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Working Paper
Dual Employment Protection Legislation: A Framework for Analysis

IZA Discussion Papers, No. 1564

Provided in Cooperation with:
IZA – Institute of Labor Economics

Suggested Citation: Dolado, Juan José; Jansen, Marcel; Jimeno, Juan F. (2005) : Dual Employment Protection Legislation: A Framework for Analysis, IZA Discussion Papers, No. 1564, Institute for the Study of Labor (IZA), Bonn

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IZA DP No. 1564

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ABSTRACT

Dual Employment Protection Legislation: A Framework for Analysis∗

In many countries, Employment Protection Legislation (EPL) establishes different regulations for certain groups of workers who face more disadvantages in the labor market (young workers, women, unskilled workers, etc.) with the aim of improving their employability. Well-known examples are the introduction of atypical employment contracts (e.g. temporary and determined-duration contracts) which ease firing restrictions for some, but not all, workers. This paper discusses the effects of EPL varying among workers of different skills on the level and composition of unemployment, job flows, productivity and welfare. By using an extension of Mortensen-Pissarides’ (1994) search model where heterogeneous workers compete for the same jobs, we are able to identify several key channels through which changing firing costs for some groups of workers affects hiring and firing of all workers and, hence, may have a different impact on aggregate labor market variables than reducing firing costs across the board. Some analytical and simulation results also show that these effects of differentiated firing costs by workers’ skills may be different depending upon the initial state of the labor market.

JEL Classification: J64, J63

Keywords: firing costs, unemployment, matching

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∗ We are very grateful to Samuel Bentolila, Simon Burgess, Claudio Sapelli, seminar participants at the 7th Conference of the Central Bank of Chile on Labor Markets and Institutions (Santiago), University of Bristol (DAEUP/CEPR Conference), EEA 2004 (Madrid) and Banco de España, and to an anonymous referee for their insightful comments on preliminary drafts of this paper. This paper has been produced as part of a CEPR research network on “A Dynamic Approach to Europe’s Unemployment Problem” funded by the EC under the contract HPSE-CT-2001-00071.
1. Introduction

In many European countries labor market reforms are often framed as employment promotion policies aimed at favoring particularly disadvantaged groups in the labor market. A well-known example is the use of employment subsidies (see, for instance, Drèze and Malinvaud, 1994) targeted at specific population groups like, e.g., young, low-skilled or long-term unemployed workers (see, for instance, Drèze and Malinvaud, 1994). Another, perhaps less well-known, example has been provided by a number of recent reforms in Employment Protection Legislation (EPL, hereafter) whereby the availability of flexible contracts for hiring (part-time, fixed-term, seasonal, etc.) has been restricted to workers belonging to specific categories (related to occupations, skills, age or educational attainments) which typically exclude prime-age workers.\(^1\) Our paper focuses on the effects of the latter reforms which we claim are pervasive across the EPL regulations in many countries.

While there may be good political economy reasons for reforming the labor market through two-tier schemes (see Saint-Paul, 1996, 2000), the economic consequences of allowing for targeted EPL regulations are less well understood. To the best of our knowledge, most papers analyzing the effects of firing costs have generally overlooked the fact that severance payments differ for workers with different skills. This paper aims at filling this gap in the literature by providing a useful analytical framework where to examine the effects of differentiated employment policies in frictional labor markets with heterogeneous workers. We analyze their effects on a number of relevant dimensions of the labor market, such as equilibrium unemployment (and its distribution among workers of different types), job turnover, productivity and welfare.

\(^1\) See Booth, Dolado and Frank (2002).
Our approach builds upon a growing literature on equilibrium unemployment in labor markets with workers and jobs heterogeneity starting with the seminal paper by Mortensen and Pissarides (1994). More specifically, our contribution complements the available studies on the effects of firing costs (e.g. Mortensen and Pissarides, 1999, and Ljungqvist, 2002) and studies of partial reforms focusing on the conversion of fixed-term employment contracts into permanent ones (e.g., Blanchard and Landier, 2002, Cahuc and Postel-Vinay, 2002).

While the literature on dual labor markets has highlighted the consequences of having the option to convert one type of labor contract into another one, as regards to employment and job turnover, it ignores another important feature of dual labor markets, namely the fact that employment policies are often targeted to specific group of workers and that regulations pertaining to one specific segment of the labor market may affect other segments as well. For example, an important channel for these spillover effects stems from changes in the overall labor market tightness, which determines both the exit rate out of unemployment for all workers and the profits of firms from opening vacancies. Furthermore, in as far as these changes in labor market tightness affect workers’ outside option values we may also expect changes in firms’ hiring and firing decisions. An exhaustive analysis of dual EPL therefore

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2 In contrast to Mortensen and Pissarides (1994) where two-sided heterogeneity is considered, we only allow for workers’ heterogeneity. However, whereas in their model the labour market is completely segmented, our contribution here is to relax that result by allowing workers with different characteristic to compete for the same jobs.

3 For instance, both studies find that, after a reduction of firing costs in entry-level jobs, firms find attractive to hire more workers. However, they also become more reluctant to convert them into regular permanent employment contracts as, with low firing costs, taking the chance of matching with another worker may become an attractive option. This leads to a high workers’ turnover and, if the gap in severance pay is sufficiently large, to a rise in unemployment.

4 Belot, Boone and van Ours (2002) analyze the trade-off between productivity and flexibility that may also influence the firm’s decision to convert a temporary job into a permanent one when job stability is productivity-enhancing.

5 Some theoretical analyses of fixed-term contracts (e.g. Cahuc and Postel-Vinay, 2002, and Nunziata and Staffolani, 2001) assume that there are some restrictions on the use of fixed-term contracts and impose a maximum value for the proportion of fixed-term employees that firms can hire. Note, however, that this restriction does not capture the targeted nature of “employment promotion” contracts.
requires a model with endogenous job creation and job destruction, possibly in line with the one we propose here.

One of the main motivations for this paper comes from some previous work by us on the functioning of labor markets with heterogeneous jobs and workers (see Dolado, Jansen and Jimeno, 2003) where it was shown that differentiated firing costs might reduce equilibrium unemployment in labor markets with pervasive mismatch and on-the-job search. Our earlier analytical framework relied upon a matching model with two-sided heterogeneity (skilled and unskilled jobs and low-educated and high-educated workers), where high-educated workers can be mismatched (i.e., can occupy unskilled jobs) and, if so, on-the-job search is exerted. Mismatch of overeducated workers in low-skilled jobs implies a negative externality on firms opening unskilled vacancies when both types of workers are equally productive at this type of jobs since, having a higher quit rate, on-the-job seekers make those jobs more unstable and therefore firms are less prone to open them. Thus, to the extent that larger firing costs for workers in skilled jobs reduces job turnover in these jobs, there might be situations where this type of targeted EPL policy reduces mismatch and the unemployment rates of both types of workers in the presence of skilled-biased technological change.7

In this paper, however, we will abstract from on-the-job search and restrict the analysis to a single type of job which can be filled with either low-productivity workers or high-productivity workers, which may be entitled to different firing costs. This analytical shortcut will allow us to focus on the interactions between both types of workers in the same labor

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6 There is also a positive externality on the supply of unskilled vacancies since more workers are looking for those jobs. However, it can be shown that the negative externality dominates.

7 There are other papers using search equilibrium models with worker and/or job heterogeneity to analyze the effects of some policy measures. For instance, Acemoglu (2001) shows that unemployment benefits and minimum wages increase welfare in a model with heterogeneous jobs in segmented markets. Albrecht and Vroman (2002) analyze a labour market in which low and high-educated workers can be hired for unskilled jobs
market so as to learn about the effects of dual EPL. In particular, building upon Mortensen and Pissarides’ (1994) model, we provide a simple framework where to quantify the efficiency gains and the impact on workers’ welfare of the following alternative policies regarding firing costs: (i) a targeted reduction for the low-skilled workers, (ii) a targeted reduction for the high-skilled workers, and (iii) a comprehensive reduction for all workers.

Another important simplifying assumption is that the firing costs are assumed to be pure waste as, for example, those stemming from judicial red-tape costs, etc. in the process of dismissals (see Burda, 1992). In a more general model in which severance payments could play some insurance or productivity-enhancing role, the losses originating from their reduction ought to be weighed against the gains obtained under our assumption. Yet, the effects stressed here are likely to remain the same.

After performing numerical simulations of the model, our main findings are that i) targeted reductions of firing costs may have different aggregate effects than commensurate reductions across the board and that ii) the effects on unemployment rates and welfare of the above-mentioned targeted reductions of firing costs may qualitatively depend on the initial state of the labor market, the shape of the matching function and the distribution of shocks across workers. By introducing less churning when markets are sclerotic, a targeted reduction of firing costs for low-skilled workers yields higher welfare gains than a commensurate reduction for firing costs for all workers. A targeted reduction of firing costs for low-skilled workers achieves the greatest welfare gains when: (i) the initial labor market tightness is low so that firms fill rapidly the newly created vacancies induced by the reduction in labor costs, (ii) the elasticity of the matching rate of workers with respect to tightness is high (i.e., the matching function is not too concave) so that the process by which unemployed workers

while only high-educated workers can perform skilled jobs, without allowing for on-the-job-search as in Dolado, Jansen and Jimeno (2003).
match with the new vacancies is fast; and (iii) when the volatility of the shocks affecting low-skilled workers is higher since lower firing costs imply larger savings for firms which would have to pay firing costs more frequently given the higher volatility of jobs.

The plan for the rest of the paper is as follows. Section 2 motivates the analysis by describing some recent labor market reforms in several countries which share the common feature of using different EPL regulations for workers with different skills. As will be illustrated, in many countries, not just in Western European, notice periods, procedures for dismissals and severance payments vary across workers’ occupations. Moreover, recent reforms typically amount to targeted reductions of firing costs through the introduction of “atypical contracts”, etc., albeit only for workers with worse employment prospects. Next, in order to search for some empirical evidence, Section 3 summarizes the empirical findings about the effects of dual EPL, both considering cross-country evidence and case studies pertaining to specific country experiences. Section 4 contains the theoretical analysis of the effects of firing costs in labor markets with heterogeneous workers competing for identical jobs. In Section 5 we present numerical simulations of the model and discuss their robustness to changes in some of the key parameters. Lastly, Section 6 concludes. Appendix A presents some comparative-statics results of the model, whereas Appendix B offers the derivation of the ergodic distribution of productivity which underlies the welfare analysis.

2. How EPL differs among workers

It is well-known that EPL varies significantly across countries. However, less attention has been devoted to the fact that EPL also varies within countries depending on firms’ and worker’s characteristics such as firm size, existence of collective agreement, tenure, skill,
educational level, etc.⁸ As regards worker’s skills, there are two sources of variation in the enforcement of EPL. First, procedural requirements for dismissals, notice and severance pay provisions, and prevailing standards of penalties for unfair dismissals are usually stricter for white-collar workers. Secondly, high-skill workers are not always entitled to be hired under “atypical” employment contracts involving less strict EPL provisions.

Examples of countries with EPL provisions that are less strict for blue-collar workers are Austria, Belgium, Denmark, France, Greece and Italy. With the exception of France, the required notice period in the other five countries is shorter for blue-collar workers than for white-collar workers. Typically, severance pay for individual dismissals is similar for both blue-collar and white-collar workers, except in Denmark and Greece, where the former are entitled to lower indemnities. Compensation pay for unjustified dismissal is also lower for blue-collar workers in Belgium and Greece.⁹

With regard to partial reforms based on the introduction of “atypical” employment contracts, Spain provides a paradigmatic case study. After the surge of the proportion of temporary employees in total (salaried) employment (35% in 1995) following the 1984 reform, there have been a series of countervailing labor market reforms during the 1990s (1994, 1997 and 2001) aimed at reducing that share by providing a less stringent EPL for permanent contracts and considerable restrictions on the use of fixed-term contracts.¹⁰

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⁸ OECD (1994) presents a detailed and comprehensive description of EPL in several countries and its variation by worker skills, tenure, the existence of collective agreements, and firm size. For a justification and the implications of variable enforcement of EPL by firm size, see Boeri and Jimeno (2005).

⁹ More institutional details of EPL in these countries are in OECD (1994), Annex 2.A. The information in the text refers to the end of the 1990s.

¹⁰ See Dolado, García-Serrano and Jimeno (20032) for a detailed description of those reforms.
From the viewpoint of this paper, probably the most important reform was the one taking place in 1997 when an agreement to reform the system of labor contracts was reached. The agreement called for the creation of new permanent contracts in case of “unfair dismissals” entailing a mandatory firing cost which was lower than that pertaining to the old permanent contracts (33 days of wages per year of seniority with a maximum of 24 months-wages against 45 days of wages and 42 months-wages, respectively). However, the eligible groups were limited to young workers (aged 18-29), long-term unemployed registered at the public employment office for at least twelve months, unemployed above 45 years of age, disabled people and workers whose contract were transformed from temporary into permanent ones. By contrast, prime-aged workers in the age bracket 30-45 with unemployment spells shorter than a year were excluded. In the 2001 reform, in an attempt to extend the use of the new contacts, the government managed to add young workers between 16 and 30 years of age, long-term unemployed registered for at least six months, and unemployed women of any age working in sectors where they were under-represented.

Yet, Spain is not the only country that has liberalized atypical employment contracts or reduced firing costs contingent on some workers’ characteristics. In 1984 Italy also introduced “employment promotion contracts” (Contratti di Formazione e Lavoro) aimed at the hiring and firm-based training of young workers (between 15 and 29 years of age). Moreover, fixed-term contracts were first introduced in France in 1979 but their scope was very much reduced by the socialist government in 1982. After a reform in 1990, these contracts can only be used for seasonal activities, the replacement of an employee on leave, the hiring of staff on short-term projects, or the replacement of an employee on leave.

11 The reason for this restricted eligibility criterion was that it is against the Spanish constitutional rights to have to identical workers holding an, otherwise, identical open-ended contract except for their severance payments. Thus, the government in accord with the parties in the agreement, made the new contracts only available for specific targeted groups for which it was legal to provide those contracts.
temporary increases in activity and for facilitating employment for targeted groups, ranging from the young to the long-term unemployed workers (see Blanchard and Landier, 2002).

In Latin America as well there have been dual labor market reforms, some aimed at decreasing firing costs (Colombia and Peru at the end of the 1980s) and others at increasing them (Brazil, Venezuela, Chile, the Dominican Republic, Nicaragua, and Panama). However, the only country which significantly liberalized the use of atypical contracts targeted on some demographic groups was Argentina, where a reform in 1991 introduced fixed-term contracts and training contracts for young workers, while a new reform in 1995 introduced special contracts to promote employment of certain population groups.

3. Empirical evidence on the effects of targeted EPL

Broadly speaking, there are two streams in the empirical literature on the labor market effects of institutions. First, there are cross-country studies that use some quantitative or qualitative indicators representing those institutions to explain international differences in labor market outcomes, such as employment and unemployment rates. Within this literature, a large number of recent studies have looked at the interactions between institutions and shocks and to the different impact of institutions on the labor market outcomes of different population groups, such as youths and females. Nonetheless, in most studies, targeted employment policies or partial labor market reforms are considered, if anything, in the construction of the overall institutional indexes regarding EPL strength, but not separately as an institutional

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12 See IDB (2003), chapter 7.
13 See Nickell and Layard (1999).
14 On interactions, see Blanchard and Wolfers (2000). On the different impact of labor market institutions across population groups, see Bertola, Blau, and Kahn (2003), Jimeno and Rodriguez-Palenzuela (2002), and Neumark
feature on its own. This approach can be fairly restrictive since, as will be discussed below, a general reduction of firing costs has not the same labor market effects as a commensurate reduction in the firing costs of a certain group of workers.

Among the studies that estimate the labor market impact of targeted employment policies (e.g., those based on temporary contracts) separately from aggregate indexes of EPL, Jimeno and Rodriguez-Palenzuela (2002) find that a less strict regulation of fixed-term employment contracts tends to reduce youth unemployment rates without any impact on the prime-age male unemployment rate. Likewise, using an unbalanced panel of nine OECD countries during the late 1980s and first half of the 1990s, Nunziata and Staffolani (2001) also try to estimate the effects of EPL by distinguishing three types of regulations: EPL regarding dismissals of permanent employees, regulations regarding fixed-term employees, and temporary work agencies (TWAs) regulations. They find that less stringent fixed-term contract regulations had a significant positive impact on temporary and total employment particularly when the economy is recovering from a recession. In the case of young workers (15-24 years of age), however, they find an increase of both temporary and permanent employment. Lastly, as regards the use TWAs, they find again that less stringent regulations tend to have an incremental effect on both temporary and total employment in downturns. Yet, in the case of young workers, the favorable effects on permanent employments are negligible.

The second stream of the empirical literature looks at specific country episodes in order to measure the effect of reforms by analyzing labor market outcomes before and after the reform, along the lines of the “differences-in-differences” evaluation approach. Studies of this...

kind are, for instance, Kugler, Jimeno and Hernanz (2003) on the Spanish 1997 reform, Blanchard and Landier (2002) on France, and Hopenhayn (2001) on the Argentinian reform. In the Spanish case, Kugler, Jimeno and Hernanz (2003) find that the reduction of firing costs (and payroll taxes) for young, older and long-term unemployed workers had a positive effect on hiring for young workers, with little effect on dismissals, while it increased both dismissals and hiring for older men. Blanchard and Landier (2002), looking at transitions between temporary and permanent employment, observe increased turnover since 1983 in France, specially at younger cohorts, for whom the probability of holding a fixed-term job has increased a lot while their probabilities of staying or becoming unemployed show no clear trend. Finally, Hopenhayn (2001) also finds that the introduction of fixed-term contracts in Argentina has had a very strong impact on labor turnover, inducing an increase in hiring but also some strong substitution of permanent jobs by temporary jobs.

In sum, the available empirical evidence points out that the effects of targeted EPL reforms seemingly depend on the phase of the business cycle when they are implemented and that there exist spillover effects of either sign on those groups not directly affected by the regulations. In the sequel, we develop a simple analytical framework where to understand these phenomena.

4. **A model of firing costs with heterogeneous workers**

Our model draws on Mortensen and Pissarides (1994) with two extensions. First, we allow for heterogeneity in workers’ skills. And, secondly, we assume that the initial productivity of jobs is random. The first extension gets at how reforms aimed at easing firings of one type of workers affects unemployment, productivity and welfare of all workers, both those affected
and those unaffected by the reform. The second extension allows for a more detailed analysis of how the hiring of different types of workers depends crucially on the structure of firing costs. Firing costs are modeled as pure waste (not as a transfer to the worker) and, hence, do not play any efficiency role.\textsuperscript{15}

As is conventional, the model is in continuous time and only steady states are considered. The economy is populated by a continuum of workers of measure one. Workers are risk neutral, infinitely lived, and are of two types depending on their productivity (low, L, and high, H). Firms know the worker’s type, the arrival rate of productivity shocks and the distributions from which productivity is drawn from each worker’s type. The mass of workers of type L is $\alpha$. The income obtained while unemployed is $z_i$ ($i=L,H$), which may differ for low skilled and high skilled workers.

The number of firms is endogenously determined. Each firm offers one job. The cost of keeping a job vacancy unfilled is $c$. Vacancies are created until the exhaustion of any rents from vacancy creation. When a worker and a firm with a job vacancy meet, they realize the value of the match. The productivity of the match is a random draw from a c.d.f. $F^i(\varepsilon)$ with support $[0, 1]$, ($i=L,H$), such that $F^L(\varepsilon)>F^H(\varepsilon)$ for all $\varepsilon$. Thus, the distribution of productivity of H-type workers stochastically dominates the distribution of productivity of L-type workers. Wages are determined by continuous Nash bargaining.

Job termination is endogenous. There are $i.i.d.$ productivity shocks with Poisson arrival rates $\lambda_i$ ($i=L,H$). To terminate the job, firms must pay pure-waste dismissal costs $K_i$ ($i=L,H$).

\textsuperscript{15}More plausibly, there could be some efficiency and insurance roles that could justify an optimal level of firing costs different from zero. As discussed in the Introduction, we abstract from those to keep the analysis manageable.
Further, we assume that all separations involve dismissal costs. By allowing for different termination costs for different types of workers we aim at capturing the “targeted/dual” nature of EPL discussed at length in the previous sections. Our intuition is that there are direct and indirect effects of reducing the firing costs for L-type workers. First, the direct effect stems from the fact that the productivity threshold at which L-type workers are dismissed (hired) is higher (lower) the lower $K_L$ is. The indirect effects, in turn, arise through the determination of the value of jobs filled with by H-type workers which also changes when $K_L$ is reduced.

**Matching, hiring, and firing**

Job vacancies and unemployed workers meet according to a conventional CRS matching function $m(v,u)$, where $v$ and $u$ denote, respectively, the masses of job vacancies and of unemployed workers. The matching function is increasing in both arguments and homogeneous of degree one. Labor market tightness is denoted by $\theta = v/u$. Given the matching function, firms meet with L-type unemployed workers with probability $δq(θ)$ and with H-type unemployed workers with probability $(1-δ)q(θ)$, where $δ$ is the proportion of unemployed workers of type L and $q(θ)=m(1,1/θ)$ being $q'(θ)<0$. The matching rate of workers is $θq(θ)$.

After meeting a worker and knowing the match-specific productivity, employers face a hiring decision. Since the surplus of the match is increasing in productivity, there are two productivity thresholds $(ε^b_L, ε^b_H)$ above which hiring takes place. As for the firing decision, after the match is hit by a productivity shock, employers decide whether or not to terminate the job. For each worker’s type there are again two productivity thresholds $(ε'^d_L, ε'^d_H)$ below which jobs are terminated.
**Flows**

Given the matching probabilities and the hiring and firing rules, the flow equations are given by:

\[
(1 - F^L(e^h_L))\theta q(\theta)\delta u = \lambda^L F^L(e^d_L)e_L, \tag{1}
\]

\[
[1 - F^H(e^h_H)]\theta q(\theta)(1 - \delta)u = \lambda^H F^H(e^d_H)e_H, \tag{2}
\]

where \(e_L\) and \(e_H\) are the masses of L and H-type employed workers, respectively. The left-hand-sides of (1) and (2) give the outflows from unemployment while the right-hand-sides give the inflows to unemployment (i.e., outflows from employment) for L and H-type workers, respectively.

Since \(\delta u + e_L = \alpha\) and \((1 - \delta)u + e_H = 1 - \alpha\), the steady state unemployment rates of both types of workers are:

\[
ur_L = \frac{\delta u}{\alpha} = \frac{\lambda^L F^L(e^h_L)}{[(1 - F^L(e^h_L))\theta q(\theta) + \lambda^L F^L(e^d_L)]}, \tag{1'}
\]

\[
ur_H = \frac{(1 - \delta)u}{1 - \alpha} = \frac{\lambda^H F^H(e^h_H)}{[1 - F^H(e^h_H)]\theta q(\theta) + \lambda^H F^H(e^d_H)}. \tag{2'}
\]

**Bellman equations**

Let \(U_i\) and \(W_i(\epsilon)\) denote, respectively, the value of unemployment and the value of employment with productivity \(\epsilon\), for workers of type \(i (=L,H)\). Then, the corresponding Bellman equations are given by:

\[
rU_i = z_i + \theta q(\theta)\int_{\epsilon^i}^{1} [W_i(x) - U_i]dF^i(x) \tag{3}
\]
where \( r \) is the interest rate, \( z \) is the flow utility while unemployed (interpreted here as home production or leisure and, thus, not to be financed), and \( w \) is the wage. Notice that, since there is continuous renegotiation, wages depend on productivity and change instantly every time a productivity shock occurs.

As regards the employers, the value functions of an unfilled vacancy (\( V \)) and the value functions of filled vacancies with worker of type \( i \) (\( J_i \)) are given by the following Bellman equations:

\[
\begin{align*}
    rV &= -c + \delta q(\theta) \frac{1}{\lambda} \int_{\varepsilon}^{} [J_i(x) - V] dF^i(x) + (1 - \delta)q(\theta) \frac{1}{\lambda} \int_{\varepsilon}^{} [J_{ii}(x) - V] dF^{ii}(x) \\
    rJ_i(\varepsilon) &= \varepsilon - w_i(\varepsilon) + \frac{1}{\lambda} \int_{\varepsilon}^{} [J_i(x) - J_i(\varepsilon)] dF^i(x)
\end{align*}
\]

Wage determination

When a match is formed, wages are determined by symmetric Nash bargaining with continuous renegotiation, where the bargaining power for each party is equal to 0.5. This implies:

\[
J_i(\varepsilon) - V + K_i = W_i(\varepsilon) - U_i
\]

Hence, in our setup, workers get insider power for firing costs since the beginning of the match to extract the rents from firing costs. In other words, the possibility of undoing the detrimental effect of firing costs on firm’s profits by the worker accepting a wage cut at the
beginning of the match is excluded. As shown by Ljungqvist (2002), this assumption is crucial for the analysis of the employment effects of firing costs. When firing costs are assumed to reduce the firm’s threat point in the initial match, as in equation (7), they have a significantly impact on hiring and tend to increase equilibrium unemployment while, by contrast, they tend to increase employment when the worker’s relative share of match surplus is assumed to remain invariant as the severance pay varies.\textsuperscript{16} Under our assumption of continuous bargaining, the wage determination condition in equation (7) determines both the relative split of the match surplus when firms bargain with not yet hired workers, and in bargains with hired workers in consecutive renegotiations. Thus, the effects of firing costs on wages are internalized upon the initial match.

**Equilibrium**

The productivity thresholds at which the hiring process starts to take place are those at which the value of a filled vacancy is equal to the value of an unfilled vacancy. Since there is free entry, \( V=0 \) in the steady-state equilibrium. Likewise, jobs are terminated when the value of the job is equal to the value of an unfilled vacancy minus termination costs. Thus,

\[ J_i(e_i^h) = V = 0 \]  \hspace{1cm} (8)

\[ J_i(e_i^{ud}) + K_j = V = 0 \]  \hspace{1cm} (8')

\textsuperscript{16} Mortensen and Pissarides (1999) propose alternative specifications of the bargaining process in which the worker extract rents from firing costs in continuing matches but not in the first match. Ljungqvist (2002) shows that this kind of a two-tier wage system is formally equivalent to assuming that the relative split of the match surplus is unaffected by firing costs throughout the employment relationship. This equivalence arises by imposing a wage profile under the Mortensen and Pissarides (1999) set up under which new workers post a bond equal to their share of any future expected firing costs. A version of the model where the outside option of the firm for newly created jobs is simply \( V \), and not \( V + K \), has also been calibrated with qualitatively similar results and is available upon request.
Solving the model

The surplus of a job of productivity \( \varepsilon \) occupied by a worker of type \( i \) is

\[
S_i(\varepsilon) = J_i(\varepsilon) - V + W_i(\varepsilon) - U_i
\]

Equations (4) and (6) can be rewritten as follows:

\[
(r + \lambda_i)[W_i(\varepsilon) - U_i] = w_i(\varepsilon) - z_i + \frac{1}{\varphi_i} \int_{\varepsilon}^z [W_i(x) - U_i]dF_i(x) - \theta q(\theta) \int_{\varepsilon}^z [W_i(x) - U_i]dF(x) \quad (4')
\]

\[
(r + \lambda_i)[J_i(\varepsilon) - V + K_i] = \varepsilon - w_i(\varepsilon) + \frac{1}{\varphi_i} \int_{\varepsilon}^z [J_i(x) - V + K_i]dF_i(x) - r(V - K_i) \quad (6')
\]

Hence, adding up those two equations and using (7) yields:

\[
(r + \lambda_i)S_i(\varepsilon) = \varepsilon - z_i + \frac{1}{\varphi_i} \int_{\varepsilon}^z S_i(x)dF_i(x) - \frac{\theta q(\theta)}{2} \int_{\varepsilon}^z S_i(x)dF(x) - r(V - K_i) \quad (9)
\]

Further, noting that \( S_i'(\varepsilon) = \frac{1}{r + \lambda_i} \) and integrating by parts implies that:

\[
\int_{\varepsilon}^z S_i(x)dF(x) = [1 - F_i(\varepsilon)]S_i(\varepsilon) + \frac{1}{r + \lambda_i} \int_{\varepsilon}^z [1 - F_i(x)]dx \quad \text{for all } \varepsilon
\]

Thus,

\[
(r + \lambda_i)S_i(\varepsilon) = \varepsilon - z_i + \frac{\lambda_i}{r + \lambda_i} \int_{\varepsilon}^z [1 - F_i(x)]dx - \frac{\theta q(\theta)}{2(r + \lambda_i)} \int_{\varepsilon}^z [1 - F_i(x)]dx -
\]

\[
- \frac{\theta q(\theta)}{2} [1 - F_i(\varepsilon)]S_i(\varepsilon) - r(V - K_i) \quad (10)
\]

This equation gives the productivity thresholds values for hiring and firing.

Since \( S_i(\varepsilon_i^d) = 0 \) and \( S_i(\varepsilon_i^h) = 2K_i \), and in equilibrium the value of an unfilled vacancy is nil, then:
\[\varepsilon^d_i = z_i - \frac{\lambda_i}{r + \lambda_i} \int_{\varepsilon_i^d}^1 \left[ \frac{1}{2} S_L(x) - K_L \right] dx + \frac{\theta q(\theta)}{2(r + \lambda_i)} \int_{\varepsilon_i^d}^1 \left[ \frac{1}{2} S_H(x) - K_H \right] dx + \left[ \theta q(\theta)[1 - F^L(\varepsilon_i^d)] - \frac{\lambda_i}{2} \right] K_i \] \tag{11}

\[\varepsilon^h_i = z_i - \frac{\lambda_i}{r + \lambda_i} \int_{\varepsilon_i^h}^1 \left[ \frac{1}{2} S_L(x) - K_L \right] dx + \frac{\theta q(\theta)}{2(r + \lambda_i)} \int_{\varepsilon_i^h}^1 \left[ \frac{1}{2} S_H(x) - K_H \right] dx + \left[ \theta q(\theta)[1 - F^L(\varepsilon_i^h)] + r + 2\lambda_i \right] K_i \] \tag{12}

so that \(\varepsilon^h_i - \varepsilon^d_i = 2(r + \lambda_i)K_i\). Equations (11) and (12) give the job creation and job destruction rules. Notice that all of them depend on labor market tightness. Thus, the rules for hiring and firing each type of worker depend on labor market tightness, which, in hand, is determined by the job flows implied by those hiring and firing rules.

Finally, in equilibrium the supply of vacancies is determined by the free-entry condition, \(V = 0\), which can be written as follows:

\[\frac{c}{q(\theta)} = \delta \int_{\varepsilon_i^d}^1 \left[ \frac{1}{2} S_L(x) - K_L \right] dF^L(x) + (1 - \delta) \int_{\varepsilon_i^h}^1 \left[ \frac{1}{2} S_H(x) - K_H \right] dF^H(x) = \frac{\delta}{2(r + \lambda_L)} \int_{\varepsilon_i^d}^1 [1 - F^L(x)]dx + \frac{1 - \delta}{2(r + \lambda_H)} \int_{\varepsilon_i^h}^1 [1 - F^H(x)]dx \] \tag{13}

A graphic illustration

The set of equations above can be grouped in three blocks. The two pairs of equations in (11) and (12) give the relationships between the hiring and firing thresholds, on the one hand, and labor market tightness, on the other. As tightness rises, workers have a higher reservation value and, hence, the firing productivity thresholds increase (see equation A.1 in Appendix A). Further, insofar as job destruction is higher, the firms initially hire worker at higher levels of productivity, so that the hiring threshold also increases with tightness. The increasing relationships between the firing thresholds and tightness are represented, respectively, by the upwards sloping loci JD in Panels I (low skilled workers) and II (high skilled workers) of
Figure 1. Also, the increasing relationships between the hiring thresholds and tightness are represented, respectively, by the upwards sloping loci JH in Panels III (low skilled workers) and IV (high skilled workers) of the same Figure.

The effects of firing costs on the hiring and firing thresholds are standard. For given tightness, the higher firing costs are, the lower is the firing threshold, and the higher is the hiring threshold (see equations A.2 and A.3 in Appendix A). Hence, when firing costs are reduced there is more job destruction and more hiring, as is conventionally found in the literature. This means that, in Figure 1, the loci JD shifts to the left and the loci JH shifts to the right as firing costs fall.

The second block of the model is the job creation condition given by equation (13). This condition captures the relationships between tightness, the hiring thresholds and the skill composition of the unemployment pool. First, as the firing threshold rises the expected duration of the match is shorter and, hence, the expected surplus from filled vacancies falls. Thus, vacancy creation is lower and tightness decreases (see equation A.4 in Appendix A). This relationship is plotted as the downwards-sloping loci JC in Panels I to IV of Figure 1. Secondly, as the proportion of low skill unemployed changes, tightness also changes in a way that depends on the difference between the expected surplus from hiring low-skilled workers and the expected surplus from hiring high-skilled workers. Plausibly, the expected surplus from jobs filled by high-skilled workers is larger and tightness decreases with the proportion of low-skilled unemployed (see equation A.6 in Appendix A). As for the effects of firing costs, there two effects on job creation. First, there is the direct effect (last two terms in equation (13)) of making the expected surplus of the match lower and, therefore, induce firms

17 Mortensen and Pissarides (1994) assume that all jobs start at the same productivity level, so that in their model there is no hiring rule.
to open fewer vacancies. Secondly, by making firing and hiring less frequent, they also decrease the expected gains from vacancy creation and reduce tightness. This means that the locus JC shifts upwards as firing costs are reduced (see equations A.7 and A.8 in Appendix A).

Finally, equations (1’) and (2’) show how tightness and thresholds determine the size and the composition of the unemployment pool, giving the so-called Beveridge curve, which we plot as the downwards-sloping locus BC in Panel V of Figure 1. The number of unemployed workers of each group rises with the corresponding job destruction threshold. Thus, unemployment is decreasing in labor market tightness, as workers receive job offers more often when the labor market is tighter. This locus shifts to the right as the destruction thresholds rises following a reduction of firing costs. Panels VI and VII of Figure 1 represent the determination of the skill composition of unemployment by yielding the number of L and H-type unemployed, respectively, given the lines from the origin with slopes \( \delta \) and 1- \( \delta \).
Figure 1. The model
Figure 2, in turn, depicts the effects of lowering firing costs for the L-type workers ($K_L$ is decreased) while Figure 3 represents the effects of a comprehensive reduction of firing costs (both $K_L$ and $K_H$ fall). A reduction in the firing costs of L-type workers shifts the job creation loci (13) upwards in Panels I, II, III and IV. As for the hiring and firing rules, the locus representing the firing decision of L-type workers, (JD) in Panel I, shifts to the left, while the locus representing the hiring decision of low skill workers, (JC) in Panel III, shifts to the right. Notice that the loci representing the firing and hiring decisions of H-type workers remain invariant as we are only considering a reduction of firing costs for the L-type workers.

In the new equilibrium, tightness increases, the job destruction thresholds are higher for both types of workers, the hiring threshold for H-type worker also rises, whereas the effect on the hiring threshold for L-type workers has an ambiguous sign. Panels V, VI and VII complete the representation of the new equilibrium, plotting the corresponding unemployment rate and the proportion of L-type workers. In Panel V, the BC locus shifts upwards as job destruction thresholds increase. The change in the proportion of L-type unemployed has an ambiguous sign. In Figure 2, only for illustrative purposes, we assume that this proportion falls, so that in Panel VI the slope of the line from the origin falls, while the corresponding line for H-type workers in Panel VII becomes steeper. As can be observed, total unemployment may either rise or fall depending on the relative size of the shift of the BC curve and of the rise in tightness. Assuming that both aggregate unemployment and the proportion of low skilled unemployed fall, the unemployment rate of the L-type workers decreases while the unemployment rate of high skilled workers may rise (as in Figure 2) or fall depending on the relative size of the changes in total unemployment and in the composition of the pool of unemployed.
Figure 3 represents the effects of a comprehensive reduction in firing costs (both $K_L$ and $K_H$ are decreased). As in the previous case, the job creation locus (13) shifts upwards, while the loci representing the firing rules (JD) shift to the left, and those representing the hiring rules (JH) shift to the right for both types of workers. Thus, as in the case of a targeted reduction, both tightness and job destruction increase, but now the effect of the overall reduction of firing costs on the hiring thresholds of all workers has an ambiguous sign. Notice also that, as before, the skill composition of unemployment may change either way. Assuming again that both total unemployment and the proportion of L-type unemployed fall, the unemployment rate of low skilled workers also falls, while the sign of the change in the unemployment rate of high skilled workers is ambiguous.

We now turn to some numerical simulations of the model in order to grasp further insights on the magnitude of the effects at work.
Figure 2. A reduction in firing costs of low skilled workers ($K_L$)
Figure 3. A comprehensive reduction in firing costs
5. Numerical examples

To solve the model we must find a vector of variables \((\delta, u, \theta, \varepsilon_L^h, \varepsilon_L^d, \varepsilon_H^h, \varepsilon_H^d)\) satisfying equations \((1')\), \((2')\), \((11)\), \((12)\) and \((13)\). Note that equations \((11)\) and \((12)\) come in pairs, so that we have seven equations in seven unknowns. The solution to this system cannot be found analytically, and so one has to resort to numerical simulations in order to grasp some understanding of the effects of reductions of firing costs. To perform simulations, we assume that the productivity of L-type workers, \(\varepsilon_L\), is uniformly distributed in \([0,1]\) and that the productivity of H-type workers, \(\varepsilon_H\), is uniformly distributed in \([\varepsilon_H^{\min}, 1]\), with \(\varepsilon_H^{\min} > 0\). Thus,

\[
F^i(x) = \frac{x - \varepsilon_i^{\min}}{1 - \varepsilon_i^{\min}} \quad \varepsilon_i^{\min} \leq x \leq 1
\]

with \(\varepsilon_L^{\min} = 0\) and \(\varepsilon_H^{\min} > 0\).

Under the assumption of uniform distributions, the system of seven equations to be solved is as follows:

\[
ur_L^i = \frac{\lambda_L \varepsilon_L^d}{(1 - \varepsilon_L^h) \theta q(\theta) + \lambda_L \varepsilon_L^d}
\]

\[
ur_H^i = \frac{\lambda_H (\varepsilon_H^d - \varepsilon_H^{\min})}{(1 - \varepsilon_H^h) \theta q(\theta) + \lambda_H (\varepsilon_H^d - \varepsilon_H^{\min})}
\]

\[
\varepsilon_L^d = z_L - \frac{\lambda_L (1 - \varepsilon_L^d)^2}{2(r + \lambda_L)} + \frac{\theta q(\theta) (1 - \varepsilon_L^h)^2}{4(r + \lambda_L)} + \left[\frac{\theta q(\theta) (1 - \varepsilon_L^h)}{(1 - \varepsilon_L^{\min})} - r\right] K_L
\]

\[
\varepsilon_H^d = z_H - \frac{\lambda_H (1 - \varepsilon_H^d)^2}{2(r + \lambda_H)(1 - \varepsilon_H^{\min})} + \frac{\theta q(\theta) (1 - \varepsilon_H^h)^2}{4(r + \lambda_H)(1 - \varepsilon_H^{\min})} + \left[\frac{\theta q(\theta) (1 - \varepsilon_H^h)}{(1 - \varepsilon_H^{\min})} - r\right] K_H
\]

\[
\varepsilon_L^h - \varepsilon_L^d = 2(r + \lambda_L) K_L
\]

\[
\varepsilon_H^h - \varepsilon_H^d = 2(r + \lambda_H) K_H
\]

---

18 This assumption simplifies the computation but can be restrictive. In effect, by assuming uniform distributions for productivity we minimize the employment changes after variations in the hiring and destruction thresholds which would be significantly higher with more skewed distributions.
Throughout the set of simulations presented below we choose parameter values following previous calibration exercises in the received literature (e.g. Mortensen and Pissarides, 1999, and Ljungqvist, 2002), with some variations to perform some robustness analysis and comparisons across labor markets with different structural conditions. Since targeted employment policies with dual EPL are most often observed in “sclerotic” labor markets, we start by performing a numerical example of the effects of reducing firing costs in a market characterized by high unemployment, and low tightness and job turnover. Then, we will choose alternative parameter values to calibrate a “tight” labor market, with low unemployment, and high tightness and job turnover. In a following subsection we discuss robustness of the results.

The model period is one quarter. Following Ljungqvist (2002) and Mortensen and Pissarides (1994), we set the quarterly interest rate \( r = 0.01 \). As for the matching function, we take

\[
m(u, v) = \frac{huv}{\left(0.5u^\gamma + 0.5v^\gamma\right)^{\frac{1}{\gamma}}}
\]

where \( h > 0 \) is a shift parameter and \( \gamma \) is a parameter capturing the decreasing returns in the elasticity of the job finding-rate w.r.t. tightness so that for higher values of \( \gamma \) concavity increases.\(^\text{19}\) Initially, we choose \( h = 0.5 \) and \( \gamma = 1 \), which implies a strong concavity (SC) of the matching function. In our baseline simulations of a model with homogeneous workers, this matching function delivers quarterly exit rates of unemployment ranging from 14.5% to 26.1%, which imply that the expected average duration of an

\[ \frac{c}{q(\theta)} = \frac{\delta(1-\varepsilon^h_l)^2}{4(r + \lambda_l)} + \frac{(1-\delta)(1-\varepsilon^h_y)^2}{4(r + \lambda_y)(1-\varepsilon^{max}_y)} \]

\(^\text{19} \)This functional form has been proposed by Den Haan, Ramey and Watson (2000). Note that when \( \gamma \uparrow 0 \), it becomes the Cobb-Douglas matching function, \( m(u, v) = h \left( u^{\delta^2} + v^{\delta^2} \right)^{\frac{1}{\delta}} \). Furthermore, the elasticity of the matching rate of workers, \( \theta q(\theta) \), with respect to \( \theta \) for this functional form is \( (1+\theta^\gamma)^{-1} \). Thus, the higher is \( \gamma \), the lower will be that elasticity. In the Cobb-Douglas case, the elasticity is \( \frac{1}{2} \) and, thus, constant.
unemployment spell is between 11 and 21 months, values in line with those observed in the real world.

The rest of the parameter values are also chosen following previous calibration exercises in the literature and trying to match some of the characteristics of a sclerotic labor market. The flow cost of posting a vacancy is set at $c=1/3$, namely about $2/3$ of average worker’s productivity. The flow utility for being unemployed is set at $z_L = z_H = 0.25$, namely half of worker’s average productivity. The arrival rates of productivity shocks, $\lambda_L$ and $\lambda_H$, are initially set at $\lambda_L = \lambda_H = 0.081$, as in Mortensen and Pissarides (1994), which, for homogeneous workers with a uniform distribution of productivity in [0,1], yields quarterly firing rates ranging from 3.2% (for high firing costs) to 4% (nil firing costs). The proportion of high skilled workers and their minimum value of the productivity are set at $\alpha=1/2$ and $\varepsilon_{H\min}=1/3$, which for the highest value of firing costs yields that the proportion of low skilled unemployed is 74.9%.

We compute the stationary equilibria corresponding to alternative values of firing costs ranging form 0 to a highest value of $K_L=K_H= 1$, which is roughly of the same order of magnitude as six month of an average worker’s production. We look at the effects of changing the firing costs for L-type workers ($K_L$) in the range [0, 1] on: i) labor market tightness ($\theta$), ii) unemployment rates, iii) productivity thresholds levels for hiring and firing, iv) asset values for each type of worker and employment status, and v) average productivity and total production (net of vacancy costs). Asset values, average productivity and production are computed using the ergodic distribution of employment, which is derived in Appendix B. We compare these results to the effects of a reform reducing firing costs for high-skilled
workers (that is, changing the firing costs for H-type workers \(K_{H}\) in the range \([0, 1]\)), and to the effects of a comprehensive reform (namely, a similar reduction of both \(K_L\) and \(K_H\)).

Simulation I: A sclerotic labor market

Given a labor market characterized for the set of parameter values discussed above, Figures 4a, 4b and 4c present, respectively, the steady state values of the main variables under i) a targeted reduction of firing costs of low skilled workers, ii) a targeted reduction of firing costs of high skilled workers, and iii) a comprehensive reform of firing costs for all workers. Table 1 gathers the results for three extreme cases (\(K_L = K_H = 1\), \(K_L = 0\) and \(K_H = 1\), and \(K_L = 1\) and \(K_H = 0\)). We now comment on the main salient features of these results and discuss the economic forces at work.

In this sclerotic labor market, there is initially a very low value of labor market tightness (\(\theta\) is around 0.54) and high unemployment rates of L-type (17.9%) and H-type workers (6.0%). This means that initially only 24.1% of the unemployed are high-skilled. As observed in panel 1 of Figure 4a, a reduction of \(K_L\) from 1 (the benchmark value) to 0 increases tightness, giving rise to a large reduction of the unemployment rate of L-type workers of about 4.2 percentage points, while the unemployment rate of H-type workers increases by about 2.1 percentage points (panel 2).\(^{20}\) Also, as \(K_L\) falls and the labor market becomes tighter and both the hiring and the firing rates of L-type workers increase. By contrast, as discussed before, we observe a parallel increase in the hiring and firing thresholds for H-type workers (as \(K_H\) remains unchanged), giving rise to an increase in their unemployment rate (panels 2 and 4). Notice also that as \(K_L\) falls, average productivity rises, since the less productive matches are

\(^{20}\) Note that, in order to follow the correct direction of changes as \(K_L\) decreases, the Figures should be looked from right to left since the horizontal axis of the panels display increasing values of \(K_L\).
destroyed, and therefore total production increases. Welfare of both types of workers increases, regardless of their employment status (panel 5), the reason being that their wages (conditional on having a job) are higher, because of the higher value of being unemployed (higher $0q(0)$), and the higher average productivity of employed workers (panel 6).

Next, Figure 4b plots the effects of a similar reform, this time targeted at reducing only the firing costs of H-type workers, instead of L-type workers. Although the results are qualitatively similar to those discussed before, the quantitative effects are significantly different, as (i) the increase in tightness is smaller; (ii) the fall in unemployment rate of L-type workers and the rise in the unemployment rate of H-type workers are almost negligible, and iii) the welfare gains are significantly lower than under a targeted reduction of firing costs of low skilled workers.

Finally, the effects of the two above-mentioned targeted reforms are also quantitatively different to those of a comprehensive reform resulting in a simultaneous reduction of the firing costs for all workers by the same amount. Since both types of workers have the same weight in total population, we compare the above-mentioned targeted reforms where firing costs go from 1 to 0 for each group to a comprehensive reform where severance payments go down from 1 to 0.5 for all workers. Figure 4c plot the steady state values of the main labor market variables, while Table 1 provides a summary of the comparison with the two previous targeted reductions of firing costs commented above. The main conclusions to be drawn are the following. Labor market tightness is highest and the unemployment rate of low skilled workers is lowest under a targeted reduction of firing costs for L-type workers. Total production and average productivity are highest under a targeted reduction of firing costs for H-type workers, since in this case the hiring threshold for this type of workers is lowest and
the destruction rate is highest. All workers, regardless of their skills and employment status, are better off under a targeted reduction of firing costs for L-type workers. Thus, this type of EPL policy achieves the largest welfare gains in sclerotic labor markets. It unambiguously reduces the unemployment rate of L-type workers while that of H-type workers does not increase by much or, as will be discussed below, may even decrease if one allows for a faster response of the job-finding rate as the labor market gets tighter. In this last case, any increase in expected profits stemming from lower firing costs for less productive workers will therefore translate into both a strong increase in job creation and in the number of matches, which may end up cutting not only the unemployment of those L-type workers directly affected by the reform but also the unemployment rate of the H-type workers. Hence, this targeted reform turns out to be the more welfare-enhancing for all workers in the sense that the asset values of being employed or unemployed raise by a larger amount (because the higher unemployment, if any, of H-type workers is more than compensated by their higher wages whilst both wages and employment of L-type workers raise) and total production (net of costs of opening vacancies) also achieves the largest increase.\(^\text{21}\)

\(^{21}\) As mentioned in note 13, we are not considering other efficiency and insurance reasons which may yield a decreasing welfare effect of reducing firing costs.
Figure 4a. Reducing firing costs of low-skilled workers in a sclerotic labor market.
Figure 4b. Reducing firing costs of high-skilled workers in a sclerotic labor market
Figure 4c. A comprehensive reduction of firing costs in a sclerotic labor market
Table 1. Comparison of the effects of several EPL reforms in a sclerotic labor market

<table>
<thead>
<tr>
<th></th>
<th>Initial situation</th>
<th>Lower firing costs for low-skill workers</th>
<th>Lower firing costs for high-skill workers</th>
<th>Comprehensive reform</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_l=K_h=1$</td>
<td>$K_l=0, K_h=1$</td>
<td>$K_l=1, K_h=0$</td>
<td>$K_l=K_h=0.5$</td>
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<tr>
<td>Tightness, $\theta$</td>
<td>0.54</td>
<td>0.88</td>
<td>0.71</td>
<td>0.78</td>
</tr>
<tr>
<td>Proportion of low-skilled unemployed, $\delta$ (%)</td>
<td>74.9</td>
<td>63.0</td>
<td>73.6</td>
<td>68.1</td>
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<td>Unemployment rates (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Low-skilled workers, $u_{l_L}$</td>
<td>17.9</td>
<td>13.7</td>
<td>17.7</td>
<td>15.8</td>
</tr>
<tr>
<td>High-skilled workers, $u_{l_H}$</td>
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<td>8.1</td>
<td>6.4</td>
<td>7.4</td>
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<td>Thresholds levels</td>
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<td></td>
<td></td>
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<tr>
<td>Low-skilled workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Destruction, $\epsilon_{l}^d$</td>
<td>0.397</td>
<td>0.479</td>
<td>0.429</td>
<td>0.458</td>
</tr>
<tr>
<td>Hiring, $\epsilon_{l}^h$</td>
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<td>0.479</td>
<td>0.611</td>
<td>0.549</td>
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<tr>
<td>High-skilled workers</td>
<td></td>
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</tr>
<tr>
<td>Destruction, $\epsilon_{h}^d$</td>
<td>0.438</td>
<td>0.497</td>
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<td>0.507</td>
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<td>Hiring, $\epsilon_{h}^h$</td>
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<td>0.679</td>
<td>0.506</td>
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<td>Exit rates from unemployment (%)</td>
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<tr>
<td>Low skilled workers</td>
<td>14.8</td>
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<td>High skilled workers</td>
<td>20.0</td>
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<tr>
<td>Firing rates (%)</td>
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<tr>
<td>Low skilled workers</td>
<td>3.2</td>
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<td>High skilled workers</td>
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<td>2.1</td>
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<td>Asset values</td>
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<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Unemployed</td>
<td>40.9</td>
<td>42.5</td>
<td>41.7</td>
<td>42.2</td>
</tr>
<tr>
<td>Employed</td>
<td>45.4</td>
<td>46.3</td>
<td>45.9</td>
<td>46.2</td>
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<tr>
<td>High-skilled workers</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>41.9</td>
<td>43.2</td>
<td>42.6</td>
<td>43.0</td>
</tr>
<tr>
<td>Employed</td>
<td>46.7</td>
<td>47.3</td>
<td>46.9</td>
<td>47.2</td>
</tr>
<tr>
<td>Total production</td>
<td>0.644</td>
<td>0.662</td>
<td>0.669</td>
<td>0.667</td>
</tr>
<tr>
<td>Average productivity</td>
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<td>0.743</td>
<td>0.761</td>
<td>0.755</td>
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<tr>
<td>Workers’ welfare (weighted asset values)</td>
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<tr>
<td>Low-skilled workers</td>
<td>41.76</td>
<td>43.13</td>
<td>42.45</td>
<td>42.86</td>
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<tr>
<td>High-skilled workers</td>
<td>46.77</td>
<td>47.25</td>
<td>46.89</td>
<td>47.14</td>
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<tr>
<td>Difference with respect to initial situation (%)</td>
<td>--</td>
<td>3.28</td>
<td>1.65</td>
<td>2.63</td>
</tr>
<tr>
<td>Low-skilled workers</td>
<td>--</td>
<td>1.03</td>
<td>0.26</td>
<td>0.79</td>
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<tr>
<td>High-skilled workers</td>
<td>--</td>
<td></td>
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</tbody>
</table>

Notes: Steady state values for simulations with the following set of parameter values: $r=0.01$, $h=0.5$, $\gamma=1$, $c=1/3$, $\alpha=1/2$, $\epsilon_{H}^{min}=1/3$, $z_{L} = z_{H} = 0.25$, $\lambda_{L} = \lambda_{H} = 0.081$. 

36
The previous results provide some relevant insights on the effects of EPL reforms in labor markets with heterogeneous workers. First, the turnover rates of all workers are affected by any reform, regardless of the group at which the reduction of firing costs is targeted. Secondly, there are several effects of the EPL reforms which may have opposite welfare implications. In principle, lower firing costs imply more job destruction but also higher labor market tightness and a higher outflow rate of unemployment. Moreover, a tighter labor market implies higher wages for employed workers. Third, lower firing costs increase the value of a filled job and the value of an unfilled vacancy and, hence, job creation rises. Under the assumptions of the previous simulation, it turns out that the sum of all these effects is a welfare-enhancing result, particularly when the reduction of firing costs affects only the L-type workers. This is so because in a sclerotic labor market the job creation effect dominates the negative effects of higher job destruction on workers’ welfare.

To gain further insights on this intuition, we now report the simulation results of the three types of EPL reform considered above in a labor market in which tightness is initially high. Thus we change some parameter values to simulate a labor market in which initially the unemployment rate is about half of the average unemployment rate in Simulation I above. We increase the scale matching parameter to $h=2$ and reduce the unemployment flow incomes to $z_L = z_H = 0.1$, the flow cost of posting a vacancy to $c = 0.1$ and the arrival rates of productivity shocks to $\lambda_L = \lambda_H = 0.04$.\footnote{Notice that asset values and welfare cannot be compared across simulations, as we are imposing different utility values of unemployment in each case.} Figures 5a and 5b present, respectively, the effects of reducing the firing costs of L-type and H-type workers, while Figure 5c present a similar set of results.
regarding a simultaneous reduction of firing costs for both types of workers. Table 2 gives the comparison of three reductions of firing costs of the same order of magnitude.

In this tight labor market, initially (i.e., when $K_L = K_H = 1$) the vacancy-unemployment ratio is 2.72, the unemployment rates are 6.9% and 5.5% for L-type and H-type workers, respectively, so that the proportion of H-type unemployed is 55.8%. Both targeted reforms, increase tightness, and as in the case of a sclerotic labor market, this rise in tightness is larger when the targeted reduction affects L-type workers. After targeted reductions of firing costs, only the unemployment rate of the affected group falls, while the impact on the unemployment rate of the other groups is negligible. In contrast with the previous simulation of a sclerotic labor market, where all workers, regardless of their skills and employment status, get the largest welfare gains under a targeted reduction of firing costs of the low skilled, in this simulation of a tight labor market, employed workers, regardless of their skills, are better off after a comprehensive reform or after a targeted reduction of firing costs of high skilled workers, while unemployed workers are better off after a comprehensive reform or after a targeted reduction of firing costs of low skilled workers. Hence, the impact of unemployment and the support for alternative reductions of firing costs seem to change depending on the type of labor market being considered.
Figure 5a. Reducing firing costs of low-skilled workers in a tight labor market
Figure 5b. Reducing firing costs of high-skilled workers in a tight labor market
Figure 5c. A comprehensive reduction of firing costs in a tight labor market
Table 2. Comparison of the effects of several EPL reforms in a tight labor market

<table>
<thead>
<tr>
<th></th>
<th>Initial situation</th>
<th>Lower firing costs for low-skill workers</th>
<th>Lower firing costs for high-skill workers</th>
<th>Comprehensive reform</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_L=K_H=1$</td>
<td>$K_L=0, K_H=1$</td>
<td>$K_L=1, K_H=0$</td>
<td>$K_L=K_H=0.5$</td>
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<tr>
<td><strong>Tightness, $\theta$</strong></td>
<td>2.72</td>
<td>4.54</td>
<td>4.31</td>
<td>4.33</td>
</tr>
<tr>
<td><strong>Proportion of low-skilled unemployed, $\delta$ (%)</strong></td>
<td>55.8</td>
<td>45.1</td>
<td>66.9</td>
<td>56.1</td>
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<tr>
<td>Unemployment rates (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-skilled workers, $u_{rL}$</td>
<td>6.9</td>
<td>4.4</td>
<td>6.8</td>
<td>5.5</td>
</tr>
<tr>
<td>High-skilled workers, $u_{rH}$</td>
<td>5.5</td>
<td>5.4</td>
<td>3.4</td>
<td>4.3</td>
</tr>
<tr>
<td><strong>Thresholds levels</strong></td>
<td></td>
<td></td>
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<tr>
<td>Low-skilled workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Destruction, $\varepsilon_{L}^{d}$</td>
<td>0.760</td>
<td>0.792</td>
<td>0.769</td>
<td>0.785</td>
</tr>
<tr>
<td>Hiring, $\varepsilon_{L}^{h}$</td>
<td>0.860</td>
<td>0.792</td>
<td>0.869</td>
<td>0.835</td>
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<tr>
<td>High-skilled workers</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Destruction, $\varepsilon_{H}^{d}$</td>
<td>0.792</td>
<td>0.800</td>
<td>0.825</td>
<td>0.818</td>
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<tr>
<td>Hiring, $\varepsilon_{H}^{h}$</td>
<td>0.892</td>
<td>0.900</td>
<td>0.825</td>
<td>0.868</td>
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<td><strong>Exit rates from unemployment (%)</strong></td>
<td></td>
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</tr>
<tr>
<td>Low skilled workers</td>
<td>40.8</td>
<td>68.2</td>
<td>42.4</td>
<td>53.5</td>
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<tr>
<td>High skilled workers</td>
<td>47.3</td>
<td>49.0</td>
<td>85.1</td>
<td>64.2</td>
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<tr>
<td><strong>Firing rates (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low skilled workers</td>
<td>3.0</td>
<td>3.2</td>
<td>3.1</td>
<td>3.1</td>
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<tr>
<td>High skilled workers</td>
<td>2.8</td>
<td>2.8</td>
<td>3.0</td>
<td>2.9</td>
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<tr>
<td><strong>Asset values</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Low-skilled workers</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Unemployed</td>
<td>44.7</td>
<td>45.5</td>
<td>45.0</td>
<td>45.4</td>
</tr>
<tr>
<td>Employed</td>
<td>46.4</td>
<td>46.7</td>
<td>47.2</td>
<td>47.2</td>
</tr>
<tr>
<td>High-skilled workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>45.0</td>
<td>45.7</td>
<td>45.3</td>
<td>45.7</td>
</tr>
<tr>
<td>Employed</td>
<td>46.7</td>
<td>47.0</td>
<td>47.4</td>
<td>47.4</td>
</tr>
<tr>
<td><strong>Total production</strong></td>
<td>0.862</td>
<td>0.852</td>
<td>0.879</td>
<td>0.867</td>
</tr>
<tr>
<td><strong>Average productivity</strong></td>
<td>0.919</td>
<td>0.897</td>
<td>0.926</td>
<td>0.914</td>
</tr>
<tr>
<td><strong>Workers’ welfare (weighted asset values)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Low-skilled workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference with respect to initial situation (%)</td>
<td>44.93</td>
<td>45.67</td>
<td>45.28</td>
<td>45.65</td>
</tr>
<tr>
<td>High-skilled workers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference with respect to initial situation (%)</td>
<td>46.71</td>
<td>47.00</td>
<td>47.40</td>
<td>47.39</td>
</tr>
</tbody>
</table>

Notes: Steady state values for simulations with the following set of parameter values: $r=0.01$, $h=2$, $\gamma=1$, $c=0.1$, $\alpha=1/2$, $\varepsilon_{H}^{min}=1/3$, $z_L=z_H=0.1$, $\lambda_L=\lambda_H=0.04$.

Further discussion: Alternative assumptions

The results from the simulations above obviously depend on some of the assumptions made. Among them, the specification of the matching function, the assumption of the same rate of arrival of productivity shocks to all workers, and the steady-state condition which equals
inflows and outflows of unemployment for each type of workers have some implications which we now discuss.

As discussed above, the effects of EPL on job creation depend crucially on the elasticity of the matching function with respect to vacancies. In the previous simulations we have used a matching function featuring strong concavity, namely, where the number of consummated matches responds slowly to increasing vacancy-unemployment ratio. Had we used a Cobb-Douglas matching function, with $\gamma \uparrow 0$ instead of $\gamma = 1$, decreasing returns in the job-finding rate would not arise so quickly (see footnote 21). Hence, job creation could become so strong (relative to the strongly-concave case) that also ends up favoring the employment outcomes of those workers not directly affected by the partial reform. Thus, in this case, the reduction of unemployment rates and the rise of the exit rates from unemployment and welfare gains are higher after any reduction of firing costs.

This matching elasticity is also crucial to understand the different qualitative results from reforms in a sclerotic labor market and in a tight labor market. The incentives to open new vacancies depend on the skill composition of the unemployment pool and the matching rate with the unemployed. In a sclerotic labor market, the proportion of L-type unemployed workers is typically large and the probability of filling the vacancy is high. On the contrary, in our characterization of the tight labor market, the proportion of H-type workers is higher but the probability of filling a vacancy is lower. Hence, the response of job creation to reducing firing costs may be lower than in the sclerotic labor market.

Differences in the volatilities of productivity across groups, captured by changes in the Poisson rate of arrival of the shocks ($\lambda_H$ and $\lambda_L$), also affect the impact of reductions of firing
costs. When, as it seems realistic, there is higher volatility in the productivity of matches with L-type workers than with, a reduction in $K_L$ leads to larger welfare gains for any reduction of firing costs, but, in particular, for targeted reductions on the low skilled.\(^{23}\)

Finally, it is important to remark that our results refer only to a comparison of steady-states for different values of the firing costs. By imposing that inflows and outflows of unemployment are equal for each type of worker, we are obtaining the corresponding equilibrium unemployment rates for different sets of parameter values. Thus, Figures 4 and 5 should not be read as providing the dynamic effects of EPL reforms and our welfare analysis only concern comparison of asset values in steady state. Likewise, panels 5 of the Figures cannot provide any hint about the changes in the asset values of L and H-type employed and unemployed workers in the transition paths between steady states.

6. Concluding remarks

One relevant feature of employment policies and labor market reforms is that they are very often targeted at some demographic groups, particularly at those facing more difficulties in finding jobs (youth, female, long-term unemployed, etc). Some empirical studies trying to estimate the effects of this type of policies conclude that the impact on the labor market outcomes for different population groups can be very different, and do not always move in the same direction.

\(^{23}\) The results of the simulations with different arrival rates of productivity shocks for low-skilled and high-skilled workers are available from the authors upon request.
In this paper we have presented a version of Mortensen and Pissarides’ (1994) search equilibrium model, with heterogeneous workers competing for the same jobs, which illustrates why it may be difficult to pin down the consequences of this type of reforms. According to some simulation results, the impact of targeted reductions of firing costs on unemployment and welfare of different groups of workers may depend on the initial state of the labor market (more or less tight), on the volatility of the productivity on continuing jobs, and on the elasticity of the matching function with respect to vacancies. An interesting outcome of our analysis is that support for targeted reforms is likely (subject to our parameter choice) to be larger in sclerotic labor markets than in tight ones, in accord with the evidence presented by Saint-Paul (1996) about the timing of these reforms.

Finally, although we have centered our analysis on the reduction of firing costs for different types of workers, it is plausible that the effects of other targeted employment policies (like targeted reductions of non-wage costs or differentiated minimum wages) are also contingent on the initial characteristics of the labor market being analyzed. Hence, our future research agenda will examine whether the proposed analytical framework could prove as well useful in examining the effects of those policies.
Appendix A: Some comparative statistics of the model

a) The hiring and the firing rules

Differentiating equation (11) using Leibniz’s rule, and making use of the result that
\[ \text{equation (11)} \]
and the result that
\[ \text{result that} \]
yields:

\[
\frac{\partial \varepsilon_i^b}{\partial \theta} = \frac{\int_{\varepsilon_i}^{1} \left[ 1 - F'(x) \right] dx + 2(r + \lambda_i) \left[ 1 - F'(\varepsilon_i^b) \right] K_i}{2 \left( r + \lambda_i F'(\varepsilon_i^b) \right) + \theta \left[ 1 - F'(\varepsilon_i^b) \right] + 2(r + \lambda_i) \left[ 1 - F'(\varepsilon_i^b) \right] K_i} > 0
\]  
(A.1)

\[
\frac{\partial \varepsilon_i^d}{\partial \theta} = \frac{-2r(r + \lambda_i) \left[ r + 2(r + \lambda_i) \theta f'(\varepsilon_i^b) K_i \right]}{2 \left( r + \lambda_i F'(\varepsilon_i^b) \right) + \theta \left[ 1 - F'(\varepsilon_i^b) \right] + 2(r + \lambda_i) \theta f'(\varepsilon_i^b) K_i} < 0
\]  
(A.2)

\[
\frac{\partial \varepsilon_i^b}{\partial K_i} = \frac{2(r + \lambda_i) \left[ r + 2\lambda_i F'(\varepsilon_i^d) + \theta \left[ 1 - F'(\varepsilon_i^b) \right] \right]}{2 \left( r + \lambda_i F'(\varepsilon_i^d) \right) + \theta \left[ 1 - F'(\varepsilon_i^b) \right] + 2(r + \lambda_i) \theta f'(\varepsilon_i^b) K_i} > 0
\]  
(A.3)

b) The job creation condition

Partial differentiation of equation (13) yields:

\[
\frac{\partial \theta}{\partial q(\theta)} = \frac{\delta q(\theta)^2 \left[ 1 - F'(\varepsilon_L^b) \right]}{2cq'(\theta)(r + \lambda_L)} < 0
\]  
(A.4)

\[
\frac{\partial \theta}{\partial q''(\theta)} = \frac{(1 - \delta) q(\theta)^2 \left[ 1 - F''(\varepsilon_h^b) \right]}{2cq'(\theta)(r + \lambda_H)} < 0
\]  
(A.5)

\[
\frac{\partial \theta}{\partial \gamma} = \frac{q(\theta)^2 \left[ \int_{\varepsilon_i}^{1} \frac{1 - F'(x)}{2(r + \lambda_H) dx} \right] - \left[ \int_{\varepsilon_i}^{1} \frac{1 - F'(x)}{2(r + \lambda_L) dx} \right]}{cq'(\theta)}
\]  
(A.6)

The effect of firing costs on tightness in the job creation condition has a negative sign, as indicated by:

\[
\frac{\partial \theta}{\partial K_L} = \frac{\delta [1 - F'(\varepsilon_L^b)] q(\theta)^2}{cq'(\theta)} < 0
\]  
(A.7)
\[ \frac{\partial \theta}{\partial K_H} = \frac{(1-\delta)[1-F(h)]}{c q'(\theta)^2} < 0 \]  

(A.8)

c) The Beveridge curve

Equations (1') and (2') give how tightness and thresholds determine the size and composition of the unemployment pool. Differentiation of equations (1') and (2') yields:

\[
du_i = \Phi_i d\varepsilon^d_i - \Psi_i d\bar{\theta} \\
\Phi_i = \frac{\alpha \lambda \varepsilon_i \left\{ f'(\varepsilon^d_i) \left[ 1 - F'(\varepsilon^h_i) \right] + f'(\varepsilon^h_i) F'(\varepsilon^d_i) \right\}}{\left\{ 1 - F'(\varepsilon^h_i) \bar{\theta} + \lambda_i F'(\varepsilon^d_i) \right\}^2} \\
\Psi_i = \frac{\alpha \lambda \varepsilon_i F'(\varepsilon^d_i) \left[ 1 - F'(\varepsilon^h_i) \right]}{\left\{ 1 - F'(\varepsilon^h_i) \bar{\theta} + \lambda_i F'(\varepsilon^d_i) \right\}^2} 
\]

(A.9)

Hence, the number of unemployed workers of each group rises with the corresponding job destruction threshold. It is decreasing in labor market tightness, as the arrival rate of job offer falls. Signing the effects of tightness and thresholds on the skill composition of the unemployed is more cumbersome as:

\[
d\delta = \frac{(1-\delta)du_L}{u_L + u_H} - \frac{\delta du_H}{u_L + u_H} 
\]

(A.10)
Appendix B: The ergodic distribution of productivity

To derive the ergodic distribution of productivity let us just consider one type of worker. Let $G(x)$, $\epsilon^d \leq x \leq 1$ be the ergodic distribution of productivity, $x$, where $\epsilon^d$ is the level of productivity at which jobs are destroyed, and let $F(x)$ be the distribution function from which productivity shocks are drawn. Workers are hired from the pool of unemployed when $x \geq \epsilon^h$, being $\epsilon^d \leq \epsilon^h$.

Consider first the support of the distribution $\epsilon^d \leq x \leq \epsilon^h$. In any infinitesimally small time interval, $dt$, the mass of employed workers $\lambda dt$ receive a productivity shock, hence, the number of jobs with productivity $x' \leq x$, increases by the number of jobs with productivity above $x$ downgrading their productivity between $x$ and the destruction threshold $\epsilon^d$:

$$(1-u)[1-G(x)][F(x) - F(\epsilon^d)]\lambda dt, \quad \text{for } \epsilon^d \leq x \leq \epsilon^h$$

The number of jobs of productivity $x' \leq x$, for $\epsilon^d \leq x \leq \epsilon^h$, being destroyed in any infinitesimally small time interval, $dt$, is given by the number of jobs which upgrade their productivity above $x$ and those being destroyed with productivity below the destruction threshold $\epsilon^d$:

$$(1-u)G(x)[1-[F(x) - F(\epsilon^d)]]\lambda dt, \quad \text{for } \epsilon^d \leq x \leq \epsilon^h$$

Now, consider the rest of the support of the distribution, $x > \epsilon^h$. In any infinitesimally small time interval, $dt$, the mass of employed workers $\lambda dt$ receive a productivity shock, while a mass of $uq(\theta)dt$ are hired. Thus, the number of jobs with productivity $x' \leq x$, increases by additions from firms with productivity above $x$ and firms form the lower segment of the distribution, $\epsilon^d \leq x \leq \epsilon^h$, changing their productivity between $x$ and $\epsilon^h$, and by the new hires:

$$(1-u)[1-G(x)][F(x) - F(\epsilon^d)]\lambda dt + u\theta q(\theta)[F(x) - F(\epsilon^h)]dt, \quad \text{for } x > \epsilon^h$$

On the other hand, the number of jobs of productivity $x' \leq x$, for $x > \epsilon^h$, being destroyed is given by the jobs which upgrade their productivity below $x$ and the jobs which downgrade their productivity below the hiring threshold $\epsilon^h$:

$$(1-u)G(x)[1-[F(x) - F(\epsilon^d)]]\lambda dt, \quad \text{for } x > \epsilon^h$$

Thus, the ergodic distribution must satisfy

$$[1-G(x)][F(x) - F(\epsilon^d)] = G(x)[1-[F(x) - F(\epsilon^d)]], \quad \text{for } \epsilon^d \leq x \leq \epsilon^h$$

$$[1-G(x)][F(x) - F(\epsilon^d)] + \frac{F(\epsilon^d)}{1-F(\epsilon^h)}[F(x) - F(\epsilon^h)] = G(x)[1-[F(x) - F(\epsilon^d)]], \quad \text{for } x > \epsilon^h$$

where, to derive the second expression, we make use of the steady state condition for unemployment inflows and outflows which yields

48
\[
\frac{u\theta q(\theta)}{\lambda(1-u)} = \frac{F(\epsilon^d)}{1 - F(\epsilon^h)}
\]

These two equations give, respectively

\[
G(x) = F(x) - F(\epsilon^d), \quad \text{for} \quad \epsilon^d \leq x \leq \epsilon^h
\]

\[
G(x) = F(x) - F(\epsilon^d) + \frac{F(\epsilon^d)}{1 - F(\epsilon^h)}[F(x) - F(\epsilon^h)], \quad \text{for} \quad x > \epsilon^h
\]

Hence, the ergodic distribution is

\[
G(x) = \begin{cases} 
0, & \text{for } x < \epsilon^d \\
F(x) - F(\epsilon^d), & \text{for } \epsilon^d \leq x \leq \epsilon^h \\
F(x) - F(\epsilon^d) + \frac{F(\epsilon^d)}{1 - F(\epsilon^h)}[F(x) - F(\epsilon^h)], & \text{for } x > \epsilon^h
\end{cases}
\]

Notice that if there are no firing costs, so that \( \epsilon^d = \epsilon^h \), then

\[
G(x) = \frac{F(x) - F(\epsilon^h)}{1 - F(\epsilon^h)}, \quad \text{for } x \geq \epsilon^h
\]

In the case of \( F(x) \) being uniform in the interval \( [\epsilon_{\min},1] \), with \( \epsilon_{\min} < \epsilon^d \), so that \( F(x) = \frac{x - \epsilon_{\min}}{1 - \epsilon_{\min}} \), the ergodic distribution is

\[
G(x) = \begin{cases} 
0, & \text{for } x < \epsilon^d \\
\frac{x - \epsilon^d}{1 - \epsilon_{\min}}, & \text{for } \epsilon^d \leq x \leq \epsilon^h \\
\frac{x - \epsilon^d}{1 - \epsilon_{\min}} + \frac{\epsilon^d - \epsilon_{\min}}{1 - \epsilon^h}, & \text{for } x > \epsilon^h
\end{cases}
\]

Finally, in our model with a mass \( \alpha \) of low-skilled workers and a mass \((1 - \alpha)\) of high-skilled workers, the ergodic distribution of productivity, conditioned on employment, is

\[
(\alpha - \delta u)G^L(x) + [1 - \alpha - (1 - \delta)u]G^H(x)
\]

where \( G^i(x) (i=L,H) \) is the ergodic distribution related to \( F^i(x) \) as indicated above. Using this distribution we compute average productivity

\[
\text{avprod} = \frac{\int_{\epsilon^d}^{\epsilon^h} x dG^L(x) + [1 - \alpha - (1 - \delta)u] \int_{\epsilon^d}^{\epsilon^h} x dG^H(x)}{1 - u}
\]
and total production, net of vacancy costs, given by:

$$(1-u)\text{avprod} - \theta uc$$
References


