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Price Customization and Targeting in Platform Markets*

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Abstract

Recent technologies enable matching intermediaries to engage in unprecedented levels of *targeting*, whereby matches finely depend on the agents' characteristics, but also favor *customized* (i.e., match-specific) pricing. Yet, novel regulations on the transfer of personal data, as well as a renewed trend towards market decentralization, are expected to hinder price customization and favor *uniform* pricing (whereby the price of a match charged to agents on a given side of a market is invariant in the agents' observable characteristics). To assess the impact of these developments, we build a matching model in which agents' preferences are both vertically and horizontally differentiated. Mirroring current practices, we show how, absent regulations, platforms maximize profits through price customization, link the latter to structural elasticities, and assess the targeting effects of market power. Perhaps surprisingly, we show that uniform pricing may either increase or decrease targeting levels and consumer welfare, depending on testable properties of demand. The analysis has implications for online shopping, ad-exchanges, and media platforms.

JEL classification: D82

Keywords: Many-to-many matching, asymmetric information, platforms, incentives, price discrimination

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1 Introduction

Over the last two decades, new technologies have permitted the development of matching intermediaries of unprecedented scale engaging in unparalleled level of targeting. Notable examples include ad exchanges, matching publishers with advertisers, business-to-business platforms, matching firms with mutually beneficial commercial interests, and dating websites, matching agents with common passions. The same advances in technology that favored high levels of targeting also enabled greater price customization, whereby the price of a match finely depends on observable characteristics of the matching partners.

For instance, in advertising exchanges, the assignment of, and payments from, advertisers depend on scores that summarize the compatibility of the ads with each publisher's content.[1] A similar trend can be found in other markets, not traditionally analyzed through the lens of matching. In online shopping, for example, it is common practice among retailers to use customers' personal data to set personalized prices. In one of the most publicized cases, Orbitz, an online travel agency, reportedly used information about customers' demographics to charge targeted customers higher hotel fees.[2] Similarly, Safeway, an online grocery chain, often proposes individualized price offers and quantity discounts to customers with certain profiles.[3] The retailers' knowledge about consumers' characteristics often comes from data brokers, who collect and sell personal information (in the form of demographics, geolocation, browsing history, etc).[4]

In the markets mentioned above, price customization is easy to enforce, as the agents' "horizontal" characteristics are observable (for instance, in ad exchanges, the advertisers' profile is often revealed by the ads' content) or can be learnt from third parties (for example, in online retailing, information about consumers can often be obtained from data brokers or affiliated websites). In other markets, instead, the agents' horizontal characteristics that are relevant for price customization have to be indirectly elicited, and this may require bundling.[5] A case in point is that of media markets (for instance, satellite/cable TV providers), that use sophisticated pricing strategies that condition payments on the entire bundle of channels selected by the subscribers.[6]

[1] See, for example, https://support.google.com/adxseller/answer/2913506?hl=en&ref_topic=3376095. Moreover, ad exchanges use advertiser-specific reservation prices which are easily automated using proxy-bidding tools. Ad exchanges also price discriminate on the publisher side, by making the payments to the publishers depend on the publishers' profile and on the volume of impressions.

[2] See the article "On Orbitz, Mac Users Steered to Pricier Hotels," the *Wall Street Journal*, August 23, 2012.

[3] See https://www.bloomberg.com/news/articles/2013-11-14/2014-outlook-supermarkets-offer-personalized-pricing.

[4] According to *The New York Times*, the data broker industry's revenue reached $156 billion in 2013 (see the article "The Dark Market for Personal Data," August 16, 2014).

[5] For instance, Ad exchanges have recently developed new contractual arrangements that allows them to bundle different ads as a way of screening the publishers' unobservable preferences (see, Mirrokni and Nazerzadeh (2017)).

[6] Most satellite/cable TV providers price discriminate on the viewer side by offering viewers packages of channels whereby the baseline configuration can be customized by adding channels at a cost that depends on the baseline configuration originally selected (see, among others, Crawford (2000), and Crawford and Yurukoglu (2012)). For example, in the US, Direct TV offers various vertically differentiated (i.e., nested) packages (both in English and in

While having a long history in the policy debate, price customization has attracted renewed attention due to the two-sided nature of matching intermediaries, and the amount of information now available for target pricing.[7,8] The concern is that, by leveraging the platforms' market power, price customization hinders the efficiency gains permitted by better targeting technologies. Recent regulations speak directly to these issues. In the European Union, for example, the General Data Protection Regulation (GDPR) and the ePrivacy Regulation (ePR) mandate that businesses ask for consumers' consent prior to collecting and transmitting personal data. Such regulations hinder price customization based on data from third parties.[9,10]

Market decentralization poses yet another challenge to price customization. The same technological progress of the last few years that has facilitated the growth of matching markets is now expected to favor a gradual transition of such markets from a centralized structure, where matching is controlled by platforms, to a more decentralized structure where one side (typically, the "seller" side) posts stand-alone prices, while the other side (typically, the "buyer" side) then constructs the matching sets. For example, in the market for media content, several analysts believe that the increase in the speed of fiber-optic and broadband internet connection will favor a gradual transition to a market structure whereby viewers will pay directly the content providers, bypassing the intermediation of media platforms (such as satellite or cable TV providers).[11] In decentralized markets, price customization is often hindered by the absence of data about consumers' tastes (for instance, due to privacy regulation), along with the difficulty of tracking consumers' purchases with other vendors. This is in contrast to centralized matching markets where price customization can often be implemented through bundling (see footnote 6).

The aim of this paper is to understand how targeting and price customization shape the matching opportunities offered by profit-maximizing platforms, and study the impact on targeting and consumer welfare of uniform pricing (whereby payments to the platforms do not depend on the "horizontal characteristics" of the agents' profiles), be it a result of regulation or market decentralization.

Spanish). It then allows viewers to add to these packages various (horizontally differentiated) premium packages, which bundle together channels specialized in movies, sports, news, and games. In addition, viewers can further customize the packages by adding individual sports, news, and movie channels.

[7] In the US, the first law to regulate price discrimination is the Robinson-Patman Act of 1936.

[8] In the case of media markets, see, for example, the Federal Communications Commission 2004 and 2006 reports on the potential harm of price customization through bundling. In the case of online retailing, see the UK Office of Fair Trading 2010 eponymous report on online targeting in advertising and pricing.

[9] See Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the processing of personal data and on the free movement of such data.

[10] In the US, the Federal Trade Commission (FTC) recommended in 2014 legislation increasing the transparency of the data broker industry and giving consumers greater control over their personal information. See https://www.ftc.gov/news-events/press-releases/2014/05/ftc-recommends-congress-require-data-broker-industry-be-more.

[11] A similar trend is also taking place in online advertising, where many publishers now prefer employing direct-sales channels so as to avoid the commissions charged by the platforms. See, for instance, https://digiday.com/media/advertisers-are-often-left-flying-blind-in-ad-exchanges/.

To examine these issues, we develop a model where agents' preferences exhibit elements of both vertical and horizontal differentiation. Certain agents value being matched with agents from the other side of the market uniformly more than others (vertical differentiation). At the same time, agents from the same side may disagree on the relative attractiveness of any two agents from the opposite side (horizontal differentiation). We capture the two dimensions of differentiation by letting agents' types be located on a cylinder, where the height represents the vertical dimension, while the radial position determines the horizontal preference. Each agent's utility from interacting with any other agent from the opposite side increases with the agent's vertical dimension. Fixing the vertical dimension, each agent's utility is single-peaked with respect to the horizontal dimension. More specifically, we identify each agent's radial position with his bliss point. Accordingly, each agent's utility for interacting with any other agent from the opposite side decreases with the distance between the agent's bliss point (his radial position) and the partner's location (the partner's radial position). Such preference structure, in addition to its analytical convenience, mirrors the one in the "ideal-point" models typically used in the empirical literature on media and advertising markets (see, for example, Goettler and Shachar 2001).

A key element of our analysis is the focus on *matching tariffs*, which describe how the payments asked by the platform vary with the matching sets demanded by the agents. A tariff exhibits *uniform pricing* if all agents from a given side face the same price schedule for different quantities of the matches with agents at a given location on the opposite side. Formally, uniform tariffs are tariffs that do not condition an agent's payment to the platform on the agent's *own radial position* (i.e, the horizontal dimension of the agent's preferences). A particularly simple type of uniform pricing often proposed as a potential regulatory remedy to the market power enjoyed by media platforms is stand-alone linear pricing (for a discussion, see Crawford and Yurukoglu 2012).

Absent any regulation, platforms typically offer *customized* tariffs on both sides, whereby the customization is obtained by means of menus of *matching plans*. Each plan is defined by its baseline configuration (i.e., a baseline set of partners from the opposite side), a baseline price, and a collection of prices describing the cost to the subscriber of customizing the plan by adding extra matches. The cost of the customization is typically non-linear in the volume of matches of any given type added to the plan (second-degree price discrimination). Importantly, the cost of the customization is also a function of the baseline plan selected by the subscriber. Because different plans are targeted to agents from different "locations," such tariffs thus also display a form of third-degree price discrimination. As the analysis reveals, when neither the vertical nor the horizontal dimensions defining the agents' preferences are observable by the platform, information about such dimensions is elicited through a careful design of the baseline plans and the associated price-customizing schedules.

Our first main result derives conditions under which the profit-maximizing tariffs exhibit price customization. Equipped with these conditions, we then offer a convenient representation of the optimal price schedules. The representation yields a formula describing the price each agent from each given location has to pay to include in his matching set any feasible amount of matches from

any location on the opposite side of the market. The formula links location- and volume-specific prices to the various local elasticities of the demands on the two sides of the market. In this sense it constitutes the analog in a matching market of the familiar Lerner-Wilson formula of optimal non-linear pricing. The formula differs from the traditional one in that it accounts for (a) the reciprocity of the matching, and (b) the fact that the platform combines second-degree price discrimination (higher vertical types self-select into larger matching plans) with third-degree price discrimination (the total price paid by each subscriber depends on the horizontal dimension of the subscriber's preferences, both when the latter are observed by the platform and when they are indirectly elicited through self-selection).

Our characterization of the profit-maximizing matching tariffs reveals interesting patterns of cross-subsidization, unique to matching environments. Namely, optimal tariffs induce a form of *negative assortative matching at the margin*: at any given location, agents with a low value for matching (a low vertical dimension) are matched only to those agents from the other side whose value for matching is sufficiently high. This form of negative assortativeness naturally takes into account the agents' mutual attractiveness, as determined by their joint locations. As a result, the matching sets of any two agents from the same side are nested only if the two agents share the same dimension of horizontal preferences.

We also leverage on this characterization to study the interplay between targeting and market power. Specifically, we derive conditions on primitives (on the agents' utilities and distributions of their preferences) under which the under-provision of matches under profit maximization (relative to the efficient level) is either magnified or alleviated as the distance of a partner's location from an agent's bliss point increases. This analysis has no parallel in the standard models of price discrimination, which are not amenable to study the effects of market power on targeting levels.

Our second set of results investigates the effects on prices, the composition of the matching demands, and consumer welfare of uniform pricing on a given side of the market. Analogously to the generalized Lerner-Wilson formula discussed above, we provide a novel representation of the optimal price schedules that uses local elasticities to describe the prices agents on each side have to pay per quantity of matches from each location on the opposite side. Relative to the case of customized pricing, this new pricing formula identifies the relevant aggregate elasticities in environments where location-specific pricing is not possible. The typical marginal revenue and marginal cost terms (which determine the optimal cross-subsidization pattern) are now averages that take into account not only the uniformity of prices, but also how the procurement costs of matches are affected by the horizontal component of the agents' preferences. From a more theoretical perspective, the characterization contributes to the mechanism design literature by developing a novel technique to handle constraints on the transfer rule employed by the principal (as opposed to the familiar constraints on the allocation rules, which are typically easier to analyze using standard techniques).

We then put this characterization to work, revealing how uniform pricing affects targeting and welfare. Intuition might suggest that uniform pricing should increase targeting by preventing plat-

forms from charging higher prices for the matches involving the most preferred partners. This simple intuition, however, may fail to account for the fact that platforms re-optimize their entire price schedules to respond to aggregate elasticities. Perhaps surprisingly, uniform pricing can either decrease or increase the equilibrium level of targeting, depending on how match-demand elasticities vary with location. We derive sufficient conditions in terms of primitives (on the modularity and convexity of the agents' utilities, as well as the distributions of vertical preferences) under which targeting is higher (alternatively, lower) under uniform pricing (alternatively, customized pricing). The usefulness of these results is illustrated in three applications (online shopping, online advertising, and media markets), where we specialize the model to identify testable properties of demand under which targeting is higher (alternatively, lower) under uniform pricing.

We then use such conditions to look into the welfare effects of uniform pricing. Exploiting a novel connection between uniform pricing in matching markets and the literature on third-degree price discrimination, we show how to adapt the elegant analysis in Aguirre et al. (2010) to the matching markets under examination. The results identify sufficient conditions for uniform pricing to increase consumer surplus (in the side where prices are uniform). These results, once combined with appropriate empirical work, can guide the design of regulatory interventions in platform markets where price customization is a concern.

Lastly, we show that the same analysis that permits us to uncover the welfare effects of uniform pricing can be adapted to shed light on the welfare effects of a transition to a decentralized market structure (where sellers independently post stand-alone prices), thus contributing a novel angle to the policy debate over whether or not such transition should be encouraged.

Outline of the Paper. The rest of the paper is organized as follows. Below, we close the introduction by briefly reviewing the pertinent literature. Section 2 presents the model. Section 3 identifies conditions under which the profit-maximizing matching tariffs are customized, and derives properties of the associated pricing schedules. Section 4 studies the effects of uniform pricing and of the transition from a centralized to a decentralized structure. Section 5 concludes. All proofs are in the Appendix at the end of the document.

Related Literature

This paper studies many-to-many matching (with monetary transfers) in markets where the agents' preferences are both vertically and horizontally differentiated. Particularly related are Jeon, Kim and Menicucci (2017) and Gomes and Pavan (2016). The first paper studies the provision of quality by a platform in a setting where quality provision enhances match values. The second paper studies the inefficiency of profit maximization in many-to-many matching markets. Both papers abstract from the possibility that agents' preferences be horizontally differentiated (in tastes and match values), thus ignoring the issues of targeting and price customization that are the heart of the analysis in the present paper. Fershtman and Pavan (2017) considers many-to-many matching in a model with a

rich preference structure similar to the one in the present paper, combining elements of vertical and horizontal differentiation. Contrary to the present paper, however, it focuses on dynamic markets in which agents learn the attractiveness of the partners and experience shocks to their preferences over time.

Related are also Jullien and Pavan (2017), and Tan and Zhou (2017). The former paper studies platform competition in markets where agents' preferences for the different platforms are heterogenous but where agents value homogeneously the interactions with agents from the opposite side of the market. The latter paper studies price competition in a model where multiple platforms compete by offering differentiated services to the various sides of the market, and where agents' preferences are heterogenous with both within-side and across-sides network effects. While considering rich preference structures, both papers abstract from price discrimination. The latter is examined in Damiano and Li (2007) and Johnson (2013). Contrary to this paper, these works consider markets where matching is one-to-one and agents' preferences are differentiated along a vertical dimension only.

This paper considers a many-to-many matching market where agents might disagree on the relative attractiveness of any two agents from the other side (horizontal differentiation). Similar preferences structures are examined in the matching literature surveyed in Roth and Sotomayor (1990) (for a more recent treatment, see Hatfield and Milgrom (2005)), and in the literature on the school assignment problem (see, for example, Abdulkadiroglu, Pathak and Roth (2005a, 2005b) and Abdulkadiroglu and Sonmez (2003)). These literatures are methodologically distinct from the current paper, in that they focus on solution concepts such as stability and do not allow for transfers.

More broadly, markets where agents purchase access to other agents are the focus of the broad literature on two-sided markets (see Belleflamme and Peitz (2017) for the most up-to-date overview). Most of this literature, however, restricts attention to a single network, or to mutually exclusive networks. Ambrus, Calvano and Reisinger (2016) relax this structure by proposing a model of competing media platforms with overlapping viewerships (i.e., multi-homing). By contrast, we stick to a monopolistic market, but introduce a richer preferences structure (displaying horizontal tastes for matches), what enables us to study targeting and price customization in such markets.

Our discussion of market decentralization is related to Loertscher and Niedermayer (2017), who compare the agency and wholesale business models by a platform that faces competition from an independent bilateral exchange. To focus on the substitutability between market places, that paper, however, abstracts from targeting and price-customization.

The study of price customization is related to the literature on price discrimination. In the case of second-degree price discrimination, Mussa and Rosen (1978), Maskin and Riley (1983), and Wilson (1997) study the provision of quality/quantity in markets where agents possess private information about a vertical dimension of their preference. Our study of price customization in many-to-many matching markets introduces two novel features relative to the standard monopolistic screening problem. First, the platform's feasibility constraint (namely, the reciprocity of the matches) has no equivalent in markets for commodities. Second, agents' preferences are differentiated along both

a vertical and a horizontal dimension. This richer preferences structure calls for a combination of second- and third-degree price discrimination, and leads to novel cross-subsidization patterns.[12]

The paper also contributes to the literature on third-degree price discrimination. In addition to the paper by Aguirre et al. (2010) mentioned above, see Bergemann, Brooks, and Morris (2015) for an excellent overview and for recent developments. The latter paper characterizes all combinations of producer and consumer surplus that arise from different information structures about the buyers' willingness-to-pay (alternatively, from different market segmentations). Our setup differs from theirs in many respects. First, the informational costs incurred by the intermediary are endogenous, and depend on its price-customization strategies on both sides of the market (the information structure is exogenous in Bergemann, Brooks, and Morris (2015)). Second, the preferences structure is different, reflecting specific features of the applications under consideration.

Related is also the literature on bundling (see, among others, Armstrong (2013), Hart and Reny (2015), and the references therein). The present work differs from that literature in two important aspects. First, while preferences are multi-dimensional both in the present paper and in that literature, in our setting, preferences can be orthogonally decomposed into a vertical and a horizontal dimension. The bundling literature, instead, assumes a more general preference structure, which, however, hinders the characterization of the optimal price schedules, except in certain special cases. Second, reflecting the practices of many-to-many matching intermediaries, we assume that sales are monitored, so that prices can condition on the entire matching set of each agent. The bundling literature, by contrast, typically assumes that purchases are anonymous.

Lastly, the paper contributes to the literature on targeting in advertising markets (see, for example, Bergemann and Bonatti (2011, 2015) and Cox et al. (2017) and the references therein). Our work contributes to this literature by introducing a richer class of (non-linear) pricing strategies and by comparing the matching outcomes that emerge under a decentralized structure to their counterparts in platform markets where the matching between the advertises and the publishers (or content providers) is mediated. Contrary to some of the papers in this literature, however, we abstain from platform competition. Importantly, we also assume that agents can perfectly communicate their preferences, and face no informational frictions regarding the desirability of the matches. Eliaz and Spiegler (2016) relax these assumptions, and consider the mechanism design problem of a platform that wants to allocate firms into search pools created in response to noisy preference signals provided by the consumers. Relatedly, Eliaz and Spiegler (2017) consider the problem of a profit-maximizing advertising platform who wants to elicit the advertisers' profiles so as to match them to consumers with preferences for diversity. These papers do not investigate the effects of uniform pricing, but rather focus on the incentives of firms to truthfully reveal their "ideal audiences."

[12]Related is also Balestrieri and Izmalkov (2015). That paper studies price discrimination in a market with horizontally differentiated preferences by an informed seller who possesses private information about its product's quality (equivalently, about the "position" of its good in the horizontal spectrum of agents' preferences). The focus of that paper is information disclosure, while the focus of the present paper is matching, targeting, and price customization.

2 Model

A monopolistic platform matches agents from two sides of a market. Each side $k \in \{a, b\}$ is populated by a unit-mass continuum of agents. Each agent from each side $k \in \{a, b\}$ has a bi-dimensional type $\theta_k = (v_k, x_k) \in \Theta_k \equiv V_k \times X_k$ which parametrizes both the agent's preferences and the agent's attractiveness.

The parameter x_k, which describes the agent's location, captures horizontal differentiation in preferences. For convenience, we assume that agents are located on a circle of perimeter one, in which case $X_k = [0, 1]$, $k = a, b$. The parameter $v_k \in V_k \equiv [\underline{v}_k, \overline{v}_k] \subseteq \mathbb{R} \cup \{+\infty\}$, on the other hand, captures heterogeneity in preferences along a vertical dimension. It controls for the intensity of the agent's matching utility across locations (i.e., the overall utility the agent derives from interacting with a generic agent from the opposite side, before doing any profiling). Hereafter, we let $\text{Int}[V_k]$ denote the interior of the set V_k.

We assume that the vertical dimensions v_k are the agents' private information. As for the horizontal dimensions, i.e., the locations x_k, we consider both cases where they are publicly observable, as well as cases where they are the agents' private information. In particular, we consider the following four scenarios:

- *Scenario (i)*: Locations are publicly observable on both sides;
- *Scenario (ii)*: Locations are private information on side a and publicly observable on side b;
- *Scenario (iii)*: Locations are publicly observable on side a and private information on side b;
- *Scenario (iv)*: locations are private information on both sides.

Agents derive higher utility from being matched to agents whose locations are closer to their own. Their utility also increases, over all locations, with their vertical type. We assume the utility that an agent from side $k \in \{a, b\}$ with type $\theta_k = (v_k, x_k)$ derives from being matched to an agent from side $l \neq k$ with type $\theta_l = (v_l, x_l)$ is represented by the function

$$u_k(v_k, |x_k - x_l|),$$

where $|x_k - x_l|$ is the circular (minimal) distance between the two agents' locations. The function u_k is Lipschitz continuous, bounded, strictly increasing in v_k, and weakly decreasing in $|x_k - x_l|$. To make things interesting, we assume u_k is strictly decreasing in $|x_k - x_l|$ on at least one side. The following examples illustrate the type of preferences covered by the aforementioned specification.

Example 1. ***(online shopping)*** The platform is an online intermediary matching consumers from side a with sellers from side b. We identify with each location x_b one product variety, and let x_a describe a consumer's most preferred variety. The utility that a consumer of type $\theta_a = (v_a, x_a)$ derives from purchasing a product of variety x_b is given by

$$u_a(v_a, |x_a - x_b|) = v_a - \psi\left(|x_a - x_b|\right),$$

where $v_a > 0$ is the consumer's utility from her most preferred variety, and the increasing function $\psi(\cdot)$, satisfying $\psi(0) = 0$, measures the loss of utility from purchasing varieties other than the most preferred one. Each consumer is interested in buying only one good from each seller (unit-demand), but is interested in purchasing multiple goods of the same variety from different sellers. Each seller is characterized by the product variety it produces, x_b, and by her constant marginal cost of production v_b. Accordingly, the gross profit that each seller of type $\theta_b = (v_b, x_b)$ derives from selling each unit of her good is equal to $u_b(v_b, |x_b - x_a|)) = v_b < 0$ (being therefore invariant to the buyers' type). While each seller's product variety x_b is publicly observable, each consumer's most preferred variety (her bliss point) might be known to the platform (for example, when the latter has access to data about consumers' characteristics), which corresponds to Scenario (i), or be the consumer's private information, which corresponds to Scenario (ii). $\diamondsuit$

The following example is also of particular interest.

Example 2. ***(ad exchange)*** The platform is an ad exchange matching advertisers from side a to publishers from side b. The expected profit that an advertiser of type $\theta_a = (v_a, x_a)$ obtains from an impression at the website of a publisher of type $\theta_b = (v_b, x_b)$ is given by

$$u_a(v_a, |x_a - x_b|) = v_a \phi(|x_a - x_b|),$$

where v_a is the advertiser's profit per sale and where the strictly decreasing function $\phi : [0, \frac{1}{2}] \to [0, 1]$ describes how the probability of a conversion (i.e., the probability the ad view turns into a sale) varies with the distance between the publisher's profile, x_b, and the advertiser's ideal audience, x_a. By contrast, publishers can be viewed (to a first approximation) as indifferent with respect to the kind of advertisement displayed at their websites. The matching (dis)utility of a publisher reflects the opportunity cost of not using the advertisement space to sell its own products, or from not selling the ad slot outside of the platform. Accordingly, the profit that a publisher of type $\theta_b = (v_b, x_b)$ derives from displaying the ad of an advertiser of type $\theta_a = (v_a, x_a)$ is given by $u_b(x_b, |x_a - x_b|) = v_b \leq 0$, all $x_a, x_b \in [0, 1]$. The informational environment that best suits such market is Scenario (i), as both an advertiser's ideal type of audience and a publisher's profile are typically observable by the ad exchange. $\diamondsuit$

Another example that shares the preference structure of Example 2 is that of a lobbying firm (platform) matching interest groups from side a with public officials from side b (see Kang and You (2016) for a detailed description of this market). The value of the connection to each interest group is the product of its gain from a policy change, v_a, and the valence (or influence) of the public official, $\phi(|x_a - x_b|)$. The latter decreases with the distance between the interest group's core business, x_a, and the public official's policy field, x_b. The public official's (electoral or legal) costs of capture are captured by v_b, which can be viewed (to a first approximation) as independent of the characteristics of the interest group. In this example, it seems natural to assume that each interest

group's core business as well as each public official's policy field are observable by the lobbying firm, which corresponds to Scenario (i) above.

Example 3. ***(media platform)*** The platform is a media outlet matching viewers from side a with content providers from side b. The utility that a viewer of type $\theta_a = (v_a, x_a)$ derives from having access to the content of a provider of type $\theta_b = (v_b, x_b)$ is given by the constant-elasticity-of-substitution (CES) function

$$u_a(v_a, |x_a - x_b|) = \left[\alpha \cdot (v_a)^\delta + (1-\alpha) \cdot \phi\left(|x_a - x_b|\right)^\delta\right]^{\frac{1}{\delta}},$$

where $\alpha \in [0,1]$ captures the relative importance of the viewer's vertical and horizontal preferences, with $V_a \subset \mathbb{R}_+$. In turn, the strictly decreasing function $\phi : [0, \frac{1}{2}] \to \mathbb{R}_+$ describes how the viewer's utility declines with the distance between the viewer's ideal type of content, x_a, and the provider's content, x_b. Finally, $\delta \in \mathbb{R}/\{0\}$ measures the elasticity of substitution between the vertical and horizontal dimensions. By contrast, to a first approximation, content providers might be viewed as indifferent with respect to the profile of the viewers that access their content. The matching (dis)utility of a content provider reflects the extra revenue from advertisers (as advertisers typically pay more to content providers with a higher exposure to viewers), or the expenses from broadcasting rights paid to third parties (which are proportional to the audience reached). In this case, for any $v_b \in V_b$, any $x_a, x_b \in [0,1]$, $u_b(v_b, |x_b - x_a|)) = v_b$, with $v_b > 0$ under the first interpretation, and $v_b < 0$ under the second interpretation. In this case, the informational environment that best suits this market is that of Scenario (ii), as each viewer's ideal type of content is likely to be his own private information, whereas each content provider's profile (the type of content provided) is observable by the platform. $\diamondsuit$

The preference structure of Example 3 follows closely the ones typically assumed in the empirical literature on media markets (see, for example, Goettler and Shachar 2001).

The type $\theta_k = (v_k, x_k)$ of each agent from each side $k \in \{a, b\}$ is an independent draw from the absolutely continuous distribution F_k with support Θ_k. We denote by F_k^v (alternatively, F_k^x) the marginal distribution of F_k with respect to v_k (alternatively, x_k), and by $F_k^{v|x}$ the distribution of v_k conditional on x_k. We denote by f_k^v the density of F_k^v and by $\lambda_k^v \equiv f_k^v/[1 - F_k^v]$ its hazard rate. We use analogous notation for the densities and hazard rates of the conditional distributions $F_k^{v|x}$.

Let $\Sigma(\Theta_l)$ be the Borel sigma algebra associated with the set Θ_l. The total payoff that an agent from side $k \in \{a, b\}$ with type $\theta_k = (v_k, x_k)$ obtains from being matched, at a price p, to a set of types $\mathbf{s}_k \in \Sigma(\Theta_l)$ from side $l \neq k$ is given by

$$\pi_k(\mathbf{s}_k, p; \theta_k) = \int_{\mathbf{s}_k} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - p, \tag{1}$$

whereas the payoff that the same agent obtains outside of the platform (where no matches are formed and no payments are made) is equal to zero.[13]

[13]The representation in (1) assumes the agent is matched to all agents from side $l \neq k$ whose type is in $\mathbf{s}_k$. That

Remark 1. The specification in (1) implies that the utility that each agent derives from each match is independent of who else the agent is matched with. It also implies that such utility is invariant in each partner's vertical type. In a previous version, we considered a more general setting where such assumptions are relaxed. In particular, we assumed that (a) each agent's attractiveness depends on both the agent's location and other traits that are correlated with the agent's vertical dimension, and (b) that the utility that each agent derives from each matching set $\mathbf{s}_k$ is given by a concave (alternatively, convex) transformation of $\int_{\mathbf{s}_k} u_k(\cdot)dF_l(\theta_l)$, where the concavity/convexity of the transformation captures decreasing/increasing returns to match quality. Because both the exposition and the analysis are significantly more tedious under such richer specifications, we opted here for the convenience of the representation in (1).

Tariffs and Matching Demands

The platform offers matching tariffs to each side $k \in \{a, b\}$. A *matching tariff* T_k specifies the (possibly negative) total payment $T_k(\mathbf{s}_k)$ that each agent from side k must pay to the platform for being matched to the set of types $\mathbf{s}_k \in \Sigma(\Theta_l)$ from side $l \neq k$.

Given the tariff T_k, the *matching demand* of each agent from side k with type $\theta_k = (v_k, x_k)$ is given by the set

$$\hat{\mathbf{s}}_k(\theta_k; T_k) \in \arg\max_{\mathbf{s}_k \in \Sigma(\Theta_l)} \left\{ \int_{\Theta_l} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - T_k\left(\mathbf{s}_k\right) \right\}. \tag{2}$$

To guarantee participation by all agents, we require that $T_k(\mathbf{s}_k) = 0$ if $\mathbf{s}_k = \emptyset$.

Definition 1. The tariffs T_k, $k = a, b$, are *feasible* if, for all $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l \in \{a, b\}$, $l \neq k$,

$$\theta_l \in \hat{\mathbf{s}}_k(\theta_k; T_k) \Longleftrightarrow \theta_k \in \hat{s}_l(\theta_l; T_l). \tag{3}$$

A pair of feasible matching tariffs thus induces *reciprocal* demands. That is, if an agent from side k with type θ_k finds it optimal to be matched to all agents from side $l \neq k$ with type θ_l, then all agents from side l with type θ_l find it optimal to be matched to all agents from side k with type θ_k.

Given any matching set $\mathbf{s}_k \in \Sigma(\Theta_l)$, any location x_l, we denote by $q_{x_l}(\mathbf{s}_k)$ the "mass" of agents from side l located at x_l included in the matching set $\mathbf{s}_k$.[14]

Definition 2. The tariff T_k offered by the platform to the side-k agents, $k \in \{a, b\}$, is consistent with *uniform pricing* if there exists a collection of *price schedules* $p_k : [0, 1]^2 \to \mathbb{R}$, one for each location

matching sets are described by the types from the opposite side an agent has access to, as opposed to the identities of the agents in the matching set, reflects the property that, under both the welfare- and the profit-maximizing tariffs, each agent from each side $k = a, b$ who decides to include in his matching set some agent from side $l \neq k$ whose type is θ_l optimally chooses to include in his matching set *all* agents from side l whose type is θ_l.

[14] Hereafter, we abuse of terminology by referring to the density of agents of a certain type as the "mass" of agents of that type.

$x_l \in [0,1]$, such that the total payment asked by the platform for each matching set $\mathbf{s}_k \in \Sigma(\Theta_l)$ is given by

$$T_k(\mathbf{s}_k) = \int_0^1 p_k(q_{x_l}(\mathbf{s}_k)|x_l)dx_l. \tag{4}$$

Conversely, the tariff T_k is *discriminatory if* there are no price schedules such that T_k is consistent with Condition (4), for all $\mathbf{s}_k$.

Hence, under uniform pricing, the tariff offered by the platform to the side-k agents consists of a collection of non-linear price schedules, $(p_k(\cdot|x_l))_{x_l\in[0,1]}$, with each schedule $p_k(q|x_l)$ specifying the total price each side-k agent has to pay to be matched to q agents from side $l \neq k$ located at $x_l \in [0,1]$. Importantly, the price $p_k(q|x_l)$ is independent of the agent's own characteristics, θ_k, and of the mass of agents from other locations included in the matching set $\mathbf{s}_k$.

The next definition describes a type of discriminatory tariffs that plays an important role in the analysis below.

Definition 3. The tariff T_k is *customized* if there exists a collection of matching plans

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot;x_k), \mathbf{S}_k(x_k)) : x_k \in [0,1]\},$$

one for each side-k location $x_k \in [0,1]$, such that each side-k agent selecting the basic plan $\underline{\mathbf{s}}_k(x_k)$ and then selecting the customization $\mathbf{s}_k \in \mathbf{S}_k(x_k) \subseteq \Sigma(\Theta_l)$ is asked to make a total payment equal to[15]

$$\underline{T}_k(x_k) + \int_0^1 \rho_k(q_{x_l}(\mathbf{s}_k)|x_l;x_k)dx_l, \tag{5}$$

with $\rho_k\left(q_{x_l}\left(\underline{\mathbf{s}}_k(x_k)\right)|x_l;x_k\right) = 0$ all $x_l \in [0,1]$.

Under customized tariffs, the platforms thus offer to the side-k agent a menu of matching plans, one for each side-k location x_k. Each plan specifies (a) a baseline configuration, formally captured by the default set of types $\underline{\mathbf{s}}_k(x_k) \subseteq \Theta_l$ from side $l \neq k$ included in the package, (b) a baseline price $\underline{T}_k(x_k)$, (c) a collection of possible customizations $\mathbf{S}_k(x_k) \subseteq \Sigma(\Theta_l)$, and (d) a collection of non-linear schedules $\rho_k(q|x_l;x_k)$, one for each location $x_l \in [0,1]$, that jointly define the cost of customizing the plan. As in the case of uniform pricing, each non-linear schedule $\rho_k(q|x_l;x_k)$ specifies the price charged to the side-k agents for being matched to q agents from side $l \neq k$ located at x_l. Contrary to the case of uniform pricing, though, the price depends on the plan selected by the side-k agent,

[15] The payment specified by the tariff for any matching set $\mathbf{s}_k \notin \{\cup\mathbf{S}_k(x_k) : x_k \in [0,1]\}$ can be taken to be arbitrarily large to guarantee that no type finds it optimal to select any such set. The existence of such payments is guaranteed by the assumption that u_k is bounded, $k = a, b$. Furthermore, in case locations are private information on side k, the collection of matching plans is required to have the property that for any set $\mathbf{s}_k \in \mathbf{S}_k(x_k) \cap \mathbf{S}_k(x'_k)$, the total payment associated with $\mathbf{s}_k$ is the same no matter whether the set is obtained by selecting the plan x_k or the plan x'_k. When, instead, locations are public, the collection of matching plans $\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot;x_k), \mathbf{S}_k(x_k)) : x_k \in [0,1]\}$ may entail multiple prices for the same matching set $\mathbf{s}_k$. However, because, in this case, each agent can be constrained to choosing the plan designed for his location, de facto each agent faces a tariff specifying a single price for each set.

which is conveniently indexed by the side-k location, x_k. A menu of customized tariffs thus combines elements of *second-degree* price discrimination (each price function $\rho_k(q|x_l;x_k)$ is possibly non-linear in q) with elements of *third-degree* price discrimination (each non-linear price function $\rho_k(q|x_l;x_k)$ depends on the plan, and hence the location, of the side-k agents). That $\rho_k(q_{x_l}(\underline{\mathbf{s}}_k(x_k))|x_l;x_k)=0$, all $x_l \in [0,1]$, in turn means that an agent making no changes to a baseline plan is asked to make no further payments to the platform beyond $\underline{T}_k(x_k)$. Customized tariffs capture important features of real-world matching plans offered by platforms such as cable TV providers, ad Exchanges, and online retailers.

The platform's problem consists in choosing a pair of feasible matching tariffs T_k, $k=a,b$, that maximizes its profits, which are given by

$$\sum_{k=a,b}\int_{\Theta_k} T_k(\hat{\mathbf{s}}_k(\theta_k;T_k))dF_k(\theta_k). \tag{6}$$

Hereafter, we denote by T_k^* the profit-maximizing tariffs and by $\mathbf{s}_k^*$ the induced matching sets, $k=a,b$.

3 Customized Tariffs

We start by studying the platform's problem when no restrictions are imposed on the matching tariffs it can offer. Before describing the results, we introduce a few conditions that play an important role in the analysis below.

Condition 1. [R] Regularity: For any $k,l \in \{a,b\}$, $l \neq k$, $(\theta_k,\theta_l) \in \Theta_k \times \Theta_l$, the virtual values

$$\varphi_k(\theta_k,\theta_l) \equiv u_k(v_k,|x_k-x_l|) - \frac{1-F_k^{v|x}(v_k|x_k)}{f_k^{v|x}(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}(v_k,|x_k-x_l|)$$

are continuous, non-decreasing in v_k, and non-increasing in $|x_k-x_l|$.

Condition R imposes that "virtual values" $\varphi_k(\theta_k,\theta_l)$ have the same monotonicity properties of the true values, which is the natural analog of standard regularity conditions (see, e.g., Myerson (1981)) in matching environments. We assume that this condition holds throughout the entire analysis.

Condition 2. [I$_k$] Independence on side k: for any $\theta_k=(v_k,x_k)\in\Theta_k$, $F_k(\theta_k)=F_k^x(x_k)F_k^v(v_k)$.

Condition I$_k$ requires that the vertical parameters v_k be drawn independently from the locations x_k. In the media market application, this condition implies that knowing a viewer's "bliss point", i.e., his preferred type of content, carries no information about the overall importance the viewer assigns to media content.

Condition 3. [S$_k$] Symmetry on side k: for any $\theta_k=(v_k,x_k)\in\Theta_k$, $F_k(\theta_k)=x_kF_k^v(v_k)$.

Condition S_k strengthens the independence condition by further requiring that locations be uniformly distributed over $X_k = [0,1]$, as typically assumed in models of horizontal differentiation.[16] As we show below, this condition disciplines the agents' matching demands on side k, when locations are private information on side $l \neq k$.

We then have the following result:

Lemma 1. ***(properties of the optimum)*** *Suppose the environment satisfies the properties of one of the following four cases: Scenario (i); Scenario (ii) along with Conditions I_a and S_b; Scenario (iii) along with Conditions S_a and I_b; Scenario (iv) along with Conditions S_a and S_b. Then, under the profit-maximizing tariffs, for any $k \in \{a,b\}$,*

1. *the matching tariff T_k^* is customized;*
2. *the matching sets $\mathbf{s}_k^*$ exhibit negative assortativeness at the margin: there exist functions $t_k^* : \Theta_k \times [0,1] \to V_l$ such that*

$$\mathbf{s}_k^*(\theta_k) = \{(v_l, x_l) \in \Theta_l : \ v_l > t_k^*(\theta_k, x_l)\},$$

with the threshold function t_k^* non-increasing in v_k and non-decreasing in $|x_k - x_l|$. Finally, when locations are public on side $k \in \{a,b\}$, without loss of optimality, the side-k customized tariffs do not need to restrict the set of possible customizations, i.e., for each $x_k \in [0,1]$, $\mathbf{S}_k(x_k) = \Sigma(\Theta_l)$.

The conditions in Lemma 1 guarantee that the platform can price discriminate along the agents' locations, without leaving the agents rents for the private information the agents may possess regarding their locations. That is, in Scenarios (ii)-(iv), these conditions guarantee that the platform obtains the same profits as when locations are publicly observable, as in Scenario (i).

To gain some intuition, consider first Scenario (ii). Under Conditions I_a and S_b, the platform's pricing problem on side a is symmetric across any two locations. This is because of two reasons. First, the location of any agent from side a provides no information about the agent's vertical preferences (this is guaranteed by Condition I_a). Second, when the platform offers the same tariffs as in Scenario (i), the gross utility that each type $\theta_k = (v_a, x_a)$ obtains from the matching set $\mathbf{s}_a^*(\theta_k)$ coincides with the gross utility obtained by type $(v_a, x_a + d)$ from choosing the matching set $\mathbf{s}_a^*(v_a, x_a + d)$, $d \in [0, 1/2]$. This occurs because the matching set $\mathbf{s}_a^*(v_a, x_a + d)$ is a *parallel translation* of the matching set $\mathbf{s}_a^*(v_a, x_a)$ by d units of distance, along the horizontal dimension (this is guaranteed by Condition S_b). As a result, when, in Scenario (ii), the platform offers the same matching plans and tariffs as in Scenario (i), the matching sets demanded by any two agents with types (v_a, x_a) and $(v_a, x_a + d)$ are parallel translations of one another, and are priced identically. Note that, to guarantee that agents reveal their locations when the latter are the agents' private information, the platform

[16] Similar assumptions are typically made also in the targeting literature; see, for example, Bergemann and Bonatti (2011, 2015), and Cox et al. (2017).

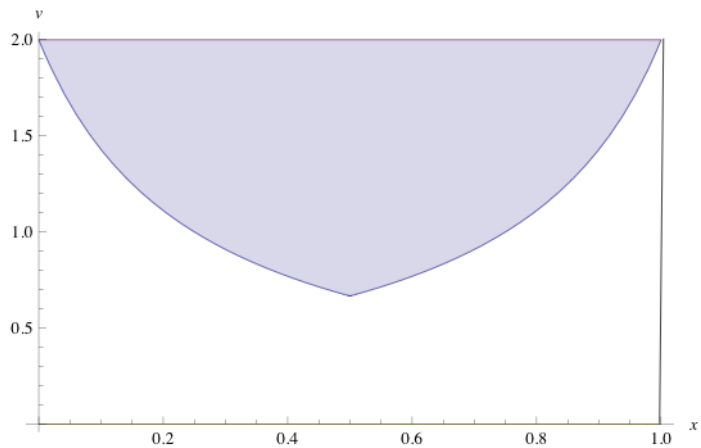

Figure 1: Matching sets under profit-maximizing tariffs. The shaded area in the figure describes the matching set for an agent from side a located at $x_a = 1/2$.

may need to restrict the set of possible customizations at each location, $\mathbf{S}_k(x_k)$, to coincide with the matching sets demanded under Scenario (i).[17] The above properties, together with a judicious restriction on the set of possible customizations $\mathbf{S}_k(x_k)$, imply that the matching demands and payments induced in Scenario (i) are implementable also under Scenario (ii). A symmetric situation applies to Scenario (iii).

Arguments similar to the ones above for Scenarios (ii) and (iii) imply that, in Scenario (iv), where locations are private information on both sides, when the platform offers the same matching plans and tariffs as in Scenario (i) to each side, agents continue to choose the same matching sets as in Scenario (i), provided that Condition S_k holds on both sides of the market.

Importantly, under the optimal tariffs of Lemma 1, for any given location x_k, the matching sets demanded by those agents with higher vertical types are supersets of those demanded by agents with lower vertical types. In this sense, the induced matching sets $\mathbf{s}_k^*$ exhibit *negative assortativeness at the margin*. Side-l agents located at x_l with a low vertical type v_l are included in the matching sets of the side-k agents located at x_k only if the latter's vertical types v_k are large enough.

To gain further intuition for the result in Lemma 1, consider Scenario (i), bearing in mind that the same conclusions apply to Scenarios (ii)-(iv) under the additional conditions in the lemma. Because locations are observable, the marginal profits the platform obtains by matching type $\theta_l = (v_l, x_l)$ from side l to type $\theta_k = (v_k, x_k)$ from side k are positive if, and only if,

$$\varphi_k(\theta_k, \theta_l) + \varphi_l(\theta_l, \theta_k) \geq 0. \tag{7}$$

Echoing Bulow and Roberts (1989), the above condition can be interpreted as stating that two agents are matched if, and only if, their *joint* marginal revenue to the platform is weakly positive

[17] In the absence of such restrictions, an agent of type $\theta_k = (v_k, x_k)$ misrepresenting his location to be $x_k' \neq x_k$ may find it optimal to select a matching set that no agent located at x_k' would have demanded under Scenario (i).

(we elaborate on this point further in the next subsection). Condition R guarantees that $\varphi_l(\theta_k, \theta_l)$ is non-decreasing in v_l. This implies existence of a threshold $t_k^*(\theta_k, x_l)$ such that Condition (7) is satisfied if, and only if, $v_l \geq t_k^*(\theta_k, x_l)$. Moreover, the threshold $t_k^*(\theta_k, x_l)$ is non-increasing in v_k and non-decreasing in the distance $|x_k - x_l|$. This means that, as v_k increases, the matching set of type θ_k expands to include new agents with lower vertical types. Moreover, as v_k increases, the vertical type of the marginal agents located at x_l added to the matching set $\mathbf{s}_k^*(v_k, x_k)$ are higher the "farther" away the location x_l is from x_k. Figure 1 illustrates the above properties by depicting the matching set of a representative agent from side a located at $x_a = .5$.

Remark 2. Lemma 1 describes symmetry conditions under which customized tariffs are offered by profit-maximizing platforms under imperfect information about the agents' locations. These conditions rule out the optimality of bunching and other complications (such as lotteries over tariffs and matching sets), while enabling us to highlight the role of price customization. These conditions can be relaxed by considering a model with a finite number of locations equidistant in the unit-circle. In a discrete setup, the agents' incentive to lie over the horizontal dimensions are diminished. We opted, here, for a continuum of locations for its greater analytical convenience, especially for pricing, as we show below.

3.1 Lerner-Wilson formula for matching schedules

We now derive further properties of the customized tariffs that maximize the platform's profits. To facilitate the exposition, hereafter, we assume that locations are public on both sides (that is, the environment satisfies the properties of Scenario (i)). As explained in the proof of Lemma 1, in this case, because each side-k agent located at x_k is constrained to choosing the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$, designed for him, there is no need for the platform to restrict the set of possible customizations $\mathbf{S}_k(x_k)$ each x_k-agent can choose from; that is, without loss of optimality, the platform can set $\mathbf{S}_k(x_k) = \Sigma(\Theta_l)$, for all $x_k \in [0, 1]$. To ease the notation, hereafter, we then drop the sets $\mathbf{S}_k(x_k)$ from the specification of the matching plans.

Next, consider the problem of a side-k agent of type $\theta_k = (v_k, x_k)$ under the plan

$$(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k)).$$

The mass of agents located at x_l demanded by type θ_k is given by

$$\hat{q}_{x_l}(\theta_k) \in \arg \max_{q \in [0, f_l^x(x_l)]} \left\{ u_k(v_k, |x_k - x_l|) \cdot q - \rho_k(q|x_l; x_k) \right\}.$$

Assuming the price schedule $\rho_k(\cdot|x_l; x_k)$ is convex and differentiable, with derivative $\rho_k'(\cdot|x_l; x_k)$, it follows that $\hat{q}_{x_l}(\theta_k)$ is a solution to the following first-order condition[18]

$$u_k(v_k, |x_k - x_l|) = \rho_k'(\hat{q}_{x_l}(\theta_k)|x_l; x_k) \tag{8}$$

[18]The strict convexity of the price function $\rho_k(\cdot|x_l; x_k)$ over the set of quantities purchased in equilibrium is a direct implication of the supermodularity of the agents' payoffs $u_k(v_k, |x_k - x_l|) \cdot q$ in (v_k, q).

whenever $\hat{q}_{x_l}(\theta_k) \in (0, f_l^x(x_l))$. For future reference, for any pair of locations $x_k, x_l \in [0,1]$, and any marginal price

$$\rho'_k \in [u_k(\underline{v}_k, |x_k - x_l|), u_k(\bar{v}_k, |x_k - x_l|)],$$

let $\hat{v}_{x_l}(\rho'_k|x_k)$ denote the value of v_k that solves the equation

$$u_k(v_k, |x_k - x_l|) = \rho'_k. \tag{9}$$

If, instead, $\rho'_k \notin [u_k(\underline{v}_k, |x_k - x_l|), u_k(\bar{v}_k, |x_k - x_l|)]$, we let $\hat{v}_{x_l}(\rho'_k|x_k) = \underline{v}_k$ for all $\rho'_k < u_k(\underline{v}_k, |x_k - x_l|)$, and $\hat{v}_{x_l}(\rho'_k|x_k) = \bar{v}_k$ for all $\rho'_k > u_k(\bar{v}_k, |x_k - x_l|)$.

Note that, because the price function $\rho_k(\cdot|x_l; x_k)$ is strictly convex over the range of quantities purchased in equilibrium, the marginal price ρ'_k uniquely identifies the quantity q. Furthermore, because agents with higher vertical types demand larger matching sets, the *demand* for the q-th unit of the x_l-agents by the x_k-agents, at the *marginal price* ρ'_k, is given by[19]

$$D_k\left(\rho'_k|x_l; x_k\right) \equiv \left[1 - F_k^{v|x}\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right)|x_k\right)\right] f_k^x(x_k), \tag{10}$$

where, as above, we dropped the arguments $(q|x_l; x)$ of the marginal price to lighten the notation. Accordingly, $D_k(\rho'_k|x_l; x_k)$ coincides with the mass of agents from side k located at x_k whose vertical type exceeds $\hat{v}_{x_l}(\rho'_k|x_k)$.

Using (10), we then define the elasticity of the demand by the side-k agents located at x_k (in short, the x_k-demand) for the q-th unit of the x_l-agents with respect to its marginal price ρ'_k by (once again, the arguments of the marginal price ρ'_k are dropped to ease the notation)

$$\varepsilon_k\left(\rho'_k|x_l; x_k\right) \equiv -\frac{\partial D_k\left(\rho'_k|x_l; x_k\right)}{\partial\left(\rho'_k\right)} \cdot \frac{\rho'_k}{D_k\left(\rho'_k|x_l; x_k\right)}. \tag{11}$$

Note that, when $\rho'_k \in [u_k(\underline{v}_k, |x_k - x_l|), u_k(\bar{v}_k, |x_k - x_l|)]$,

$$\varepsilon_k\left(\rho'_k|x_l; x_k\right) = \lambda_k^{v|x}\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right)|x_k\right) \cdot \left[\frac{\partial u_k}{\partial v}\left(\hat{v}_{x_l}\left(\rho'_k|x_k\right), |x_k - x_l|\right)\right]^{-1} \cdot \rho'_k, \tag{12}$$

where $\lambda_k^{v|x}(v_k|x_k)$ is the hazard-rate of the conditional distribution $F_k^{v|x}$. The next proposition characterizes the price schedules associated with the profit-maximizing customized tariffs of Lemma 1 in terms of the location-specific elasticities of the demands on both sides of the market.

Proposition 1. ***(Lerner-Wilson price schedules)*** *Suppose the environment satisfies the properties of Scenario (i). The price schedules $\rho_k^*(\cdot|x_l; x_k)$ associated with the profit-maximizing customized tariffs T_k^* are differentiable and convex over the equilibrium range $[q_{x_l}(\mathbf{s}_k(\underline{v}_k, x_k)), q_{x_l}(\mathbf{s}_k(\overline{v}_k, x_k))]$,*

[19] By the *demand* for the q-th unit of the x_l-agents by the x_k-agents we mean the mass of agents from side k located at x_k who demand at least q matches with the x_l-agents.

$k = a, b$. *Moreover,* ρ_a^* *and* ρ_b^* *jointly satisfy the following Lerner-Wilson formulas for all pairs* (x_a, x_b) *and* (q_a, q_b) *such that* $q_a = D_b\left(\rho_b^{*\prime}(q_b|x_a;x_b)|x_a;x_b\right)$ *and* $q_b = D_a\left(\rho_a^{*\prime}\left(q_a|x_b;x_a\right)|x_b;x_a\right)$*:*

$$\underbrace{\rho_a^{*\prime}(q_a|x_b;x_a)\left(1-\frac{1}{\varepsilon_a\left(\rho_a^{*\prime}(q_a|x_b;x_a)|x_b;x_a\right)}\right)}_{\textit{net effect on side-a profits}} \tag{13}$$

$$+ \quad \underbrace{\rho_b^{*\prime}(q_b|x_a;x_b)\left(1-\frac{1}{\varepsilon_b\left(\rho_b^{*\prime}(q_b|x_a;x_b)|x_a;x_b\right)}\right)}_{\textit{net effect on side-b profits}} \quad = \quad 0.$$

The Lerner-Wilson formulas (13) jointly determine the price schedules on both sides of the market. Intuitively, these formulas require that the marginal contribution to profits from adding to the matching sets of the x_k-agents the q_k-th unit of the x_l-agents coincide with the marginal contribution to profits from adding to the matching sets of the x_l-agents the q_l-th unit of the x_k-agents, where q_k and q_l are jointly related by the reciprocity condition in the Proposition (that is, $q_a = D_b\left(\rho_b^{*\prime}(q_b|x_a;x_b)|x_a;x_b\right)$ and $q_b = D_a\left(\rho_a^{*\prime}\left(q_a|x_b;x_a\right)|x_b;x_a\right)$. As for the standard Lerner-Wilson formula for monopoly/monopsony pricing, on each side, the marginal contribution to profits of such an adjustment has two terms: the term $\rho_k^{*\prime}(q_k|x_l,x_k)$ captures the marginal benefit from adding the extra agents, whereas the semi-inverse-elasticity term $\rho_k^{*\prime}(q_k|x_l,x_k)\left[\varepsilon_k\left(\rho_k^{*\prime}(q_k|x_l;x_k)|x_l;x_k\right)\right]^{-1}$ capture its associated infra-marginal losses.

Importantly, as anticipated above, the quantities q_k and q_l at which the conditional price schedules are evaluated have to clear the market, as required by the reciprocity condition (3). The result in the proposition uses the fact that the demands under the optimal tariffs satisfy the threshold structure in Lemma 1 to establish that the mass of x_k-agents that, at the marginal price $\rho_k^{*\prime}(q_k|x_l,;x)$, demand q_k agents or more of type x_l coincide with the mass $D_k\left(\rho_k^{*\prime}\left(q_k|x_l;x_k\right)|x_l;x_k\right)$ of x_k-agents with vertical type above $\hat{v}_{x_l}\left(\rho_k^{*\prime}|x_k\right)$, where recall that $\hat{v}_{x_l}\left(\rho_k^{*\prime}|x_k\right)$ is the vertical type at which the utility of the x_k agents for the x_l matches equals the marginal price ρ_k^*, as defined in (9). Together with reciprocity, Lemma 1 then also implies that the mass q_k of x_l-agents that, at the marginal price $\rho_l^{*\prime}(q_l|x_k,x_l)$, demand $q_l = D_k\left(\rho_k^{*\prime}\left(q_k|x_l;x_k\right)|x_l;x_k\right)$ or more of the x_k-agents coincides with the mass of x_l-agents with vertical type above $\hat{v}_{x_k}\left(\rho_l^{*\prime}|x_l\right)$.

Finally, that the price schedules $\rho_k^*(q_k|x_l;x_k)$ are convex in q_k reflects the fact that the matching demands of the x_k-agents for the x_l-agents are increasing in the vertical types v_k. As a result, the marginal price $\rho_k^{*\prime}(q_k|x_l,x_k)$ for the q_k-unit of the x_l-agents has to increase with q_k.

The formulas in (13) also reveal how profit-maximizing platforms optimally cross-subsidize interactions among agents from multiple sides of the market while accounting for heterogeneity in preferences along both vertical and horizontal dimensions. In particular, the price schedules offered at any two locations x_k and x_l are a function of the *location-specific demand elasticities* $\varepsilon_k\left(\cdot|x_l;x_k\right)$ at these locations. This reflects the fact that, at the optimum, platforms make use of information about horizontal preferences to offer matching tariffs that extract as much surplus as possible from

agents from both sides. As we show below, the ability to tailor price schedules to locations (a form of third-degree price discrimination) has important implications for the composition of the demands prevailing under optimal tariffs.

A platform with knowledge of the distribution of preferences in the market can use Proposition 1 to construct the profit-maximizing price schedules, or, alternatively, identify the thresholds that describe the optimal matching demands (as in Lemma 1). Conversely, Proposition 1 can also be used in empirical work, as it provides a system of structural equations that allows the econometrician to recover the distribution of the agents' preferences from price schedules and match volumes. The work by Kahn and You (2016) follows a related approach in the matching market for lobbying, but abstracting from horizontal differentiation. Proposition 1 might help in extending their empirical analysis to markets where horizontal differentiation is expected to play an important role.

3.2 Distortions and Horizontal Differentiation

We now investigate how distortions in the supply of matching opportunities (relative to efficiency) due to market power vary with the agents' horizontal preferences. In particular, we are interested in whether distortions increase or decrease as one considers locations farther away from an agent's bliss point. The analysis in this section, which, to the best of our knowledge, has no parallel in the price discrimination literature, has important implications for how policy makers may consider regulating mediated matching markets.

Let

$$\sum_{k=a,b} \int_{\Theta_k} \int_{\hat{\mathbf{s}}_k(\theta_k;T_k)} u_k\left(v_k, |x_k - x_l|\right) dF_j(\theta_j) dF_k(\theta_k), \tag{14}$$

denote the welfare associated with a feasible pair of matching tariffs T_k, $k = a, b$. It is straightforward to see that a pair of tariffs T_k^e, $k = a, b$, maximizes social welfare if, and only if, the induced matching demands satisfy the following property: For any two agents with types $\theta_k = (v_k, x_k)$ and $\theta_l = (v_l, x_l)$,

$$\theta_l \in \hat{\mathbf{s}}_k(\theta_k; T_k^e) \quad \Longleftrightarrow \quad u_k\left(v_k, |x_k - x_l|\right) + u_l\left(v_l, |x_k - x_l|\right) \geq 0.$$

In turn, this means that, given any pair of welfare-maximizing tariffs T_k^e, $k = a, b$, there must exist threshold functions $t_k^e\left(\theta_k, x_l\right)$ such that $\theta_l \in \hat{\mathbf{s}}_k(\theta_k; T_k^e)$ if, and only if, $v_l \geq t_k^e\left(\theta_k, x_l\right)$. Arguments similar to those establishing Lemma 1 and Proposition 1 then imply that the welfare-maximizing tariffs T_a^e and T_b^e are customized, and their associated marginal price schedules $(\rho_k^e)'$, jointly solve

$$\rho_a^{e\prime}(q_a|x_b; x_a) + \rho_b^{e\prime}(q_b|x_a; x_b) = 0,$$

at any pair q_a and q_b such that $q_a = D_b\left(\rho_b^{e\prime}(q_b|x_a; x_b)|x_a; x_b\right)$ and $q_b = D_a\left(\rho_a^{e\prime}\left(q_a|x_b; x_a\right)|x_b; x_a\right)$.

The following example illustrates the differences between the matching sets sustained under welfare maximization and their counterparts under profit maximization, when preferences are as in Example 1.

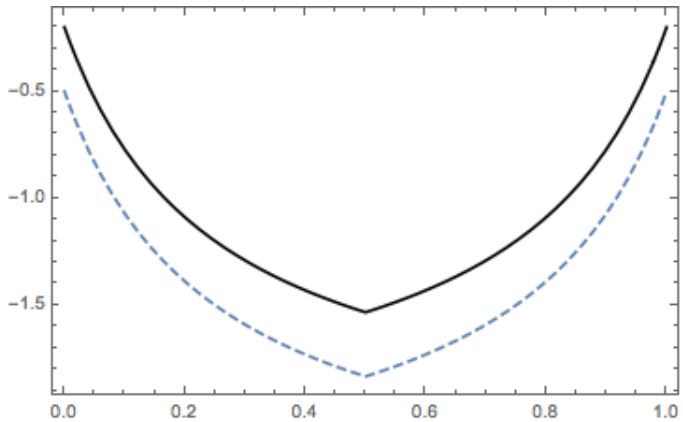

Figure 2: The welfare-maximizing demand threshold $t_a^e(\theta_a, x_b)$ (dashed blue curve) and the profit-maximizing demand threshold $t_a^*(\theta_a, x_b)$ (solid black curve) for agents on side a located at $x_a = .5$ under the preferences of Example 1, when $\psi(|x_a - x_b|) = \exp\{|x_a - x_b|\}$, and vertical dimensions v_b are drawn from a shifted exponential distribution.

Example 4. (***online shopping* - continued**) Consider the market described in Example 1, and assume that the environment satisfies the properties of Scenario (i), and Condition I_k holds, for $k = a, b$. Consider a consumer with preferences $\theta_a = (v_a, x_a)$ and a product of variety x_b for which the profit- and the welfare-maximizing allocations are interior (that is $t_a^*(\theta_a, x_b), t_a^e(\theta_a, x_b) \in \mathrm{Int}[V_b]$). Then, the distortions due to market power are captured by the discrepancy

$$t_a^*(\theta_a, x_b) - t_a^e(\theta_a, x_b) = \frac{1}{\lambda_b^v(t_a^*(\theta_a, x_b))} + \frac{1}{\lambda_a^v(v_a)}.$$

Such discrepancy is invariant in the distance $|x_a - x_b|$ if v_b is drawn from a shifted exponential distribution, as illustrated in Figure 2.[20] By contrast, distortions are decreasing (resp., increasing) with distance if the hazard rate $\lambda_b(v_b)$ is increasing (resp., decreasing) in v_b. $\diamondsuit$

The above example is special, in that the match utility of the side-a agents is modular (i.e., the cross-derivative of u_a in v_a and $d = |x_a - x_b|$ is zero), and the match utility of the side-b agents is modular and linear in v_b. To obtain more general conclusions, we first need to introduce a definition:

Definition 4. (distortions and distance) Distortions on side $k \in \{a, b\}$ decrease (alternatively, increase) with distance if, and only if, for any $\theta_k \in \Theta_k$,

$$u_l(t_k^*(\theta_k, x_l), |x_l - x_k|) - u_l(t_k^e(\theta_k, x_l), |x_l - x_k|)$$

decreases (alternatively, increases) with $|x_k - x_l|$.

Hence, fixing the type $\theta_k = (v_k, x_k)$ of a side-k agent, distortions increase with distance when the difference between the minimal utility asked by a profit-maximizing platform and a welfare-maximizing platform to each x_l-agent to be matched with type θ_k increases with the distance between

[20] That is, $F_b^v(v_b) = 1 - exp\left\{\tilde{\lambda}_b(v_b - K)\}\right\}$ for $K < 0$ and $\tilde{\lambda}_b < 0$, with $V_b = [K, +\infty)$.

the two agents' locations. Note that the difference in utilities, as in Definition 4, reduces to the difference in the thresholds, as in Example 4, when the side-l's preferences are invariant to locations.

Proposition 2. ***(distortions under customized pricing)*** *Suppose the environment satisfies the properties of Scenario (i), and Conditions I_k hold, $k = a, b$.*[21] *Consider any $(\theta_k, x_l) \in \Theta_k \times [0, 1]$ for which $t_k^*(\theta_k, x_l), t_k^e(\theta_k, x_l) \in Int[V_l]$.*[22] *The following statements are true for $k, l = a, b$, $l \neq k$:*

1. *If u_k is submodular, u_l is submodular and concave in v_l, and F_l^v has an increasing hazard rate, distortions on side k decrease with distance;*

2. *If u_k is supermodular, u_l is supermodular and convex in v_l, and F_l^v has a decreasing hazard rate, distortions on side k increase with distance.*[23]

The next two examples specialize the conclusions of Proposition 2 to advertising and media markets. To avoid repetitions, we assume in these examples that the environment satisfies the properties of Scenario (i) and that Conditions I_k hold, $k = a, b$. As explained above, the conclusions in all these examples extend to environments in which locations are private on one, or both, sides of the market, provided that condition I_k is replaced with condition S_k on the side of the market opposite to the one where locations are private information, as per Lemma 1.

Example 5. (***ad exchange* - continued**) Consider the advertising market of Example 2. Note that the agents' preferences under this specification are consistent with the conditions of Part 1 in Proposition 2 (u_a is strictly submodular, whereas u_b is linear in v_b and invariant in $|x_a - x_b|$). Therefore, when F_b^v has an increasing hazard rate, under profit maximization, advertisers (on side a) are more often matched (relative to efficiency) to those publishers whose profile is more distant from their ideal audience. Figure 3 illustrates this situation. $\diamondsuit$

Example 6. (***media platform* - continued**) Consider the media market of Example 3. When the elasticity of substitution $\delta \leq 1$, the viewers' preferences on side-a are submodular. In this case, distortions decrease with distance provided F_b^v has an increasing hazard rate. This means that, under profit maximization, the media consumption of viewers is distorted by excluding primarily those types of content that viewers like the most. Conversely, when $\delta \geq 1$, the viewers' preferences

[21] The conclusions in the proposition continue to hold if the environment satisfies the properties of Scenarios (ii), (iii), or (iv), and Condition I_k is replaced by S_k on the side of the market opposite to the one in which locations are private information, as per Lemma 1.

[22] The results for the case in which either $t_k^*(\theta_k, x_l)$, or $t_k^e(\theta_k, x_l)$, coincide with the extreme points of V_l are not particularly interesting. When $t_k^e(\theta_k, x_l) = \bar{v}_l$, $t_k^*(\theta_k, x_l) = \bar{v}_l$. In this case, distortions are invariant in the distance. If, instead, $t_k^e(\theta_k, x_l) = \underline{v}_l$, then distortions are weakly increasing in the distance. Finally, if $t_k^*(\theta_k, x_l) = \bar{v}_l$ then clearly distortions are weakly decreasing in distance. In these cases, the monotonicity of the difference in the utilities does not depend on the hazard rate of the distributions, nor on the modularity of the match values.

[23] Strictly so, if at least one of the conditions is strict (in Parts 1 and 2).

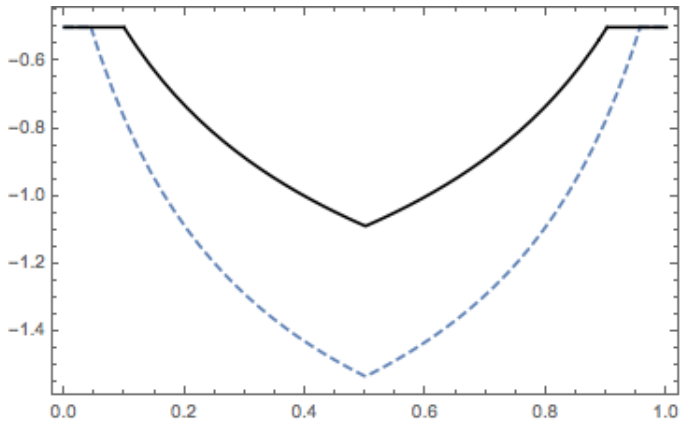

Figure 3: The welfare-maximizing demand threshold $t_a^e(\theta_a, x_b)$ (dashed blue curve) and the profit-maximizing demand threshold $t_a^*(\theta_a, x_b)$ (solid black curve) for agents on side a located at $x_a = .5$ under the preferences specification of Example 2.

are supermodular, and distortions increase with distance provided F_b^v has a decreasing hazard rate.[24] ♢

Proposition 2 identifies natural conditions on primitives (match utilities and distributions) that lead to location-specific elasticities that increase or decrease with distance. To understand the result, consider Part 1 first. When u_a is submodular, the effect of vertical differentiation on match utility declines with the distance in locations. This implies that the location-specific semi-elasticity $\varepsilon_a(\rho_a'|x_b; x_a)/\rho_a'$ of the side-a demand, evaluated along the matching demand of type θ_a, is higher at locations x_b that are farther from x_a (as implied by equation (12)). Holding ρ_b' fixed, this property of elasticities contributes to marginal prices on side a being lower (relative to their efficient counterparts) at locations x_b that are farther from x_a, as implied by the Lerner-Wilson formulas (13). Likewise, when F_b^v has an increasing hazard rate and u_b is submodular and concave in v_l, the side-b location-specific semi-elasticity $\varepsilon_b(\rho_b'|x_a; x_b)/\rho_b'$, evaluated along the matching demand of type θ_a, is also higher at locations x_b that are farther from x_a (as implied by equation (12)). Holding ρ_a' fixed, this property of elasticities thus contributes to marginal prices on side b being lower (relative to their efficient counterparts) at locations x_b that are farther from x_a (as implied by the Lerner-Wilson formulas (13)). Because matchings is reciprocal, the side-a marginal prices then also decrease (relative to their efficient counterparts) as distances increase.

The two effects described above point in the same direction: Namely, the discrepancy between the side-a profit-maximizing marginal prices and their efficient counterparts are relatively lower at locations x_b that are farther from x_a. When this is the case, distortions decrease with distance.

[24] To see how the results follow from parts 1 and 2 in Proposition 2, recall that, in this example, the preferences of the side-b agents are linear in v_b and invariant in $|x_a - x_b|$; hence they are both weakly submodular and concave in v_b, and weakly supermodular and convex in v_b.

Mutatis mutandis, a similar explanation applies to the environment considered in Part 2 in the proposition.

4 Uniform Pricing

Stringent regulations on the transfer of personal data, as well as a renewed trend towards market decentralization, are expected to hinder the customization of prices and favor instead uniform pricing. In this section, we study platforms' behavior under uniform pricing. After doing so, we discuss how our results can be adapted to study the effects of market decentralization, whereby matches are unmediated and prices are *de facto* uniform on a given side of the market.

Uniform Pricing and Aggregate Demand Elasticities

Suppose the platform is forced to adopt a uniform price schedule $p_a(\cdot|x_b)$ on side a (with marginal schedule $p'_a(\cdot|x_b)$). Recall that, for each location $x_b \in [0,1]$, and each quantity $q \in [0, f_b^x(x_b)]$, such a schedule specifies the price that the side-a agents have to pay to be matched to q agents from side b located at x_b. Under such schedule, the aggregate demand (over all locations x_a) for the q-th unit of the x_b-agents at the marginal price $p'_a(q|x_b)$ is equal to

$$\bar{D}_a\left(p'_a|x_b\right) \equiv \int_0^1 D_a\left(p'_a|x_b; x_a\right) dx_a = \int_0^1 \left\{1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p'_a|x_a\right)|x_a\right)\right\} f_a^x(x_a) dx_a,$$

where, as in the previous section, $D_a\left(p'_a|x_b; x_a\right)$ denotes the mass of agents located at x_a that demand q units or more of the x_b-agents, and where, as in the previous section, the arguments $(q|x_b)$ of the marginal prices $p'_a(q|x_b)$ have been dropped, to ease the exposition.

The elasticity of the aggregate demand for the q-th unit of the x_b-agents with respect to its marginal price $p'_a(q|x_b)$ is then equal to

$$\bar{\varepsilon}_a\left(p'_a|x_b\right) \equiv -\frac{\partial \bar{D}_a\left(p'_a|x_b\right)}{\partial\left(p'_a\right)} \cdot \frac{p'_a}{\bar{D}_a\left(p'_a|x_b\right)} = \mathbb{E}_{\bar{H}(\tilde{x}_a|x_b, p'_a)}\left[\varepsilon_a\left(p'_a|x_b; \tilde{x}_a\right)\right],$$

where $\varepsilon_a\left(p'_a|x_b; x_a\right)$ is as defined in (11), and where the expectation is over $X_a = [0,1]$, under the distribution $\bar{H}\left(\cdot|x_b, p'_a\right)$ whose density is equal to

$$\bar{h}\left(x_a|x_b, p'_a\right) \equiv \frac{D_a\left(p'_a|x_b; x_a\right)}{\int_0^1 D_a\left(p'_a|x_b; x'_a\right) dx'_a}.$$

Hereafter, we refer to $\bar{\varepsilon}_a(\cdot|x_b)$ as to the *aggregate elasticity* of the side-a demand for the x_b-matches. This elasticity measures the percentage variation in the mass of agents from side a that demand at least q matches with the side-b agents located at x_b in response to a percentage change in the marginal price for the q-th unit of the x_b-agents. It is also equal to the average (over the side-a locations) elasticity of the x_a-demands for the q-th unit of the x_b-agents with respect to the marginal price p'_a, where the average is under a distribution that assigns to each location x_a a weight

proportional to the mass of agents $D_a\left(p_a'|x_b,x_a\right)$ located at x_a demanding q units, or more, of the x_b-agents.

The next proposition derives the optimal tariffs employed by a platform that is constrained to not engage in third-degree price discrimination on side a (equivalently, to price uniformly on side a). Consistently with the previous two propositions, we ease the exposition by assuming that locations are public information on both sides (that is, the environment satisfies the properties of Scenario (i)), with the understanding that the results extend to the other scenarios under the extra assumptions of Lemma 1. We denote by T_a^u and T_b^u the profit-maximizing tariffs on sides a and b, respectively, when the platform is constrained to price uniformly on side a.

Proposition 3. ***(uniform pricing)*** *Suppose the environment satisfies the properties of Scenario (i) and that the platform is constrained to price uniformly on side a (but is free to offer a customized tariff on side b). The profit-maximizing price schedules $p_a^u(\cdot|x_b)$ and $\rho_b^u(\cdot|x_a;x_b)$ are differentiable and convex over the equilibrium ranges,*[25] *and jointly satisfy the following optimality conditions for all $x_b\in[0,1]$, all $q\leq f_b^x(x_b)$,*

$$\underbrace{p_a^{u\prime}(q|x_b)\left(1-\frac{1}{\bar{\varepsilon}_a\left(p_a^{u\prime}(q|x_b)\right)}\right)}_{\textit{net effect on side-a profits}}$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p_a')}\left[\underbrace{\rho_b^{u\prime}(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)\left(1-\frac{1}{\varepsilon_b\left(\rho_b^{u\prime}(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)|\tilde{x}_a;x_b\right)}\right)}_{\textit{net effect on side-b profits}}\right]=0, \tag{15}$$

where $H\left(x_a|x_b,p_a'\right)$ is the distribution over $X_a=[0,1]$ whose density is given by

$$h\left(x_a|x_b,p_a'\right)\equiv\frac{\partial D_a\left(p_a'|x_b;x_a\right)}{\partial\left(p_a'\right)}\left(\frac{\partial\bar{D}_a\left(p_a'|x_b\right)}{\partial\left(p_a'\right)}\right)^{-1},$$

and where $\hat{q}_b(q;x_a;x_b)\equiv D_a\left(p_a^{u\prime}(q|x_b)|x_b;x_a\right)$ is the mass of side-a agents located at x_a that demand q or more matches with the side-b agents located at x_b when the (uniform) marginal price for the q-th unit of the x_b-agents is equal to $p_a^{u\prime}(q|x_b)$.

The result in the proposition provides structural equations similar to those corresponding to the Lerner-Wilson formulas in (13), but adapted to account for the imposition of uniform pricing on side a. Such structural conditions jointly determine the price schedules on both sides of the market. Under uniform pricing, the price schedule on side a for the sale of the side-b matches cannot condition on the location of the side-a agents. As a result, the markup for the sale of the q-th unit of the x_b-matches is constant across all side-a locations x_a. The relevant elasticity for determining this markup is then the aggregate elasticity $\bar{\varepsilon}_a(\cdot|x_b)$, rather than the location-specific elasticities $\varepsilon_a\left(\cdot|x_b;x_a\right)$ in

[25] Namely, at any $q_a\in[q_{x_b}(\mathbf{s}_a(\underline{v}_a,x_b+.5)),q_{x_b}(\mathbf{s}_a(\overline{v}_a,x_b))]$ and $q_b\in[q_{x_a}(\mathbf{s}_b(\underline{v}_b,x_b)),q_{x_a}(\mathbf{s}_a(\overline{v}_b,x_b))]$.

the Lerner-Wilson formula (13). Interestingly, even if the platform can price discriminate on side b by offering a customized tariff to the side-b agents, when it is constrained to price uniformly on side a, the cost of procuring the side-b agents is the average (mark-up augmented) price

$$\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)}\left[\underbrace{\rho_b^{u\prime}(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)\left(1-\frac{1}{\varepsilon_b\left(\rho_b^{u\prime}(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)|\tilde{x}_a;x_b\right)}\right)}_{\text{net effect on side-}b\text{ profits}}\right]$$

charged to the x_b-agents for their interactions with the various x_a-agents demanding q, or more, x_b-matches.

Also note that, by virtue of the reciprocity condition (3), the quantities q_a and q_b at which the conditional price schedules are evaluated have to clear the market for any pair of locations (x_a, x_b). For this to be possible, it is important that the platform be able to employ a customized tariff on side b, as the latter ensures that the platform has enough price instruments to procure the side-b matches demanded by the side-a agents, while respecting reciprocity.

Finally, as in the case where price customization is allowed on both sides, the convexity of the price schedules $p_a^u(\cdot|x_b)$ and $\rho_b^u(\cdot|x_a;x_b)$ in q reflects the fact that the matching demands of those agents with a higher vertical type are supersets of those with a lower vertical type.

As revealed by the pricing formulas (13) and (15), the effects of the imposition of uniform pricing on side a on the composition of the matching sets on both sides hinge on the comparison between the aggregate inverse-elasticity $1/\bar{\varepsilon}_a$ and the location-specific inverse-elasticities $1/\varepsilon_a(\cdot|x_b,x_a)$ on side a, as well as the comparison between the average inverse-elasticity $\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)}\left[1/\varepsilon_b(\cdot|\tilde{x}_a;x_b)|\tilde{x}_a;x_b)\right]$ of the x_b-demands for the various x_a-matches and the inverse-elasticities $1/\varepsilon_b(\cdot|\tilde{x}_a;x_b)|\tilde{x}_a;x_b)$ of the same demands for the specific matches. In turn, such comparisons naturally reflect how the average virtual valuations on both sides compare to their location-specific counterparts. To see this, first note that

$$\frac{1}{\bar{\varepsilon}_a\left(p_a^{u\prime}(q|x_b)\right)}=\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)}\left[\frac{1}{\varepsilon_a\left(p_a^{u\prime}(q|x_b)|x_b;\tilde{x}_a)\right)}\right]. \tag{16}$$

That is, the inverse aggregate elasticity of the side-a demand for the q-th unit of the x_b-matches is equal to the average of the various location-specific inverse elasticities of the side-a agents for the same unit of the same x_b-matches, where the average is under the same measure $H(x_a|x_b,p'_a)$ introduced in the proposition. Let $\check{v}_b(q;x_b)$ be implicitly defined by

$$\left(1-F_b^{v|x}(\check{v}_b(q;x_b)|x_b)\right)f_b^x(x_b)=q.$$

Note that $\check{v}_b(q;x_b)$ denotes the value of the vertical dimension of the x_b-agents such that the mass of x_b-agents with a vertical type higher than $\check{v}_b(q;x_b)$ is equal to q. Using the characterization in

Lemma 1, we then have that the optimality condition (15) can be re-written as

$$\underbrace{\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)}\left[\varphi_a\left(\left(\hat{v}_{x_b}\left(p'_a|\tilde{x}_a\right),\tilde{x}_a\right),(\check{v}_b(q;x_b),x_b)\right)\right]}_{\text{net effect on side-}a\text{ profits}}+$$
$$\underbrace{\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a)}\left[\varphi_b\left(\left(\check{v}_b(q;x_b),x_b\right),\left(\hat{v}_{x_b}\left(p'_a|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]}_{\text{net effect on side-}B\text{ profits}}=0. \tag{17}$$

Hereafter, we assume that the left-hand side of (17) is monotone in the marginal price p'_a, which amounts to quasi-concavity of the platform's profit function with respect to the marginal price, after accounting for the cost of procuring the x_b-agents, as explained in the proof of Proposition 3. The above property implies that the necessary condition in (17) is also sufficient for optimality.

Now recall that, under price customization (on both sides), the platform matches a pair of agents $\theta_a=(v_a,x_a)$ and $\theta_b=(v_b,x_b)$ if, and only if, type-θ_a's virtual value for interacting with type θ_b is large enough to compensate for the virtual value that type θ_b derives from interacting with type θ_a (formally, θ_a and θ_b are matched if, and only if, $\varphi_a(\theta_a,\theta_b)+\varphi_b(\theta_b,\theta_a)\geq 0$). Under uniform pricing (on side a), instead, the platform matches the above pair of agents if, and only if, the following is true: if all side-a agents with the same *true value* for interacting with type θ_b as type θ_a were to be matched to type θ_b, the *average virtual value* among such agents for the match with type θ_b would compensate for the *average virtual value* that type θ_b derives from being matched with all such agents. Formally, under uniform pricing (on side a), types θ_a and θ_b are matched if, and only if,

$$\mathbb{E}_{H(\tilde{x}_a|x_b,u_a(v_a,|x_b-x_a|))}\left[\varphi_a\left(\left(\hat{v}_{x_b}\left(u_a(v_a,|x_b-x_a|)|\tilde{x}_a\right),\tilde{x}_a\right),\theta_b\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,u_a(v_a,|x_b-x_a|))}\left[\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(u_a(v_a,|x_b-x_a|)|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]\geq 0.$$

This observation plays an important role in determining how targeting and welfare are affected by the imposition of uniform pricing, as we show below.

4.1 Targeting under Uniform and Customized Pricing

Digital technology is often praised for its ability to increase match precision (or targeting) in a variety of markets. Yet, technology alone is no guarantee of large targeting gains, as the matching demands enjoyed by agents obviously depend on the pricing practices followed by platforms. Price customization allows a platform to charge agents prices that directly depend on their horizontal preferences. To the extent that agents value the most those matches of higher proximity, one might expect price-customization to hinder targeting, as it permits platforms to set higher prices for those matches the agents like the most. Without further inquiry, this observation seems to lend support to policies that impose uniform pricing. Indeed, recent proposals, requiring stringent protection of consumer privacy (de facto banning price customization), stand-alone pricing for media content, or anonymous pricing for advertising slots, appear to follow this line of reasoning. This intuition is,

however, incomplete, as it ignores the (endogenous) changes in prices that the platform undertakes in response to uniform pricing. The analysis below provides some guidelines when it comes to the comparisons of uniform and customized pricing on targeting.

Definition 5. (targeting) Customized pricing (on both sides) leads to more targeting than uniform pricing (on side a) if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} < 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ > 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

Conversely, uniform pricing on side a leads to more targeting than customized pricing on both sides if, for each $\theta_a = (v_a, x_a)$, there exists $d(\theta_a) \in (0, \frac{1}{2})$ such that

$$t_a^*(\theta_a, x_b) - t_a^u(\theta_a, x_b) \begin{cases} > 0 & \text{if} \quad |x_a - x_b| < d(\theta_a) \\ < 0 & \text{if} \quad |x_a - x_b| > d(\theta_a). \end{cases}$$

Intuitively, customized pricing (on both sides) leads to more targeting than uniform pricing (on side a) if, under the profit-maximizing customized tariffs, agents demand more matches close to their ideal points, and less matches far from their ideal points, relative to what they do under uniform pricing. Accordingly, the threshold function $t_a^*(\theta_a, x_b)$ under customized pricing is below the corresponding threshold function $t_a^u(\theta_a, x_b)$ for nearby matches (i.e., for locations x_b such that $|x_a - x_b| < d(\theta_a)$), and above $t_a^u(\theta_a, x_b)$ for more distant matches (for which $|x_a - x_b| > d(\theta_a)$). Figure 4 illustrates the situation captured by the above definition.

Note that, because matching is reciprocal, the above definition has an analogous implication for side b. Namely, when customized pricing (on both sides) leads to more targeting than uniform pricing (on side a), then the side-b threshold function under customized pricing, $t_b^*(\theta_b, x_a)$, also single-crosses its counterpart under uniform pricing, $t_b^u(\theta_b, x_a)$, only once, and from below, as a function of the distance $|x_a - x_b|$.

The next proposition identifies primitive conditions under which uniform pricing on side a (for short, uniform pricing) leads to more targeting that customized pricing on both sides (for short, customized pricing).

Proposition 4. ***(comparison: targeting)*** *Suppose the environment satisfies the properties of Scenario (i), Condition I_a holds, and preferences on side b are location-invariant, that is, for all $x_a, x_b \in [0, 1]$, all $v_b \in V_b$, $u_b(v_b, |x_b - x_a|) = v_b$. Then the following statements are true.*

1. If the match value function u_a is submodular and concave in v_a, and F_a^v has an increasing hazard rate, then uniform pricing leads to more targeting than customized pricing.

2. If the match value function u_a is supermodular and convex in v_a, and F_a^v has a decreasing hazard rate, then customized pricing leads to more targeting than uniform pricing.

Proposition 4 helps understanding the effects of price customization on targeting in a variety of markets. In the examples below, we assume the environment satisfies the properties of Scenario (i) and Condition I_a holds.

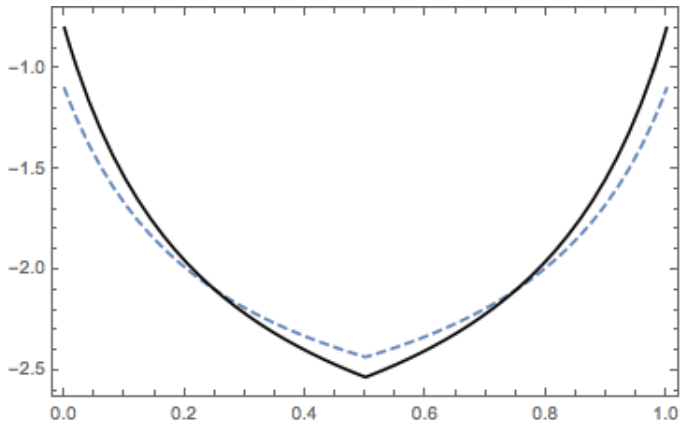

Figure 4: The threshold function $t_a^*(\theta_a, x_b)$ under customized pricing (solid black curve) and uniform pricing $t_a^u(\theta_a, x_b)$ (dashed blue curve) when customized pricing (on both sides) leads to more targeting than uniform pricing (on side a).

Example 7. (***online shopping*** **- continued**) Consider the online shopping market of Example 1. Price customization leads to more (respectively, less) targeting than uniform pricing when F_a^v has a decreasing (respectively, increasing) hazard rate. Therefore, the targeting consequences of stringent consumer privacy regulations (de facto banning price customization) depend on the distribution of consumers' vertical tastes, but not on the intensity of niche preferences determining the disutility from the mismatches (formally captured by the ψ function). The same conclusions remain valid if the environment satisfies the properties of Scenario (ii) and Condition S_b also holds. $\diamondsuit$

Example 8. (***ad exchange*** **- continued**) Consider the ad exchange market of Example 2. Price customization leads to less targeting than uniform pricing (as the match function u_a is submodular and linear in v_a) when F_a^v has an increasing hazard rate (e.g., a uniform or exponential cdf). Under this condition, anonymous pricing for advertising slots (e.g., as a result of regulation) makes advertisers be more often matched (relative to laissez-faire) to those publishers whose profile is closer to their ideal audience. $\diamondsuit$

Example 9. (***media platform*** **- continued**) Consider the media market of Example 3. Price customization leads to more targeting than uniform pricing when the viewers' elasticity of substitution is high (namely, when $\delta > 1$) and F_a^v has a decreasing hazard rate (e.g., a Pareto cdf). By contrast, price customization leads to less targeting than uniform pricing when the viewers' elasticity of substitution is low (namely, when $\delta < 1$) and F_a^v has an increasing hazard rate (e.g., a uniform cdf).[26] The effects on targeting of regulation requiring stand-alone pricing for media content therefore *jointly* depend on the viewers' elasticity of substitution and the distribution of their vertical preferences.

[26] When the hazard rate is constant (that is, F_a^v is a shifted exponential distribution), the effect of uniform (as opposed to customized) pricing on targeting solely depends on the viewers' elasticity of substitution.

The same conclusions remain valid if the environment satisfies the properties of Scenario (ii) and Condition S_b also holds. $\diamondsuit$

The primitive conditions in Proposition 4 have implications for how location-specific elasticities on both sides of the market compare to the average elasticities. To obtain intuition, consider first Part 1 and fix the side-b location x_b. Let ρ_a' be a short-cut for the marginal price $\rho_a'(q|x_b;x_a)$ for the q-th unit of the x_b-agents charged to the side-a agents located at x_a under customized pricing. To make things interesting, assume $\rho_a' \in (u_a(\underline{v}_a, |x_b - x_a|), u_a(\bar{v}_a, |x_b - x_a|))$. We then have that, under customized pricing, the semi-price elasticity[27]

$$\varepsilon_a\left(\rho_a'|x_b;x_a\right)(\hat{\rho})^{-1} = \lambda_a^v\left(\hat{v}_{x_b}\left(\rho_a'|x_a\right)\right)\cdot\left[\frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(\rho_a'|x_a\right), |x_b - x_a|\right)\right]^{-1}$$

of the demand for the q-th unit of the x_b-agents by the side-a agents is increasing in the distance $|x_b - x_a|$. By contrast, under uniform pricing, the aggregate counterpart of the above semi-elasticity, which is given by

$$\bar{\varepsilon}_a\left(p_a^{u\prime}|x_b\right)\left(p_a^{u\prime}\right)^{-1},$$

where p_a^u is a short-cut for $p_a^u(q|x_b)$, is constant in x_a (and therefore in the distance $|x_b - x_a|$), as the marginal price p_a^u for the q-th unit of x_b-matches is the same for all x_a-locations. As a consequence of the above property, under the assumptions in Part 1, the relevant semi-elasticity in the customized-pricing regime is lower (alternatively, higher) than in the uniform-pricing regime when the distance $|x_b - x_a|$ is small (alternatively, large). This implies that the marginal price $\rho_a^{*\prime}$ for the q-th unit of x_b-matches charged to the x_a-agents under the customized-pricing regime is lower than the corresponding price p_a^u under the uniform-pricing regime when locations are far apart, whereas the opposite is true at nearby locations. Accordingly, there is more targeting under uniform pricing than under customized pricing.

Mutatis mutandis, the assumptions in Part 2 in the proposition imply that, under customized pricing, the semi-price elasticity of the demand for the q-th unit of the x_b-agents by the side-a agents is strictly decreasing in $|x_b - x_a|$, in which case customized pricing leads to more targeting than uniform pricing.

Remark 3. To simplify the exposition, Proposition 4 assumes that, on side b, preferences are location-invariant and that, on side a, the vertical and horizontal dimensions are independent (that is, condition I_a applies). The result, however, applies more generally. To see this, let $t_k^*(\cdot)$ be the threshold functions describing the matching sets optimally induced by the platform when it can offer customized tariffs on both sides of the market. Let $L : [0,1] \times \Theta_b \to \mathbb{R}$ be the function defined by

$$L(x_a|\theta_b) \equiv \varphi_b\left(\theta_b, (t_b^*(\theta_b, x_a), x_a)\right) - \frac{1 - F_a^{v|x}\left(t_b^*(\theta_b, x_a)|x_a\right)}{f_a^{v|x}\left(t_b^*(\theta_b, x_a)|x_a\right)}\cdot\frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_a - x_b|\right)$$

[27]Recall that, in this case, $\hat{v}_{x_b}\left(\rho_a'|x_a\right)$ is defined by the unique solution to $u_a\left(\hat{v}_{x_b}\left(\rho_a'|x_a\right), |x_b - x_a|\right) = \rho_a'$.

for all $(x_a, \theta_b) \in [0,1] \times \Theta_b$. Then uniform pricing (on side a) leads to more (respectively, less) targeting than customized pricing, on both sides, if, for any θ_b, the function $L(\cdot|\theta_b)$ is nondecreasing (respectively, non-increasing) in $|x_b - x_a|$. The conditions in Parts 1 and 2 of Proposition 4 are sufficient for the function $L(\cdot|\theta_b)$ to be monotone in $|x_b - x_a|$ for any θ_b, (nondecreasing and non-increasing, respectively). The proof in the Appendix deals with the more general case described above.

4.2 Welfare under Uniform and Customized Pricing

The result in Proposition 4 can also be used to study the welfare implications of price customization. To see this, suppose the market satisfies the conditions in Part 1 in the proposition. Then, under uniform pricing on side-a, the side-a agents face lower marginal prices $p'_a(q|x_b)$ for the x_b-agents they like the most and higher marginal prices for those side-b agents whose location is far from their bliss point.

The above findings permit us to adapt results from the third-degree price discrimination literature to the matching environment under consideration here to identify conditions under which welfare of the side-a agents increases with the imposition of uniform pricing on side a. Formally, recall that, under uniform pricing, the demand by the x_a-agents for each q-th unit of the x_b-matches is given by

$$D_a\left(p'_a|x_b; x_a\right) = \left[1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p'_a|x_a\right)|x_a\right)\right] f_a^x(x_a)$$

where, to ease the notation, we dropped $(q|x_b)$ from the arguments of the marginal price $p'_a(q|x_b)$. Now let

$$CD_a\left(p'_a|x_b; x_a\right) = -\frac{\partial^2 D_a\left(p'_a|x_b; x_a\right)}{\partial\left(p'_a\right)^2}\left(\frac{\partial D_a\left(p'_a|x_b; x_a\right)}{\partial\left(p'_a\right)}\right)^{-1} p'_a$$

denote the convexity of the demand by the x_a-agents for the q-th unit of the x_b-agents with respect to the marginal price p'_a.[28] Before proceeding, we have to impose the following regularity condition.

Condition 4. [IR] Increasing Ratio: For any $(x_a, x_b) \in [0,1]^2$, any $q \in [0, f_b^x(x_b)]$, the function

$$z_a\left(p'_a|x_b; x_a\right) \equiv \frac{p'_a}{2 - CD_a\left(p'_a|x_b; x_a\right)}$$

is nondecreasing in the marginal price p'_a for the q-th unit of the x_b-agents.

We then have the following result:

Proposition 5. ***(comparison: welfare)*** *Suppose the environment satisfies the properties of Scenario (i), Condition IR holds, and either one of the following alternatives is satisfied:*

1. The assumptions in Part 1 of Proposition 4 hold and, for any p'_a and x_b, the convexity $CD_a\left(p'_a|x_b; x_a\right)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents declines with the distance $|x_a - x_b|$.

[28] Note that $CD_a(p'_a|x_b; x_a)$ is also the elasticity of the marginal demand $\partial D_a(p'_a|x_b; x_a)/\partial p'_a$ with respect to the marginal price p'_a.

2. The assumptions in Part 2 of Proposition 4 hold and, for any p'_a and x_b, the convexity $CD_a(p'_a|x_b;x_a)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents increases with the distance $|x_a - x_b|$.

Then welfare of the side-a agents is higher under uniform pricing on side a than under customized pricing on both sides.

The next examples apply Proposition 5 to assess the welfare effects of uniform pricing in different applications. We assume that, in addition to the conditions of Proposition 4, Condition IR holds across all the considered specifications.

Example 10. (***online shopping* - continued**) Consider the online shopping market of Example 1. When F_a^v has a increasing hazard rate and the density $f_a^v(v_a)$ is log-convex, shoppers' welfare is higher under uniform pricing, and so is the level of targeting. When, instead, F_a^v has a decreasing hazard rate and the density $f_a^v(v_a)$ is log-concave, uniform pricing leads to lower targeting but shoppers' welfare is higher under uniform pricing than under customized pricing.[29] $\diamondsuit$

Example 11. (***ad exchange* - continued**) Consider the ad exchange market of Example 2. Advertisers' profits are higher under uniform pricing, and so is the level of targeting, when F_a^v has an increasing hazard rate and its convexity function

$$CF_a^v(v_a) \equiv -\frac{d^2F_a^v}{dv_a^2}\left(\frac{dF_a^v}{dv_a}\right)^{-1} v_a = -\frac{f_a^{v\prime}(v_a)}{f_a^v(v_a)}v_a$$

is decreasing. The latter condition is weaker than the requirement that the density $f_a(v_a)$ is log-convex. $\diamondsuit$

Condition IR, as well as the convexity properties of the demand functions in Proposition 5, parallel those in Aguirre et al (2010). The value of the proposition is in showing how our results about the connection between targeting and customized pricing also permit us to apply to the environment under examination here the welfare results from the third-degree price discrimination literature. Note that Proposition 4 is key to the result in Proposition 5. It permits us to identify "stronger markets," in the sense of Aguirre et al. (2010), with those for matches involving agents from closer locations (Part 1) or more distant locations (Part 2). Once the connection between targeting and price customization is at hand, the welfare implications of customized pricing then naturally parallel those in the third-degree price discrimination literature.

Also note that the result in Proposition 5 is just an illustration of the type of welfare results that Proposition 4 permits. Paralleling the analysis in Proposition 2 in Aguirre et al. (2010), for example, we can also identify primitive conditions under which welfare is higher under price customization than under uniform pricing, as well as conditions under which price customization impacts negatively one side of the market and positively the other.

[29] The conclusions remain valid if the environment satisfies the properties of Scenario (ii) and, in addition, Condition S_b holds.

4.3 Centralized vs Decentralized Markets

In the market for media content, several analysts believe that the increase in the speed of fiber-optic and broadband internet connection will favor a gradual transition of the market to a structure whereby viewers will pay directly the content producers, thus bypassing current market intermediaries such as cable TV providers. This corresponds to a decentralized market (where matching is governed by agents on one side of the market posting prices for their matches with the other side), as opposed to the centralized markets considered so far in this paper (where matching and pricing are controlled by a platform). As it turns out, the results above also permit us to assess the effects of market decentralization on matching allocations and welfare.

To illustrate, consider a matching market where side a is populated by "buyers" and side b is populated by "sellers". Unlike in the baseline model, suppose sellers post prices independently and sell the "match" directly to the buyer (i.e., the market is decentralized). Absent data about consumers' tastes (for instance, due to privacy regulation), we can assume that buyers' locations are private information, whereas sellers' locations (their product variety) is observable by the buyers. In this environment, provided sellers are not able to monitor the purchases that each buyer makes with the other sellers, customized pricing on side a is infeasible. This is to be contrasted to a centralized matching market, where, as described in Lemma 1, location-dependent prices can be implemented by the platform (even when locations are private information) through customized matching plans (a form of bundling), that induce buyers to reveal their locations while receiving no further informational rents.

Therefore, a move from a centralized to a decentralized market is akin to a uniform-pricing requirement, in that sellers have to set (for lack of information and pricing instruments) the same prices for all buyers. But there is more: Decentralization also brings a reduction in the prices set by the sellers, due to the elimination of the monopsony mark-up applied by the platform in a centralized market. Combining this extra effect with the ones identified in Propositions 4 and 5 permits us to conclude that, whenever uniform pricing increases the welfare of the side-a agents, so does market decentralization. The conditions in Proposition 5 are therefore sufficient to conclude that consumer surplus (i.e., the welfare of the side-a agents) is higher in a decentralized market than in a centralized one. This observation may help policymakers identify markets where the transition to a decentralized structure should be promoted, for example through fiscal incentives and/or direct subsidies.

5 Concluding Remarks

This paper studies many-to-many matching in markets in which agents' preferences are both vertically and horizontally differentiated. The analysis delivers the following results. First, it identifies primitive conditions under which profit-maximizing platforms engage in price customization. That is, they offer agents the possibility to customize their matching set by including partners of different

profiles based on their horizontal preferences, with the price for such customization varying with the configuration of the baseline plan. We show that the optimal tariffs induce negative assortative matching at the margin. As the matching sets expand, the marginal agents from each location included in the set are always those with the lowest value for matching. The composition of the pool of marginal agents, however, naturally respects horizontal differences in preferences, with most of the marginal agents coming from "locations" close to the subscriber's bliss-point. We then provide a formula relating the optimal prices to location-specific elasticities of the demands on both sides of the market that can be used in empirical work for testing and structural estimation and that permits us to study how the distortions in the provision of matching services vary with the horizontal dimension of the agents' preferences.

Second, the paper studies the effects on prices, the composition of matching sets, and welfare, of uniform-pricing obligations that hinder platforms' possibility to condition prices on subscribers' "locations", as in the case of privacy regulations preventing online retailers to condition buyers' prices on their age, gender, of physical location. Finally, the paper contributes to the policy debate about the desirability of mediation in matching markets by offering a new angle relating the welfare effects of decentralization to the targeting and price-customization implications of different market structures.

We believe the results have useful implications for various markets. Consider, for example, online shopping. As mentioned in the introduction, recent regulation requiring consumers' consent for the diffusion of personal information is likely to hinder price customization when third-party data is needed. Perhaps surprisingly, our analysis shows that this may either increase or decrease targeting levels, depending on testable characteristics of consumer demand. Related conditions can also be used to evaluate whether or not the pricing effects of privacy regulation are welfare-increasing.

Another natural application of our framework is the market for online advertising (see, among others, Bergemann and Bonatti (2011) for an overview of such markets). Ad exchanges such as AppNexus, AOL's Marketplace, Microsoft Ad Exchange, OpenX, Rubicon Project Exchange, and Smaato, use sophisticated pricing algorithms where prices depend not only on volumes but on advertisers' and publishers' profiles. Such algorithms thus enable price-customization practices that appear similar, at least in spirit, to those studied in the present paper. While such algorithms have initially been praised for the customization possibilities they offer, more recently they have been associated with targeting and price-discriminatory practices often seen with suspicion by consumers and regulators. The policy debate about the desirability of regulations imposing uniform pricing, or about the pros and cons of market decentralization, lacks a formal model shedding light on how matching demands and welfare are affected by such changes. Our paper contributes to such debate by offering a stylized yet flexible framework that one can use to study both the distortions associated with price customization, as well as the market outcomes under uniform pricing.

Next, consider the market for cable TV. Most providers price discriminate on the viewer side by offering viewers packages of channels whereby the baseline configuration can be customized by

adding channels at a cost that depends on the baseline configuration originally selected (see, among others, Crawford (2000), and Crawford and Yurukoglu (2012)). Such industry is evolving fast and many analysts predict a transition to a market structure whereby viewers will purchase content directly from the channels, thus bypassing the intermediation of the current providers. Our analysis sheds some light on how prices set by individual channels compare to their counterparts in markets where the interactions between the channels and the viewers are mediated by cable companies, and identifies conditions under which the transition to a decentralized structure is advantageous to the viewers.

We conclude by discussing a few limitations of our analysis and venues for future research. First, our analysis abstracts from platform competition. Second, and related, it assumes platform have the power to set prices on both sides of the market. While these assumptions are a natural starting point, there are many markets where multiple platforms compete on multiple sides and their ability to set prices is hindered by their lack of bargaining power. For example, the market for cable TV is populated by multiple providers. Furthermore, as indicated in Crawford and Yurukoglu (2012), large channel conglomerates enjoy nontrivial bargaining power vis-a-vis cable TV providers, which suggests that prices are likely to be negotiated on the channel side instead of being set directly by the platforms. Extending the analysis to accommodate for platform competition and limited bargaining power on one, or multiple, sides of the market is expected to provide further insights on the bundling and pricing strategies of many platforms.

Furthermore, certain platforms, most notably B2B platforms, have recently expanded their services to include e-billing and supply-management. These additional services open the door to more sophisticated price-discriminatory practices that use instruments other than the composition of the matching sets. Extending the analysis to accommodate for such richer instruments is another interesting direction for future research (see, e.g., Jeon, Kim, and Menicucci (2016)).

Lastly, in future work, it would be desirable to extend the analysis to accommodate for "within-side" network effects (e.g., congestion and limited attention) and dynamics.

6 Appendix

Proof of Lemma 1. We establish the result using mechanism design techniques. Let

$$(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$$

denote a direct revelation mechanism, where agents are asked to report their types and where $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ denotes the allocation (matching set and total transfer) specified by the mechanism for each side-k agent reporting θ_k.

By familiar envelope arguments, a necessary condition for each type $\theta_k = (v_k, x_k) \in \Theta_k$, $k = a, b$, to prefer reporting truthfully to lying with respect to the vertical dimension v_k while reporting truthfully the horizontal dimension x_k is that transfers satisfy the envelope conditions

$$\begin{aligned} p_k(\theta_k) = & \int_{\mathbf{s}_k(\theta_k)} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y, x_k)} \frac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dF_l(\theta_l) dy, \\ & - U_k(\underline{v}_k, x_k), \end{aligned} \tag{18}$$

where $U_k(\underline{v}_k, x_k)$ is the payoff of a side-k agent with type $(\underline{v}_k, x_k)$.

Using (18), the platform's profits under any incentive-compatible mechanism can then be written as

$$\sum_{k=a,b} \int_{\Theta_k} \left\{ \begin{array}{c} \int_{\mathbf{s}_k(\theta_k)} \left[u_k\left(v_k, |x_k - x_l|\right) - \frac{1 - F_k^v(v_k|x_k)}{f_k^v(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}\left(v_k, |x_k - x_l|\right) \right] dF_l(\theta_l) \\ -U_k(x_k, \underline{v}_k) \end{array} \right\} dF_k(\theta_k).$$

Using the definition of the virtual-value functions $\varphi_k(\theta_k, \theta_l)$ in the main text, we then have that the platform's profits are maximal when $U_k(\underline{v}_k, x_k) = 0$ for all $x_k \in X_k$, $k = a, b$, and when the matching sets are chosen so as to maximize

$$\sum_{k=a,b} \int_{\Theta_k} \left\{ \int_{\mathbf{s}_k(\theta_k)} \varphi_k\left(\theta_k, \theta_l\right) dF_l(\theta_l) \right\} dF_k(\theta_k) \tag{19}$$

subject to the reciprocity condition

$$\theta_l \in \mathbf{s}_k(\theta_k) \Longleftrightarrow \theta_k \in \mathbf{s}_l(\theta_l), \quad l, k \in \{a, b\},\ k \neq l. \tag{20}$$

Hereafter, we first describe the matching sets that maximize (19) subject to the above reciprocity condition and then show that, under the assumptions in the proposition, the platform can implement the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$, where the functions $s_k(\cdot)$ are those that maximize (19) subject to (20), and where the functions $p_k(\cdot)$ are as in (18), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$.

Define the indicator function $m_k(\theta_k, \theta_l) \in \{0, 1\}$ taking value one if and only if $\theta_l \in \mathbf{s}_k(\theta_k)$, that is, if and only if the two types θ_k and θ_l are matched. Then define the following measure on the Borel sigma-algebra over $\Theta_k \times \Theta_l$:

$$\nu_k(E) \equiv \int_E m_k(\theta_k, \theta_l) dF_k(\theta_k) dF_l(\theta_l). \tag{21}$$

Reciprocity implies that $m_k(\theta_k, \theta_l) = m_l(\theta_l, \theta_k)$. As a consequence, the measures ν_k and ν_l satisfy $d\nu_k(\theta_k, \theta_l) = d\nu_l(\theta_l, \theta_k)$. Equipped with this notation, the expression in (19) can be rewritten as

$$\begin{aligned}&\sum_{k,l=a,b,\, l\neq k} \int_{\Theta_k\times\Theta_l} \varphi_k\left(\theta_k, \theta_l\right) d\nu_k(\theta_k, \theta_l)\\ &= \int_{\Theta_k\times\Theta_l} \triangle_k(\theta_k, \theta_l) m_k(\theta_k, \theta_l) dF_k(\theta_k) dF_l(\theta_l), \end{aligned} \tag{22}$$

where, for $k, l = a, b$, $l \neq k$,

$$\triangle_k(\theta_k, \theta_l) \equiv \varphi_k\left(\theta_k, \theta_l\right) + \varphi_l\left(\theta_l, \theta_k\right).$$

Note that the functions $\triangle_a(\theta_a, \theta_b) = \triangle_b(\theta_b, \theta_a)$ represent the marginal effects on the platform's profits of matching types θ_a and θ_b. It is then immediate that the rule $(m_k(\cdot))_{k=a,b}$ that maximizes the expression in (22) is such that, for any $(\theta_k, \theta_l) \in \Theta_k \times \Theta_l$, $k, l = a, b$, $l \neq k$, $m_k(\theta_k, \theta_l) = 1$ if and only if

$$\triangle_k(\theta_k, \theta_l) \geq 0.$$

Next, observe that, under Condition R, the function

$$\varphi_k\left(\theta_k, \theta_l\right) \equiv u_k\left(v_k, |x_k - x_l|\right) - \frac{1 - F_k^v(v_k|x_k)}{f_k^v(v_k|x_k)} \cdot \frac{\partial u_k}{\partial v}\left(v_k, |x_k - x_l|\right)$$

is strictly increasing in v_k, $k, l = a, b$, $l \neq k$. We conclude that the matching rule that maximizes (19) subject to the reciprocity condition (20) can be described by means of a collection of threshold functions $t_k^* : \Theta_k \times X_l \to V_l$, $k, l = a, b$, $l \neq k$, such that, for any $\theta_k = (v_k, x_k)$, any $\theta_l = (v_l, x_l)$, $\theta_l \in \mathbf{s}_k(\theta_k)$ if, and only if, $v_l \geq t_k^*\left(\theta_k, x_l\right)$. The threshold functions $t_k^*\left(\cdot\right)$ are such that, for any $\theta_k \in \Theta_k$, any $x_l \in [0, 1]$, $t_k^*\left(\theta_k, x_l\right) = \underline{v}_l$ if $\triangle_k(\theta_k, (\underline{v}_l, x_l)) > 0$, $t_k^*\left(\theta_k, x_l\right) = \bar{v}_l$ if $\triangle_k(\theta_k, (\bar{v}_l, x_l)) < 0$, and $t_k^*\left(\theta_k, x_l\right)$ is the unique solution to $\triangle_k\left(\theta_k, \left(t_k^*\left(\theta_k, x_l\right), x_l\right)\right) = 0$ if

$$\triangle_k(\theta_k, (\underline{v}_l, x_l)) \leq 0 \leq \triangle_k(\theta_k, (\bar{v}_l, x_l)).$$

Condition R also implies that, for ay $x_k, x_l \in [0, 1]^2$, the threshold $t_k^*\left(\theta_k, x_l\right)$ is decreasing in v_k and non-decreasing in $|x_l - x_k|$.

Equipped with the above result, we now show that, in each of the environments described by the conditions in the proposition, the platform can implement the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k\in\Theta_k}^{k=a,b}$, where $\mathbf{s}_k(\theta_k)$ are the matching sets described by the above threshold rule, and where the payment functions $p_k(\theta_k)$ are the ones in (18), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$.

First observe that the payoff that each type θ_k obtains in the above direct revelation mechanism when reporting truthfully is equal to

$$U_k(\theta_k) = \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y, x_k)} \frac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dF_l(\theta_l) dy.$$

That $U_k(\theta_k) \geq 0$ follows directly from the fact that u_k is non-decreasing in v_k. This means that the mechanism is individually rational (meaning that each type θ_k prefers participating in the mechanism

and receiving the allocation $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ to refusing to participate and receiving the allocation $(\emptyset, 0)$ yielding a payoff equal to zero).

Below we show that either the above direct mechanism is also incentive-compatible (meaning that each type θ_k prefers the allocation $(\mathbf{s}_k(\theta_k), p_k(\theta_k))$ designed for him to the allocation $(\mathbf{s}_k(\theta'_k), p_k(\theta'_k))$ designed for any other type θ'_k), or it can be turned, at no cost to the platform, into a mechanism implementing the same allocations as the above ones which is both incentive compatible and individually rational.

Definition 6. (nested matching) A matching rule $\mathbf{s}_k(\theta_k)$ is nested if, for any pair $\theta_k = (v_k, x_k)$ and $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ such that $x_k = \hat{x}_k$, either $\mathbf{s}_k(\theta_k) \subseteq \mathbf{s}_k(\hat{\theta}_k)$, or $\mathbf{s}_k(\theta_k) \supseteq \mathbf{s}_k(\hat{\theta}_k)$. A direct revelation mechanism is nested if its matching rule is nested.

Clearly, the direct mechanism defined above where the matching rule is described by the threshold function $t^*_k(\theta_k, x_l)$ is nested. Now let $\Pi_k(\theta_k; \hat{\theta}_k)$ denote the payoff that type θ_k obtains in a direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))^{k=a,b}_{\theta_k \in \Theta_k}$ by mimicking type $\hat{\theta}_k$.

Definition 7. (ICV) A direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))^{k=a,b}_{\theta_k \in \Theta_k}$ satisfies incentive compatibility along the v dimension (ICV) if, for any $\theta_k = (v_k, x_k)$ and $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ with $x_k = \hat{x}_k$, $U_k(\theta_k) \geq \Pi_k(\theta_k; \hat{\theta}_k)$.

The following property is then true (the proof is standard and hence omitted):

Property 1. *A nested direct revelation mechanism* $(\mathbf{s}_k(\theta_k), p_k(\theta_k))^{k=a,b}_{\theta_k \in \Theta_k}$ *satisfies ICV if, and only if, the following conditions jointly hold:*

1. *for any* $\theta_k = (v_k, x_k)$ *and* $\hat{\theta}_k = (\hat{v}_k, \hat{x}_k)$ *such that* $x_k = \hat{x}_k$, $v_k > \hat{v}_k$ *implies that* $\mathbf{s}_k(\theta_k) \supseteq \mathbf{s}_k(\hat{\theta}_k)$*;*
2. *the payment functions* $p_k(\theta_k)$ *satisfy the envelope formula (18).*

Clearly, the direct revelation mechanism where the matching rule is the one corresponding to the threshold functions $t^*_k(\cdot)$ described above and where the payment functions $p_k(\theta_k)$ are the ones in (18), with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$, is not only nested but satisfies the two conditions in the lemma. It follows that such a mechanism satisfies ICV.

Equipped with the above results, we now show that, in each of the environments corresponding to the combination of conditions described in the proposition, the above direct revelation mechanism is either incentive-compatible, or it can be augmented to implement the same allocations prescribed by $(\mathbf{s}_k(\theta_k), p_k(\theta_k))^{k=a,b}_{\theta_k \in \Theta_k}$ at no extra cost to the platform.

Consider first Scenario (i). Recall that, in this case, locations are public on both sides. That the mechanism is ICV implies that any deviation along the vertical dimension is unprofitable. Furthermore, because locations are public on both sides, any deviation along the horizontal dimension is detectable. It is then immediate that the platform can augment the above direct revelation mechanism by adding to it punishments (in the form of large fines) for those agents lying along the

horizontal dimension. The augmented mechanism is both individually rational and incentive compatible and implements the same allocations as the original mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ at no extra cost to the platform.

Next suppose the environment satisfies the properties of Scenario (ii) and, in addition, Conditions I_a and S_b hold. Again, because locations are public on side b, incentive compatibility on side b can be guaranteed by augmenting the mechanism as described above for Scenario (i). Thus consider incentive compatibility on side a. The latter requires that

$$U_a(v_a, x_a) \geq \Pi_a((v_a, x_a); (\hat{v}_a, \hat{x}_a)),$$

for all $(x_a, \hat{x}_a, v_a, v_a) \in X_a^2 \times V_a^2$. The above inequality is equivalent to

$$\int_{\underline{v}_a}^{v_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_a}{\partial v}(y, |x_a - x_b|)\, dF_b(\theta_b) dy \geq \int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,\hat{x}_a)} \frac{\partial u_a}{\partial v}(y, |\hat{x}_a - x_b|)\, dF_b(\theta_b) dy \tag{23}$$
$$+ \int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} [u_a(v_a, |x_a - x_b|) - u_k(\hat{v}_a, |\hat{x}_a - x_b|)] dF_b(b).$$

It is easy to see that, for any $\theta_a = (v_a, x_a) \in \Theta_a$,

$$\int_{\mathbf{s}_a(v_a,x_a)} \frac{\partial u_a}{\partial v}(v_a, |x_a - x_b|)\, dF_b(\theta_b) = \int_{d \in [0,1/2]} \frac{\partial u_a(v_a, d)}{\partial v} dW(d; \theta_a), \tag{24}$$

where $W(d; \theta_a)$ is the measure of agents whose distance from x_a is at most d included in the matching set $\mathbf{s}_a(v_a, x_a)$ of type θ_a under the proposed mechanism. It is also easy to see that, under Conditions I_a and S_b, the expression in (24) is invariant in x_a. That is, $W(d; \theta_a) = W(d; \theta_a')$ for any $d \in [0, 1/2]$, any $\theta_a, \theta_a' \in \Theta_a$ with $v_a = v_a'$.[30] This means that

$$\int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,\hat{x}_a)} \frac{\partial u_a}{\partial v}(y, |\hat{x}_a - x_b|)\, dF_b(\theta_b) dy = \int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_a}{\partial v}(y, |x_a - x_b|)\, dF_b(\theta_b) dy.$$

By the same arguments,

$$\int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} u_a(\hat{v}_a, |\hat{x}_a - x_b|)\, dF_b(\theta_b) = \int_{\mathbf{s}_a(\hat{v}_a,x_a)} u_a(\hat{v}_a, |x_a - x_b|)\, dF_b(\theta_b).$$

Furthermore, under Condition R, the threshold functions $t_k^*(\theta_k, x_l)$ and non-decreasing in the distance $|x_l - x_k|$. In turn, this implies that

$$\int_{\mathbf{s}_a(\hat{v}_a,\hat{x}_a)} u_a(v_a, |x_a - x_b|)\, dF_b(\theta_b) \leq \int_{\mathbf{s}_a(\hat{v}_a,x_b)} u_a(v_a, |x_b - x_a|)\, dF_b(\theta_b).$$

It follows that the right hand side of (23) is smaller than

$$\int_{\underline{v}_a}^{\hat{v}_a} \int_{\mathbf{s}_a(y,x_a)} \frac{\partial u_k}{\partial v}(y, |x_a - x_b|)\, dF_b(\theta_b) dy$$
$$+ \int_{\mathbf{s}_a(\hat{v}_a,x_a)} [u_a(v_a, |x_a - x_b|) - u_a(\hat{v}_a, |x_a - x_b|)] dF_b(\theta_b),$$

[30] Conditions I_k, $k = a, b$, suffice to guarantee that the function $\Delta_k(\theta_k, \theta_l)$ depends only on v_k, v_l, and $|x_l - x_k|$. The strengthening of Condition I_b to S_b is, however, necessary to guarantee that the mass of agents of a given distance d included in the matching sets of any pair of types $\theta_a, \theta_a' \in \Theta_a$ with $v_a = v_a'$ is the same.

which is the payoff that type $\theta_a = (v_a, x_a)$ obtains by announcing $(\hat{v}_a, x_a)$ (that is, by lying about the vertical dimension but reporting truthfully the horizontal one). That the inequality in (23) holds then follows from the fact that the direct revelation mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ satisfies ICV.

The arguments for an environment satisfying the properties of Scenario (iii) along with Conditions I_b and S_a are symmetric to those for an environment satisfying the properties of Scenario (ii) along with Conditions I_a and S_b, and hence the proof is omitted.

Finally, consider an environment satisfying the properties of Scenario (iv) along with Conditions S_a and S_b. That the proposed mechanism is incentive compatible follows from the same arguments as for Scenario (ii) above, now applied to both sides of the market.

We conclude that, in each of the environments considered in the proposition, the allocations $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$, where the matching sets $\mathbf{s}_k(\theta_k)$ are the ones specified by the threshold functions $t_k^*(\cdot)$ described above, and where the payments are the ones in (18) with $U_k(\underline{v}_k, x_k) = 0$, all $x_k \in X_k$, $k = a, b$ can be sustained in a mechanism that is both individually rational and incentive compatible. The result in the proposition then follows from the fact that (a) such allocations are profit-maximizing among those consistent with the rationality of the agents (i.e., satisfying the IC and IR constraints), and (b) can be induced by offering customized tariffs

$$\{(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k)) : x_k \in [0,1]\}$$

satisfying the properties described below. For each plan $x_k \in [0,1]$, the baseline configuration is given by

$$\underline{\mathbf{s}}_k(x_k) = \mathbf{s}_k(\underline{v}_k, x_k),$$

the baseline price is given by

$$\underline{T}_k(x_k) = p_k(\underline{v}_k, x_k) = \int_{\mathbf{s}_k(\underline{v}_k, x_k)} u_k\left(\underline{v}_k, |x_k - x_l|\right) dF_l(\theta_l),$$

the set of possible customizations is given by

$$\mathbf{S}_k(x_k) = \{\mathbf{s}_k(v_k, x_k) : v_k \in V_k\},$$

and the price schedules $\rho_k(q|x_l; x_k)$ are such that, for $q = q_{x_l}\left(\mathbf{s}_k(\underline{v}_k, x_k)\right)$, $\rho_k(q|x_l; x_k) = 0$, while for $q \in \left(q_{x_l}\left(\mathbf{s}_k(\underline{v}_k, x_k)\right), q_{x_l}\left(\mathbf{s}_k(\bar{v}_k, x_k)\right)\right]$,

$$\rho_k(q|x_l; x_k) = q u_k\left(v_k(q; x_k, x_l), |x_k - x_l|\right) - \int_{\underline{v}_k}^{v_k(q; x_k, x_l)} q_{x_l}\left(\mathbf{s}_k(y, x_k)\right) \frac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dy - \underline{T}_k(x_k) \tag{25}$$

where

$$v_k(q; x_k, x_l) = \inf\left\{v_k \in V_k : q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q\right\}.$$

Any agent selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ and then choosing a matching set $\mathbf{s}_k \notin \mathbf{S}_k(x_k)$ is charged a fine large enough to make the utility of such a set, net of the payment,

negative for all types. Likewise, when locations are public on side k, any side-k agent selecting a plan other than $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ is charged a large enough fine to make the choice unprofitable for any type. Note that the existence of such fines is guaranteed by the assumption that u_k is bounded, $k = a, b$.

That the above customized tariff implements the same allocations as the direct mechanism $(\mathbf{s}_k(\theta_k), p_k(\theta_k))_{\theta_k \in \Theta_k}^{k=a,b}$ then follows from the following considerations. Each type $\theta_k = (v_k, x_k)$, by selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ designed for agents with the same location as type θ_k and then choosing the customization $\mathbf{s}_k(v_k, x_k)$ specified by the direct mechanism for type θ_k is charged a total payment equal to

$$\underline{T}_k(x_k) + \int_0^1 \left[q_{x_l}(\mathbf{s}_k(v_k, x_k)) u_k\left(v_k, |x_k - x_l|\right) - \int_{\underline{v}_k}^{v_k} q_{x_l}(\mathbf{s}_k(y, x_k)) \tfrac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dy \right] dx_l - \underline{T}_k(x_k)$$

$$= \int_{\mathbf{s}_k(\theta_k)} u_k\left(v_k, |x_k - x_l|\right) dF_l(\theta_l) - \int_{\underline{v}_k}^{v_k} \int_{\mathbf{s}_k(y, x_k)} \tfrac{\partial u_k}{\partial v}\left(y, |x_k - x_l|\right) dF_l(\theta_l) dy$$

$$= p_k(\theta_k),$$

exactly as in the direct mechanism. That each type θ_k maximizes his payoff by selecting the plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ and then choosing the customization $\mathbf{s}_k(v_k, x_k)$ specified for him by the direct mechanism then follows from the fact that (a) the direct mechanism is incentive compatible, (b) the payment associated with any other plan $(\underline{\mathbf{s}}_k(\hat{x}_k), \underline{T}_k(\hat{x}_k), \rho_k(\cdot|\cdot; \hat{x}_k), \mathbf{S}_k(\hat{x}_k))$ followed by the selection of a set $\mathbf{s}_k$ is either equal to the payment specified by the direct mechanism for some report $(\hat{v}_k, \hat{x}_k)$, or is so large to make the net payoff of such selection negative.

Finally, to see that, when locations are public on side k, without loss of optimality, the side-k customized tariff does not need to restrict the agents' ability to customize their matching sets (that is, $\mathbf{S}_k(x_k) = \Sigma(\Theta_l)$, all x_k) recall that, in this case, each side-k agent located at x_k can be induced to select the matching plan $(\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ designed for agents located at x_k by setting the fee associated with the selection of any other plan sufficiently high. The separability of the agents' preferences then implies that, once the plan $\underline{\mathbf{s}}_k(x_k), \underline{T}_k(x_k), \rho_k(\cdot|\cdot; x_k), \mathbf{S}_k(x_k))$ is selected, even if $\mathbf{S}_k(x_k) = \Sigma(\Theta_l)$, because the price schedules $\rho_k(\cdot|\cdot; x_k)$ satisfy (25), type θ_k prefers to select $q_{x_l}(\mathbf{s}_k(v_k, x_k))$ agents from each location x_l to any other mass of agents from the same location x_l, irrespective of the mass of agents from other locations type θ_k includes in his matching set. Q.E.D.

Proof of Proposition 1. Fix a pair of locations $x_a, x_b \in [0, 1]$. From Lemma 1, the profit-maximizing tariffs are customized and induce agents to select matching sets satisfying the threshold property of Lemma 1. Furthermore, from the proof of Lemma 1, for any $\theta_k = (v_k, x_k)$, any $x_l \in [0, 1]$, the threshold t_k^* is such that $t_k^*(\theta_k, x_l) = \underline{v}_l$ if $\triangle_k(\theta_k, (\underline{v}_l, x_l)) > 0$, $t_k^*(\theta_k, x_l) = \bar{v}_l$ if $\triangle_k(\theta_k, (\bar{v}_l, x_l)) < 0$, and $t_k^*(\theta_k, x_l)$ is the unique solution to $\triangle_k(\theta_k, (t_k^*(\theta_k, x_l), x_l)) = 0$ if

$$\triangle_k(\theta_k, (\underline{v}_l, x_l)) \leq 0 \leq \triangle_k(\theta_k, (\bar{v}_l, x_l)).$$

This means that, for any $q_k \in (0, f_l^x(x_l))$, either there exists no $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$, or there exists a unique $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$. Now take any $q_k \in (0, f_l^x(x_l))$ for which there exists $v_k \in V_k$ such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$. As explained in the main text, for any such q_k, the unique value of v_k such that $q_{x_l}(\mathbf{s}_k(v_k, x_k)) = q_k$ is also the unique value of v_k that solves

$$u_k\left(v_k, |x_k - x_l|\right) = \rho_k'\left(q_k|x_l; x_k\right). \tag{26}$$

Now let $\hat{v}_{x_l}\left(\rho_k'|x_k\right)$ be the unique solution to (26) and $v_l'(q_k; x_l)$ be the unique solution to

$$\left[1 - F_l^{v|x}\left(v_l'(q_k; x_l)|x_l\right)\right] f_l^x(x_l) = q_k.$$

That the demands under the profit-maximizing tariffs satisfy the threshold structure of Lemma 1 implies that

$$t_k^*\left(\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), x_k\right), x_l\right) = v_l'(q_k; x_l)$$

and that

$$\varphi_k\left(\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), x_k\right), (v_l'(q_k; x_l), x_l)\right) + \varphi_l\left(v_l'(q_k; x_l), x_l), \left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), x_k\right)\right) = 0. \tag{27}$$

Lastly, observe that, for any such q_k,

$$\varepsilon_k\left(\rho_k'|x_l; x_k\right) = \frac{f_k^{v|x}(\hat{v}_{x_l}\left(\rho_k'|x_k\right)|x_k)}{1 - F_k^{v|x}(\hat{v}_{x_l}\left(\rho_k'|x_k\right)|x_k)}\left[\frac{\partial u_k}{\partial v}\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), |x_k - x_l|\right)\right]^{-1} \rho_k'\left(q_k|x_l; x_k\right). \tag{28}$$

Using the definition of φ_k from the main text together with (26) and (28), we then have that, for any such q_k,

$$\varphi_k\left(\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), x_k\right), (v_l'(q_k; x_l), x_l)\right) = \rho_k'\left(q_k|x_l; x_k\right)\left[1 - \frac{1}{\varepsilon_k\left(\rho_k'|x_l; x_k\right)}\right]. \tag{29}$$

Likewise, when $q_l = \left[1 - F_k^{v|x}\left(\hat{v}_{x_l}\left(\rho_k'|x_k\right)|x_k\right)\right] f_k^x(x_k)$,

$$\varphi_l\left(\left(v_l'(q_k; x_l), x_l\right), \left(\hat{v}_{x_l}\left(\rho_k'|x_k\right), x_k\right)\right) = \rho_l'\left(q_l|x_k; x_l\right)\left[1 - \frac{1}{\varepsilon_l\left(\rho_l'|x_k; x_l\right)}\right]. \tag{30}$$

Combining (29) and (30) with (27), we obtain the result in the proposition. Q.E.D.

Proof of Proposition 2. Take any $(\theta_k, x_l) \in \Theta_k \times [0, 1]$ for which $t_k^*\left(\theta_k, x_l\right), t_k^e\left(\theta_k, x_l\right) \in Int[V_l]$. Recall that, in this case, $t_k^*\left(\theta_k, x_l\right)$ is given by the unique solution to

$$\varphi_k\left(\theta_k, \left(t_k^*\left(\theta_k, x_l\right), x_l\right)\right) + \varphi_l\left(\left(t_k^*\left(\theta_k, x_l\right), x_l\right), \theta_k\right) = 0$$

whereas $t_k^e\left(\theta_k, x_l\right)$ is given by the unique solution to

$$u_k\left(v_k, |x_k - x_l|\right) + u_l\left(t_k^e\left(\theta_k, x_l\right), |x_l - x_k|\right) = 0.$$

This means that

$$u_l\left(t_k^*\left(\theta_k,x_l\right),|x_l-x_k|\right)-u_l\left(t_k^e\left(\theta_k,x_l\right),|x_l-x_k|\right)$$
$$=\frac{1-F_k^v(v_k)}{f_k^v(v_k)}\frac{\partial u_k}{\partial v_k}\left(v_k,|x_k-x_l|\right)+\frac{1-F_l^v\left(t_k^*(\theta_k,x_l)\right)}{f_l^v\left(t_k^*(\theta_k,x_l)\right)}\frac{\partial u_l}{\partial v_l}\left(t_k^*\left(\theta_k,x_l\right),|x_k-x_l|\right). \tag{31}$$

Now fix $\theta_k=(v_k,x_k)$, let $\mu(x_k,x_l)=|x_k-x_l|$ and, without loss of generality, assume $x_l>x_k$ (the results for the case $x_l<x_k$ are analogous to those for the case $x_l>x_k$, after the obvious change in sign due to the fact that, in this case, increasing distance means decreasing x_l). Differentiating the expression in the right-hand-side of (31) with respect to x_l at some x_l for which $\mu(x_k,x_l)\in\left(0,\frac{1}{2}\right)$, we obtain that

$$\frac{\partial}{\partial x_l}\left[u_l\left(t_k^*\left(\theta_k,x_l\right),|x_l-x_k|\right)-u_l\left(t_k^e\left(\theta_k,x_l\right),|x_l-x_k|\right)\right]=\frac{1-F_k^v(v_k)}{f_k^v(v_k)}\cdot\frac{\partial^2 u_k(v_k,|x_k-x_l|)}{\partial v\partial\mu}$$
$$-\frac{\left(\lambda_l^v\right)'\left(t_k^*(\theta_k,x_l)\right)}{\lambda_l^v\left(t_k^*(\theta_k,x_l)\right)^2}\frac{\partial t_k^*}{\partial x_l}\left(\theta_k,x_l\right)\frac{\partial u_l\left(t_k^*(\theta_k,x_l),|x_k-x_l|\right)}{\partial v_l}$$
$$+\frac{1}{\lambda_l^v\left(t_k^*(\theta_k,x_l)\right)}\left(\frac{\partial^2 u_l\left(t_k^*(\theta_k,x_l),|x_k-x_l|\right)}{\partial v_l^2}\frac{\partial t_k^*(\theta_k,x_l)}{\partial x_l}+\frac{\partial^2 u_l\left(t_k^*(\theta_k,x_l),|x_k-x_l|\right)}{\partial v_l\partial\mu}\right).$$

By Lemma 1, $\frac{\partial t_k^*}{\partial x_l}\left(\theta_k,x_l\right)\geq 0$. Therefore, under the assumptions of Part 1, the expression above is negative. Conversely, under the assumptions of Part 2, the expression above is positive. The result in the proposition then follows from the above properties. Q.E.D.

Proof of Proposition 3. The platform's problem consists in choosing a collection of side-a uniform price schedules $p_a(\cdot|x_b)$, one for each side-b location $x_b\in[0,1]$, along with a collection of side-b price schedules $\rho_b(\cdot|x_a;x_b)$, one for each pair $(x_a,x_b)\in[0,1]^2$, that jointly maximize its profits, which can be conveniently expressed as

$$\int_0^1\int_0^{f_b^x(x_b)}\bar{D}_a\left(p_a'(q|x_b)|x_b\right)p_a'(q|x_b)dqdx_b$$
$$+\int_0^1\int_0^1\int_0^{f_a^x(x_a)}D_b\left(\rho_b'(q|x_a;x_b)|x_a;x_b\right)\rho_b'(q|x_a;x_b)dqdx_adx_b,$$

subject to the feasibility constraint (3).

For any x_b , $q\leq f_b^x(x_b)$, and $p_a'(q|x_b)$, let

$$\hat{v}_{x_b}\left(p_a'|x_a\right)=\begin{cases}v_a\ s.t.\ u_a(v_a,|x_a-x_b|)=p_a' & \text{if } p_a'\in[u_a(\underline{v}_a,|x_a-x_b|),u_a(\bar{v}_a,|x_a-x_b|)]\\ \underline{v}_a & \text{if } p_a'<u_a(\underline{v}_a,|x_a-x_b|)\\ \bar{v}_a & \text{if } p_a'>u_a(\bar{v}_a,|x_a-x_b|).\end{cases} \tag{32}$$

Given the above definition, we have that the demand by the x_a-agents for the q-th unit of the x_b-agents at the marginal price $p'_a(q|x_b)$ is equal to

$$D_a\left(p'_a(q|x_b)|x_b;x_a\right) = \left[1 - F_a^{v|x}\left(\hat{v}_{x_b}\left(p'_a|x_a\right)|x_a\right)\right] f_a^x(x_a).$$

Also, for any $q \leq f_b^x(x_b)$, recall that $v'_b(q;x_b)$ is the unique solution to $\left[1 - F_b^{v|x}\left(v'_b(q;x_b)|x_b\right)\right] f_b^x(x_b) = q$. Reciprocity, along with optimality, implies that the most profitable way to deliver q units of x_b-agents to each x_a-agent demanding to be matched to q units of x_b-agents is to match the x_a-agent to every x_b-agent whose vertical type exceeds $v'_b(q;x_b)$. In other words, the optimal tariffs induce matching demands with a threshold structure, as in the case where tariffs are customized on both sides of the market (cfr Lemma 1). Now for each $x_a, x_b \in [0,1]$, each $q \leq f_b^x(x_b)$, let

$$\hat{q}_b(q;x_a;x_b) \equiv D_a\left(p'_a(q|x_b)|x_b;x_a\right).$$

Given $p'_a(q|x_b)$, the platform thus optimally selects customized prices for the x_b-agents for each quantity $\hat{q}_b(q;x_a;x_b)$ of the x_a-agents equal to

$$\rho'_b(\hat{q}_b(q;x_a;x_b)|x_a;x_b) = u_b(v'_b(q;x_b), |x_b - x_a|). \tag{33}$$

Such prices guarantee that, for each $x_a \in [0,1]$, $D_b\left(\rho'_b(\hat{q}_b(q;x_a;x_b)|x_a;x_b)|x_a;x_b\right) = q$, thus clearing the market.

The function $p'_a(q|x_b) : \mathbb{R} \times [0,1] \to \mathbb{R}$ thus uniquely defines the matching sets on both sides of the market. Now, from the arguments in the proof of Lemma 1, we know that the maximal revenue the platform receives from the side-b agents when each x_b-agent with vertical type v_b is assigned a matching set equal to $\mathbf{s}_b(v_b, x_b)$ is given by

$$\int_{\Theta_b}\left\{\int_0^1\left\{u_b\left(v_b, |x_b - x_a|\right) - \frac{1 - F_b^{v|x}(v_b|x_b)}{f_b^{v|x}(v_b|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v_b, |x_b - x_a|\right)\right\} q_{x_a}(\mathbf{s}_b(v_b, x_b))dx_a\right\} dF_b(\theta_b).$$

In turn, this means that the platform's problem can be re-casted as choosing a function $\frac{dp_a}{dq}(q|x_b) : \mathbb{R} \times [0,1] \to \mathbb{R}$ that maximizes

$$\int_0^1\int_0^{f_b^x(x_b)}\left\{\bar{D}_a\left(p'_a(q|x_b)|x_b\right)p'_a(q|x_b) - \mathcal{C}\left[p'_a(q|x_b)\right]\right\} dq dx_b$$

where, for each $x_b \in [0,1]$, each $q \leq f_b^x(x_b)$, the function

$$\mathcal{C}\left[p'_a(q|x_b)\right] \equiv$$

$$-\int_0^1\left\{u_b\left(v'_b(q;x_b), |x_b - x_a|\right) - \frac{1 - F_b^{v|x}(v'_b(q;x_b)|x_b)}{f_b^{v|x}(v'_b(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v'_b(q;x_b), |x_b - x_a|\right)\right\} D_a\left(p'_a(q|x_b)|x_b;x_a\right) dx_a$$

captures the "procurement costs" of clearing the matching demands of all side-a agents that demand at least q matches with the x_b-agents. This problem can be solved by point-wise maximization of

the above objective function, i.e., by selecting for each $x_b \in [0,1]$, $q \leq f_b^x(x_b)$ (equivalently, for each $(x_b, v_b) \in [0,1] \times V_b$), $p'_a(q|x_b)$ so as to maximize

$$\bar{D}_a\left(p'_a(q|x_b)|x_b\right) p'_a(q|x_l) - \mathcal{C}\left[p'_a(q|x_b)\right].$$

The first-order conditions for such a problem are given by

$$p'_a(q|x_b)\frac{\partial \bar{D}_a\left(p'_a(q|x_b)|x_b\right)}{\partial\left(p'_a\right)}\left[1-\frac{1}{\bar{\varepsilon}_a\left(p'_a(q|x_b)|x_b\right)}\right] - \mathcal{C}'\left[p'_a(q_a|x_b)\right] = 0,$$

where

$$\mathcal{C}'\left[p'_a(q_a|x_b)\right]$$

$$= -\int_0^1 \left\{u_b\left(v'_b(q;x_b), |x_b - x_a|\right) - \frac{1-F_b^{v|x}(v'_b(q;x_b)|x_b)}{f_b^{v|x}(v'_b(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v'_b(q;x_b), |x_b - x_a|\right)\right\} \frac{\partial D_a(p'_a(q|x_b)|x_b;x_a)}{\partial(p'_a)} dx_a.$$

Now observe that (33) implies that

$$u_b\left(v'_b(q;x_b), |x_b - x_a|\right) - \frac{1-F_b^{v|x}(v'_b(q;x_b)|x_b)}{f_b^{v|x}(v'_b(q;x_b)|x_b)} \cdot \frac{\partial u_b}{\partial v}\left(v'_b(q;x_b), |x_b - x_a|\right)$$

$$= \rho'_b(\hat{q}_b(q;x_a;x_b)|x_a;x_b)\left(1 - \frac{1}{\varepsilon_b\left(\rho'_b(\hat{q}_b(q;x_a;x_b)|x_a;x_b)|x_a;x_b\right)}\right).$$

This means that the above first-order conditions can be rewritten as

$$p'_a(q|x_b)\left[1 - \frac{1}{\bar{\varepsilon}_a(p'_a(q|x_b)|x_b)}\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p'_a(q|x_b))}\left[\rho'_b(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)\left(1 - \frac{1}{\varepsilon_b\left(\rho'_b(\hat{q}_b(q;\tilde{x}_a;x_b)|\tilde{x}_a;x_b)|\tilde{x}_a;x_b\right)}\right)\right] = 0,$$

where $H(x_a|x_b, q)$ is the distribution over $X_a = [0,1]$ whose density is given by

$$h_a\left(x_a|x_b, p'_a(q|x_b)\right) \equiv \frac{\frac{\partial D_a(p'_a(q|x_b)|x_b;x_a)}{\partial(p'_a)}}{\frac{\partial \bar{D}_a(p'_a(q|x_b)|x_b)}{\partial(p'_a)}}.$$

The above properties imply the result in the proposition. Q.E.D.

Proof of Proposition 4. As explained in the main text, the proof below is for the more general case in which (a), on side b, the agents' preferences may depend on the locations, and (b) on side a, the vertical and horizontal dimensions are possibly correlated.

Fix $\theta_b = (v_b, x_b)$ and let $q = f_b^x(x_b)\left[1 - F_b^v(v_b)\right]$. The result in Proposition 3 implies that, under uniform pricing on side a and customized pricing on side b, for any $x_a \in X_a$ such that $t_b^u(\theta_b, x_a) \in \text{Int}[V_a]$, $t_b^u(\theta_b, x_a)$ is such that

$$u_a(t_b^u(\theta_b, x_a), |x_b - x_a|) - \mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\frac{1-F_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}{f_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)} \cdot \frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), |\tilde{x}_a - x_b|\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\varphi_b\left(\theta_b, \left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right), \tilde{x}_a\right)\right)\right] = 0, \tag{34}$$

where $H\left(x_a|x_b,p_a^{u\prime}\right)$ is the distribution over $X_a=[0,1]$ whose density is given by

$$h\left(x_a|x_b,p_a^{u\prime}\right)\equiv\frac{\frac{\partial D_a(p_a^{u\prime}|x_b;x_a)}{\partial(p_a^{u\prime})}}{\frac{\partial \bar{D}_a(p_a^{u\prime}|x_b)}{\partial(p_a^{u\prime})}},$$

and where $p_a^{u\prime}$ is a shortcut for $p_a^{u\prime}(q|x_b)$ with the latter equal to $p_a^{u\prime}(q|x_b)=u_a(t_b^u(\theta_b,x_a),|x_a-x_b|)$. Note that, to arrive at (34), we used the result in Proposition 3 along with the property in (16) and the fact that, for any x_a such that $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\notin Int[V_a]$, $h\left(x_a|x_b,p_a^{u\prime}\right)=0$, whereas for any x_a such that $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\in Int[V_a]$,

$$\frac{p_a^{u\prime}}{\varepsilon_a\left(p_a^{u\prime}|x_b;x_a\right)}=\frac{1-F_a^v\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\right)}{f_a^v\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\right)}\cdot\frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right),|x_a-x_b|\right).$$

We also used the fact that, for any x_a such that $h\left(x_a|x_b,p_a^{u\prime}\right)>0$ (equivalently, $\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right)\in Int[V_a]$),

$$\rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b)\left(1-\frac{1}{\varepsilon_b\left(\rho_b'(\hat{q}_b(q;x_a;x_b)|x_a;x_b)|x_a;x_b\right)}\right)$$

$$=\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(p_a^{u\prime}|x_a\right),x_a\right)\right),$$

as shown in the proof of Proposition 3.

On the other hand, under customized pricing on both sides, for any such $\theta_b=(v_b,x_b)$, any $x_a\in X_a$ such that $t_b^*(\theta_b,x_a)\in Int[V_a]$, the threshold $t_b^*(\theta_b,x_a)$ is such that

$$u_a(t_b^*(\theta_b,x_a),|x_b-x_a|)-\frac{1-F_a^v\left(t_b^*(\theta_b,x_a)\right)}{f_a^v\left(t_b^*(\theta_b,x_a)\right)}\cdot\frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b,x_a),|x_a-x_b|\right)+\varphi_b\left(\theta_b,\left(t_b^*(\theta_b,x_a),x_a\right)\right)=0.$$

It is then immediate that, for any x_a such that

$$-\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\frac{1-F_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}{f_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}\cdot\frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),|x_b-\tilde{x}_a|\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]$$

$$\leq-\frac{1-F_a^v\left(t_b^*(\theta_b,x_a)\right)}{f_a^v\left(t_b^*(\theta_b,x_a)\right)}\cdot\frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b,x_a),|x_b-x_a|\right)+\varphi_b\left(\theta_b,\left(t_b^*(\theta_b,x_a),x_a\right)\right)$$

we have that $t_b^u(\theta_b,x_a)\geq t_b^*(\theta_b,x_a)$, whereas, for any x_a such that

$$-\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\frac{1-F_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}{f_a^v\left(\hat{v}_{x_b}(p_a^{u\prime}|\tilde{x}_a)\right)}\cdot\frac{\partial u_a}{\partial v}\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),|x_b-\tilde{x}_a|\right)\right]$$

$$+\mathbb{E}_{H(\tilde{x}_a|x_b,p_a^{u\prime})}\left[\varphi_b\left(\theta_b,\left(\hat{v}_{x_b}\left(p_a^{u\prime}|\tilde{x}_a\right),\tilde{x}_a\right)\right)\right]$$

$$\geq-\frac{1-F_a^v\left(t_b^*(\theta_b,x_a)\right)}{f_a^v\left(t_b^*(\theta_b,x_a)\right)}\cdot\frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b,x_a),|x_b-x_a|\right)+\varphi_b\left(\theta_b,\left(t_b^*(\theta_b,x_a),x_a\right)\right)$$

we have that $t_b^u(\theta_b,x_a)\leq t_b^*(\theta_b,x_a)$.

The results in the proposition then follow from the monotonicity of the function

$$L(x_a|\theta_b) \equiv \varphi_b\left(\theta_b, (t_b^*(\theta_b, x_a), x_a)\right) - \frac{1 - F_a^{v|x}\left(t_b^*(\theta_b, x_a)|x_a\right)}{f_a^{v|x}\left(t_b^*(\theta_b, x_a)|x_a\right)} \cdot \frac{\partial u_a}{\partial v}\left(t_b^*(\theta_b, x_a), |x_a - x_b|\right)$$

in the distance $|x_a - x_b|$ (holding θ_b fixed), along with the fact that, by virtue of reciprocity, $t_b^u(\theta_b, x_a) \leq t_b^*(\theta_b, x_a)$ if and only if

$$t_a^u((t_b^*(\theta_b, x_a), x_a), x_b) \leq t_a^*((t_b^*(\theta_b, x_a), x_a), x_b)$$

and, likewise, $t_b^u(\theta_b, x_a) \geq t_b^*(\theta_b, x_a)$ if and only if

$$t_a^u((t_b^*(\theta_b, x_a), x_a), x_b) \geq t_a^*((t_b^*(\theta_b, x_a), x_a), x_b).$$

Q.E.D.

Proof of Proposition 5. The proof follows from the combination of the results in Proposition 4 with the results in Proposition 1 in Aguirre et al (2010). When the environment satisfies the conditions in Part 1 of Proposition 4, starting from uniform pricing on side a, the introduction of customized pricing on side a leads to an increase in prices for nearby locations and a reduction in prices for distant locations. Proposition 1 in Aguirre et al (2010), along with the fact that the environment satisfies Condition IR and that, for any x_b and p'_a, the convexity $CD_a\left(p'_a|x_b; x_a\right)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents declines with the distance $|x_a - x_b|$, then implies that welfare of the side-a agents is higher under uniform pricing. Likewise, under the conditions in Part 2 of Proposition 4, that welfare of the side-a agents is higher under uniform pricing follows from the fact that, starting from uniform pricing on side a, the introduction of customized pricing on side a leads to an increase in prices for distant locations and a reduction in prices for nearby locations. The welfare implications of such price adjustments then follow again from Proposition 1 in Aguirre et al (2010), along with the fact that Condition IR holds and that, for any x_b and p'_a, the convexity $CD_a\left(p'_a|x_b; x_a\right)$ of the demands by the x_a-agents for the q-th unit of the x_b-agents increases with the distance $|x_b - x_a|$. Q.E.D.

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