_{i t, j}$ component (plot $c$ and $d$ ), corresponding to $\chi_{8 t} \hat{\beta}_{8 t}$ common indicator.

## 5 Concluding remarks

The paper develops empirical implementations to conduct inference in time-varying coefficients by using a structural unconstrained Panel Bayesian Multicountry VAR model, in order to deal with confounding and latent effects. The aim of the project is to assess an analytical contribution to the recent literature on idiosyncratic business cycles and policy making, minimizing omitted biases that occurr when dealing with multicountry data. Bayesian computations are used to allow cross-unit interdependencies and unit-specific dynamics and restrict the coefficients to have a low-dimensional time-varying factor structure. The econometric model uses hierarchical priors for the vector of components in order to permit exchangeability, time variations, and endogeneity in the innovations in the factors. The identified setup only requires minimal prior input from the researcher. In this way, an overparametrized VAR can be transformed into a parsimonius Normal Linear model, where the regressors are all (directly) observed and the loadings are the time-varying coefficient factors. Generalized Conditional IRFs and Conditional Forecasts are obtained with the output of an MCMC routine and a variant of Gibbs sampling.

An empirical application to a pool of the current members of the European Monetary Union illustrates the functioning of the model, with particular attention on the recent recession and successive consolidations. Three interesting macroeconomic-financial issues left even now unanswered have been addressed. First, how length and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. The analysis is consistent and robust with the more recent literature on idiosyncratic business cycles, which recognizes the importance to separate common shocks from propagation of country- and variable-specific shocks when studying economic-financial linkages.

From a modelling perspective, I demonstrate that growth schocks spill over in a heterogeneous way across countries, although a significant common component held, mainly during the crisis period and even more during post-crisis consolidations. Overall period, commonalities are stronger in financial dimension where the shock transmission is more intense. The results prove the presence of higher interdependencies among variables and that consolidations occurred simultaneously behind more coordinated fiscal actions across member states. Accounting for variable-specific shocks,
the spreading of spillovers show a homogeneous pattern across countries and deeper divergences whether economic-institutional linkages are involved.

From a policy perspective, despite a currency union, different country-specific developments of competitiveness, consumption, investment, and production affecting national economy should be designed in order to shrink growth divergencies across countries. With the advent of the financial crisis, fiscal expansion has been associated with smaller output growth loss and national fiscal actions have been used in some sort of coordinated way. After gradual economic recovery began to be observed, several countries started to put in practice a fiscal consolidation package. Nevertheless, even though the macroeconomic policy mix with a discretionary fiscal expansion and a neutral monetary policy were likely to mitigate output growth during recession and successive consolidation, without coordination efforts going beyond what already exists in the set of rules given in the Maastricht Treaty, eurozone imbalances and different degrees of productivity growth would tend to persist in the future.

## References

Beetsma, R. and Jensen, H. (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. Journal of International Economics, 67(2):320-352.

Blanchard, O. and Perotti, R. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes on output. Quarterly Journal of Economics, 117(4):1329-1368.

Buti, M., Franco, D., and Ongena, H. (1998). Fiscal discipline and flexibility in emu: The implementation of the stability and growth pact. Oxford Review of Economic Policy, 14(3):8197.

Canova, F. and Ciccarelli, M. (2009). Estimating multicountry var models. International Economic Review, 50(3):929-959.

Canova, F. and Marrinan, J. (1997). Sources and propagation of international output cycles: Common shocks or transmission? Journal of International Economics, 46(1):133-166.

Chib, S. (1995). Marginal likelihood from the gibbs output. Journal of the American Statistical Association, 90(432):1313-1321.

Chib, S. (1996). Calculating posterior distributions and model estimates in markov mixture models. Journal of Econometrics, 75(1):79-97.

Chib, S. and Greenberg, E. (1995). Understanding the metropolis-hastings algorithm. The American Statistician, 49(4):327-335.

Chib, S. and Jeliazkov, I. (2001). Marginal likelihood from the metropolis-hastings output. Journal of the American Statistical Association, 96(453):270-281.

Ciccarelli, M., Ortega, E., and Valderrama, M. T. (2012). Heterogeneity and cross-country spillovers in macroeconomic-financial linkages. Working Paper 1498.

Ciccarelli, M. and Rebucci, A. (2007). Measuring contagion using a bayesian time-varying coefficient model. Journal of Financial Econometrics, 5(2):285-320.

Crespo-Cuaresma, J. and Fernandez-Amadorb, O. (2013). Business cycle convergence in emu: A first look at the second moment. Journal of Macroeconomics, 37(C):265-284.

Degiannakis, S., Duffy, D., Filis, G., and Livada, A. (2016). Business cycle synchronisation in emu: Can fiscal policy bring member-countries closer? Economic Modelling, 52(B):551-563.

Doan, T., Litterman, R., and Sims, C. (1984). Forecasting and conditional projection using realistic prior distributions. Econometric Review, 3(1):1-100.

Eichengreen, B. and Wyplosz, C. (1998). The stability pact: More than a minor nuisance? Economic Policy, 13(26):65-113.

Facchini, F., Melki, M., and Pickering, A. (2017). Labour costs and the size of government. Oxford Bulletin of Economics and Statistics, 79(2):251-275.

Geweke, J. (2001). Bayesian econometrics and forecasting. Journal of Econometrics, 100(1):11-15.

Giannone, D., Banbura, M., and Reichlin, L. (2009). Large bayesian vector auto regressions. Journal of Applied Econometrics, 25(1):71-92.

Gordon, R. H. and Bovenberg, L. A. (1996). Why is capital so immobile internationally? possible explanations and implications for capital income taxation. The American Economic Review, 86(5):1057-1075.

Imbs, J. (2006a). The real effects of financial integration. Journal of International Economics, 68(2):296-324.

Imbs, J. (2006b). Trade, finance, specialization and synchronization. Review of Economics and Statistics, 86(3):723-734.

Kadiyala, R. K. and Karlsson, S. (1997). Numerical methods for estimation and inference in bayesian var models. Journal of Applied Econometrics, 12(2):99-132.

Koop, G. (1996). Parameter uncertainty and impulse response analysis. Journal of Econometrics, 72(1-2):135-149.

Lane, P. R. and Milesi-Ferretti, G. M. (2007). The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 1970-2004. Journal of International Economics, 73(2):223-250.

Litterman, R. B. (1986). Forecasting with bayesian vector autoregressions five years of experience. Journal of Business and Economic Statistics, 4(1):25-38.

Mastrogiacomo, M., Bosch, N. M., Gielen, M. D., and Jongen, E. L. (2017). Heterogeneity in labour supply responses: Evidence from a major tax reform. Oxford Bulletin of Economics and Statistics, 79(4):769-796.

Mundell, R. A. (1961). A theory of optimum currency areas. The American Economic Review, 51(4):657-665.

Pesaran, H. M. (2006). Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica, 74(4):967-1012.

Pesaran, H. M., Dees, S., Mauro, F. D., and Smith, V. (2007). Exploring the international linkages of the euro area: a global var analysis. Journal of Applied Econometrics, 22(1):1-38.

Pesaran, H. M. and Shinb, Y. (1998). Generalized impulse response analysis in linear multivariate models. Economics Letters, 58(1):17-29.

Raftery, A. E., Ciccio, T. D., Kass, R. E., and Wasserman, L. (1997). Computing bayes factors by combining simulation and asymptotic approximations. Journal of the American Statistical Association, 92(439):903-915.

Raftery, A. E. and Lewis, S. M. (1997). Estimating bayes factors via posterior simulation with the laplace-metropolis estimator. Journal of the American Statistical Association, 92(438):648-655.

Reinhart, C. M. and Rogoff, K. S. (2009). The aftermath of financial crises. American Economic Review, 99(2):466-72.

Sorensen, B. E. and Yosha, O. (1998). International risk sharing and european monetary unification. Journal of International Economics, 45(2):211-238.

Tabellini, G. (1990). Domestic politics and the international coordination of fiscal policies. Journal of International Economics, 28(3-4):245-265.

# Supplementary Materials on 'Panel Bayesian VAR Modeling for Policy and Forecasting when dealing with confounding and latent effects' 

This version: September, 2018

## 1 Dynamic Analysis

### 1.1 Conditional Generalized Impulse Responses

Conditional impulse responses are generally computed as the difference between two realizations of $y_{t+\tau}$, with $\tau=1,2, \ldots$, which are identical up to time $t$. Thus, between $t+1$ and $t+\tau$, one can assume two time impulses in the $j$ th component of $e_{t+\tau}$. First, one that occurs only at time $t+1$. Second, the other that no shocks take place at all dates between $t+1$ and $t+\tau$. In a model with time-varying coefficients, the approach is inadequate since it overlooks that between $t+1$ and $t+\tau$, structural coefficients may also change. Therefore, impulse responses are obtained as the difference between two conditional expectations of $y_{t+\tau}$. In both cases, they are conditioned on the history of the data $Y_{t}$ and of the factors $\beta^{t}$, the parameters of the law of motion of the coefficients, and all future shocks. However, impulse responses are conditioned on a random draw for the current shocks, whereas in the other the current shocks is set to its unconditional value. Hence, they are worked out on future shocks instead of integrating them out because, computationally, such a choice gives more stable responses, even though this makes standard error bands larger than in the case where future shocks are integrated out. There are two potential types of impulses. First, one to the variables of the system. Second, one to the factors.

Here, the reparametrized NLR is:

$$
\begin{equation*}
y_{t}=\chi_{t} \beta_{t}+\left(E_{t}+X_{t} u_{t}\right) \quad \text { with } \quad \beta_{t}=\beta_{t-1}+v_{t} \tag{1}
\end{equation*}
$$

where $\beta_{t}=\left[\beta_{1 t}^{\prime}, \beta_{2 t}^{\prime}, \ldots, \beta_{F t}^{\prime}\right]^{\prime}, \chi_{t}=\left[\chi_{1 t}, \ldots, \chi_{F t}\right], \chi_{i t}=D X_{t}, X_{t}=\left[Y_{t-1}, W_{t}, W_{t-1}, Z_{t}, Z_{t-1}\right]$. Let
$\mho=\left[\left(E_{t}+X_{t} u_{t}\right)^{\prime}, v^{\prime}\right]^{\prime}$ be the vector of reduced-form shocks and $P_{t}=\left[L_{t}^{-1}\left(E_{t}+X_{t} u_{t}\right)^{\prime}, L_{t}^{-1} v_{t}^{\prime}\right]^{\prime}$ be the vector of structural shocks where $E_{t}=L_{t} v_{t}, L_{t} L_{t}^{\prime}=\Omega$ so that $\operatorname{var}\left(v_{t}\right)=\sigma^{2} I_{k}$ and $L_{t}=J \cdot K_{t}$ where $K_{t} K_{t}^{\prime}=I$ and $J$ is a matrix that orthogonalizes the VAR shocks.

Here, a Choleski system is obtained setting $K_{t}=I, \forall t$, and choosing $J$ to be lower triangular whereas more structural identification schemes are obtained letting $J$ be an arbitrary square root matrix and $K_{t}$ a matrix implementing certain theoretical restrictions. The identification matrix $K_{t}$ is allowed to be time-varying since, when recursive estimations are used, estimates of $\Omega$ depends on $t$.

Let $\mathbb{P}_{t}=\left(\Omega, \sigma^{2}, H_{t}, \Phi\right)$, let $\bar{P}_{j, t}$ be a particular realization of $P_{j, t}$ and $P_{-j, t}$ indicate the structural shocks, excluding the one in the $j$ th component. Let $F_{t}^{1}=\left\{Y^{t-1}, \beta^{t}, \mathbb{P}_{t}, L_{t}, P_{j, t}\right\}$, with $P_{j, t}=\left\{\bar{P}_{j, t}, P_{-j, t} \cdot \mho_{t+1}^{t+\tau}\right\}$, and $F_{t}^{2}=\left\{Y^{t-1}, \beta^{t}, \mathbb{P}_{t}, L_{t}, P_{j, t}\right\}$, with $P_{j, t}=\left\{E P_{j, t}, P_{-j, t} \cdot \mho_{t+1}^{t+\tau}\right\}$ be two conditioning sets. Thus, responses to a shock at $t$ in the $j$ th component of $P_{t}$ are obtained as:

$$
\begin{equation*}
I R(t, t+\tau)=E\left(Y_{t+\tau} \mid F_{t}^{1}\right)-E\left(Y_{t+\tau} \mid F_{t}^{2}\right) \quad t=1,2, \ldots \tag{2}
\end{equation*}
$$

In order to see what definition equation 2 involves, rewrite the original EPBVAR model in a companion form, without intercept:

$$
\begin{equation*}
Y_{t+\tau}=A_{t+\tau} Y_{t+\tau-1}+B_{t+\tau} W_{t+\tau-1}+C_{t+\tau} Z_{t+\tau-1}+E_{t+\tau} \tag{3}
\end{equation*}
$$

and let

$$
\begin{equation*}
\gamma_{t+\tau}=D\left[\beta_{t+\tau-1}+v_{t+\tau}\right]+u_{t+\tau} \tag{4}
\end{equation*}
$$

where $\gamma_{t+\tau}=\left[\operatorname{vec}\left(A_{1 t+\tau}\right)\right.$, vec $\left(B_{t+\tau}\right)$, vec $\left.\left(C_{t+\tau}\right)\right]$ and $A_{1 t+\tau}$ is the first row of $A_{t+\tau}$. Taking $Y^{t-1}=$ $\left(Y_{t-1}, Y_{t-2}, \ldots, W_{t-1}, W_{t-2}, \ldots, Z_{t-1}, Z_{t-2}, \ldots\right), A^{t}=\left(A_{t}, A_{t-1}, \ldots\right), B^{t}=\left(B_{t}, B_{t-1}, \ldots\right), C^{t}=$ $\left(C_{t}, C_{t-1}, \ldots\right)$, and $L_{t+\tau}=L_{t}$ for $\forall \tau$ as given. Solving backward, equations 3 and 4 can be rewritten as:

$$
\begin{align*}
Y_{t+\tau}= & \left(\prod_{k=0}^{\tau} A_{t+\tau-k}\right) Y_{t-1}+B_{t+\tau} W_{t+\tau-1}+C_{t+\tau} Z_{t+\tau-1}+\sum_{l=1}^{\tau}\left(\prod_{k=0}^{l-1} A_{t+\tau-k}\right) . \\
& \cdot B_{t-\tau-l} W_{t+\tau-l-1} \cdot C_{t-\tau-l} Z_{t+\tau-l-1}+L_{t-\tau} \eta_{t+\tau}+\sum_{l=1}^{\tau}\left(\prod_{k=0}^{l-1} A_{t+\tau-k}\right) . \\
& \cdot L_{t+\tau-l} \eta_{t+\tau-l} \tag{5}
\end{align*}
$$

and as

$$
\begin{equation*}
\gamma_{t+\tau}=D \beta_{t-1}+D \sum_{k=0}^{\tau} v_{t+\tau-k}+u_{t+\tau} \tag{6}
\end{equation*}
$$

Consider first the case of a ( $m+1$ )-period impulse in the $j$ th component of $v$. For example: $v_{j, t+k}=\bar{v}_{j, t+k} ; v_{-j, t+k}, k=0,1,2, \ldots, m$ and $v_{t+m^{\prime}}$, with $\forall m^{\prime}>m$, are restricted. Then,

$$
\begin{align*}
I R_{t, t+\tau}= & E_{t}\left[Y_{t+\tau} \mid Y_{t-1}, A^{t}, B^{t}, C^{t}, \mathbb{P}_{t}, L_{t},\left\{\bar{\eta}_{j t+m}\right\}_{k=o}^{m},\left\{\eta_{-j t+k}\right\}_{k=0}^{m},\left\{\eta_{t+k}\right\}_{k=m+1}^{\tau}\right]- \\
& -E_{t}\left[Y_{t+\tau} \mid Y^{t-1}, A^{t}, B^{t}, C^{t}, \mathbb{P}_{t}, L_{t},\left\{\eta_{t+k}\right\}_{k=0}^{\tau}\right]= \\
= & E_{t}\left[\left(\prod_{k=0}^{\tau-1}\right)^{j} H_{t}^{j}\left(\bar{\eta}_{j t}-E \eta_{j t}\right)+\left(\prod_{k=0}^{\tau-2} A_{t+\tau-k}\right)^{j} \cdot L_{t+1}^{j}\left(\bar{\eta}_{j t+1}-E \eta_{j t+1}\right)+\ldots\right. \\
& \left.\cdots+\left(\prod_{k=0}^{\tau-m-1} A_{t+\tau-k}\right)^{j} \cdot L_{t+m}^{j}\left(\bar{\eta}_{j t+m}-E \eta_{j t+m}\right)\right] \tag{7}
\end{align*}
$$

where the superscript $j$ refers to the jth column of the matrix. It is easy to see that, when $A_{t}=A$, $B_{t}=B$ and $C_{t}=C, \forall t$, equation 7 reduces to standard impulse responses and, when $E_{t}$ and $v_{t}$ are correlated (that is both the sign and the size of the shocks matter a shock in $v_{t}$ ), may induce changes in $A_{t}, B_{t}$ or $C_{t}$.

Given 2, responses in the NLR model can be computed as follows:

1. Choosing $t$, $\tau$, and $J_{t}$. Drawing $\Omega^{\delta}=L_{t}^{\delta}\left(L_{t}^{\delta}\right)^{\prime},\left(\sigma^{2}\right)^{\prime}$ from their posterior distribution and $u_{t}^{\delta}$ from $N\left(0,\left(\sigma^{2}\right)^{2} I \otimes L_{t}^{\delta}\left(L_{t}^{\delta}\right)^{\prime}\right)$. Computing $y_{t}^{\delta}=\chi_{t} \beta_{t}+L_{t} \bar{\eta}_{t}+X_{t} u_{t}^{\delta}$.
2. Drawing $\Omega=L_{t+1}^{\delta}\left(L_{t+1}^{\delta}\right)^{\prime},\left(\sigma^{2}\right)^{\delta}, \mathcal{H}_{t+1}^{\delta}, \phi^{\delta}$. Drawing $\eta_{t+1}^{\delta}$ from their posterior distribution. Using the law of motion of the factors to compute $\beta_{t+1}^{\delta}, \delta=1,2, \ldots, \Delta$, and the definition of $D$ to compute $\chi_{t+1}$. Drawing $u_{t+1}^{\delta}$ from $N\left(0,\left(\sigma^{2}\right)^{\delta} I \otimes L_{t+1}^{\delta}\left(L_{t+1}^{\delta}\right)^{\prime}\right)$ and computing $y_{t+1}^{\delta}=$ $\chi_{t+1} \beta_{t+1}+L_{t+1} \bar{\eta}_{t+1}+X_{t+1} u_{t+1}, \delta=1,2, \ldots, \Delta$.
3. Repeating Step 2 and computing $\beta_{t+k}^{\delta}, y_{t+k}^{\delta}, k=1,2, \ldots, \tau$.
4. Repeating Steps 1-3 by setting $\eta_{t+K}=E\left(\eta_{t+1}\right), k=1,2, \ldots, m$ and using the draws for the shocks in 1-3.

Responses to structural shocks to the law of motion of the factors can be computed in the same way. An impulse in $v_{t}=\bar{v}$ lasting $(m+1)$ periods implies from equation 6 that:

$$
\begin{equation*}
E\left(\bar{\gamma}_{t+\tau}-\gamma_{t+\tau}\right)=D \sum_{k=0}^{m} H_{t+k}\left(\bar{\eta}_{t+\tau-k}-E \eta_{t+\tau}-k\right) \tag{8}
\end{equation*}
$$

and

$$
\begin{align*}
I R_{t, t+\tau}= & E_{t}\left[\prod_{k=0}^{\tau}\left(\bar{A}_{t+1, \tau-k}-A_{t+\tau-k}\right) Y_{t+1}+\sum_{l=1}^{\tau} \prod_{k=0}^{l-1}\left(\bar{A}_{t+1, \tau-k}-A_{t+\tau-k}\right) \cdot B_{t+\tau-l-1} .\right. \\
& \left.\cdot C_{t+\tau-l-1}+\sum_{l=1}^{\tau} \prod_{k=0}^{l-1}\left(\bar{A}_{t+1, \tau-k}-A_{t+\tau-k}\right) L_{t+\tau-l} \eta_{t+\tau-l}\right] \tag{9}
\end{align*}
$$

### 1.2 Conditional Forecasts

There are two types of conditional forecasts one can compute in this framework. Those involving displacement of the endogenous variables $W_{t}$ and $Z_{t}$ from their unconditional path, and those involving a particular path for a subset of the endogenous variables from their conditional path.

Both types of conditional forecasts can be constructed using the output of the Gibbs sampler routine.

Consider first displacing the endogenous variables from their expected future path for $m+1$ periods. Calling the new path $\bar{W}_{t+k}$ and $\bar{Z}_{t+k}, k=0,1, \ldots, m$. Then, the response of $\mathrm{Y}_{t+\tau}$ is:

$$
\begin{align*}
I R_{t, t+\tau}= & E_{t}\left[\left(\prod_{k=0}^{\tau-2} A_{t+\tau-k}\right) B_{t+1}\left(\bar{W}_{j t}-W_{j t}\right) C_{t+1}\left(\bar{Z}_{j t}-Z_{j t}\right)+\left(\prod_{k=0}^{\tau-3} A_{t+\tau-k}\right) \cdot\right. \\
& \cdot B_{t+2}\left(\bar{W}_{j t+1}-W_{j t+1}\right) \cdot C_{t+2}\left(\bar{Z}_{j t+1}-Z_{j t+1}\right)+\ldots+\left(\prod_{k=0}^{\tau-2-m} A_{t+\tau-k}\right) . \\
& \left.\cdot B_{t+m+1}\left(\bar{W}_{j t+m}-W_{j t+m}\right) \cdot C_{t+m+1}\left(\bar{Z}_{j t+m}-Z_{j t+m}\right)\right] \tag{10}
\end{align*}
$$

Thus, to compute conditional forecasts of this type in the NLR model, one need to:

1. Choosing $t, \tau$, and a path $\left\{\bar{W}_{t+k}, \bar{Z}_{t+k}\right\}_{k=0}^{m}$. Drawing $\Omega^{\delta},\left(\sigma^{2}\right)^{\delta}$ from their posterior, drawing $E_{t}^{\delta}+X_{t} u_{t}^{\delta}$ and computing $y_{t}^{\delta}$.
2. Drawing $\left(H_{t}\right)^{\delta}, \Psi^{\delta}$ from their posterior distribution; drawing $v_{t+1}^{\delta}$ and using the law of motion of the factors to draw $\beta_{t+1}^{\delta}, \delta=1,2, \ldots, \Delta$ and the definition of $D$ to compute $\chi_{t+1}$. Then, $E_{t+1}^{\delta}+X_{t+1} U_{t+1}^{\delta}$ are drawn to compute $y_{t+1}^{\delta}=\chi_{t+1} \beta_{t+1}^{\delta}+\left(E_{t+1}^{\delta}+X_{t+1} u_{t+1}^{\delta}\right)$, $\delta=1,2, \ldots, \Delta$.
3. Repeating Step 2 in order to compute $\beta_{t+k}^{\delta}, y_{t+k}^{\delta}, k=1,2, \ldots, \tau$.
4. Repeating Steps 1-3. In this way, it sets $W_{t+k}=E\left(W_{t+k}\right), Z_{t+k}=E\left(Z_{t+k}\right), k=0,1, \ldots, m$, using the draws for the shocks in 1-3.

Finally, considering the case in which the future path of a subset of $Y_{t}$ 's is fixed. For example, in a system with real and financial linkages, one would like to work out on a given path for the future growth spillovers. Hence, partioning $Y_{t}=A_{t} Y_{t-1}+B_{t} W_{t-1}+C_{t} Z_{t-1}+E_{t}$ in two blocks, let $Y_{2 t+k}=\bar{Y}_{2 t+k}$ be the fixed variables and $Y_{1 t+k}$ those allowed to adjust, the Impulse Responses
are:

$$
\begin{align*}
I R_{t, t+\tau}= & E\left[L_{t}^{1}\left(\prod_{k=0}^{t-1} A_{t+\tau-k}\right)^{1}\left(\bar{\eta}_{2 t}-\eta_{2 t}\right)+L_{t+1}^{1}\left(\prod_{k=0}^{t-2} A_{t+\tau-k}\right)^{1}\left(\bar{\eta}_{2 t+1}-\eta_{2 t+1}\right)+\ldots\right. \\
& \left.\ldots+L_{t+m}^{1}\left(\prod_{k=0}^{t-1-m} A_{t+\tau-k}\right)^{1}\left(\bar{\eta}_{2 t+m}-\eta_{2 t+m}\right)\right] \tag{11}
\end{align*}
$$

where $\bar{\eta}_{2 t+k}=\bar{Y}_{2 t+k}-A_{21, t+k} Y_{1 t-k-1}-A_{22, t+k} Y_{2 t-k-1}-B_{2 t+k} W_{t+k-1}-C_{2 t+k} Z_{t+k-1}$ and the superscript 1 refers to the first row of the matrix. Hence, to compute this type of conditional forecasts one need to:

1. Partitioning $y_{t}=\left(y_{1 t}, y_{2 t}\right)$, choosing $t$ and a path $\left\{y_{2 t+k}\right\}_{k=0}^{\tau}$. Using the model to solve for the $\bar{\eta}_{2 t}$ that gives $y_{2 t}=\bar{y}_{2 t}$, backing out the implied $y_{1 t}^{\delta}$ once draws for $E_{1 t}^{\delta}$, and computing $u_{t}^{\delta}$ from their posterior distribution. Thus, $v_{t+1}^{\delta}$ can de drawn using the law of motion of the factors to obtain $\beta_{t+1}^{\delta}$, with $\delta=1,2, \ldots, \Delta$, and the definition of $D$ to compute $\chi_{t+1}$.
2. Using the model to solve for $\bar{\eta}_{2 t}$ that gives $y_{2 t+1}=\bar{y}_{2 t+1}$, backing out the implied $y_{1 t+1}^{\delta}$ once draws for $E_{1 t+1}^{\delta}$, and computing $u_{t+1}^{\delta}$ as above. Hence, once can draw $v_{t+2}^{\delta}$, using the law of motion of the factors to compute $\beta_{t+2}^{\delta}$, with $\delta=1,2, \ldots, \Delta$, and the definition of $D$ to compute $\chi_{t+2}$.
3. Repeating Step 2 and computing $\beta_{t+k}^{\delta}, y_{t+k}^{\delta}, k=2,3, \ldots$.
4. Repeating Steps 1-3, once can set $\eta_{2 t+k}^{\delta}=E\left(\eta_{t+k}\right), \forall k$ using the draws for the shocks in 1 $-3$.

In Step 2 of all algorithms, it has implicitly assumed that selecting a path for the shocks does not alter neither the law of motion of the factors nor the beliefs about the true structural shocks.

## 2 Data Description

Table 1: Descriptive Statistics

| VARIABLES | DESCRIPTION |
| :--- | :--- |
| General government spending | Financial accounts for general government spending <br> weighted for the GDP. |
| Gross fixed capital formation | Investments of fixed assets at current prices weighted <br> for the GDP. |
| GDP growth rate | It is calculated as: Log $\left(\frac{G D P_{i t, j}}{G D P_{i t-1, j}}\right)$. <br> Neneral government debt <br> Non-financial accounts for general government debt <br> as percentage of GDP. <br> Non-financial accounts for general government net <br> borrowing/lending as percentage of GDP. <br> EMU convergence criterion |
| Enterest rate | series. |
| Bilateral flows of trade | Exports and imports by Member States of the EU <br> at current prices weighted for the GDP. |
| Bilateral flows of capital | Financial transactions on the total economy at <br> current prices weighted for the GDP. |
| Net international investments | Total international investment position in million of <br> euro weighted for the GDP. |
| Nominal labour cost | It is defined as the ratio of labour costs to labour <br> productivity. |
| General government consumption |  |
| Total general government expenditure at current |  |
| prices weighted for the GDP. |  |

## 3 Summary Statistics

Table 2: System Contributions accounting for $\chi_{1 t} \hat{\beta}_{1 t} \& \chi_{2 t} \hat{\beta}_{2 t}$ Indicators

| Country | GOV | GDPG | GFCF | INT | DEBT | CURR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I T$ | -0.380 | 0.028 | 0.128 | 0.281 | 0.060 | -0.282 |
| $E S$ | 0.360 | -0.188 | -0.070 | -0.259 | 0.103 | 0.545 |
| $F R$ | 0.223 | 0.073 | -0.417 | 0.119 | 0.307 | -0.209 |
| $A T$ | -0.215 | 0.899 | 0.205 | -0.565 | -0.556 | -0.175 |
| DE | -0.121 | -0.301 | 0.667 | 0.364 | 0.287 | 0.372 |
| $I E$ | -0.283 | -0.354 | -0.129 | 0.237 | 0.243 | 0.082 |
| $G R$ | -0.001 | -0.084 | -0.156 | -0.138 | -0.182 | -0.274 |
| PT | 0.417 | -0.074 | -0.228 | -0.038 | -0.262 | -0.060 |
| St. Dev. | 0.302 | 0.392 | 0.333 | 0.313 | 0.307 | 0.311 |

The Table refers to the conditional generalized IRFs from 1999q1 to 2016q1, absorbing outcomes from conditional forecasts computed on the time frame of 9 quarters. It describes, in terms of standard deviations, the systemic contributions of the productivity given a $1 \%$ shoch to real and financial dimension. They correspond to $\chi_{1 t} \hat{\beta}_{1 t}$ and $\chi_{2 t} \hat{\beta}_{2 t}$ cross-country indicators.

Table 3: System Contributions accounting for $\chi_{3 t} \hat{\beta}_{3 t} \& \chi_{4 t} \hat{\beta}_{4 t}$ Indicators

| Country | GOV | GDPG | GFCF | INT | DEBT | CURR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I T$ | 0.274 | -0.410 | 0.416 | 0.285 | 0.372 | 0.181 |
| $E S$ | -0.072 | -0.115 | 0.280 | -0.144 | 0.112 | 0.508 |
| $F R$ | 0.035 | 0.045 | -0.091 | -0.538 | -0.278 | -0.136 |
| $A T$ | -0.284 | 0.838 | 0.174 | 0.268 | 0.328 | -0.275 |
| DE | -0.010 | -0.234 | -0.091 | -0.224 | -0.213 | -0.142 |
| $I E$ | -0.634 | -0.200 | -0.792 | -0.094 | -0.427 | -0.447 |
| $G R$ | 0.020 | -0.041 | 0.130 | 0.429 | 0.189 | 0.193 |
| PT | 0.671 | 0.117 | -0.027 | 0.018 | -0.082 | 0.118 |
| St. Dev. | 0.381 | 0.377 | 0.367 | 0.318 | 0.294 | 0.306 |

The Table refers to the conditional generalized IRFs from $1999 q 1$ to 2016q1, absorbing outcomes from conditional forecasts computed on the time frame of 9 quarters. It describes, in terms of standard deviations, the systemic contributions of the productivity given a $1 \%$ shoch to real and financial dimension. They correspond to $\chi_{3 t} \hat{\beta}_{3 t}$ and $\chi_{4 t} \hat{\beta}_{4 t}$ cross-country indicators.

Table 4: System Contributions accounting for $\chi_{5 t} \hat{\beta}_{5 t} \& \chi_{6 t} \hat{\beta}_{6 t}$ Indicators

| Country | GOV | GDPG | GFCF | INT | DEBT | CURR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I T$ | -0.527 | 0.419 | 0.151 | -0.094 | -0.363 | -0.046 |
| $E S$ | 0.468 | -0.514 | -0.013 | 0.490 | 0.400 | 0.554 |
| $F R$ | -0.072 | -0.044 | 0.133 | 0.107 | 0.036 | 0.354 |
| $A T$ | 0.071 | -0.130 | -0.282 | 0.143 | 0.369 | -0.082 |
| $D E$ | -0.122 | 0.375 | -0.084 | 0.157 | 0.195 | 0.026 |
| $I E$ | 0.246 | 0.204 | 0.716 | -0.167 | -0.221 | -0.579 |
| $G R$ | 0.214 | 0.001 | -0.619 | -0.739 | -0.232 | 0.066 |
| PT | -0.279 | -0.312 | -0.003 | 0.104 | -0.183 | -0.293 |
| St. Dev. | 0.318 | 0.325 | 0.382 | 0.357 | 0.294 | 0.352 |

The Table refers to the conditional generalized IRFs from 1999q1 to 2016q1, absorbing outcomes from conditional forecasts computed on the time frame of 9 quarters. It describes, in terms of standard deviations, the systemic contributions of the productivity given a $1 \%$ shoch to real and financial dimension. They correspond to $\chi_{5 t} \hat{\beta}_{5 t}$ and $\chi_{6 t} \hat{\beta}_{6 t}$ cross-country indicators.

Table 5: System Contributions accounting for $\chi_{7 t} \hat{\beta}_{7 t} \& \chi_{8 t} \hat{\beta}_{8 t}$ Indicators

| Country | wreal | wfin | wireal | wifin | wcommon | icommon |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I T$ | 0.249 | 0.442 | -0.200 | -0.377 | 0.097 | -0.224 |
| $E S$ | 0.188 | -0.081 | -0.319 | 0.488 | 0.095 | -0.776 |
| $F R$ | 0.038 | -0.393 | -0.157 | 0.140 | -0.521 | 0.066 |
| $A T$ | -0.266 | 0.171 | 0.250 | 0.168 | 0.219 | 0.123 |
| $D E$ | -0.082 | -0.103 | -0.324 | 0.102 | -0.089 | 0.004 |
| $I E$ | -0.539 | -0.371 | 0.505 | 0.102 | -0.167 | 0.010 |
| $G R$ | -0.114 | 0.387 | 0.229 | -0.341 | 0.589 | 0.563 |
| $P T$ | 0.525 | -0.051 | 0.016 | -0.282 | -0.223 | 0.234 |
| St. Dev. | 0.328 | 0.313 | 0.303 | 0.304 | 0.332 | 0.386 |

The Table refers to the conditional generalized IRFs from 1999q1 to 2016q1, absorbing outcomes from conditional forecasts computed on the time frame of 9 quarters. It describes, in terms of standard deviations, the systemic contributions of the productivity given a $1 \%$ shoch to global economy. They correspond to $\chi_{7 t} \hat{\beta}_{7 t}$ variable-specific indicator and $\chi_{8 t} \hat{\beta}_{8 t}$ common indicator.

Figure 1 draws the general government spending ( $\operatorname{plot} 1 \mathrm{aa}$ ) and structures component ( $\operatorname{plot} 1 \mathrm{~b}$ ). The former proves that fiscal policy would have been countercyclical during the recent recession and the second period of the consolidation (from 2011q1), and procyclical in the first period of the consolidation (from 2009q1). The structures component shows an irregular path over time and the latter would be increased in the last decade proving the huge economic-institutional constraints which national authorities have to conform to follow domestic policies.


Figure 1: General Government Spending (plot $a$ ) and structures $_{i t, j}$ component (plot $b$ ) are drawn spanning the period from $1999 q 1$ to $2013 q 4$.

## 4 Conditional GIRFs - Graphic Representation



Figure 2: Impulse responses of the productivity given a $1 \%$ shock to economy (solid lines) and $60 \%$ confidence bands (dashed lines) are drawn in standard deviation of the variables in the system, spanning the period $1999 q 1$ to $2016 q 1$. They account for $\chi_{3 t} \hat{\beta}_{3 t}$ (plot $a$ ) and $\chi_{4 t} \hat{\beta}_{4 t}$ (plot $b$ ) cross-country indicators.


Figure 3: impulse responses of the productivity given a $1 \%$ shock to economy (solid lines) and $60 \%$ confidence bands (dashed lines) are drawn in standard deviation of the variables in the system, spanning the period 1999q1 to $2016 q 1$. They account for $\chi_{5 t} \hat{\beta}_{5 t}$ (plot $a$ ) and $\chi_{6 t} \hat{\beta}_{6 t}$ (plot b) cross-country indicators.


Figure 4: Country-specific indicators are shown in standard deviation from the historical average of annual growth rates, spanning the period $1999 q 1$ to $2016 q 1$. They correspond to $\chi_{3 t} \hat{\beta}_{3 t}$ (plot $a$ ) and $\chi_{4 t} \hat{\beta}_{4 t}$ (plot b) cross-country indicators.


Figure 5: Country-specific indicators are shown in standard deviation from the historical average of annual growth rates, spanning the period $1999 q 1$ to $2016 q 1$. They correspond to $\chi_{5 t} \hat{\beta}_{5 t}$ (plot $a$ ) and $\chi_{6 t} \hat{\beta}_{6 t}$ (plot b) cross-country indicators.


Figure 6: Variable-specific (a) and common (b) indicators across all countries for real and financial dimension are shown in standard deviation from the historical average of annual growth rates, spanning the period $1999 q 1$ to $2016 q 1$. They correspond to $\chi_{7 t} \hat{\beta}_{7 t}$ variable-specific indicator (plot $a$ ) and $\chi_{8 t} \hat{\beta}_{8 t}$ (plot b) common indicator.


[^0]:    Terms of use:
    Documents in EconStor may be saved and copied for your personal and scholarly purposes.

    You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

    If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

[^1]:    ${ }^{1}$ Latent or hidden factors are variables that are not directly observed but are rather inferred from other variables that are observed and, hence, directly measured.
    ${ }^{2}$ Confounding factors are variables that affect both dependent and independent variables causing spurious association. This latter cannot be described and, hence, evaluated in terms of correlations or associations being a causal concept.
    ${ }^{3}$ See e.g., Chib (1995), Chib (1996), Pesaran and Shinb (1998), and Chib and Jeliazkov (2001).

[^2]:    ${ }^{4}$ The vec operator transforms a matrix into a vector by stacking the columns of the matrix one underneath the other.

[^3]:    ${ }^{5}$ The Wishart distribution is a multivariate extension of $\chi^{2}$ distribution and, in Bayesian statistics, corresponds to the conjugate prior of the inverse covariance-matrix of a multivariate normal random vector.
    ${ }^{6}$ To be more precise, it would correspond to the parsimonious Seemingly Unrelated Regression (SUR) model developed in literature. See for istance, Canova and Ciccarelli (2009) and Ciccarelli et al. (2012).
    ${ }^{7}$ See e.g., Kadiyala and Karlsson (1997).

[^4]:    ${ }^{8}$ For istance, the Minnesota priors are based on an approximation which involves replacing $\Omega$ with an estimate, $\hat{\Omega}$. See Doan et al. (1984) and Litterman (1986).

[^5]:    ${ }^{9}$ These implementations do not allow to use the Minnesota prior since its covariance matrix is written in terms of blocks which vary across equations.

[^6]:    ${ }^{10}$ See e.g., Chib and Greenberg (1995).

[^7]:    ${ }^{11}$ See e.g., Mundell (1961), Imbs (2006b), Imbs (2006a), and Tabellini (1990).
    ${ }^{12}$ See e.g., Sorensen and Yosha (1998), Blanchard and Perotti (2002), and Beetsma and Jensen (2005).

[^8]:    ${ }^{13}$ For istance, I refer to some main studies, such as: Lane and Milesi-Ferretti (2007); Mastrogiacomo et al. (2017); Facchini et al. (2017); Degiannakis et al. (2016); Crespo-Cuaresma and Fernandez-Amadorb (2013); Canova and Marrinan (1997); Reinhart and Rogoff (2009); and Ciccarelli and Rebucci (2007).

[^9]:    ${ }^{14}$ The weights $s_{i t, j}$ component corresponds to the sum of rweights ${ }_{i t, j}$ and fweights $_{i t, j}$.

[^10]:    ${ }^{15} \mathrm{O}$ wn computations.

[^11]:    ${ }^{16}$ The conditional projection for output growth is the one the model would have obtained over the same period conditionally on the actual path of unexpected shock for that period.
    ${ }^{17}$ The unconditional projection is the one the model would obtain for output growth for that period based only on historical information and consistent with a model-based forecast path for the other variables.

[^12]:    ${ }^{18}$ See e.g., Eichengreen and Wyplosz (1998) and Buti et al. (1998).
    ${ }^{19}$ They correspond to Portugal, Ireland, Italy, and Greece (IMF, 2011).

[^13]:    ${ }^{20}$ See e.g. Gordon and Bovenberg (1996) and Sorensen and Yosha 1998.

