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Working Paper
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IZA Discussion paper series, No. 389

Provided in Cooperation with:
IZA – Institute of Labor Economics

Suggested Citation: Beissinger, Thomas; Egger, Hartmut (2001) : Dynamic Wage Bargaining if Benefits are Tied to Individual Wages, IZA Discussion paper series, No. 389, Institute for the Study of Labor (IZA), Bonn

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November 2001
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Discussion Paper No. 389
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**ABSTRACT**

**Dynamic Wage Bargaining if Benefits are Tied to Individual Wages**

In dynamic wage bargaining models it is usually assumed that individual unemployment benefits are a fraction of the average wage level. In most countries, however, unemployment benefits are instead tied to the previous level of individually earned wages. We show how the analysis has to be modified if this fact is taken into account and compare our findings for the wage-setting curve with outcomes under other unemployment compensation schemes. In particular, we show that the widely used vertical wage-setting curve relies on more restrictive assumptions than usually considered. We also demonstrate that a reduction of unemployment benefits of those who get unemployed after the bargaining period leads to higher equilibrium unemployment.

**JEL Classification:** J51, J64, E24

**Keywords:** Dynamic wage bargaining, wage curve, unemployment benefits

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* We are grateful to Lutz Arnold, Josef Falkinger and Laszlo Goerke for valuable comments and suggestions.
1 Introduction

The literature dealing with dynamic wage bargaining on the firm level in a general equilibrium framework usually makes the following assumptions:\(^1\) (i) Unemployment benefits are not lump-sum transfers but a function of the wage level. (ii) The functional relationship is such that individual unemployment benefits are a fraction of the average wage level. (iii) Unemployment benefits are the same for persons being already unemployed in the bargaining period and persons getting unemployed in later time periods.

Due to the presumed large number of firms and labor unions, assumption (ii) implies that the bargaining parties consider unemployment benefits to be exogenous. As a further consequence benefits are identical for all persons getting unemployed in the same period independent of their previous workplace. Assumption (iii) implies that unemployment benefits do not depend on the time period in which a worker becomes unemployed. This would be the case if unemployment benefits of all unemployed are related to the *actual* average wage level.

We find assumption (i) to be plausible for most economies (one exception being Great Britain), but argue that assumptions (ii) and (iii) have to be modified. For many real-world economies these assumptions are not a valid description of the institutional setup, since unemployment benefits are usually tied to the previous level of individually earned wages.\(^2\) In this case one could expect that labor unions will not consider unemployment benefits to be exogenous but will take account of the fact that higher wage claims today imply higher unemployment benefits tomorrow. Moreover, unemployment benefits of those being unemployed in the bargaining period will differ from benefits of those who get unemployed in later periods.

In this paper we show how the analysis and the results are modified if assumptions

\(^1\)Cf. Layard and Nickell (1990), Manning (1993) and Altenburg and Straub (1998).

(ii) and (iii) are altered. In the comparison of our results with those of the literature, we focus on the case usually considered in the literature, namely the case with risk-neutral workers and Cobb-Douglas technology. In section 2.1 the theoretical framework is developed and the aggregate wage-setting curve is derived. Section 2.2 compares our results for the wage-setting curve with those obtained under other unemployment compensation systems. Section 2.3 describes the general equilibrium and section 3 concludes.

2 The Model

2.1 Derivation of the Aggregate Wage-Setting Curve

We consider a closed economy with monopolistic competition in the goods market and wage bargains taking place at the firm level. There are $n$ identical firms and an equal number of identical unions. All firms are unionized and bargain with their own union. Since the number of firms and unions is large, the bargaining parties neglect the consequences of their decisions for the rest of the economy. The values of the objective functions of firms and unions in the case of a successful bargain in period $t$ are denoted as $\Pi_t$ and $Z_t$, respectively. If an agreement cannot be reached the corresponding values are $\bar{\Pi}_t$ and $\bar{Z}_t$. The bargaining outcome is obtained by maximizing the asymmetric Nash product $\Omega_t = (Z_t - \bar{Z}_t)^\gamma (\Pi_t - \bar{\Pi}_t)^{1-\gamma}$, where $\gamma$ describes the bargaining power of labor unions ($0 < \gamma < 1$). After the determination of wages employers unilaterally choose the level of employment which guarantees the highest profits. This “right to manage” of employers is taken into account in the wage bargain.

Consider first the employer’s side of the wage bargain. If profits are earned at the end of each period, a firm’s discounted profits at the beginning of period $t$ are

$$\Pi_t = \frac{1}{1 + \rho} [R (N_t) - W_t N_t - F_t + \Pi_{t+1}],$$

(1)
where $\rho$ is the discount rate of firms, $N_t$ is employment and $W_t$ and $F_t$ denote (real) wages and fixed costs, respectively. $\Pi_{t+1}$ denotes discounted profits at the beginning of period $t + 1$. For the revenue function $R$ it is assumed that $R_N > 0$ and $R_{NN} < 0$, where $R_N$ and $R_{NN}$ are the first and second derivatives of the revenue function with respect to employment. Now assume that in a bargaining period $t$ wages are negotiated for $T$ periods, implying $W_t = W_{t+1} = \ldots = W_{t+T-1}$ and $N_t = N_{t+1} = \ldots = N_{t+T-1}$. In this case the bargained wage $W_t$ will affect $\Pi_t$, $\ldots$, $\Pi_{t+T-1}$, but not $\Pi_{t+T}$ onwards. If fixed costs are constant for all periods, i.e. $F_t = F$, the same period profits are obtained in all $T$ periods in which the wage contract is binding. This leads to

$$
\Pi_t = \frac{1-\nu}{\rho} [R (N_t) - W_t N_t - F] + \nu \Pi_{t+T}, \quad \text{with} \quad \nu \equiv 1 / (1 + \rho)^T. \quad (2)
$$

If no agreement is reached, firms have to incur losses caused by the fixed costs until a new bargaining round takes place. It is assumed that the timing as well as the outcome of the next bargaining round are not affected by the fact that no agreement has been reached in period $t$. Discounted profits of the firm in the no-agreement case are then given by\(^3\)

$$
\Pi_t = -\frac{1-\nu}{\rho} F + \nu \Pi_{t+T}. \quad (3)
$$

With eqs. (2) and (3) the firm’s contribution to the Nash bargain is

$$
\Pi_t - \Pi_t = \frac{1-\nu}{\rho} [R (N_t) - W_t N_t]. \quad (4)
$$

Turning to the union’s side of the wage bargain, it is assumed that the contribution of labor unions to the Nash maximand is given by

$$
Z_t - \mathcal{Z}_t = N_t \left( V_t - \tilde{V}_t \right), \quad (5)
$$

where $V_t$ describes the individual welfare associated with a job within the firm and $\tilde{V}_t$ is the individual welfare associated with being unemployed (both variables

\(^3\)In Manning (1991) similar expressions for $\Pi_t$ and $\Pi_t$ are derived.
referring to period \( t \)).

To determine the outcome of the Nash bargain, \( V_t - \bar{V}_t \) has to be specified. The model under consideration is deterministic in nature, i.e. stochastic shocks are neglected. Therefore, an exogenous quit rate is introduced into the model to allow for job turnover in the steady state. Let \( \delta \) denote the proportion of employees who leave employment and enter unemployment in each period. With the unemployment rate \( u \) being constant in the steady state, the probability of an unemployed person for finding a new job, \( a \), is endogenously determined by

\[
a = \frac{\delta (1 - u)}{u}.
\]

An increase in the unemployment rate reduces the probability of getting a new job, i.e. \( \partial a / \partial u < 0 \), whereas a rise in the quit rate enhances the reemployment prospects, i.e. \( \partial a / \partial \delta > 0 \). It is assumed that wages and employment are both determined at the beginning of the period but payments (wages or unemployment benefits) are made at the end of the period. Benefits for an unemployed person depend on the respective wage earned in his last occupation. In the steady state \( (u \text{ constant}) \), at the beginning of any period \( t \) the expected lifetime utility \( V_t \) of an individual who remains employed in period \( t \) in the firm under consideration is given by the following present value expression:

\[
V_t = \frac{1}{1 + r} \left[ U(W_t) + \delta \bar{V}_{t+1} + (1 - \delta) V_{t+1} \right],
\]

where \( r \) is the discount rate of workers, which may differ from the discount rate of firms \( \rho \), and \( U \) denotes the utility function of income. \( \bar{V}_{t+1} \) is the expected lifetime utility of a person (at the beginning of period \( t + 1 \)) who gets unemployed at the beginning of period \( t + 1 \) and has previously worked in the firm under consideration.

---

\(^4\)Eq. (5) is a standard formulation in the literature, which, for instance, is chosen by Layard and Nickell (1990) and Manning (1991, 1993). The latter author uses a slightly more general specification by allowing for an employment weight which may be different from one.
It is defined as

\[ V_{t+1} = \frac{1}{1 + r} \left[ U (B_{t+1}) + aV_{t+2} + (1 - a) V_{t+1} \right] , \]  

(8)

where \( B \) denotes (real) unemployment benefits. A subscript 1 is used to distinguish utility levels and wages (or benefits) of individuals who work in an occupation elsewhere (or have previously worked elsewhere and are now unemployed) from the respective values of individuals who work (or have previously worked) in the firm under consideration. Thus, \( V_1 \) is the expected lifetime utility of a person being employed elsewhere and is (for period \( t \)) defined by

\[ V_{1t} = \frac{1}{1 + r} \left[ U (W_{1t}) + \delta V_{1t+1} + (1 - \delta) V_{1t+1} \right] , \]  

(9)

with

\[ V_{1t+1} = \frac{1}{1 + r} \left[ U (B_{1t+1}) + aV_{1t+2} + (1 - a) V_{1t+2} \right] . \]  

(10)

\( W_1 \) denotes the wage paid in firms different from the one under consideration. \( B_1 \) are unemployment benefits for persons who previously worked at a firm different from that under consideration. \( V_1 \) is the expected lifetime utility of an unemployed person whose last workplace was at a firm elsewhere. In eq. (8) it is implicitly assumed that a person leaving the pool of unemployed will never again get a job in the firm under consideration, since the probability of this event is virtually zero due to the large number of firms. Furthermore, in this model it is assumed that a direct switch from one firm to another is not possible. Workers who want to change their employer first have to enter the pool of the unemployed.

As explained in the introduction we consider an economy where unemployment benefits of individuals are related to their own previous earnings, thereby modifying assumption (ii) which states that benefits are a function of the average wage level. Hence, for those getting unemployed in period \( t+1 \) their own wage of period \( t \) is relevant for the determination of benefit levels. This has to be taken into account in equations (8) and (10) implying that \( B_{t+1} = bW_t \) and \( B_{1t+1} = bW_{1t} \), where \( b \) is
the exogenous replacement ratio \((0 < b < 1)\). Since at the firm level a distinction is made between wages paid by different firms, this modification implies that unemployment benefits may vary between unemployed persons who were previously employed in different firms. Making unemployment benefits dependent on previous individual earnings also implies that assumption (iii) has to be altered. Since wage levels before the bargaining period may differ from wages prevailing afterwards, it follows that also unemployment benefits may be different across time periods.

To show the consequences of a modification of assumptions (ii) and (iii), we focus on the infinite bargain case with \(T \to \infty\), which has been considered by Layard, Nickell (1990) and Hoel (1991).\(^5\) It is assumed that a symmetric equilibrium prevailed before the bargaining period \(t = t_0\) implying that wages and unemployment benefits were the same for all individuals. At the beginning of period \(t_0\) in each firm unique wage and employment levels are chosen for this and all future periods. As a result the (new) steady state will already be reached in period \(t_0\). Omitting time indices for steady state values, equations (7) to (10) become:

\[
V = \frac{1}{1 + r} \left[ U(W) + \delta V + (1 - \delta) V \right], \tag{11}
\]

\[
\overline{V} = \frac{1}{1 + r} \left[ U(bW) + aV_1 + (1 - a) \overline{V} \right], \tag{12}
\]

\[
V_1 = \frac{1}{1 + r} \left[ U(W_1) + \delta \overline{V}_1 + (1 - \delta) V_1 \right], \tag{13}
\]

\[
\overline{V}_1 = \frac{1}{1 + r} \left[ U(bW_1) + aV_1 + (1 - a) \overline{V}_1 \right]. \tag{14}
\]

Additionally one obtains

\[
\tilde{V} = \frac{1}{1 + r} \left[ U(B_{t_0}) + aV_1 + (1 - a) \tilde{V} \right], \tag{15}
\]

where \(B_{t_0}\) denotes the level of unemployment benefits for those persons, who entered unemployment before or in the bargaining period. Since benefits depend on

\(^5\)In the finite bargain case additional problems arise since the steady state concept applied by Layard and Nickell (1990) and Manning (1991) cannot be used if future wage levels are influenced by today’s bargain.
wages paid by the previous employer, one must distinguish the expected lifetime utility of a person who gets unemployed after the bargaining period and has previously worked in the firm under consideration, \( \bar{V} \), from the expected lifetime utility of a person who gets unemployed before or in the bargaining period, \( V \). The reason is that wage levels and hence unemployment benefits before and after the wage bargain may differ. From equations (11) to (15) and eq. (6) it can be deduced that

\[
V - \bar{V} = \frac{1}{r + \delta} \left[ U(W) - \bar{U}(W) \right],
\]

where

\[
\bar{U}(W) \equiv [1 - \tau(u) - \psi(u) + \lambda(u)] U(W_1) + \tau(u) U(B_{W_1}) + \psi(u) U(bW_1) - \lambda(u) U(bW).
\]

The parameters in this expression are defined as

\[
\tau(u) \equiv \frac{(r + \delta) u}{(1 - u) \delta + ru}, \quad \psi(u) \equiv \frac{(1 - u) u \delta^2}{(ru + \delta)((1 - u) \delta + ru)},
\]

\[
\lambda(u) \equiv \frac{\delta u}{(1 - u) \delta + ru}, \quad \omega(u) \equiv \tau(u) + \psi(u) - \lambda(u) = \frac{(r + \delta) u}{ru + \delta}.
\]

Taking account of eqs. (4), (5) and (16), the right-to-manage bargaining result is obtained by maximizing the Nash product

\[
O_{W_0} = \left\{ N \left[ U(W) - \bar{U}(W) \right] \right\}^\gamma \{ R(N) - WN \}^{1-\gamma}, \quad \text{s.t.} \quad R_N - W = 0,
\]

with respect to \( W \) and \( N \). The constraint in eq. (19) reflects the fact that firms choose a point on the labor demand curve. After some transformations the following first order condition is obtained:

\[
\frac{[U_W - \bar{U}_W] W}{U(W) - \bar{U}(W)} = \mu, \quad \text{where} \quad \mu \equiv \varepsilon_{N,W} + \frac{1 - \gamma}{\gamma} \frac{\varepsilon_{R,N}}{1 - \varepsilon_{R,N}}.
\]

\( U_W \) and \( \bar{U}_W \) denote the derivatives of \( U \) and \( \bar{U} \) with respect to \( W \). \( \varepsilon_{N,W} \equiv -R_N/(NR_{NN}) \) is the elasticity of labor demand with respect to wages (in absolute

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\( ^6 \)The constant factors \( 1/\rho \) and \( 1/(r + \delta) \) are omitted, since they have no influence on the Nash solution (note that in the infinite bargain case \( \lim_{\nu \to \infty} \nu = 0 \).
values), and \( \varepsilon_{R,N} \equiv (NR_N) / R \) denotes the elasticity of revenues with respect to employment.

In the literature the analysis is usually restricted to the case with an isoelastic goods demand function of the Blanchard, Kiyotaki (1987) type. Defining \( \kappa \) as \( \kappa \equiv (\eta - 1) / \eta \), where \( \eta \) is the constant elasticity of demand \( (\eta > 1) \), and describing technology by means of the production function \( F(N,K) \), where \( K \) is the fixed stock of capital, one obtains

\[
R_N = \frac{\kappa RF_N}{F} \quad \text{and} \quad R_{NN} = R_N \frac{F_{NN}}{F_N} - (1 - \kappa) R_N \frac{F_N}{F}; \quad \text{with} \ 0 < \kappa < 1.
\]

\( F_N > 0 \) and \( F_{NN} < 0 \) denote the first and second partial derivative of \( F \) with respect to \( N \). The higher the degree of competition on the goods market (described by a higher elasticity of the demand for goods) the higher is the parameter \( \kappa \). If it is assumed that the production function is linear-homogenous the expression \( \mu \) defined in eq. (20) can be written as

\[
\mu = \frac{1}{(1 - \kappa) \varepsilon_{Y,N} + \frac{1}{\sigma} (1 - \varepsilon_{Y,N})} + \frac{1 - \gamma}{\gamma} \frac{\kappa \varepsilon_{Y,N}}{1 - \kappa \varepsilon_{Y,N}}.
\] (21)

In this equation \( \varepsilon_{Y,N} \) denotes the elasticity of output with respect to employment and \( \sigma \) the elasticity of substitution between employment and capital. In the Cobb-Douglas case with \( F(N,K) = N^\alpha K^{1-\alpha} \), it follows that \( \varepsilon_{Y,N} = \alpha \) and \( \sigma = 1 \). Hence, the expression \( \mu \), which in general depends on \( W \) and \( N \), becomes constant if the production function is of the Cobb-Douglas type. In order to derive a solution which is directly comparable with the respective results in the literature, we restrict our analysis to the case with isoelastic goods demand and Cobb-Douglas technology. Furthermore, we assume that workers are risk-neutral, which implies that \( U(W) = W \) and \( U(bW) = bW \).

In this case \( \mathcal{U}(W) \), which is defined in eq. (17), can be written as:

\[
\mathcal{U}(W) \equiv [1 - \omega(u)] W_1 + \tau(u) B_0 + \psi(u) bW_1 - \lambda(u) bW.
\] (22)

\footnote{The following analysis would also be valid in a somewhat more general model variant with isoelastic utility functions of workers \( U(W) = W^\beta / \beta \), which has also been considered in the literature. To simplify the exposition we restrict the analysis to the case with \( \beta = 1 \).}
Due to eq. (20) the following wage is chosen on the firm level:

\[
W = \frac{\mu}{\mu - 1} \left\{ [1 - \omega(u)]W_1 + \tau(u)B_{t_0} + \psi(u)bW_1 \right\},
\]

(23)

where \( \mu \) is constant in the Cobb-Douglas case. For \( W > 0 \) the condition \( \mu > 1 \) must hold.

As next step the aggregate wage-setting curve will be derived. Since all bargaining units are identical, in the general equilibrium \( W_1 = W \) must hold. In appendix A.1 it is shown that the following aggregate wage-setting equation results:

\[
W = W(u) = \frac{\mu \tau(u)}{[1 + \lambda(u)b][\mu \omega(u) - 1]} B_{t_0}.
\]

(24)

For \( W > 0 \) it must hold that \( \mu \omega(u) - 1 > 0 \). Hence, the wage-setting curve is only defined for \( u \in (\underline{u}, 1] \), where \( \underline{u} = \delta/ [\mu \delta + (\mu - 1) r] \) determines the lower bound for the unemployment rate. It can be shown that \( W > B_{t_0} \) for all permissible values of \( u \). A lower \( \mu \) implies that on the firm level the wage mark-up on the outside alternative income rises. For this reason a lower \( \mu \) cet. par. leads to a higher wage-setting curve in real wage-unemployment space.\(^8\)

In the current debate about remedies for the unemployment problem it is often claimed that unemployment benefits must be reduced to lower wage pressure and hence unemployment. With respect to \( B_{t_0} \), which are unemployment benefits of those who are unemployed in the bargaining period, such claims seem to be confirmed by eq. (24), since a reduction of \( B_{t_0} \) leads to a lower wage-setting curve. Interestingly, the opposite result is obtained if the replacement ratio \( b \) is changed and unemployment benefits \( B_{t_0} \) are held constant. Such a situation may arise if in the bargaining period the government announces that unemployment benefits of those who get unemployed after the bargaining period will be reduced, whereas the payments to the currently unemployed remain unchanged. As can be seen from

\(^8\)We also scrutinized the implications of a changing \( \delta \) or \( r \) for the location and shape of the wage-setting curve. However, the resulting expressions turned out to be rather complicated and difficult to interpret. Due to a different focus of this paper we omit a discussion of the impact of these parameters on the wage-setting curve.
eq. (24), a reduction in the replacement ratio \( b \) with given \( B_{t_0} \) leads to higher wage pressure. The reason for this surprising result is that a reduction of \( b \) lowers the costs of a wage increase more than the corresponding gains, thus provoking higher wage claims at a given unemployment rate.\(^9\)

Next we compute the slope of the wage-setting curve, which due to eq. (24) leads to

\[
\frac{dW(u)}{du} = \frac{\mu \tau'(u) q(u) - \mu \tau(u) q'(u)}{[q(u)]^2}, \tag{25}
\]

where \( q(u) \equiv [1 + \lambda(u) b] [\mu \omega(u) - 1] > 0 \). In appendix A.2 it is demonstrated that

\[
\frac{dW}{du} \lesssim 0 \quad \text{iff} \quad \mu(1 - b) \lesssim g(u), \tag{26}
\]

where \( g(u) \equiv (\delta + ru)^2 \big/ [\delta u^2 (\delta + r)] \). Since \( g'(u) < 0 \) it follows that the right-hand side of eq. (26) declines with increases in \( u \). Due to the assumption that \( \mu > 1 \) it is easy to show that \( \mu(1 - b) < g(u) \big|_{u=u} \). Hence, for \( u \to u \) the wage-setting curve has a negative slope, i.e. \( dW(u)/du < 0 \). Since for \( u \to u \) the expression \([\mu \omega(u) - 1] \) approaches zero, it follows from eq. (25) that for \( u \to u \) the slope of the wage-setting curve approaches \( -\infty \).

For \( u = 1 \) one obtains \( g(u) = (\delta + r) / \delta \). Thus, two cases must be distinguished. If \( \mu(1 - b) \leq (\delta + r) / \delta \) the wage-setting curve does not have a positive slope at \( u = 1 \). Together with \( g'(u) \big|_{u=u} < 0 \) it can be concluded that the wage-setting curve falls over the whole range \((u, 1]\). However, if \( \mu(1 - b) > (\delta + r) / \delta \) the wage-setting curve has a positive slope at \( u = 1 \). In this case there exists a negatively and positively sloped bough of the wage-setting curve.

\(^9\)The costs and gains of a wage increase are given by \([V(W) - \bar{V}]N'(W)\) and \(V'(W)N\), respectively, both expressions depending on \( b \). The decline in costs is due to the fact that \( V \) declines by more than \( \bar{V} \) which is due to different discount factors (the probability for getting unemployment benefits \( bW \) is higher for an employed person than for a person who is unemployed in the bargaining period and must be reemployed before benefits \( bW \) are obtained in the following unemployment spells). The decline in the gains of a wage increase is due to the fact that future benefits, which depend on current wages, are rising less if \( b \) is lower.
To obtain some intuition for the slope of the wage-setting curve consider first a situation with low unemployment. An increase in the unemployment rate then implies worse outside opportunities at the firm level (at given wages $W_1$), thereby reducing wage pressure. In the general equilibrium this effect is reinforced by the fact that wages $W_1$ in all firms decline and that future unemployment benefits, which are linked to wages, will be lower. However, the unemployment rate may eventually reach a critical level at which unions drive up their wage demands again if the unemployment rate increases. Intuitively this can be explained in the following way: On the one hand, a higher unemployment rate increases the weight of current unemployment benefits $B_{t_0}$ (in the consideration of unions), and also increases the weight of future benefits $bW$ of those who remain at work in the bargaining period. On the other hand, future unemployment benefits decline if real wages determined by the bargaining parties go down. If real wages are low and the unemployment rate is high enough, the welfare loss of an additional unemployed member may become lower than the welfare gain of higher wages for those who remain at work (remember that $B_{t_0}$ is exogenously given). In this case wage demands of unions increase. Since all unions behave identical, average wages go up improving the outside opportunities of workers. This gives rise to higher wage demands again. As a result, the aggregate wage-setting curve may have a positively sloped bough.

2.2 Comparison with other Unemployment Compensation Schemes

a) Earnings-related benefits as a function of the actual average wage level:

It is useful to contrast the wage-setting curve implied by eq. (24) with the wage-setting curve which results under the usual assumptions (ii) and (iii) of the literature. These assumptions correspond to an unemployment compensation scheme in which benefits of all unemployed are always related to the actual average wage.
level. In this case unemployment benefits are the same across individuals and time periods, i.e. \( B_{t_0} = bW = bW_1 \) in the above equations. Eq. (22) can then be simplified to
\[
U \equiv [1 - \omega(u)] W_1 + \omega(u) bW_1, \quad (22a)
\]
with \( \omega(u) \) being defined in eq. (18).\(^{10}\) Hence, \( U \) does not depend on individual wages and does not include an exogenous benefit component \( B_{t_0} \). Using eq. (22a) instead of eq. (22) in the above analysis would lead to the following aggregate wage-setting equation:
\[
1 = \mu (1 - b) \omega(u). \quad (24a)
\]
This equation has completely different implications compared to the one derived in eq. (24). Since in eq. (24a) only the unemployment rate shows up as endogenous variable, the wage-setting curve is vertical. This implies that the wage-setting equation alone already determines the equilibrium rate of unemployment \( u^* \), with
\[
u^* = \frac{\delta}{(r + \delta)(1 - b)\mu - r}. \quad (27)
\]
A permissible solution requires that \( u^* \in [0, 1] \) and therefore \( (1 - b)\mu \geq 1.\(^{11}\) Due to eq. (27) a reduction in \( b \) leads to lower equilibrium unemployment. The reason is that in this model variant a lower \( b \) not only reduces future unemployment benefits but also benefits of the currently unemployed.

A vertical wage-setting curve has often been used in the literature since it is in line with the stylized fact that labor demand shifts caused by technological change have no (long-run) impact on the level of unemployment.\(^{12}\) To obtain a vertical curve not only must be assumed that workers have isoelastic utility functions and that production is of the Cobb-Douglas type, but, in addition, it is also necessary

\(^{10}\)Eq. (22a) results, for instance, in the model of Layard and Nickell (1990), p. 781.

\(^{11}\)With \( \delta > 0 \) the solution \( u^* = 0 \) is not possible. A comparison with the lowest permissible rate of unemployment \( \underline{u} \) of the general model reveals that \( u^* > \underline{u} \).

\(^{12}\)See, for instance, Layard et al. (1991).
to restrict the analysis to a model variant with a special institutional setup of the unemployment compensation system.

b) Earnings-related benefits as a function of the average wage level which prevailed when getting unemployed:

The crucial assumption for a vertical wage-setting curve is assumption (iii) which requires that unemployment benefits are the same for persons being already unemployed in the bargaining period and persons getting unemployed in later time periods. The importance of this assumption can be seen by analyzing a model variant where this assumption is not made, but assumption (ii) is maintained. This amounts to an unemployment compensation scheme in which unemployment benefits are related to the average wage level which prevailed during the last employment spell of the respective person. In this case in eq. (22) the term \( bW \) must be replaced by \( bW_1 \), whereas \( B_{t_0} \) denotes benefits which are related to the previous average wage level (prevalent before the wage bargain took place), i.e.

\[
U(W) \equiv [1 - \omega(u)] W_1 + \tau(u) B_{t_0} + [\psi(u) - \lambda(u)] bW_1.
\]

In this case the aggregate wage-setting equation can be written as

\[
W = \widetilde{W}(u) = \frac{\mu \tau(u)}{\mu[\tau(u) + (\psi(u) - \lambda(u))(1 - b)] - [1 + \lambda(u)b] + \lambda(u)b} B_{t_0}.
\]

Taking account of the explanations in appendix A.1, the wage-setting equation can be transformed to:

\[
W = \widetilde{W}(u) = \frac{\mu \tau(u)}{[1 + \lambda(u)b][\mu \omega(u) - 1] + \lambda(u)b} B_{t_0}.
\]

For \( \widetilde{W}(u) > 0 \) the denominator of eq. (24b) must be positive. For this to hold it is sufficient that \( u > \underline{u} \), since in this case \( \mu \omega(u) - 1 > 0 \).\(^{13}\) From a comparison

\(^{13}\)Although the wage-setting equation in eq. (24) is also valid for unemployment rates lower than \( \underline{u} \) we only consider the intervall \((\underline{u}, 1]\), in which a comparison with the other wage-setting curves is possible.
of eqs. (24) and (24b) in the intervall \((u, 1]\) it follows that wage pressure is \(ceteris\) \(paribus\) higher if unemployment benefits are a function of individual wages instead of the average wage level. Since the denominator in eq. (24) is smaller, for all permissible values of \(u\) the respective wage-setting curve lies strictly above the wage-setting curve implied by eq. (24b). The intuition for this result is that unions take into account that higher wages today mean higher benefits for its members tomorrow if unemployment benefits depend on individual wages. This leads to higher wage pressure. Similar to eq. (24) it also follows from eq. (24b) that a reduction in the future replacement ratio \(b\) at current benefit levels \(B_{t_0}\) increases wage pressure. It can therefore be concluded that for this surprising result to hold it is not necessary that unemployment benefits depend on individual wages.

Differentiating \(\tilde{W}(u)\) with respect to \(u\) leads to

\[
\frac{d\tilde{W}(u)}{du} = \frac{\mu\tau'(u)\tilde{q}(u) - \mu\tau(u)\tilde{q}'(u)}{[\tilde{q}(u)]^2}B_{t_0},
\]

where \(\tilde{q}(u) \equiv [1 + \lambda(u)b][\mu\omega(u) - 1] + \lambda(u)b\). Performing the same analysis as described in appendix A.2 and using \(\lambda'(u)\tau(u) = \lambda(u)\tau'(u)\), it can be seen that the same condition for the slope of the wage-setting curve as in eq. (26) is obtained, i.e.

\[
\frac{d\tilde{W}}{du} \geq 0 \quad \text{iff} \quad \mu(1 - b) \geq \frac{g(u)}{g(u)},
\]

where the function \(g(u)\) is defined after eq. (26). As a result, in the intervall \((u, 1]\) the slope of the wage-setting curve \(\tilde{W}(u)\) has the same sign as the slope of the wage-setting curve \(W(u)\) determined by eq. (24).

\(c)\) \textbf{Intertemporally constant flat-rate unemployment benefits:}

For some countries, e.g. Great Britain, a flat-rate unemployment compensation scheme, where all unemployed earn a constant payment \(B\), is typical. Substituting \(B\) for \(bW\), \(bW_1\) and \(B_{t_0}\) in eq. (22) leads to

\[
\mathcal{T} \equiv [1 - \omega(u)]W_1 + \omega(u)B.
\]
In this case the following aggregate wage-setting equation is obtained:

\[ W = \frac{\mu \omega(u)}{\mu \omega(u) - 1} B. \]  

(24c)

For \( W(u) > 0 \) it is required that \( \mu \omega(u) - 1 > 0 \). Hence, the corresponding wage-setting curve is defined for the same interval \((u, 1]\) as the wage-setting curve in eq. (24). The wage-setting curve implied by eq. (24c) falls over the whole range \( u \in (u, 1] \) since unemployment benefits do not depend on real wages determined in the bargaining process. If it is assumed that \( B = B_0 \), it can be derived that the wage-setting curve defined by eq. (24c) lies strictly below \( W(u) \) defined by eq. (24).\(^{14}\) The reason is that rising wages do not lead to higher unemployment benefits, which \textit{cet. par.} leads to lower wage pressure.

2.3 The General Equilibrium

In this section we consider the general equilibrium which results with the wage-setting curve derived in eq. (24). For the general equilibrium also the price-setting curve must be taken into account. With iselastic goods demand, Cobb-Douglas technology and the assumption of a symmetric equilibrium the price-setting equation is

\[ W = H(u) = \alpha \kappa \left( \frac{K}{L(1-u)} \right)^{1-\alpha}, \]  

(28)

where \( K \) and \( L \) denote the total stock of capital and the labor supply. It is evident that the price-setting curve has a positive slope in real wage-unemployment space, with \( \lim_{u \to 1} H'(u) = \infty \). Since the wage-setting curve of eq. (24) has a finite value at \( W(u) \big|_{u=1} \) and since \( \lim_{u \to u} W'(u) = -\infty \) it follows that an equilibrium exists. If \( W(u) \) is falling over the whole range \((u, 1]\), the uniqueness of equilibrium is guaranteed. If the wage-setting curve has a positive bough over some part of the interval, multiple equilibria \textit{may exist} from a theoretical point of view (depending on the parameter constellation). However, if in simulation experiments plausible

\(^{14}\)This can be seen by taking into account that \( \tau(u) - \omega(u) = \omega(u)\lambda(u) \).
ranges for the various parameters are chosen, it turns out that a unique equilibrium is obtained.

[Figure 1]

Consider as an example figure 1(a) where two wage-setting curves with different replacement ratios $b$ are plotted. As can be seen from figure 1(a) both wage-setting curves have a positive bough. Furthermore, the wage-setting curve with the lower replacement ratio $b = 0.3$ lies above the wage-setting curve with the higher replacement ratio $b = 0.6$. In figure 1(b) the price-setting curve is depicted. It can be seen that the slope of this curve is getting very steep with high unemployment rates and for $u \to 1$ approaches infinity. Figure 1(c) shows only a small range for the unemployment rate where wage-setting curve and price-setting curve intersect in the general equilibrium. Since to the right of the intersection point the price-setting curve lies above the wage-setting curve and since with higher unemployment rates the slope of the price-setting curve increases more than the slope of the wage-setting curve (along the positive bough), a unique equilibrium is obtained. Figure 1(c) confirms the analytical result that a lower replacement ratio for future unemployment benefits leads to higher wage pressure and hence higher unemployment. However, the effect of a significant reduction in $b$ from 0.6 to 0.3 on the unemployment rate is rather small.  

3 Summary and Conclusions

We show how the analysis in a dynamic wage bargaining model has to be modified if in contrast to the literature it is assumed that firstly unemployment benefits are

\[ r = (1 - \alpha) \kappa ((K/L)/(1 - \bar{u}))^{-\alpha} \] and set $\bar{u} = 0.06$.  

\[ \alpha \kappa = 0.7, \gamma = 0.25, B_0 = 0.4 \]

and $\delta = 0.07$. To obtain consistent values for $K/L$ in the price-setting equation we used the relationship $r = (1 - \alpha) \kappa ((K/L)/(1 - \bar{u}))^{-\alpha}$ and set $\bar{u} = 0.06$.  

In point B the unemployment rate is equal to 9.9 percent compared to 9.67 percent in point A.
tied to the previous level of individually earned wages and secondly unemployment
benefits of those already unemployed at the beginning of the bargaining period
differ from benefits of those who get unemployed in later periods. For this aim we
develop a dynamic wage bargaining framework which encompasses “conventional”
models of the literature as special cases. Focusing on a model with Cobb-Douglas
production technology and risk-neutral workers, we demonstrate that in our model
the result of a vertical wage-setting curve is not obtained. It is shown that a crucial
prerequisite for a vertical wage-setting curve is the assumption that unemployment
benefits do not depend on the time period in which a worker becomes unemployed.
If this assumption is abandoned, the wage-setting curve exhibits one of the following
shapes depending on the parameter values: either the curve has a negative slope
over the whole range of (permissible) unemployment rates or the curve exhibits
a negative slope for low unemployment rates and has a positive bough for high
ones. As a further result it follows from our analysis that wage claims are cet.
par. higher if unemployment benefits are a function of individual wages instead
of the average wage level. The reason is that labor unions take into account that
higher wages today imply higher benefits for its members tomorrow which leads to
higher wage pressure. We also examine how the wage-setting curve is affected by
a variation in the parameters. For instance, we consider the consequences of labor
market reforms which reduce unemployment benefits of those who get unemployed
after the bargaining period, but keep the benefits of the currently unemployed
unchanged. We show that such a policy leads to higher wage pressure and thus
increases the equilibrium rate of unemployment.
A Appendix

A.1 Derivation of the aggregate wage-setting curve

With $W_1 = W$ it follows from eq. (23) that
\[ W = \frac{\mu \tau(u)}{\mu \left[ \tau(u) + (1 - b) (\psi(u) - \lambda(u)) \right] - (1 + \lambda(u)b) B_0}. \] (A.1)

The denominator of this equation can be written as
\[ (1 + \lambda(u)b) \left\{ \frac{\mu \omega(u) \left[ 1 - \lambda(u)b \frac{\psi(u) - \lambda(u)}{\lambda(u)\omega(u)} \right]}{1 + \lambda(u)b} - 1 \right\}. \] (A.2)

Due to the definitions in eq. (18) it holds that
\[ \frac{\psi(u) - \lambda(u)}{\lambda(u)\omega(u)} = -1. \] (A.3)

If this is taken into account in eq. (A.2), the aggregate wage-setting curve can be written as in eq. (24).

A.2 The slope of the wage-setting curve

Since $q(u) \equiv [1 + \lambda(u)b] [\mu \omega(u) - 1]$, eq. (25) can be rewritten as:
\[ \frac{dW}{du} = q(u)^{-2} \mu \left\{ \tau'(u) [1 + b \lambda(u)] [\mu \omega(u) - 1] \right. 
\[ - \tau(u) [\lambda'(u) b (\mu \omega(u) - 1) + (1 + \lambda(u)b) \mu \omega'(u)] \bigg\}. \] (A.4)

Hence
\[ \frac{dW}{du} = q(u)^{-1} \mu \left\{ \tau'(u) - \tau(u) \left[ \frac{\lambda'(u)b}{1 + b\lambda(u)} + \frac{\mu \omega'(u)}{\mu \omega(u) - 1} \right] \right\}. \] (A.5)

The derivatives $\tau'$, $\lambda'$ and $\omega'$ are:
\[ \tau'(u) = \frac{(r + \delta) \delta}{[(1 - u) \delta + ru]^2} > 0, \quad \lambda'(u) = \frac{\delta^2}{[(1 - u) \delta + ru]^2} > 0, \]
\[ \omega'(u) = \frac{\delta(r + \delta)}{(ru + \delta)^2} > 0. \] (A.6)

Eq. (A.5) can be simplified by noting that
\[ \tau(u) \lambda'(u) = \tau'(u) \lambda(u) = \frac{\delta^2 u(r + \delta)}{[(1 - u) \delta + ru]^3}, \] (A.7)

which leads to
\[ \frac{dW}{du} = q(u)^{-2} \mu \left\{ \tau'(u) (\mu \omega(u) - 1) - \mu \tau(u) \omega'(u) (1 + b\lambda(u)) \right\}. \] (A.8)
The sign of \( dW/du \) depends on the sign of the expression in brackets. Taking the definition of the parameters in eq. (18) and the derivatives in eq. (A.6) into account it follows that

\[
\tau'(u)(\mu \omega(u) - 1) - \mu \tau(u) \omega'(u)(1 + b \lambda(u)) = \\
\frac{\mu (1 - b) (r + \delta)^2 u^2 \delta^2 - \delta (r + \delta) (ru + \delta)^2}{[(1 - u) \delta + ru]^2 (ru + \delta)^2}. \quad (A.9)
\]

The sign of \( dW/du \) depends on the sign of the numerator of this expression, which leads to condition (26).
References


(a) Wage-setting curves with different replacement ratio

(b) Price-setting curve

(c) General Equilibrium

Figure 1: The consequences of a decrease in the replacement ratio
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