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Abstract

The aim of the Internal Ratings Based Approach (IRBA) of Basel II was that capital suffices for unexpected losses with at least a 99.9% probability. However, because only a fraction of the required regulatory capital (a quarter to a half) had to be loss absorbing capital, the actual bank solvency probabilities may have been much lower, as the global financial crisis illustrates. Our estimates suggest that under Basel II IRBA the loss-absorbing capital of an average-quality portfolio bank suffices for unexpected losses with a 95%-99% probability. This translates into an expected bank failure rate as high as once in twenty years. Even if the bank's interest income is incorporated into our model, the expected failure rate is still substantial. We show that the expected failure rate increases with loan portfolio riskiness. Our calculations may be viewed as a measure of regulatory "self-delusion" included in Basel II capital requirements.

Keywords: capital requirements, Internal Ratings Based Approach, Basel II, financial crisis

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1. Introduction

The Basel II capital adequacy framework, introduced in most European countries by 2008, required that the amount of bank capital should not be smaller than 8% of the *risk-weighted assets* of the bank. It allowed banks to choose between a standardized approach with rather crude risk weights, and an internal rating based (IRB) approach, in which banks were allowed to use their own estimates of default risks for evaluating the risk weights. The method with which the risk weights are calculated in the latter approach is motivated by an asymptotic single risk factor (ASRF) model for loan risks, and when the model is correct, the capital required by the IRB requirements covers the unexpected loan losses of the bank with a 99.9% probability each year.

The Basel II framework is currently being replaced with a revised Basel III framework, which is to be implemented in a gradual manner by 2019. The issues addressed with the reform include problems of procyclicality, moral hazard, model uncertainty, and the insufficient quality (i.e., loss absorbing capacity) of instruments which qualify as regulatory capital.

The *quality* of the capital which banks are required to have in the Basel II framework has been viewed as unsatisfactory because according to the Basel II regulations only one half (i.e., 4% of the risk-weighted assets) of bank capital is required to consist of Tier I capital, and the rest of bank capital is allowed to contain Tier 2 capital,¹ which may contain items such as *subordinated debt*. As also the inability of the bank to meet its obligations to its subordinated debt holders might lead to a bank failure, Tier 2 capital does not guarantee bank stability in the same way as Tier 1 capital does. In other words, although the Basel II IRB capital requirements guarantee that the assets of the bank suffice for covering its liabilities to its depositors and senior debt holders with the probability of 99.9% each year when the ASRF model behind the requirements is correct, the requirements do not guarantee that the assets suffice

¹Basel Committee on Banking Supervision (2006), p. 244. Cf. Caruana (2010), p. 2.

for all the liabilities with 99.9% probability. In addition, only one half of the required Tier 1 capital (i.e. 2% of the risk-weighted assets) has been required to consist of capital in the narrowest sense of the word (i.e. of common equity), and the other half has been allowed to consist of disclosed reserves (cf. Caruana, 2010, p. 2).

In this paper we analyze the implications of the low quality of Basel II capital for the true solvency of banks. We put forward a simple model of the banking sector which contains Basel IRB requirements and the ASFR model behind them as its elements, and in which the amount of the capital that actually promotes bank stability is less than the amount prescribed by the Basel IRB requirements. More specifically, in our model we shall introduce a parameter ξ , which measures the share of the capital that promotes bank stability within the total amount of capital of the bank. We assume that $0 < \xi \leq 1$, and we have a particular interest in two values of ξ , the value $\xi = 1/2$ which corresponds to the case in which only the Tier 1 capital promotes bank stability analogously with equity capital, and the value $\xi = 1/4$ which corresponds to the case in which only common equity promotes bank stability.² We find that the true solvency implied by the IRBA minimum requirement is dramatically lower than the intended 99.9%.

More specifically, we consider the effects of lowered capital standards both theoretically and by calibrating our model to the actual loan portfolios of banks which have been reported in Gordy (2000), and we address two distinct research questions. Firstly, we shall investigate the relation between the quality of capital, as measured by ξ , and what we shall call the capital insufficiency probability, i.e. the probability with which bank capital does not suffice for the purpose that is specified for it in the Basel IRB regulations. This consists of covering for unexpected losses of the bank. The capital insufficiency probability would have the value 0.1% for all portfolios, if

²The emphasis in Basel III on considerably raising the minimum requirements of common equity indicates that the Basel Committee puts most trust in common equity and motivates our interest in the case $\xi = 1/4$, in which only common equity has the stability-promoting characteristics of bank capital.

the Basel IRB regulations functioned in the way that they are supposed to function. The difference between its actual value and 0.1% may be viewed as a quantitative measure of "regulatory self-delusion" which took place in the implementation of Basel II, and which allegedly contributed to the onset of the global financial crisis. When $\xi = 1/4$ the capital insufficiency probability turns out to have a value of about 5% for an average-quality loan portfolio, corresponding to insufficiency once in twenty years, rather than in one thousand years as Basel II requirements prescribe.

Secondly, we also investigate the dependence between ξ and the probability with which the bank becomes insolvent and fails. The insolvency probability of a bank is different from its capital insufficiency probability (as we use the term) for two reasons. Firstly, bank capital is required to cover only the bank's unexpected losses with 99.9% probability in the Basel II framework, while the expected losses are supposed to be covered "on an ongoing basis, e.g. by provisions and write-offs" (Basel Committee on Banking Supervision, 2005, p. 7). This implies that actual failure rates of banks will depend also on the probability with which the mechanisms for covering the expected losses work in the way they should. Secondly, banks have the obligation to use also their interest income from non-defaulting loans for covering the losses from defaulting loans, implying that a bank does not necessarily become insolvent and fail when its unexpected losses exceed its capital.

This implies that estimates of capital insufficiency probability will have an essentially broader range of validity than estimates of bank insolvency probability: the capital insufficiency probability values that we present below are valid in all settings in which loan default probabilities are governed by the ASFR model behind Basel II IRB requirements, but bank insolvency probabilities will depend on the specific assumptions that are made concerning mechanisms for covering the expected losses, and concerning the market structure of the banking sector (because the market structure affects the interest rates). Below we shall give a favorable reading to the Basel II requirements by assuming that there are provisions which always suffice for covering

the expected losses, and we shall assume that there is a perfect competition in the banking sector. Obviously, the estimated bank failure rates would be increased if we assumed that also the mechanisms for covering expected losses might fail, and decreased if we assumed a less competitive market structure (which would correspond to higher interest rates) for the banking sector.³ Overall, the amount of "regulatory self-delusion" that we observe is lessened, but remains very significant, when we take bank interest income into account.

The rest of the paper is organized as follows. In Section 2 we present the main features of our model from the perspective of a single bank without discussing the market structure of the banking sector in detail. In Section 3 we present the ASRF model behind Basel II, explain how it can be applied to the setting of Section 2, and define the parameter ξ which measures the quality of the items which qualify as capital in the Basel II IRB framework. In Section 4 we investigate the probability with which bank capital suffices for the unexpected losses for different values of ξ both theoretically and in the context of some calibrated bank portfolios. In each case we will be interested in the connection between the probability of capital insufficiency, the riskiness of the portfolio of the bank (as measured by the default probabilities of its loans), and the value of ξ . In Section 5 we put forward one particular model which has the structure discussed in Section 2. We study the consequences of the model in the limit in which the number of banks increases and the banking sector becomes perfectly competitive, and solve for interest rates in this case. In Section 6 we once more consider both the theoretical example of the specialized bank and banks with calibrated portfolios, this time investigating the interconnections between bank insolvency probabilities, the value of the quality parameter ξ , and the riskiness of the

³See also footnote 5 below. In the context of the model of this paper, the fact that decreased competition in the banking sector would correspond to increased bank stability is illustrated by the formulas (27) and (28), which according to Lemma 1 characterize the equilibrium of an oligopoly of M banks. Formula (27) implies that smaller values of M must correspond to greater values of the function V_i , and according to (21) and (26) greater values of V_i must correspond to larger interest rates r_i and smaller bank failure rates.

loans in the portfolio of the bank. Section 7 concludes.

2. The Basic Features of the Model

The structure of our model is illustrated by Figure 1.⁴ The agents of the model are banks which get funding from their owners and from depositors, firms, and a financial supervisor. There are just two periods, $T = 0$ and $T = 1$. At time $T = 0$ each bank first collects funds from its owners and deposits from depositors, and lends them out to the firms as small loans, so that the portfolio of the bank is well-diversified. The firms are divided into types $i = 1, \dots, n$. The firms of type i make investments into projects of type i , and each project of each type may succeed or fail. The projects of different types differ with respect to their failure probability, the failure probability of each project of type i being \bar{p}_i . The projects are of size 1, and their only source of funding are the bank loans. Hence, also each loan is of size 1. The interest rate for firms of type i will be denoted by \bar{r}_i . The share of the loans of type i in the bank's portfolio is denoted by α_i , so that

$$\sum_{i=1}^n \alpha_i = 1 \tag{1}$$

The result of each investments projects (i.e. success or failure) is realized in period $T = 1$. A successful project of type i ($i = 1, \dots, n$) produces $1 + a_i$ (where $a_i > \bar{r}_i$) of which the bank receives $1 + \bar{r}_i$, but if a project is unsuccessful, it produces only $1 - \lambda$. In this case the loan defaults and the bank receives $1 - \lambda$. Hence, λ expresses the loss given default of the bank. In the next step, the deposits are withdrawn and the bank dissolves. The bank will fail if and only if its assets do not suffice for covering

⁴Cf. Repullo and Suarez (2004), in which Repullo and Suarez put forward a simple model of a banking sector that is subject to Basel II capital requirements. The structure of Repullo-Suarez model can be illustrated with Figure 1 when one sets $n = 2$, i.e. when one assumes that there are just two kinds of firms.

the deposits. The probability of bank insolvency will, obviously, depend both on the extent to which the defaults of the individual loans are correlated and the amount of capital that the owners of the bank have invested in it. The correlations between project failures are determined by ASRF model which we shall discuss in a detailed manner in Section 3 below.

[Figure 1]

A financial supervisor provides deposit insurance and determines the minimum level for the funds that the owners of the bank are required to provide. Wishing to model the Basel II capital regulations, which draw a distinction between the expected and unexpected losses of a bank, we shall assume that the supervisor deals with the expected and unexpectedly losses with separate pieces of regulation. As Basel Committee on Banking Supervision (2005) explains, the argument behind IRB requirements presupposes that financial institutions view the expected losses as a cost component of their business, and as already explained they are supposed to cover the expected losses on an ongoing basis by e.g. provisions and write-offs (ibid., p. 7), while the capital requirements aim at promoting bank stability against unexpected losses.

Clearly, the expected loss per loan of the considered bank amounts up to

$$L_E = \lambda \sum_{i=1}^n \alpha_i \bar{p}_i \tag{2}$$

By assumption, the supervisor requires that the expected loss L_E is covered by provisions, and puts forward the capital requirement according to which the capital that the owners provide as a protection against unexpected losses as at least K per loan. (We shall present and discuss the definition, of K , (12) in the next section). In the simplified world of our two-period model, the only legitimate source for the provisions are the funds of the bank owners, implying that they are quite analogous with bank

capital.⁵ Hence, the results of our analysis would not be changed if we assumed that the bank was subject to the capital requirement $K + L_E$, with no provisions being required.

3. The Basel IRB Framework and the ASRF Model behind it

The Basel IRB capital requirements aim at covering the unexpected losses with a probability of 99.9% each year, and the minimum amount of capital which suffices for this aim depends, of course, on the model which connects the default probabilities of loans with probability distribution of the loss of the bank. Motivated by the wide range of countries and institutions to which the IRB framework is to be applied, the Basel Committee has chosen to make use of a model which yields portfolio invariant capital requirements (Basel Committee on Banking Supervision, 2005, p.4) This means that the capital requirement that applies to a financial institution may be calculated simply as the sum of the capital requirements that correspond to its individual exposures.

The requirement of portfolio invariance puts surprisingly strong restrictions on the mathematical models that may be used as a basis of IRB capital requirements. As Gordy (2003) shows, only asymptotic single risk factor models satisfy the requirement of portfolio invariance. Here the word "asymptotic" captures the assumption that the portfolio of each bank consists of a large number of small exposures. An asymptotic model is concerned with limit in which the size of the exposures of the bank approaches

⁵The explanatory document Basel Committee on Banking Supervision (2005) also states that financial institutions view the expected losses as a cost component of their business, which they manage in a number of ways, "including through the pricing of credit exposures and through provisioning" (ibid., p. 2). Hence, under another interpretation of the Basel capital regulations banks would be allowed to compensate for the expected losses from the defaulting loans by interest income from the non-defaulting loans, in which case they would not be required to have the funds L_E in the form of provisions. This interpretation would, obviously, yield a model in which bank insolvency probabilities would be larger than those reported in Section 6 below.

zero and their number approaches infinity, and by a single risk factor model one means a model in which there is - in addition to the idiosyncratic risks that are associated with each loan - just a single systematic risk factor which affects the default rates of all loans of all banks.

The asymptotic single risk factor (ASRF) model behind the Basel IRB requirements is due to Merton (1974) and Vasicek (2002). The notation which we use for presenting this model resembles closely the notation of Repullo-Suarez (2004). In our notation, the default probability of each loan j of type i ($i = 1, ..n$) is characterized the random variable x_j which is defined by

$$x_j = \mu_i + \sqrt{\rho_i}z + \sqrt{1 - \rho_i}\varepsilon_j \quad (3)$$

The loan j defaults if $x_j > 0$. Here μ_i is a constant which characterizes the loans of type i , $z \sim N(0, 1)$ is the systematic risk factor, which applies to all loans of all types, and the random variables $\varepsilon_j \sim N(0, 1)$ are independent of each other and of z . We refer to the parameter ρ_i , which measures the extent to which the default rates of the loans of type i depend on the systematic risk factor, as the correlation parameter, since ρ_i can also be thought of as a measure of the correlation between the defaults of different loans. We immediately observe that also the random variable $\sqrt{\rho_i}z + \sqrt{1 - \rho_i}\varepsilon_j$ follows a normalized normal distribution, implying that the probability \bar{p}_i of default of a loan of type i is given by

$$\bar{p}_i = P(x_j > 0) = P(\sqrt{\rho_i}z + \sqrt{1 - \rho_i}\varepsilon_j > -\mu_i) = 1 - \Phi(-\mu_i) = \Phi(\mu_i)$$

where Φ is the cumulative distribution function of the normalized normal distribution.

The IRB requirements are based on the idea that the bank is able to estimate the default probability \bar{p}_i . We now observe that once \bar{p}_i is known, the parameter μ_i can be calculated as

$$\mu_i = \Phi^{-1}(\bar{p}_i) \quad (4)$$

whereas the correlation parameter ρ_i is assumed to be given by⁶

$$\rho_i = 0.12 \left(2 - \frac{1 - e^{-50\bar{p}_i}}{1 - e^{-50}} \right) \quad (5)$$

As our next step, we consider a given value of the systematic risk factor z and calculate the default probability of an individual loan j of type i for the given z . We denote this conditional probability by $p_i(z)$. Clearly, $p_i(z)$ is the probability that for the given z

$$\mu_i + \sqrt{\rho_i}z + \sqrt{1 - \rho_i}\varepsilon_j > 0$$

and, equivalently, the probability that

$$\varepsilon_j > -\frac{\mu_i + \sqrt{\rho_i}z}{\sqrt{1 - \rho_i}}$$

As ε_j is a random variable following the normalized normal distribution, this probability is equal with the probability that

$$\varepsilon_i < \frac{\mu_i + \sqrt{\rho_i}z}{\sqrt{1 - \rho_i}}$$

and it is given by

$$p_i(z) = \Phi \left(\frac{\mu_i + \sqrt{\rho_i}z}{\sqrt{1 - \rho_i}} \right) \quad (6)$$

As already explained, the random variables ε_j which correspond to the individual loans are independent. In the limit in which the number of the type i loans of the bank increases towards infinity, the distribution of share of defaulting type i loans among all the type i loans of the bank becomes, by the law of large numbers, increasingly concentrated at the probability that each individual loan defaults, i.e. $p_i(z)$. In the idealized world of an asymptotic single risk factor model, the share of the defaulting loans of type i will be taken to have exactly the value $p_i(z)$ with probability 1.

In our setting in which the loss given default is λ and each loan is of size 1, this implies that for each z the bank's loss per loan of type i is $\lambda p_i(z)$. The expected loss per loan for type i loans is, of course, $\lambda \bar{p}_i$, and using (4) and (6) the unexpected loss

⁶The definition (5) of ρ is equivalent with the definition of the correlation parameter R in Basel Committee on Banking Supervision (2006), p. 64. For a discussion of the motives for choosing the ρ value (5), see Basel Committee on Banking Supervision (2005), pp. 12-14.

per loan of type i is seen to be

$$\lambda p_i(z) - \lambda \bar{p}_i = \lambda \Phi \left(\frac{\Phi^{-1}(\bar{p}_i) + \sqrt{\rho_i} z}{\sqrt{1 - \rho_i}} \right) - \lambda \bar{p}_i$$

Each side of this equation is an increasing function of z , a fact which yields a simple connection between the amount of capital and the probability of its sufficiency for the unexpected losses. If the bank has the amount of capital $\lambda k_{B2}(\bar{p}_i)$, where the function $k_{B2}(\bar{p}_i)$ is for some z^* given by

$$k_{B2}(\bar{p}_i) = \Phi \left(\frac{\Phi^{-1}(\bar{p}_i) + \sqrt{\rho_i} z^*}{\sqrt{1 - \rho_i}} \right) - \bar{p}_i$$

the probability that this capital suffices for unexpected loan losses from a unit mass of loans of type i will be simply the probability that $z < z^*$, i.e. $\Phi(z^*)$. This probability will have some particular value $\bar{\theta}$ when one puts

$$z^* = \Phi^{-1}(\bar{\theta}) \tag{7}$$

In this case the function $k_{B2}(\bar{p}_i)$ equals

$$k_{B2}(\bar{p}_i) = \Phi \left(\frac{\Phi^{-1}(\bar{p}_i) + \sqrt{\rho_i} \Phi^{-1}(\bar{\theta})}{\sqrt{1 - \rho_i}} \right) - \bar{p}_i \tag{8}$$

If one puts $\bar{\theta} = 0.999$, it is easy to see that the requirement according to which the amount of bank capital should have at least the value $\lambda k_{B2}(\bar{p}_i)$ is identical with IRB requirements of the Basel II documents, when these are applied to a unit mass of loans with default probability \bar{p}_i , loss given default λ , and maturity $M = 1$ (Basel Committee for Banking Supervision 2006, p. 64). This capital equals 8% of risk-weighted assets in the sense of Basel Committee definition of risk weights (ibid.). Hence, the Basel IRB requirements yield the value 99.9% to the capital sufficiency probability, provided that the model behind the requirements is correct, and provided that all capital is of a proper quality.

In our setting, in which the share of the loans of each type i in the bank's portfolio

is denoted by α_i , the total losses of the bank per loan are seen to be

$$L = \lambda \sum_{i=1}^n \alpha_i p_i(z) \quad (9)$$

and, as the expected losses of the bank per loan are given by (2), the unexpected losses per loan will be given by

$$L_U = L - L_E = \sum_{i=1}^n \lambda \alpha_i p_i(z) - \lambda \sum_{i=1}^n \alpha_i \bar{p}_i \quad (10)$$

On the other hand, the total IRB capital requirement of the bank is calculated as the sum of the capital requirements that apply to its individual exposures. Given the bank's portfolio, the requirement for a unit mass of loans is

$$K^* = \lambda \sum_{i=1}^n \alpha_i k_{B2}(\bar{p}_i) \quad (11)$$

We can now observe that this capital requirement has also the desired portfolio invariance property. Remembering that $z^* = \Phi^{-1}(\bar{\theta})$, it is seen that if $z \leq z^*$, the capital that corresponds in (11) to the loans of a type i , $\lambda \alpha_i k_{B2}(\bar{p}_i)$, suffices for covering the losses from the loans of type i , but if $z > z^*$, the capital for every loan type i is insufficient for the loans of type i . Hence, also the probability that the capital K suffices for all the loan losses has the value $\Phi(z^*) = \bar{\theta}$, where $\bar{\theta} = 0.999$ in the actual Basel II framework.

We shall now give a rigorous formulation to the assumption that supervisor may let items of insufficient quality count as capital in its capital requirements, and that only some part of the capital K^* promotes bank stability in the way capital is supposed to do. In our simplified model, in which the only possible liabilities of the bank are the funds of its owners and its deposits, this assumption may best be interpreted as

the postulate that the bank is subject to the capital requirement

$$K = \xi K^* \tag{12}$$

where K^* is given by (11) and ξ has some fixed value for which $0 < \xi \leq 1$. In this case capital will be sufficient for the unexpected losses if and only if

$$L_U < \xi K^* \tag{13}$$

Using (10) and (11), this condition is seen to be equivalent with

$$\sum_{i=1}^n \alpha_i p_i(z) < \sum_{i=1}^n \alpha_i \bar{p}_i + \xi \sum_{i=1}^n \alpha_i k_{B2}(\bar{p}_i)$$

The right-hand side of this inequality is independent of z , and we may conclude from (6) that its left-hand side is an increasing function of z . Hence, capital will suffice for unexpected loan losses if and only if

$$z < \bar{z}(\xi)$$

where the function $\bar{z}(\xi)$ is defined by the condition

$$\sum_{i=1}^n \alpha_i p_i(\bar{z}(\xi)) = \sum_{i=1}^n \alpha_i \bar{p}_i + \xi \sum_{i=1}^n \alpha_i k_{B2}(\bar{p}_i) \tag{14}$$

Putting $\bar{z} = \bar{z}(\xi)$, the capital sufficiency probability (i.e., the probability with which bank capital is sufficient to cover the unexpected losses) is now simply the probability that $z < \bar{z}$, i.e. $\Phi(\bar{z})$, and the capital insufficiency probability is

$$P_I = 1 - \Phi(\bar{z}) \tag{15}$$

In the next section we shall discuss the connection between the capital insufficiency probability P_I , the capital quality parameter ξ , and the portfolio of the bank.

4. Quality of Capital and the Probability of Capital Insufficiency

We shall begin our discussion of the comparative statics of capital insufficiency with some theoretical results. For the purposes of the theoretical analysis we shall view the number $\bar{\theta}$, which expresses the required probability of capital sufficiency and which has the value $\bar{\theta} = 0.999$ in the Basel IRB framework, as an arbitrary constant. Analogously with the numerical value $\bar{\theta} = 0.999$, also the connection (5) between the correlation parameter ρ_i and the default probability \bar{p}_i is a restrictive assumption which is not inherent in the Merton-Vasicek model behind the Basel II IRB framework. Accordingly, in the theoretical part of our analysis we will not assume (5) to be valid, but shall view default probability \bar{p}_i and the correlation parameters ρ_i as independent parameters.

We observe that the loss given default parameter λ does not appear in our definition of the capital insufficiency probability P_I , which consists of (15) and the implicit definition of $\bar{z} = \bar{z}(\xi)$, (14). Intuitively, this is because the lowered capital requirement, i.e. ξ times the Basel IRB requirement, and the capital that the bank actually needs for covering its unexpected loan losses are both proportional to λ . Since according to (6) the function $p_i(z)$ is increasing in z , it is also clear from (14) and (15) that - as long as the Basel II capital requirement is positive⁷ - an increase in ξ must correspond to an increase in $\bar{z}(\xi)$ and a decrease in P_I . We formulate these observations as a separate theorem.

Theorem 1. (a) The capital insufficiency probability P_I is independent of the loss given default λ .

⁷To see why the Basel IRB requirement is not necessarily positive if the parameters \bar{p}_i , $\bar{\theta}$, and ρ_i are not calibrated in an economically sensible manner, but viewed as arbitrary constants, one should keep in mind that the Basel IRB requirement expresses the amount of capital that the bank needs *in addition to* the provisions (which are meant for covering the expected losses) for avoiding capital insufficiency with the probability $\bar{\theta}$. Accordingly, the Basel IRB requirement will be negative if the provisions suffice by themselves for this purpose. This can, of course, be the case if the value of $\bar{\theta}$ is sufficiently low.

(b) When the Basel IRB capital requirement (8) is positive, the capital insufficiency probability P_I is a decreasing function of the quality of capital, as measured by ξ .

An analysis of the comparative statics of capital insufficiency should address also the connection between the capital insufficiency probability P_I and the default probabilities \bar{p}_i of the loans, and the connection between P_I and the correlation parameter values ρ_i (as we are now viewing the correlation parameter values as independent of the probabilities \bar{p}_i). There is a simple connection between the changes in \bar{p}_i and P_I when one restricts attention to specialized banks (i.e. banks whose loans are all of the same type i and have the same default probability $\bar{p} = \bar{p}_i$). Putting $\rho = \rho_i$ and $\mu = \Phi^{-1}(\bar{p})$, and using (4), (6), (7), and (8), the condition (14) which defines the function $\bar{z} = \bar{z}(\xi)$ is in the case of a specialized bank seen to be equivalent with

$$\Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}(\xi)}{\sqrt{1-\rho}}\right) = \xi\Phi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) + (1-\xi)\Phi(\mu) \quad (16)$$

The formula (16) has the following implication.

Theorem 2. Assume $\xi < 1$, consider a specialized bank all of whose loans have the same default probability \bar{p} , and assume that the Basel IRB requirement $k_{B2}(\bar{p})$ is positive. Now an increase in the default probability \bar{p} of the loans will increase the capital insufficiency probability P_I of the bank.

Our numerical analyses have revealed that there is no analogous theorem that would be concerned with a change in the correlation parameter ρ_i , although it has turned out that, for the reasonably calibrated versions of the model, an increase of ρ_i will mostly increase the capital insufficiency probability.⁸

⁸More specifically, it has turned out that for low values the default probability \bar{p} an increase in ρ will typically increase capital insufficiency probability, but the opposite may well be the case for a bank with junk bonds. E.g., if one considers a specialized bank whose loans are all of the same type, puts $\bar{p} = 0.1$ and $\xi = 0.6$, and lets ρ vary in the region $[0.12, 0.24]$ in which it is postulated to vary in the actual Basel IRB framework, it will turn out that capital sufficiency probability will be an increasing function of ρ for the smaller values of ρ and decreasing function for larger values.

To get further results, we move to a discussion of a calibrated version of the model. In this discussion we shall assume that (5) is valid, i.e. that the default probability \bar{p}_i fixes the correlation parameter ρ_i in accordance with the Basel IRB regulations, and we give the parameter $\bar{\theta}$, which expresses the capital sufficiency probability when $\xi = 1$, the value $\bar{\theta} = 0.999$.

We shall first consider a specialized bank whose loans have the same default probability \bar{p} . Figure 2 depicts the capital insufficiency probability P_I of the bank as a function of \bar{p} for three values of the parameter ξ , the values $\xi = 1/4$, $\xi = 1/2$, and $\xi = 1$. Here the value $\xi = 1/4$ corresponds to the amount of common equity required by the Basel II framework, and the value $\xi = 1/2$ corresponds to the amount of Tier 1 capital required by the Basel II framework. The value $\xi = 1$ corresponds to the total amount of required capital, and as the the definition of Basel II IRB requirements implies, for this value of ξ the capital insufficiency probability has the constant value 0.001.

[Figure 2]

We observe from the figure that, although the insufficiency of capital for covering unexpected loan losses becomes for all values of \bar{p} more likely as ξ decreases, the effects of the decrease in capital is more dramatic when the default risks of loans are large. E.g., a bank with a homogenous portfolio of 0.1% default probability would have a capital insufficiency probability of 0.5% when $\xi = 1/2$ and of 1.7% when $\xi = 1/4$, but if the default probability is 1% in the homogenous portfolio, the corresponding figures are 0.9% when $\xi = 1/2$ and 3.8% when $\xi = 1/4$. In the theoretical case of a bank with a homogenous portfolio of 10% default probability, we would get the much more extreme values 2.7% when $\xi = 1/2$ and 11.5% when $\xi = 1/4$.

| | Portfolio credit quality | | | | |
|-----|--------------------------|-------------|---------|--------------|---------------|
| | High (%) | Average (%) | Low (%) | Very Low (%) | \bar{p} (%) |
| AAA | 3.8 | 2.9 | 1.0 | 0.5 | 0.01 |
| AA | 5.9 | 5.0 | 1.5 | 1.0 | 0.02 |
| A | 29.3 | 13.4 | 3.7 | 3.2 | 0.06 |
| BBB | 38.0 | 31.2 | 16.5 | 13.2 | 0.18 |
| BB | 19.1 | 32.4 | 38.1 | 35.6 | 1.06 |
| B | 2.7 | 11.1 | 32.4 | 37.0 | 4.94 |
| CCC | 1.3 | 4.0 | 6.8 | 9.5 | 19.14 |

Table 1. Portfolio qualities of US banks, as reported by Gordy (2000)

However, it is more interesting to investigate the probability with which the capital of a bank with a realistically calibrated loan portfolio is insufficient for covering the yearly unexpected loan losses. Table 1 reproduces data from Gordy (2000), and its columns "High", "Average", "Low", and "Very low" corresponds to a classification of US banks used by Gordy. These columns show the average shares of loans of various ratings in the portfolios of the relevant banks.⁹ As its right-most column, the table contains an estimated default probability \bar{p} as a percentage for the loans of each rating, also reported in *ibid.* (p. 134).

[Figure 3]

We have calibrated our model by putting $n = 7$ and by choosing the loans of the different categories $i = 1, \dots, 7$ to have the default probabilities $\bar{p}_1, \bar{p}_2, \dots, \bar{p}_7$ that are presented as the default probabilities of the loans with ratings AAA, AA, A, BBB, BB, B, and CCC in Table 1. We have considered four different choices for the parameters $\alpha_1, \alpha_2, \dots, \alpha_7$. These choices correspond to the four portfolios reported in the columns "High", "Average", "Low", and "Very Low" in Table 1. The functions which express

⁹ More precisely, the columns "High", "Average", and "Low" correspond to actual averages in a sample of US banks, whereas the column "Very Low" is a theoretical construct. See Gordy (2000), p. 130.

the capital insufficiency probability P_I as a function of ξ for these portfolios will below be denoted by $P_{IH}(\xi)$, $P_{IA}(\xi)$, $P_{IL}(\xi)$, and $P_{IVL}(\xi)$, respectively. These functions are depicted in Figure 3 in the interval $1/4 \leq \xi \leq 1$. Table 2 shows the capital insufficiency probability for the considered portfolios as a percentage for the three values of ξ that are of special interest, $\xi = 1/4$, $\xi = 1/2$, and $\xi = 1$.

| ξ | $P_{IH}(\xi)$ (%) | $P_{IA}(\xi)$ (%) | $P_{IL}(\xi)$ (%) | $P_{IVL}(\xi)$ (%) |
|-------|-------------------|-------------------|-------------------|--------------------|
| 1/4 | 3.58 | 5.46 | 7.17 | 7.83 |
| 1/2 | 0.83 | 1.18 | 1.55 | 1.70 |
| 1 | 0.1 | 0.1 | 0.1 | 0.1 |

Table 2. The capital insufficiency probability for selected values of ξ .

Table 2 illustrates the dramatic consequences that the low quality of capital can have for bank stability. Its lower-most row correspond to banks with the minimum amount of capital allowed by Basel II IRB requirements, and since we have assumed that the model which underlies these requirements is correct, in the lower-most row the probability of capital insufficiency is 0.1% for all portfolios. The row $\xi = 1/2$ may be thought of as corresponding to banks which have the minimum amount of Tier 1 capital, and whose Tier 2 capital consists of items which cannot save the bank from failing in the way equity capital would do. We observe that the probability with which the capital of the bank fails to fulfill the purpose prescribed by Basel II IRB regulations increases more than eightfold from the value 0.1% intended by regulators if the bank has a high-quality portfolio, and 17-fold if it has a very low quality portfolio. If the bank has the minimum amount equity capital, i.e. one fourth of the total capital required by IRB requirements, and if the rest of the "capital" is of an unsatisfactory quality, the considered probability turns out to be almost 36 times as large as intended for a high quality bank, and more than 78 time as large for a very low quality bank.

5. A Model with a Competitive Banking Sector

As we saw above, the probability with which bank capital is insufficient for the purpose that has been specified for it by the Basel IRB regulatory framework is distinct from the probability with which the bank becomes insolvent and fails, as the bank insolvency probabilities are influenced also by the interest income that banks receive from their non-defaulting loans. Above in Section 2 we stated that the interest income from each loan of type i is \bar{r}_i , but did not yet explain how \bar{r}_i was determined. This was because the results that we have presented so far are independent of the values \bar{r}_i ($i = 1, \dots, n$) and hence, also independent of the market structure of banking sector. However, in order to analyse bank insolvency probabilities, we must fill in the missing details of the model, postulate some particular market structure for the banking sector, and solve for the interest rates in the Nash equilibrium of considered banking sector.

We wish to study the case in which the banking sector is competitive, and we also wish to be able calibrate our model to actual bank portfolios such as the ones shown in Table 1. However, as Repullo-Suarez (2004, p. 503) demonstrate, perfectly competitive banks have an incentive to specialize to loans of a single type, when loan defaults are governed by the ASRF model behind Basel IRB requirements.¹⁰ This forces us to make an extra twist in our analysis: we shall not consider a perfectly competitive banking sector, but an oligopoly with M banks, and we shall take the limit in which $M \rightarrow \infty$ to represent a perfectly competitive banking sector.

More specifically, the model that we have chosen is a Salop circle model¹¹ with M banks, indexed by $m = 1, \dots, M$, which are located at equal distances on a unit circle (i.e. a circle with the circumference 1). The firms whose loans the banks are financing are located on the same circle, the density of the firms of type i ($i = 1, \dots, n$) being α_i . This implies that (1) is valid, i.e. that

¹⁰We shall present an intuitive explanation for this specialization result in footnote 12 below.

¹¹Cf. Chipori et al (1995), which applies a Salop circle model of competition in the banking sector to the regulation of *deposit rates* (rather than that of the amount of bank capital). For a more recent application, see Hauswald and Marquez (2006).

$$\alpha_1 + \dots + \alpha_n = 1.$$

Just like before, our model has the structure shown in Figure 1, the loans are of size 1, their loss given default is λ , and the default probabilities of the loans are determined by the ASRF model behind Basel II, in the way which was elaborated in Section 3 above. We normalize the deposit interest rate to zero.

At time $T = 0$, the banks and firms are involved in a Bertrand-type game in which each bank m ($m = 1, \dots, M$) first sets an interest rate r_{im} for the loans of each type i ($i = 1, \dots, n$), and each firm chooses a bank in order to maximize its profit in period $T = 1$, knowing the interest rates. As it was explained in Section 2, if the project of a firm of type i fails, the firm goes bankrupt and is able to pay only the amount $1 - \lambda$ to the bank. If the project of a firm of type i succeeds and the interest rate for its loan has been r_{im} , the firm receives the revenue $1 + a_i$ from the project and pays $1 + r_{im}$ to its bank. We now postulate that the firm has, in addition to costs which are caused by its loan, an extra cost $\tilde{\beta}_i y_m$ when it borrows funds from a bank whose distance is y_m from it. This implies that the expected profit of the firm is

$$Q_m = (1 - \bar{p}_i) [(1 + a_i) - (1 + r_{im})] - \tilde{\beta}_i y_m = (1 - \bar{p}_i) (a_i - r_{im}) - \tilde{\beta}_i y_m \quad (17)$$

The firm will choose the bank m which maximizes Q_m .

We wish to find the symmetric equilibrium of our model, and with this aim in mind, we assume that all other banks except for bank m charge the interest rate \bar{r}_i for loans of type i , and investigate the optimal choice of the interest rate $r_i = r_{im}$ by bank m . Below it will be handy to make use of the vector notations $\bar{\mathbf{r}} = (\bar{r}_1, \bar{r}_2, \dots, \bar{r}_N)$ for the interest rates of the other banks, and $\mathbf{r} = (r_1, r_2, \dots, r_N)$ for the interest rates of bank m .

We conclude from (17) that a firm of type i which is between banks m and $m + 1$ and located at the distance y from bank m , will be indifferent between loans from the banks m and $m + 1$ if

$$(1 - \bar{p}_i)(a_i - r_i) - \tilde{\beta}_i y = (1 - \bar{p}_i)(a_i - \bar{r}_i) - \tilde{\beta}_i \left(\frac{1}{M} - y\right)$$

When one puts $\beta_i = \tilde{\beta}_i / (1 - \bar{p}_i)$, this condition is seen to be equivalent with

$$y = \frac{1}{2M} + \frac{1}{2\beta_i} (\bar{r}_i - r_i)$$

An analogous calculation is valid for the firms between banks $m-1$ and m . Altogether, the demand $d_i(r_i, \bar{r}_i)$ for loans of type i from bank m is $2\alpha_i y$, i.e.

$$d_i(r_i, \bar{r}_i) = \frac{\alpha_i}{M} + \frac{\alpha_i}{\beta_i} (\bar{r}_i - r_i) \quad (18)$$

We denote the funds that the regulator demands for each loan of type i from the bank owners by k_i . As it was explained in Section 3, k_i is the sum of the provisions for expected loss $\lambda \bar{p}_i$ and the capital requirement and it is given by

$$k_i = \lambda \bar{p}_i + \xi(\lambda k_{B2}(\bar{p}_i)) \quad (19)$$

where the function k_{B2} is defined by (8).

Remembering that we have normalized the deposit interest rate to zero, we may express the net worth of the considered bank m after the systematic risk factor z has been realized as

$$\pi(\mathbf{r}, \bar{\mathbf{r}}, z) = \sum_{i=1}^n d_i(r_i, \bar{r}_i) \pi_i(r_i, z) \quad (20)$$

where $\pi_i(r_i, z)$ is the difference between the expected repayment for each loan of type i and the deposits that correspond to it. This difference is given by

$$\begin{aligned} \pi_i(r_i, z) &= (1 - p_i(z))(1 + r_i) + p_i(z)(1 - \lambda) - (1 - k_i) \\ &= k_i + r_i - p_i(z)(\lambda + r_i) \end{aligned} \quad (21)$$

We assume that the opportunity cost for bank capital is δ . This opportunity cost can be viewed as the riskless interest rate that the capital of the bank would earn elsewhere in the economy. The net value of the bank is the difference between the investment that the bank owners make into the bank in period $T = 0$ and its discounted expected

net worth in period $T = 1$, and it is given by

$$V(\mathbf{r}, \bar{\mathbf{r}}) = - \sum_{i=1}^n d_i(r_i, \bar{r}_i) k_i + \frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}(\mathbf{r}, \bar{\mathbf{r}})} \pi(\mathbf{r}, \bar{\mathbf{r}}, z) d\Phi(z) \quad (22)$$

where $\hat{z}(\mathbf{r}, \bar{\mathbf{r}})$ is the value of z for which integrand becomes zero. In other words, $\hat{z}(\mathbf{r}, \bar{\mathbf{r}})$ is defined by

$$\pi(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}(\mathbf{r}, \bar{\mathbf{r}})) = 0 \quad (23)$$

Intuitively, if $z < \hat{z}(\mathbf{r}, \bar{\mathbf{r}})$, the liabilities of the bank are smaller than its assets, the bank does not fail, and the positive value $\pi(\mathbf{r}, \bar{\mathbf{r}}; z)$ expresses the net worth of the bank to its owners. However, if $z > \hat{z}(\mathbf{r}, \bar{\mathbf{r}})$, the bank becomes insolvent and fails and is of a zero net worth (rather than negative net worth) to its owners.¹² When the interest rates $r = (r_1, r_2, \dots, r_N)$ and $\bar{r} = (\bar{r}_1, \bar{r}_2, \dots, \bar{r}_N)$ have been fixed, the value $\hat{z} = \hat{z}(\mathbf{r}, \bar{\mathbf{r}})$ that corresponds to them determines the failure, or insolvency probability P_F of the bank as

$$P_F = 1 - \Phi(\hat{z}) \quad (24)$$

For the purposes of the discussion below it turns out to be handier to express the net value (22) in the form

$$V(\mathbf{r}, \bar{\mathbf{r}}) = \sum_{i=1}^n d_i(r_i, \bar{r}_i) V_i(r_i, \hat{z}(\mathbf{r}, \bar{\mathbf{r}})) \quad (25)$$

¹²The formula (22) allows us to understand intuitively the specialization result by Repullo and Suarez to which we referred, i.e. the result that banks have an incentive to specialize to a single loan category in a model with perfect competition. Under perfect competition there is just a single interest rate $r_i = \bar{r}_i$ for the loans of each type i , and each bank takes the interest rates as given. If a bank has, say, loans of types 1 and 2 in its portfolio, and if we denote \hat{z} values that correspond to specialized banks with type 1 and 2 loans by \hat{z}_1 and \hat{z}_2 (for the given interest rates), it will as a rule be the case that $\hat{z}_1 \neq \hat{z}_2$. Whenever the realized z value is between \hat{z}_1 and \hat{z}_2 , one of the specialized banks fails but the other one does not. In such cases, the mixed-portfolio bank will be of a smaller net worth than the two specialized financial institutions, since the non-specialized bank must use the income from its positive-net-worth component for paying the losses of its negative-net-worth component, while two separate institutions have no analogous obligation towards each other. It should be observed that as we are using a Salop circle model with a fixed number M of banks, we are implicitly assuming that a bank cannot split itself into many separate banks which all would have the same location.

where the function V_i is given by

$$V_i(r_i, \hat{z}) = -k_i + \frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}} \pi_i(r_i, z) d\Phi(z) \quad (26)$$

Intuitively, the function V_i expresses the net value of a loan of type i . The result (25) states that the net value of the bank may be calculated as the sum of the net values of its individual loans. The net values of different loan categories are interconnected only via the parameter \hat{z} which specifies the circumstances under which the bank fails. The parameter \hat{z} depends on all the interest rates, and according to (26), the profits and losses from loans of type i are included in the net value of the bank only when it does not fail.

The symmetric equilibrium of the model is the case in which it is optimal for the bank m to choose the interest rates $\bar{r}_1, \dots, \bar{r}_n$ that the other banks have chosen. The symmetric equilibrium is characterized by the following lemma.

Lemma 1. The model has a symmetric equilibrium. In a symmetric equilibrium the interest rates $\bar{\mathbf{r}} = (\bar{r}_1, \dots, \bar{r}_N)$ satisfy

$$V_i(\bar{r}_i, \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})) = \frac{\beta_i}{(1 + \delta)M} \int_{-\infty}^{\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})} (1 - p_i(z)) d\Phi(z) \quad (27)$$

where borderline value $\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})$ of the systematic risk factor z , which is defined by (23), satisfies the condition

$$\sum_{i=1}^N \alpha_i \pi_i(\bar{r}_i, \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})) = 0 \quad (28)$$

The value V_i which is given by (27) may also be viewed as the net value of a unit mass of loans of type i . Since the total net value of all the loans of type i of is $\alpha_i V_i$, and since M appears in the denominator the left-hand side of (27), we may conclude that the total net value of the loans of each type i approaches zero as M increases and competition becomes more severe.

We shall view the interest rates that the rates \bar{r}_i characterized by Lemma 1 approach when $M \rightarrow \infty$ as the representation of the interest rates in a perfectly competitive market. These interest rates are given by Lemma 2.

Lemma 2. When the equilibrium interest rates \bar{r}_i , that are determined by (27) are viewed as functions of M , it must be the case that

$$\lim_{M \rightarrow \infty} \frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})} \pi_i(\bar{r}_i; z) d\Phi(z) = k_i \quad (29)$$

where $\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})$ is given by (28).

Motivated by Lemma 2, we shall take the situation in which the limiting relation is valid precisely, i.e. the situation in which

$$\frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})} \pi_i(\bar{r}_i; z) d\Phi(z) = k_i \quad (30)$$

to be the representation of perfect competition in the context of our model. Intuitively, (30) states that the expected net worth of a loan of type i must be identical with its total capital costs k_i for the bank (i.e., the costs of required provisions and the costs due to capital requirements) when the expectation is calculated restricting attention to the range of z values for which the bank does not fail. This condition differs from the corresponding condition of a specialized bank in a competitive market only in so far that according to (28) the value $\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})$ depends on the net worth of all the loans that bank has in its portfolio, and is not necessarily identical with the value of z for which $\pi_i(\bar{r}_i, z)$ becomes zero.

6. The Quality of Bank Capital and Bank Insolvency Probability

We now turn to a discussion of the probability with which a bank becomes insolvent and fails when the condition (30) is valid. Together with (28), the formula (30) characterizes the interest rates \bar{r} in a competitive banking sector and also the value $\hat{z} = \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})$, which further determines the bank insolvency, or failure probabilities P_F in accordance with (24).

We begin our analysis by considering a specialized bank with a homogeneous loan portfolio. We study the relationship between the default probability of its loans and its insolvency probability both theoretically and by making a calibration, by considering once more the values $\xi = 1/4$, $\xi = 1/2$, and $\xi = 1$. As the second part of our analysis, we let ξ vary continuously for the four actual bank portfolios of Table 1.

Our model can be applied to a specialized bank by putting $N = 1$ and $\alpha = 1$. Using (21), the condition (30) which determines the equilibrium interest rates can in this case be written as

$$\frac{1}{1 + \delta} \int_{-\infty}^{\hat{z}} (k_1 + (1 - p_1(z)) \bar{r}_1 - \lambda p_1(z)) d\Phi(z) = k_1 \quad (31)$$

where \hat{z} is the value for which the integrand is zero. This equilibrium condition is identical with that of the simpler model that was put forward in Repullo - Suarez (2004, p. 504). Repullo and Suarez assume that all the capital required by the Basel IRB requirements is equity capital, so that their analysis corresponds in our notation to the case $\xi = 1$. They show that in the calibrated version of their model the failure rate of the bank, which is still given by (24), is a decreasing function of the default probability of its loans (ibid., p. 507, Table 1).

In order to understand this result intuitively, it is useful to think of the probability of bank non-failure (i.e. solvency) $1 - P_F$, as the sum of the probabilities of two

mutually exclusive events, the event of (what we have called) capital sufficiency and the event that capital and provisions are insufficient for covering the losses of the bank, but the bank is nevertheless saved from failing by the interest income from its non-defaulting loans. The probability of the former event is $1 - P_I$, and it has by construction the value 0.999 in the Repullo-Suarez model, and the probability of latter event is $P_I - P_F$. Hence, Repullo and Suarez's result shows that the value of $P_I - P_F$ is in their calibrations larger for banks with riskier loans.

This can be understood intuitively by noting that when capital sufficiency probability $1 - P_I$ is kept fixed, an increase in the default probabilities of loans affects the bank insolvency probability in three ways. Firstly, the direct effect of a larger number of loan defaults is to decrease the interest income and hence, the net worth of the bank. This effect tends to increase the insolvency probability. According to (5), the larger default probability is associated with a decreased correlation between the defaults of the individual loans, and this should, secondly, decrease the insolvency probability of the bank. Thirdly, an increased default probability is also associated with higher interest rates in equilibrium. (As one can easily see from (31), a bank with more risky loans must charge higher interest rates both because of the higher capital costs k_1 and because of the higher default rate of loans for each z , $p_1(z)$). Repullo and Suarez's result shows that in realistically calibrated versions of the model the two latter stabilizing effects are stronger than the first destabilizing one. Intuitively, banks with risky loans are more stable than low-risk loan banks because of the higher interest rates that they charge and because of the smaller correlation between the defaults of their loans (and despite of the fact that a larger fraction of their loans default) when the capital sufficiency probability is kept fixed.

We now turn to the analysis of the case in which $\xi < 1$, still thinking of the probability of bank solvency as the sum of the probabilities of the mutually exclusive events of capital sufficiency and solvency despite of capital insufficiency. An increase in the default probability of the loans of a specialized bank will affect both components

of the bank solvency probability. In Section 4 we saw that if the riskiness of the loans of a specialized bank increases in a realistically calibrated version of the model, the capital sufficiency probability $1 - P_I$ will decrease. However, Repullo and Suarez's results suggest that in this case the probability $P_I - P_F$, i.e. the probability that the bank does not fail despite of capital insufficiency, should increase. Bank solvency probability is the sum

$$1 - P_F = (1 - P_I) + (P_I - P_F)$$

and the sign of its change will depend on the relative strengths of the two opposing effects.

[Figure 4]

Figure 4 shows the sum of these two effects for a specialized bank and for the three values of the quality parameter ξ which are of a special interest, i.e. $\xi = 1/4$, $\xi = 1/2$, and $\xi = 1$. Analogously with Figure 2, Figure 4 shows bank insolvency probability for each of these values as a function of the default probability \bar{p} when \bar{p} varies from $\bar{p} = 0.001$ to $\bar{p} = 0.1$. As it is seen from the figure, for the smaller values of \bar{p} the bank insolvency probability is an increasing function of \bar{p} both when $\xi = 1/4$ and when $\xi = 1/2$. This should be interpreted as meaning that the destabilizing effect of the increase in capital insufficiency probability dominates the stabilizing effect that increased interest income and decreased correlation between loan defaults have on the bank. However, the opposite is the case for the larger values of \bar{p} , since for those values bank insolvency probability decreases as \bar{p} increases.

[Figure 5]

We now turn to the more realistic case of a bank with one of the portfolios of Table 1. We denote the insolvency probability of a bank as a function of ξ for these portfolios by $P_{FH}(\xi)$, $P_{FA}(\xi)$, $P_{FL}(\xi)$, and $P_{FVL}(\xi)$, respectively. Figure 5 shows the graphs of these four functions. Comparing Figures 3 and 5, it is observed that the portfolio of the bank has an essentially smaller effect on its insolvency probability than on its capital sufficiency probability. This is, of course, because the greater interest income

of banks with riskier loans partially compensates for their greater capital insufficiency probability. The curves that correspond to the banks with low quality portfolio and very low quality portfolio are almost indistinguishable in Figure 5, but otherwise the figure also illustrates the point that when ξ , i.e. the share of "proper" capital within the required capital, is small, banks with riskier portfolios will have larger insolvency probabilities. This is due to the greater capital insufficiency probability of banks with risky loans. On the other hand, when ξ is close to 1 (and almost all capital is proper), the order of the curves is reversed. This result confirms the earlier finding by Repullo-Suarez (2004) that we discussed above, and is due to the larger interest income of the banks with risky portfolios, and to the smaller correlation between the defaults of their loans. Table 3 is analogous with Table 2, and it shows the bank insolvency probabilities for the ξ values of special interest, $\xi = 1/4$, $\xi = 1/2$, and $\xi = 1$.

| ξ | $P_{FH}(\xi)$ (%) | $P_{FA}(\xi)$ (%) | $P_{FL}(\xi)$ (%) | $P_{FVL}(\xi)$ (%) |
|-------|-------------------|-------------------|-------------------|--------------------|
| 1/4 | 1.90 | 2.15 | 2.33 | 2.33 |
| 1/2 | 0.50 | 0.54 | 0.57 | 0.56 |
| 1 | 0.07 | 0.05 | 0.04 | 0.04 |

Table 3. The bank insolvency probability for selected values of ξ .

7. Concluding Remarks

In this paper we have analyzed the true solvency of banks under the Basel II IRB minimum capital requirements. We have taken into account the possibility that only Tier 1 capital, or perhaps even only common equity as part of Tier 1 capital, might provide loss absorbing capacity for the bank. This may lead to a lower actual bank solvency probability than the probability of 99.9% over a one-year horizon, the official target of Basel II. Assuming the model behind Basel II IRB capital requirements to

be valid, and drawing a distinction between capital sufficiency and bank solvency, we observed that for a bank with an average-quality portfolio, bank capital might fail to suffice for its stated aim (i.e., covering the unexpected losses) with a probability of between 1.2% and 5.5%, rather than with the probability of 0.1%, if only a part of bank capital promotes bank stability. Besides the capital insufficiency probability, the insolvency rates of banks will depend also on the capability of banks to cover for their expected losses and on the interest income of banks. Our calculations of insolvency probabilities were based on a favorable view of the methods of covering the expected losses (as we assumed that these methods never fail) and on the assumption of a competitive banking sector, in which banks cannot earn economic profits from their interest income. Under these assumptions, we obtained yearly insolvency probabilities between 0.5% and 2.2% for an average quality bank. We also observed that both capital insufficiency probability and insolvency probability increase with the riskiness of the bank's portfolio.

Our work can be generalized to a variety of directions. E.g., an obvious application would be to study its implications for the new Basel III framework, in which the minimum and "required" amounts of common equity have been substantially increased. Systemically important financial institutions (SIFIs) will in normal times be required to have 9.5% common equity of their risk-weighted assets by the 2019 full implementation deadline of Basel III, on top of which further national counter-cyclical buffers might be imposed. Our framework would readily lend itself to calculating the corresponding new theoretical capital sufficiency and solvency levels, which would now obviously exceed the 99.9% level, and - since in our calculations the improvements of capital quality had a stronger effect on the stability of banks with riskier portfolios - our results suggest that after the implementation of Basel III banks with low-quality portfolios might be more stable than high-quality portfolio banks.

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APPENDIX. PROOFS.

Proof of Theorem 1. Theorem 1 is an immediate consequence of (6), (14), and (15) .

Our proof of Theorem 2 will appeal to a well-known result concerning normal distributions, which we shall present as Lemma A, and which we shall prove for the sake of completeness. Here φ is the density function of the normalized normal distribution.

Lemma A. If $a < b$,

$$-b < \frac{\varphi(b) - \varphi(a)}{\Phi(b) - \Phi(a)} < -a. \tag{32}$$

Proof. By definition,

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Hence,

$$\varphi'(x) = \frac{-x}{\sqrt{2\pi}} e^{-x^2/2} = -x\varphi(x)$$

so that whenever $a < x < b$

$$-b\varphi(x) < \varphi'(x) < -a\varphi(x)$$

Integrating from a to b , this result implies that

$$-b \int_a^b \varphi(x) dx < \int_a^b \varphi'(x) dx < -a \int_a^b \varphi(x) dx$$

However, this is equivalent with

$$-b(\Phi(b) - \Phi(a)) < \varphi(b) - \varphi(a) < -a(\Phi(b) - \Phi(a))$$

and further with

$$-b < \frac{\varphi(b) - \varphi(a)}{\Phi(b) - \Phi(a)} < -a$$

which is the result which was to be proved.

Proof of Theorem 2. Remembering that $\mu = \Phi^{-1}(\bar{p})$, we begin by concluding from (7) and (8) that the Basel II capital requirement that applies to each loan of the considered bank is

$$\lambda k_{B2}(\bar{p}) = \lambda \left(\Phi \left(\frac{\mu + \sqrt{\rho} z^*}{\sqrt{1-\rho}} \right) - \Phi(\mu) \right)$$

and that, since this requirement is by assumption positive, it must be the case that

$$\Phi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) > \Phi(\mu) \quad (33)$$

Viewing ξ as a constant and putting $\bar{z} = \bar{z}(\xi)$, we observe that according to (16), $\Phi\left(\frac{(\mu + \sqrt{\rho}\bar{z})}{\sqrt{1-\rho}}\right)$ is a weighted average of $\Phi\left(\frac{(\mu + \sqrt{\rho}z^*)}{\sqrt{1-\rho}}\right)$ and $\Phi(\mu)$. Together with (33) and the fact that Φ is an increasing function, this implies that

$$\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}} > \frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}} > \mu \quad (34)$$

We may express the condition (16) in the form

$$\Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu) = \xi \left(\Phi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) \right) + \xi \left(\Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu) \right)$$

Dividing both sides of this equation by its left-hand side, the equation is seen to be further equivalent with

$$G(\mu, \bar{z}) = 1/\xi - 1 \quad (35)$$

where

$$G(\mu, \bar{z}) = \frac{\Phi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)}{\Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu)} \quad (36)$$

As the condition (35) defines implicitly \bar{z} as a function of μ , the function G may be used for investigating the dependence of \bar{z} and μ and - since μ is an increasing function of the default probability \bar{p} of the loans and the capital insufficiency probability P_I . is a decreasing function of \bar{z} - also the dependence of \bar{p} and P_I . As our next step, we observe that (36) and the fact that Φ is an increasing function immediately imply that

$$\frac{\partial G}{\partial \bar{z}} < 0 \quad (37)$$

We wish to find out the sign of $\partial G/\partial \mu$. With this aim in mind, we first observe that

$$\frac{\partial G(\mu, \bar{z})}{\partial \mu} = G(\mu, \bar{z}) \left[\frac{1}{\sqrt{1-\rho}} \frac{\varphi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \varphi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)}{\Phi\left(\frac{\mu + \sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)} - \frac{\varphi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)/\sqrt{1-\rho} - \varphi(\mu)}{\Phi\left(\frac{\mu + \sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu)} \right]$$

$$< \frac{1}{\sqrt{1-\rho}} G(\mu, \bar{z}) \left[\frac{\varphi\left(\frac{\mu+\sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \varphi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)}{\Phi\left(\frac{\mu+\sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)} - \frac{\varphi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \varphi(\mu)}{\Phi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu)} \right]$$

Remembering (34) and putting $a = (\mu + \sqrt{\rho}\bar{z}) / \sqrt{1-\rho}$ and $b = (\mu + \sqrt{\rho}z^*) / \sqrt{1-\rho}$,

Lemma A implies that

$$\frac{\varphi\left(\frac{\mu+\sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \varphi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)}{\Phi\left(\frac{\mu+\sqrt{\rho}z^*}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right)} < -\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}$$

and, by putting $a = \mu$ and $b = (\mu + \sqrt{\rho}\bar{z}) / \sqrt{1-\rho}$ and applying Lemma A once more, also that

$$-\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}} < \frac{\varphi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \varphi(\mu)}{\Phi\left(\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}}\right) - \Phi(\mu)}$$

Combing our last three results, we may now conclude that

$$\frac{\partial G(\mu, \bar{z})}{\partial \mu} < \frac{1}{\sqrt{1-\rho}} G(\mu, \bar{z}) \left[\left(-\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}} \right) - \left(-\frac{\mu+\sqrt{\rho}\bar{z}}{\sqrt{1-\rho}} \right) \right]$$

i.e. that

$$\frac{\partial G}{\partial \mu} < 0. \quad (38)$$

The equation (35) may be viewed as defining \bar{z} as a function $\bar{z}(\mu)$ of μ . This implicit function satisfies the condition

$$G(\mu, \bar{z}(\mu)) = 1/\xi - 1 \quad (39)$$

Forming the total derivative of (39) with respect to μ and solving for $d\bar{z}/d\mu$ yields

$$\frac{d\bar{z}}{d\mu} = -\frac{\partial G(\mu, \bar{z})/\partial \mu}{\partial G(\mu, \bar{z})/\partial \bar{z}}$$

However, now we may conclude from (37) and (38) that

$$\frac{d\bar{z}}{d\mu} < 0 \quad (40)$$

By definition, $\mu = \Phi^{-1}(\bar{p})$, so that μ is an increasing function of the default probability \bar{p} , and according to (15) the capital insufficiency probability $P_I = 1 - \Phi(\bar{z})$ is a monotonously decreasing function of z . Hence, we may conclude from (40) that an increase in the default probability \bar{p} will decrease \bar{z} and increase the capital insufficiency probability P_I . This completes the proof.

Proof of Lemma 1. Assume that all banks except for bank m charge the interest rates $\bar{\mathbf{r}} = (\bar{r}_1, \dots, \bar{r}_N)$. As before, we denote the interest rates charged by bank m by $\mathbf{r} = (r_1, \dots, r_N)$. The interest rates $\bar{\mathbf{r}}$ constitute a symmetric Nash equilibrium if in the considered case it is optimal for the bank m to choose the interest rates $\mathbf{r} = \bar{\mathbf{r}}$. We wish to prove that this is the case when (27) is valid.

Consider the problem of optimizing the values of r_i ($i = 1, \dots, N$) for the given $\bar{\mathbf{r}}$. When the values r_i are chosen optimally it must be the case for each $i = 1, \dots, N$ that

$$\frac{\partial}{\partial r_i} V(\mathbf{r}, \bar{\mathbf{r}}) = 0$$

For the purposes of the proof of this lemma it turns out to be handy to introduce the function

$$\tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}) = \sum_{i=1}^N d_i(r_i, \bar{r}_i) V_i(r_i, \hat{z}) \quad (41)$$

Clearly, the net value of the bank m , which we earlier defined by (25), satisfies

$$V(\mathbf{r}, \bar{\mathbf{r}}) = \tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}(\mathbf{r}, \bar{\mathbf{r}}))$$

According to (25), a change in r_i affects $V(\mathbf{r}, \bar{\mathbf{r}})$ in two ways, via the change in \hat{z} via the direct dependence of $V(\mathbf{r}, \bar{\mathbf{r}})$ on r_i (when \hat{z} is kept fixed). We may write

$$\frac{\partial}{\partial r_i} V(\mathbf{r}, \bar{\mathbf{r}}) = \frac{\partial}{\partial r_i} \left(\tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}) \right)_{\hat{z}=\hat{z}(\mathbf{r}, \bar{\mathbf{r}})} + \frac{\partial \hat{z}(\mathbf{r}, \bar{\mathbf{r}})}{\partial r_i} \left(\frac{\partial \tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z})}{\partial \hat{z}} \right)_{\hat{z}=\hat{z}(\mathbf{r}, \bar{\mathbf{r}})} \quad (42)$$

However, according to (41), (26), and (20)

$$\tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}) = \sum_{i=1}^N d_i(r_i, \bar{r}_i) V_i(r_i, \hat{z}) = - \sum_{i=1}^N d_i(r_i, \bar{r}_i) k_i + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}} \pi(\mathbf{r}, \bar{\mathbf{r}}, z) d\Phi(z)$$

and since by definition $\pi(\mathbf{r}, \bar{\mathbf{r}}; \hat{z}(\mathbf{r}, \bar{\mathbf{r}})) = 0$, we may now conclude that

$$\left(\frac{\partial \tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z})}{\partial \hat{z}} \right)_{\hat{z}=\hat{z}(\mathbf{r}, \bar{\mathbf{r}})} = \frac{\varphi(\hat{z}(\mathbf{r}, \bar{\mathbf{r}}))}{1+\delta} \pi(\mathbf{r}, \bar{\mathbf{r}}; \hat{z}(\mathbf{r}, \bar{\mathbf{r}})) = 0.$$

Now (42) may be put in the simpler form

$$\frac{\partial}{\partial r_i} V(\mathbf{r}, \bar{\mathbf{r}}) = \frac{\partial}{\partial r_i} \left(\tilde{V}(\mathbf{r}, \bar{\mathbf{r}}, \hat{z}) \right)_{\hat{z}=\hat{z}(\mathbf{r}, \bar{\mathbf{r}})} \quad (43)$$

The result (43) states, intuitively, that as one form the derivative of $V(\mathbf{r}, \bar{\mathbf{r}})$ with respect to an interest rate r_i , one does not have to pay attention to the change in \hat{z} that the change in the interest rate causes. Using (43), (41), (18), and (26), and putting $\hat{z} = \hat{z}(\mathbf{r}, \bar{\mathbf{r}})$, we may now calculate that

$$\begin{aligned} \frac{\partial}{\partial r_i} V(\mathbf{r}, \bar{\mathbf{r}}) &= \frac{\partial}{\partial r_i} (d_i(r_i, \bar{r}_i) V_i(r_i, \hat{z})) \\ &= \left(\frac{\partial}{\partial r_i} d_i(r_i, \bar{r}_i) \right) V_i(r_i, \hat{z}) + d_i(r_i, \bar{r}_i) \left(\frac{\partial}{\partial r_i} V_i(r_i, \hat{z}) \right) \\ &= -\frac{\alpha_i}{\beta_i} V_i(r_i, \hat{z}) + \frac{d_i(r_i, \bar{r}_i)}{1+\delta} \left(\frac{\partial}{\partial r_i} \int_{-\infty}^{\hat{z}} \pi_i(r_i; z) d\Phi(z) \right) \end{aligned}$$

Applying (21), this further implies that

$$\frac{\partial}{\partial r_i} V(\mathbf{r}, \bar{\mathbf{r}}) = -\frac{\alpha_i}{\beta_i} V_i(r_i, \hat{z}) + \frac{d_i(r_i, \bar{r}_i)}{1+\delta} \left(\int_{-\infty}^{\hat{z}} (1 - p_i(z)) d\Phi(z) \right)$$

The interest rates $\bar{\mathbf{r}} = (\bar{r}_1, \dots, \bar{r}_N)$ constitute a symmetric equilibrium if this derivative is zero for each $i = 1, \dots, N$ when $\mathbf{r} = \bar{\mathbf{r}}$. Since according to (18)

$$d_i(\bar{r}_i, \bar{r}_i) = \frac{\alpha_i}{M}$$

a symmetric equilibrium obtains if for each $i = 1, \dots, N$

$$-\frac{\alpha_i}{\beta_i} V_i(\bar{r}_i, \hat{z}) + \frac{\alpha_i}{(1+\delta)M} \left(\int_{-\infty}^{\hat{z}} (1 - p_i(z)) d\Phi(z) \right) = 0$$

However, this is equivalent with (27), since we are now considering the case in which $\mathbf{r} = \bar{\mathbf{r}}$ and $\hat{z} = \hat{z}(\mathbf{r}, \bar{\mathbf{r}})$.

Finally, $\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})$ is according to (23) and (20) determined by

$$\pi(\bar{\mathbf{r}}, \bar{\mathbf{r}}, \hat{z}) = \sum_{i=1}^N d_i(\bar{r}_i, \bar{r}_i) \pi_i(\bar{r}_i, \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})) = 0$$

According to (18), this is equivalent with

$$\sum_{i=1}^N \frac{\alpha_i}{M} \pi_i(r_i; \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})) = 0$$

which is further equivalent with (28). This completes the proof.

Proof of Lemma 2. Taking the limit $M \rightarrow \infty$ on both sides of (27), it follows that the equilibrium interest rates $\bar{\mathbf{r}} = (\bar{r}_1, \bar{r}_2, \dots, \bar{r}_N)$ satisfy

$$\lim_{M \rightarrow \infty} V_i(\bar{r}_i, \hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})) = 0.$$

This is according to definition (26) equivalent with

$$\lim_{M \rightarrow \infty} \left(-k_i + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}(\bar{\mathbf{r}}, \bar{\mathbf{r}})} \pi_i(\bar{r}_i, z) d\Phi(z) \right) = 0$$

and further, also equivalent with

$$\lim_{M \rightarrow \infty} \frac{1}{1+\delta} \int_{-\infty}^{\widehat{z}(\bar{\mathbf{r}}, \bar{\mathbf{F}})} \pi_i(\bar{r}_i; z) d\Phi(z) = k_i$$

which is the result which was to be proved.

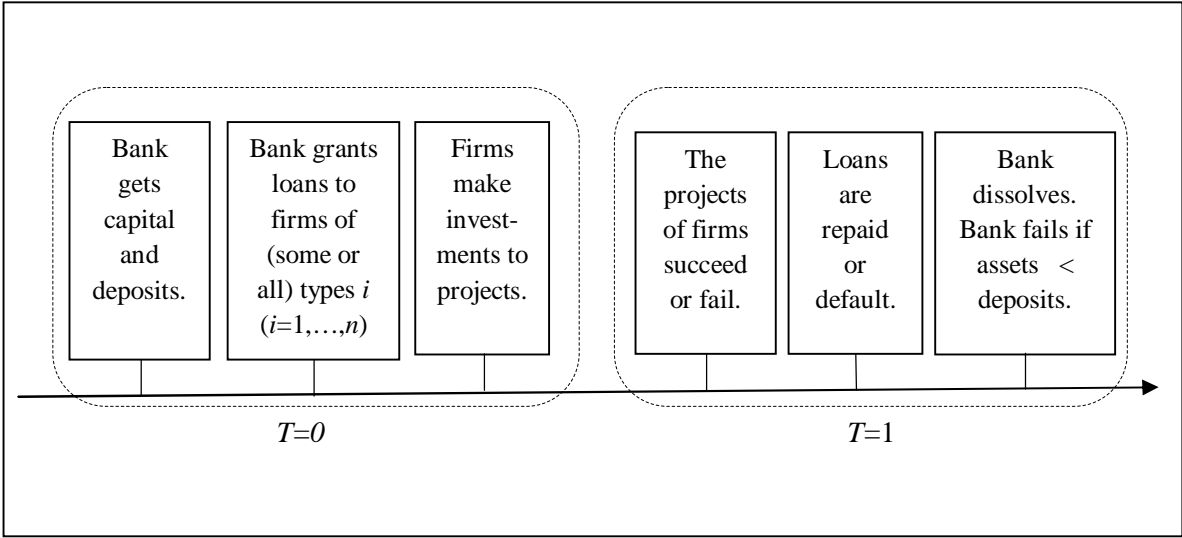


Figure 1. Time line.

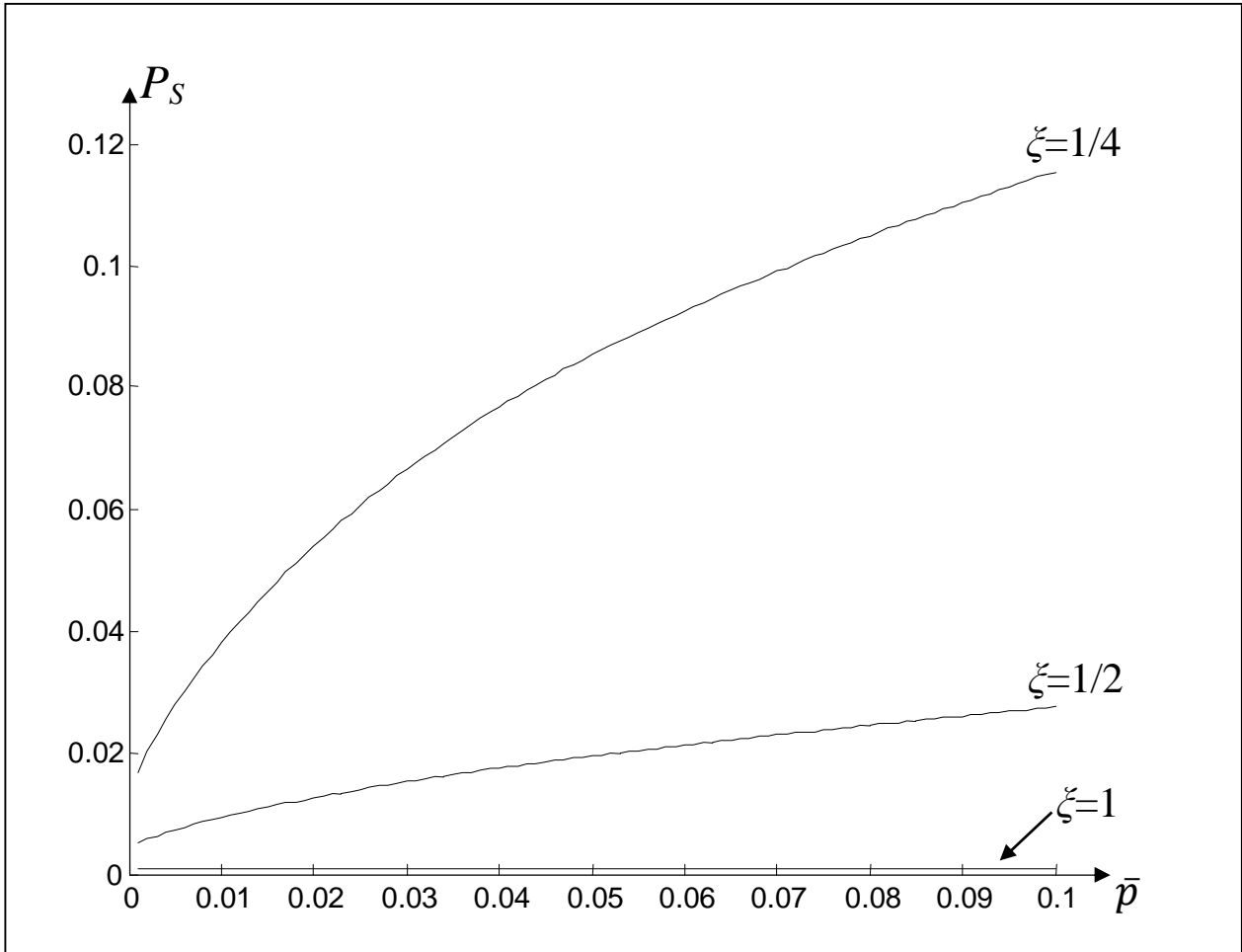


Figure 2. The probability of capital insufficiency of a specialized bank as a function of loan default a probability for selected values of ξ .

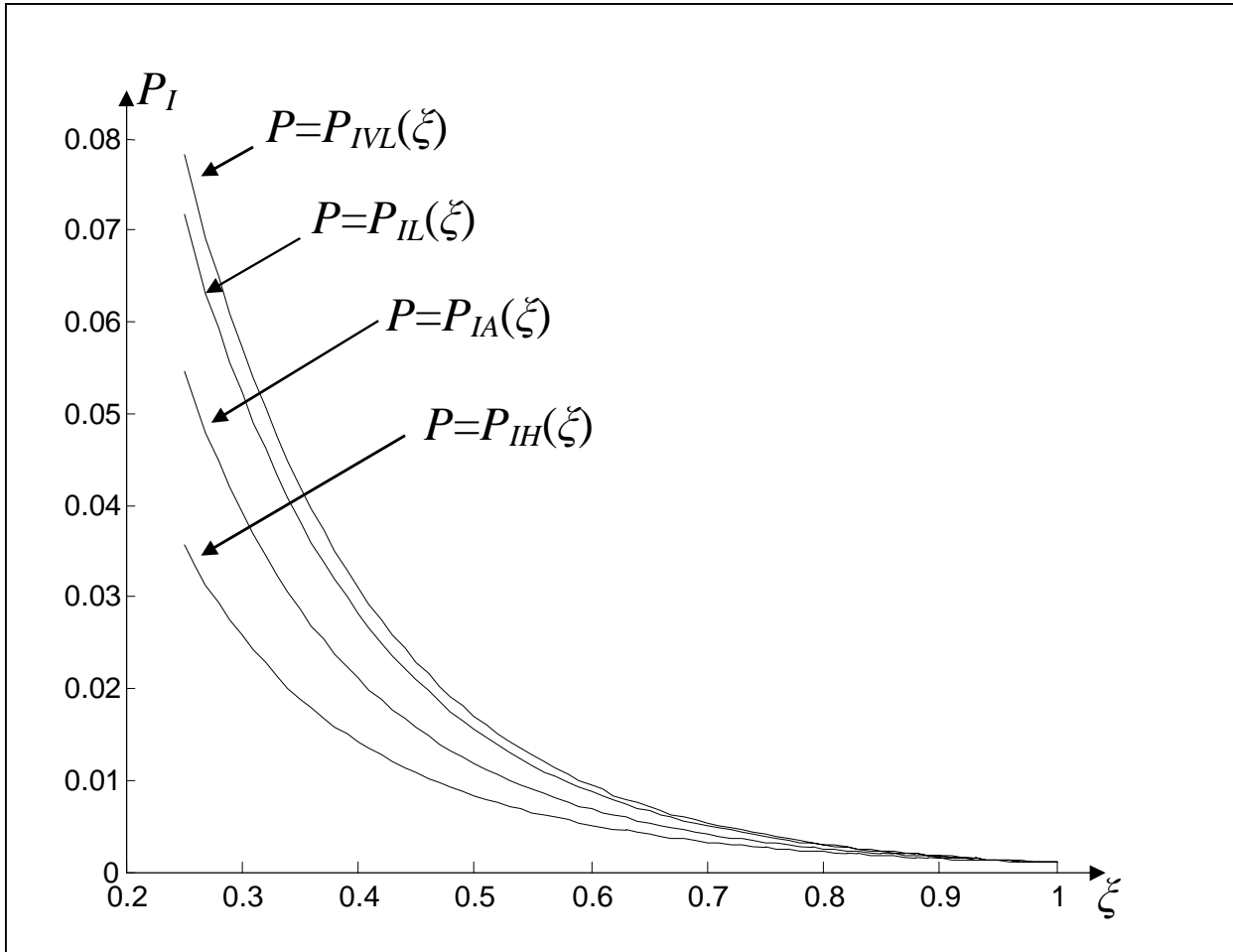


Figure 3. The probability of capital insufficiency as a function of ξ for selected portfolios.

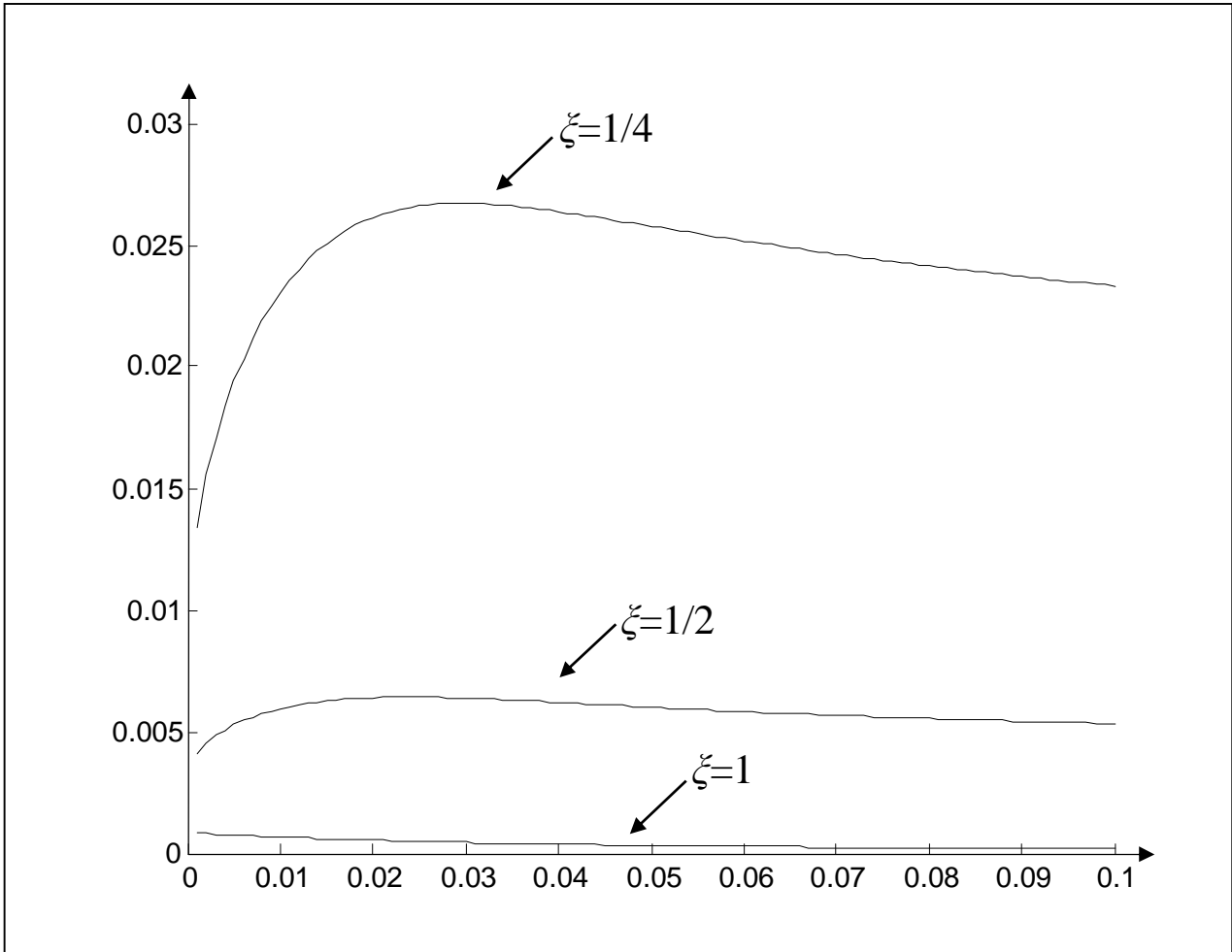


Figure 4. The failure probability of a specialized bank as a function of loan default probability for selected values of ξ .

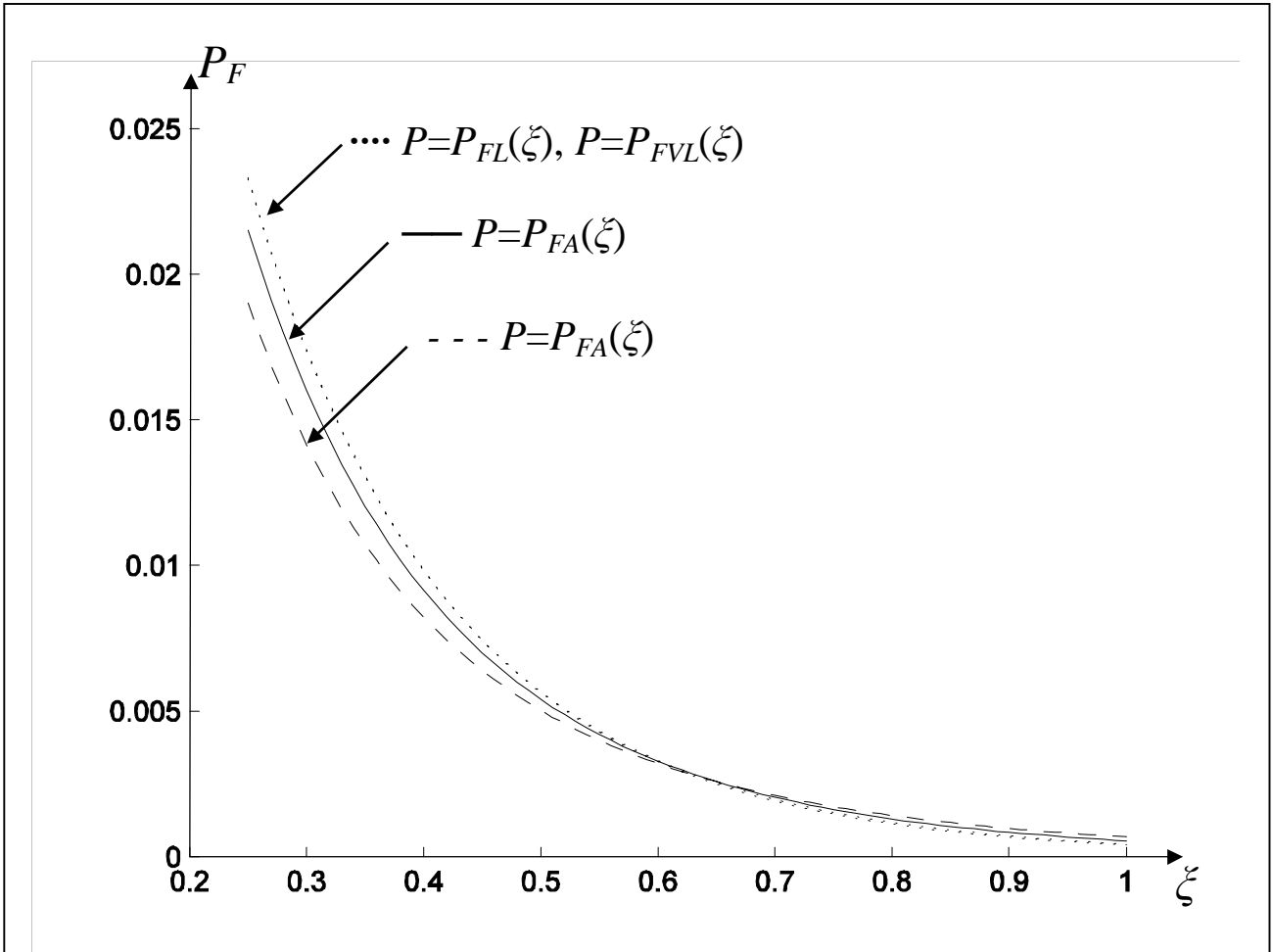


Figure 5. The probability of bank insolvency as a function of ζ for selected portfolios.

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