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**Working Paper**

## Monetary policy transmission in China: A DSGE model with parallel shadow banking and interest rate control

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# Online appendix

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## Online appendix

This online appendix contains some additional details about the derivations that are behind key equations in the paper. In particular, we provide the complete set of equilibrium conditions.

The business cycle DSGE model aiming to capture the development of the shadow banking sector in China is solved via perturbation methods as we approximate the model around its steady state up to the first order. For this, we employ the DYNARE package using external MATLAB functions for the steady state values. Last but not least, we use Guerrieri and Iacoviello's (2014) linear first-order piecewise perturbation algorithm in order to analyze the effects of interest rate controls. For ease of notation, we adopt the following notation introduced by Verona et al. (2013):

$$q_t = \frac{Q_{\bar{K},t}}{P_t}$$

$$\lambda_{n,t} = \lambda_t P_t$$

$$w^{e,re} = \frac{W^{e,re}}{P_t}$$

$$w^{e,se} = \frac{W^{e,se}}{P_t}$$

$$w^{sb} = \frac{W^{sb}}{P_t}$$

$$n_{t+1}^{se} = \frac{N_{t+1}^{se}}{P_t}$$

$$n_{t+1}^{re} = \frac{N_{t+1}^{re}}{P_t}$$

$$n_{t+1}^{sb} = \frac{N_{t+1}^{sb}}{P_t}$$

## Intermediate good producers

The arbitrage condition for the choice of capital services implies

$$(1) \quad \frac{r_t^{k,re}}{r_t^{k,se}} = \left( \frac{u_t^{re} \tilde{K}_{t-1}^{re}}{u_t^{se} \tilde{K}_{t-1}^{se}} \right)^{\rho-1}$$

Deriving the marginal cost of intermediate good producer yields

$$A = \left( - \frac{r_t^{k,re} (\eta (u_t^{re} \tilde{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \tilde{K}_{t-1}^{se})^\rho)^{1-\frac{1}{\rho}}}{\alpha (u_t^{re} \tilde{K}_{t-1}^{re})^{\rho-1} \left( \frac{h_t}{(\eta (u_t^{re} \tilde{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \tilde{K}_{t-1}^{se})^\rho)^{\frac{1}{\rho}}} \right)^{1-\alpha}} \right)$$

$$(2) \quad B = \rho \left( \frac{\tilde{w}_t}{1-\alpha} \right)^{1-\frac{\alpha}{\rho+\alpha-\rho\alpha}} \left( (u_t^{re} \tilde{K}_{t-1}^{re})^{\rho-1} \frac{\alpha}{r_t^{k,re}} \right)^{\frac{-\alpha}{\rho+\alpha-\rho\alpha}}$$

$$C = \left\{ \left[ (\eta (u_t^{re} \tilde{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \tilde{K}_{t-1}^{se})^\rho)^{\frac{1}{\rho}} \right]^\alpha h_t^{1-\alpha} \right\}^{\frac{(\rho-1)\alpha}{\rho+\alpha-\rho\alpha}} \frac{1}{\rho+\alpha-\rho\alpha}$$

$$A B C = 0 .$$

Solving for an expression for the total amount of capital services gives us

$$(3) \quad K_t = \left[ \eta (u_t^{re} \tilde{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \tilde{K}_{t-1}^{se})^\rho \right]^{\frac{1}{\rho}}$$

As is common in the literature, we employ a Cobb-Douglas production function which is defined as

$$(4) \quad Y_t = \exp(a_t) h_t^{1-\alpha} K_{t-1}^\alpha .$$

## Capital producers

The first-order condition with respect to investment yields

$$(5) \quad \lambda_{n,t} q_t \left( 1 - \frac{s''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{s'' I_t}{I_{t-1}} \right) - \lambda_{n,t}$$

$$+ \beta \lambda_{n,t+1} q_{t+1} S \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) = 0$$

and the law of motion for the evolution of capital stock is

$$(6) \quad \eta \tilde{K}_t^{re} + (1 - \eta) \tilde{K}_t^{se} - (1 - \delta) (\eta \tilde{K}_t^{re} + (1 - \eta) \tilde{K}_t^{se}) - I_t \left( 1 - \frac{S''}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) = 0.$$

## Shadow banks and high-risk firms

The law of motion for optimism in the shadow banking sector is given by

$$(7) \quad \chi_t^{sb} = \rho_\chi^{sb} \chi_{t-1}^{sb} + (1 - \rho_\chi^{sb}) \alpha^{sb} (n_t^{re} - \bar{n}^{re}).$$

The first-order condition with respect to capital utilization of high-risk firm is

$$(8) \quad r_t^{k,re} = a'(u_t^{re}).$$

The rate of return to capital of high-risk firm can be written as

$$(9) \quad 1 + R_t^{k,re} = \frac{\pi_t}{q_{t-1}} \{ [u_t^{re} r_t^{k,re} - a(u_t^{re})] + (1 - \delta) q_t \}.$$

The debt contract between the high-risk firm and the shadow bank requires

$$(10) \quad E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1}^a)] \frac{1+R_t^{k,re}}{1+r_t^E} + \frac{\Gamma'(\bar{\omega}_{t+1}^a)}{\Gamma'(\bar{\omega}_{t+1}^a) - \mu G'(\bar{\omega}_{t+1}^a)} [\Gamma_t(\bar{\omega}_t^a) - \mu G_t(\bar{\omega}_t^a)] \frac{1+R_t^{k,re}}{1+r_t^E} - 1 \right\} = 0$$

The associated profit condition of the shadow bank is

$$(11) \quad [\Gamma_t(\bar{\omega}_t^a) - \mu G_t(\bar{\omega}_t^a)] \frac{q_{t-1} \bar{K}_t^{re}}{n_t^{re}} \frac{1+R_t^{k,re}}{1+r_t^E} = \frac{q_{t-1} \bar{K}_t^{re}}{n_t^{re}} - 1$$

As a result, the law of motion of high-risk firm's net worth is

$$(12) \quad n_{t+1}^{re} = \gamma^{re} \frac{q_{t-1}}{\pi_t} \tilde{K}_t^{re} \left[ R_t^{k,re} - r_t^E - \mu \int_0^{\omega_t^a} \omega dF_{t-1}(\omega) (1 + R_t^{k,re}) \right] \\ + \gamma^{re} \frac{n_t^{re}}{\pi_t} (1 + r_t^E) + w^{e,re}$$

and the external finance premium is determined as

$$(13) \quad P_t^{ext,re} = \frac{\bar{K}_t^{re} q_t \bar{\omega}_{t+1}^a (1+R_{t+1}^{k,re})}{q_t \bar{K}_t^{re} - n_t^{re}} - (1 + r_t^E)$$

The no-default lending rate on high-risk firm's debt is

$$(14) \quad R_t^{sb} = \frac{\tilde{K}_t^{re} q_t \bar{\omega}_{t+1}^a (1 + R_{t+1}^{k,re})}{q_t \tilde{K}_t^{re} - n_t^{re}}$$

The arbitrage condition for shadow bank savings deposits is

$$(15) \quad r_t^E = \frac{1 + r_t^d}{1 - \phi_t} - 1.$$

The ex-post default threshold value of high-risk firms is

$$(16) \quad \bar{\omega}_t^b = \bar{\omega}_t^a (1 + \chi_t^{re})$$

The law of motion of shadow bank's net worth is given by

$$(17) \quad n_t^{sb} = (1 - \phi_{t-1})n_{t-1}^{sb} + [1 - F_t(\bar{\omega}_t)]R_t^{sb}L_t^{re} \\ + (1 - \mu) \int_0^{\bar{\omega}_t^b} \omega dF(\omega)(1 + R_t^{k,re})Q_{\tilde{K},t-1} \tilde{K}_t^{re} - (1 + r_t^E)L_t^{re} + w^{sb}$$

The shadow bank's capital ratio is defined as

$$(18) \quad \kappa_t^{sb} = \frac{n_t^{sb}}{L_t^{re}}.$$

Finally, the shadow bank's default probability

$$(19) \quad \phi_t = cdf(\kappa_t^{sb}, \sigma^{sb}).$$

## Commercial banks and low-risk firms

The law of motion for optimism in the formal banking sector takes the form

$$(20) \quad \chi_t^{rb} = \rho_\chi^{rb} \chi_{t-1}^{rb} - \alpha^{rb} (1 - \rho_\chi^{rb}) (n_t^{se} - \bar{n}^{se})$$

The time-varying interest elasticity due to optimism is

$$(21) \quad \varepsilon_t^{l,op} = \varepsilon^l (1 + \chi_t^{rb}).$$

The first-order condition with respect to capital utilization of low-risk firms yields

$$(22) \quad r_t^{k,se} = a'(u_t^{se})$$

and the associated rate of return to capital of low-risk firm is

$$(23) \quad 1 + R_t^{k,se} = \frac{\pi_t}{q_{t-1}} \{ [u_t^{se} r_t^{k,se} - a(u_t^{se})] + (1 - \delta)q_t \}.$$

The low-risk firm chooses capital to maximize profits, so the first-order conditions is

$$(24) \quad R_{t+1}^{rb} - R_{t+1}^{k,se} - 1 + \frac{1}{\beta} = 0.$$

The associated law of motion of low-risk firm's net worth is

$$(25) \quad n_t^{se} = q_{t-1} \tilde{K}_{t-1}^{re} \frac{\gamma^l}{\pi_t} (R_t^{k,se} - r_{t-1}^l) + \frac{\gamma^l}{\pi_t} (1 + r_{t-1}^l) n_{t-1}^{se} + w^{e,se}.$$

The relationship between commercial bank's deposits and loans is

$$(26) \quad D_t \frac{1-\nu}{\nu} = L_t^{se}.$$

Solving for the commercial bank's deposit rate yields

$$(27) \quad (1 + r_t^d) = \frac{\varepsilon^d}{\varepsilon_{t-1}^d} (1 + R_t^d)$$

The optimal rule for setting the commercial bank's lending rate is<sup>1</sup>

$$(28) \quad 1 + r_{t+1}^l = \frac{1}{\varepsilon_{t+1}^{l,op} - 1 + \kappa^l} [\varepsilon_{t+1}^{l,op} (1 + R_{t+1}^l) + \kappa^l (1 + r_{t+1}^{l,cb})]$$

The relationship between PboC's policy rate and the deposit rate of the wholesale branch of the commercial bank becomes

$$(29) \quad R_t^d = R_t + \frac{c_d D_t}{\bar{Y} \nu}$$

<sup>1</sup>Since the lending benchmark rate is not necessarily binding, it is important to allow for the possibility that banks set a lending rate lower than the floor determined by the central bank. In order to do that we identify two regimes for equation (28) by means of Guerrieri and Iacoviello's (2014) algorithm:

$$k^l = \begin{cases} x & \text{if } r_t^l < r_t^{l,cb} \\ 0 & \text{if } r_t^l \geq r_t^{l,cb} \end{cases}$$

where  $x$  takes on different values depending on the tightness of the lending rate regulation.

Furthermore, the relationship between PboC's policy rate and the lending rate of wholesale branch of the commercial bank

$$(30) \quad R_t^l = R_t + k^w (L_t^{se} - L_t^{cb}) + L_t^{se} \frac{c_l}{\bar{Y}}$$

## Households

The first-order condition with respect to time deposits is

$$(31) \quad (-\lambda_{n,t}) + \frac{\lambda_{n,t+1} \beta (1+r_t^d)}{\pi_{t+1}} = 0$$

and the first-order condition with respect to consumption yields

$$(32) \quad \lambda_{n,t} - (C_t - b C_{t-1})^{(-\sigma_c)} + \beta b (C_{t+1} - C_t b)^{(-\sigma_c)} = 0 .$$

## Aggregate resource constraint

The aggregate resource constraint can be written as

$$(33) \quad C_t + I_t + \eta \mu \int_0^{\bar{\omega}_t^b} \omega dF(\omega) (1 + R_t^{k,re}) \frac{Q_{\bar{K},t-1} \bar{K}_t^{re}}{P_t} \\ + \eta a (u_t^{re}) \bar{K}_t^{re} + (1 - \eta) a (u_t^{se}) \bar{K}_t^{se} = Y_t .$$

## First-order conditions associated with Calvo sticky prices and wages

The relevant equations are:

$$(34) \quad \lambda_{n,t} Y_t + \beta \theta_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} = 0$$

$$(35) \quad \frac{r_t^{k,re} (\eta (u_t^{re} \bar{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \bar{K}_{t-1}^{se})^\rho)^{1-\frac{1}{\rho}}}{\alpha (u_t^{re} \bar{K}_{t-1}^{re})^{\rho-1} \left( \frac{h_t}{(\eta (u_t^{re} \bar{K}_{t-1}^{re})^\rho + (1-\eta) (u_t^{se} \bar{K}_{t-1}^{se})^\rho)^{\frac{1}{\rho}}} \right)^{1-\alpha}} \lambda_{n,t} Y_t \lambda_f \\ + \beta \theta_p K_{p,t+1} \left( \frac{\pi_t^{1-\iota_1}}{\pi_{t+1}} \right)^{\frac{(-\lambda_f)}{\lambda_f-1}} - K_{p,t} = 0$$



$$(36) \quad \frac{h_t ((C_t - b C_{t-1})^{(-\sigma c)} - \beta b (C_{t+1} - C_t b)^{(-\sigma c)})}{\lambda_w} + \frac{\beta \theta_w (\pi_t^{1-\iota_w, 1})^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} \left( \frac{1}{\frac{\pi_{t+1} \tilde{w}_{t+1}}{\tilde{w}_t}} \right)^{\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} - F_{w,t} = 0$$

$$(37) \quad h_t^{1+\sigma_l} + \beta \theta_w \left( \frac{\pi_t^{1-\iota_w, 1}}{\frac{\pi_{t+1} \tilde{w}_{t+1}}{\tilde{w}_t}} \right)^{\frac{\lambda_w(1+\sigma_l)}{1-\lambda_w}} K_{w,t+1} - K_{w,t} = 0$$

$$(38) \quad K_{p,t} - F_{p,t} \left( \frac{1 - \theta_p \left( \frac{\pi_t^{1-\iota_1}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \theta_p} \right)^{1-\lambda_f} = 0$$

$$(39) \quad K_{w,t} - \frac{\tilde{w}_t F_{w,t}}{\psi_l} \left( \frac{1 - \theta_w \left( \frac{\pi_t^{1-\iota_w, 1}}{\frac{\tilde{w}_t \pi_t}{\tilde{w}_{t-1}}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \theta_w} \right)^{1-\lambda_w(1+\sigma_l)} = 0$$

## Aggregate variables

The expression for aggregate net worth is

$$(40) \quad n_t^{ag} = n_t^{re} \eta + n_t^{se} (1 - \eta) .$$

The total amount of low-risk firm's loans is given by

$$(41) \quad L_{t+1}^{se} = q_t \tilde{K}_{t+1}^{se} - n_{t+1}^{se} ,$$

while the total amount of high-risk firm's loans is

$$(42) \quad L_{t+1}^{re} = q_t \tilde{K}_{t+1}^{re} - n_{t+1}^{re} .$$

The leverage of low-risk and high-risk firms is

$$(43) \quad lev_t^{se} = \frac{q_t \tilde{K}_t^{se}}{n_t^{se}}$$

and

$$(44) \quad lev_t^{re} = \frac{q_t \bar{K}_t^{re}}{n_t^{re}},$$

respectively. Substituting yields average leverage

$$(45) \quad lev_t^{ag} = \eta lev_t^{re} + (1 - \eta) lev_t^{se},$$

and the aggregate loan amount

$$(46) \quad L_t^{ag} = \eta L_t^{re} + (1 - \eta) L_t^{se}.$$

## Monetary policy

The PBoC's Taylor-rule for setting the policy rate is

$$(47) \quad R_t = \tilde{\rho}(R_{t-1}) + (1 - \tilde{\rho})[\bar{R} + \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_y(Y_t - \bar{Y})] + \varepsilon_t^{MP}.$$

The PBoC's deposit rate ceiling is determined according to

$$(48) \quad r_t^{d,cb} = \bar{r}^d,$$

while the PBoC's lending rate floor is

$$(49) \quad r_t^{l,cb} = \bar{r}^l.$$

Finally, PBoC's window guidance policy follows the Taylor-type rule

$$(50) \quad L_t^{CB} = \phi_i^{cb}(L_{t-1}^{CB}) + (1 - \phi_i^{cb})(\bar{L}^{se} + \phi_l^l[L_t - \bar{L}^{se}] + \phi_l^\pi[\pi_t - \bar{\pi}] + \phi_l^y[Y_t - \bar{Y}]).$$

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