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SUOMEN PANKKI
Bank of Finland

BANK OF FINLAND STUDIES E:13 • 1998

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Abstract

In order to study the role of money in an inflation-targeting regime for monetary policy, we compare the interest rate and money as monetary policy instruments. The theoretical part of the study builds on a dynamic stochastic general equilibrium model that combines the money-in-the-utility-function approach with sticky prices. Preference and technology shocks are the driving forces of the economy. We show that conditioning the interest rate on the expected future cost change can be used to achieve constant inflation or constant inflation expectations. The assumed adjustment costs in 'money demand' lead to an equilibrium in which inflation can be controlled by money growth without having information on the current state of the economy. The tradeoff between money and the interest rate as a monetary policy instrument depends on the parameter stability of the technology change process relative to that of the 'money demand' function.

We experiment with the parameter stability of the demand for money using Finnish monthly data for 1980 – 1995. The steady-state — utility function — parameters of the model of narrow money (M1), estimated with cointegration techniques, are stable; whereas in the model of harmonized M3 (M3H) the parameters are *not* stable. The theoretical model fits the M1 data. The adjustment cost parameters of the M1 model describing the dynamics of the demand for money could indicate the occurrence of technological improvements in banking and payments during the sample period. These results suggest that from the Finnish viewpoint M1 would be a more appropriate intermediate target for monetary policy than harmonized M3. Due to small sample problems, we compare parameters of the theoretical model estimated using the Generalized Method of Moments and Full Information Maximum Likelihood method. The process driving the forcing variables is approximated with vector autoregression. Both the GMM and FIML parameter estimates are reasonable and the differences are negligible. The cross-equation restrictions implied by the rational expectations hypothesis are clearly rejected.

Keywords: demand for money, monetary transmission, money-in-the-utility-function, sticky prices, technology shock, GMM, FIML

Tiivistelmä

Tutkimuksessa selvitetään rahan roolia, kun rahapolitiikalla on suora inflaatiotavoite. Lisäksi vertaillaan rahan määrää ja korkoja rahapolitiikan ohjausvälineinä ja tarkastellaan eri tilastollisin menetelmin rahan kysynnän vakautta Suomessa. Talousteoreettisena tarkastelukehikkona on dynaamisen raha hyötyfunktiossa -mallin muunnelma, jota on täydennetty yrityssektorin epätäydelliseen kilpailuun perustuvilla hintajäykkyyksillä. Teknisen kehityksen ja rahan tuottamien likviditeettipalvelusten vaihtelun oletetaan olevan talouden dynamiikan taustalla. Tutkimuksessa osoitetaan, että sekä inflaatio että inflaatio-odotukset voidaan vakauttaa ehdollistamalla korkopäätökset odotetulle teknisen kehityksen kasvuvauhdille. Lisäksi siinä osoitetaan, että rahan määrän sopeutumiskustannusten vuoksi inflaatio voidaan vakauttaa myös rahan määrää hallinnoimalla ja vielä ilman tietoa talouden nykyhetken häiriöistä. Kannattaako rahapolitiikan ohjausvälineenä käyttää rahan määrää vai korkoja, riippuu keskeisesti siitä, osataanko tuleva tekninen kehitys arvioida riittävän tarkasti ja tunnetaanko rahan kysyntään liittyvät mallin parametrit.

Tutkimuksen empiirisessä osassa tarkastellaan rahan kysynnän vakautta. Rahan määrän mittareiksi on valittu suppea raha (M1) ja yhdenmukaistettu lavea raha (M3H). Tarkasteluajanjaksona on tammikuu 1980 – joulukuu 1995. Empiirisissä tarkasteluissa käy ilmi, että suppean rahan kysyntä on melko vakaa koko tarkasteluperiodin ajan, kun taas lavean rahan kysyntä on epävakaa. Rahapolitiikan kannalta suppea raha onkin siten ongelmattomampi rahan määrän mittari kuin lavea raha. Pienotosongelmien vuoksi tutkimuksessa vertaillaan lisäksi eri menetelmin estimoidun suppean rahan kysynnän parametriestimaatteja. Vertailtavat menetelmät ovat yleistetty momenttimenetelmä (GMM) ja täyden informaation suurimman uskottavuuden menetelmä (FIML), jota täydennetään oletuksella, että taloudenpitäjien odotuksia voidaan arvioida vektoriautoregressiivisen mallin avulla. Tutkimus osoittaa, että eri menetelmin saadut rahan kysynnän parametriestimaatit ovat hämmästyttävän lähellä toisiaan ja että odotuksia ei onnistuta arvioimaan kovinkaan tarkasti.

Asiasanat: rahan kysyntä, rahapolitiikan välittyminen, raha hyötyfunktiossa, hintajäykkyydet, tekninen kehitys, GMM, FIML

Acknowledgements

This thesis stems from a desire to link my previous empirical work more closely to the theoretical economic models. I felt that my licenciate thesis, which was built solely on the empirical approach, was not expandable. When I joined the Bank of Finland's Research Department in 1996, the head of the department, Juha Tarkka, guided me to the world of dynamic stochastic macroeconomic models. In his enthusiastic manner he has supported my research in many ways — not least by rising very difficult and well founded questions. Jouko Vilmunen, a research supervisor in the Research Department, remotely supervised my work already at the start when he still stationed in Brussels. We exchanged piles of faxes. Equiped with his very positive and frank — Carelian — character and with deep skills in both macro and micro economics and even with econometrics, Jouko has been able to pull me out of the swamps and misery of various technical difficulties. I can only join the long line of doctoral candidates that Jouko and Juha have 'produced' and to thank them sincerely.

Professor Matti Virén has commented the sections of the thesis at all parts of its life cycle. He was a member of the supervising group and an official examiner, and he will be one of the opponents in the defence of the thesis. As an official examiner, professor Markku Rahiala discovered some fundamental problems with the thesis — only some of which have I been able to fix. He also guided me to the fascinating world of state-space models. I want to thank their valuable and timely efforts in examining the thesis.

Many professors and colleagues have discussed with me the problems of the thesis and have commented on parts of it. Among them, Anne Brunila, Pekka Ilmakunnas, Jarmo Kontulainen, Pentti Saikkonen and Anders Vredin truly deserve my warmest thanks. The library of the Bank of Finland is simply superb; the staff's ability to provide fast and prompt service is amazing. Glenn Harma has been of invaluable help in patiently improving my English (and economic thinking) in the many texts produced during the research project. I also would like to thank Marja Hirvesalo-Niini for handling the printing.

When I was writing my master's thesis our eldest son, Pietari, was born; Aleksi's turn came during the licenciate thesis; and again and — hopefully — finally, Vili was born during the work on this thesis. All of them have become used to missing a busy father in their early

childhood. My dear wife, Hanna, carried the burden of caring for the children in their early days and, later, doing all the home chores in order to give me time to play with the kids. Although I have not worked at home, the moods of the current state of the research have followed me home and I have not always been an easy person to live with. I owe them very much.

Kerava, June 1998
Antti Ripatti

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Chapter 1

Introduction

Contents

1.1 Motivation

The demand for money is an important part of any macroeconomic model that is used to study issues related to monetary policy. This is related to the fact that monetary policy is modelled by the money supply. Money demand has also been a popular subject in applied economics. In addition to monetary policy relevance, there may be a practical reason for this: it is a popular benchmark application for new econometric methods.

Despite the compact theory of money demand and its importance from the monetary theory standpoint, it seems that empirical work has rarely made use of the theoretical money demand concept. This means that the parameter estimates typically reported in money demand studies are almost useless from the standpoint of theoretical money demand models. The empirical equations are difficult to interpret as the rational response of economic agents to changes in their environment. This thesis attempts to narrow the gap between the theoretical and applied money demand models and to analyse the role of money in the dynamic stochastic general equilibrium model.

During the late 1970s and early 1980s the focus of demand for money studies shifted from the estimation of static money demand equations to the estimation of dynamic relationships. The static models tended to 'overpredict'[1] money growth in the late 1970s. Financial innovations were modelled by allowing for richer dynamics in the empirical model. This was supported by advances in time series analysis and computer technology. Unfortunately there has not been much feedback from the empirical studies to the macroeconomic models of monetary policy.

The rise of unit root and cointegration literature shifted the focus toward modelling the long run. The 'old' static money demand equation came back in a cointegration vector. The parameters of the theoretical demand for money models of the 1950s took the form of steady-state parameters. However, the short-run dynamics are still typically modelled using the purely empirical approach. The short run and the long run have deep statistical links and restrictions on one part have a significant impact on other part. One needs economic theory to yield restrictions on both the short run and the long run.

[1] This period was called the 'missing money' period and is well documented by Judd and Scadding (1982) and Goldfeld and Sichel (1990).

The theoretical demand for money models of Miller and Orr (1966) and Tobin (1956, 1958) still form the theoretical 'background' of the steady-state part of empirical models, whereas in Lucas (1988) and Sill (1995) these are replaced by the cash-in-advance motivation for money and in Poterba and Rotemberg (1987) by the money-in-the-utility-function approach. None of these approaches aims at explaining the resulting residual autocorrelation. That is, the theoretical models do not yield an explanation of the 'dynamics of the money'.

We may summarize the problems with the demand for money studies as follows. The empirical money demand research has increasingly focused on estimating the long-run relationships between the variables included and on *ad hoc* modelling of the short-run dynamics. The economic theory — based on the microfoundations money models of the 1950s[2] or the cash-in-advance or money-in-the-utility-function models — is fairly ignorant of the dynamics of the demand for money. This thesis attempts to bridge the atheoretical empirical approach and the theoretical models by adding autocorrelation producing features to the theoretical model. It also proposes an econometric framework for estimating the parameters of the derived models and devices that enables evaluation of the parameter stability in order to study the usefullness of the models in the conduct of inflation-targeting monetary policy. Money demand, with the addition of the dynamics arising from adjustment costs, obtains a new role as part of a dynamic stochastic general equilibrium (DSGE) model with price rigidities. It is also shown that, given the parameters of money demand, money can serve as an instrument of an inflation-targeting monetary policy. The derived DSGE model also sets forth the conditions under which inflation-targeting monetary policy can omit information on money demand. The suggested model and econometric framework are extensively applied to Finnish monthly data for the period 1980–1995.

[2] See also recent dynamic extensions such as Smith (1986) and Romer (1986).

1.2 Approaches

Money-in-the-utility-function

This thesis builds on a discrete time version of the stochastic money-in-the-utility-function (MIUF) framework initiated by Sidrauski (1967). The MIUF has proved to be a usefull approach for introducing money into the economy. It provides a flexible modelling device for studing the monetary policy and the role of money in a dynamic context. We enrich the usual MIUF specification with stochastic preferences for the liquidity services provided by money. These preference changes are one of the driving forces of the economy.

MIUF is a device which — in a particular and abstract way — models the liquidity services provided by money. However, it separates the issue of money existence from the issue of the dynamic characteristics of money. One is able to combine the transactions and portfolio demands for money within a single framework. The well-known drawback of the MIUF model is that it does not answer questions such as what people do with money or why they want to hold money. There are other dynamic stochastic approaches in modelling money demand like the shopping-time model of McCallum and Goodfriend (1987), the dynamic Baumol-Tobin model of Smith (1986) and Romer (1986) and the cash-in-advance (CIA) model of Lucas and Stokey (1987). Feenstra (1986) studies the functional equivalence of liquidity constraints (CIA models in particular) and MIUF models. The functional equivalence of shopping time and MIUF models is studied by Croushore (1993). These alternative choices are not without difficulties either. For example, Blanchard and Fisher (1989) points out that '*... specifying nature of the Clower constraint [CIA models] for each type of transaction can quickly become cumbersome as well as analytically intractable*'. The CIA and MIUF models produce a similar relationship between money, prices, consumption and interest rates if one chooses the proper form of utility function and allows for credit goods in the CIA economy. Both models can also be augmented with the adjustment costs. The empirical advantage of the MIUF approach is that it does not require the division of consumption into money and credit goods.

The dynamics of money demand are modeled by incorporating the adjustment costs of changing money holdings in the household's budget constraint. At first sight it might seem strange to consider the

adjustment costs of exchanging an asset that should be the most inexpensive to adjust. However, there are several justifications for doing this. One may argue for the existence of adjustment costs by appealing to the fact that since most of the money measures include bank accounts, there are certainly costs involved in changing money balances. Furthermore, the conversion of bonds into money balances is not without costs, and adjustment costs might reflect these. The form that these adjustment costs take is very much an empirical question. Another argument is that adjustment cost is a valuable device for modelling the persistence of observed money growth.

Sticky Prices

In modeling firms' behaviour — the supply side — we assume that prices are somewhat sticky. We believe that it is the supply side of the economy which entails frictions rather than the demand side of the economy. In order to keep our model analytically tractable, the supply side is fairly stylized. From the set of alternative approaches, we follow the cost-of-changing-prices approach of Rotemberg (1982). As noted by Rotemberg (1987), the price dynamics produced by dynamic adjustment costs are observationally equivalent to those produced by Calvo (1983) contracts. Most of the Phillips curves derived from sticky price models can be written in a form in which current inflation depends on the expected next-period inflation, possibly lagged inflation and on a demand factor such as output, output gap or consumption. These kind of supply functions have been discussed and estimated by Roberts (1995). An alternative view is represented by the island model of Lucas (1972) which relies on information imperfections. In that model, informational imperfections lead people to mistakingly believe that overall price changes are relative price changes. Given suitable information assuptions, this lead to an aggregate supply equation similar to ours.

It is also assumed that markets are characterized by imperfect (monopolist) competition. This is due to the intent to introduce demand effects. Monopolistic competition has the advantage that profits are zero in the long run and so there is no need to include shares in the model. We also make these same assumptions. Although our model inherits many features of Rotemberg model, our contribution is to explicitely incorporate the demand side, ie household behaviour, in the framework. We augment Rotemberg's supply side with the aggregate techonology shock. This is the main driving force of our economy and of inflation in particular.

There is currently an expanding literature on dynamic stochastic general equilibrium models with sticky prices. These models are used to study economic fluctuations like the real business cycle (RBC) models. Technology shocks are the driving force of the economy, and sticky prices are introduced to analyse the effects of monetary policy and to improve the empirical fit of the models. While our model belongs to this field of research, the monetary policy questions analysed here are different. RBC models are designed more to study the business cycles than is our model, which is designed to motivate the role of money and to study the information requirements of the choice of monetary policy instrument.

Monetary Policy

Our DSGE model can be considered a microfoundation for the Barro and Gordon (1983*b*, 1983*a*) type models. A significant part of the monetary-policy-time-consistency literature more or less disregards forward-looking aspects[3], which are explicitly set out in our framework. Another theme ignored by the time consistency literature and the currently expanding literature on discretionary rules is the role of technology shocks[4], which forms the main building block of the RBC models and, in our DSGE model, turns out to be one of the most important factors in the choice of monetary policy instrument.

Our study of the monetary policy instrument rules builds on the ideas of the seminal work by Poole (1970). We consider money and the one-period interest rate as instrument choices. Hence, in contrast to the Bundesbank approach we must assume that the central bank can, in principle, directly control money. This is the compromise that is necessary in order to distinguish the instrument role of money from the interest rate instrument.

We concentrate only on inflation targeting monetary policy. This is due to the fact that the central banks that have explicitly announced their targets, have expressed them in terms of inflation. None of them are targeting a price level or nominal income. In discussing monetary policy rules, we even avoid specifying an explicit objective function for the central bank. However, the derived rules are optimal rules in the case where the central bank minimizes quadratic losses of inflation deviations from the target. To enable derivation of analytical results,

[3] Important exceptions are Svensson (1997) and Svensson (1998).

[4] A notable exception is Ireland (1997).

it is also assumed that no credibility problems exist. The central bank is able to commit to the announced policy. The introduction of credibility issues would lead us into numerical analysis of the model (see eg Backus and Driffill 1986).

Since we linearize the first order conditions, the optimal monetary policy rules are also linear in the case of quadratic monetary policy preferences. Shocks enter the model in linear form, which simplifies the choice of monetary policy instrument. In a nonlinear world, we could face the problem that it would not necessarily be optimal to fully accommodate the shocks, as is illustrated by Brainard (1967). Consequently, the choice of monetary policy instrument would not necessarily be as straightforward as in our (linear) case.

Linearization

We (log)linearize the nonlinear first order conditions of our model, using the first order Taylor approximation around the steady-state. By linearizing the first order conditions, we benefit from being able to solve the model analytically. An alternative approach is to introduce quadratic preferences to obtain linear first order conditions. We prefer linearizing the first order conditions because we can thus retain the original interpretations of the deep parameters. A drawback is the fact that the first order Taylor approximation treats the steady-state as a linearization point. This set of extra parameters does not create any problem in the purely theoretical context but, when combined with the nonstationarity (in the mean) assumption, leads to interpretational difficulties. This is because for nonstationary variables the steady-state defines an attractor that is not a point, as assumed in the Taylor approximation, and because the linearization points enter the set of parameters to be estimated. We have no solution to this problem, and this is the drawback of allowing the nonstationarity.

Econometrics

In analysing monetary policy empirically, one cannot avoid taking expectations explicitly into account. The traditional conditional models fail to answer questions like how would the private sector react to an announced interest rate path. We are obliged to use econometric techniques when we consider the cross-equation restrictions or the moment restriction implied by the theoretical model. In the empirical part, we also benefit from linearization of the theoretical model since the set of linear models includes a wider spectrum of estimators.

We believe that it is important to assess the empirical performance of forward-looking structural monetary models since these models provide a meaningful framework in which to pose questions about the choice of monetary policy strategy and monetary policy instruments.

The basic alternative approaches are the Generalized Method of Moments (GMM) and — for solved models — Full Information Maximum Likelihood (FIML). Since the pathbreaking research program culminated in Hansen (1982) and Hansen and Singleton (1982), the GMM has been widely applied. The novel advantage of GMM is in the utilization of the moment condition implied by the rational expectations hypothesis. The estimation criterion is based on these moment conditions and in the overindentified case these conditions can be tested simply with the χ^2 test. The GMM criterion can be based on the first order conditions implied by the theoretical model. The likelihood-based approach relies on the solution of the model. The stochastic characteristics of forcing variables, ie variables determined outside the model, must be specified. The closed form solution and likelihood function can be derived and the model can be estimated using numerical optimization methods. If the stochastic characteristics of forcing variables are correct, this method is fully efficient, whereas the GMM is a limited information method when information on the stochastic characteristics of forcing variables is not utilized.

The choice of econometric approach is directly connected with Lucas' critique. The core of Lucas' critique is that if the parameters of the marginal model, ie the model of forcing variables, change, this will induce parameter non-constancy also in the behavioural equation. This is due to the cross-equation restrictions implied by rational expectations between the behavioural equation and marginal models. The parametric changes in the marginal model could be due eg to the change in monetary policy. Non-constancy of the marginal model would jeopardize the parameter constancy of the behavioural equation. Consequently, there is danger in applying the full information estimation to rational expectations model without taking into account the structural change. Parameter constancy is an important subject for study. The Euler equation estimation, which is a limited information method and which does not rely on information on the marginal model, is less vulnerable to Lucas' critique. The minimized criterion is based solely on the moment condition implied by rational expectations. The parameters that are estimated are deep parameters, typically preferences and technology parameters. It is worth noting that

these too can be subject to change and so their constancy should also be studied.

We study parameter constancy by testing and by recursive estimation. These are applied at every stage of the estimation. A wellknown problem with the test of overidentification restrictions in GMM is that it lacks power against structural changes in parameters. Hence some alternative tests of parameter constancy within the GMM framework are applied. We solve the information problems of the GMM approach by estimating the model also by the full information method. We approximate the marginal model of forcing variables with vector autoregression (VAR). Consequently, we are able to compare estimates of the deep parameters. However, we do not compare the methods *per se*. That is a task of another thesis. We are interested in the differences in the parameter estimates.

There are yet other approaches available for estimating rational expectations models. Watson (1989) recommends that one estimate and solve Euler equations in the state space form. This method can be applied only to the class of linear models and requires a specification of the marginal model. Gallant and Tauchen (1996) shift GMM toward the full information method by deriving the moment conditions from the score function of the marginal model. Their method is computationally not as burdensome as the simulated method of moments estimator suggested by Duffie and Singleton (1993) and Ingram and Lee (1991). Gouriéroux, Monfort and Renault (1993) and Smith (1993) use the parameters of the marginal model to define the GMM criterion.

We believe that nonstationarities in the macroeconomic data are not due solely to nonlinearities. Hence we need methods for analysing nonstationary data and we cannot directly use the above-mentioned new methods, which are based on the stationarity assumption. The time series methods for nonstationary data are derived for linear models. That is the econometric reason why we linearize our theoretical models. By linearizing we are able to distinguish the nonstationary part, ie the variables-in-levels part, from the stationary part of the model. We estimate the level part of the model with cointegration methods and the stationary part with the above-mentioned version of GMM and FIML. Thus the estimation is (legitimately) separated into two steps. We rely extensively on the super-consistency of the parameters of cointegration in the belief that this is a fruitfull approach. Even in the case of failure of the rational expectations restrictions, ie

the moment restrictions in the GMM and the cross-equation restrictions in FIML, we obtain some results concerning the parameters of the theoretical model. It also turns out that the cointegration part of the model reflects mainly the preference parameters and the dynamics part of the model reflects mainly the technology parameters, ie the adjustment cost parameters. In the case that the latter are subject to structural change, the former can still be constant. One should also note that the theoretical model does not imply cross-equation restrictions on the parameters of the cointegration part.

Data

We use Finnish monthly data covering the years 1980–1995 to illustrate the proposed methods. During the 1980s Finland's economic institutions changed in many respects[5]: financial markets were deregulated, deposit and loan rates were liberalized, credit rationing was abolished, money markets were created and capital movements were deregulated[6].

We study two money measures: narrow money (M1) and harmonized[7] broad money (M3H). The first is a liquid, transaction-oriented measure. The second, a very broad measure, is a candidate as an intermediate target in the Economic and Monetary Union (EMU). These aggregates may be difficult for the central bank to control, but they are well defined and widely used. Browne, Fagan and Henry (1997) explore studies on the demand for EU-wide money, giving the impression that the demand for the broad aggregate is more 'stable' than the narrow one.

It might be usefull to disaggregate the monetary aggregates by sector (firms and households), and this is supported by the fact that our economic model represents household behaviour (see eg Thomas 1997*a*, Thomas 1997*b*). Unfortunately, we are not able to disaggregate the deposits by sector. It is also important from the monetary policy standpoint that we are able to model the aggregate demand for money.

The deregulation did not affect the components of narrow money (M1). Narrow money includes currency held by the public plus

[5] The financial deregulation was quite similar in other OECD countries as well.

[6] The deregulation process and consequent asset price boom and bust are analysed in detail eg by Bordes, Currie and Söderström (1993).

[7] It is probably not the final version of the harmonized broad money. The definition may change before the third stage of EMU starts in 1999.

chequable and transactions accounts, ie those bank accounts that can be spent by cheque or debit/credit card. The payment system was quite advanced already in the early 1980s. Although the payment medium changed from cheque to debit card, the included accounts were the same. Cheque accounts, the most volatile part of M1, are not interest bearing. Harmonized broad money (M3H) contains M1 plus all other bank accounts, certificates of deposits (CDs), repos and certain other instruments for which there are no secondary markets. The CDs part of the aggregate was severely affected by the financial deregulation. They did not exist until 1983 and their issuance was regulated — actually rationed — until 1987. The taxation of deposits changed in 1991-1992 when the withholding tax was introduced.

The price measure used in the study is the consumer price index. We do not have a monthly consumption measure and so we use the monthly GDP indicator instead. However, this is a very noisy time series. The indicator is constructed by aggregating different monthly indicators with GDP weights. In a small open economy like Finland GDP may not be a very good surrogate for consumption. The GDP share of exports is quite large and exports usually lead consumption. The cross-correlation between consumption and GDP growth indicates that GDP leads consumption by up to $1\frac{1}{2}$ years. The current correlation, 0.48, is the highest. The phase shift between consumption and GDP might worsen our estimation results. It is, however, the only monthly measure at hand.

Finally, since the Finnish money markets were created in 1987 when the Bank of Finland started its money market operations, we can use the one month money market rate from that period onwards. For the pre-1987 period, we use an interest rate that is computed from the one-month forward and spot rates for the Finnish markka vs the US dollar, ie from the covered interest parity. At that time, the Bank of Finland did not intervene in the forward exchange markets. Thus the computed interest rate measure corresponds to the market determined Euro-rate of the markka for pre-1987 period.

Monthly data has the advantage that it is published with the same frequency as the monetary policy executed. Hence, the empirical model might have a practical link to monetary policy decisions. Also, the frequency of the monthly model is possibly much closer to the frequency of decisions made by economic agents. Consequently, the problems of temporal aggregation are much smaller. The drawback with monthly data is that it is much more noisy than quarterly data.

1.3 Summary of the Essays and the Contribution

In this section we summarize the contents of the essays. In short, we derive the dynamic stochastic general equilibrium model in the first essay. The model implies that it is the parameters for money demand that should be stable so as to anable the use of money as a monetary policy instrument, and the parameters of the technology process should be known in order to steer inflation with the interest rate instrument. In the second essay we test for parameter stability of the money demand function derived in the first essay using Finnish data for M1 and M3H. In the third essay we compare the limited information and full information approaches in the estimation of the money demand equation derived in the first essay.

1.3.1 Inflation Targeting and the Role of Money in a Model with Sticky Prices and Sticky Money

The first essay develops a stochastic dynamic general equilibrium model. It models households' behaviour assuming that they include money in their utility functions. The firms are assumed to face costs in changing the prices of their products. The essay studies the role of money in inflation-targeting monetary policy by comparing the interest rate and money as a monetary policy instrument.

The representative household is assumed to obtain utility from consumption and real money balances. The utility function is the constant relative risk aversion (CRRA) form for both consumption and real money balances. The weight of the real money balances in the utility function is assumed to be stochastic, ie the liquidity services that money provides are stochastic. That is called velocity shock. A household may allocate its income to consumption, real money balances or bonds. Whenever it adjusts the real money balances it has to pay adjustment costs. Consumption is a composite good, aggregated from individual goods assuming constant elasticity of substution (CES). The first order conditions are linearized.

Monopolistic competition introduces demand effects to our model. Due to CES preferences, each monopolistic firm faces a demand function in which the demand for its product depends on the

relative price of its product and aggregate consumption. The costs of production are quadratic in quantities and they also depend on the stochastic aggregate technology parameter. A monopolistic firm faces costs in changing the price of its product. The aggregated version of the first order condition, which is linear, implies that current inflation depends on expected future inflation and the levels aggregate consumption and technology.

Despite the adjustment costs in money and in prices, the steady state of the model is very classical. The interest rate is equal to the time preference; consumption is determined by a constant and the level of technology; real money balances are determinate; and neither the price level nor nominal money balances is determinate. It is shown that inflation is finite if the mean-reversion of the linear combination of interest rate and expected technology change is strong enough. Inflation and inflation expectations can be targeted using the interest rate instrument alone. This leads to the instrument rule where the interest rate should be related to the degree of time preference and the expected technology change. We also experiment with interest rate rules by approximating the technology by the total factor productivity. Given the estimated structural time series model, the monthly changes in total factor productivity are very difficult to forecast, and the resulting interest rate rule suggests only very modest variation in the short-term interest rates.

The model also has an equilibrium where money predicts inflation. This is due to the adjustment costs of money. If the parameters of the demand for money are known, inflation can be controlled using money as the instrument. Money also has informational advantages in the sense that only the realization of previous-period velocity and technology shocks has to be known. In the first essay we are able to shift the analysis of Poole (1970) from shocks to parameter uncertainty.

1.3.2 Stability of the Demand for M1 and Harmonized M3 in Finland

In the second essay we extend the households problem to allow for interest bearing money. The purpose of the essay is to study the parameter stability of M1 and M3H. Cointegration techniques are used

to estimating the nonstationary part of the model and GMM to estimate the parameters involved in the dynamics of the money demand. It is concluded that the parameters for M1 demand are stable whereas the parameters for M3H demand are not stable.

Since harmonized broad money (M3H) is interest bearing, we augment the orginal model with the own-yield of money. This changes the cointegration implications of the model. We also derive the first order conditions in the case where the adjustment cost function is more complicated than in the first essay. This involves lagged money changes as well.

We estimate the cointegration part of the model using the FIML of Johansen (1991). We simulate the small sample critical values of the cointegration test statistics. The cointegration parameters that are related to the preference parameters of the theoretical model are stable in the M1 system but not in the M3H system. Given the cointegration parameters, the short-run dynamics implied by the Euler equation are estimated using GMM. The parameters for the short-run dynamics are associated with the adjustment cost parameters in the theoretical model. The adjustment cost parameters of the M1 model describing the dynamics of the demand for money are stable over the sample period. There are no big differences in the subperiods for M1. However, some indication of decreasing adjustment costs can be observed.

1.3.3 Limited and Full Information Estimation of the Rational Expectations Demand for Money Model: Application to Finnish M1

The third essay re-estimates the model of M1 using the full information maximum likelihood method and compares the parameter estimates to corresponding GMM estimates. The parameter differences are illustrated with a numerical experiment. The third essay also tests the cross-equation restrictions implied by the theoretical model.

We concentrate solely on the M1 model and solve the Euler equation with respect to the forcing variables. It is assumed that the log-difference of the forcing variables can be approximated by VAR. We derive the cross-equation restrictions and estimate the model parameters. The FIML estimates of the deep parameters are reasonable and do not differ from the corresponding GMM estimates. This is con-

firmed by the numerical experiments. The biggest differences are in standard errors of the parameters.

The cross-equation restrictions implied by the rational expectations hypothesis are clearly rejected as is typical for these kinds of models; exogeneity restrictions are rejected as well. The empirical fit of the demand for money equation is not very good. The empirical model might benefit in relaxing some of the very restrictive assumptions of the theoretical model.

1.3.4 Contribution

This thesis contributes in several fields of macroeconomics. Our model shares the feature of Woodford (1998), where within the dynamic stochastic general equilibrium model it is shown that inflation can be controlled using the interest rate instrument and money has no necessary role. This contribution is extended by the inclusion of technology shocks. This is typically omitted in the credibility literature or in discussions of interest rate rules. It clearly has an important role in instrument setting. The specific instrument role of money, stemming from adjustment costs, can also be considered as a contribution. In the context of the sticky price models, this is a unique feature. In general, the theoretical part of the thesis clearly enhances our understanding of inflation targeting monetary policy.

In the empirical part we combine the existing econometric toolbox with the theoretical model in a coherent way. On the contrary to the empirist approach to money demand, we are able to theoretically motivate the dynamics of the demand for money. The clear link between the theoretical and empirical models helps us to narrow the controversy between the economic theory and the empirics of money demand. Finland is an illustrative example of how some of the economic relationships survive in a turbulent economic environment. We test for the robustness of the results and concentrate on the important issue of parameter stability. We are not satisfied with just one estimate of the parameters of interest but expend much effort in searching for other, possibly more efficient, estimators.

According to the empirical results in the light of the theoretical model, Finland is in no great danger in choosing narrow money (M1) as a monetary policy instrument. The parameter estimates are plausi-

ble, fairly stable and robust with respect to the choice of estimator[8]. The inclusion of Finnish M3H in EMU-wide M3H would increase the aggregation bias in the EMU-wide M3H because the parameter estimates of the demand for M3H are far from the EMU average. The supply side of the model is fairly stylized and might not be empirically applicable without fine tuning. Thus the experiment in deriving the inflation stabilizing interest rate path is more illustrative than normative.

The model has some weaknesses that might influence the instrument choice in practice and which provide a challenge for future research. They are mainly related to the theoretical model. We have excluded the credibility issue. In the context of the forward-looking model these issues have to be handled with numerical methods (see Svensson 1998, Backus and Driffill 1986). Consequently, the parameters of the model must be known. Furthermore, the strong assumption of information symmetry as the expected technology growth between the central bank and the private sector guarantees that the inflation target can be achieved. However, informational asymmetries could alter outcome of the theoretical.

[8] According to our experience, one may use FIML in the estimation of demand for money models augmented with the adjustment costs if the adjustment cost function is simple enough.

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Chapter 2

Inflation Targeting and the Role of Money in a Model with Sticky Prices and Sticky Money

Contents

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Abstract

In order to study the role of money in an inflation targeting regime for monetary policy, we compare the interest rate and money as monetary policy instruments. Our dynamic stochastic general equilibrium model combines the money-in-the-utility-function approach with sticky prices. We allow for time-varying preferences for real money balances, ie for velocity shocks and stochastic 'technology' shocks in prodution. We show that conditioning the interest rate on the expected future technology change can be used to achieve constant inflation or constant inflation expectations. However, the prediction of technology growth could be a heroic task. The assumed adjustment costs in 'money demand' lead to an equilibrium in which inflation can be controlled by money growth without having information on the current state of the economy. Finally, the tradeoff between money and the interest rate as a monetary policy instrument depends on the parameteric stability of the technology change process relative to that of the 'money demand' function.

Keywords: monetary transmission mechanism, money-in-the-utility-function model, sticky prices, technology shock, monetary policy strategy

JEL classification: E31, E41, E52

2.1 Introduction

Central banks are increasingly arguing that they can control only price level in the long run. Consequently, they should be responsible for price stability, ie they should set their policy instrument so as to target the price level or inflation. There are of course nuances concerning how to implement such a target. A number of countries, eg New Zealand, the United States, Canada and more recently the United Kingdom, Sweden, Finland and Spain, have specified inflation as the direct target of their policies. However, the German Bundesbank still uses a monetary aggregate as an intermediate target while aiming at price stability.

More recently (see eg EMI 1997) it has been discussed whether the forthcoming European System of Central Banks (ESCB) should engage in direct inflation targeting or monetary targeting. Without using any explicit model, EMI (1997) argues that both are based on the same final objective (price stability), both are forward-looking, and both employ a wide range of indicators.

The purpose of this study is to analyse inflation targeting with a fully articulated model. The model should permit us to discuss the choice of the monetary policy instrument and the information requirements of instruments and instrument rules.

EMI (1997) defines inflation targeting as a monetary policy strategy whereby the central bank uses an interest rate instrument to directly control inflation, ie the very strategy used by the countries mentioned above. Monetary targeting is defined in conformity with the Bundesbank's approach whereby money serves as an intermediate target; inflation is the ultimate goal of such a policy and interest rates are used as instruments. In this study we use the same definition of inflation targeting but we assume that the money stock can be set at exactly the desired level. Thus money stock could be regarded as an instrument rather than an intermediate target.

An instrument is a tool that the central bank can directly control. There are no control errors; the instrument variable can for all practical purposes be set exactly. Examples are the monetary base and the discount rate. When an instrument can be set precisely and there is no uncertainty in the model or in the parameters, no instrument choice problem exists. The target can be achieved precisely with any instrument. However, when the model incorporates uncertainty, we face the problem of ordering uncertainty and actions. If deviations from the

target are possible, the uncertainty must occur *after* the instrument is set. Otherwise the target again can be fully controlled.

We investigate the instrument rules. Instrument rules are instrument settings that are conditional on the state of the economy and that lead to either constant inflation or constant inflation expectations. The instrument choices under consideration are the interest rate and money. Since Poole (1970), there has been a long tradition of studying the optimal choice of a monetary policy instrument. We evaluate the instruments with respect to their ability to meet the inflation or inflation expectations target and with respect to their requirement concerning information on the structure of economy.

Inflation targeting means that the central bank either wants to restrict inflation or inflation expectations to a certain targeted level. Svensson (1996) labels the latter case inflation forecast targeting. The difference between these two approaches lies in the information setup: If the central bank perceives current period shocks to the economy, it can control inflation. However, if it has to move first, ie determine the value of an instrument before the shocks hit, it can fix only the expected inflation rate.

Despite the short-term nonneutrality of monetary policy in our model, we analyse monetary policy only in the case where the central bank is able to commit to the announced policy. Consequently, We study what Svensson (1997) calls *strict inflation targeting*.

The model at hand describes the dynamics between money, interest rates and inflation. We use the log-linearized version of the money-in-the-utility-function (MIUF) model augmented with adjustment costs, as in Ripatti (1998), to derive the dynamic relationship between money, prices, consumption and interest rates. Firms' behaviour is modelled as in Rotemberg (1981, 1982), where the monopolistic firm faces costs (eg menu costs) when it changes the price of its product. The demand function faced by the monopolistic firm is based on constant elasticity of substitution of goods. Our approach is to combine the adjustment cost-augmented MIUF and Rotemberg's sticky price model and augment them with stochastic preferences for money and stochastic aggregate technology shocks. The model is analysed in section 2.2.

Our choice for introducing money into the economy is the money-in-the-utility-function approach. This approach is applied by Obstfeld and Rogoff (1995), Hairault and Portier (1993) and Blanchard and Kiyotaki (1987), among others. Another motivation for the existence

money is to introduce the cash-in-advance constraint. This was done by Carlstrom and Fuerst (1995), Rotemberg (1987), Svensson (1986), Yun (1996), Baba (1997) and Obstfeld and Rogoff (1996). A third route was chosen by King and Wolman (1996), who follow McCallum and Goodfriend (1987) by assuming that money reduces shopping time. Money simplifies transactions in the model by Reinhart (1992). Our system is designed to capture the persistence of money growth by introducing costs in the adjustment of money holdings. Carlstrom and Fuerst (1995) augment their model with the constraint that part of the household's cash is to be deposited in a financial intermediary, which is also a type of adjustment cost.

Price rigidity is introduced into our model via menu (adjustment) costs as suggested by Rotemberg (1982). Other alternatives, which show up in real business cycles models with price rigidities, include staggered price changes (Calvo 1983) and prefixed prices (eg Svensson 1986). As Rotemberg (1987) notes, the first two choices are observationally equivalent. Price adjustment *à la* Rotemberg (1982) is also used by Baba (1997) and Rotemberg (1987). Hairault and Portier (1993), which follows the MIUF approach, also includes quadratic price adjustment costs, while differing in detail from Rotemberg (1981). Even though the quadratic adjustment cost approach is easily critized (see footnote 6), it is a very simple modelling device and has the advantage of yielding log-linear approximation. Another alternative for a price adjustment mechanism would be based on the model by Calvo (1983), where price changes are geometrically (discrete case) or exponentially (continuous case) distributed. This has been used by Yun (1996), Rotemberg and Woodford (1997), Reinhart (1992), Kollmann (1997) and King and Wolman (1996). Svensson (1986) assumes that goods prices are predetermined, ie that they adjust only to the lagged state of the market. To simplify their models, Obstfeld and Rogoff (1995), Obstfeld and Rogoff (1996) and Rotemberg and Woodford (1997) assume that the representative household is both consumer and producer and thus include production in the utility function. This of course simplifies the welfare analysis. They do not need to assume any price rigidities in their models and yet demand has short-run effects. The seminal paper by Blanchard and Kiyotaki (1987) uses a static approach and relies on the assumption of incomplete markets, which has also been critized (see eg Carlton 1996).

We introduce two kinds of driving forces into our model. The first one, labelled velocity shock, captures the representative household's

stochastic preferences for real money balances. It also allows for velocity shocks that are independent of interest rates. We study how changes in preferences for holding money, ie for the liquidity services produced by money, influence computed equilibrium. In the spirit of RBC models, we also introduce a technology shock into the cost function of the representative monopolist.

As noted at the end of section 2.2, it turns out that we are able to characterize the equilibrium relationship between inflation and expected interest rates without knowledge of 'money demand' parameters and velocity shocks. This corresponds to the situation where the central bank has a direct inflation target and uses interest rates as its monetary policy instrument. In such a setup, money is recursively determined by expected exogenous shocks and interest rates.

We study the feasible interest rate paths, ie the interest rate paths that yield finite inflation, in section 2.3. This gives us a kind of benchmark equilibrium. We continue by studying more ambitious monetary policies. We cover the cases where the central bank targets either inflation or inflation expectations. As a sufficient condition, we propose two interest rate rules where the interest rate is conditioned on the expected future technology change. These rules are compared to the standard rule whereby the interest rate is conditioned on current inflation. The main lesson from this exercise is that inflation cannot be stabilized without considering expectations on regarding technology change. We present some preliminary empirical results on the expected technology change. These results support the view that forecasting future technology growth is a difficult task.

A novel feature of our model is that we are able to ascribe a role to money. The role arises from the adjustment costs. Due to the adjustment costs, money growth captures the expectations regarding future interest rates and technology changes. We get an equilibrium in which money is the only *current* determinant of inflation. Consequently we are able to propose a money rule, where money is conditioned on period $t-1$ state variables, ie variables that the central bank can observe. In order to utilize this framework, the central bank must know more of the parameters of the model.

Finally, in the last section, we discuss the choice of monetary policy strategy, which in our case is the choice of a monetary policy instrument. We conclude that the choice between money or an interest rate as the monetary policy instrument is largely an empirical question.

2.2 A Model with Sticky Money and Sticky Prices

Although Lucas (1988) prefers to base the 'money demand model', or rather the relationship between money, consumption and interest rates, on the cash-in-advance constraint, we have chosen to include real balances directly in the utility function. Other options for modelling the demand for money would be transactions costs and shopping time models. However, the money-in-the-utility-function (MIUF) has the advantage of analytical simplicity and it allows us to illustrate the dynamics of the relationship.

The strong persistence[1] of nominal balances and its growth rate suggests that changes in nominal balances might involve adjustment costs. Consequently, we augment the usual MIUF model with adjustment costs for changing money holdings. Adjustment costs for changing money holdings are somewhat artificial since money should be the asset that is most cheaply exchanged. On the other hand, one can imagine that there are costs involved in adjusting eg bank accounts[2], ie shoe sole costs. However, we believe that adjustment costs comprise an approximate modelling device for incorporating dynamics in 'money demand' analysis. On the aggregate level they could, for example, mimic more complex dynamics, eg (s, S) behaviour. The MIUF model is derived in section 2.2.1.

The motivation for modelling prices as sticky also relates to the empirical argument that inflation seems to be so persistent that it could even be approximated by an integrated process. In addition to price stickiness, we assume monopolistic competition, in order to introduce demand effects into our model. The Euler equation expressing the model's supply characteristics is introduced in section 2.2.3.

Since the purpose of the model is to analyse the choice of monetary policy and to estimate the model parameters, we linearize the model[3]. We mainly utilize the first order Taylor approximation around the — hopefully cointegrated and thus stationary — steady state. The log-linearization is separated in section 2.2.2. Although the model is

[1] See the extensive literature on demand for money studies based on cointegration techniques.

[2] Bank accounts are usually at least partly incorporated in money measures.

[3] The estimation and testing theory for models with (possibly) nonstationary variables is fairly well established for linear models.

not entirely classical, its steady state properties are very much in the classical mode. The steady state is examined in section 2.2.4. Section 2.2.5 discusses possible equilibria, and we leave most of the monetary policy issues to section 2.3.

2.2.1 Household Preferences

The economy is inhabited by infinitely lived households, firms and the monetary authority (central bank). It also includes a continuum of differentiated goods that are produced by monopolistically competitive firms. The firms and goods are indexed by $z \in [0, 1]$. The differentiated goods are aggregated to produce a single composite good, which yields utility to the household.

The *household* optimizes the discounted sum of expected utility from consumption and real money balances:

$$\max E_0 \sum_{t=0}^{\infty} \delta^t \left[u(C_t) + \mathfrak{Z}_t v\left(\frac{M_t}{P_t}\right) \right].$$

The household allocates its 'phantom' income y (ie periodic exogenous endowment) and other earnings among (composite) consumption (C_t: real value of consumption), bonds (B_t: real value of bonds denominated in units of time t consumption) which pay a gross real return of $1 + r_t$ (from time t to time $t + 1$), and real money balances, $\frac{M_t}{P_t}$, which pay a gross return $\frac{P_t}{P_{t+1}}$. Whenever it adjusts its money balances between period $t - 1$ and period t, the household suffers losses amounting in real terms to $a(M_t, M_{t-1})/P_t$. The household's budget constraint is

$$C_t + B_t + \frac{M_t}{P_t} + \frac{a(M_t, M_{t-1})}{P_t} \leq y + \frac{M_{t-1}}{P_t} + (1 + r_{t-1})B_{t-1}. \tag{2.1}$$

We also assume that the periodic utility functions obtained from consuption and real money balances are concave, ie $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$ and $v''(\cdot) < 0$. In this stylized model households' income, y, is created by firms' profits. The profits are taxed (in a nondistortionary manner) by the government, which distributes the profits randomly to the households. This interpretation simplifies the technicalities and closes the model.

C_t is the number of units of the composite good available at period t:

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \tag{2.2}$$

where $\theta > 1$ is the constant elasticity of substitution (CES). The price deflator for nominal money balances is the consumption-based money price index implicit in (2.2). It is obtained as a solution of the cost minimization problem subject to the aggregator (2.2):

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}. \tag{2.3}$$

Note that the utility function is additive and the weight of utility gained by the representative household from real money balances is stochastic. Since we utilize the representative household approach, it is quite natural to allow real money balances to yield stochastic liquidity services. Consequently, the stochastic variable $\beth_t$ is introduced to allow for stochastic transaction technology. Another way of interpretating this is that the velocity shock allows the heterogeneity of households to vary over time. Stochastic preferences for liquidity services yielded by money can be motivated so as to allow for shocks to money demand — or to velocity — that are independent of monetary policy. Hence by construction, velocity cannot be controlled via monetary policy. When we iterate the budget constraint, we obtain the following transversality condition:

$$\lim_{T\to\infty} E_t \prod_{j=0}^{T} \frac{1}{1+r_{t+j}} \left(B_{t+1+T} + \frac{M_{t+1+T}}{P_{t+1+T}}\right) = 0. \tag{2.4}$$

This states that the present value of financial assets held in period T (real bonds and money) tends to zero as time T tends to infinity. That is to say the expected growth rate for financial assets is restricted to stay below the real rate, r.

The first-order conditions for bonds and nominal money balances are

$$\delta E_t\{(1+r_t)u'(C_{t+1})\} = u'(C_t) \tag{2.5}$$

$$1+a'_{M_t}(M_t, M_{t-1}) =$$

$$\delta E_t\left\{\frac{P_t}{P_{t+1}}\frac{u'(C_{t+1})}{u'(C_t)}\left[1-a'_{M_t}(M_{t+1}, M_t)\right]\right\} + \mathfrak{Z}_t\frac{v'\left(\frac{M_t}{P_t}\right)}{u'(C_t)}. \tag{2.6}$$

We assume that a nominal bond exists in the economy. The condition (2.5) for the nominal bond is given by

$$(1+i_t)\delta E_t\left\{\frac{1}{P_{t+1}}u'(C_{t+1})\right\} = \frac{1}{P_t}u'(C_t). \tag{2.7}$$

If we combine the additional assumption that

$$\mathrm{cov}_t\left(\frac{P_t}{P_{t+1}}\frac{u'(C_{t+1})}{u'(C_t)}, \left[1-a'_{M_t}(M_{t+1}, M_t)\right]\right) = 0 \tag{2.8}$$

with the equation (2.7), the condition (2.6) for nominal money can be written as

$$\mathfrak{Z}_t\frac{v'(\frac{M_t}{P_t})}{u'(C_t)} = a'_{M_t}(M_t, M_{t-1}) + \frac{1}{I_t}E_t a'_{M_t}(M_{t+1}, M_t) + 1 - \frac{1}{I_t}, \tag{2.9}$$

where $I_t \equiv 1+i_t$. The covariance condition holds for example if consumers are risk neutral and inflation is deterministic or if the net own-yield of money, $1-a'_{M_t}(M_{t+1}, M_t)$, is deterministic. The left-hand side of equation (2.9) is the marginal rate of substitution of consumption for real balances. The right hand side is the rental cost, in terms of the consumption good, of holding an extra unit of real balances for one period. Note that the rental cost differs from the usual $1-1/I_t$. Depending on the functional form, the model might yield multiple equilibria.

Assumption 1 (Utility function and adjustment costs). Next we parameterize the utility function to the CRRA form as

$$u(C_t) = \begin{cases} \frac{1}{1-\rho}\left(C_t^{1-\rho}-1\right) & \text{if } \rho \neq 1 \\ \log C_t & \text{if } \rho = 1 \end{cases},$$

$$v\left(\frac{M_t}{P_t}\right) = \begin{cases} \frac{1}{1-\omega}\left[\left(\frac{M_t}{P_t}\right)^{1-\omega}-1\right] & \text{if } \omega \neq 1 \\ \log\left(\frac{M_t}{P_t}\right) & \text{if } \omega = 1 \end{cases}$$

and the adjustment cost function $a(\cdot)$ as:

$$a(M_t, M_{t-1}) = \frac{\kappa}{2}(M_t - M_{t-1})^2, \tag{2.10}$$

where κ is the adjustment cost parameter.

The adjustment cost function expresses the notion that it is the growth rate of money that affects costs, as in Cuthbertson and Taylor (1987, 1990). The chosen functional form allows for persistence in the level of money balances. Its motivation relies on our empirical experience with Finnish monetary aggregates. It is however difficult to connect these parameters to any specific payment technology.

From (2.9) one obtains

$$\Delta M_t = \frac{1}{I_t} E_t \Delta M_{t+1} - \frac{1}{\kappa I_t}(I_t - 1) + \frac{\mathfrak{Z}_t}{\kappa I_t} I_t C_t^{\rho} \left(\frac{M_t}{P_t}\right)^{-\omega}. \tag{2.11}$$

The household's problem also implies the following transversality conditions:

$$\lim_{T\to\infty} \delta^T E_t \left(\frac{M_{t+T}}{P_{t+T}}\right)^{-\omega} \frac{M_{t+T}}{P_{t+T}} = 0 \tag{2.12}$$

$$\lim_{T\to\infty} \delta^T E_t C_{t+T}^{-\rho} C_{t+T} = 0. \tag{2.13}$$

The first condition (2.12) states that expected real money balances cannot grow with a rate faster than $(1-\omega)/\delta$. Correspondingly, the condition (2.13) states that consumption growth rate cannot exceed $(1-\rho)/\delta$. This means that exactly all the household's resources are exhausted as t approaches infinity.

2.2.2 Log-linear Approximation

In order to log-linearize equation (2.11), we find the steady state for equation (2.11) and then use the first-order Taylor approximation around the steady state. For the steady state, the adjustment costs are zero and $P_t = P$, $C_t = C$, $I_t = I$, $I_t = I$, $M_t = M$ $(\forall t \geq 0)$. We denote logarithmic variables in the lower case (eg $\log C \equiv c$) and $I \equiv 1 + i$. Note that $i \approx \log(1+i)$ and $\zeta_t \equiv \log(\mathfrak{Z}_t)$. In the steady state, equation (2.11) reduces to

$$\frac{M}{P} = \left[\mathfrak{Z} C^{\rho} \left(\frac{I}{I-1}\right)\right]^{\frac{1}{\omega}}, \tag{2.14}$$

which resembles the standard demand-for-money function. From the first-order Taylor approximation around the (log of) the steady state, we obtain the log-linear Euler equation

$$I\Delta m_t = E_t\Delta m_{t+1} + \frac{i\omega}{\kappa M}\Big[\Big(m - p - \frac{\rho}{\omega}c + \frac{1}{i\omega}i - \frac{1}{\omega}\zeta\Big) - m_t + p_t + \frac{\rho}{\omega}c_t - \frac{1}{i\omega}i_t + \frac{1}{\omega}\zeta_t\Big]. \quad (2.15)$$

The Euler equation (2.15) can also be written as

$$\left[L^{-2} - \left(I + 1 + \frac{i\omega}{\kappa M}\right)L^{-1} + I\right] E_t m_{t-1} = \underbrace{-\frac{i\omega}{\kappa M}\left(m - p - \frac{\rho}{\omega}c + \frac{1}{i\omega}i - \frac{1}{\omega}\zeta\right)}_{\equiv m^*} - \frac{i\omega}{\kappa M}\left(p_t + \frac{\rho}{\omega}c_t - \frac{1}{i\omega}i_t + \frac{1}{\omega}\zeta_t\right). \quad (2.16)$$

The parameterized version of Euler equation (2.7) is

$$1 = \delta E_t\left\{(1+i_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right\}. \quad (2.17)$$

The right-hand side contains the conditional expectation for a non-linear function of random future consumption. Therefore, because of Jensen's inequality, a first-order Taylor series approximation is inadequate. The second-order Taylor approximation of Euler equation (2.17) gives us

$$\begin{aligned} 1 &= \delta E_t\left\{\exp\left[\log\left((1+i_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right)\right]\right\} \\ &= \delta\exp\left\{E_t\left[\log(1+i_t) - \Delta\log P_{t+1} - \rho\log C_{t+1} + \rho\log C_t\right]\right. \\ &\quad \left. + \frac{1}{2}\mathrm{Var}_t\left[\log\left((1+i_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right)\right]\right\}. \end{aligned}$$

Taking logs of both sides yields

$$-\rho c_t = \log(\delta) + i_t - E_t\Delta p_{t+1} - \rho E_t c_{t+1} + v,$$

where $i_t \approx \log(1+i_t)$, $v \equiv \frac{1}{2}\mathrm{Var}_t\left[\log\left((1+i_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right)\right]$ (which is assumed to be constant). This approximation is exact if $(1+i_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}$ is lognormally distributed.

Therefore, the log-linearized Euler equation is

$$\rho E_t \Delta c_{t+1} = \log(\delta) + v + i_t - E_t \Delta p_{t+1}. \tag{2.18}$$

Our assumption of a constant conditional covariance might be misleading. Modelling of the variance term is nowadays an important field of research in consumption-based asset pricing theory.

2.2.3 Monopolistic Firms

We assume monopolistic competition[4] in order to introduce demand effects into our model. In tracking the price stickiness, we follow Rotemberg (1981) except that we use the demand function derived from equation (2.11). Another widely[5] used representation is Calvo (1983). He obtains observationally similar aggregate price dynamics in a setting in which individual firms have an exogenous probability of being permitted to change the price in a given period (Rotemberg 1987).

Each monopolistic firm produces a distinct nonstorable good. The number of monopolistic firms is large and each individual firm produces such a small part of the aggregate that it need not take into account the effects of its production and pricing decisions on the aggregate price level or aggregate demand.

Household consumption, C_t, is a real consumption index. Assuming CES preferences for consumption goods, a monopolistic firm faces a demand function of the form

$$C_t(z) = A(z)\left(\frac{P_t(z)}{P_t}\right)^{-\theta} C_t, \quad \theta > 1, \tag{2.19}$$

where $C_t(z)$ is the quantity of good z demanded at time t, which can be obtained from equation (2.11). $A(z)$ is a firm-specific constant,

[4] See Carlton (1996) for a critical assesment of the role of imperfect competition in macromodels.

[5] See eg Yun (1996), Rotemberg and Woodford (1997), Reinhart (1992), Kollmann (1997) and King and Wolman (1996).

$P_t(z)$ is the price of good z at time t and P_t the general price level. The quantity demanded of any particular good depends not only on the relative price of the good but also the aggregate real consumption.

Each monopolist's cost function is quadratic:

$$K(C_t(z), z) = U(z)\mathcal{T}_t^{-1}P_tC_t(z)^2/2.$$

The cost of producing the quantity $C_t(z)$ at time t is $K(C_t(z), z)$. $U(z)$ is a firm-specific, small and positive parameter. The production function contains an aggregate stochastic technology parameter, $\mathcal{T}_t$. Positive changes in $\mathcal{T}$ reflect advances in aggregate technology. In the spirit of real business cycle models, we assume that the technology shock is a key driving force in economic fluctuations. Without shocks, the modelled economy converges to a balanced growth path and then grows smoothly. In standard real business cycle models with the cash-in-advance constraint, like Cooley and Hansen (1989), the optimal policy makes no attempt to respond to technology shocks. In models with nominal rigidities, like Cho and Cooley (1995) and Yun (1996), nominal shocks affect the business cycle as well, and it is important to take the technology shock into account in designing optimal monetary policy. Ireland (1997) shows how a technology shock influences the optimal monetary policy in a model of sticky prices.

Since we assume a large number of monopolistic firms, ie that the economy is atomistic, the aggregate price level and aggregate consumption are given for each monopolistic firm. Hence, the monopolistic firm maximizes nominal profits with respect to the price of its good, $P_t(z)$

$$\pi(P_t(z)) = P_t(z)C_t(z) - U(z)\mathcal{T}_t^{-1}P_tC_t(z)^2/2.$$

The first-order condition for maximization requires that $P_t(z)$ be equal to $P_t(z)^*$, where

$$P_t(z)^* = \frac{\theta}{\theta - 1}U(z)\mathcal{T}_t^{-1}P_tC_t(z). \tag{2.20}$$

The first factor, $\theta/(\theta-1)$, is the firm's markup. The higher the value of θ, the smaller the markup and the smaller the firm's monopoly power. The limiting case, $\theta \to \infty$, corresponds to perfect competition. The profit maximizing price level, $P(z)$, is the same for nominal and real profits since the individual firm's price setting does not influence the aggregate price, ie $\partial P_t/\partial P_t(z) = 0$.

Rotemberg (1981) derives the second-order Taylor approximation for profits around $P_t(z)^*$ and assumes that the monopolistic firm faces costs of changing prices[6]. The monopolist's problem can then be approximated by

$$\max_{\{p_t(z)\}} -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \delta^t \left\{[p_t(z) - p_t(z)^*]^2 + d[p_t(z) - p(z)_{t-1}]^2\right\}. \tag{2.21}$$

The first-order condition is

$$\delta d E_t \Delta p_{t+1}(z) - d\Delta p_t(z) = p_t(z) - p_t(z)^*, \tag{2.22}$$

where a lowercase letter denotes the logarithm of the variable denoted by the corresponding uppercase letter and $\tau_t \equiv \log \mathcal{T}_t$. Note that the time preference parameter, δ, is the same for consumers and producers. The current change in the price of product z is determined by the optimal (in the absence of adjustment costs) price level rise compared to the present level and the expected future level. Due to the adjustment costs in changing price, the firm must take into account the expected future optimal prices.

The maximization problem also involves a transversality condition:

$$\lim_{T\to\infty} E_t \delta^T \left[(p_{t+T}(z) - p_{t+T}(z)^*) + d(p_{t+T}(z) - p_{t-1+T}(z))\right] = 0. \tag{2.23}$$

In the aggregation we utilize the demand function faced by the firm.

[6] Arguments for costs in changing prices can be classified into two categories (see eg Barro 1972, Sheshinski and Weiss 1977): administrative and similar costs (menu costs; see Levy, Bergen, Dutta and Venable (1997) for empirical relevance) and costs due to unfavourable reactions by customers. Rotemberg (1987) illustrates that increasing prices could, for example, upset customers. However, the quadratic adjustment cost approach assumes symmetry in the costs of raising or lowering prices. Prescott (1987, page 113) criticizes the menu cost approach: 'I have no answer to the question of how to measure these menu change costs, but these theories will never be taken seriously until an answer is provided'. Nevertheless, we treat the assumption of menu costs as merely a simplifying device to introduce price inflexibility into our model. On a more fundamental level, the problem remains as to how to explain the apparently significant costs of changing prices.

The aggregated[7] version of equation (2.22) is

$$\delta d E_t p_{t+1} - (d + \delta d) p_t + d p_{t-1} = -S - D(c_t - \tau_t), \tag{2.24}$$

where $S \equiv \int_0^1 \frac{1}{\theta+1} h(z) \left[\log\left(\frac{\theta}{\theta-1}\right) + u(z) + a(z)\right] dz$ and $D \equiv \frac{1}{\theta+1}$. Let λ denote the stable root of the corresponding characteristic equation $\frac{1}{\delta} + 1 = \lambda + \frac{1}{\lambda}$. The Euler equation (2.24), in the first difference form, is

$$d\delta E_t \Delta p_{t+1} - d\Delta p_t = -S - D(c_t - \tau_t).$$

It is also assumed that all monopolistic firms have homogenous expectations as to all the random variables in the system, ie c_t and τ_t. Consequently, the forward solution is

$$p_t = \frac{S}{1-\lambda} + \frac{1}{\lambda\delta} p_{t-1} + \frac{D}{d\delta} E_t \sum_{j=0}^{\infty} \left(\frac{1}{\lambda}\right)^j (c_{t+j} - \tau_{t+j}) \tag{2.25}$$

or in difference form

$$\Delta p_t = \frac{S}{d(1+\delta)} + \frac{D}{d\delta} E_t \sum_{j=0}^{\infty} \delta^j (c_{t+j} - \tau_{t+j}). \tag{2.26}$$

According to the solution, a firm must consider the past price level and forecast the future aggregate demand as well as the technology shocks in order to determine its current price level. The aggregate demand links the consumption decision of the representative household and the pricing decisions of the monopolistic firms.

The quadratic adjustment cost approach has the advantage of yielding equations that can be easily estimated. Calvo's approach shares this quality. The drawback of the model is that the adjustment cost parameter is fixed; even in a regime of large and sudden increases in aggregate demand, the prices adjust rather slowly. In Calvo's model, individual price changes can be large while the price level adjusts sluggishly. There is also a mapping from the adjustment cost parameter of the present model to the probability of price changes in Calvo's model.

[7] In the aggregation we approximate the CES consumption price index with a Cobb-Douglas price index using the weights $h(z)$. In the limiting case, $\theta \to 1$, the approximation is exact.

Due to the price stickiness the profits of the firms may fluctuate. In order to close the model, we assume that the government levies a nondistortive lump sum tax on profits and redistributes the revenue randomly to the households. Thus profits are the only source of household income.

One could also augment the model with *competitive* labour markets without altering the results obtained in this section (Rotemberg 1981). This would result merely in parametric changes in the Euler equation of prices. These results hold even in the case where each firm is a monopsonistic buyer of its type of labour and the production function exhibits constant returns to scale.

Another implicit assumption concerning the adjustment costs of price changes is the symmetry and state-independence of the cost parameter, d. There is however only weak evidence of asymmetries in GDP (Hess and Iwata 1997). State-independence is possibly a more restrictive assumption. For example, it might be the case that in a high inflation regime the adjustment costs are larger than in the case of a low inflation regime.

2.2.4 Steady State

To construct the steady state and to derive the equilibrium, we write our Euler equations as

$$\left[L^{-1} - \left(1 + I + \frac{\omega i}{\kappa M}\right) + IL\right] E_t m_t = m^* - \frac{\omega i}{\kappa M} E_t \left(p_t + \frac{\rho}{\omega} c_t - \frac{1}{\omega i} i_t + \frac{1}{\omega} \zeta_t\right), \tag{2.27}$$

$$\left[L^{-1} - \left(\frac{1}{\delta} + 1\right) + \frac{1}{\delta} L\right] E_t p_t = -\frac{S}{d\delta} - \frac{D}{d\delta} E_t \left(c_t - \tau_t\right), \tag{2.28}$$

$$(L^{-1} - 1) E_t p_t + \rho (L^{-1} - 1) E_t c_t = \log \delta + v + E_t i_t, \tag{2.29}$$

where $L^{-j} E_t x_t = E_t x_{t+j}$. Utilizing equations (2.27)–(2.29) and the steady state relationship (2.14), we obtain steady state solutions by letting $L = 1$. We use 'bar' notation for steady state values for each

variable. The steady state is determined by the following equations:

$$\bar{c} = \bar{\tau} - \frac{S}{D}, \tag{2.30}$$

$$\bar{i} = -\log\delta \quad \text{and} \tag{2.31}$$

$$\bar{p} = \bar{m} - \frac{\rho}{\omega}\bar{\tau} - \frac{1}{\omega}\bar{\zeta} + \frac{\rho S}{\omega D} + \frac{\log\delta + \log(-\log\delta)}{\omega}. \tag{2.32}$$

The steady state properties of the model are very classical. The steady state consumption level is determined by supply factors, ie the steady state technology shock and the aggregated parameters of the monopolistic firms' cost functions. Since there is no inflation in the steady state, the nominal interest rate equals the real rate of interest. Since the conditional covariance, v, is zero in the steady state, the real and nominal rate equals $1/\delta - 1$, ie the degree of time preference. As in any classical model, the price level is left undetermined. However, the real balances are determined by the steady state technology and preference shocks and all the other parameters of the model. If the money stock is given, the price level is determinated by the model and *vice versa*. The higher the level of aggregate technology, the lower the real money balances. The more weight that households give to real money balances, ie the more liquidity services money yields, the higher the steady state level of real money balances. The steady state also corresponds to the temporal equilibrium of the model in the situation where the adjustment costs for both money and prices are zero.

2.2.5 Equilibrium

We combine the Euler equations (2.18) and (2.24) in order to solve for inflation and consumption growth, and we discuss the possible equilibria that these equations yield. The solution for money can then be obtained by using the solutions for inflation and consumption growth.

Equilibrium consumption is characterized by the following equation:

$$c_t = \frac{1}{\delta\lambda_p} c_{t-1} + \frac{1}{\delta\lambda_p} (\log\delta + v + i_{t-1})$$
$$- \frac{1}{\lambda_p}\left(1 - \frac{1}{\delta\lambda_p}\right) E_t \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_p}\right)^j (\log\delta + v + i_{t+j})$$
$$+ \frac{D}{d\delta\rho\lambda_p} E_t \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_p}\right)^j \left(\tau_{t+j} + \frac{S}{D}\right) \quad (2.33)$$

where $|\lambda_p| > 1$ is one of the roots[8] of $\frac{\delta+1}{\delta} + \frac{D}{d\delta\rho} = \lambda + \frac{1}{\delta\lambda}$. Clearly the dynamic solution to the consumption displays saddle path dynamics; current consumption depends on past consumption as well as on the entire anticipated future path of interest rates and technology changes. This property of a rational expectations solution is general and shows up in many types of rational expectations models. Past interest rates have a positive effect on current consumption, whereas present and future interest rates affect it negatively. The term $\frac{1}{\lambda_p}\left(1 - \frac{1}{\delta\lambda_p}\right)$ has a positive sign. This is a reflection of the familiar consumption smoothing behaviour of households under well-functioning capital markets.

The dynamic relationship between inflation and interest rates is not as straightforward as the relationship between consumption and interest rates. As above, we combine the Euler equations (2.18) and (2.24) and, then, substitute for consumption and obtain

$$E_t\Delta p_{t+2} - \left(1 + \frac{1}{\delta} + \frac{D}{d\delta\rho}\right) E_t\Delta p_{t+1} + \frac{1}{\delta}\Delta p_t =$$
$$- \frac{D}{d\delta\rho}\left[i_t + (\log\delta + v) - \rho E_t\Delta\tau_{t+1}\right]. \quad (2.34)$$

The characteristic equation is as above. Then we solve forwardly the unstable root, λ_p:

$$\left(1 - \delta\lambda_p L^{-1}\right) E_t\Delta p_t = -\frac{D\lambda_p}{d\rho} E_t \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_p}\right)^j$$
$$\times \left(i_{t+j} + \log\delta + v - \rho\Delta\tau_{t+1+j}\right). \quad (2.35)$$

[8] It can be shown that the roots are $|\lambda_p| > 1$ and $|1/(\delta\lambda_p)| < 1$.

Note however that we still have the stable root, $|\delta\lambda_p| > 1$. This is a well known problem in macroeconomic models where the 'IS function' contains an expected value of an endogenous variable (see eg Kerr and King 1996 and references therein). The main point here is that the future inflation has a greater than one-for-one effect on current inflation. We discuss two possible candidates for equilibria.

With the first candidate, the solution displays saddle-path dynamics and can be written explicitely as

$$\Delta p_t = \frac{1}{\delta\lambda_p}\Delta p_{t-1} + \frac{D}{d\delta\rho\lambda_p} E_t \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_p}\right)^j \left(i_{t-1+j} + \log\delta + v - \rho\Delta\tau_{t+j}\right).$$

This solution however yields a *positive relationship between future interest rates and inflation.* For example, increases in the current nominal interest rate, while keeping the path of (anticipated) future interest rates constant, increase current inflation. Although, it is well known that rational expectations solutions can display non-standard dynamics in certain model types[9], we conjencture that in the present context this 'perverse' dynamic relationship reflects unstable dynamics.

The second candidate for the forward solution of (2.35) clarifies the feasibility of the possible choices of interest rate paths. We iterate it T periods forward:

$$\begin{aligned}\Delta p_t = \frac{D}{d\rho}\Bigg\{ & \frac{1}{\lambda_p - \frac{1}{\delta\lambda_p}} E_t \sum_{j=0}^{T} \left[(\delta\lambda_p)^j\lambda_p - \lambda_p^{-j}(\delta\lambda_p)^{-1}\right] \\ & \times \left(i_{t+j} + \log\delta + v - \rho\Delta\tau_{t+1+j}\right) \\ & + \frac{1}{\delta(\lambda_p - \frac{1}{\delta\lambda_p})}\left[(\delta\lambda_p)^{T+1} - \left(\frac{1}{\lambda_p}\right)^{T+1}\right] E_t \sum_{j=T+1}^{\infty} \left(\frac{1}{\lambda_p}\right)^{j-(T+1)} \\ & \times \left(i_{t+j} + \log\delta + v - \rho\Delta\tau_{t+1+j}\right)\Bigg\} + (\delta\lambda_p)^{T+1} E_t\Delta p_{t+T+1}. \quad (2.36)\end{aligned}$$

[9] See eg the (overlapping generations) model of inflation analysed by Sargent (1993), where increased government deficits reduce inflation in a stable rational expectations equilibrium. The reason is that the economy finds itself in the wrong side of the 'Laffer' curve in the inflation tax rate.

The dynamics of the equilibrium inflation have Wicksellian features. Any interest rate peg that differs sufficiently from the degree of time preference and expected technology change will destabilize inflation. Current inflation is dominated by the long-term expectations for future inflation. Consequently, interest rates should be set to yield zero inflation on the average in the long run. This also means that the central bank must on average relate the nominal interest rate to the expected next-period technology shock, $\rho\Delta\tau_{t+1}$, and the steady-state interest rate, $\log\delta + v$.

The last term, $(\delta\lambda_p)^{T+1}E_t\Delta p_{t+T+1}$, which represents the aggregated transversality condition (2.23) of inflation, must be zero. The first partial sum in the braces, which describes the expected future up to period T, is in general bounded only with finite T. The discount factor, $(\delta\lambda_p)^j\lambda_p - \lambda_p^{-j}(\delta\lambda_p)^{-1}$, is greater than unity and thus restricts the speed of mean reversion of the linear combination of interest rates and technology changes. For certain paths of $e_t \equiv i_{t+j} + \log\delta + v - \rho\Delta\tau_{t+1+j}$, the first term will be bounded even in the limiting case as T approaches infinity. The second partial sum represents the expectations from the period $T+1$ onwards. Its discount factor is less than unity but the expectation term is multiplied by a factor that is greater than unity and raised to the power $T+1$. This puts restrictions on the very distant path of process e_t. The second partial sum must approach zero as T approaches infinity.

Note that in the above equilibria, the greater the cost of changing prices, d, the less the influence of monetary policy (ie the interest rate) on the inflation. This is due to the rigidity of changing prices. In this sense, the model is not very classical in the short run. The price rigidity is exogenous with respect to monetary policy and with respect to the level of inflation. This is probably an unrealistic assumption for a high inflation regime. In the more realistic case, the adjustment cost would be linked to the level of inflation. King and Wolman (1996) discuss this possibility in the context of Calvo's price setting.

Finally, the solution for money growth is given by the following equation:

$$
\begin{aligned}
\Delta m_t = & \frac{i(\log\delta + v)\left[D(\omega-1) - d\delta\rho\right]}{\kappa M d\rho\delta(1-\lambda_m)(1-\lambda_p)} \\
& + \left(\frac{I}{\lambda_m} + \frac{1}{\delta\lambda_p}\right)\Delta m_{t-1} - \frac{I}{\delta\lambda_p\lambda_m}\Delta m_{t-2} \\
& + \frac{1}{\kappa M\left(\frac{\lambda_m}{\lambda_p} - 1\right)} E_t \sum_{j=2}^{\infty}\left[\left(\frac{1}{\lambda_m}\right)^j - \left(\frac{1}{\lambda_p}\right)^j\left(\frac{\lambda_p}{\lambda_m}\right)\right] \\
& \times \left\{L - \left(\frac{1}{\delta} - i + \frac{D}{d\delta\rho}\right)L^2\right. \\
& \left. + \left[\frac{D}{d\delta\rho}(1+\omega i) + i + 1 + \frac{1}{\delta}(2+i)\right]L^3 - \frac{1}{\delta}(1+i)L^4\right\} i_{t+j} \\
& - \frac{i}{\kappa M\left(\frac{\lambda_m}{\lambda_p} - 1\right)} E_t \sum_{j=2}^{\infty}\left[\left(\frac{1}{\lambda_m}\right)^j - \left(\frac{1}{\lambda_p}\right)^j\left(\frac{\lambda_p}{\lambda_m}\right)\right] \\
& \times \left[L - \left(\frac{1}{\delta} + 1 + \frac{D}{d\delta\rho}\right)L^2 + \frac{1}{\delta}L^3\right] \times \Delta\zeta_{t+j} \qquad (2.37) \\
& - \frac{iD}{\kappa M d\delta\left(\frac{\lambda_m}{\lambda_p} - 1\right)} E_t \sum_{j=2}^{\infty}\left[\left(\frac{1}{\lambda_m}\right)^j - \left(\frac{1}{\lambda_p}\right)^j\left(\frac{\lambda_p}{\lambda_m}\right)\right] \\
& \times (\omega L^2 - L^3)\Delta\tau_{t+j},
\end{aligned}
$$

where $|\lambda_m| > 1$ is one of the roots of $1 + I + \frac{\omega i}{\kappa M} = \lambda + \frac{I}{\lambda}$. Money growth does not feedback to the solutions for inflation and consumption growth. Consequently, it does not have any role in an inflation-targeting monetary policy using an interest rate instrument. Equilibrium is determined by expected future interest rates, technology shocks and velocity shocks. The existence of velocity shocks adds an extra complication to monetary targeting. In addition to money demand parameters, the central bank will need to know the stochastic characteristics of the velocity shocks.

2.3 Inflation Targeting and Monetary Policy Strategy

Our model describes the dynamic relationship between money, prices, consumption and interest rates. The model includes two types of exogenous shock: a preference shock in the money-in-the-utility-function and an aggregate technology shock to production. So far we have treated interest rates as being predetermined, ie we have analysed the equilibria given an exogenous path of expected interest rates. In this section we give a more profound characterization of the equilibrium. In section 2.3.1 we characterize the possible interest rate path given the preference for finite inflation. Section 2.3.2 gives more precision to the concept of inflation targeting. We compare direct inflation targeting to inflation-expectations targeting. In section 2.3.3 we experiment with interest rate rules by approximating the technology with total factor productivity. Given the estimated structural time series model, the monthly changes in the total factor productivity are very difficult to forecast and the resulting interest rule suggests only very modest variation in the short-term interest rate. Then in section 2.3.4 we utilize the Euler equation for money to study the equilibrium, where inflation is determined by current money growth and certain other lagged variables.

The model has interesting implications for monetary policy strategy. Inflation (see equation 2.36) is determined by the discounted sum of *expected future interest rates* and discounted expected future technology changes. If interest rates are controllable by the central bank[10], there is a direct channel from the monetary policy instrument to inflation.

The equilibrium inflation is characterized by knife-edge dynamics: if expected interest rates and technology changes diverge by too much for too long from each other and from the steady-state interest rate, the result will be destabilizing inflation. The equilibrium exhibits the kind of behaviour that was studied by Knut Wicksell already in the early part of the century (see Wicksell 1936). Since the discount factor for the forward sum is greater than unity, it determines the speed at which the linear combination of interest rate and technology change

[10] Most of the central banks do control short-term interest rates: the two-week repo rate (Bundesbank, Swedish Riksbank), two-week tender rate (Bank of Finland), FED funds rate (Federal Reserve), etc.

converges to zero. We discuss the feasibility of interest rate paths in section 2.3.1.

We extend the analysis to the case where the central bank aims to stabilize inflation or inflation expectations. In the former case inflation is fixed at the target rate, whereas in the latter case shocks cause inflation to diverge from the targeted rate. The differences are due to information differences between households and firms *versus* the central bank. The central bank must make the first move. Different cases arise depending on whether nature moves before or after the central bank, ie whether shocks occur before or after the central bank moves. We compare these rules to the classical case in which interest rates are conditioned on current inflation. It turns out that, in contrast ot the other rules, the standard case does not necessarily lead to a constant inflation rate.

Finally we are able to show that money has a particular feature in our setup. Because of the adjustment costs on money balances, we find an equilibrium where money can be used as an anchoring device for inflation. The central bank can even use a money rule whereby money growth is conditioned on period $t-1$ information on the state variables. This gives the central bank a device for controling inflation with a lagged information set. However, the central bank needs to know the parameters of the 'money demand' equation.

2.3.1 Feasible Interest Rate Paths

In this section we discuss the possible interest rate paths for given stochastic specifications of the technology process, given the aim of finite inflation. We discuss first some general results and then the restrictions on the interest rate process for the case in which the expected technology change is a constant.

The forward solution (2.36) can be written as

$$\Delta p_t = -\frac{D}{d\rho(\delta\lambda_p - \lambda_p^{-1})} E_t \sum_{j=0}^{\infty} \left[(\delta\lambda_p)^j - \left(\frac{1}{\lambda_p}\right)^j \right] \times (i_{t-1+j} + \log\delta + v - \rho\Delta\tau_{t+j}). \quad (2.38)$$

Since $|\delta\lambda_p| > 1$ and $|1/\lambda_p| < 1$, the discount factor, $(\delta\lambda_p)^j - \left(\frac{1}{\lambda_p}\right)^j$, is greater than one. The discount factor restricts the feasible paths

of the interest rate and technology growth processes. Assuming an autoregressive process for the weighted sum of the interest rate and technology change processes $e_t \equiv i_t + \log\delta + v - \rho\Delta\tau_{t+1}$, ie $\mathfrak{a}(L)e_t = \varepsilon_t^z$ (where ε_t^z is identically and indepently distributed with zero mean) we can prove the following proposition:

Proposition 1. Assuming $e_t \equiv i_t + \log\delta + v - \rho\Delta\tau_{t+1}$, it follows that for an autoregressive process $\mathfrak{a}(L)e_t = \varepsilon_t^z$, with ε_t^z independently and identically distributed with mean zero, inflation will be finite if the absolute value roots of $|\mathfrak{a}(L)| = 0$ are greater than $\delta\lambda_p$.

The proof follows from the discount term, which is at most of the order $\delta\lambda_p$. However, in general there is no need to be restricted to the class of linear interest rate processes.

The proposition is not readily applicable and does not lead to unconditional[11] interest rate rules unless we parameterize the process driving technology changes. It asserts that the expected technology change and interest rate should not diverge but should instead converge at least with a rate that is limited by $1/\delta\lambda_p$. The absolute requirement for inflation-stabilizing monetary policy, given the technology process, is to choose the parameters of the interest rate process so that they satisfy the condition in proposition 1. Hence, by the very nature of rational expectations, the policy is a sequence of the relevant control variable — not a single point at a single point of time. Accordingly, the policy is generally not defined by a single point for the interest rate but rather by the parameters of the interest rate process. If the monetary policy is aimed at stabilizing inflation, policymakers must choose the parameters of the interest rate process according to proposition 1. Thus the parameters of the interest rate process depend on the parameters of the technology change process.

Consider, for example, the case where $E_t\Delta\tau_{t+1} = \tau^\star$ and e_t is assumed to follow an AR(1) process

$$i_t = -(\log\delta + v) + \rho\tau^\star + \iota(i_{t-1} + \log\delta + v - \rho\tau^\star) + \varepsilon_t^\iota,$$

where ε_t^ι is a zero-mean independent process[12]. The j-period conditional forecast is then $E_t i_{t+j} = \iota^j(i_t + \log\delta + v - \rho\tau^\star)$. The expectation

[11] By the term *unconditional rule* we mean a rule that is independent of the state of the economy, eg a constant money growth rule.

[12] We need not assume homoscedasticity. Note that it is not necessary to define the policy shocks, ε_t^ι.

part of equation (2.38) is then $E_t \sum_{j=1}^{\infty}[(\delta\lambda_p)^j - (1/\lambda_p)^j]\iota^j(i_t + \log\delta + v - \rho\tau^\star)$, which is finite if $|\iota| < 1/\delta\lambda_p$. Monetary policy in a world of constant technology growth is fairly simple. In order to keep inflation bounded, the central bank must choose the parameters of the interest rate process so that its degree of mean reversion is sufficiently high. The central bank need not to react to innovations in technology. All it must do is to fix the parameters of the interest rate process so that they satisfy the condition of proposition 1. In the above AR(1) example, the change in policy implies a different choice of ι.

In the general case, where the expected technology change is not a constant, monetary policy must react to innovations in the expected growth rate of technology but not to innovations in the level of technology. Due to this fact the interest rate must be changed for every period when a shock occurs to the growth rate of $\Delta\tau_t$. However, a change in the interest rate is not a monetary policy change since it is the parameters of the interest rate process that define the monetary policy. It is also implicitly assumed that the central bank can commit itself to an interest rate rule.

2.3.2 Targeting Inflation or Inflation Expectations

In this section we study three cases. In the first case the central bank (monetary authority) targets a constant inflation rate, $\Delta p_t = \pi^\star$. This leads to a sufficient condition for determining the interest rate rule. The policy rule that results is optimal in the case where the central bank has a direct inflation target and a quadratic objective function with respect to deviations of inflation from target. The second case assumes that in each period the central bank must set the interest rate path before the technology and velocity shocks occur, ie such shocks are not known by the central bank when it makes its interest rate decision. Svensson (1996) refers to this as inflation forecast targeting, $E_{t-1}\Delta p_t = \pi^\star$. Finally we investigate the pure feedback rule proposed by Kerr and King (1996), where the interest rates are set on the basis of feedback from current inflation.

To fix inflation at a given rate, we replace the actual inflation in equation (2.38) with targeted inflation, $\pi^\star$, and solve for the current

interest rate:

$$i_t = \rho E_t \Delta\tau_{t+1} + \frac{\log\delta + v}{(\delta\lambda_p - \lambda_p^{-1})^2} - \frac{d\delta}{D}\pi^\star$$
$$- \frac{1}{\delta\lambda_p - \lambda_p^{-1}} E_t \sum_{j=2}^{\infty} \left[(\delta\lambda_p)^j - \lambda_p^{-j}\right] (i_{t-1+j} - \rho\Delta\tau_{t+j}). \quad (2.39)$$

According to the resulting interest rate rule, the interest rate should be set in line with the expected technology change and targeted inflation rate, $\pi^\star$. In addition to this, the central bank ties its hands for the future by announcing the interest rate path that is in line with targeted inflation. The convergence results hold for this infinite sum, as in section 2.3.1.

The central bank could, for example in the case of expected technological improvements, announce that in order to reach the targeted inflation today it will raise interest rates only in the future. It could even lower the current interest rates today if it can commit to raising them in the future. By means of such an announcement, it could not only postpone the interest rate change but also achieve inflation of $\pi^\star$. However, this policy cannot be pursued forever because the transversality condition (2.23) restricts the possibility of postponing the decision. In general, the model allows the central bank to choose any kind of inflation path (within the limits of the transversality condition) by announcing a suitable interest rate path with respect to expected technology changes.

A sufficient condition for achieving the targeted inflation, $\pi^\star$, can be obtained by replacing the expected and actual inflation rates in Euler equation (2.34) by targeted inflation rate $\pi^\star$, and solving for the current interest rate. We obtain the interest rate rule

$$i_t = \pi^\star - (\log\delta + v) + \rho E_t \Delta\tau_{t+1}. \quad (2.40)$$

The outcome of the inflation-targeting policy is that the central bank must set the interest rate according to expected technology change and inflation target minus the steady state interest rate in order to keep inflation at the targeted rate, $\pi^\star$. If we assume a constant technology growth rate of $\tau^\star$, the constant interest rate $i_t = \pi^\star - (\log\delta + v) + \rho\tau^\star$ stabilizes inflation at $\pi^\star$. This however is not the case when the technology process is not a martingale with drift.

We consider the following example, where the technology change follows the first order autoregressive process $\Delta\tau_t = \beta\Delta\tau_{t-1} + \varepsilon_t^\tau$.

The interest rate rule is then $i_t = \pi^\star - (\log\delta + v) + \rho\beta\Delta\tau_t$. Hence the interest rates vary over time as the technology changes vary. It is also clear that there must not be any parameter uncertainty, ie the technology change process must be known. Note that the above discussion on shocks also applies here. The central bank need not react to shocks to the level of technology if these do not change the growth rate of technology.

We resuffle the sequence of decisions by assuming that the interest rate is set before the current technology and velocity shocks take place. Thus the interest rate can be conditioned on period $t-1$ information only. The conditional interest rate rule can be obtained from (2.40) as

$$E_{t-1}i_t = \pi^\star - (\log\delta + v) + \rho E_{t-1}\Delta\tau_{t+1}. \tag{2.41}$$

We note that here the central bank must forecast even the current technology change. The realization of the period t technology change may deviate from the expected one and thus the current inflation may differ from $\pi^\star$. We denote the update of the expectations as to technology change as $\epsilon_t^{\Delta\tau} \equiv E_t\Delta\tau_{t+1} - E_{t-1}\Delta\tau_{t+1}$. Since the central bank cannot know the shocks for period t, it cannot directly control inflation. Inflation is then determined by the target and the expectations update, ie $\Delta p_t = \pi^\star + \epsilon_t^{\Delta\tau}$.

We may interprete the above equation as the optimal rule under strict inflation targeting when the central bank uses the inflation forecast as the intermediate target (see Svensson 1996). Households and firms know the current level of technology when they make their decisions on money, consumption and prices. Consequently they do not make mistakes in setting the current inflation. Given the above example, where technology change follows a first-order autoregressive process, we note that the central bank observes the interest rate rule $E_{t-1}i_t = \pi^\star - (\log\delta + v) + \rho\beta^2\Delta\tau_{t-1}$. In such an autoregressive case, it can replace the period $t-1$ technology change with that period's inflation. The rule $E_{t-1}i_t = \pi^\star - (\log\delta + v) + \rho\beta(\Delta p_{t-1} - \pi^\star) + \rho\beta^3\Delta\tau_{t-2}$ is equivalent with the above rule. When we iterate the rule backwards,

we end up with the following distributed lag version of the rule:

$$E_{t-1}i_t = \pi^\star - (\log\delta + v) + \rho\sum_{j=1}^{t}\beta^j(\Delta p_{t-j} - \pi^\star)$$
$$= (1-\beta)\pi^\star - (1-\beta)(\log\delta + v) + \beta\rho(\Delta p_{t-1} - \pi^\star) - \beta E_{t-2}i_{t-1}. \quad (2.42)$$

Hence the central bank can condition the current interest rate on the whole inflation history instead of on the expected technology change. Another interpretation is that the setting of the current interest rate must be conditioned on the last deviation from target and the last interest rate setting. This is due to the autoregressive technology change process and is not a general result. Also, in this particular case the technology process parameters (here β) must be known.

Third, we consider the following situation wherein the central bank operates a feedback rule of the form

$$i_t = \underline{i} + g(\Delta p_t - \pi^\star), \text{ where } g \geq 0. \quad (2.43)$$

This rule is analysed eg by Parkin (1978), McCallum (1981) and Kerr and King (1996). We append this 'nominal anchor' to the equilibrium relationship (2.38). The result is that Euler equation (2.34) reduces to

$$E_t\Delta p_{t+2} - \left(1 + \frac{1}{\delta} + \frac{D}{d\delta\rho}\right)E_t\Delta p_{t+1} + \left(\frac{1}{\delta} + \frac{D}{d\delta\rho}g\right)\Delta p_t =$$
$$- \frac{D}{d\delta\rho}\left(\log\delta + v + \underline{i} - g\pi^\star - \rho E_t\Delta\tau_{t+1}\right). \quad (2.44)$$

It can be shown that the roots of the characteristic polynomial of Euler equation (2.44) are both greater than unity if $g > 1$ or they are on opposite sides of the unit circle if $0 \leq g < 1$. When $g > 1$, the central bank reacts forcefully to deviations of inflation from target. In this case the inflation is bounded if the technology changes are bounded. Without the aggregate technology process, this is the same result as in Kerr and King (1996). What differs is the fact that *the exact inflation target can be achieved with this rule only in the case where the expected technology change is a constant.* From the above, it follows that the central bank must explicitely take into account the expected technology change in deciding on the interest rate rule. With an autoregressive technology process, the central bank can condition

the interest rate on past inflation but it must also condition the interest rate on past technology changes. There is no escape from conditioning the interest rate rule on technology information. Our result is very much like the analysis of Ireland (1997).

2.3.3 Forecasting the Technology Growth

It is interesting to know the stochastic properties of the technology growth. In this section we outline the estimation methodology of the technology process and discuss some empirical results that are reported in detail in Ripatti (1998*a*). We approximate the technology process by the total factor productivity observed annually[13]. We base the estimation procedure on the Euler equation (2.24). The other data is monthly Finnish data (1980–1995). Thus the within-year changes in τ_t are unknown.

In the following we denote the periodicity of the data by s. We replace the expectational term by the realization and the expectated error, $E_t\Delta p_{t+1} = \Delta p_{t+1} - \varepsilon^p_{t+1}$, and assume the independence and the normality of the residuals, $\varepsilon^p_t \sim$NID$(0, \sigma^2_p)$. Thus after the above replacement and shifting $t \to t-1$, equation (2.24) can be written in the form

$$\Delta p_t = -\frac{S}{d\delta} + \frac{1}{\delta}\Delta p_{t-1} - \frac{D}{d\delta}\left(c_{t-1} - \tau_{t-1}\right) + \varepsilon^p_t. \tag{2.45}$$

We assume that τ_t follows the structural time series process

$$\tau_t = \sum_{i=1}^{k} \gamma_i \tau_{t-i} + \mu_t + \eta^\tau_t, \quad \eta^\tau_t \sim \text{NID}(0, \sigma^2_\tau), \tag{2.46}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta^\mu_t, \quad \eta^\mu_t \sim \text{NID}(0, \sigma^2_\mu), \tag{2.47}$$

$$\beta_t = \xi\beta_{t-1} + \beta^\star + \eta^\beta_t, \quad \eta^\beta_t \sim \text{NID}(0, \sigma^2_\beta), \quad |\xi| \leq 1. \tag{2.48}$$

The component β_t is the slope of the trend μ_t. The irregular component, η^τ_t, the level disturbance, η^μ_t, and the slope disturbance, η^β_t, are mutually uncorrelated, ie $\eta^\tau_t \perp \eta^\mu_t \perp \eta^\beta_t$. η^μ_t allows the trend to shift up and down and η^β_t allows the slope to change.

[13] See Statistics Finland (1997) for a detailed description of the compilation of total factor productivity.

Since the structural equation (2.24) contains an expectational term, there is a danger that the residuals in equation (2.45) are correlated with the explanatory variables. Watson (1989) illustrates how to resolve the issue in the state-space framework. There is, however, a drawback: one needs to parameterize the stochastics of the consumption process as well. We will take a short cut and ignore the problem. Appendix 2.4 describes the state-space representation and estimation of the system.

We study the two major interest rate rules stated above: in the first case the central bank knows period t technology and velocity shocks (rule based on equation 2.40; $\Omega_t = I_t$) and in the second case (rule based on equation 2.41; $\Omega_t = I_{t-1}$) on previous period, $t-1$, shocks. Hence, the interesting part of the rule,

$$i_t = \pi^\star - (\log\delta + v) + \rho E(\Delta\tau_{t+1}|\Omega_t), \tag{2.49}$$

is the conditional expectation of the future technology change $E(\Delta\tau_{t+1}|\Omega_t)$. That is our main motivation for estimating the technology process, τ_t.

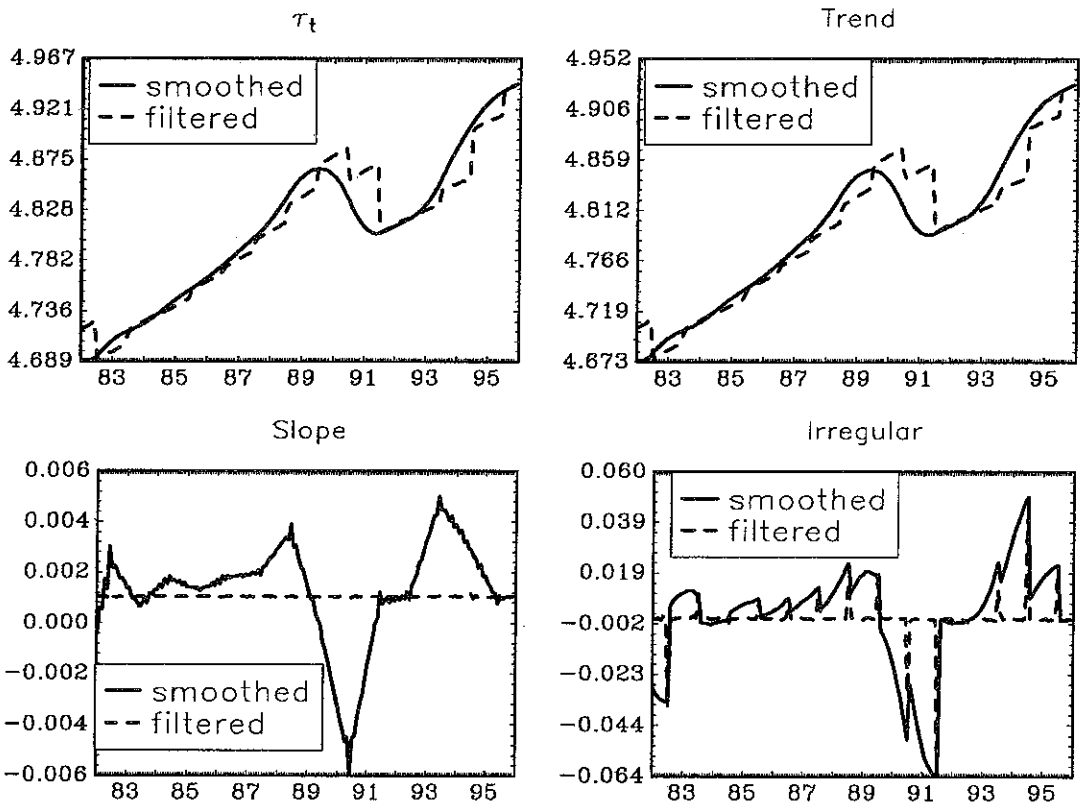
The expected technology growth, which can be derived from equations (2.46) – (2.48), is

$$E(\Delta\tau_{t+1}|\Omega_t) = \sum_{i=1}^{k} \gamma_i \Delta\tau_{t+1-i|\Omega_t} + \beta_{t|\Omega_t} - \eta_{t|\Omega_t}. \tag{2.50}$$

Since τ_t and β_t are time-varying and hence part of the state vector in the state-space representation of the system, we must use both filtered and smoothed values in order to mimic the problem of the central bank. In the first case we use filtered values for the current variables, ie $\tau_{t|t}$, $\beta_{t|t}$, $\eta^{\tau}_{t|t}$, and smoothed values for the lagged τs, ie $\tau_{t-1|t}, \ldots, \tau_{t-k+1|t}$. Following Anderson and Moore (1979), we apply *fixed lag* smoothing and obtain the smoothed values as a byproduct of the Kalman filter, since we have lagged τs in the state vector. Thus no extra computing is needed. In the second case we need predictions for $\tau_{t|t-1}$ and $\beta_{t|t-1}$, filtered values for $\tau_{t-1|t-1}$ and smoothed values for the lagged τs, ie $\tau_{t-2|t-1}, \ldots, \tau_{t-k+1|t-1}$. The irregular term is zero in the second case, ie $\eta_{t|t-1} = 0$.

We repeat the preliminary result of Ripatti (1998*a*). The estimated components of the τ_t process are shown in figure 2.1. The parameter estimates of equations (2.45), (2.46) – (2.48) are quite poor with respect to their standard errors. It is however important to use even

Figure 2.1 **Components of τ_t**



this weak information in the estimation of the τ_t process. Figure 2.1 shows that the stochastic trend dominates τ_t and the stochastic slope term $\Delta\tau_t$. The smoothed values of these components look reasonable. However, it is the filtered value of the slope component that more closely corresponds to the central bank's forecasting problem. According to these preliminary results, the filtered value of the slope term is practically constant. If our model contained all the information that the central bank has at hand, the resulting interest rate path would be almost constant[14]. The linearity tests reported by Ripatti (1998*a*) suggest that there might even exist nonlinearity in the form of a regime shift.

The difference between the two cases lies mainly in the treatment of the irregular term. If the central bank knew the period t shocks, there would be spikes in the interest rate path due to the spikes in the filtered irregular term. In the other rule, the irregular term is zero and no such spikes exist. These results clearly demonstrate that forecasting the technology growth could be very difficult task. They also

[14] Note that we have assumed that v is constant over time.

suggest that the supply side should be modelled in more detail if the model is to form basis for empirical application. Nevertheless, even a simple version of the model exhibits the basic insight that aggregate technology plays a crucial role in the formulation of monetary policy.

2.3.4 Money as Nominal Anchor

In this section we explore the case where the central bank uses money as a nominal anchor. Money has a special role in our framework. We show how money replaces expectations as to future technology shocks and discuss how the central bank can use the current growth of nominal balances as a monetary policy instrument. Since money growth incorporates the above-mentioned expectations, it follows that money can be a robust monetary policy instrument.

We solve for consumption from the Euler equation for prices (2.29) and substitute the solution into the Euler equation for money (2.27), to obtain the following stochastic difference equation:

$$(L^{-1} - 1/\delta)E_t\Delta p_t = -\frac{1}{d\delta}\left(S + \frac{D\kappa M}{\rho i}m^*\right) + \frac{D\kappa M}{d\delta\rho i}\left[L^{-1} - \left(1 + I + \frac{\omega i}{\kappa M}\right) + IL\right]E_t m_t + \frac{D\omega}{d\delta\rho}E_t\left(p_t - \frac{1}{\omega i}i_t + \frac{\rho}{\omega}\tau_t + \frac{1}{\omega}\zeta_t\right).$$

Note that $|1/\delta| > 1$. This equation has the following interesting deterministic backward solution:

$$\Delta p_t = -\frac{1}{d\delta}\left(S + \frac{D\kappa M}{\rho i}m^*\right) + \frac{1}{\delta}\Delta p_{t-1} + \frac{D\kappa M}{d\delta\rho i}\Delta m_t + \frac{D\kappa MI}{d\delta\rho i}\Delta m_{t-1} - \frac{D\omega}{d\delta\rho}\left(m_{t-1} - p_{t-1} + \frac{1}{\omega i}i_{t-1} - \frac{\rho}{\omega}\tau_{t-1} - \frac{1}{\omega}\zeta_{t-1}\right). \quad (2.51)$$

The coefficient of current money changes determines the positive relationship between money growth and inflation. This relationship enables the control of inflation by controlling money growth. The negative sign of the loading coefficient, $-\frac{D\omega}{d\delta\rho}$, motivates the results for the

estimation of the money demand system, as in Ericsson and Sharma (1996).

Another interpretation of the relationship is that money serves as an important information variable on inflation. It replaces the discounted sum of expected interest rates and expected future technology changes. Thus the relationship is also useful to a central bank with an interest rate instrument and a direct inflation target. In fact, if the central bank is uncertain about the precise parameterization of the stochastic process for technology change or there is simply a great deal of uncertainty as to future technology shocks, the bank can use money as an indicator of public expectations as to future technology changes and interest rates.

This particular contemporaneous relationship arises from the adjustment costs of changing money holdings. In the case of zero adjustment costs ($\kappa = 0$), the money growth terms vanish and the equation contains only lagged inflation and the error correction term, ie the last term in parentheses. In such a case the inflation could not be directly controlled by the money instrument. Hence the contemporaneous relationship between inflation and money growth is a unique feature of the model at hand, not a feature of the MIUF model in general.

Replacing current inflation, Δp_t, with targeted inflation, π^*, we obtain the following money growth rule:

$$\Delta m_t = \frac{\rho i}{D\kappa M}\left(S + \frac{D\kappa M}{\rho i}m^* + d\delta\pi^*\right) - \frac{d\rho i}{D\kappa M}\Delta p_{t-1} + I\Delta m_{t-1} + \frac{\omega i}{\kappa M}\left(m_{t-1} - p_{t-1} + \frac{1}{\omega i}i_{t-1} - \frac{1}{\omega}\zeta_{t-1} - \frac{\rho}{\omega}\tau_{t-1}\right). \quad (2.52)$$

We see that the central bank can precisely determine the period t inflation rate by relying solely on information known at the end of period $t-1$. Only the period $t-1$ realizations of τ_{t-1} and ζ_{t-1} need be known. One need not know the stochastic specification of the shocks. Hence the money rule of equation (2.52) is robust with respect to the stochastic processes for τ_t and ζ_t. The central bank can control inflation exactly even if the money supply is set at the start of period t, ie before period t shocks occur.

There is no free lunch here either: equation (2.51) can be utilized only in the case where the parameters of the relationship (2.27) are known. The parameters that must be known, in addition to the parameters in the interest rate rules, are the level of adjustment costs, κM,

the mean of steady state, m^*, and the linearization point of the interest rate, i.

The form of the relationship described by equation (2.52) is, in a sense, a formulation of the Bundesbank approach[15]. The German Bundesbank sets its money growth target as the sum of growth rates (targets) for prices, velocity and potential output. Here $\frac{1}{i\omega}i_t - \frac{1}{\omega}\zeta_t$ presents the velocity term, which is partly independent of monetary policy and depends on the time-varying liquidity services that money provides; positive changes in $\frac{\rho}{\omega}\tau_t$ represent advances in potential output. The relationship between real money balances, interest rates, and preference and technology shocks represents the steady state relationship to which the system converges. The rest of the equation represents the adjustment dynamics for inflation and money.

2.4 Demand for Money and Monetary Policy

We conclude the study by discussing the role of the demand for money in a monetary policy framework that entails an inflation target. To some extent, this discussion is connected with that on the choice of monetary policy instrument initiated by Poole (1970).

Our model is based on the money-in-the-utility-function approach augmented with a sticky price supply side equation. The special characteristic of the model is the adjustment costs of money balances. The driving forces of the system arise from aggregate technology shocks as well as shocks to preferences regarding real money balances. The Euler equation for money determines either money balances or the interest rate. Consumption is determined by the Euler equation for bonds. Finally, the adjustment costs of changing prices lead to the Euler equation for inflation.

The equilibria determined by the last two Euler equations fully characterize the relationship between inflation and interest rates given the expected technology change. In this respect, money is not needed in the model in order to control inflation. In such a case, the first Euler equation simply determines recursively the money holdings. An interest rate rule that targets constant inflation or inflation expectations must be conditioned on the expected technology change. This

[15] See eg Deutsche Bundesbank (1997) or any other January issue of its monthly report.

is one of the main findings of the study. In the more realistic situation when the shocks for period t are not known by the central bank, inflation cannot be targeted at all. In that case only the inflation expectations can be targeted. The only exception is the simplest case where the technology change is constant over time. However, in general, precise estimation of the technology process may be impossible or at least very difficult, and stability of the technology process is a very heroic assumption. In such a case the inflation targeting rule cannot be determined. In summary, one can say that it is possible to reach the inflation target using the interest rate instrument only if the technology process is known.

We also show that in the case of adjustment costs for nominal money balances there exists an equilibrium in which current money growth determines current inflation. This is also the outcome for models with no friction in prices or money. However, this classical result is not generally present in sticky price models. The introduction of adjustment costs in changing money holdings leads to this result in our setup. Monetary policy can be based on money as an instrument or money as an intermediate target. This approach is robust in two respects: First, the proposed money rule is based only on the information of period $t-1$. Hence, the rule is robust to any choice of the ordering of actions. Second, no information is needed on the stochastic process governing technology changes or velocity changes. This robustness relieves the central bank of the task of estimating the technology change process. The cost of using money as an information variable for expected technology changes and expected interest rates is that some additional parameters must be known. These parameters are related to the 'money demand' parameters. We should emphasize that this kind of equilibrium exists only because of adjustment costs and it is not a general feature of a dynamic stochastic money-in-the-utility-function model combined with sticky prices.

Although central banks cannot possibly precisely control a wide monetary aggregate, they can easily control the monetary base[16]. It has been argued (see eg Goodhart 1994) that even controlling the monetary base is not only undesirable but also infeasible. The reason for this is the resulting increased volatility of interest rates. However, this is not necessarily a problem for central banks that use averaging of the required reserves. McCallum (1997) argues that 'neither inter-

[16] The money measure is not however restricted in this study.

est rate nor monetary base instruments are infeasible,' and concludes that it is a question of desirability.

Our model exhibits the basic insight of Poole (1970): If there is uncertainty about the parameters of the 'money demand' function (money demand shocks in Poole's terminology), the interest rate is suitable as the monetary policy instrument. On the other hand, if productivity shocks occur or — in our setup — the parameters of the technology changes are not known, nominal money balances are a suitable monetary policy instrument. In this sense, our model shifts Poole's analysis from shocks to parameter uncertainty.

It is clear that the tradeoff between these two instrument choices is the parameter uncertainty concerning 'money demand', on the one hand, and parameter uncertainty concerning the technology process, on the other hand. The third dimension is the issue of whether current or expected inflation is to be targeted — ultimately this is the issue of ordering decisions in our economy. It is clear that the choice of a monetary policy instrument is very much an empirical and economy-specific question.

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Appendix State Space Form

The annual observation of τ_t is the average of the monthly values. Hence, τ is a flow variable from the statistical point of view. We define a variable τ_t^A as follows

$$\tau_t^A = \begin{cases} \text{the annual observation} & \text{if } t-5 = s, 2s, \ldots, N, \\ 0 & \text{otherwise,} \end{cases}$$

where N is the sample size and -5 in time index is due to the fact that the annual observation is yearly average and is to be located to the middle point[17] of the year. Let $m = \max(k, s)$. In order to estimate the system defined by equations (2.45) – (2.48), we write the model into the following state space form

$$\begin{aligned} y_t &= Z_t \alpha_t + \phi X_t + \varepsilon_t, \quad \varepsilon_t \sim \text{NID}(0, H_t) \\ \alpha_t &= T\alpha_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}(0, Q_t) \end{aligned}, \tag{2.53}$$

where

$$y_t = \begin{bmatrix} \Delta p_t \\ \tau_t^A \end{bmatrix}, \quad \alpha_t' = \begin{bmatrix} \tau_t & \ldots & \tau_{t-m+1} & \mu_t & \beta_t & 1 \end{bmatrix},$$

$$X_t = \begin{bmatrix} \Delta p_{t-1} \\ c_{t-1} \\ 1 \end{bmatrix}, \quad \phi = \begin{bmatrix} 1/\delta & -D/d\delta & -S/d\delta \\ 0 & 0 & 0 \end{bmatrix}$$

$$Z_t = \begin{cases} \begin{bmatrix} 0 & \frac{D}{d\delta} & 0 & \ldots\ldots\ldots & 0 \\ 1/s & \ldots\ldots & 1/s & 0 & \ldots & 0 \end{bmatrix} & t-5 = s, 2s, \ldots, N, \\ \begin{bmatrix} 0 & \frac{D}{d\delta} & 0 & \ldots & 0 \\ 0 & \ldots\ldots\ldots & & & 0 \end{bmatrix} & \text{otherwise,} \end{cases}$$

[17] We prefer July instead of June.

$$T = \begin{bmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_k & 0 & \dots & 0 & 1 & 0 & 0 \\ 1 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & 1 & 1 & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & \xi & \beta^\star & \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 1 & \end{bmatrix}$$

$$H_t = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} \varepsilon_t^p \\ 0 \end{bmatrix}, \quad \eta_t' = \begin{bmatrix} \eta_t^\tau & 0 & \dots & 0 & \eta_t^\mu & \eta_t^\beta & 0 \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_\tau^2 & \dots & 0 & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \sigma_\mu^2 & 0 & 0 \\ 0 & \dots & \dots & 0 & \sigma_\beta^2 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix}.$$

The parameters of interest are $\gamma_1, \dots, \gamma_k$, $\beta^\star$, $1/\delta$, $-D/d\delta$, $-S/d\delta$, ξ, σ_p^2, σ_τ^2, σ_μ^2 and σ_β^2. Model can be easily augmented with, for example, seasonal dummies. Note also that no measurement error is allowed in the aggregation equation. The state-space model can be estimated using maximum likelihood principle. The Kalman filter can be used produce the likelihood function.

Chapter 3

Stability of the Demand for M1 and Harmonized M3 in Finland

Contents

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Abstract

We derive a theoretical model for the demand for money using the adjustment cost -augmented money-in-the-utility-function approach. The steady state — utility function — parameters of the model of narrow money (M1), estimated with cointegration techniques, are stable over the foreign exchange rate regime shift; whereas in the model of harmonized M3 (M3H) they are *not* stable. The theoretical model fits the M1 data. The adjustment cost parameters of the M1 model describing the dynamics of the demand for money might indicate technological improvements in banking and payments during the sample period. These results suggest that from the Finnish viewpoint M1 would be a more appropriate intermediate target for monetary policy than harmonized M3.

Keywords: money-in-the-utility-function model, structural breaks, demand for money, narrow money, harmonized M3

JEL classification: C22, C52, E41

3.1 Introduction

One possible choice for the monetary policy strategy of the future European System of Central Banks (ESCB) is to use money as an intermediate target (see, eg EMI 1997). This requires a stable demand for money relationship. The stability of different money measures could vary across European countries. The main candidates under investigation are narrow liquid money (M1) and harmonized broad money (M3H)[1]. From the perspective of the economic and monetary union (EMU), it is important to find a money measure whose demand is *stable* in all the participating countries and for which the national money demand parameters are as close as possible to average EMU values.

The aim of this study is to analyse the stability of the demand for these two money measures in Finland. We build on the recent European tradition of empirical research[2] on the demand for money by linking the time series econometrics of nonstationary variables to the theoretical model. We estimate and test the stability of the preference and techonology parameters of the theoretical model. In contrast to the pure time series approach, we have the possibility of relating possible structural changes to the preference or technology part of the model.

In section 3.2, we derive the demand for money from the money-in-the-utility-function approach. To estimate the parameters in the presence of integrated variables, we log-linearize the first-order condition. The first-order condition is then linear in the levels of the variables but nonlinear in the parameters. The steady-state part, ie the preference parameters, of the first-order condition can be estimated with cointegration techniques and the other part, ie the technology parameters, with the generalized method of moments estimator for given estimates of the steady state. The econometrics is overviewed in section 3.3 and the estimates[3] are reported and the stability eval-

[1] The European Monetary Institute (EMI) is still working on harmonization rules for various money measures. The present measure of M3H will probably not be the final measure.

[2] Examples are Cuthbertson and Taylor (1990), Hendry and Ericsson (1991), Baba, Hendry and Starr (1992), Juselius (1996), Fase (1994), Wolters, Teräsvirta and Lütkepohl (1996).

[3] The cointegration estimation is done with CATS in RATS by Hansen and Juselius (1995) except for the small sample simulations, which are performed with

uated in section 3.4. The final section concludes and discusses the policy implications of the empirical results.

3.2 Theoretical Background: Money-in-the-Utility-Function Model

Although Lucas (1988) prefers to base the 'money demand model', or rather the relationship between money, consumption and interest rates, on the cash-in-advance constraint, we have chosen to include real balances directly in the utility function. Feenstra (1986) demonstrates the functional equivalence between using real balances as an argument in the utility function and liquidity cost models, while Croushore (1993) demostrates the equivalence between shopping time and money-in-the-utility-function (MIUF) models. We are also interested in the dynamics of this relationship. The strong persistence of nominal balances — and in the growth rate — might imply that changes in nominal balances involve transaction costs. One may argue for the existence of adjustment costs by the fact that the conversion of bonds into money balances is not without costs. Furthermore, since most of the money measures contain bank accounts, there are certainly costs involved in adjusting money balances. Alternatively, adjustment costs can be viewed as an analytically convenient modelling device. To be able to study the dynamics of money demand, we incorporate adjustment costs into our model. In the case of adjustment costs, the MIUF approach is also analytically simpler for our purposes.

There are not many empirical money demand studies in which the estimated parameters are based on an explicit theoretical model. The following studies are quite close to ours: Poterba and Rotemberg (1987) base their empirical investigation on the liquidity cost approch; and Lucas (1988) and Sill (1995) rely on the 'cash-in-advance' constraint.

Gauss utilizing the CIA code by Paolo Paruolo. The GMM estimation is done with Gauss, with part of the coding being based on the Hansen/Heaton/Ogaki GMM package by Ogaki (1993). I thank Paolo Paruolo and Masao Ogaki letting me use their code.

We start with an MIUF model in which the *household* maximizes the discounted sum of expected utility from consumption and money:

$$\max E_0 \sum_{t=0}^{\infty} \delta^t \left(u(C_t) + \mathfrak{Z}_t v\left(\frac{M_t}{P_t}\right)\right). \tag{3.1}$$

The household allocates its real income, y, and other earnings among consumption goods (C_t: real value of consumption); bonds (B_t: real value of bonds denominated in units of time t consumption), which pay a gross real return $1+r_t$ (from time t to time $t+1$); and real money balances, M_t/P_t, which pay a gross return of P_t/P_{t+1}. For some definitions of money, money also pays a nominal return (own-yield of money) of $O_t \equiv 1 + o_t$. Whenever it adjusts its money balances between period $t-1$ and period t, the household suffers losses (in real terms) of $a(M_t, M_{t-1}, M_{t-2})/P_t$. $\mathfrak{Z}_t$ is a stochastic weight on the real money balances in the utility function. It allows shocks to the liquidity services of money, ie velocity or money demand shocks. The household's budget constraint is

$$C_t + B_t + \frac{M_t}{P_t} + \frac{a(M_t, M_{t-1}, M_{t-2})}{P_t} \leq y + \frac{O_{t-1}M_{t-1}}{P_t} + (1 + r_{t-1})B_{t-1}. \tag{3.2}$$

The first-order conditions of the household's optimization problem (3.1) subject to (3.2) are

$$\delta E_t\{(1 + r_t)u'(C_{t+1})\} = u'(C_t) \tag{3.3}$$

$$\begin{aligned}\mathfrak{Z}_t v'\left(\frac{M_t}{P_t}\right) =& u'(C_t)\left[1 + a'_{M_t}(M_t, M_{t-1}, M_{t-2})\right] \\ &- \delta E_t\left\{\frac{u'(C_{t+1})P_t}{P_{t+1}}\left[O_t - a'_{M_t}(M_{t+1}, M_t, M_{t-1})\right]\right\} \\ &+ \delta^2 E_t\left[\frac{u'(C_{t+2})P_t}{P_{t+2}} a'_{M_t}(M_{t+2}, M_{t+1}, M_t)\right].\end{aligned} \tag{3.4}$$

We assume that a nominal bond exists in our generic economy. The condition (3.3) for the nominal bond is given by

$$(1 + i_t)\delta E_t\left\{\frac{1}{P_{t+1}}u'(C_{t+1})\right\} = \frac{1}{P_t}u'(C_t), \tag{3.5}$$

where $I_t \equiv 1+i_t$ is a gross return on the nominal bond. If we combine the additional assumptions

$$\mathrm{cov}_t\left(\frac{P_t}{P_{t+1}}\frac{u'(C_{t+1})}{u'(C_t)}, \left[O_t - a'_{M_t}(M_{t+1}, M_t, M_{t-1})\right]\right) = 0 \text{ and}$$
$$\mathrm{cov}_t\left(\frac{P_t}{P_{t+2}}\frac{u'(C_{t+2})}{u'(C_t)}, a'_{M_t}(M_{t+2}, M_{t+1}, M_t)\right) = 0$$

with the equation (3.5), the condition (3.4) for nominal money can be written as

$$\begin{aligned}\beta_t\frac{v'(\frac{M_t}{P_t})}{u'(C_t)} &= a'_{M_t}(M_t, M_{t-1}, M_{t-2}) + 1 - \frac{O_t}{I_t} \\ &+ \frac{1}{I_t}E_t a'_{M_t}(M_{t+1}, M_t, M_{t-1}) + E_t\frac{1}{I_t I_{t+1}}a'_{M_t}(M_{t+2}, M_{t+1}, M_t).\end{aligned} \tag{3.6}$$

The covariance conditions hold for example if consumers are risk neutral and inflation is deterministic or if the net own-yield of money, $O_t - a'_{M_t}(M_{t+1}, M_t, M_{t-1})$ and the expected marginal adjustment costs are deterministic. The left-hand side of equation (3.6) is the marginal rate of substitution of consumption for real balances. The right hand side is the rental cost, in terms of the consumption good, of holding an extra unit of real balances for one period. Due to the adjustment costs, the rental cost differs from the usual $1 - O_t/I_t$.

Next we parametrize the utility function to the constant relative risk aversion (CRRA) form as

$$u(C_t) = \begin{cases}\frac{1}{1-\rho}\left(C_t^{1-\rho} - 1\right) & \text{if } \rho \neq 1 \\ \log C_t & \text{if } \rho = 1\end{cases},$$
$$v\left(\frac{M_t}{P_t}\right) = \begin{cases}\frac{1}{1-\omega}\left[\left(\frac{M_t}{P_t}\right)^{1-\omega} - 1\right] & \text{if } \omega \neq 1 \\ \log\left(\frac{M_t}{P_t}\right) & \text{if } \omega = 1\end{cases}$$

and the adjustment cost function $a(\cdot)$ as

$$a(M_t, M_{t-1}, M_{t-2}) = \frac{\kappa}{2}\left[(M_t - M_{t-1}) - \nu(M_{t-1} - M_{t-2})\right]^2, \tag{3.7}$$

where κ and ν are adjustment cost parameters. The adjustment cost function expresses the notion that it is differences in the growth rate

of money that affect costs, not the growth rate itself, as is typical (Cuthbertson and Taylor 1987, Cuthbertson and Taylor 1990, among others). However, if the parameter ν is zero, the adjustment cost function is the typical one. The chosen functional form allows for persistence not only in the level of money balances but also in the growth rate of money. Hence it tracks autocorrelation in the money growth. Tinsley (1993) argues that use of the simple quadratic form of adjustment costs in levels is the reason why the cross-equation restrictions implied by the rational expectations hypothesis are usually rejected in empirical excercises. He suggests that one should use a temporally richer specification of the adjustment cost function. A drawback of our parameterization is that there might exist offsetting changes in money balances, which would imply zero adjustment costs[4]. This is also true in the case of positive ν with the adjustment cost specification that is quadratic in changes. However, our formulation is slightly more general but contains the above-mentioned drawback.

It is standard practice to estimate such first-order conditions with generalized method of moments (GMM) estimators. However, what is sometimes overlooked — typically in the studies of the early 1980s — is the problem of non-stationarity of the mean. Stationarity of stochastic processes is the key assumption of GMM. If that is rejected, as is often the case for macroeconomic time series, one should use other estimators, which unfortunately exist for linear models only. Thus, it is necessary to linearize the first-order conditions. We use the first-order Taylor approximation around the steady state. In the steady state, the stochastic processes should have finite variance, which is not the case if any of the variables in the model are $I(1)$. It is however possible that a linear combination of $I(1)$ variables is stationary. If so, the variables are cointegrated. We think that the linearized version of the steady-state solution of the model should represent the stationary linear combination of the variables. This would make it possible to linearize this model also.

In order to log-linearize equation (3.6), we first seek the stationary equilibrium for equation (3.6) and then use the first-order Taylor approximation around the stationary equilibrium. For the stationary equilibrium, the adjustment costs are zero and $C_t = C$, $I_t = I$, $M_t = M$, $O_t = O$, $P_t = P$, $\mathfrak{Z}_t = \mathfrak{Z}$ $(\forall t \geq 0)$. We denote logarithmic variables in the lower case (eg $\log C \equiv c$), $\zeta_t = \log \mathfrak{Z}_t$ and $I \equiv 1 + i$

[4] This was pointed out by a referee.

and $O \equiv 1 + o$. In the stationary equilibrium, equation (3.6) reduces to

$$1 - \frac{O}{I} = 3C^{\rho} \left(\frac{M}{P}\right)^{-\omega}, \tag{3.8}$$

which is like the usual, static demand for money function. From the first-order Taylor approximation around the (log of) stationary equilibrium, we obtain the following log-linear Euler equation:

$$\begin{aligned} \Delta m_t = [I + \nu(1+\nu)]^{-1} \Big\{ & \frac{(i-o)\omega}{\kappa M} \left(m - p - \frac{\rho}{\omega} c\right) + \frac{O}{\kappa M}(i-o) \\ & - I^{-1}\nu E_t \Delta m_{t+2} + \left[1 + \nu(1 + I^{-1})\right] E_t \Delta m_{t+1} + I\nu \Delta m_{t-1} \\ & - \frac{(i-o)\omega}{\kappa M} \left(m_t - p_t - \frac{\rho}{\omega} c_t\right) - \frac{O}{\kappa M}(i_t - o_t) + (1 - O/I)(\zeta_t - \zeta) \Big\}, \end{aligned} \tag{3.9}$$

where the last term, $(1 - O/I)(\zeta_t - \zeta)$, is the deviation of the log-linearized velocity shock from its steady-state value. In the case of integrated (of order one) variables, the left hand side of equation (3.9) is stationary. In order to have a stationary right hand side, the levels of the variables on the right hand side, $z_t = [m_t, p_t, c_t, i_t, o_t]'$, should be cointegrated. This particular parameterization (3.9) suggests two cointegration vectors. The first is the net opportunity cost of money[5], $i_t - o_t$, and the second is the 'adjusted' velocity, $m_t - p_t - \frac{\rho}{\omega} c_t$. However, other parameterizations with different numbers of cointegration vectors are also possible. When there are five integrated (of order one) variables, there can be at most four cointegrating vectors. Thus one should test for the cointegration rank and then apply the restrictions implied by the theoretical model to identify the cointegration vectors.

One can for example combine the two levels terms on the right hand side of (3.9). Such parameterization corresponds to the single cointegrating vector case. If we also assume that the own-yield is zero, ie $o = 0$ or equivalently $O = 1$, we obtain the following version

[5] If the opportunity cost and own-yield of money are cointegrated, then the banks' interest rate margin is stationary.

of equation (3.9):

$$\Delta m_t = [I + \nu(1+\nu)]^{-1} \left\{ \frac{i\omega}{\kappa M} \left(m - p - \frac{\rho}{\omega} c + \frac{1}{i\omega} i \right) \right.$$
$$- I^{-1}\nu E_t \Delta m_{t+2} + \left[1 + \nu(1 + I^{-1})\right] E_t \Delta m_{t+1} + I\nu \Delta m_{t-1}$$
$$\left. - \frac{i\omega}{\kappa M} \left(m_t - p_t - \frac{\rho}{\omega} c_t + \frac{1}{i\omega} i_t \right) + (1 - I^{-1})(\zeta_t - \zeta) \right\}. \quad (3.10)$$

This is the form we will use in the analysis of M1 data.

However, there is a drawback to such linearization. If the variables of the model are integrated of order one, the linearization point defines an attractor that is not a point, contrary to the assumption in the Taylor approximation. As can be seen from the linearized first order conditions, the linearization points enter the set of parameters to be estimated. The real-business-cycle (RBC) litterature attempts to resolve the issue by assuming a linear trend in the steady state. But one still cannot solve the problem by appending the drift term. We do not have a solution to this problem.

3.3 Econometric Setup

The econometric methods are briefly described in the following sections. The principal tools used in the following statistical analysis of the demand for money are the Euler equation estimation by GMM and cointegration analysis in the ML framework of Johansen (1991). Under the assumption of nonstationary variables, the theoretical model yields restrictions on the cointegration vectors. Given the estimated cointegration vectors, the estimation of the rest of the parameters of the Euler equation (3.9) relies on the GMM approach of Hansen (1982).

To illustrate how the estimation can be performed in two steps, we write equation (3.9) as

$$\Delta m_t = [I + \nu(1+\nu)]^{-1} \left\{ m^\star - I^{-1}\nu E_t \Delta m_{t+2} \right.$$
$$\left. + \left[1 + \nu(1 + I^{-1})\right] E_t \Delta m_{t+1} + I\nu \Delta m_{t-1} - \gamma\beta' z_t + (1 - O/I)\zeta_t \right\} \quad (3.11)$$

where $m^{\star} \equiv \frac{(i-o)\omega}{\kappa M}\left(m - p - \frac{\varrho}{\omega}c\right) + \frac{O}{\kappa M}(i-o) - (1 - O/I)\zeta$, $\gamma \equiv [\frac{O}{\kappa M} \quad \frac{\omega(i-o)}{\kappa M}]$ and

$$\beta = \begin{bmatrix} 0 & -1 \\ 0 & 1 \\ 0 & \frac{\varrho}{\omega} \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad z_t = \begin{bmatrix} m_t \\ p_t \\ c_t \\ i_t \\ o_t \end{bmatrix}.$$

If the variables in z_t are integrated of order one, $I(1)$, the model can be interpreted as a sort of forward-looking error correction model, where β represents the cointegration vectors and the rest of the parameters come from the short-run dynamics. Due to the non-stationarity, one cannot estimate the parameters of the system by GMM, which assumes stationarity of the stochastic processes. According to Dolado, Galbraith and Banerjee (1991), if the forcing variables are integrated of order d ($\sim I(d)$), the endogenous variable m_t is also integrated of the same order. In the case of quadratic adjustment costs, they propose a two-step estimation procedure:

1. The parameters in β can be estimated using the ML method of Johansen (1991), which will be described in the following section. Since the parameters of the cointegration vectors of β are superconsistent one can treat the estimates of $\hat{\beta}$ as fixed in the second stage[6].

2. In the second step all the variables of the model are stationary. In this case, one can estimate the rest of the parameters by GMM.

The next section summarizes ML estimation of the cointegration vectors and the following section discusses the GMM estimation of the model with special emphasis on the stability of the parameters.

[6] Superconsistency means that the estimated parameters of the cointegration vectors converge much faster to the true values than eg do the parameters of ordinary least squares regression of stationary variables. Due to this fact, one is able to treat the parameters of the cointegration vectors as (asymptotically) fixed in the subsequent analysis of stationary variables.

3.3.1 Johansen's VAR model

We present the FIML estimation within the VAR of the cointegration relations and methods for testing the long-run structural hypothesis. The following presentation is based on the papers of Johansen (1988, 1991, 1995) and Johansen and Juselius (1990). The p-dimensional VAR process in levels, $A(L)z_t = \mu + \Psi D_t + \varepsilon_t$ ($\varepsilon_t \sim \text{NID}(0, \Sigma)$), can be written in the difference form

$$\Delta z_t = \Pi z_{t-1} + \Gamma_1 \Delta z_{t-1} + \cdots + \Gamma_{k-1} \Delta z_{t-k+1} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1, \ldots, T, \quad (3.12)$$

where $\Pi = -I_1 + \sum_{i=1}^{k} A_i$, $\Gamma_i = -(\sum_{j=i+1}^{k-1} A_j)$, μ is constant and D_t is a vector of deterministic variables. The Π matrix has a reduced rank in the case of cointegration ($\text{rank}(\Pi) < p$). Any reduced rank matrix can be presented as a product of two full-rank matrices $\Pi = \alpha\beta'$.

Johansen (1988) and Johansen (1991) show that the ML estimator of the space spanned by β is the space spanned by r canonical variates reflecting the r largest squared canonical correlations between residuals of the least squares regressions of contemporaneous differences on lagged differences and regressions of levels on lagged differences. It is important to note that one can estimate only the space spanned by β, not the individual cointegration vectors.

Johansen (1988) derives a likelihood ratio test for testing the number of cointegration vectors, ie the rank of Π. Osterwald-Lenum (1992) has simulated the critical values of these test statistics for $p = 12$. It has been shown in some simulation studies (Eitrheim 1991, Toda 1995, Haug 1996) that using the asymptotic tables might be misleading for small samples. For this study, we have simulated the model under the null, in order to obtain empirical critical values for the trace tests.

3.3.2 GMM Estimation of the Euler Equation and Tests of Parameter Stability

Since the parameters $\hat{\beta}$, estimated by cointegration methods, are superconsistent, one can estimate the rest of the model parameters, $\Theta = \{m^\star, I, O, \nu, \omega, \kappa M\}$, using the GMM of Hansen (1982) taking $\hat{\beta}$ as given. We define the 5-dimensional vector of variables

$w_t = [\Delta m_{t+2}, \Delta m_{t+1}, \Delta m_t, \Delta m_{t-1}, \hat{\beta}' z_t]'$. The total number of parameters is $j \equiv \dim(\Theta)$.

Given the instrument[7] set $\mathbf{x}_t$ (l-dimensional vector; see table 3.3) we define the orthogonality conditions — implied by the Euler equation (3.9) — as

$$h(\Theta, w_t) = \big\{ -[I + \nu(1+\nu)]\,\Delta m_t + m^\star - I^{-1}\nu\Delta m_{t+2} + \left[1 + \nu(1 + I^{-1})\right]\Delta m_{t+1} + I\nu\Delta m_{t-1} - \alpha\hat{\beta}' z_t \big\}\mathbf{x}_t, \quad (3.13)$$

where $h(\Theta, w_t)$ is a $l \times 1$ vector-valued function. Note that we estimate the constant term, $m^\star$, as a separate, unrestricted parameter. This takes into account the growth component of the variables, which is only implicitly accounted for in the linearization of the model. Let Θ^* denote the true value of Θ such that $E(h(\Theta^*, w_t))$=0, $\mathcal{W}_T \equiv [w_1, \ldots, w_T]$ and $g(\Theta, \mathcal{W}_T) \equiv \frac{1}{T}\sum_{t=1}^{T} h(\Theta, w_t)$. The idea behind GMM is to choose Θ so as to make the sample moment $g(\Theta, \mathcal{W}_T)$ as close as possible to the population moments. Thus, the GMM estimate $\hat{\Theta}$ is the value of Θ that minimizes

$$Q(\Theta, \mathcal{W}_T) = [g(\Theta, \mathcal{W}_T)]'\, \hat{S}_T^{-1}\, [g(\Theta, \mathcal{W}_T)]\,. \quad (3.14)$$

Due to the two period forecast and velocity shock, the error term[8] $I^{-1}\nu\varepsilon_{t+2} - [1 + \nu(1 + I^{-1})]\,\varepsilon_{t+1} + (1 - I^{-1})\zeta_t$ is an MA(2) process. This is the fact that has to be taken into account in the estimation of the asymptotic covariance matrix S. We use the VARHAC estimator by den Haan and Levin (1996) and the quadratic spectral kernel estimator by Newey and West (1994).

Due to the financial deregulation, we test for the stability[9] of the parameters. The financial deregulation culminated at the end of 1986 when the major part of the restrictions on the deposit and lending rates were abolished. At the start of 1987 the Bank of Finland began its open market operations. We test for a structural change at that time.

The total sample size is T. Let T_0 denote the possible break point, $\mathcal{W}_{T_0} \equiv [w_1, \ldots, w_{T_0}]$, $\mathcal{W}_{T-T_0} \equiv [w_{T_0+1}, \ldots, w_T]$, $g(\Theta_0, \mathcal{W}_{T_0}) \equiv$

[7] Instruments should be chosen so as to correlate as highly as possible with Δm_{t+2} and Δm_{t+1} but not with the forecast error. The lagged error correction terms, for example, typically contain much information on the endogenous variables involved.

[8] We denote $\Delta m_{t+j} = E_t \Delta m_{t+j} + \varepsilon_{t+j}$.

[9] Hamilton (1994) and Oliner, Rudebusch and Sichel (1996) survey structural stability tests using the GMM approach. See also Hoffman and Pagan (1989), Ghysels and Hall (1990*a*) and Dufour, Ghysels and Hall (1994).

$\frac{1}{T_0}\sum_{t=1}^{T_0} h(\Theta_0, w_t)$, $g(\Theta_1, \mathcal{W}_{T-T_0}) \equiv \frac{1}{T-T_0}\sum_{t=T_0+1}^{T} h(\Theta_1, w_t)$ and Θ_0 and Θ_1 are the first and second subsample parameters. One can consider, for example, January 1987 ($= T_0$) as a possible break point. According to Hamilton (1994), one approach is to use the first subsample to estimate $\hat{\Theta}_0$ by minimizing

$$Q(\Theta_0, \mathcal{W}_{T_0}) = [g(\Theta_0, \mathcal{W}_{T_0})]' \hat{S}_0^{-1} [g(\Theta_0, \mathcal{W}_{T_0})],$$

where $\hat{S}_0$ is the (first subsample) estimate of the covariance matrix. Hansen (1982) shows that

$$\sqrt{T_0}\left(\hat{\Theta}_0 - \Theta_0^*\right) \xrightarrow{d} N(0, V_0).$$

$\hat{V}_0$ can be estimated from

$$\hat{V}_0 = \left(\hat{D}_0 \hat{S}_0^{-1} \hat{D}_0'\right)^{-1}, \text{ where } \hat{D}_0 \equiv \frac{\partial g(\hat{\Theta}_0, \mathcal{W}_{T_0})}{\partial \Theta_0'}.$$

One also computes the analogous measures for the second subsample, of size $T - T_0$. We denoting $\pi \equiv \frac{T_0}{T-T_0}$. Andrews and Fair (1988) suggest the test statistic

$$\text{AF} = T(\hat{\Theta}_0 - \hat{\Theta}_1)' \left\{\pi^{-1}\hat{V}_0 + (1-\pi)^{-1}\hat{V}_1\right\}^{-1} (\hat{\Theta}_0 - \hat{\Theta}_1) \qquad (3.15)$$

to test the null hypothesis $\Theta_0 = \Theta_1$. The test statistic $\text{AF} \xrightarrow{d} \chi^2(j)$. In case the date of the possible structural break is not known, the test can be repeated for different values of T_0, so that T_0 can be chosen to produce the largest value for the test statistic. Andrews (1993) derives the asymptotic distribution of such a test. The test setup entails the limitation that each of the subsample sizes should approach infinity. This is also a drawback of the Ghysels and Hall (1990*b*) setup.

Ghysels and Hall (1990*b*) propose a test whereby they estimate the model using the first subsample and then examine whether the orthogonality conditions of the model are satisfied over the second subsample using the parameter estimates obtained from the first subsample. The null and alternative hypotheses for the test are

$$\begin{aligned} H_0 &: E\left(h(\Theta_0, w_t)\right) = 0,\ t = 1, \ldots, T_0 \quad \text{and} \\ &\quad E\left(h(\Theta_0, w_t)\right) = 0,\ t = T_0 + 1, \ldots, T \\ H_1 &: E\left(h(\Theta_0, w_t)\right) = 0,\ t = 1, \ldots, T_0 \quad \text{and} \\ &\quad E\left(h(\Theta_0, w_t)\right) \neq 0,\ t = T_0 + 1, \ldots, T. \end{aligned}$$

The test statistic is defined as

$$\mathrm{GH} = (T - T_0) \left[g(\hat{\Theta}_0, \mathcal{W}_{T-T_0}) \right]' \hat{V}_1^{-1} \left[g(\hat{\Theta}_0, \mathcal{W}_{T-T_0}) \right],$$

where

$$\hat{V}_1 = \hat{S}_1 + (\pi)^{-1} \tilde{D}_1 \left(\hat{D}_0' \hat{S}_0^{-1} \hat{D}_0 \right)^{-1} \tilde{D}_1'$$

$$\hat{D}_0 = \frac{\partial g(\hat{\Theta}_0, \mathcal{W}_{T_0})}{\partial \Theta'} \qquad \tilde{D}_1 = \frac{\partial g(\hat{\Theta}_0, \mathcal{W}_{T-T_0})}{\partial \Theta'}.$$

The test statistic $\mathrm{GH} \xrightarrow{d} \chi^2(l)$. Oliner et al. (1996) study different choices of weighting matrix V_1. Matrices S_i $(i = 0, 1)$ can be consistently estimated for each subsample using covariance matrix estimators eg by Newey and West (1994) or den Haan and Levin (1996). In addition to the subsample estimates, one candidate is the full sample estimate.

3.4 Estimation Results

In the following two subsections, we present the results from estimation of the parameters of the theoretical model. First we estimate the steady-state part of the theoretical model. Parameters in the steady-state part of the model reflect the parameters of the utility function[10]. That is, we test for cointegration and estimate the restricted[11] cointegration space β implied by the theoretical model. In order to evaluate the stability of the utility function parameters, we test recursively whether the estimated restricted full-sample cointegration space lies within the space estimated recursively for the period 1985–1995.

We proceed with the given cointegration vectors (estimated from the full sample) and estimate the rest of the parameters of the Euler equation (3.9). The rest of the parameters in the Euler equation are related to the adjustment cost function. We also test for the stability of these parameters.

[10] The scale elasticity is ρ/ω; In the M1 model, the opportunity cost semi-elasticity is $\frac{1}{\omega i}$.

[11] We test for the restrictions implied by the theoretical model.

3.4.1 Data

The data is Finnish monthly data[12] covering January 1980 – December 1995. Narrow money (M1) contains cash held by the public and transactions accounts at the banks. Harmonized broad money (M3H) contains M1 plus all other accounts (including foreign currency) at the banks and money market deposits and repos at the banks. Prices are measured by the consumer price index (1990=100). Consumption is replaced by the monthly GDP volume indicator, which is a combined index of various indicators such as industrial production, retail sales, consumption of electricity, etc. The opportunity cost of money is the covered 1-month Eurodollar rate for the markka for the pre-1987 period and the 1-month HELIBOR (money market rate) for the later period.

We do not have a measure for the own-yield of M1. Some part of it (cash and chequable accounts) has zero yield. The yield for the rest is impossible to evaluate since the interest is usually paid on the minimum balance for the month and we do not have data on intramonth deposits. We believe that zero (or constant) own yield is a fairly good approximation for the period at hand. In the theoretical model, this means that we have the restriction $O = 1$ ($o = 0$).The own-yield of M3H is a weighted average of the after-tax deposit rates. We use current weights. The drawback of using monthly data is that they contain many exogenous shocks which are usually smoothed out in annual or quarterly data. We try to model the most important ones: the seasonal pattern of the GDP volume indicator is changed by the construction cycle (JULY). The same variable was influenced by the harbour workers strike in June 1991 (TRAF). Money balances were influenced by several exogenous factors: The timing of tax rebates was changed in 1991–1995 (REBATE). Devaluation speculation (DSPEC) is visible in the money market rate. That variable is also a measurement of currency substitution. Capital gains taxation was changed in 1988–1989 (CGAINT). Bank office workers went on strike in February 1990 (BSTRIKE1 and BSTRIKE2). The withholding tax was introduced in January 1991 (WTAX). The dummies are impulse dummies, ie they take the value unity in the indicated period and zero otherwise. They are used in the difference part of the error correction models.

[12] The data are from the Bank of Finland database. The M3H data are unofficial estimates.

The set of deterministic dummy variables differs between the M1 and M3H models (see table 3.3). The M3H system is augmented with the dummy MFREST, which is unity for the pre-1987 period, during which the Ministry of Finance restricted banks' certificates of deposit (CD) issues and the Bank of Finland did not use CDs in its open market operations, and zero otherwise. That dummy enters into the cointegration space and is restricted to enter only into the cointegration relations between own-yield and opportunity cost of money.

3.4.2 Estimates of Steady-state Parameters

We impose the price homogeneity restriction[13] on the model by analyzing real money in the steady state. The adjustment cost function in the theoretical model is parameterized to allow lag length three; $k = 3$ in equation (3.12). This lag length is long enough to yield zero residual autocorrelations. The vector error correction model is augmented with the centred seasonal dummies and with the set of intervention dummies. These are listed in table 3.3.

Table (3.1) reports the trace tests for cointegration rank. According to the trace test and reported 95 per cent *empirical*[14] fractiles, there exists one cointegration vector in the M1 system, as is predicted by the theory. The empirical significance level for the null of no cointegration is less than 0.01.

The determination of the cointegration rank of the M3H system is more problematic. The difference between empirical and asymptotic critical values is quite small[15]. Comparison of the trace tests with the empirical critical values indicates that the cointegration rank is one. If we include the dummy variable MFREST in the cointegration space, the trace test value for $r = 0$ is 57.91 while the *asymptotic* 95 per cent fractile is 55.67. For the null hypothesis $r \leq 1$ the trace test value is 21.22 and the the *asymptotic* 95 per cent fractile is 35.71 (the

[13] Note, however, that we introduce this price homogeneity also into the short-run dynamics. Ripatti (1994) cannot reject long-run price homogeneity.

[14] The empirical fractiles of the trace test are based on 10 000 replications under the null.

[15] The asymptotic critical values are obtained from Johansen (1995), table 15.3. One should note that the asymptotic critical values are not the correct ones since we have a set of noncentred dummies in the model. However, they are the ones that are given by econometric software packages such as PC-FIML or CATS in RATS.

significance level is approximately 0.5). This leads to the conclusion that the cointegration rank is one.

Table 3.1 **Trace Tests of Cointegration Rank**

M1[a]					M3H[b]			
λ	Trace test	95 % frac-tile[c]	95 % asymp-to-tic frac-tile[d]	H_0	λ	Trace test	95 % frac-tile	95 % asymp-totic frac-tile
0.186	42.16	31.22	29.38	$r = 0$	0.173	53.81	51.55	47.21
0.013	3.19	15.51	15.34	$r \leq 1$	0.072	17.93	31.63	29.38
0.003	0.64	4.41	3.84	$r \leq 2$	0.016	3.73	17.53	15.34
				$r \leq 3$	0.004	0.73	5.14	3.84

[a] Since we have no measure of the own-yield of money, the dimension of the M1 model is three instead of four.

[b] The dummy MFREST has been included in the deterministic part of the M3H model in the estimation and the simulation of the test statistic.

[c] See footnote 14.

[d] See footnote 15.

The normality of residuals is violated in the interest rate equations (table 3.2). This is due to the excess kurtosis. The autocorrelation figures show only slight residual autocorrelation in the 12th lag in the equation for Δc_t in the M3H system.

Next we test for the restrictions on the β-space implied by the Euler equations (3.9) and (3.10). For the M1 model, there are no restrictions in the cointegration space. However, we test for the unit scale elasticity since the free estimate is very close to one (0.95). The restriction is not rejected (p-value= 0.49). The restricted cointegration vector is

$$\hat{\beta}'_{\mathrm{M1}} z_t = \left[\; (m-p)_t - c_t + 1.807 i_t \; \right]. \tag{3.16}$$

The results contradict the results of Ripatti (1994), where the estimated scale elasticity was significantly below one and the interest rate semi-elasticity only sightly above one. The unit scale elasticity implies that the risk aversion parameters in the utility function are equal, ie $\hat{\rho} = \hat{\omega}$. The recursive estimates of the scale elasticity and opportunity cost semi-elasicity are given in figure 3.1. The graphs indicate that the parameters have been fairly stable during the past ten

Table 3.2 **Residual Diagnostics**

M1[a]					M3H			
ARCH(3)	Au-toc.	Norm.	R^2	Equation	ARCH(3)	Au-toc.	Norm.	R^2
0.03	0.17	3.12	0.77	$\Delta(m-p)_t$	2.16	0.13	2.09	0.34
9.81	0.09	0.04	0.95	Δc_t	10.60	0.04	6.45	0.95
1.11	0.21	47.54	0.36	Δi_t	6.88	0.90	80.27	0.56
				Δo_t	0.59	0.59	87.45	0.31

[a] The test statistics ARCH(3) for no ARCH of the third degree is $\chi^2(3)$ distributed; Autoc. is the p-value of the LM test for up to 12^{th} order autocorrelation of residuals; the Jarque–Bera normality (as null) test statistic is $\chi^2(2)$ distributed.

years. However, the scale elasticity has increased slightly during the 1990s which explains the differences in Ripatti (1994) and this study. The left panel of figure 3.2 clearly supports the conclusion that the parameter estimates of the steady state are stable[16].

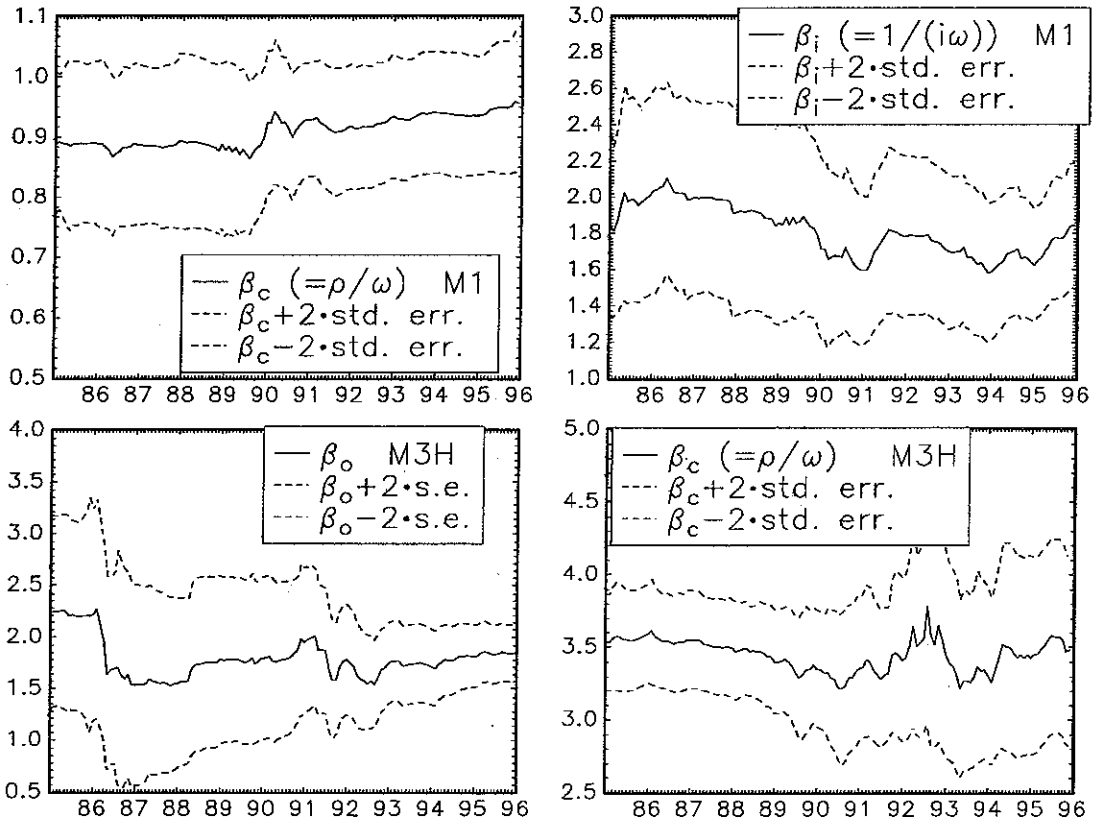
In the estimation of the M3H model with no restrictions, the first cointegration vector might be interpreted as the spread between the opportunity cost and the own-yield of money and the second cointegration vector (possibly nonstationary) as the velocity equation with scale elasticity greater than unity. The dummy variable MFREST is restricted to the first cointegration vector. The coefficients of real money and GDP do not differ significantly from zero in the first cointegration vector. According to the theoretical model, one should include the second cointegration vector in the analysis. The trace test does not indicate stationarity, although it is assumed so in the following procedures.

I restrict the cointegration space in the following way (as implied by the theoretical model):

$$\hat{\beta}'_{\text{M3H}} z_t = \begin{bmatrix} 0 & +0 & +i_t & -o_t & -0.029\text{MFREST} \\ (m-p)_t & -3.26c_t & +0 & +0 & +0 \end{bmatrix},$$

[16] Hansen and Juselius (1995) provides an attractive way to test the stability of the parameters of the cointegration vector. We estimate the cointegration space using the full sample and test recursively whether the estimated subsample cointegration space (β_τ, $\tau = T_f + 1, \ldots, T$, where T_f is starting point of recursive testing) contains the full-sample cointegration space $\hat{\beta}_{T_0}$, ie $\mathcal{H}_{\beta_\tau}: \quad \hat{\beta}_{T_0} \in \text{sp}(\beta_\tau), \quad \tau = T_f, \ldots, T$.

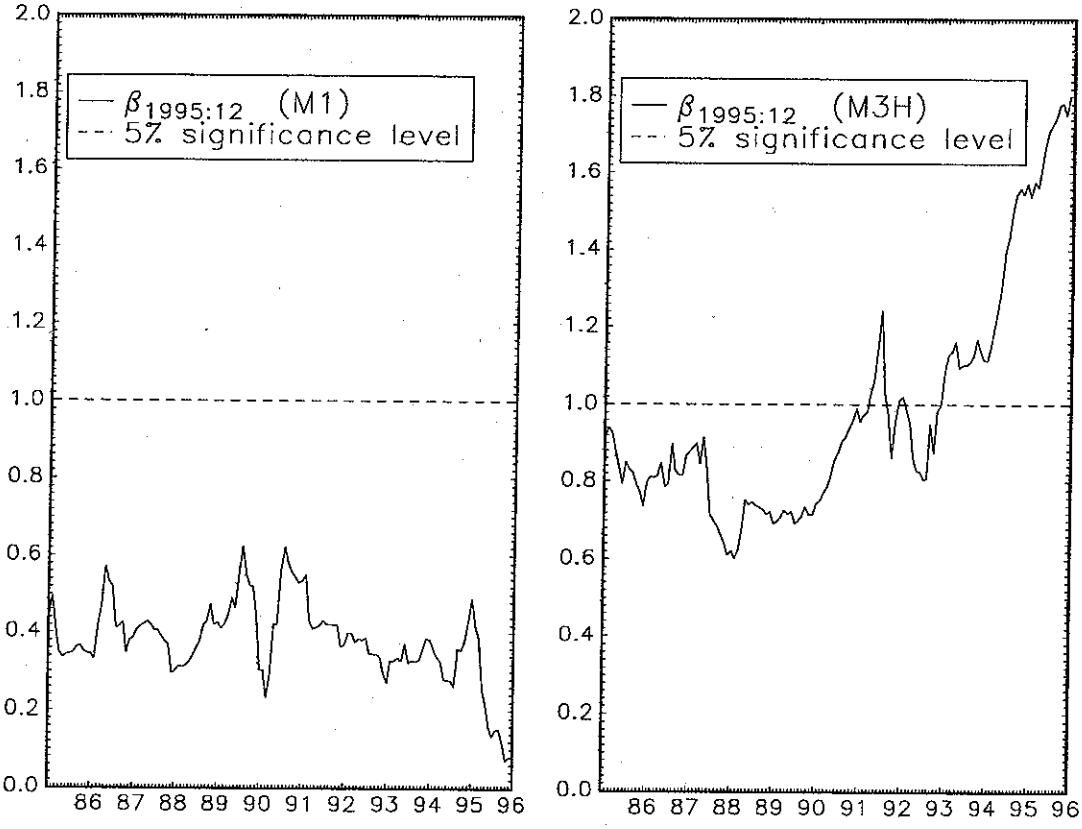
Figure 3.1 **Recursive Estimates of the Scale Elasticity (β_c) and the Opportunity Cost Semi-elasticity (β_i) of M1 and the Own-Yield Semi-elasticity (β_o) and the Scale Elasticity (β_c) of M3H**



where the coefficients of MFREST in the first vector and c_t in the second vector are estimated freely. These restrictions are not supported by the data (p-value$<$ 0.001). If we estimate the own-yield semi-elasticity freely (1.8 times the opportunity cost semi-elasticity[17]), the restrictions are not rejected (p-value= 0.2). If we assume (as the trace test indicates) that there is only one cointegration vector in the M3H model, the test results concerning the first cointegration vector are almost identical. The recursive test statistic for the hypothesis that the estimated full sample cointegration space lies within the cointegration space for the subsamples ending in 1985 onwards (figure 3.2)

[17] We use the after-tax own-yield of money but the ordinary opportunity cost of money. The results do not differ when the after-tax opportunity cost of money is used — the coefficient is 1.6. We have chosen to use the ordinary opportunity cost of money instead of the after-tax opportunity cost since firms have the possibility of subtracting interest expenditures from taxes and they can also use foreign subsidiaries in order to avoid paying taxes on interest income.

Figure 3.2 **Recursive Tests for Restricted $\hat{\beta}_{1995:12} \in \mathbf{sp}(\hat{\beta}_\tau)$,** $\tau = 1985:1, \ldots, 1995:12$



The 5% significance level of a single test scaled to unity. The coefficients of the pre-determined variables (dummies etc) and short-run dynamics are the full sample estimates computed before the recursive test. Note that the null hypothesis in M1 case also contains unit scale elasticity.

indicates serious instabilities during the 1990s part of the recursive period.

The coefficient of the dummy MFREST indicates that the banks' interest rate margin was on the average three percentage points higher during the period of regulation of CD issues, before 1987. The deregulation significantly reduced the banks' margin by boosting the average cost of liabilities. The scale elasticity in the second cointegration vector is much too high to be reliable. It implies that the risk aversion measure for real money is three times as large as for consumption.

The recursive estimates of the M3H model further illustrate the problem. The own-yield elasticity varies between 1.5 and 2.3 during the recursive period (lower left panel of figure 3.1); the scale elasticity varies between 3.2 and 3.9 and the confidence interval actually *widens* during the recursive period.

Finally, we augment the M3H model with the variable that is the logarithmic difference between M3H and M1. If the coefficient of the variable in such a modification of the M3H model is unity, the model is a genuine M1 model and the aggregation from M1 to M3H is not valid. The estimated cointegration space with the own-yield of money restricted to zero is as follows (standard errors in parentheses below the coefficients):

$$\hat{\beta}'[z_t' \quad (m3h_t - m1_t)]' =$$
$$\begin{bmatrix} (m3h-p)_t & -0.94c_t & +1.75i_t & +0 & -0.98(m3h_t - m1_t) \\ & (0.13) & (0.19) & & (0.05) \end{bmatrix}.$$

It is clear that the estimated model is the same as the M1 model. This suggests that the theoretical model that is consistent with the M1 model is not consistent with the M3H model[18].

We summarize this section by the fact that the steady-state parameters, ie the utility function parameters of the M1 model, are stable. We can continue on to the estimation of the adjustment cost parameters of the M1 model, ie the dynamics of the M1 system. The parameters of the M3H model are neither stable nor of plausible size.

3.4.3 Estimates of Adjustment Cost Parameters

We proceed with the GMM estimation of the first-order condition (3.10). The M3H model does not fulfil the key assumptions of GMM: stationarity is violated by the error correction terms. Thus there is no point in estimating the Euler equation for M3H. We do not estimate the deterministic variables, such as seasonal, strike and other dummies, with the GMM; that would be computationally burdensome and

[18] We have tried several other specifications of the M3H system. The deterministic trend in the cointegration space — restricted to the second cointegration vector — yields plausible parameter estimates for the fixed exchange rate period (1980-1992). According to the test results, the velocity seems to be trend stationary. However, the forecasting performance of such a model is very unpleasant. The trend does not fit the cointegration space at all for the floating exchange rate regime. We have also augmented the original variable set with some other variables that might capture the financial deregulation of the 1980s and the broken trend in the decline of velocity in the 1990s. An example of this kind of variable is the stock of CDs issued by the banks and the Bank of Finland. The parameter estimates of such models are not plausible and those kinds of variables are not consistent with the theoretical model.

would increase the number of instruments needed. However, before the GMM estimation we run extra OLS regressions in which we condition on the variables listed in table 3.3. The instrument sets used in the GMM estimation are also listed in table 3.3. The test of overinden-tification restrictions (J-test) yields very low p-values when the instrument set is augmented with the current and one-lag variables. This is obvious since the household makes current consumption decision at the same time as its money holding decision. The same is possibly true for the price and interest rate decisions. Hence, with that instrument set, the moment conditions are violated. In the iteration of the GMM objective (3.14), we follow the guidance of Hansen, Heaton and Yaron (1996). Since the weighting matrix in the objective function is also a function of the parameters, we iterate that as well. This of course increases the computational burden.

Table 3.3 **Deterministic Variables and Instruments**

	M1	M3H
Δm_t and cointegration relations adjusted[a] for	CGAINT, BSTRIKE1, BSTRIKE2, TRAF, DSPEC, REBATE, JULY, WTAX and 11 centred seasonals	CGAINT, BSTRIKE1, BSTRIKE2, TRAF, DSPEC, REBATE, JULY and 11 centred seasonals
Instruments	Constant, Δm_{t-j}, Δp_{t-j}, Δc_{t-j}, Δi_{t-j} $(j = 2, 3)$ and $(m_{t-j} - p_{t-j} - \hat{\beta}_1 c_{t-j} + \hat{\beta}_2 i_{t-j})$ $(j = 1, 2, 3)$	No GMM estimation.

[a] These are the variables that are used in the separate regressions in order to condition on the seasonality and various tax and strike effects.

In the GMM estimation of the M1 model, we faced numerical problems in estimating the linearization point of the opportunity cost of money, I. Therefore we decided to fix it at the level of 1.05. This influences the estimate of the preference parameters, ρ and ω, since they are derived from the long-run elasticities and from the lineariza-

Table 3.4 **Parameter Estimates of the Euler Equations for M1**

Parameter[a]	1980:5 – 1995:12	1980:5 – 1986:12[b]	1987:1 – 1995:12[c]
$I = (1+i)$	1.05	1.05	1.05
ν	−0.45	−1.76	−0.30
	(0.09)	(0.34)	(0.05)
κM	9.36	0.47	7.72
	(4.59)	(0.15)	(3.79)
ω	11.07	11.07	11.07
ρ^d	11.07	11.07	11.07
Δm_{t+2}	0.54	0.70	0.34
	(0.24)	(0.12)	(0.07)
Δm_{t+1}	−0.33	0.61	−1.77
	(0.23)	(0.09)	(0.11)
Δm_{t-1}	1.37	0.45	1.31
	(0.13)	(0.13)	(0.08)
Coefficient of the error correction[e] term	−0.07	−0.49	−0.08
	(0.04)	(0.22)	(0.04)
p-value of the J-test	0.29	0.003	0.12
p-value of the parameter stability tests		AF[f]:<0.001; GH[g]: >0.99	

[a] Standard errors are in parentheses below the parameter value. The standard error of the 'derived' parameters, ie parameters that are computed from the original free parameters, are based on the delta method. However, they do not account for the uncertainty of the cointegration parameters.

[b] Period of the financial deregulation.

[c] Period of free capital markets.

[d] In the M1 system, $\rho = \omega$ due to the unit scale elasticity.

[e] For M1, this is the loading of the single cointegration vector, ie $m_t - p_t - \frac{\varrho}{\omega} c_t + \frac{1}{\omega i} i_t$.

[f] Andrews and Fair (1988) test statistics.

[g] Ghysels and Hall (1990*b*) test statistics, based on the weighting matrices of each subsample.

tion point of the opportunity cost of money[19]. By fixing I we also fix the risk aversion parameters at the fairly high level[20]. The risk aversion of 11.1 is fairly high according to the estimates that have been previously obtained, eg by Braun et al. (1993) and Roy (1995).

The full sample estimate of the level of adjustment costs, κM, differs significantly from zero. The lagged adjustment cost, ν, is also significant. Given the present level of M1 and the average monthly growth rate during past years, the full sample estimates of the adjustment cost parameters imply monthly adjustment costs of FIM 50 million[21]. This is roughly 0.025 per cent of M1, which is fairly low, but since money is the asset that is the cheapest to adjust, we consider these numbers fairly realistic. The data support our specification of the adjustment cost function since both of the adjustment cost parameters differ significantly from zero. The usual quadratic-in-levels specification of adjustment cost function is too restrictive. The test for overidentification restrictions (J-test) does not reject the validity of instruments for the full sample. The residuals show second order autocorrelation, which is taken into account in the design of the weighting matrix and in the standard errors.

In the estimation, we test for a structural break at the end of 1986, the end of the period of financial deregulation. The Bank of Finland started open market operations in March 1987, at which time the bank quotas for CD issues were abolished. The parameter estimates and the test statistic for the structural stability tests are presented in table 3.4. The parameter stability tests, described in section 3.3.2, give conflicting results. The Andrews and Fair test statistic indicates structural change while the Ghysels and Hall test statistic does not.

[19] That is also the reason why the standard error of the estimates of ρ and ω are not computed.

[20] The risk aversion parameters of the capital asset pricing models are typically in the range 0.5–4.0. For example, the multicountry (Germany, Japan, USA) estimates of Roy (1995) are typically close to the lower bound of the range in the models in which bonds are the only type of asset. When the set of assets is augmented with shares, the risk aversion parameter tends to get estimates between 2 and 6. Braun, Constantinides and Ferson (1993) extend the approach, by relaxing the time separability of the utility function, to allow for habit persistence. Their point estimates for the risk aversion parameter for six large industrial countries vary between 0.35 (Japan) and 12 (Canada). Unfortunately, such studies have not been implemented with Finnish data.

[21] Assuming that there are approximately 5 million bank accounts in Finland and that the monthly service fee is FIM 10 per account per month, we end up with the estimated level of adjustment costs.

The last two columns of table 3.4 give parameter estimates from the financial deregulation period and the free capital markets period respectively. The estimates of the adjustment cost parameters clearly vary. In the first subsample the estimate of the level parameter, κM, is essentially lower while the estimate of the lagged change parameter, ν, is much higher. For the first subsample, the moment conditions are rejected by the J-test. Thus the estimation results concerning the first subsample are unreliable. The parameter estimates of the second subsample are closer to their full sample counterparts. The lower estimates for the adjustment cost parameters might reflect the advances[22] in payment technology for transaction accounts and in banking in general that have occurred since the latter part of the 1980s.

The recursive estimates of κM, ν and the constant term are not very convincing[23] (see figure 3.3). The lagged adjustment cost parameter, ν, varies considerably. The p-values of the J-test show that the last years of the sample period are influential with respect to the moment condition.

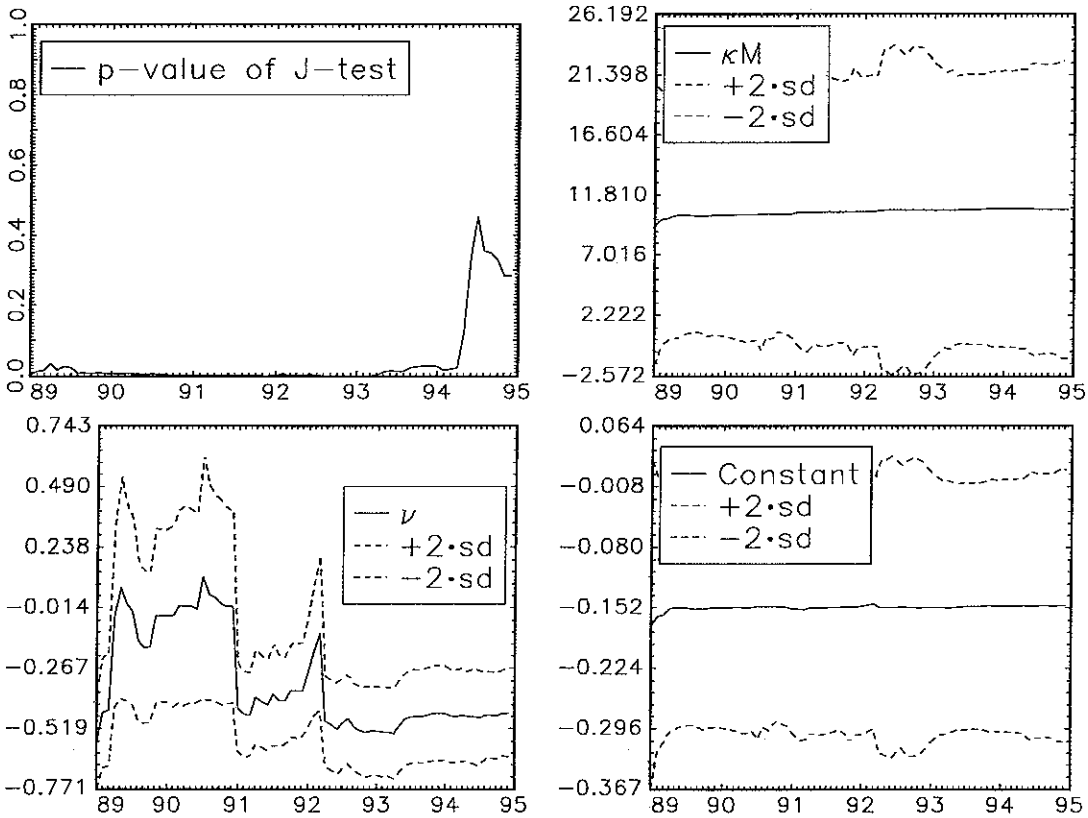
3.5 Discussion

Starting from the dynamic money-in-the-utility-function model and assuming adjustment costs of changing money holdings, we derived the first-order condition describing the demand for money. For integrated (of order one) variables, the log-linearized version of the first-order condition leads to the hypothesis of two cointegration vectors and to the restrictions on those cointegration vectors. The theoretical model is designed for the analysis of the harmonized monetary aggregate, M3H, but it can also be used in the analysis of narrow money, M1.

[22] In Finland the number of automatic teller machines (ATM) per capita is among highest in the world. Also other electronic payment systems are very highly developed in Finland. The share of debit card payments and electronic funds transfers at point of sale (EFT-POS) is very high. Giro payments are the most important form of funds transfer. On the other hand, the shares of cheque and currency payments are very low. Cheques are presently used mainly in large-value payments.

[23] Due to the computational burden we cut the iteration in recursive estimation at a quite 'early' stage. For this reason the standard errors and estimates do not correspond those reported in table 3.4.

Figure 3.3 **Recursive Estimates of Some Parameters of the Euler Equation for M1**



The estimates of the steady-state parameters of the first-order conditions of the M1 model are stable. The test for cointegration rank supports the single cointegration vector. The unit scale elasticity implies that the risk aversion parameters of consumption and money are identical. The interest rate semi-elasticity is reasonable, 1.8. The recursive estimation of these parameters and the recursive test of the constancy of the full sample cointegration space displays no instability. The GMM estimation of the Euler equation of M1 produces parameters of reasonable size and sign when the linearization point of the opportunity cost of money is fixed. The system might indicate instabilities in the adjustment cost parameters, which may reflect advances in the banking, payment and transfer technologies during the sample period or the impact of financial deregulation.

The test statistics for the M3H system do not support the restrictions on the utility function parameters implied by the model: First, the empirical and asymptotic critical values imply a single cointegration vector. Second, this cointegration vector relates the opportunity cost of money and the own-yield of money, but not their difference as the theoretical model predicts. Third, when it is assumed that there

exist two cointegration vectors, the second cointegration vector implies a scale elasticity of about three, which is a very large value compared to typical international estimates of 1–2. Finally, the recursive estimation of the scale elasticity betrays significant unsteadiness. The hope for a proper aggregation from M1 to M3H is ruined by the fact that adding the difference between M3H and M1 to the cointegration space of the M3H model leads exactly to the model of M1. Since the M3H model does not satisfy the presumption of GMM estimation there is no use in appling the GMM method to the Euler equation of M3H.

The non-existence of cointegration between price level and M3H implies that the levels of M3H and consumer prices might not be related. The long-run income elasticity of M3H is approximately twice the magnitude of the European aggregates[24]. Hence, the inclusion of the Finnish M3H would increase aggregation bias in the demand for Europe-wide M3H. The steady-state parameters in the demand for M1 in Finland are much closer to their EU-wide counterparts. Consequently, the aggregation of Finnish M1 to the EU-wide M1 is on much more solid ground. The results suggest that from the Finnish point of view M1 would be a more appropriate intermediate target for monetary policy than harmonized M3.

[24] See van Riet (1993) and Browne et al. (1997) for a survey of the demand for money in Europe and Papi and Monticelli (1995) for some recent results.

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Chapter 4

Limited and Full Information Estimation of the Rational Expectations Demand for Money Model: Application to Finnish M1

Contents

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Abstract

We compare parameter estimates of the intertemporal money-in-the-utility-function model estimated using the Generalized Method of Moments and Full Information Maximum Likelihood method. The process driving the forcing variables is approximated with vector-autoregression. Both the GMM and FIML parameter estimates are reasonable, and their difference is negligible. This is confirmed by the numerical experiments. However, the standard errors of the parameters differ widely. The cross-equation restrictions implied by the rational expectations hypothesis are clearly rejected, as is typical for these kinds of models; exogeneity restrictions are rejected as well.

Keywords: money-in-the-utility-function model, demand for money, narrow money, Generalized Method of Moments, Full Information Maximum Likelihood

JEL classification: C22, C32, C52, E41

4.1 Introduction

In the companion study, Ripatti (1998), we presented an intertemporal money-in-the-utility-function model and estimated the log-linearized first order conditions in two steps by cointegration techniques and the Generalized Method of Moments (GMM) estimator. This is an example of the *limited information* approach to the estimation of 'deep' parameters, since we make no special assumptions on the process driving the forcing variables[1]. We used two money measures: narrow money (M1) and broad harmonized money (M3H). In contrast to the M3H model, estimation of the M1 model resulted in stable parameters. The estimates of the deep parameters are within a reasonable range.

In this paper we extend the analysis of M1 in two directions. First, we approximate the processes of the forcing variables by a finite order vector autoregression and estimate the same demand for money parameters as in the companion study, using the Full Information Maximum Likelihood (FIML) method. This gives us an exceptional opportunity to compare the GMM and FIML parameter estimates. Second, as a byproduct of the FIML approach, we can test the cross-equation restrictions implied by the theoretical model.

Although the GMM provides consistent estimates of the 'deep' parameters of preferences and technology, it is used as a limited information technique in the sense that all the moment restrictions of the theoretical model are otherwise utilized, but the process driving the forcing variables is not restricted[2]. Of course, this particular feature of the approach may prove advantageous, since it provides at least a partial hedge against the Lucas critique. Furthermore, tests of overidentification restrictions serve as a diagnostic tool to check whether the moment restrictions implied by the theoretical model are valid[3].

However, even if we knew something about the process driving the forcing variables, we would not be able to utilize that information in the above GMM approach[4]. The FIML estimation takes into

[1] We even relax the usual stationarity assumption.

[2] The GMM assumes stationarity of the variables. In this paper we relax that assumption and use cointegration techniques to estimate the parameters of the steady state.

[3] There are several caveats to GMM estimation and the test for overidentification restrictions; see Newey (1985) and Hall (1993) and cited references.

[4] Note that in many cases maximum likelihood estimates of the parameters can

account this kind of information, but a specific parameterization and certain distributional assumptions on the process of forcing variables are needed. If, in the estimation period, structural changes in the processes of forcing variables have occurred and we do not explicitly take them into account, the resulting parameter estimates are subject to the Lucas critique. Hence, there is a tradeoff between the two approaches. In this study, we approximate the process of the forcing variables with a vector autoregression.

By applying both approaches in this study, we are able to compare the parameter estimates produced by the limited and full information methods. This comparison could shed some light on the tradeoff between the GMM and FIML methods. However, the present study will not give a systematic account of this experiment (as does West 1986) eg by means of Monte Carlo simulations (see also Fuhrer, Moore and Schuh 1995). Instead, it illustrates the differences in the parameter estimates by conducting two policy simulations and forecasting experiments, since this is the preferred context for application of the estimated model.

Once the process of the forcing variables is specified, one can test the cross-equation restrictions implied by the theoretical model. Within the FIML framework, one can use the likelihood ratio test and avoid the invariance problem of the nonlinear Wald test used in the approach proposed by Campbell and Shiller (1987).

Section 4.2 introduces the intertemporal money-in-the-utility-function model and presents its main features. The GMM estimation is introduced in section 4.3.1. A reparameterization of the model and testable restrictions and the FIML approach are derived in section 4.3.2. Section 4.4 presents the estimates of cointegration space and the GMM and FIML parameter estimates[5] and compares them. Section 4.5 illustrates their differences via two simulation experiments. The final section concludes.

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also be viewed as GMM estimates. In this respect our use of terminology is a bit loose. GMM is not in general a limited information estimator. However, the typical application of GMM to Euler equation estimation is limited information estimation.

[5] The computations were done with PC-FIML 9.0 (see Doornik and Hendry 1997) and Gauss 3.2.11 with the CO and CML libraries.

4.2 Money-in-the-Utility-Function Model

Although Lucas (1988) prefers to base the "money demand model", or rather the relationship between money, consumption and interest rates, on the cash-in-advance constraint, we have chosen to include real balances directly in the utility function. Other possible choices for modelling the demand for money are transactions costs, and shopping time models. However, the money-in-the-utility-function (MIUF) has the advantage of being analytically simple and it allows us to illustrate the dynamics of the relationship.

In the MIUF model, the household optimizes the discounted sum of expected utility from consumption and money (for details, see Ripatti 1998):

$$\max E_0 \sum_{t=0}^{\infty} \delta^t \left(u(C_t) + \zeta v \left(\frac{M_t}{P_t} \right) \right)$$

subject to the following budget constraint:

$$C_t + B_t + \frac{M_t}{P_t} + \frac{a(M_t, M_{t-1}, M_{t-2})}{P_t} \leq y + \frac{M_{t-1}}{P_t} + (1 + r_{t-1}) B_{t-1},$$

where y is exogenous — 'phantom' — income, C_t the real value of consumption, B_t the real value of bonds denominated in units of time-t consumption, r_t the real return on bonds, M_t the money holdings, P_t the price level and $a(\cdot)$ the adjustment costs. The parameter ζ defines the weight of the real money balances in the utility function.

The strong persistence[6] of nominal balances — and its growth rate — suggest that nominal balances might involve adjustment costs. Thus we augment the usual MIUF model with the adjustment costs of changing money. Adjustment costs of changing money holdings are slightly artificial since money should be the asset that is most cheaply exchanged. On the other hand, one could imagine that there are costs involved in adjusting eg bank accounts[7], ie shoe sole costs. However, we feel that adjustment costs provide an approximate modelling device for incorporating dynamics into the 'money demand' analysis. On the aggregate level they could, for example, mimic more complex

[6] See the large literature of demand for money studies based on cointegration techniques.

[7] Bank accounts are usually, at least partly, incorporated in money measures.

dynamics, eg (s, S) behaviour. The MIUF model is derived in the first subsection.

When we iterate the budget constraint, we obtain the following transversality condition:

$$\lim_{T\to\infty} E_t \prod_{j=0}^{T} \frac{1}{1+r_{t+j}} \left(B_{t+1+T} + \frac{M_{t+1+T}}{P_{t+1+T}} \right) = 0. \tag{4.1}$$

This states that the present value of the financial assets held in period T (real bonds and money) approaches zero as time as T approaches infinity. That is to say, the expected growth of financial assets is restricted to stay below the real rate, r.

We specify the utility function in the constant-relative-risk-aversion form (CRRA):

$$u(C_t) = \begin{cases} \frac{1}{1-\rho}\left(C_t^{1-\rho} - 1\right) & \text{if } \rho \neq 1 \\ \log C_t & \text{if } \rho = 1 \end{cases},$$

$$v\left(\frac{M_t}{P_t}\right) = \begin{cases} \frac{1}{1-\omega}\left[\left(\frac{M_t}{P_t}\right)^{1-\omega} - 1\right] & \text{if } \omega \neq 1 \\ \log\left(\frac{M_t}{P_t}\right) & \text{if } \omega = 1 \end{cases}$$

and the adjustment cost function $a(\cdot)$ as follows:

$$a(M_t, M_{t-1}, M_{t-2}) = \frac{\kappa}{2}(M_t - M_{t-1})^2 + \frac{\nu}{2}(M_{t-1} - M_{t-2})^2,$$

where κ and ν are adjustment cost parameters. The specification of adjustment cost function differs from the one used in Ripatti (1998). This parameterization does not allow for correlation in the money growth process and hence leads to a dynamically much simpler form.

We assume that nominal bonds exists in our generic economy and that the following conditional covariance conditions apply:

$$\operatorname{cov}_t\left(\frac{P_t}{P_{t+1}}\frac{u'(C_{t+1})}{u'(C_t)}, \left[1 - a'_{M_t}(M_{t+1}, M_t, M_{t-1})\right]\right) = 0 \text{ and}$$

$$\operatorname{cov}_t\left(\frac{P_t}{P_{t+2}}\frac{u'(C_{t+2})}{u'(C_t)}, a'_{M_t}(M_{t+2}, M_{t+1}, M_t)\right) = 0.$$

The first covariance condition holds eg if consumers are risk neutral and inflation is deterministic or if the net own-yield of money,

$1 - a'_{M_t}(M_{t+1}, M_t, M_{t-1})$, is deterministic and the second covariance condition holds eg if $a'_{M_t}(M_{t+2}, M_{t+1}, M_t)$ is deterministic. The first order condition for nominal bonds can then be written as

$$E_t C_t^{-\rho} = \delta E_t \left\{ (1 + i_t) \frac{P_t}{P_{t+1}} C_{t+1}^{-\rho} \right\}.$$

The right hand side contains the conditional expections of a nonlinear function of random future consumption. The condition for nominal money can be written as

$$\zeta \frac{\left(\frac{M_t}{P_t}\right)^{-\omega}}{C_t^{-\rho}} = 1 - \frac{1}{I_t} + (\kappa + \nu / I_t) \Delta M_t - \frac{1}{I_t} E_t (\kappa + \nu / I_{t+1}) \Delta M_{t+1}, \tag{4.2}$$

where $I_t \equiv 1 + i_t$. The left-hand side of equation (4.2) is the marginal rate of substitution of consumption for real balances. The right-hand side is the rental cost, in terms of the consumption good, of holding an extra unit of real balances for one period. Note that the rental cost differs from the usual one, $1 - 1/I_t$, due to the adjustment costs.

Due to the possibly nonstationary variables, the GMM is not suitable for estimation of the equation (4.2). We must use estimators that can be applied to models with nonstationary variables and loglinearize the equation (4.2). When we loglinearize the first order conditions around the steady state, we obtain the following log-linear Euler equation:

$$\begin{aligned} \Delta m_t = {} & \frac{i\omega}{IM(\kappa + \nu/I)} \left(m - p - \frac{\rho}{\omega} c + \frac{1}{i\omega} i \right) + \frac{1}{I} E_t \Delta m_{t+1} \\ & - \frac{i\omega}{IM(\kappa + \nu/I)} \left(m_t - p_t - \frac{\rho}{\omega} c_t + \frac{1}{i\omega} i_t \right), \end{aligned} \tag{4.3}$$

where the variable names without subscript are linearization points of the equation. Given that the variables of the first order condition (4.3) are integrated of order one (I(1)), the Euler equation (4.3) suggests one cointegration vector. Other parameterizations of equation (4.3) might imply up to three cointegration vectors.

4.3 Limited and Full Information Estimators and Tests for Cross-Equation Restrictions

4.3.1 GMM Estimation of the Parameters

West (1988) and Sims, Stock and Watson (1990) show that for linear models[8] with nonstationary variables — like our's (4.3) — the parameters can be estimated with instrumental variables techniques and that the variance-covariance matrix can be estimated in the usual way, given that the nonstationary variables and instrument variables are mutually cointegrated and that the first differences of the nonstationary variables have nonzero drift terms. However, in practice this approach is misguided since the finite sample distribution is not invariant with respect to the values of the drift-term parameters. This approach also leads to tests whose power goes to zero as the sample size increases[9].

We choose the following two step approach suggested by Dolado et al. (1991): first we estimate the cointegration vector implied by the last term in parentheses in equation (4.3), using the FIML approach of Johansen (1991). Given these cointegration vectors, we use the GMM to estimate the stationary part of equation (4.3). The details of the approach are described in Ripatti (1998).

In the GMM estimation we derive the orthogonality conditions from equation (4.3). Since the expectation error must be independent of the period-t information, we can form the following *unconditional* moment condition. Let $\mathbf{x}_t$ be the l dimensional[10] vector of instruments. The orthogonality conditions are then

$$h(\Theta, w_t) = \left[-\Delta m_t + m^* + \frac{1}{I} E_t \Delta m_{t+1} \right. \\ \left. - \frac{i\omega}{IM(\kappa + \nu/I)} \left(m_t - p_t - \frac{\rho}{\omega} c_t + \frac{1}{i\omega} i_t \right) \right] \mathbf{x}_t, \quad (4.4)$$

where $\Theta \equiv (I, M(\kappa + \nu/I), m^*)$ is the parameter vector and $w_t \equiv (\Delta m_{t+1}, \Delta m_t, m_t, p_t, c_t, i_t)'$ the vector of variables observed by the

[8] They consider linear models in variables. Nagaraj and Fuller (1991) extends the analysis to linear models that are nonlinear in parameters.

[9] See the references above and Campbell and Perron (1991).

[10] s is equal or greater than the number of parameters to be estimated

econometrician. The sample mean of these conditions is

$$g(\Theta; w_1, \ldots, w_T) \equiv \frac{1}{T} \sum_{t=1}^{T} h(\Theta, w_t),$$

and the GMM objective function to be minimized is

$$Q(\Theta) = g(\Theta; w_1, \ldots, w_T)' \hat{S}_T^{-1} g(\Theta; w_1, \ldots, w_T). \tag{4.5}$$

Due to the one-period forecast, the error term[11] $I^{-1}\varepsilon_{t+1}$ is white noise. One may however argue like Ripatti (1998) that the inclusion of the stochastic preference shock might yield higher order autocorrelation in the orthogonality condition. Another possible source of higher order autocorrelation is the temporal aggregation of the data. This fact must be taken into account in the estimator of the covariance matrix:

$$S = \lim_{T \to \infty} (1/T) \sum_{t=1}^{T} \sum_{v=-\infty}^{\infty} E\left[h(\Theta^*, w_t) h(\Theta^*, w_{t-v})'\right],$$

where Θ^* denotes the true value of Θ. We use the VARHAC estimator by den Haan and Levin (1996) and quadratic the spectral kernel estimator by Newey and West (1994).

If the number of orthogonality conditions, l, exceeds the number of parameters, j, the model is overidentified. Hansen (1982) shows that it is possible to test the overidentification restrictions (J-test), since

$$\left[\sqrt{T} g(\hat{\Theta}, w_1, \ldots, w_T)\right]' \hat{S}_T^{-1} \left[\sqrt{T} g(\hat{\Theta}, w_1, \ldots, w_T)\right] \xrightarrow{d} \chi^2(l - j),$$

where $\xrightarrow{d}$ denotes convergence in distribution.

In the GMM estimation, we encounter the problem of defining the instrument set, $\mathbf{x}_t$. The GMM estimator varies with the choice of instruments. According to the simulation experiments of Tauchen (1986) and Kocherlakota (1990), increasing the number of instruments reduces the estimators' variance but increases the bias in small samples. In the iteration of the GMM objective (4.5), we follow the guidence of Hansen et al. (1996). Since the weighting matrix in the objective function is also a function of the parameters, we iterate that as well. This of course increases the computational burden.

[11] We denote $\Delta m_{t+j} = E_t \Delta m_{t+j} + \varepsilon_{t+j}$.

4.3.2 FIML Estimation

Campbell and Shiller (1987) combine rational expectations present value models and the cointegrated VAR model. Their idea relies on approximation of the processes of forcing variables using VAR and incorporating that information in the Euler equation. The approach is applicable only to linear (in variables) models. We use the FIML approach instead. Our aim is to estimate the parameters (other than cointegration parameters) of the model given the process of the forcing variables. We can apply the likelihood ratio test statistic to test the cross-equation restrictions implied by the rational expectations hypothesis. We can also specify the process of the forcing variable in such a way that we can perform the policy experiments discussed in the introduction.

We write our Euler-equation (4.3) in the error correction form with forward-looking dynamics:

$$\Delta m_t = \frac{m^*}{1-\lambda_1} - (1 - I/\lambda_1)\left(m_{t-1} + \frac{\alpha}{\lambda_1 - I - 1}\beta' X_{t-1}\right) - \frac{\alpha}{\lambda_1 - 1} E_t \sum_{j=0}^{\infty} \lambda_1^{-j} \beta' \Delta X_{t+j} \quad (4.6)$$

where

$$\alpha \equiv \frac{i\omega}{M(\kappa + \nu/I)}, \quad \beta \equiv \begin{bmatrix} 1 \\ \rho/\omega \\ -1/i\omega \end{bmatrix}, \quad X_t \equiv \begin{bmatrix} p_t \\ c_t \\ i_t \end{bmatrix}.$$

The parameter λ_1 represents the stable root of the characteristic equation

$$\lambda^2 - (1 + I + \alpha)\lambda + 1 = 0, \quad (4.7)$$

which has been factored as $(L^{-1} - \lambda_1)(1 - \lambda_2 L)$. L is usual the lag operator, ie $L^{-1}x_{t-1} = x_t$. With the reasonable parameter values, ie $\alpha > 0$ and $I > 1$, the roots have the properties $|\lambda_1| < 1$ and $|\lambda_2| > 1$. This means that we can solve the root λ_1 forward and the root λ_2 backward. In this case $\lambda_1 + \lambda_2 = 1 + I + \alpha$ and $\lambda_1\lambda_2 = I$. Equation (4.6) is based on that information and represents the solution in such case.

The first differences of the forcing variables, ΔX_t, in equation (4.6) are assumed to be stationary. Any stationary process has Wold decomposition, which can be approximated in small samples by a finite order autoregressive process:

$$\Delta X_t = \mu + \sum_{i=1}^{k} \upsilon_i \Delta X_{t-i} + \epsilon_t. \tag{4.8}$$

Since ΔX_t is (3×1) vector, each υ_i is a (3×3) matrix. Equation (4.8) can be written in the companion form, as above, as

$$V_t = \Upsilon V_{t-1} + \varsigma_t, \tag{4.8'}$$

where $V_t = [\Delta X_t' \ \cdots \ \Delta X_{t-k+1}' \ \mu]$ is a $[(3k+1) \times 1]$ vector, Υ a $[(3k+1) \times (3k+1)]$ matrix and $\varsigma_t = [\varsigma_{1,t} \ \varsigma_{2,t} \ \varsigma_{3,t} \ 0 \ \cdots \ 0]'$ a $[(3k+1) \times 1]$ vector. As above, we use the $[(3k+1) \times 1]$ selection matrix $h = [\mathbf{I}_3 \ \mathbf{0}_3 \cdots \mathbf{0}_3]'$ to pick up the component ΔX_t from V_t, ie

$$\Delta X_t = h' V_t.$$

We also define the information set of the econometrician $H_t = \{\Delta X_t, \Delta X_{t-1}, \dots\}$ as above. The information set of the econometrician is strictly smaller than that of the economic agent, Ω_t (here, the representative household), ie $H_t \subset \Omega_t$. Also $E(V_{t+i}|H_t) = \Upsilon^i V_t$ applies.

Finally, when

$$\begin{aligned} E\left\{\sum_{j=0}^{\infty} \lambda_1^{-j} \beta' \Delta X_{t+j} \middle| H_t\right\} &= E\left\{\sum_{j=0}^{\infty} \lambda_1^{-j} \beta' h' V_{t+j} \middle| H_t\right\} \\ &= \sum_{j=0}^{\infty} \lambda_1^{-j} \beta' h' \Upsilon^j V_t \\ &= \sum_{j=0}^{\infty} \beta' h' (\Upsilon/\lambda_1)^j V_t \\ &= \beta' h' \left(\mathbf{I}_{3k+1} - \Upsilon/\lambda_1\right)^{-1} V_t, \end{aligned}$$

equation (4.6) can be written in the form

$$\begin{aligned} \Delta m_t = \frac{m^*}{1-\lambda_1} &- (1 - I/\lambda_1)(m_{t-1} - \tilde{\beta}' X_{t-1}) \\ &+ \frac{\lambda_1 - 1 - I}{\lambda_1 - 1} \tilde{\beta}' h' \left(\mathbf{I} - \Upsilon/\lambda_1\right)^{-1} V_t + \eta_t, \end{aligned} \tag{4.9}$$

where

$$\eta_t \equiv \frac{\lambda_1 - 1 - I}{\lambda_1 - 1} \sum_{j=0}^{\infty} \lambda_1^{-j} \tilde{\beta}' \left\{ E[\Delta X_{t+j} | \Omega_t] - E[\Delta X_{t+j} | H_t] \right\} \text{ and}$$

$$\tilde{\beta} \equiv -\frac{\alpha}{\lambda_1 - I - 1} \beta.$$

The error term, η_t, in equation (4.9) arises from the difference between the information sets of the econometrician and the household. Our equation to be estimated is in the same form as in Binder and Pesaran (1995) or Blanchard (1983). We write our equations in vector form in order to illustrate various restrictions implied by the model. Let us assume that $k = 2$ and $\mu = 0$. Then equations (4.8) and (4.9) can be stacked as

$$\underbrace{\begin{bmatrix} 1 & \varrho_{11} & \varrho_{12} & \varrho_{13} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_0} \underbrace{\begin{bmatrix} \Delta m_t \\ \Delta p_t \\ \Delta y_t \\ \Delta i_t \end{bmatrix}}_{\Delta v_t} = \underbrace{\begin{bmatrix} 0 & \varrho_{21} & \varrho_{22} & \varrho_{23} \\ 0 & \upsilon_{111} & \upsilon_{112} & \upsilon_{113} \\ 0 & \upsilon_{121} & \upsilon_{122} & \upsilon_{123} \\ 0 & \upsilon_{131} & \upsilon_{132} & \upsilon_{133} \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} \Delta m_{t-1} \\ \Delta p_{t-1} \\ \Delta y_{t-1} \\ \Delta i_{t-1} \end{bmatrix}}_{\Delta v_{t-1}}$$

$$+ \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \upsilon_{211} & \upsilon_{212} & \upsilon_{213} \\ 0 & \upsilon_{221} & \upsilon_{222} & \upsilon_{223} \\ 0 & \upsilon_{231} & \upsilon_{232} & \upsilon_{233} \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} \Delta m_{t-2} \\ \Delta p_{t-2} \\ \Delta y_{t-2} \\ \Delta i_{t-2} \end{bmatrix}}_{\Delta v_{t-2}}$$

$$+ \underbrace{\begin{bmatrix} -(1 - I/\lambda_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} 1 \\ \tilde{\beta} \end{bmatrix}' \begin{bmatrix} m_{t-1} \\ p_{t-1} \\ y_{t-1} \\ i_{t-1} \end{bmatrix}}_{e_{t-1}} + \underbrace{\begin{bmatrix} \eta_t \\ \epsilon_{p,t} \\ \epsilon_{y,t} \\ \epsilon_{i,t} \end{bmatrix}}_{\varepsilon_t}. \tag{4.10}$$

We assume that $\varepsilon_t \sim \mathsf{NID}(\mathbf{0}_{4\times 1}, \Sigma_\varepsilon)$, where Σ_ε need not be a diagonal matrix. The log-likelihood function of the system (4.10), that is to be maximized is

$$\ell(\nu, I, \kappa M, \omega, \Sigma_\varepsilon) = -\frac{T}{2} \log(|\Sigma_\varepsilon|) - \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t' \Sigma_\varepsilon \varepsilon_t.$$

We have a number of interesting hypotheses here. Since A_0 is non-singular, we can write the model in the form of a Vector Error

Correction Mechanism with two lags (VECM(2)):

$$A_0\Delta v_t = A_1\Delta v_{t-1} + A_2\Delta v_{t-2} + ae_{t-1} + \varepsilon_t \iff$$
$$\Delta v_t = A_1^*\Delta v_{t-1} + A_2^*\Delta v_{t-2} + a^*e_{t-1} + \varepsilon_t^*, \tag{4.11}$$

where $A_1^* \equiv A_0^{-1}A_1$, $A_2^* \equiv A_0^{-1}A_2$ and $a^* \equiv A_0^{-1}a$. Our theoretical model restricts ϱ_{ij} to be a highly nonlinear function of υ_{kij}. The number of restrictions is $3k$, ie six here. The restrictions are given by

$$\varrho = [-\varrho_{11} \; -\varrho_{12} \; -\varrho_{13} \; \varrho_{21} \; \varrho_{22} \; \varrho_{23}]$$
$$= \frac{\lambda_1 - 1 - I}{\lambda_1 - 1}\tilde{\beta}'h'\left(\mathbf{I} - \Upsilon/\lambda_1\right)^{-1}.$$

Since all the variables in the system are stationary, the hypothesis can be tested with the likelihood ratio test. The test statistic is asymptotically $\chi^2(3k)$ distributed. We can test our restricted model (4.11) against the unrestricted VECM(2) model with and without the cross-equation restrictions implied by the rational expectations assumption. Given the structure of A_0, the last three elements of the first column of A_1^* and A_2^* are zero and $A_0^{-1}a = a$. These features imply that in the system (4.11), the Δm_t should not Granger cause the forcing variables and that the forcing variables should be weakly exogenous with respect to the long-run parameters α and β. Jointly this means that forcing variables should be *strong exogenous*, which is also a testable hypothesis.

4.4 Empirical Results

4.4.1 Estimates of Long-Run Parameters

In the companion paper Ripatti (1998) we have estimated the cointegration part of the model using the FIML of Johansen (1988) and the dynamics part using the GMM of Hansen (1982). In that paper the dynamics of the money demand are somewhat richer than in this paper due to the more complicated adjustment cost function. The Finnish data consist of monthly observations on narrow money (M1), the consumer price index, the GDP volume indicator and the one-month money market rate. The estimation period is January 1980 – December 1995.

We impose the price homogeneity restriction[12] on the model by analysing real money in the steady state. We choose lag length three, $k = 3$. This lag length is long enough to yield zero residual autocorrelations. The vector error correction model is augmented with the centred seasonal dummies and with the set of intervention dummies. These are listed in the appendix.

Table 4.1 **Trace Tests of Cointegration Rank**

H_0	λ	Trace test	95 % fractile[a]	95 % asymptotic fractile[b]
$r = 0$	0.186	42.16	31.22	29.38
$r \leq 1$	0.013	3.19	15.51	15.34
$r \leq 2$	0.003	0.64	4.41	3.84

[a] To obtain empirical distributions for the tests, the trace tests have been calculated, under the null, for 10 000 replications.

[b] The asymptotic fractiles are from Johansen (1995) and are not the correct ones since we include some non-centred dummies in the system.

Table 4.1 reports the trace tests for cointegration rank. According to the trace test and reported 95 per cent *empirical*[13] fractiles, there exists one cointegration vector in the M1 system, as is predicted by the theory. The empirical significance level for the null of no cointegration is less than 0.01.

The normality of residuals is violated in the interest rate equations (table 4.2). This is due to the excess kurtosis. The autocorrelation figures, which are not reported here, show no residual autocorrelation.

[12] Note however that we introduce this price homogeneity also into the short-run dynamics. Ripatti (1994) cannot reject long-run price homogeneity. However, he uses a different sample period and a slightly different data set.

[13] The empirical fractiles of the trace test are based on 10 000 replications under the null.

Table 4.2 **Residual Diagnostics**

Equation	ARCH(3)[a]	Norm.	Autoc.	R^2
$\Delta(m-p)_t$	0.03	3.12	0.17	0.77
Δc_t	9.81	0.04	0.09	0.95
Δi_t	1.11	47.54	0.21	0.36

[a] The test statistics ARCH(3) for no ARCH of the third degree is $\chi^2(3)$ distributed; Autoc. is the p-value of the LM test for up to the 12^{th} order autocorrelation of residuals; the Jarque–Bera normality (as null) test statistic is $\chi^2(2)$ distributed.

Next we test for the restrictions on the β-space implied by the Euler equation (4.3). For the M1 model, there are no restrictions in the cointegration space. However, we test for the unit scale elasticity since the free estimate is very close to one (0.95). The restriction is not rejected (p-value= 0.49). The restricted cointegration vector is

$$\hat{e}_t = \left[\; (m-p)_t - c_t + 1.807 i_t \; \right]. \tag{4.12}$$

The results contradict the results of Ripatti (1994), where the estimated scale elasticity was significantly below one and the interest rate semi-elasticity only sightly above one. The unit scale elasticity implies that the risk aversion parameters in the utility function are equal, ie $\hat{\rho} = \hat{\omega}$. According to the stability studies by Ripatti (1998), the parameters have been fairly stable during the past ten years. However, the scale elasticity has increased slightly during the 1990s, which explains the differences in Ripatti (1994) and this study.

4.4.2 GMM Parameter Estimates

Given the above cointegration vector, the GMM estimates of the parameter are presented in table 4.3. The instrument set, $\mathbf{x}_t$, contains the constant, Δm_{t-j}, Δp_{t-j}, Δy_{t-j}, Δi_{t-j} ($j = 2, 3$) and $\hat{e}_{t-j}$ ($j = 1, 2, 3$). Due to the first order autocorrelation we use VARHAC estimator of the asymptotic covariance matrix where the first order

autocorrelation is taken into account. It affects the weighting matrix of GMM and the standard errors of the parameters.

The estimate of the linearization point, I, is somewhat high, 1.3. One would *a priori* expect it to be only slightly above unity. As equation (4.3) shows, it is the inverse of the coefficient of the expected next-period money growth. However, the reasonable *a priori* values of I are within the 95 per cent confidence bounds.

Table 4.3 **Parameter Estimates of the Euler Equations for M1**

Parameter[a]	Value	Standard error	t statistic
$I = (1+i)$	1.29	0.36	3.64
$M(\kappa + \nu/I)$	7.83	5.37	1.46
Constant	-0.20	0.14	1.45
ω	1.90	9190	<0.001
ρ^b	1.90	9190	<0.001
Coefficient of the lead term	0.77	847	<0.001
Coefficient of the error correction[c] term	-0.05	249	<0.001
Significance level of the test for overidentification restrictions			0.19

[a] The standard error of the 'derived' parameters, ie parameters computed from the original free parameters, are based on linear approximation with respect to the original parameters of the model. However, they do not account for the uncertainty of the cointegration parameters.

[b] In the M1 system, $\rho = \omega$ due to the unit scale elasticity.

[c] This is the loading of the single cointegration vector, ie $m_t - p_t - \frac{\rho}{\omega} c_t + \frac{1}{\omega i} i_t$.

We cannot identify the adjustment cost parameters, κ and ν. They must be estimated together with the linearization point of money holdings, $M(\kappa + \nu/I)$. The GMM estimate of this term does not differ significantly from zero. The risk aversion parameters of the utility function, ω and ρ, are in the middle of the range of multi-country comparisons[14]. Their standard errors, which are computed using the

[14] The risk aversion parameters of the capital asset pricing models are typically in the range 0.5–4.0. For example, the multicountry (Germany, Japan, USA) estimates of Roy (1995) are typically close to the lower bound of the range in the models in which bonds are the only type of asset. When the set of assets is augmented

delta method, are huge. The same problem concerns other derived parameters as well. The J-test does not indicate any violation of the orthogonality conditions.

4.4.3 FIML Parameter Estimates

In this section, we follow the Full Information Maximum Likelihood setup suggested in section 4.3.2. We estimate the model formed by equations (4.8) and (4.9); since the characteristic root of (4.9) can be solved analytically we may estimate the deep parameters directly. We also compare the FIML and GMM estimates of the deep parameters. Finally, we test for the cross-equation and exogeneity restrictions implied by our theoretical model against the vector error correction model (VECM).

The error term in equation (4.9) arises from differences in the information sets of the economic agent and the econometrician. Nothing guarantees that the error terms in equations (4.8) and (4.9) are independent. Hence system estimation is needed. First, we need to determine the lag length, k, of the process in equation (4.8). Since the estimation of (4.9) is computationally burdensome, we cannot perform system-wide stability tests. Thus we concentrate on the stability of the process of the forcing variables.

We need three lags, ie $k = 3$, in (4.8) to obtain white noise residuals. The estimated system is fairly stable (see figure 4.1). However, there might be instabilities in the interest rate change equation at the start of the floating exchange rate regime. The introduction of the VAT in July 1994 is not modelled adequately, which is reflected in the recursive Chow tests.

Table 4.4 reports the parameter estimates of Υ, the estimate of loading of the error correction term and the restricted parameters, ϱ. The standard errors of the latter two sets of variables are computed using the delta method[15]. The VAR approximation of the forcing vari-

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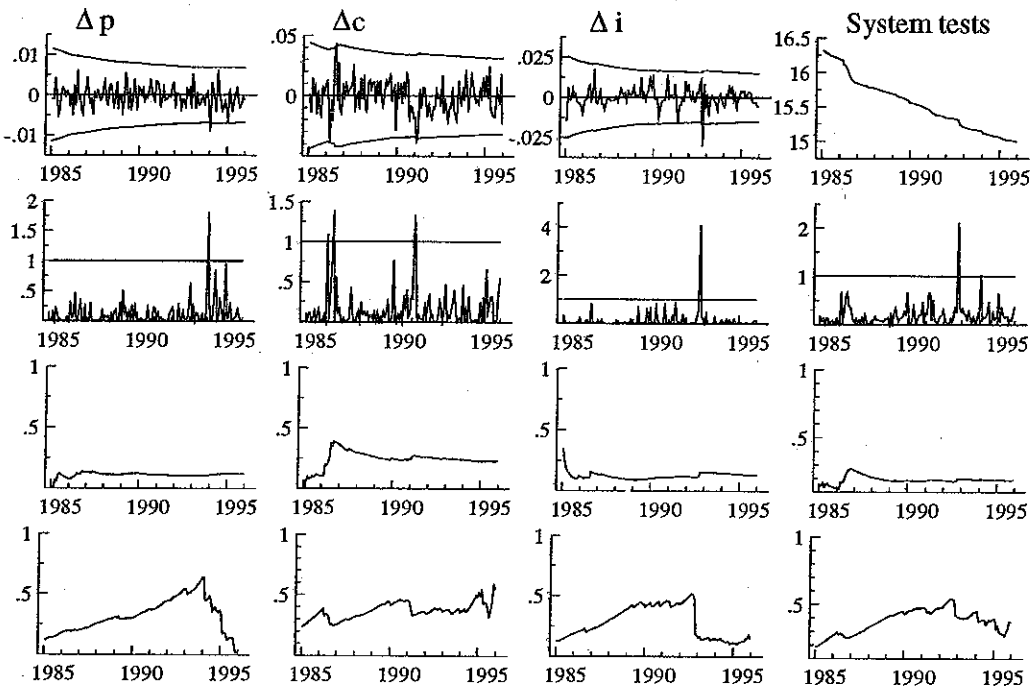
with shares, the risk aversion parameter tends to get estimates between 2 and 6. Braun et al. (1993) extend the approach, by relaxing the time separability of the utility function, to allow for habit persistence. Their point estimates for the risk aversion parameter for six large industrial countries vary between 0.35 (Japan) and 12 (Canada). Unfortunately, such studies have not been implemented with Finnish data.

[15] We estimated the coefficients of deterministic variables in the initial stage of

Figure 4.1 **Sequence of Chow Tests of the Model** (4.8)



The first row of graphs contains recursive residuals (first three columns) and recursive log-likelihood. The next row contains the sequence of Chow tests, where the model estimated using the sample ending at t is compared to the model using the sample ending at $t-1$ (t =1985M1,...,1995M12). The third row of graphs compares the full sample model with the model using the sample ending at t (t as above). In the fourth row of graphs, the model using the sample ending at 1984M12 is compared to the model using the sample ending at t (t as above). All the test statistics are scaled by one-off critical values from the F-distribution. The first column contains tests for the Δp_t equation, the second for the Δc_t, the third column for the Δi_t equation and the fourth for the system. See Doornik and Hendry (1997) for details.

ables is quite modest. The estimate of Υ shows that the interest rate path is approximated by the random walk process. The consumption growth contains only lagged inflation variables as significant regressors. The inflation equation is slightly better in terms of significant explanatory variables. The derived restricted parameters, ϱ, are interesting. All the current variables are significant. There is still high contemporaneous correlation between the residuals, which means that the system estimation applied in this study has been necessary. The

modelling. The likelihood function can be concentrated with respect to these since we have the same set of deterministic variables in every equation.

estimate of the parameter α is fairly high due to the fact that the model contains no lagged money changes. It is not significant because of the high standard errors of the deep parameters from which it is computed.

According to the residual diagnostics, the model is not quite satisfactory: there is slight autocorrelation (in lag 12) in the GDP equation. Normality is violated in the form of excess kurtosis in the price and interest rate equations. Due to these facts, the ML estimator should be considered a Quasi ML estimator.

The deep parameters to be estimated are as above, I and $M(\kappa + \nu/I)$. The risk aversion parameters have to be derived from these parameters. The FIML parameter estimates are very close to their GMM counterparts (see table refdeepesti). The major difference is that the standard errors are huge. Thus the GMM estimates are easily within any reasonable confidence bound of the FIML estimates. However, the standard errors of the risk aversion parameters are much smaller in the FIML case than in the GMM case. This standard error does not take into account the parameter uncertainty concerning the cointegration parameters. The FIML estimate of risk aversion, $\hat{\omega} = 1.82$, differs significantly from unity. The GMM estimate of the same parameter is very close to the FIML estimate. This is due to the fact the GMM and FIML estimates of the interest rate linearization points are also very close each other, as are the adjustment cost parameters.

Let us denote the various (nested) restrictions as follows:

$\mathcal{H}_{\text{VECM}}$ unrestricted VECM(k),

$\mathcal{H}_{\text{EXO}}$ VECM(k) with Granger non-causality (plus weak exogeneity of cointegration parameters) restrictions,

$\mathcal{H}_{\text{RE}}$ cross-equation restrictions implied by the theoretical model.

The hypotheses are nested as $\mathcal{H}_{\text{RE}} \subseteq \mathcal{H}_{\text{EXO}} \subseteq \mathcal{H}_{\text{VECM}}$. They are illustrated by equations (4.10) and (4.11) (with the assumption that the lag length is two). Our test setups and results are presented in table 4.6.

The message of the likelihood ratio test statistics is typical: the cross-equation restrictions implied by the theoretical model are clearly rejected. The Cambell-Shiller discussion of the interpretation of the result is valid here also. The rejection of the cross-equation restrictions might well be due to factors that are not economically important, like measurement errors etc. However, one should also note that the rejection of the exogeneity restriction is also on the border-

Table 4.4 **FIML Estimates of the Parameters**

	Equation			
Variable[a]	Δm_t	Δp_t	Δc_t	Δi_t
$\hat{e}_{t-1}$	−0.29			
	(7.75)			
Δp_t	−0.53[b]			
	(0.28)			
Δc_t	−0.52			
	(0.24)			
Δi_t	1.03			
	(0.46)			
Δp_{t-1}	0.15	0.01	0.11	−0.05
	(0.27)	(0.01)	(0.05)	(0.03)
Δc_{t-1}	−0.03	0.21	−0.08	0.15
	(0.08)	(0.07)	(0.34)	(0.17)
Δi_{t-1}	−0.02	0.03	−0.21	0.01
	(0.05)	(0.02)	(0.07)	(0.03)
Δp_{t-2}	0.05	0.01	−0.41	0.03
	(0.08)	(0.04)	(0.19)	(0.08)
Δc_{t-2}	0.03	0.20	−0.05	0.01
	(0.06)	(0.08)	(0.25)	(0.15)
Δi_{t-2}	−0.02	0.03	−0.01	−0.01
	(0.03)	(0.02)	(0.08)	(0.03)
Δp_{t-3}		0.09	−0.07	0.09
		(0.04)	(0.18)	(0.08)
Δc_{t-3}		0.13	0.19	0.21
		(0.06)	(0.26)	(0.12)
Δi_{t-3}		0.05	−0.01	−0.01
		(0.01)	(0.08)	(0.03)
	Residual Diagnostics			
Normality	6.09	11.81	0.14	128.24
$\chi^2_{0.05}(2) = 5.99$				
Box-Pierce 12 lags p-value	0.19	0.44	0.01	0.51
	Residual Variance and Correlation			
Δm_t	0.05480			
Δp_t	0.07266	0.00184		
Δc_t	0.48782	−0.08571	0.04485	
Δi_t	−0.62731	0.08991	−0.06009	0.00905

[a] The standard errors are in parenthesis below parameter estimates. They are computed from the inverse of the cross-product of the first derivative.

[b] The parameters (other than the one above) in this column are computed from the parameters of Υ. The standard errors are computed using the delta method.

Table 4.5 **FIML and GMM Estimates of the 'Deep' Parameters**

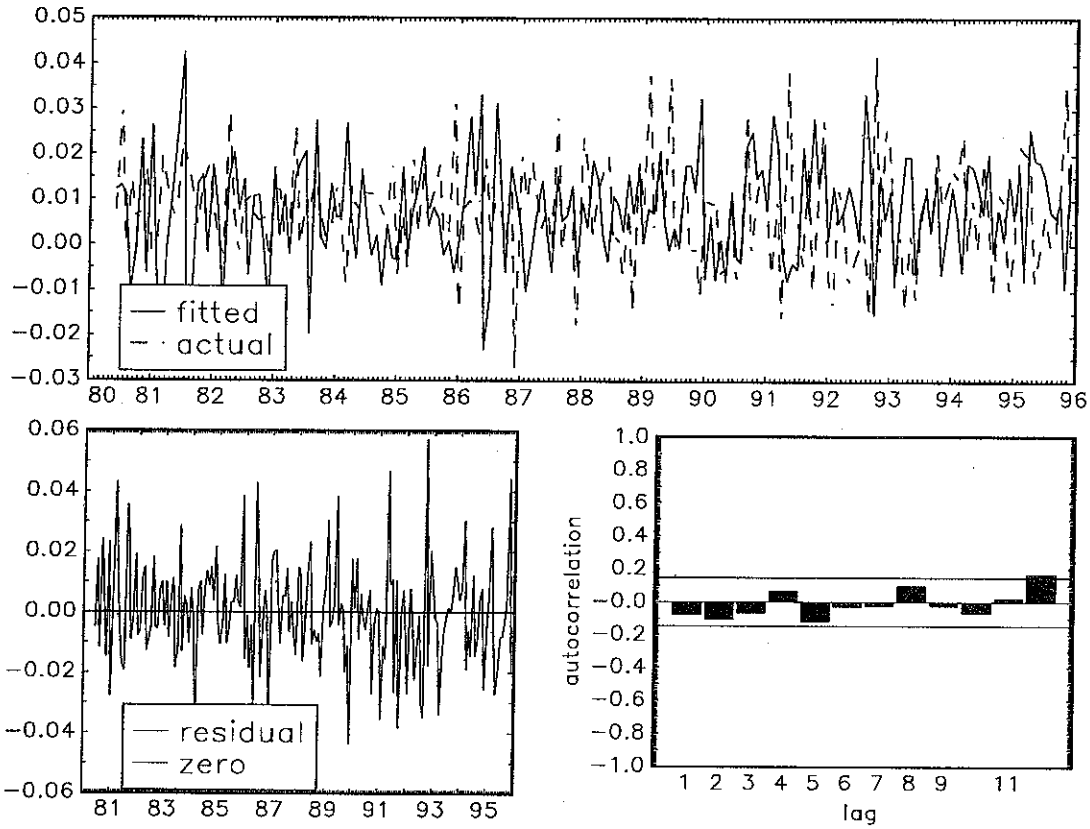
	GMM	FIML[a]
$I = (1+i)$	1.29	1.30
	(0.36)	(3.97)
$M(\kappa + \nu/I)$	7.83	6.99
	(5.37)	(191)
$\omega\ (= \rho)$	1.90	1.82
	(9190)	(0.24)

[a] The standard errors of the risk aversion parameters, ω and ρ, have been computed using the delta method.

Table 4.6 **Tests for Exogeneity and Cross-Equation Restrictions**

Hypothesis		Test	Degrees of	p-value
H_0	H_1	statistic	freedom	
$\mathcal{H}_{RE}$	$\mathcal{H}_{EXO}$	753	9	< 0.001
$\mathcal{H}_{RE}$	$\mathcal{H}_{VECM}$	782	23	< 0.001
$\mathcal{H}_{EXO}$	$\mathcal{H}_{VECM}$	29.54	15	0.014

Figure 4.2 **Actual and Fitted Money Growth**



line. This implies that our approach would benefit from modelling the behaviour of the other sectors, eg price formation and monetary policy. The model also has fairly poor fit (see figure 4.2).

4.5 Simulation Experiments

We can use our model version (4.9) for forecasting. The forecasting performance is not necessarily very good compared with the VECM since we reject the cross-equation restrictions, ie the unrestricted VECM fits the data better than the restricted model. Policy simulations serve as an alternative way of illustrating the differences between the GMM and FIML estimates of the deep parameters. We consider two forecasting and policy experiments: First, all the forcing variables are fixed at last-observation levels, ie we assume the zero growth scenario. Second, unrestricted[16] VAR is used to produce forecasts of Δp, Δc and Δi. We use both GMM and FIML estimates of the deep parameters to produce the conditional forecasts of M1. These experiments are repeated with the preannounced 5 percentage point rise in interest rates in 1999. Parameter estimates as reported in table 4.5 and equation (4.6) are used to compute the simulated paths, which are showed in figure 4.3.

The unrestricted VAR forecasts yield expanding paths for M1. The forecast based on GMM estimates of the parameters is almost the same as the forecast based on FIML estimates. This is due to the very close parameter estimates. The discount factors in the forward sum are 0.55 for GMM and 0.54 for FIML. The simulated M1 paths based on zero-growth of forcing variables converge to the same level since both techniques are based on the same steady-state estimate.

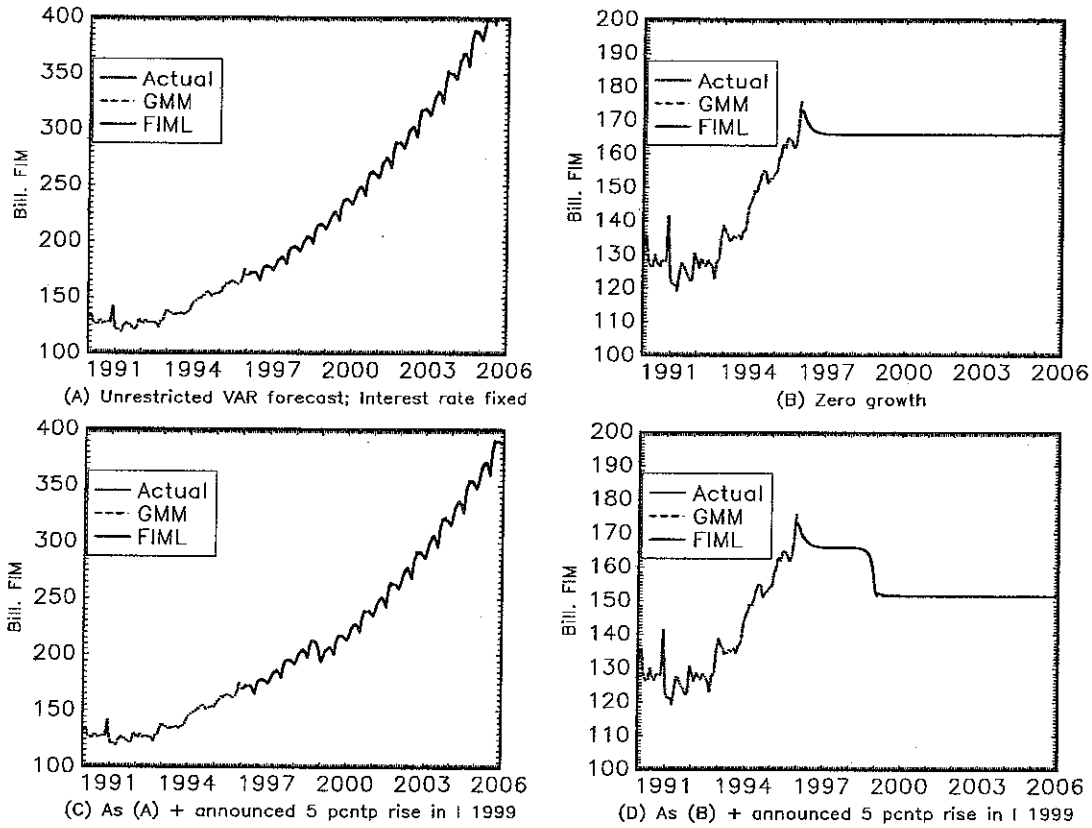
The preannounced 5 percentage point rise in interest rates in 1999 has an interesting impact. First, its discounted rise is visible only a few months earlier. That is due to the fairly low discount factors. Second, M1 converges to a level almost FIM 20 billion lower than with no change in interest rates. This clearly indicates that M1 is controllable via monetary policy. The one-month money market rate can be controlled by the Bank of Finland[17].

This study does not show which of the presented estimates is closer to the true value. One would need to conduct simulation experiments to investigate the matter — something beyond the scope

[16] Instead of unrestricted VAR, we restrict the constant to zero in the interest rate change equation. The unrestricted estimate of the constant is negative, which would imply negative interest rates over time.

[17] The main liquidity control instrument of the Bank of Finland is the tender procedure, in which the applied maturity was one month. Therefore, the Bank of Finland can control very short-term money market rates.

Figure 4.3 **Simulation Experiments with GMM and FIML Parameter Estimates**



of this study. The difference between the limited and full information estimates is very small in our case. The major differences are in the standard errors. The computational and programming burden is however much larger for the full information method (FIML) than for the limited information method (GMM). For forecasting and policy simulation purposes, the accuracy of the GMM estimates seem to be fairly close to the FIML estimates. Consequently, the choice of method depends on the user's cost function.

4.6 Conclusions

In this paper, we compare the limited information (GMM) and full information (FIML) approaches to estimating the deep parameters of an intertemporal model of money demand. We illustrate the resulting differences in the parameter estimates with two simulation experiments. We also test for the cross-equation restrictions implied by the rational expectations hypothesis against the general VECM.

The theoretical underpinnings of the paper come from an extension of an intertemporal money-in-the-utility-function to incorporate dynamic adjustment costs from adjusting money balances. The estimated form is derived by log-linearizing the appropriate Euler equations. These adjustment costs allow for persistence in the growth rate of money. In this sense the model incorporates richer dynamics eg than Cuthbertson and Taylor (1987). The cost of this extension comes, quite naturally, from more the complicated algebra and increasing computational burden as well.

We estimate the steady-state parameters of the model using cointegration methods and the rest of the parameters — ie the dynamic part of the model — using GMM and FIML. In the full information estimation, we approximate the process of the forcing variables with vector autoregression.

The GMM and FIML estimates of the parameters of the utility function are very close to each other. Larger differences occur in the estimated standard errors of these parameters. This shows up particularly well in the statistical significance of the estimated adjustment cost parameters: whereas the GMM estimates are significant, suggesting no overparameterization of the adjustment costs, the corresponding FIML estimates are clearly not significant. The similarity of the parameter estimates is illustrated by the simulations presented in figure 4.3. The differences in simulation paths based on both GMM and FIML estimates are hardly visible.

The stochastic specification of the forcing variables allows us to test the cross-equation restrictions implied by the model. They are clearly rejected at the conventional significance levels. Moreover, the model's empirical fit is unsatisfactory. This suggests that at least some of the restrictions implied by our assumptions should be relaxed and the other variables (equations) of the system should be more precisely modelled. The stability of the parameters of the stochastic specification of the forcing variables is a crucial assumption in our application of the FIML estimation. The stability of these parameters is not tested in the present paper since such a test of the whole system is computationally very demanding in the present setup. In this sense, then, we are still ignorant of the empirical validity of the Lucas critique for our FIML estimates. On the other hand, if the FIML estimate of the structure of the adjustment cost function is the correct one, then that would cast doubt on the GMM estimator. Consequently, we cannot determine which of the two approaches is superior.

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Appendix Data

The empirical counterparts for the theoretical variables are as follows:

Narrow Money: Narrow monetary aggregate M1, mill. FIM, logarithm. Includes cash held by the public and transactions accounts at banks.

Prices: Consumer price index (1990=100), logarithm, published by Statistics Finland.

Transactions: Monthly GDP volume indicator (1990=100), logarithm, published by the Statistics Finland. A combined index of various indicators such as industrial production, retail sales, consumption of electricity, etc.

Opportunity cost of money: Covered one-month Eurodollar rate for the markka for the pre-1987 period and one-month HELIBOR (money market rate) after that, divided by 100, published by the Bank of Finland. For after-tax version, see the explanation below.

Time period: January 1980 – December 1995. Graphs are presented in figure 4.4.

There are several exogenous shocks in this period also. They are modelled with the following dummy variables:

`JULY` The seasonal pattern of the GDP volume indicator has changed along with the construction cycle. An extra seasonal variable `JULY` has been added. It is the ratio of construction to total GDP, where monthly construction is measured by construction licences (Statistic Finland). The July value is multiplied by 1 and the August value by –1; the values for the rest of the year are zero.

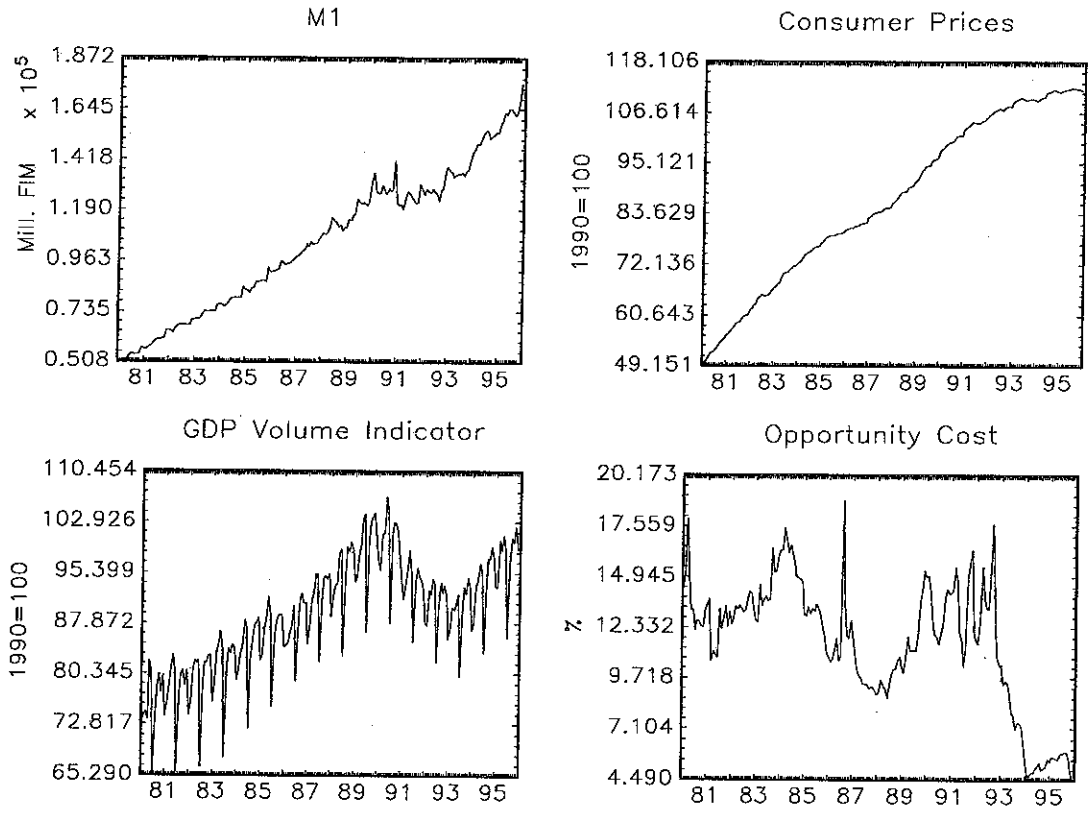
`REBATE` Tax rebates are normally paid in December. In the years 1991–1995, the pattern changed temporarily, and that is modelled by the dummy `REBATE`.

`DSPEC` Devaluation speculation raised interest rates in August 1986 and again in September – December 1991 and finally in April – November 1992, `DSPEC`. Devaluation speculation also measures the *currency substitution* effect.

`CGAINT` The increase in the capital gains tax in January 1989 is measured by the dummy `CGAINT`. It is 1 in December 1988, and –1 at end-December 1990, since the special taxfree 24-month time deposit was introduced in December 1988.

`BSTRIKE1` The strike of bank office workers in February 1990 is measured by two dummies. `BSTRIKE1` is 1 in January

Figure 4.4 **Narrow Monetary Aggregate, Consumer Price Index, GDP Volume Indicator and Opportunity Cost of Money**



1990 and –1 in March 1990, while BSTRIKE2 is 1 in February 1990. The strike increased cash held by the public and interest rates were frozen. It was not anticipated before the very end of January.

WTAX Introduction of the withholding tax for bank accounts at the start of 1991 WTAX. A 15 per cent tax on bank accounts stimulated real competition between banks.

TRAF The strike of harbour workers in June 1991 reduced industrial production during that month. The production gap was filled in the following month. That strike is modelled by the dummy TRAF.

MFREST During the pre-1987 period, the Ministry of Finance regulated banks' CD issues. MFREST has a value of unity during that period and zero otherwise.

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