

Kiema, Ilkka; Jokivuolle, Esa

**Working Paper**

## Leverage ratio requirement and credit allocation and bank stability

Bank of Finland Research Discussion Papers, No. 10/2011

**Provided in Cooperation with:**

Bank of Finland, Helsinki

*Suggested Citation:* Kiema, Ilkka; Jokivuolle, Esa (2011) : Leverage ratio requirement and credit allocation and bank stability, Bank of Finland Research Discussion Papers, No. 10/2011, Bank of Finland, Helsinki,  
<https://nbn-resolving.de/urn:nbn:fi:bof-20140807427>

This Version is available at:

<https://hdl.handle.net/10419/212189>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



Ilkka Kiema – Esa Jokivuolle

# **Leverage ratio requirement, credit allocation and bank stability**




EUROJÄRJESTELMÄ  
EUROSYSTEMET

Bank of Finland Research  
Discussion Papers  
10 • 2011

**Suomen Pankki  
Bank of Finland  
PO Box 160  
FI-00101 HELSINKI  
Finland  
☎ +358 10 8311**

**<http://www.suomenpankki.fi/en>  
E-mail: [Research@bof.fi](mailto:Research@bof.fi)**



Ilkka Kiema\* – Esa Jokivuolle\*\*

## Leverage ratio requirement, credit allocation and bank stability

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Finland.

\* Department of Political and Economic Studies, PO Box 17, 00014 University of Helsinki, Finland. E-mail: ilkka.kiema@helsinki.fi.

\*\* Aalto University School of Economics, Department of Finance and Bank of Finland, Monetary Policy and Research Department. Email: esa.jokivuolle@bof.fi. Corresponding author.

We would like to thank Ari Hyytinen, Karlo Kauko, Tuomas Takalo, Jukka Vauhkonen, Timo Vesala, Jouko Vilmunen and participants at the Bank of Finland research workshop and the Finnish Economic Association meeting 2011 for their valuable comments. An early version of this paper, ‘Leverage ratio requirement and credit allocation under Basel III’, has been available at SSRN: <http://ssrn.com/abstract=1723145>.

<http://www.suomenpankki.fi/en>

ISBN 978-952-462  
ISSN 0785-3572  
(print)

ISBN 978-952-462  
ISSN 1456-6184  
(online)

Helsinki 2011

# Leverage ratio requirement, credit allocation and bank stability

Bank of Finland Research  
Discussion Papers 10/2011

Ilkka Kiema – Esa Jokivuolle  
Monetary Policy and Research Department

## Abstract

We study the effects on credit allocation and bank stability of introducing a leverage ratio requirement (LRR) on top of risk-based capital requirements, as in Basel III. For the current 3% LRR, both low-risk and high-risk loan rates and volumes remain essentially unchanged, because banks previously specializing in low-risk lending can adapt by granting both low-risk and high-risk loans. For sufficiently high LRRs, low-risk lending rates would significantly increase and high-risk lending rates would fall. In the presence of severe ‘model risk’ concerning low-risk loans, as happened in the subprime crisis, the current 3% LRR might even reduce bank stability, counter to regulatory intentions. This is because the allocational effect caused by the LRR, which makes bank loan portfolios more alike, may turn beneficial risk spreading into harmful risk contamination. For higher levels of LRR, however, bank stability is likely to be improved even in the presence of model risk.

Keywords: bank regulation, Basel III, capital requirements, credit risk, leverage ratio

JEL classification numbers: D41, D82, G14, G21, G28

# Vähimmäisomavaraisuusaste, luottojen kohdentuminen ja pankkien vakaus

Suomen Pankin keskustelualoitteita 10/2011

Ilkka Kiema – Esa Jokivuolle  
Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tässä keskustelualoitteessa tutkitaan kalibroidulla teoreettisella mallilla vähimmäisomavaraisuusasteen (VOVA) vaikutuksia pankkiluottojen kohdentumiseen ja pankkien vakauteen. Vähimmäisomavaraisuusaste on Basel III -säännöksiin sisältyvä lisävaatimus, joka asetetaan riskiperusteisten pääomavaatimusten täydennykseksi. Tämänhetkinen 3 prosentin VOVA ei mallin mukaan juuri vaikuttaisi luottokorkoihin eikä luotonannon määriin. Tämä johtuu siitä, että pankit, jotka ovat aikaisemmin erikoistuneet joko vähäisen tai suuremman riskin luototukseen, voivat sopeutua tilanteeseen myöntämällä tasaisemmin riskiltään erilaisia luottoja. Jos vähimmäisomavaraisuusaste olisi huomattavasti korkeampi, vähäisen riskin luottojen korot nousisivat merkittävästi ja suuremman riskin luottojen korot laskisivat. Jos vähäisen riskin luottoihin sisältyy riittävän suuri ”malliriski”, kuten ns. subprime-kriisissä, 3 prosentin vähimmäisomavaraisuusaste voi jopa heikentää pankkien vakautta vastoin Baselin komitean tavoitteita. Tämä johtuu siitä, että VOVAn aiheuttama pankkien luottosalkkujen samankaltaistuminen voi tällöin toimia pikemminkin sokkien välityskanavana kuin riskien hajauttajana. Jos vähimmäisomavaraisuusaste olisi sen sijaan huomattavasti korkeampi kuin nykyinen 3 %, se todennäköisesti lisäisi pankkien vakautta myös malliriskin tapauksessa.

Avainsanat: pankkien sääntely, Basel III, pääomavaatimukset, luottoriski, omavaraisuusaste

JEL-luokittelu: D41, D82, G14, G21, G28

# Contents

Abstract.....	3
Tiivistelmä (abstract in Finnish) .....	4
<b>1 Introduction.....</b>	<b>7</b>
<b>2 The Model .....</b>	<b>11</b>
<b>3 The Equilibria.....</b>	<b>16</b>
<b>4 The Welfare Function .....</b>	<b>25</b>
<b>5 A Calibrated Version of the Model.....</b>	<b>28</b>
5.1 Model Risk .....	32
<b>6 Concluding Remarks .....</b>	<b>35</b>
Appendix.....	37
References.....	39





# 1. Introduction

The new Basel III framework contains a leverage ratio requirement, which has been added to the earlier Basel II framework to supplement risk-based minimum capital requirements on banks. According to the current leverage ratio requirement, banks must have a minimum of three percent of capital of non-risk-weighted total assets, including off-balance sheet items (see Basel Committee on Banking Supervision, 2009).<sup>1</sup>

The Basel Committee on Banking Supervision (2009) argues that the leverage ratio requirement would “help contain the build up of excessive leverage in the banking system, introduce additional safeguards against attempts to game the risk based requirements, and help address model risk”. The global financial crisis has indeed shown that many items on banks’ trading books and off-balance sheet received very low risk-weights under Basel II but turned out to have substantial risk in the crisis (see e.g. Acharya et al., 2009). Such an outcome may have been a manifestation both of “gaming” the risk-based capital requirements by shifting assets from the banking book to the trading book or off-balance sheet, and of “model risk” embedded in the theory-based risk-weights of Basel II. The leverage ratio requirement would hence set an all-encompassing “floor” to minimum capital requirements, which would limit the potential erosive effects of gaming and model risk on capital against true risks.

The leverage ratio requirement has also been criticized for interfering with the basic idea of risk-sensitive capital requirements, which is to align minimum capital requirements with banks’ true asset risks and hence promote efficient credit allocation. According to this argument, an additional leverage ratio requirement would make the effective capital requirement on low-risk assets too high<sup>2</sup>. This could lead to risk-shifting from low-risk to higher-risk assets.<sup>3</sup>

The purpose of this paper is to study the effects of the combination of a leverage ratio requirement and risk-based IRB (internal ratings based) capital requirements, already introduced in Basel II, on loan pricing and loan allocation. In addition, we consider a simple form of model risk in order to analyze the

---

<sup>1</sup>It might be more logical to talk about a capital to assets ratio requirement or an inverse of a leverage ratio requirement. For simplicity, however, we henceforth use the term leverage ratio requirement keeping in mind that it in actuality it is imposed in terms of a minimum capital to assets ratio.

<sup>2</sup>Parts of the financial industry, e.g. municipality finance in Europe, have been concerned about the effects of the leverage ratio requirement on their low-risk lending.

<sup>3</sup>A very different view is provided by Hellwig (2010) who argues for a leverage ratio requirement which would set banks’ capital at well beyond ten percent of non-risk-weighted total assets; perhaps even to the 20 to 30 percent range. Such a leverage ratio requirement should replace risk-based capital requirements which in themselves according to Hellwig (2010) spur capital arbitrage which can further spur high leverage and excessive risk-taking by banks. A sufficiently high capital to assets ratio would in contrast provide a robust buffer against even very high losses and promote good corporate governance in banks by raising bank shareholders’ stake sufficiently high. Hellwig’s (2010) view on limiting gaming the risk-based capital requirements with the leverage ratio requirement appears to be in line with the view of the Basel Committee (2009), but his recommended level of the leverage ratio requirement is much higher than the one currently opted for by the Basel Committee.

effect on bank failures of introducing the leverage ratio requirement. It turns out that in the presence of model risk the allocational effects of the leverage ratio requirement, emphasized in the current paper, have a subtle effect on the stability of the banking sector, which may well be negative.

With the exception of Blum (2008), previous literature has not, to the best of our knowledge, considered the joint effect of a leverage ratio requirement and risk-based capital requirements.<sup>4</sup> We use the framework of Repullo and Suarez (2004) who study the loan pricing and loan allocation effects of the Basel II reform in a competitive banking sector. Basel II introduced two options to banks for determining their capital requirements against loans: the IRB approach and the standardized approach. In the case of unrated customers the latter option effectively reduces to a leverage ratio type of requirement. While in the analysis of Repullo and Suarez (2004) banks choose which option to follow, in our version of their model, motivated by Basel III, banks are simultaneously subject to both the IRB requirement and a leverage ratio requirement. We analyze how the joint requirements affect loan pricing and loan allocation across different risk categories of loans as well as across banks<sup>5</sup>. Following Repullo and Suarez (2004), the model has only two loan categories, low-risk loans and high-risk loans, but it is consistent with the credit portfolio theory underlying the IRB capital requirements. Hence we can calibrate the model to data in order to obtain suggestive quantitative results.

As Repullo and Suarez (2004) show, when the IRB requirements are the only capital requirements in the model, banks have an incentive to specialize in either low-risk or high-risk lending.<sup>6</sup> When introducing the leverage ratio requirement we find three different cases (equilibria) of primary interest depending on where the leverage ratio requirement is located in between the low-risk loan's capital requirement and the high-risk loan's capital requirement. We only consider banks which under Basel II would have chosen the IRB approach to determine their capital requirements. We believe this is the most relevant case in practice because most of the large and sophisticated banks are likely to follow the IRB approach, not least because of supervisory expectations to do so.

---

<sup>4</sup>Blum (2008) presents a model in which a leverage ratio requirement can restore banks' incentives to "truth-telling" in setting the internal ratings which form the basis for risk-based capital requirements. This type of rationale might be generally used to motivate the gaming and model risk based arguments for the leverage ratio requirement, stated by the Basel Committee.

<sup>5</sup>We note that our results critically rely on the assumption that equity is a more expensive form of finance for banks than deposits which in the current model are the other source of finance for banks. This assumption is quite standard in the banking literature and the reasons for the extra premium on banks' equity are discussed e.g. by Repullo and Suarez (2004). However, recently Hellwig (2010) and Admati et al. (2010) have analyzed the reasons why this extra cost should not be exaggerated and why it is critical to make a clear distinction between the private and social costs (or benefits) of bank capital. Nonetheless, as Admati et al. (2010) point out, demand deposits can be understood as being part of a bank's "production function" and hence deposits, as opposed to equity, have a relative advantage as a form of finance for banks.

<sup>6</sup>A specialized banking market has some empirical relevance. In addition to the aforementioned municipality finance companies, for example cooperative banks, which are important in many European countries, often appear to hold quite low-risk loan portfolios.

We label the three possible types of equilibria A, B, and C. It turns out that, subject to plausible restrictions on the parameter values, one of the three kinds of equilibria exists for each value of the leverage ratio requirement between the capital requirements for the low-risk and high-risk loans. An equilibrium of type A exists when the leverage ratio requirement is above but sufficiently close to the low-risk loan's capital requirement. Similarly, an equilibrium of type C exists when the leverage ratio requirement is below but sufficiently close to the high-risk loan's capital requirement, and there must also be a range in the middle, in which an equilibrium of type B exists.

In equilibrium A, there are specialized high-risk loan banks just like in the absence of the leverage ratio requirement, and mixed portfolio banks which hold both low-risk and high-risk loans. The reason for the emergence of the mixed portfolio banks is the following. A mild leverage ratio requirement is a binding capital constraint on low-risk portfolio banks, making specialized low-risk lending unprofitable in the competitive banking sector. The banking sector can adjust to this situation as follows. The number of specialized high-risk portfolio banks is reduced and more banks start granting both low-risk and high-risk loans. So intuitively we can think that low-risk portfolio banks can restore their profitability (i.e., zero profits) by replacing some low-risk loans with high-risk loans<sup>7</sup>. The interest rate on low-risk loans also increases from the Basel II world and hence reduces the demand for low-risk loans. However, simulation results with our calibrated model indicate that these effects remain very small. High-risk loan rates and lending volume remain unchanged. In sum, our analysis suggests that a mild leverage ratio requirement, like the current 3%, leaves all lending rates and lending volumes virtually intact, provided that banks can mix both low-risk and high-risk loans in their portfolios.

However, the re-shuffling of low-risk and high-risk loans across banks may have important implications for bank stability, in particular in the presence of model risk. If there is an unanticipated positive shock to the default probability of the low-risk assets (like there seems to have been in the case of the subprime crisis), then the number of bank failures may either decrease or increase, relative to the Basel II world, depending on the size of the shock.<sup>8</sup> This results from the fact that the number of banks granting low-risk loans has increased. This helps to diversify the model risk shock across the banking sector if the shock is not

---

<sup>7</sup>It should be observed that since both the original Repullo-Suarez (2004) model and the current model describe games in which only a single round is played, one cannot strictly speaking claim that in the model a bank would change its strategy when the leverage ratio requirement is introduced. Nevertheless, one may intuitively think that the mixed-portfolio banks of equilibrium A are former low-risk loan banks which have included some high-risk loans in their portfolio. We will use this intuitive way of talking also in the discussion of other equilibria below.

<sup>8</sup>Gennaioli et al. (2011) argue that neglected risks, as a result of what they call "local thinking", and investor preference for safe assets may have led to the supply of seemingly low-risk assets such as those based on subprime loans, whose true risks are effectively unanticipated. Their theory can hence be used to motivate our simple modelling of the model risk, especially in the low-risk loans. However, we have also considered a positive shock to the default probability of high-risk loans and find that in that case the number of bank failures, relative to the Basel II world, would (almost) always increase.

too big. However, if the shock is sufficiently high, then the diversification effect turns into a contamination effect so that a larger number of banks will fail.<sup>9</sup> Simulations with a calibrated version of our model suggest that the size of the threshold value of the shock to low-risk loans' default probability, which turns the diversification effect into the contamination effect is not unreasonable, given the experience from the subprime crisis. In sum, our results suggest that it is not obvious that the argument of the Basel Committee that the leverage ratio requirement can address the model risk embedded in risk-based capital requirements, is justified. The allocational effects of the leverage ratio requirement, which the Basel III text does not really discuss, have the effect that change in bank stability can go either way, depending on whether model risk concerns low-risk or high-risk loans as well as on the size of model risk.

Our simulations suggest that with the current 3% leverage ratio requirement equilibrium of type A is the most likely. This equilibrium is possible only when the leverage ratio requirement is equal to or smaller than the average risk-based capital requirement of the whole banking sector. If this is not the case, the low-risk banks can no longer adjust to the leverage ratio requirement by taking on high-risk loans, so an equilibrium of either type B or of type C must obtain.

The nature of the equilibrium of type C is easier to grasp intuitively. In this equilibrium there are specialized low-risk loan banks whose amount of capital is determined by the leverage ratio requirement. The low-risk interest rate has risen up to the level where low-risk loan banks yield zero profits. There are also mixed-portfolio banks which one may view as former high-risk loan banks that have switched to a mixed portfolio because of the increased low-risk interest rate. When the banking sector is competitive, also these banks must make zero profits, so the increase in the low-risk loans' interest rate has the effect of lowering the high-risk loans' interest rate from the Basel II level.

There is also a region of leverage ratio requirement values for which neither of the equilibria A and C is possible (according to the calibrated version of the model, this range is quite narrow). In this range the model has a symmetric equilibrium, which we call type B, in which all the banks have the same mixed portfolio. In equilibrium B, interest rates are between the values which would correspond to the equilibria of types A and C.

For higher leverage ratio requirements, such as in equilibria B and C, it becomes increasingly likely that bank stability is improved also in the presence of model risk. This is because a high leverage ratio requirement simply provides a sufficient buffer against high losses. This direct safety benefit dominates any allocational effects which may threaten to contaminate the banking sector with unanticipated risks.

One may argue that a regulatory constellation resembling the one studied in our model prevailed in the U.S prior to the global financial crisis. Basel II was never really implemented in the U.S. and the Basel I which was still followed contained features, especially the risk-insensitive capital requirements,

---

<sup>9</sup>Our results may be generally related to the "correlated portfolios" argument as a source of bank contagion, see e.g. Acharya (2009).

which resemble the leverage ratio requirement. At the same time, markets may have expected the leading banks to reserve capital according to their internal risk models; a practice often referred to as reserving and internally allocating economic capital. As a result, markets may have imposed risk-based capital "requirements" on banks, at least with respect to "known" risks, even if regulators formally did not. We shall reflect on the US developments prior to and during the subprime crisis in the light of our model.

The rest of the paper is organized as follows. We first recapitulate the main features of the Repullo-Suarez (2004) model in Section 2 and present a generalized version of the model in which the banks are subject to a leverage ratio requirement. In Section 3 we shall discuss the three kinds of equilibria that the generalized model may have. In Section 4, we present a welfare function for our model, and in Section 5 we discuss a calibrated version of the model. In the concluding section 6 we discuss the applicability of our results to the banking sector of the United States before the subprime crisis.

## 2. The Model

There is a banking sector which finances two kinds of firms, which we shall label low-risk ( $L$ ) firms and high-risk ( $H$ ) firms. Both kinds of firms need investments of the same size for their projects. We shall refer to these projects as low-risk and high-risk projects, respectively, and we shall normalize the needed investment to 1. The portfolio of a bank is characterized by the share of the high-risk projects among all the projects that it finances, and below we shall say that a bank has portfolio  $\alpha$  when this share is  $\alpha$ .

Each bank finances the loans that it grants partially by capital and partially by deposits. The amount of capital per loan that the bank holds will below be denoted by  $k$ . We assume that deposits are publicly insured and hence earn riskless interest rate which is normalized to zero. Moreover, deposit insurance premium is assumed to be zero in the presence of capital requirements (cf. Repullo and Suarez 2004). Discussion on the cost to the government of the deposit insurance is postponed to section 4 where we study the welfare implications of our model.

By assumption, the equity capital of the banks has an expected cost  $\delta$  over the riskless interest rate. The justifications for this assumption, usually given in the literature, were discussed in footnote 5 above. The banking sector is competitive in the sense that the net value of each bank is zero.

The banks are subject to two capital requirements. One of them is a Basel II type, risk-based requirement which states that a part  $k_\eta$  of each loan of the category  $\eta$  ( $\eta = L, H$ ) must be funded by capital. For a bank with portfolio  $\alpha$ , the Basel II requirement states that the amount of capital per loan must satisfy

$$k \geq k_2(\alpha) \tag{1}$$

where

$$k_2(\alpha) = (1 - \alpha)k_L + \alpha k_H \quad (2)$$

The other one is a leverage ratio requirement which states that the bank must have the amount  $k_{lev}$  of capital per loan. Together, they amount up to the requirement that the amount of capital  $k$  per loan of the bank must satisfy

$$k \geq \kappa(\alpha) \quad (3)$$

where

$$\kappa(\alpha) = \max\{(1 - \alpha)k_L + \alpha k_H, k_{lev}\} \quad (4)$$

With simple algebra, it is observed that  $\kappa(\alpha) = k_{lev}$  if and only if  $\alpha \leq \alpha_{lev}$ , where

$$\alpha_{lev} = \frac{k_{lev} - k_L}{k_H - k_L} \quad (5)$$

and that  $\kappa(\alpha) = k_2(\alpha)$  if and only if  $\alpha \geq \alpha_{lev}$ .

**[Figure 1]**

Figure 1 shows a Basel III type capital requirements as a function of the portfolio  $\alpha$  (solid line) and contrasts it with the corresponding Basel II type requirement (broken line), which does not include a leverage ratio requirement. When  $\alpha < \alpha_{lev}$ , the leverage ratio requirement is the binding constraint and the Basel III requirement is stronger than the corresponding Basel II requirement, but when  $\alpha > \alpha_{lev}$ , the Basel II type requirement is binding. The value  $\alpha_{lev}$  given by (3) is the only value of  $\alpha$  for which both constraints are binding.

The demand  $n_\eta$  for loans of each category  $\eta$  ( $\eta = L, H$ ) is identical with the number of the firms of category  $\eta$  which choose to make an investment, and it is a non-increasing function of the interest rate  $r_\eta$  for the loans of category  $\eta$ . In other words,

$$\frac{\partial}{\partial r_\eta} n_\eta(r_\eta) \leq 0 \quad (6)$$

The projects chosen by the firms can either succeed or fail. A successful project produces  $1 + a$ , of which the bank receives  $1 + r_\eta$ , but if a project is unsuccessful, it produces only  $1 - \lambda$ . In this case the firm defaults and the bank receives  $1 - \lambda$ , so that  $\lambda$  expresses the loss given default of the bank.

The success probability of the project of a firm  $i$  is characterized the random variable  $\varepsilon_i$  which is defined by

$$x_i = \mu_i + \sqrt{\rho}z + \sqrt{1 - \rho}\varepsilon_i \quad (7)$$

and the project defaults if  $x_i > 0$ . Here  $z \sim N(0, 1)$  is the systematic risk factor, and the random variables  $\varepsilon_i \sim N(0, 1)$  are independent of each other and of  $z$ . The value of  $\mu_i$  is equal with the constant  $\mu_L$  for low-risk projects, and with the constant  $\mu_H$  for the high-risk projects.

It is easy to see that the unconditional default probability  $\bar{p}_\eta$  of the projects of type  $\eta$  ( $\eta = L, H$ ) is given by

$$\bar{p}_\eta = \Phi(\mu_\eta) \quad (8)$$

Consider now the success probabilities of the projects when the systematic risk factor  $z$  has been realized. The above assumptions imply that for a given value of  $z$  the default probability  $p_\eta$  of a project  $i$  of type  $\eta$  ( $\eta = L, H$ ) is

$$p_\eta(z) = P(\mu_\eta + \sqrt{\rho}z + \sqrt{1-\rho}\varepsilon_i > 0)$$

This is equivalent with

$$p_\eta(z) = P\left(\varepsilon_i > -\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\mu_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) \quad (9)$$

When the amount of capital that the bank has per loan has the value  $k$ , the part of the loan that the bank has funded with deposits is  $1 - k$ . In this case the net worth per loan of a bank with the portfolio  $\alpha$  is given by

$$\begin{aligned} \pi(k, \alpha, r_L, r_H; z) = & (1 - \alpha)((1 - p_L(z))(1 + r_L) + p_L(z)(1 - \lambda)) \\ & + \alpha((1 - p_H(z))(1 + r_H) + p_H(z)(1 - \lambda)) - (1 - k) \end{aligned}$$

This simplifies to the form

$$\begin{aligned} \pi(k, \alpha, r_L, r_H; z) = & k + (1 - \alpha)(r_L - p_L(z)(\lambda + r_L)) \\ & + \alpha(r_H - p_H(z)(\lambda + r_H)) \end{aligned} \quad (10)$$

Following Repullo-Suarez (2004), we shall formulate the equilibrium conditions of the model for a bank of a unit size. Given that the interest rate for bank capital is  $\delta$ , the net value of a bank of unit size with the portfolio  $\alpha$  is

$$V(k, \alpha, r_L, r_H) = -k + \frac{1}{1 + \delta} \Pi(k, \alpha, r_L, r_H) \quad (11)$$

where

$$\Pi(k, \alpha, r_L, r_H) = \int_{-\infty}^{\hat{z}_\alpha} \pi(k, \alpha, r_L, r_H; z) d\Phi(z) \quad (12)$$

and  $\hat{z}_\alpha$  is the value of  $z$  for which integrand becomes zero. Intuitively, if  $z > \hat{z}_\alpha$ , the liabilities of the bank are larger than its assets. In this case the bank will fail and be of a zero net worth (rather than negative net worth) to its owners.

Since the banking sector is competitive, the equilibrium conditions of the banking sector state that the value of  $V$  must be zero for each of the choices of  $k$  and  $\alpha$  that banks make, i.e.

$$V(k, \alpha, r_L, r_H) = 0 \quad (13)$$



that there is no legitimate choice of  $k$  and  $\alpha$  that would yield a positive net value for the bank, and that the banks cover the loan market, i.e. that the number of low-risk loans granted by the banks is  $n_L(r_L)$  and that the number of high-risk loans granted by the banks is  $n_H(r_H)$ .

It is easy to see that when the net value of a bank is given by (11), it is optimal for the banks to choose the minimum amount of capital which is allowed by the capital requirement. This result can be proved by concluding from (10), and (11), (12) that

$$\begin{aligned} \frac{\partial}{\partial k} V(k, \alpha, r_L, r_H) &= \partial \left( -k + \frac{1}{1+\delta} \Pi(k, \alpha, r_L, r_H) \right) \\ &= -1 + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}_\alpha} d\Phi(z) < -1 + \frac{1}{1+\delta} < 0 \end{aligned}$$

Hence, for each portfolio  $\alpha$ , it is in the interest of the banks to have the minimum amount of capital  $\kappa(\alpha)$  allowed by the capital requirements (cf. Repullo and Suarez, p. 502).

Repullo and Suarez prove that in the Basel II regime it is optimal for the banks to specialize in either low-risk or high-risk loans.<sup>10</sup> Hence, in the Basel II regime, each bank has either of the two portfolios  $\alpha = 0$  and  $\alpha = 1$ , and the interest rates are determined by the condition that the net value of the banks with these portfolios is zero. I.e., the interest rates have the values  $\bar{r}_L$  and  $\bar{r}_H$  which are determined by

$$\begin{cases} V_L(k_L, \bar{r}_L) = 0 \\ V_H(k_H, \bar{r}_H) = 0 \end{cases} \quad (14)$$

where  $V_L$  and  $V_H$  are the values of  $V$  which correspond to the profiles  $\alpha = 0$  and  $\alpha = 1$ . In other words, the functions  $V_L$  and  $V_H$  are given by

$$\begin{cases} V_L(k_L, r_L) = -k_L + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}_0} (k_L + r_L - p_L(z)(\lambda + r_L)) d\Phi(z) \\ V_H(k_H, r_H) = -k_H + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}_1} (k_H + r_H - p_H(z)(\lambda + r_H)) d\Phi(z) \end{cases} \quad (15)$$

so that

$$V_L(k_L, r_L) = V(k, 0, r_L, r) \text{ and } V_H(k_H, r_H) = V(k, 1, r, r_H) \text{ for all values of } r.$$

The conclusion that each bank has one of the two profiles  $\alpha = 0$  or  $\alpha = 1$  is based on the fact that under a Basel II type capital requirement the net value  $V(k_2(\alpha), \alpha, r_L, r_H)$  of a bank is a convex function of  $\alpha$ . Such convexity follows from the fact that  $k_2(\alpha)$  is a linear function of  $\alpha$ . As Figure 1 shows, the Basel III type capital requirement is a linear function of  $\alpha$  in the interval  $[0, \alpha_{lev}]$  and

<sup>10</sup> More rigorously, they show that in equilibrium the net value of a mixed-portfolio bank can never be larger than the sum of the net values of a low-risk loan bank and a high-risk loan bank with the same assets and liabilities, and that the value of the mixed-portfolio bank is smaller, with the exception of a very special choice of parameter values. (It should be observed that the net value of the mixed portfolio bank is not necessarily smaller if the number  $\hat{z}$  which appears in (6) in Repullo-Suarez (2004), 502, is identical for the low-risk loan bank and the high-risk loan bank.)

in the interval  $[\alpha_{lev}, 1]$ , when these intervals are considered separately, but not in the whole interval  $[0, 1]$ . This leads to the following analogy of Repullo and Suarez's result.

**Lemma 1.** Keeping the interest rates  $r_L$  and  $r_H$  fixed, under the Basel III capital requirement (3) the net value of a bank with portfolio  $\alpha$  for which  $0 < \alpha < \alpha_{lev}$  satisfies

$$V(\kappa(\alpha), \alpha, r_L, r_H) \leq \frac{\alpha_{lev} - \alpha}{\alpha_{lev}} V(\kappa(0), 0, r_L, r_H) + \frac{\alpha}{\alpha_{lev}} V(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H)$$

and the net value of a bank with portfolio  $\alpha$  for which  $\alpha_{lev} < \alpha < 1$  satisfies

$$V(\kappa(\alpha), \alpha, r_L, r_H) \leq \frac{1 - \alpha}{1 - \alpha_{lev}} V(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H) + \frac{\alpha - \alpha_{lev}}{1 - \alpha_{lev}} V(\kappa(1), 1, r_L, r_H)$$

These convexity results are valid with strict inequality, except for a specific combination of parameter values.<sup>11</sup> When the inequalities of Lemma 1 are valid with strict inequality, one may immediately conclude that the only profiles that the banks can have in equilibrium are  $\alpha = 0$ ,  $\alpha = \alpha_{lev}$  and  $\alpha = 1$ . (The right-hand sides of the two inequalities in Lemma 1 must be non-positive, since there cannot be portfolios with positive net value in equilibrium, and the strict inequality implies that  $V(\kappa(\alpha), \alpha, r_L, r_H) < 0$  when  $0 < \alpha < \alpha_{lev}$  or  $\alpha_{lev} < \alpha < 1$ .) If one does not wish to make the restrictive assumption of strict inequality in Lemma 1, one must rest content with the following weaker conclusion which also follows from Lemma 1.

**Corollary 1.** If the model has an equilibrium, it also has an equilibrium with the same interest rates in which the only profiles that are chosen by the banks are  $\alpha = 0$ ,  $\alpha = \alpha_{lev}$  and  $\alpha = 1$ .

Motivated by the Corollary, we shall consider only equilibria in which there are at most three kinds of banks, banks with the portfolios 0,  $\alpha_{lev}$ , and 1. We shall in what follows refer to these banks as low-risk loan banks, mixed-portfolio banks, and high-risk loan banks. In other words, by a mixed-portfolio bank we shall mean a bank with the particular mixed portfolio  $\alpha_{lev}$ , for which both the Basel II type constraint and the leverage ratio requirement are binding.

Since the leverage ratio requirement  $k_{lev}$  is a binding constraint for the low-risk banks, the net value of a low-risk bank is given by  $V_L(k_{lev}, r_L)$ , where the function  $V_L$  is given by (15). If there are low-risk loan banks on the market, the interest rate  $r_L$  must have the value for which  $V_L(k_{lev}, r_L)$  is zero. For reasons which will shortly become obvious, we shall denote this interest rate by  $r_{CL}(k_{lev})$ ; in other words,  $r_{CL}(k_{lev})$  is the interest rate for which

$$V_L(k_{lev}, r_{CL}(k_{lev})) = 0 \tag{16}$$

Both the Basel II and the Basel III regime are based on the idea that the capital of banks should cover their loan losses with a probability of at least 99,9%. When this is the case, the failure probability  $1 - \Phi(\hat{z}_0)$  of the low-risk

<sup>11</sup>The two sides are equal only when  $\hat{z}_0 = \hat{z}_1$  in (15).

bank is less than 0,001, implying that  $V_L(k_{lev}, r_{C,L}(k_{lev}))$  may be approximated by replacing  $\hat{z}_0$  with  $+\infty$  in (15). In this way, one arrives at the approximation

$$\begin{aligned} V_L(k_{lev}, r_L) &\approx -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{+\infty} (k_{lev} + r_L - p_L(z)(\lambda + r_L)) d\Phi(z) \\ &= -k_{lev} + \frac{1}{1+\delta} (k_{lev} + r_L - \bar{p}_L(\lambda + r_L)) \end{aligned}$$

The interest rate for which this approximate value is zero is the actuarially fair interest rate

$$\tilde{r}_L(k_{lev}) = \frac{\delta k_{lev} + \bar{p}_L \lambda}{1 - \bar{p}_L} \quad (17)$$

As one may expect, in the calibrated version of the model  $r_{C,L}(k_{lev})$  turns out to be very close to  $\tilde{r}_L(k_{lev})$ .

The net value of a specialized high-risk loan bank is not changed by the leverage ratio requirement and hence, if there are specialized high-risk loan banks on the market, the high-risk loan interest rate must have the value  $\bar{r}_H$  that it has under the Basel II regime. This value is determined by (14), and it can be approximated by the actuarially fair rate  $\tilde{r}_H(k_H)$  which is obtained by replacing  $\hat{z}_1$  by  $+\infty$  in the definition of the value of the high-risk loan bank, (15), and postulating that the approximate value is zero. Solving for the interest rate, one arrives at the approximation

$$\bar{r}_H \approx \tilde{r}_H(k_H) = \frac{\delta k_H + \bar{p}_H \lambda}{1 - \bar{p}_H} \quad (18)$$

Also  $\bar{r}_H$  and  $\tilde{r}_H$  turn out to be very close to each other in the calibrated version.

We also observe that - writing  $\hat{z}_{lev}$  for  $\hat{z}_{\alpha_{lev}}$  - the value of a mixed portfolio bank is given by

$$\begin{aligned} V_M(k_{lev}, r_L, r_H) &= V(k_{lev}, \alpha_{lev}, r_L, r_H) \\ &= -k_{lev} + \frac{1}{1+\delta} \int_{-\infty}^{\hat{z}_{lev}} [(1 - \alpha_{lev})((k_L + r_L - p_L(z)(\lambda + r_L))) \\ &\quad + \alpha_{lev}((k_H + r_H - p_H(z)(\lambda + r_H)))] d\Phi(z) \end{aligned} \quad (19)$$

When there are mixed portfolio banks on the market, it must be the case that

$$V_M(k_{lev}, r_L, r_H) = 0 \quad (20)$$

By itself, this condition determines neither  $r_L$  nor  $r_H$ , but it suffices to determine  $r_H$  as a function of  $r_L$  and vice versa.

### 3. The Equilibria

We begin by considering an equilibrium in which there are specialized high-risk loan banks, and which we shall label equilibrium of type A. The easiest way to

understand the the nature of this equilibrium intuitively is, perhaps, to first consider a leverage ratio requirement which is only slightly larger than the low-risk loan capital requirement  $k_L$ . If this requirement is introduced to an economy which is in equilibrium relative to the Basel II type capital requirements  $k_L$  and  $k_H$ ,<sup>12</sup> one may expect it to be irrelevant for the business model of the high-risk loan banks. If they stick to financing high-risk loans only, also the high-risk interest rate  $r_H$  must retain the value  $\bar{r}_H$  which it would have in the absence of the leverage ratio requirement.

However, the low-risk risk loan banks cannot stick to financing low-risk loans only, if the interest rate does not change from  $\bar{r}_L$ . Motivated by Corollary 1, we restrict attention to equilibria in which all low-risk loans are owned by banks with the portfolios  $\alpha = 0$  and  $\alpha = \alpha_k$ . There are two obvious scenarios which lead to these portfolios.

According to (16), the interest rate which would be needed for making it possible to stick to the portfolio  $\alpha = 0$  is  $r_{CL}(k_{lev})$ . If the low-risk interest rate raises to this value, the business model of the specialized low-risk loan banks remains possible in equilibrium, and low-risk loans might still be offered by specialized low-risk banks for which  $\alpha = 0$ . As it was explained above,  $r_{CL}(k_{lev})$  may be approximated with the actuarially fair rate,

$$r_{CL}(k_{lev}) \approx \tilde{r}_L(k_{lev})$$

which is given by (17).

On the other hand, the low-risk banks could also react to the leverage ratio requirement by becoming mixed-portfolio banks and by adding high-risk loans to their portfolio until also the risk-based requirement becomes a binding constraint for them. By definition, this will be the case when  $\alpha = \alpha_{lev}$ . A unit-size bank with the portfolio  $\alpha_{lev}$  is allowed to have the same amount of capital with a specialized low-risk loan bank, which is at the same time the same amount of capital that  $(1 - \alpha_{lev})$  low-risk loan banks and  $\alpha_{lev}$  high-risk loan banks have under the Basel II regime. Such specialized banks are both of zero net value under the Basel II interest rates  $r_L = \bar{r}_L$  and  $r_H = \bar{r}_H$ . However, a comparison of (15) and (19) shows that in this case the net value of the mixed-portfolio bank is not simply a linear combination of the net values of the corresponding specialized banks (i.e., zero), because the values  $\hat{z}_0$ ,  $\hat{z}_1$ , and  $\hat{z}_{lev}$  may differ from each other.

When  $\hat{z}_0 \neq \hat{z}_1$ , there are value of  $z$  for which one of the specialized banks fails but the other one does not, and for such values of  $z$ , the mixed-portfolio bank will have to use the income from its high-risk loans for paying the losses from its low-risk loans or vice versa. This will not be the case when the high-risk and low-risk loans are owned by separate financial institutions. Hence, for the interest rates  $r_L = \bar{r}_L$  and  $r_H = \bar{r}_H$  the mixed-portfolio bank must be of a negative net value, but - given that the probability of bank failure is below 0,1% in the Basel II and Basel III frameworks - this negative value must be quite small.

From these considerations one may conclude that an equilibrium in which

---

<sup>12</sup>Cf. footnote 8 above.

the high-risk loan banks and the mixed portfolio banks with  $\alpha = \alpha_k$  co-exist must correspond to a low-risk interest rate  $r_L$  which is larger than  $\bar{r}_L$ , but quite close to it. We shall denote this interest rate by  $r_{AL}(k_{lev})$ . The interest rate  $r_{AL}(k_{lev})$  is determined by the condition that

$$V_M(k_{lev}, r_{AL}(k_{lev}), \bar{r}_H) = 0 \quad (21)$$

The interest rate  $r_{AL}(k_{lev})$  may be approximated by applying the definition (19) to (21), and by replacing  $\hat{z}_{lev}$  by  $+\infty$  and  $\bar{r}_H$  with the corresponding actuarially fair rate  $\tilde{r}_H$  given by (18). When the resulting equation is solved for  $r_{AL}(k_{lev})$ , one arrives at the approximation

$$r_{AL} \approx \frac{\delta k_L + \bar{p}_L \lambda}{1 - \bar{p}_L} = \tilde{r}_L(k_L) \quad (22)$$

The leverage ratio requirement does not show up in this approximate value, which is simply the actuarially fair rate under the Basel II regime.

In the calibrated version of our model it has turned out that the approximations (22) and  $r_{CL}(k_{lev}) \approx \tilde{r}_L(k_{lev})$  (in which  $\tilde{r}_L$  is given by (17)) have considerable accuracy and that, as they suggest,

$$r_{AL}(k_{lev}) < r_{CL}(k_{lev}) \quad (23)$$

for all values of  $k_{lev}$  between  $k_L$  and  $k_H$ . For the rest of our analysis, we shall assume that the additional assumption (23) is valid. Intuitively, this assumption states that if a leverage ratio requirement is introduced to banking sector which is subject to Basel II regulation and if the high-risk interest rate does not change, the low-risk loan banks will rather include also high-risk loans in their profile than stick to low-risk loans only (because the interest rate which would be needed for making the former option yield zero net value for the bank is smaller than the interest rate which suffices for the latter option).

We now conclude that when there are specialized high-risk loan banks on the market, there will be also mixed-portfolio banks and the interest rates are given by

$$\begin{cases} r_L = r_{AL}(k_{lev}) \\ r_H = \bar{r}_H \end{cases} \quad (24)$$

The interest rates (24) will correspond to an equilibrium of type *A* whenever the mixed-portfolio banks and the high-risk loan banks are able to meet the demand for the two kinds of loans for the given interest rates and there are no portfolios  $\alpha \neq \alpha_{lev}, 1$  that would yield a positive value for the bank. However, the assumption (23) and Lemma 1 imply that the latter condition is always valid.

Turning to the former condition, we keep in mind that we have normalized the size of all banks to 1 and introduce the notations  $m_L$ ,  $m_M$ , and  $m_H$  for the number of low-risk loan banks, mixed-portfolio banks, and high-risk loan banks, respectively. As we just saw,  $m_L = 0$  in an equilibrium of type *A*, and all the

low-risk loans are supplied by the mixed-portfolio banks. The total number of the low-risk loans that the mixed-portfolio banks grant is  $(1 - \alpha_{lev}) m_M$ , and hence, in an equilibrium of type A

$$n_L(r_{AL}(k_{lev})) = (1 - \alpha_{lev}) m_M$$

The mixed-portfolio banks finance altogether  $\alpha_{lev} m$  high-risk loans, and the high-risk loan banks finance altogether  $m_H$  high-risk loans, and hence, it must also be the case that

$$n_H(\bar{r}_H) = \alpha_{lev} m_M + m_H$$

Solving for  $m_M$  and  $m_H$ , in an equilibrium of type A the number of the banks is given by

$$\begin{cases} m_L = 0 \\ m_M = n_L(r_{AL}(k_{lev})) / (1 - \alpha_{lev}) \\ m_H = n_H(\bar{r}_H) - (\alpha_{lev} / (1 - \alpha_{lev})) n_L(r_{AL}(k_{lev})) \end{cases} \quad (25)$$

We now observe that an equilibrium of type A is possible if and only if each of these values is non-negative, a condition which is trivially valid for  $m_L$  and  $m_M$ , and valid also for  $m_H$  if and only if

$$n_H(\bar{r}_H) \geq \frac{\alpha_{lev}}{1 - \alpha_{lev}} n_L(r_{AL}(k_{lev})) \quad (26)$$

It is clear that this condition must be valid when  $k_{lev}$  is sufficiently close to  $k_L$  (because  $\alpha_{lev} = 0$  for  $k_{lev} = k_L$ ) and that the condition cannot be valid when  $k_{lev}$  is sufficiently close to 1 (because  $\alpha_{lev} = 1$  for  $k_{lev} = k_H$ ).

Solving for  $\alpha_{lev}$ , it is observed that (26) is equivalent with

$$\alpha_{lev} \leq f_1(k_{lev}) \quad (27)$$

where the function  $f_1$  is given by

$$f_1(k_{lev}) = \frac{n_H(\bar{r}_H)}{n_L(r_{AL}(k_{lev})) + n_H(\bar{r}_H)} \quad (28)$$

The condition (27) has a simple intuitive interpretation. The value  $f_1(k_{lev})$  is the share of high-risk loans within loan demand when the banks follow the strategies that we just described, and the condition states this share is larger than or equal with the share of high-risk loans in the portfolios of the mixed-portfolio banks. This statement must, obviously, be valid if the only banks that there are on the market in addition to the mixed-portfolio banks are banks specializing in high-risk loans.

We formulate the result which we have just proved as the following proposition.

**Theorem 1.** Whenever the leverage ratio requirement  $k_{lev}$  lies in the range in which (27) is valid, there is an equilibrium in which there are high-risk banks

specializing in high-risk loans only, and mixed-portfolio banks with the portfolio  $\alpha_{lev}$ . In this equilibrium the interest rates are given by (24).

It is easy to characterize the comparative statics of the equilibrium that we have just found. It should be emphasized that the changes in interest rates and loan demands that are referred to in the following theorem turn out to be very small in the calibrated version of the model, because they are caused by the changes in the failure probabilities of banks that the leverage ratio requirement causes.

**Theorem 2.** The following statements are valid when the values of  $k_{lev}$  lies in the range in which (27) is valid:

- a) The high-risk interest rate  $r_H$  and the demand for high-risk loans are constants, and have the values that they would have in the absence of the leverage ratio requirement. However, the number of high-risk loans financed by the specialized high-risk loan banks is decreased by the leverage ratio requirement.
- b) The low-risk interest rate  $r_L$  is a non-decreasing function of  $k_{lev}$ , implying that the demand for low-risk loans is a non-increasing function of  $k_{lev}$ .
- c) The share of high-risk loans among all granted loans is larger than or equal with their share in the absence of the leverage ratio requirement, and a non-decreasing function of  $k_{lev}$ .

**[Figure 2]**

The region in which (27) is valid, so that the currently considered equilibrium is possible, has been called "Region A" in Figures 2 and 3. Figure 2 depicts  $\alpha_{lev}$  and  $f_1(k_{lev})$  as functions of  $k_{lev}$ . The value  $\alpha_{lev}$  is defined by (5), and it is a linear function of  $k_{lev}$ . Figures 2 and 3 have been drawn using a calibrated version of the model which is otherwise identical with the version discussed in Section 5, except for the fact that, for purposes of illustration, we have given unrealistically high values to the elasticity of the loan demand with respect to the interest rates.<sup>13</sup> As Figure 2 illustrates, the slope of the curve  $f_1(k_{lev})$  is quite small compared with the slope of  $\alpha_{lev}$  even for the exaggeratedly large elasticity of loan demand, implying that the two curves cross just once and the region A is an interval of the form  $[k_L, k_1]$ .

**[Figure 3]**

Figure 3 shows the number of the loans granted by the banks of each category as a function of  $k_{lev}$ . The uppermost line indicates the change in the total number of granted loans. This change remains fairly small in the whole region A, despite of the exaggerated loan demand elasticity. However, the portfolios of the banks which grant loans changes dramatically. The number of the high-risk

<sup>13</sup>In Figures 2 and 3 the interest rates are given by (49) with  $c_L = c_H = 10$ . The calibration of the other numerical parameters which affect these figures will be explained in Section 5 below.

loans increases in the portfolios of the mixed-portfolio banks as  $k_{lev}$  increases, and there will be fewer loans that are left over for the specialized high-risk loan banks. At the border-line of the regions A and B all the loans are supplied by the mixed-portfolio banks, so that the equilibrium of the model is a symmetric one. On the right side of the border line, the strategies that we just described are no longer possible because of the insufficient demand for high-risk loans.

We now investigate the question whether the model has other symmetric equilibrium besides the equilibrium at the border line of region A. Denoting the low-risk and the high-risk interest rates of the symmetric equilibrium by  $r_{BL}$  and  $r_{BH}$ , we first observe that since  $m_L = m_H = 0$  in a symmetric equilibrium, in such an equilibrium the supply of loans matches demand if and only if

$$\begin{cases} n_L(r_{BL}) = (1 - \alpha_{lev}) m_M \\ n_H(r_{BH}) = \alpha_{lev} m_M \end{cases}$$

Eliminating  $m_M$ , it follows that

$$n_H(r_{BH}) = \frac{\alpha_{lev}}{1 - \alpha_{lev}} n_L(r_{BL})$$

In addition, the interest rates of the symmetric equilibrium must be such that the net value (19) of the mixed-portfolio bank is zero. These two conditions suffice to determine the interest rates as functions of  $k_{lev}$ . In other words, the interest rates  $r_{BL}(k_{lev})$  and  $r_{BH}(k_{lev})$  are determined by

$$\begin{cases} V_M(k_{lev}, r_{BL}(k_{lev}), r_{BH}(k_{lev})) = 0 \\ \alpha_{lev} = \frac{n_H(r_{BH}(k_{lev}))}{n_L(r_{BL}(k_{lev})) + n_H(r_{BH}(k_{lev}))} \end{cases} \quad (29)$$

It is also clear that when the size of the banks is normalized to one, in a symmetric equilibrium their number is given by

$$\begin{cases} m_L = m_H = 0 \\ m_M = n_L(r_{BL}(k_{lev})) + n_H(r_{BH}(k_{lev})) \end{cases} \quad (30)$$

The equation (6) allows for a case in which loan demand is inelastic, i.e. independent of the interest rates  $r_L$  and  $r_H$ . In this case the function  $f_1$  is a constant, and the latter equation of (29) is seen to be equivalent with  $f_1(k_{lev}) = \alpha_{lev}$ . Hence, when loan demand is inelastic, there is just a single value of the leverage ratio requirement for which a symmetric equilibrium is possible. For the rest of this section, we shall assume that

$$\frac{\partial}{\partial r_\eta} n_\eta(r_\eta) < 0 \quad (\eta = L, H)$$

i.e. that loan demand is a decreasing function of the interest rates.

In order to intuitively understand the nature of the equilibrium characterized by (29), we first recall that an equilibrium of type A is impossible whenever

$$\alpha_{lev} > f_1(k_{lev})$$

If the banks in this case tried to follow the strategies of the equilibrium A, the mixed-portfolio banks would not be able to satisfy the demand for low-risk loans. Intuitively, one may expect that such excess demand of low-risk loans would increase their interest rate  $r_L$ , and this may be expected to decrease the interest rate  $r_H$  on high-risk loans, because the increase in  $r_L$  makes the mixed portfolio more attractive. Each of these effects tends to increase the share of



high-risk loans among all granted loans, and a symmetric equilibrium should be possible if the two interest rates shift to an extent which yields the value  $\alpha_{lev}$  for the share of high-risk loans in the market. More specifically, one would expect that a symmetric equilibrium was possible when a moderate increase in  $r_L$  suffices to produce the value  $\alpha_{lev}$  for the share of high-risk loans; if, however, the necessary rise in  $r_L$  is so large that it makes the specialization to low-risk loans preferable to a mixed portfolio, a symmetric equilibrium will be impossible.

More rigorously, we now determine the values of  $k_{lev}$  for which the interest rates  $r_{BL}(k_{lev})$  and  $r_{BH}(k_{lev})$ , defined by (29), correspond to a symmetric equilibrium of the model. Remembering that according to (21)

$$V_M(k_{lev}, r_{AL}(k_{lev}), \bar{r}_H) = 0$$

we observe that if

$$r_{BL}(k_{lev}) < r_{AL}(k_{lev})$$

it must be the case that  $r_{BH}(k_{lev}) > \bar{r}_H$ , implying that a specialized high-risk loan bank is of a positive net value. Hence, in this case the high-risk interest rate cannot have the value  $r_{BH}(k_{lev})$  in equilibrium. On the other hand, if

$$r_{BL}(k_{lev}) \geq r_{AL}$$

it must be the case that  $r_{BH}(k_{lev}) \leq \bar{r}_H$ , that a specialized high-risk loan bank has a non-positive net value, and - remembering Lemma 1 - also that all other banks with a portfolio  $\alpha > \alpha_{lev}$  have a non-positive net value.

Similarly, if

$$r_{BL}(k_{lev}) > r_{CL}(k_{lev})$$

a specialized low-risk loan bank will have a positive net value, so that the situation characterized by (29) cannot correspond to an equilibrium, but if

$$r_{BL}(k_{lev}) \leq r_{CL}(k_{lev})$$

a specialized low-risk loan bank will be of a non-positive value. In this case one may conclude from Lemma 1 that a bank with any portfolio  $\alpha < \alpha_{lev}$  has a non-positive net value.

Combining these results, we now conclude that the situation in which all banks are mixed-portfolio banks and the interest rates are given by (29) is an equilibrium of the model if and only if

$$r_{AL}(k_{lev}) \leq r_{BL}(k_{lev}) \leq r_{CL}(k_{lev})$$

Introducing the notation  $r_{CH}(k_{lev})$  for the high-risk loan interest rate for which

$$V_M(k_{lev}, r_{CL}(k_{lev}), r_{CH}(k_{lev})) = 0 \quad (31)$$

this condition is seen to be equivalent with

$$r_{AH}(k_{lev}) \geq r_{BH}(k_{lev}) \geq r_{CH}(k_{lev})$$

and, given (28), (29), and the assumption that loan demand is a decreasing function of the interest rate, also with

$$f_1(k_{lev}) \leq \alpha_{lev} \leq f_2(k_{lev}) \quad (32)$$

where the function  $f_2$  is given by

$$f_2(k_{lev}) = \frac{n_H(r_{CH}(k_{lev}))}{n_L(r_{CL}(k_{lev})) + n_H(r_{CH}(k_{lev}))} \quad (33)$$

We recapitulate the result that we have just proved.

**Theorem 3.** The model has a symmetric equilibrium if and only if (32) is valid. In this equilibrium each bank has the portfolio  $\alpha_{lev}$  and the interest rates satisfy (29).

It should be observed that if the demands  $n_L$  and  $n_H$  are independent of the interest rates,  $f_1$  and  $f_2$  are identical constant functions and the only value for which (32) is the value  $k_1$  for which  $f_1(k_1) = \alpha_{lev}$ , and which lies on the borderline of the region in which equilibrium A is possible. Hence, Theorem 3 is valid also in this case in a trivial form, but in this case (29) does not suffice to determine the interest rates in equilibrium.

Currently we are considering the case in which  $n_L(r_L)$  and  $n_H(r_H)$  are decreasing functions. In this case

$$f_1(k) < f_2(k)$$

for each  $k$ . When  $k_{lev}$  ranges from  $k_L$  to  $k_H$ , the values of  $\alpha_{lev}$  range from 0 to 1, but the values of  $f_1$  and  $f_2$  stay positive and smaller than 1. Hence, one may now conclude that there must be values of  $k_{lev}$  between  $k_L$  and  $k_H$  for which  $\alpha_{lev}$  is between  $f_1(k_{lev})$  and  $f_2(k_{lev})$ , i.e. for which (32) is valid. As Figure 2 illustrates, in the calibrated version of the model the curves  $f_1$  and  $f_2$  turned out to be almost horizontal, and accordingly, they cross the line  $\alpha = \alpha_{lev}$  just once. Hence, the region in which (32) is valid is an interval.

The results that we have proved lead easily to the following results concerning the comparative statics of equilibrium B.

**Theorem 4.** Assume that the demand for the loans of each kind is a decreasing function of the interest rate. The following statements are valid in a symmetric equilibrium.

- a) The low-risk interest rate is larger than in the absence of the leverage ratio requirement, and an increasing function of  $k_{lev}$ .
- b) The high-risk interest rate is smaller than in the absence of the leverage ratio requirement (except for the borderline case in which  $f_1(k_{lev}) = \alpha_{lev}$ ).
- c) The share  $\alpha$  of high-risk loans is equal with  $\alpha_{lev}$ , which is larger than their share in the absence of the leverage ratio requirement, and an increasing function of  $k_{lev}$ .

For reasons that were explained in Section 2, we have restricted attention to the equilibria in which each bank has one of the portfolios 0,  $\alpha_{lev}$ , and 1. The only equilibrium of this kind which we have not yet considered is the one in which there are specialized low-risk loan banks, and which we label an equilibrium of type C. The leverage ratio requirement is a binding constraint for a low-risk loan bank, and in accordance with (16), the low-risk interest rate must be  $r_{CL}(k_{lev})$  when there are low-risk loan banks.

We now conclude from (21), (23), and (31) that

$$r_{CH}(k_{lev}) < \bar{r}_H \tag{34}$$

implying that there cannot be specialized high-risk loan banks in an equilibrium of type C. Hence, in this equilibrium there will be mixed-portfolio banks in addition to the low-risk loan banks, and the interest rates have the values  $r_{CL}(k_{lev})$  and  $r_{CH}(k_{lev})$  which are uniquely characterized by (16) and (31), i.e.

$$\begin{cases} V_L(k_{lev}, r_{CL}(k_{lev})) = 0 \\ V_M(k_{lev}, r_{CL}(k_{lev}), r_{CH}(k_{lev})) = 0 \end{cases} \quad (35)$$

Intuitively, one may view an equilibrium of type C as an equilibrium in which the high-risk loan banks of the Basel II regime have switched to a mixed portfolio. Their motives for including low-risk loans in their portfolios can be understood intuitively as follows. When the low-risk loan interest rate has the value  $r_{CL}(k_{lev})$ , it allows the specialized low-risk loan banks to have a zero net value with the the amount  $k_{lev}$  of capital per loan. However, if a specialized high-risk loan bank adds low-risk loans to its portfolio, the amount of extra capital that it needs for financing each loan is not  $k_{lev}$  per loan, but  $k_L$  per loan, because the leverage ratio requirement is not a binding constraint for it. Hence, intuitively, adding a low-risk project to the portfolio should be profitable for the bank until the leverage ratio requirement has become a binding constraint for it, i.e. until the share of high-risk projects has sunk to  $\alpha_{lev}$ . However, the extra profits that can be earned in this way tends to lower the interest rates for high-risk loans, and accordingly, in equilibrium the high-risk interest rate sinks to  $r_{CH}$ .

The interest rates of (35) will correspond to an equilibrium of the model if two conditions are met: the low-risk loan banks and mixed portfolio banks must be able to cover the demand for loans, and it must not be the case that some portfolio  $\alpha \neq 0, \alpha_k$  would yield a positive net value for the bank. The validity of the latter condition follows from (34) and Lemma 1. Turning to the former condition, it is observed that the supply of low-risk loans matches their demand if

$$n_L(r_{CL}(k_{lev})) = m_L + (1 - \alpha_{lev})m_M$$

and that - given that there are no specialized high-risk loan banks - the supply of high-risk loans matches their demand if

$$n_H(r_{CH}(k_{lev})) = \alpha_{lev}m_M$$

This implies that in an equilibrium of type C the number of the three kinds of banks is given by

$$\begin{cases} m_L = n_L(r_{CL}(k_{lev})) - ((1 - \alpha_{lev})/\alpha_{lev})n_H(r_{CH}(k_{lev})) \\ m_M = n_H(r_{CH}(k_{lev}))/\alpha_{lev} \\ m_H = 0 \end{cases} \quad (36)$$

Equilibrium C is possible if  $m_L \geq 0$ , i.e. if the supply of low-risk loans by the mixed-portfolio banks does not exceed their demand. Given (36) and (33), the condition  $m_L \geq 0$  is seen to be equivalent with

$$\alpha_{lev} \geq f_2(k_{lev}) \quad (37)$$

We summarize the results that we have obtained so far as the following theorem.

**Theorem 5.** When the leverage ratio requirement is in the region in which (37) is valid, the model has an equilibrium in which there are just low-risk loan banks and mixed-portfolio banks. In this case the interest rates are given by (35).

We shall not present a theorem which would be analogous with theorems 2 and 4 and which would describe the comparative statics of the equilibrium that we just found. This is because it seems that little can be said about its comparative statics without making economically plausible restrictions on the parameter values.

It is clear that in an equilibrium of type C the low-risk interest rate  $r_{CL}(k_{lev})$  must be an increasing function of the leverage ratio requirement. It may be approximated with the actuarially fair rate  $\tilde{r}_L(k_{lev})$  given by (17). The interest rate  $r_{CH}(k_{lev})$  can be approximated with a procedure which is analogous with the one with which we arrived at the approximation (22). If one applies the definition (19) to the equilibrium condition

$V_M(k_{lev}, r_{CL}(k_{lev}), r_{CH}(k_{lev})) = 0$ ,  
replaces  $r_{CL}(k_{lev})$  with  $\tilde{r}_L(k_{lev})$  and  $\hat{z}_{lev}$  by  $+\infty$ , and solves for  $r_{CH}$ , one arrives at the approximation

$$r_{CH}(k_{lev}) \approx \tilde{r}_H(k_{lev}) = \frac{\delta k_{lev} + \bar{p}_H \lambda}{1 - \bar{p}_H} \quad (38)$$

This approximate value is an increasing function of the leverage ratio requirement. The approximation (38) has turned out to be quite accurate in the calibrated version of the model which we shall discuss in the subsequent sections and accordingly, in this version also  $r_{CH}(k_{lev})$  is an increasing function of the leverage ratio requirement.

## 4. The Welfare Function

Before defining a welfare function for our model, we introduce the simplifying assumption that the demand for the loans of each category is constant, i.e. independent of the interest rate. This simplifying assumption will be valid also in the calibrated version of the model, which we shall discuss in the next section.

We denote the total number of granted loans of type  $\eta$  ( $\eta = L, H$ ) under the Basel II regime by  $\bar{n}_\eta$ . When loan demand is constant, the number of low-risk and high-risk loans will be  $\bar{n}_L$  and  $\bar{n}_H$  also in all other equilibria. In this case the number of the banks of the three kinds,  $m_L$ ,  $m_M$ , and  $m_H$ , satisfy

$$\begin{cases} m_L + (1 - \alpha_k) m_M = \bar{n}_L \\ \alpha_k m_M + m_H = \bar{n}_H \end{cases} \quad (39)$$

Repullo and Suarez (2004) introduce a welfare function which is the sum of the profits of the entrepreneurs and the payoff of the government. The latter term represents the social costs of bank failure. The entrepreneurs of type  $\eta$  earn  $(a - r_\eta) \bar{R}$  if their projects succeed and nothing if their projects fail and hence, the expected profit of an entrepreneur of type  $\eta$  is given by

$$u_\eta = (1 - \bar{p}_\eta) (a - r_\eta) \quad (40)$$

so that the profits of the entrepreneurs amount up to

$$U = \bar{n}_L u_L + n_H \bar{u}_H = \bar{n}_L (1 - \bar{p}_L) (a - \bar{r}_L) + \bar{n}_H (1 - \bar{p}_H) (a - \bar{r}_H) \quad (41)$$

We follow Repullo and Suarez (2004) in letting this aggregate profit represent the welfare value of firms in our analysis.<sup>14</sup>

In our model, the payoff of the government is given by the sum of three terms, which represents the social costs of failure of the banks of each kind. The social costs of bank failure are given by the aggregate

$$G = m_L G_0 + m_M G_{\alpha_k} + m_H G_1 \quad (42)$$

where

$$G_\alpha = E \min \{ \pi(k, \alpha, r_L, r_H, z), 0 \} - s(1 - \Phi(\hat{z}_\alpha)) \quad (43)$$

is the social costs of the failure of a bank with the portfolio  $\alpha$  (so that  $\alpha = 0$  corresponds to a low-risk loan bank,  $\alpha = \alpha_k$  corresponds to a mixed-portfolio bank, and  $\alpha = 1$  corresponds to a high-risk loan bank). In (43) the first term represents the expected value of direct costs of bank failure, i.e. the liabilities that the considered bank imposes on the deposit insurance system. The latter term represents the indirect negative welfare effects that bank failures have for the economy, and the multiplier  $s$  is assumed to be constant in it.

Our welfare function will be the sum of the aggregate profits and the aggregate social costs which are due to all banks. This sum is given by

$$W = U + G \quad (44)$$

---

<sup>14</sup>However, it should be noted that our analysis would not change if we assumed that each project of type  $\eta$  had also some such positive social value  $B_\eta$  which is independent of the interest rates  $r_L$  and  $r_H$  and which is not included in the profit of the firm, and replaced  $u_\eta$  with some function

$$u'_\eta = u_\eta + B_\eta$$

so that the aggregate  $U$  would be replaced by

$$U' = U + \bar{n}_L B_L + \bar{n}_H B_H$$

This point is important because the values of the welfare function that we will shortly define might be negative, suggesting that a situation in which the considered banks and firms would not exist would be preferable to the situation of the model. However, this implausible consequence will not follow if one assumes that the welfare function contains the terms  $\bar{n}_L B_L$  and  $\bar{n}_H B_H$  which are independent of the interest rates and which represent e.g. the positive function of the firms in creating employment.

As our next step, we express this welfare function in a more intuitive form. Clearly, one may conclude from (12) that

$$\begin{aligned} E(\min\{\pi(k, \alpha, r_L, r_H, z), 0\}) \\ &= E(\pi(k, \alpha, r_L, r_H, z)) - E(\max\{\pi(k, \alpha, r_L, r_H, z), 0\}) \\ &= E(\pi(k, \alpha, r_L, r_H, z)) - \Pi(k, \alpha, r_L, r_H) \end{aligned}$$

The net value  $V(k, \alpha, r_L, r_H)$  of each bank, which is according to (11) given by

$$V(k, \alpha, r_L, r_H) = -k + \frac{1}{1+\delta} \Pi(k, \alpha, r_L, r_H)$$

must be zero in equilibrium, and hence, one may further conclude, utilizing also (10), that

$$\begin{aligned} E(\min\{\pi(k, \alpha, r_L, r_H, z), 0\}) \\ &= E(k + (1-\alpha)(r_L - p_L(\lambda + r_L)) + \alpha(r_H - p_H(\lambda + r_H))) - (1+\delta)k \\ &= (1-\alpha)(r_L - \bar{p}_L(\lambda + r_L)) + \alpha(r_H - \bar{p}_H(\lambda + r_H)) - \delta k \end{aligned}$$

for each portfolio  $\alpha$  that the banks have on the market.

With some elementary algebra, we may now conclude from (43), (42), and (39) that

$$G = \bar{n}_L(r_L - \bar{p}_L(\lambda + r_L)) + \bar{n}_H(r_H - \bar{p}_H(\lambda + r_H)) - \delta K - sD$$

where

$$D = m_L(1 - \Phi(\hat{z}_L)) + m_M(1 - \Phi(\hat{z}_M)) + m_H(1 - \Phi(\hat{z}_H)) \quad (45)$$

is the expected number of bank failures and  $K$  is the total amount of capital of the banks.

Finally, together with our definition (41) of  $U$ , our latter formula for  $G$  implies that the welfare  $W = U + G$  may also be expressed in the form

$$W = \bar{n}_L((1 - \bar{p}_L)a - \bar{p}_L\lambda)\bar{R} + \bar{n}_H((1 - \bar{p}_H)a - \bar{p}_H\lambda)\bar{R} - \delta K - sD \quad (46)$$

In (46), the first two terms are independent of capital requirements, and we may now conclude that the optimization problem of the government consists of the problem of choosing the capital requirement so that the sum

$$\delta K + sD$$

is minimized. Since in our model there are three parameters (i.e.,  $k_L$ ,  $k_{lev}$ , and  $k_H$ ) which a social planner is free to choose, a standard welfare analysis of our model would consist in finding the values of  $k_L$ ,  $k_{lev}$ , and  $k_H$  which minimize  $\delta K + sD$ .

However, we find that a welfare analysis of this kind would have little relevance for the study of actual economies. As Repullo and Suarez (2004, pp. 511-3) point out, neither the actual IRB requirements nor the "corrected" IRB requirements which they define are optimal in the sense of producing the maximal value for the welfare function  $W$  among all possible capital requirements  $k_L$  and  $k_H$ . There is no reason to expect either that the Basel III requirements, to which a leverage ratio requirement  $k_{lev}$  has been added, would maximize the welfare function (46) among all combinations of  $k_L$ ,  $k_H$  and  $k_{lev}$ .

Accordingly, we do not try to find the combination of  $k_L$ ,  $k_H$  and  $k_{lev}$  which maximizes the considered welfare function. Rather, we will investigate whether

welfare is increased or decreased when a leverage ratio requirement  $k$  is added to an economy with some fixed (not necessarily socially optimal) values of the capital requirements  $k_L$  and  $k_H$ . In an analysis of this kind, the total amount of capital  $K$  has the value

$$K = \bar{n}_L k_L + \bar{n}_H k_H$$

whenever  $k_{lev}$  lies within the region A, so that the optimal value of  $k_{lev}$  among the values in this region is simply the one which minimizes  $B$ , the expected number of defaulting banks. However, in region C the leverage ratio requirement is a binding constraint for all banks, so that the total amount of capital is given by

$$K = (\bar{n}_L + \bar{n}_H) k_{lev}$$

In this region, an increase in  $k_{lev}$  will have the negative effect of increasing capital costs, which must be weighted against its possible positive role in decreasing bank failures.

## 5. A Calibrated Version of the Model

We now present a calibrated version of the model and use it for deducing predictions concerning the variation of the interest rates and the number of bank failures, and for making a welfare analysis of the model. We also study the effects of the model risk with which the model underlying Basel II and Basel III might be associated.

We have followed Repullo-Suarez (2009, p. 16) in giving the loss given default parameter the value  $\lambda = 0.45$  and the cost of capital  $\delta$  the value  $\delta = 0.04$ .<sup>15</sup> The default rates which we postulate are based on the results by a query by the Federal Reserve System (see Gordy, 2000). Table I reproduces the default probabilities that this query yielded per rating equivalent for an average quality bank.

[Table I]

We have taken the investment grade loans (loans of the categories from AAA to BBB) to constitute the counterpart of low-risk loans in our model and correspondingly, we have viewed the non-investment grade loans as the counterpart of high-risk loans. Such an aggregation is motivated by the observation from Table I that the default probabilities of the investment grade loans are quite close to one another. There is more variation between the non-investment grades but as a whole, default probabilities are broadly speaking "polarized" between the investment grade and non-investment grade groups. From Table I, it is readily calculated that the share of the low-risk loans in the total loan portfolio of

<sup>15</sup>The size of the parameter  $\delta$  has been recently actively discussed; see e.g. Hanson et al. (2011). The estimates they refer to suggest that the 4% assumption may be somewhat high but still reasonable.

the banks is  $\bar{n}_L = 52.5\%$  and that their weighted-average default probability is  $\bar{p}_L = 0.0523\%$ . Similarly, the share of high-risk loans is  $\bar{n}_H = 47.5\%$ , and their weighted-average default probability is  $\bar{p}_H = 3.77\%$ . Further, we have normalized the demand for loans to 1 in the absence of the leverage ratio requirement, so that  $\bar{n}_L$  and  $\bar{n}_H$  express the demands for the loans of each category under the Basel II regime.

When the values of  $\bar{p}_L$  and  $\bar{p}_H$  have been fixed, the values of the parameters  $\mu_L$  and  $\mu_H$  which appear in the conditional default probability distributions are determined by the result (8), since it implies that  $\mu_L = \Phi^{-1}(\bar{p}_L) = -3.278$  and that  $\mu_H = \Phi^{-1}(\bar{p}_H) = -1.778$ .

According to Basel Committee for Banking Supervision (2006, p. 64), the capital requirement for a loan with default probability  $p$  and maturity  $M = 1$  is<sup>16</sup>

$$b(p) = \lambda \Phi \left( \frac{\Phi^{-1}(p) + \sqrt{\rho} \Phi^{-1}(0.999)}{\sqrt{1 - \rho}} \right) \quad (47)$$

where (assuming that the firm-size adjustment does not apply) the correlation parameter  $\rho$  is given by

$$\rho = 0.12 \left( 2 - \frac{1 - e^{-50p}}{1 - e^{-50}} \right) \quad (48)$$

As one may easily infer from (8), (9), and (47), these capital requirements suffice to cover loan losses with probability 99.9% when the model underlying Basel II is correct.

We have used these formulas for fixing the correlation parameters  $\rho_L$  and  $\rho_H$  and the capital requirements  $k_L$  and  $k_H$  of our model. The formula (48) implies that  $\rho_L = 0.237$  and that  $\rho_H = 0.138$ . Using these values in (47), one may conclude that  $k_L = b(\bar{p}_L) = 0.00951$  and  $k_H = b(\bar{p}_H) = 0.112$ . In other words, in the calibrated version of our model, the IRB capital requirements on the low-risk loan and the high-risk loan would be ca. 1% and 11%, respectively.

[Table II]

The above choices of parameter values have been summarized in Table II. The parameter  $s$ , which also appears in this table, will below be used only for drawing Figure 7. Figures 2 and 3, with which we illustrated the equilibria of our model above, correspond to the values in Table II. In these figures the demand functions  $n_\eta$  ( $\eta = L, H$ ) of the low-risk and the high-risk loans were assumed to have the linear form

$$n_\eta(r_\eta) = \bar{n}_\eta - c_\eta(r_\eta - \bar{r}_\eta) \quad (49)$$

<sup>16</sup> More precisely, the value of the capital requirement specified by the Basel Committee for Banking Supervision (2006, p. 64) is equal with the difference of (47) and the expected loan losses  $\lambda p$ . In the Basel II framework the expected loan losses should be covered by general loan provisions. We follow Repullo-Suarez (2004, p. 502, footnote 15) in viewing the general loan provisions as a form of capital and, accordingly, leave the term  $\lambda p$  out of (47).



Here  $\bar{n}_\eta$  ( $\eta = L, H$ ) is the demand for loans of category of  $\eta$  in the absence of the leverage ratio requirement, and  $\bar{r}_\eta$  is the corresponding interest rate. As it was explained above, The Figures 2 and 3 corresponded to an exaggeratedly large elasticity of loan demand with respect to the interest rate; these figures have been drawn assuming that  $c_L = c_H = 10$ . The rest of this section will be concerned with the case in which  $c_L = c_H = 0$  and the demands for the loans of the two categories always have the constant values  $\bar{n}_L$  and  $\bar{n}_H$ .

[Figure 4]

In Section 3 we concluded that when loan demand is constant, a symmetric equilibrium is possible only for a single value of the leverage ratio requirement. This result is illustrated by Figure 4, which constitutes the counterpart of Figure 3 (which showed the number of the banks of each kind in each of the three regions) in the current setting with constant demand for loans. As Figure 4 illustrates, the region B has now disappeared, and there are only two regions to consider: the region in which the leverage ratio requirement is small and the equilibrium is of type A, and the region with a large leverage ratio requirement and an equilibrium of type C. At the border line of the two regions, the model shows a multiplicity of equilibria.

[Figure 5]

Figure 5 depicts the low-risk and the high-risk interest rates as functions of the leverage ratio requirement. In region A, the interest rates are almost identical with the actuarially fair rates  $\tilde{r}_L(k_L)$  and  $\tilde{r}_H(k_H)$  determined by (22) and (18), and in region C, the interest rates are almost identical with the actuarially fair rates  $\tilde{r}_L(k_{lev})$  and  $\tilde{r}_H(k_L)$  determined by (17) and (38).<sup>17</sup> The relevant actuarially fair interest rates stay constant in the region A, and as Figure 5 illustrates, a leverage ratio requirement has almost no effects on interest rates in this region. The leverage ratio requirement which has been introduced as a part of the Basel III reform corresponds to the value  $k_{lev} = 0.03$  in Figure 5. This value has been indicated with a dashed vertical line in the figure. Since the value  $k_{lev} = 0.03$  lies in region A, our model predicts that the leverage ratio requirement of the Basel III framework can have only quite small effects on the interest rates, provided that the banks are free to include high-risk loans in their portfolio.

It has sometimes been suggested that a leverage ratio type of requirement should be much higher than the 3% requirement of the Basel III framework. Figure 5 illustrates also the effects of such a scenario: an increased leverage ratio requirement (beyond approximately 6%) might lead to a considerable decrease

<sup>17</sup>More specifically, in the equilibria of type A the difference between  $\tilde{r}_{L,A}$  and the value  $r_L$  deduced from our model is below 0.00043 %, and the difference between  $\tilde{r}_{H,A}$  and the value  $r_H$  deduced from the model is approximately 0.00071%. Similarly, in the equilibria of type C the difference between  $\tilde{r}_{L,C}$  and  $r_L$  is below 0.0005%, and the difference between  $\tilde{r}_{H,A}$  and  $r_H$  is below 0.01%.

in high-risk interest rates and to a considerable increase in low-risk interest rates. This will be the case when the requirement is so high that the banks which own low-risk loans are not in the position to cope with it by including high-risk loans in their portfolio.<sup>18</sup>

In Figure 4 the curve  $m = m_L(k_{lev})$  depicts the number of the specialized low-risk loan banks, and the curve  $m = m_H(k_{lev})$  depicts the number of the specialized high-risk loan banks, and as one can see from the figure, there are low-risk loan banks only in the equilibrium C, and high-risk loan banks only in equilibrium A. The portfolio  $\alpha_{lev}$  of each mixed-portfolio bank is determined by the leverage ratio requirement  $k_{lev}$ , because  $\alpha_{lev}$  is given by (5). Since we have assumed the demand for loans to be inelastic and normalized the size of the banks to 1, the number of the banks is independent of  $k_{lev}$ . Also this value has been normalized to 1, and it is indicated by the horizontal line  $m = 1$  in Figure 4. The space between this straight line and the curve  $m = m_H(k_{lev})$  (in region A) or  $m = m_L(k_{lev})$  (in region C) indicates the number  $m_M(k_{lev})$  of the mixed-portfolio banks with portfolio  $\alpha_{lev}$ .

As Figure 4 shows, in region A the number of the specialized high-risk loan banks decreases as  $k_{lev}$  increases. This is because an increasing part of the demand for high-risk loans is covered by the mixed-portfolio banks, which cope with the increased leverage ratio requirement by increasing the share of the high-risk loans in their portfolio. A change of this kind will affect the number of the defaulting banks in region A, despite of the fact that both the total amount of bank capital and the interest rates are almost invariant in this region.

As we saw above, under the Basel II requirements the capital requirements suffice to cover the loan losses with the probability 99,9%. However, the probability of bank failure is below 0.1% under Basel II requirements, because the banks use also their interest income from the non-defaulting loans for covering their loan losses. Given that the high-risk loan banks earn higher interests than the low-risk loan banks, under Basel II their failure probability is smaller than the failure probability of low-risk loan banks, i.e.

$$1 - \Phi(\hat{z}_H) < 1 - \Phi(\hat{z}_L)$$

The mixed portfolio banks have a failure probability which is between these values, and (since the Basel II requirement is a binding constraint for them), an increase in the share of the high-risk loans in their portfolios will decrease their failure probabilities. In other words,

$$1 - \Phi(\hat{z}_H) < 1 - \Phi(\hat{z}_M) < 1 - \Phi(\hat{z}_L)$$

and  $1 - \Phi(\hat{z}_M)$  decreases towards  $1 - \Phi(\hat{z}_H)$  as the share of high-risk loans increases.

In region A, the expected number of bank failures (45) receives the form

---

<sup>18</sup>Some authors have argued that there should be a flat-rate capital requirement which is much higher than a leverage ratio requirement belonging to the region C (see footnote 3 above). This would not correspond to our model where risk-weighted requirements and the leverage ratio requirement co-exist. However, we may think that the market in anycase imposes some sort of risk-based capital "requirements" on banks, as reflected in banks' internal economic capital allocation practices. Hence, our calibrated model subject to a higher leverage ratio requirement together with the IRB requirements might provide some guidance to the possible effects of such suggested reforms.

$$D = m_M (1 - \Phi(\hat{z}_M)) + m_H (1 - \Phi(\hat{z}_H))$$

In this region an increase in the leverage ratio requirement will increase the number of the riskier mixed portfolio banks (i.e. it will increase  $m_M$  and decrease  $m_H$ ), which tends to increase the number of bank failures, but it also decreases the failure probability of each mixed portfolio bank when considered separately (i.e. it decreases  $1 - \Phi(\hat{z}_M)$ ).

[Figure 6]

As Figure 6 illustrates, the result of these opposing effects turns out to be that the the expected number of bank failures decreases as a function of the leverage ratio requirement  $k_{lev}$ . The expected number of bank failures is a decreasing function of  $k_{lev}$  also in the region C, because in this region the total amount of capital of the banks increases as a function of  $k_{lev}$ . (More specifically, the capital of the specialized low-risk loan banks increases as a function of  $k_{lev}$  in region C, so that their failure probability  $1 - \Phi(\hat{z}_L)$  has a smaller value than it would have under the Basel II regime).

As it was seen in Section 4, our welfare function  $W$  may be expressed in the form (46), in which the effects of the capital requirement policy shows up only in the last two terms

$$-\delta K - sD$$

The amount of bank capital  $K$  stays constant in region A, and one may now immediately conclude from Figure 6 that in this region, welfare increases as a function of  $k_{lev}$ .

[Figure 7]

In region C, an increase in  $k_{lev}$  will not just decrease the number of bank failures, but also increase the aggregate costs of bank capital. The weights  $\delta$  and  $s$  will determine whether a raise in  $k_{lev}$  will increase welfare in region C. Figure 7 depicts the value of  $-\delta K - sD$  as a function of  $k_{lev}$ , and motivated by the experiences of the Crisis, we have chosen a fairly large value for  $s$  in this figure. We have put  $s = 10$ , meaning that the indirect social costs of bank failure are ten times the size of the balance sheet of the bank.

As it is seen from Figure 7, for our choice of  $s$  an increase in  $k_{lev}$  increases welfare also in a part of the region C, but not if  $k_{lev}$  is close to  $k_H$ . Clearly, for smaller value of  $s$  welfare would be an increasing function of  $k_{lev}$  in the whole region C, and for sufficiently large values of  $s$  welfare would a decreasing function of  $k_{lev}$  in the whole region C.

## 5.1 Model Risk

The recent crisis suggests that the Basel II framework might be subject to a considerable model risk, and that the failure rates of loans might be essentially

larger than what the regulators and banks believe them to be. It is natural to ask whether a leverage ratio requirement would help increase the stability of banks in the presence of model risk. As discussed in the introduction, this argument is one of the motivations which made the Basel Committee introduce the leverage ratio requirement.

We have investigated the effects of the leverage ratio requirement under model risk by calculating the value of our welfare function for two cases in which the actual default probabilities of loans are larger than the values  $\bar{p}_L$  and  $\bar{p}_H$  which appear in Table II. We have assumed that both the capital requirements  $k_L$  and  $k_H$  and the interest rates  $r_L$  and  $r_H$  have the values that correspond to the calibrated version of our model described by Table II, and that either the actual default probability for low-risk projects (call it  $p_{La}$ , a for 'actual') is larger than  $\bar{p}_L$ , or the actual default probability for high-risk projects (call it  $p_{Ha}$ ) is larger than  $\bar{p}_H$ . We have calculated the value of the expected number of bank failures as a function of  $p_{La}$  and of  $p_{Ha}$  for four different leverage ratio requirement regimes. The results are shown in Figures 8 and 9.

[Figure 8]

Figure 8 shows the expected number of bank failures when the default probability  $p_{La}$  varies from the value  $\bar{p}_L = 0.0523\%$ , which we have used in our calibration, up to 30%. The curve  $B = B_0(p_{La})$  shows the expected number of failing banks in the absence of leverage ratio requirement, i.e. under the Basel II regime, and the curve  $B = B_1(p_{La})$  shows the expected number of bank failures under the leverage ratio requirement which has been included in the Basel III framework, i.e.  $k_{lev} = 0.3$ . As it is seen from Figure 8, if the default probability of low-risk loans is larger than the banks and regulators believe it to be, but not too much (below ca. 17.1%), the introduction of a leverage ratio requirement of the size  $k_{lev} = 0.3$  will decrease the expected number of bank failures and increase welfare. However, the opposite is the case when  $p_{La}$  is very large.<sup>19</sup>

This result can be understood intuitively by remembering that in region A the leverage ratio requirement affects the number of bank failures in two ways. First, an increase in the leverage ratio requirement will increase both the amount of capital and the number of high-risk loans in the portfolios of the mixed-portfolio banks, and this diversification effect makes them safer. The model error tends to strengthen this effect, given that now the loans which are called "low-risk loans" are, as a matter of fact, quite risky. Secondly, an increase in the leverage ratio requirement increases also the volume of the loans that are held by the mixed-portfolio banks (and also the number of the mixed-portfolio banks, given that their size has been normalized to one). This contamination effect tends to increase the number of bank failures. Figure 5 shows that the first of these effects dominates the latter one for not too large values of the

<sup>19</sup>Note that the threshold actual default probability even as high as the 17.1% is not unreasonable given the experience from the subprime crisis. For instance, "S&P now expects the default rate on subprime loans issued in 2005, 2006, and 2007 to be 11 percent, 30 percent, and 49 percent, respectively." (7.6.2009 in The Truth About Mortgage.com)

actual default probability  $p_{La}$  whereas the latter effect dominates the first effect for sufficiently large values of  $p_{La}$ .

Since all the leverage ratio requirements in the region A correspond to the same amount of bank capital, adding a leverage ratio requirement of  $k_{lev} = 0.3$  increases welfare whenever the curve  $B = B_1(p_{La})$  is below the curve  $B = B_0(p_{La})$  in Figure 8. The curve  $B = B_2(p_{La})$  corresponds to the largest leverage ratio requirement that may be implemented into the economy without increasing the amount of bank capital. This is the value which separates the regions A and C ( $k_{lev} = 0.0582$  in Figures 4-7). (The expected number of bank failures changes discontinuously at the border line of the regions A and C, and, more rigorously, the curve  $B = B_2(p_{La})$  represents the limit of the number of bank failures when  $k_{lev}$  approaches the border line value from the left.) As Figure 8 shows, an increase of the leverage ratio requirement from  $k_{lev} = 0.3$  up to the limit of the areas A and C will decrease the expected number of bank failures and increase welfare for all the considered values of  $p_{La}$ .

Finally, the curve  $B = B_3(p_{La})$  shows the expected number of bank failures for a leverage ratio requirement of the size  $k_{lev} = k_H = 0.112$ . This limiting case has identical effects with a flat-rate (Basel I type) capital requirement of size 11.2%, and it corresponds to a considerable further decrease in the bank failure probability. However, the welfare comparisons between this regime and three other considered regimes will depend also on the relative weight that is given to the extra capital that is needed for implementing the leverage ratio requirement.

[Figure 9]

It is also interesting to study the model risks that are associated with high-risk loans. Our results concerning them turn out to be qualitatively quite different from the ones concerned with low-risk loans. Analogously with Figure 8, Figure 9 shows the expected number of bank failures as a function of the actual default probability  $p_{Ha}$  of high-risk loans when  $p_{Ha}$  varies from the value  $\bar{p}_H = 3.78\%$  to 30%. The curves  $B_0$ ,  $B_1$ ,  $B_2$ , and  $B_3$  correspond to the same capital requirement regimes with the corresponding curves of Figure 8. The curve  $B = B_0(p_{Ha})$  (which corresponds to Basel II regime) and  $B = B_3(p_{Ha})$  (which corresponds to a flat-rate capital requirement  $k_H$ ) are indistinguishable in Figure 9, because their difference stems only from the different default probability of low-risk loans, which is very small in comparison with the default probability of high-risk loans  $p_{Ha}$ .

As Figure 9 shows, if the actual value of the default probability  $p_{Ha}$  is not close to  $\bar{p}_H$  (if  $p_{Ha}$  is larger than ca. 4.79%), the leverage ratio requirement 3% of the Basel III framework, which corresponds to the curve  $B = B_1(p_{Ha})$ , will tend to increase bank failures (in comparison with Basel II), and the "borderline" leverage ratio requirement 5.82%, which corresponds to the curve  $B = B_2(p_{Ha})$ , will tend to increase bank failures even more.

To understand this result intuitively, one should keep in mind that when  $p_{Ha}$  is large, in region A an increase of  $k_{lev}$  decreases the number of the specialized

high-risk loan banks, which is a positive diversifying effect that tends to decrease bank failures, but it also increases the riskiness of each mixed-portfolio bank (because it forces the mixed-portfolio banks to include more high-risk loans in their portfolios). The latter effect is made stronger by the fact that the mixed-portfolio banks have, in addition to their high-risk loans, only loans with a low interest rate and a low capital requirement in their portfolios. As Figure 6 indicates, for the larger values of  $p_{Ha}$  the negative contaminating effect exceeds the positive diversifying effect from the decrease in the number of specialized high-risk loan banks.

To sum up, our model suggests that a leverage ratio requirement of 3% improves the stability of the banking sector when there is a model risk associated with low-risk loans, i.e. in a situation in which the loans which count as low-risk loans are risky, as long as the model error remains of a moderate size. If, however, the putative low-risk loans turn out to be so risky that a large part of them (in our calibration, more than about 17.1%) defaults, the negative contaminating effect of the leverage ratio requirement dominates its positive diversifying effect, and the leverage ratio requirement tends to increase the number of bank failures. When the model risk is associated with high-risk loans, a leverage ratio requirement tends to increase the number of bank failures already for model errors (i.e. errors in the average default probability of high-risk loans) of a much more moderate size.<sup>20</sup>

## 6. Concluding Remarks

We have studied the credit allocation and bank stability effects of introducing a leverage ratio requirement (LRR) on top of risk-based capital requirements, as in Basel III. We showed that if the LRR is above but close to the risk-based capital requirement on low-risk loans, such as the 3% LRR in Basel III, then both low-risk and high-risk loan rates and volumes remain essentially unchanged. This is because the LRR will be a binding capital constraint only on banks specializing in low-risk lending, so the banking sector can adjust by more banks granting

<sup>20</sup>It should be noted that the positive welfare effect of a 3% leverage ratio requirement in the absence of model risk, which is shown in Figure 7, was based on a rather specific feature of the Basel II framework, i.e. the fact that under the Basel II regime the banks that specialize in high-risk loans have a smaller failure **probability** than the banks that specialize in low-risk loans. If the capital requirements  $k_L$  and  $k_H$  had been chosen so that the banks of both types had precisely the same failure **probability** (cf. Repullo and Suarez 2004), a leverage ratio requirement would in Figure 7 have no welfare effects in region A. However, our simulations indicate that our results concerning model risk are not specific for the Basel II framework in a similar way. We have simulated the effects of a model risk also in cases in which the capital requirements  $k_L$  and  $k_H$  are not determined by the Basel II formula (47), but chosen so that the low-risk loan banks and the high-risk loan banks have the same failure rate. Our earlier conclusion turned out to remain qualitatively valid: a leverage ratio requirement of 3% increases the number of bank failures when there is a model risk associated with high-risk loans, and also in the presence of a model risk associated with low-risk loans, if this risk is very large.

both low-risk and high-risk loans. For counterfactually high LRRs, e.g. 6-10%, low-risk lending rates would significantly increase and high-risk lending rates would fall. Overall lending volume could drop somewhat. Bank failures would decrease because of the increased amount of bank capital. However, in the presence of model risk, modelled as an unanticipated shock to the default probability of loans, the current 3% LRR might even reduce bank stability, counter to regulatory intentions. If the model risk is associated with low-risk loans, bank stability would be reduced if the model risk were severe. This is because for a sufficiently high model risk the beneficial effect from spreading (the seemingly) low-risk loans to a larger number of banks is dominated by "contaminating" the larger number of banks by the low-risk loans that may turn out to be very risky. If the unanticipated model risk concerns high-risk loans' default probability, then the current moderate LRR (almost) always increases bank failures as a result of the contamination effect. A sufficiently high LRR would be needed to make sure the LRR reduces bank failures in the presence of model risk.

Our way of modelling the unanticipated model risk can be motivated by Gennaioli et al. (2011) who argue that a bias which they call "local thinking" may lead to neglecting rare risks. This combined with investors' preference for safe assets may have contributed to the emergence of seemingly low-risk subprime loan based assets. In other words, their theory may explain why 1) what we have called model risk can be unanticipated, and 2) why such model risk is particularly relevant in the case of (seemingly) low-risk assets.

Interestingly, our model may be used to analyze the situation which prevailed in the United States before the subprime crisis. The US never really implemented Basel II risk-based capital requirements but stayed in Basel I. In the context of our model Basel I could be interpreted as a leverage ratio requirement because there is no risk-weighting within corporate loans and overall, any risk-weighting in Basel I is quite crude. However, markets may have expected leading banks to hold capital against their true asset risks, using their internal risk models. This practice is often referred to as allocating economic capital (see also Froot et al., 1993). As a result, one may argue that the US situation resembled the one in our model where both risk-based capital requirements and a leverage ratio requirement are in effect at the same time. Consequently, our model predicts that, regardless of the specific type of the equilibrium (A-C), the spreading of low-risk assets across banks would have been wider in the US than in Europe (where only Basel II was in force<sup>21</sup>). In effect, more banks in the US than in Europe may have been exposed to the true (but unanticipated) risk of subprime loans. This may have contributed to the apparently larger shock which the subprime crisis first caused in the US banking sector compared to Europe.

---

<sup>21</sup>Indeed, Repullo and Suarez (2004) argue that the different options within Basel II further encourage banks to specialize in either low-risk or high-risk lending.

## APPENDIX. THE PROOFS OF LEMMA 1 AND COROLLARY 1.

**Proof of Lemma 1.** If  $0 < \alpha < \alpha_{lev}$ , we put

$$\beta_L = 1 - \frac{\alpha}{\alpha_{lev}}, \beta_M = \frac{\alpha}{\alpha_{lev}}, \text{ and } \beta_H = 0$$

If  $\alpha_{lev} < \alpha < 1$ , we put

$$\beta_L = 0, \beta_M = \frac{1-\alpha}{1-\alpha_{lev}}, \beta_H = \frac{\alpha-\alpha_{lev}}{1-\alpha_{lev}}$$

In each case  $\beta_L, \beta_M$ , and  $\beta_H$  are non-negative, and

$$\beta_L + (1 - \alpha_{lev})\beta_M = 1 - \alpha \text{ and } \alpha_{lev}\beta_M + \beta_H = \alpha$$

so that  $\beta_L$  low-risk banks,  $\beta_M$  mixed-portfolio banks, and  $\beta_H$  high-risk banks own together the same amount of low-risk loans and high-risk loans as a single bank with the portfolio  $\alpha$ .

By piecewise linearity (and remembering that  $\beta_H = 0$  when  $\alpha < \alpha_{lev}$  and that  $\beta_L = 0$  when  $\alpha > \alpha_{lev}$ ), the capital requirement which applies to a bank with the profile  $\alpha$  satisfies

$$\kappa(\alpha) = \beta_L \kappa(0) + \beta_M \kappa(\alpha_{lev}) + \beta_H \kappa(1)$$

Now (10) implies that

$$\begin{aligned} \pi(\kappa(\alpha), \alpha, r_L, r_H; z) &= \beta_L \pi(\kappa(0), 0, r_L, r_H; z) \\ &+ \beta_M \pi(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H; z) + \beta_H \pi(\kappa(1), 1, r_L, r_H; z) \end{aligned}$$

for each  $z$ , and that

$$\begin{aligned} \max\{\pi(\kappa(\alpha), \alpha, r_L, r_H; z), 0\} &\leq \beta_L \max\{\pi(\kappa(0), 0, r_L, r_H; z), 0\} \\ &+ \beta_M \max\{\pi(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H; z), 0\} + \beta_H \max\{\pi(\kappa(1), 1, r_L, r_H; z), 0\} \end{aligned}$$

Now one may conclude from (12) that

$$\begin{aligned} \Pi(\kappa(\alpha), \alpha, r_L, r_H) &= \int_{-\infty}^{+\infty} \max\{\pi(\kappa(\alpha), \alpha, r_L, r_H; z), 0\} d\Phi(z) \\ &\leq \beta_L \int_{-\infty}^{+\infty} \max\{\pi(\kappa(0), 0, r_L, r_H; z), 0\} d\Phi(z) \\ &+ \beta_M \int_{-\infty}^{+\infty} \max\{\pi(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H; z), 0\} d\Phi(z) \\ &+ \beta_H \int_{-\infty}^{+\infty} \max\{\pi(\kappa(1), 1, r_L, r_H; z), 0\} d\Phi(z) \\ &= \beta_L \Pi(\kappa(0), 0, r_L, r_H) + \beta_M \Pi(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H) + \beta_H \Pi(\kappa(1), 1, r_L, r_H) \end{aligned}$$

and finally from (11) that

$$\begin{aligned} V(\kappa(\alpha), \alpha, r_L, r_H) &\leq \beta_L V(\kappa(0), 0, r_L, r_H) + \beta_M V(\kappa(\alpha_{lev}), \alpha_{lev}, r_L, r_H) + \beta_H V(\kappa(1), 1, r_L, r_H) \end{aligned}$$

This contains both of the results that were to be proved as its special cases.

**Proof of Corollary 1.** In a competitive equilibrium the number of the loans of each type that are offered by the banks is fixed by the demand for loans ( $n_\eta(r_\eta)$  for the loans of type  $\eta$ ,  $\eta = L, H$ ), and the net value of each bank must be zero in it. Suppose now that in some equilibrium  $E$  there is a positive number of loans offered by banks which have portfolios  $\alpha$  for which  $0 < \alpha < \alpha_{lev}$ . Now it must be the case that

$$V(\kappa(\alpha), \alpha, r_L, r_H) = 0$$

for each of these portfolios, and one may conclude from Lemma 1 that

$$V(k_{lev}, 0, r_L, r_H) = V(k_{lev}, \alpha_{lev}, r_L, r_H) = 0$$

Now a situation  $E'$  which is similar with the equilibrium  $E$ , except for the fact that the banks with the portfolios  $\alpha$  ( $0 < \alpha < \alpha_{lev}$ ) do not exist, and their loans



have been shared by the banks with the portfolios 0 and  $\alpha_{lev}$  must also be an equilibrium.

Suppose now that in the equilibrium  $E'$  there is a positive number of loans offered by banks which have portfolios  $\alpha$  for which  $\alpha_{lev} < \alpha < 1$ . This time we conclude from Lemma 1 that

$$V(k_{lev}, \alpha_{lev}, r_L, r_H) = V(k_H, 1, r_L, r_H) = 0$$

and that a situation which is similar with the equilibrium  $E'$ , except for the fact that the banks with the portfolios  $\alpha$  ( $0 < \alpha < \alpha_{lev}$ ) do not exist, and that their loans have been shared by the banks with the portfolios  $\alpha_{lev}$  and 1 must also be an equilibrium.

## References

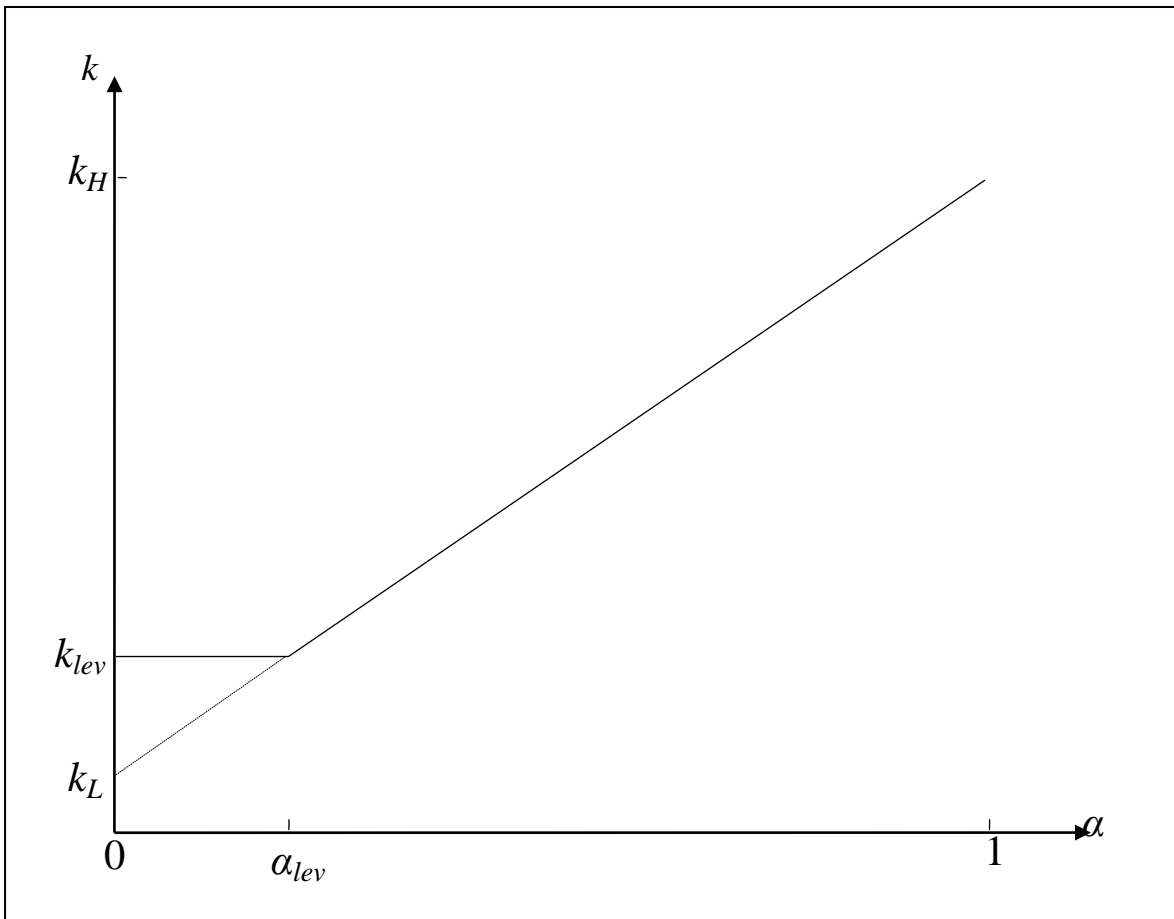
- [1] Acharya, V.V., 2009. A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability* 5(3), 224-255.
- [2] Acharya, V.V., T. Cooley, M. Richardson, I. Walter, 2009. Manufacturing Tail Risk: A Perspective on the Financial Crisis of 2007–2009. *Foundations and Trends in Finance* 4, 247–325.
- [3] Acharya, V.V., P. Schnabl, G. Suarez, 2010. Securitization without risk transfer. NBER Working Paper No. 15730.
- [4] Admati, A.R., DeMarzo, P.M., Hellwig, M.F., Pfleiderer, P., 2010. Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is *not* expensive? Stanford GSB Research Paper No. 2063.
- [5] Basel Committee on Banking Supervision, 2006, International Convergence of Capital Measurement and Capital Standards: A Revised Framework - A Comprehensive Version (Bank for International Settlements, June 2006).
- [6] Basel Committee on Banking Supervision, 2009, Strengthening the resilience of the banking sector - consultative document (Bank for International Settlements, December 2009).
- [7] Blum, J.M., 2007. Why 'Basel II' may need a leverage ratio restriction? *Journal of Banking and Finance* 32:8, 1699-1707.
- [8] Froot, K.A., D.S. Scharfstein, J.C. Stein, 1993. Risk management: Coordinating corporate investment and financing policies. *Journal of Finance* 48, 1629-1658.
- [9] Gennaioli, N., A. Schleifer, R. Vishny, 2011. Neglected risks, financial innovation, and financial fragility. forthcoming in *Journal of Financial Economics*.
- [10] Gordy, M.B., 2000. A comparative anatomy of credit risk models. *Journal of Banking and Finance* 24, 119-149.
- [11] Hanson, S.G., A.K. Kashyap, J.C. Stein, 2011. A macroprudential approach to financial regulation. *Journal of Economic Perspectives* 25: 1, 3–28.
- [12] Hellwig, M., 2010. Capital regulation after the crisis: Business as usual? Preprints for the Max Planck Institute for Research on Collective Goods, Bonn 2010/31.
- [13] Repullo, R., Suarez, J., 2004. Loan pricing under Basel capital requirements. *Journal of Financial Intermediation* 13, 496-521.
- [14] Repullo, R., Suarez, J. 2009. The procyclical effects of bank capital regulation. European Banking Center Discussion Paper No. 2010-05S.

**Table I. The shares of the loans of different categories among all granted loans and their default probabilities in the calibrated version of the model.**

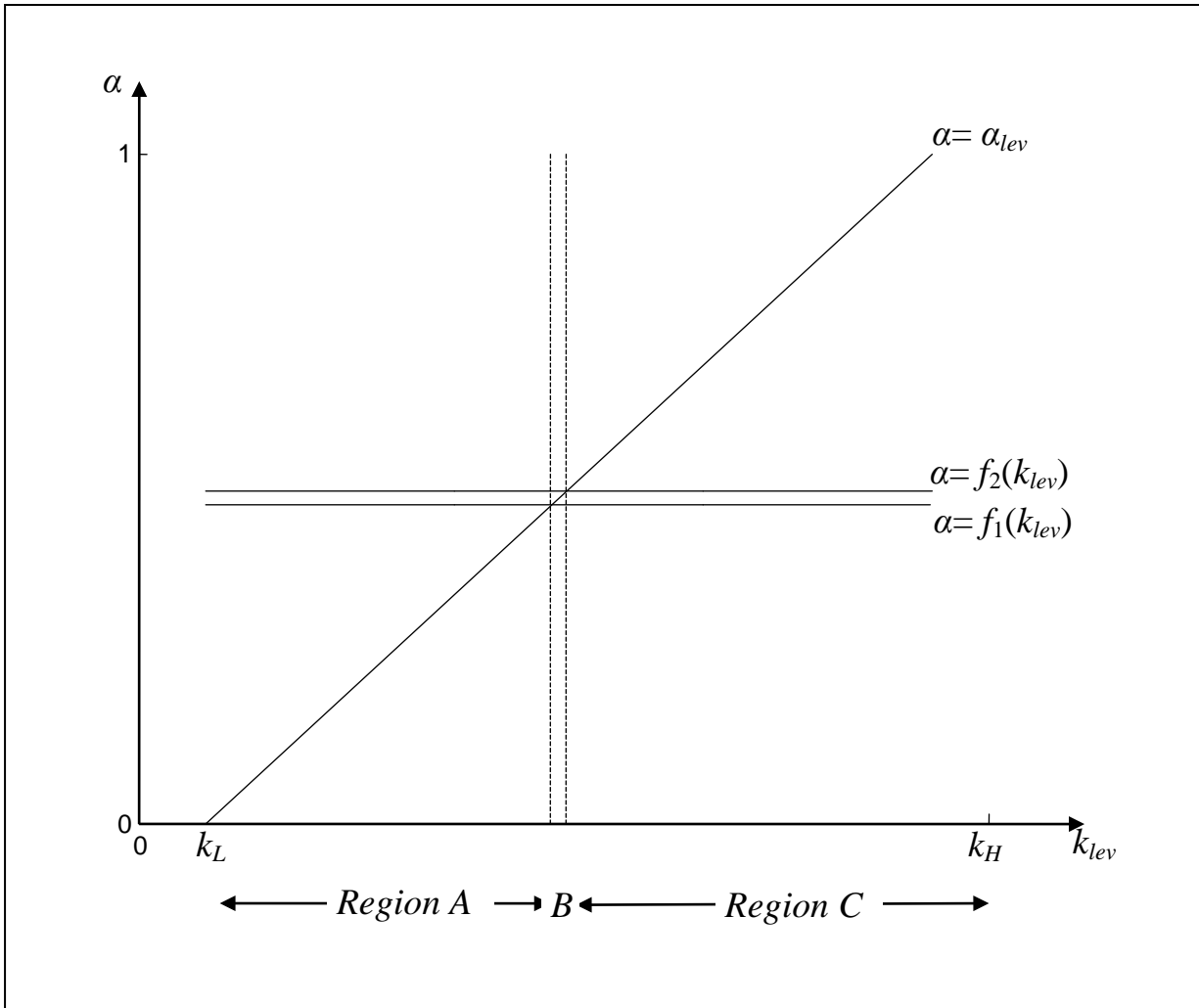
Loan Category	Share in Portfolio (%)	Default Probability (%)
AAA	2.9	0.02
AA	5.0	0.02
A	13.4	0.03
BBB	31.2	0.07
BB	32.4	1.32
B	11.1	5.58
CCC	4.0	18.6

**Table II. The parameter values of the calibrated version of the model. The value  $s$  appears in brackets, because it has been used only for drawing Figure 4.**

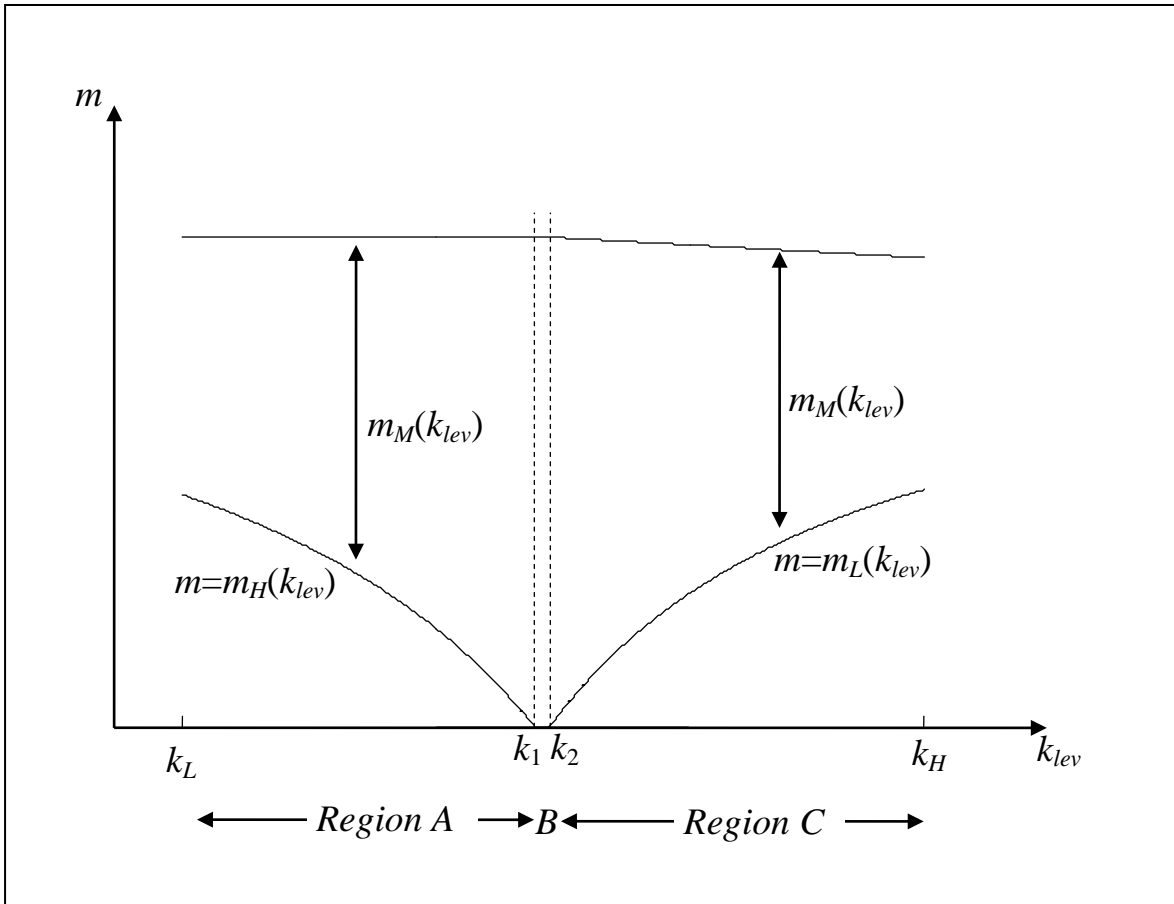
Parameter	Explanation	Value
$\delta$	Cost of equity (equity premium)	0.04
$\lambda$	Loss given default	0.45
$\bar{n}_L$	Demand for low-risk loans	0.525
$\bar{n}_H$	Demand for high-risk loans	0.475
$\bar{p}_L$	Default probability for low-risk loans	0.0523%
$\bar{p}_H$	Default probability for high-risk loans	3.77%
$\mu_L$	Parameter characterizing low-risk loan default probability distribution	-3.278
$\mu_H$	Parameter characterizing high-risk loan default probability distribution	-1.778
$\rho_L$	Correlation parameter for low-risk loans	0.237
$\rho_H$	Correlation parameter for high-risk loans	0.138
$k_L$	Basel II capital requirement for low-risk loans	0.00951
$k_H$	Basel II capital requirement for high-risk loans	0.112
$(s)$	(Social cost of bank failure)	(10)



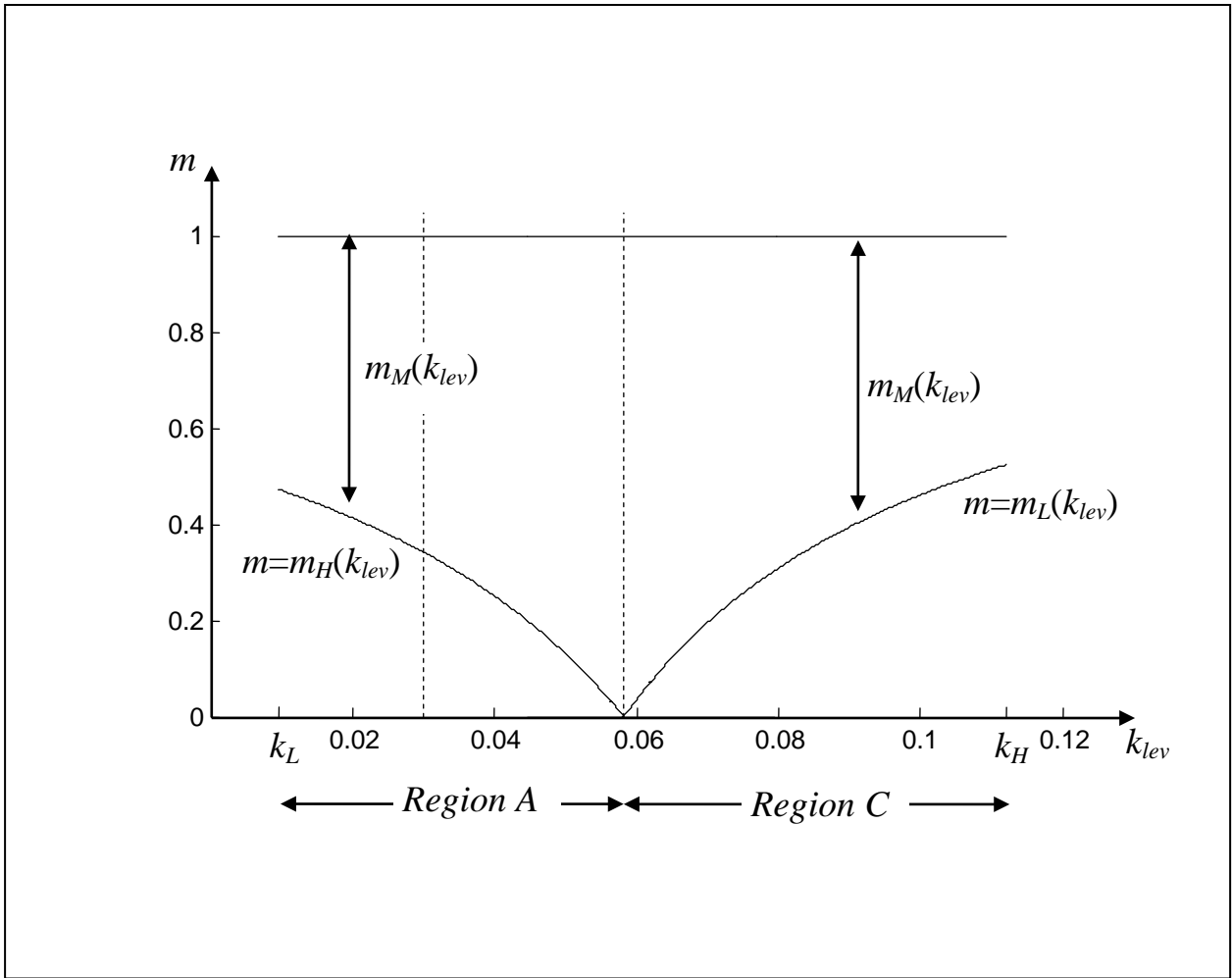
**Figure 1.** The capital requirement  $k$  per loan as a function of the share  $\alpha$  of high-risk loans in the bank portfolio.



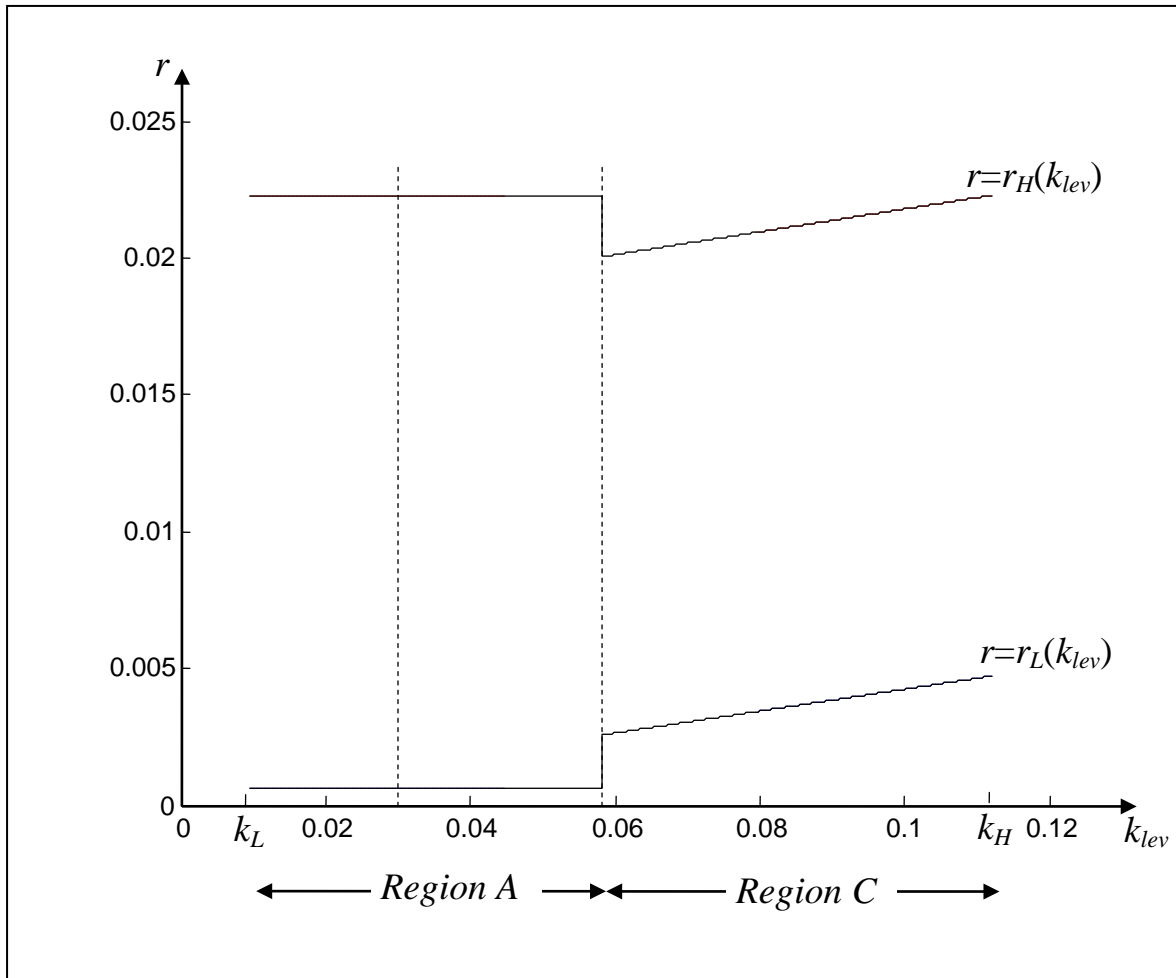
**Figure 2.** The functions  $f_1$  and  $f_2$  and the share of high-risk projects in the portfolio of a mixed-portfolio bank.



**Figure 3.** The number of high-risk loan banks, low-risk banks, and mixed portfolio banks

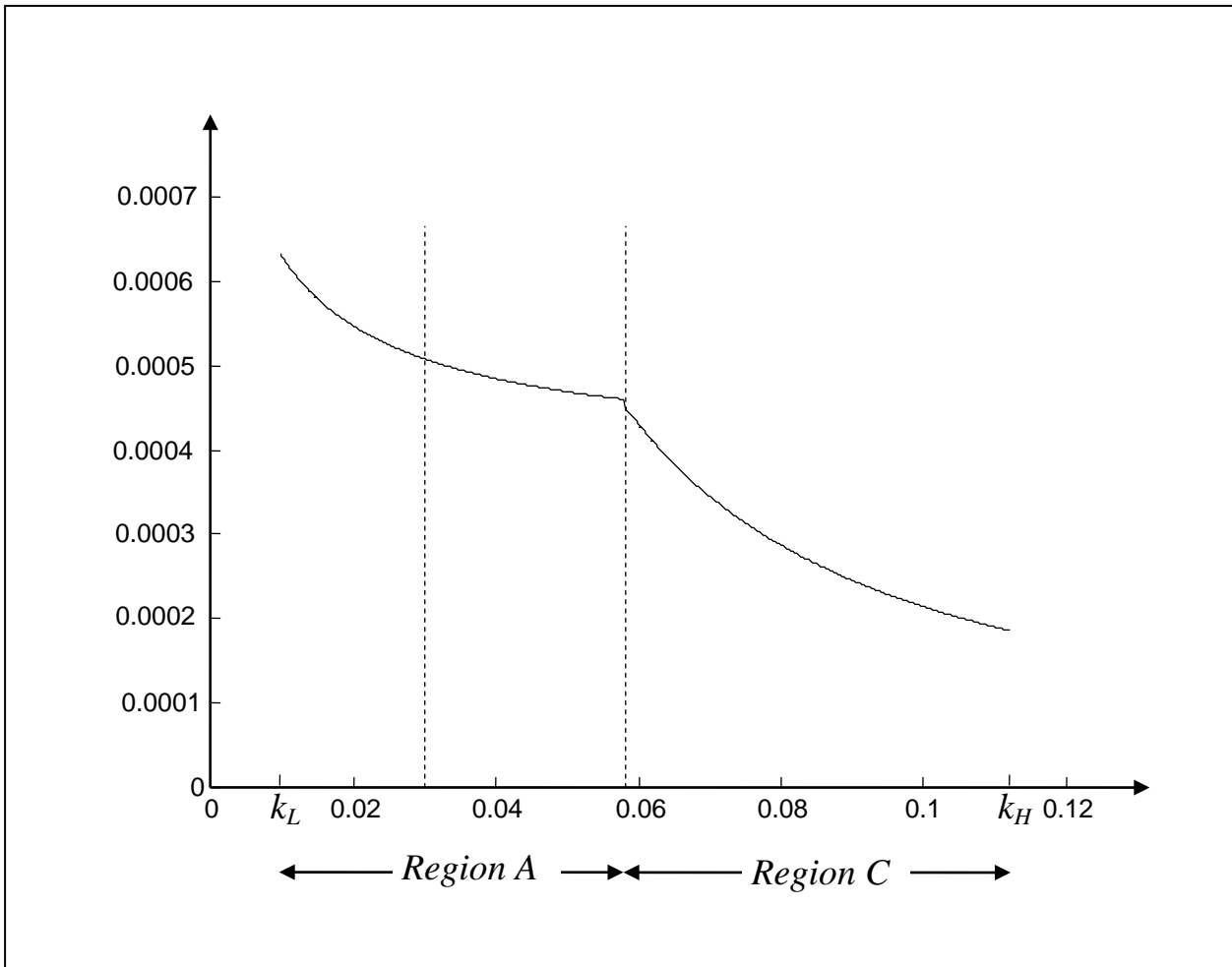


**Figure 4.** Number of high-risk loan banks, low-risk banks, and mixed portfolio banks

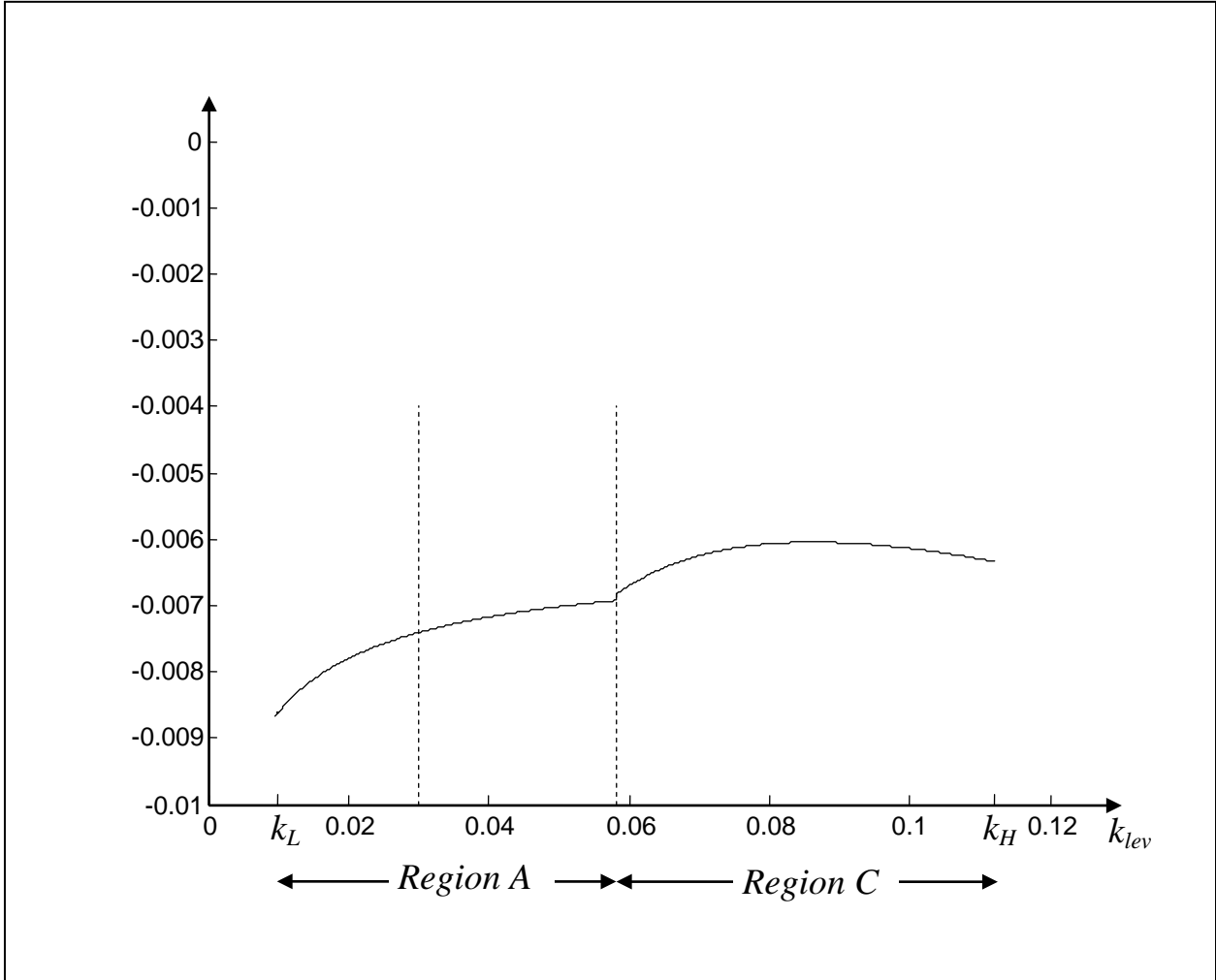


**Figure 5.** Interest rates as a function of the leverage ratio requirement

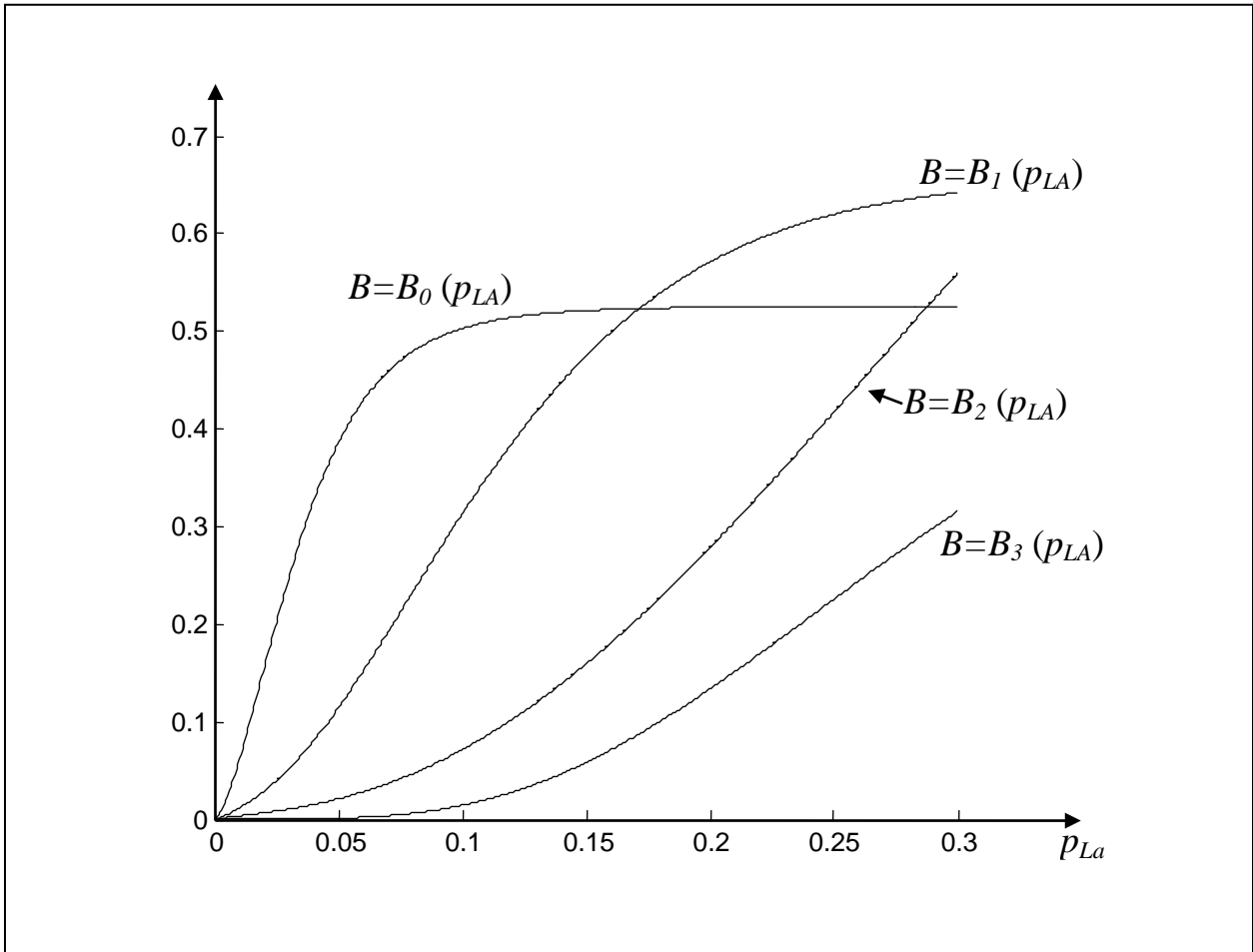




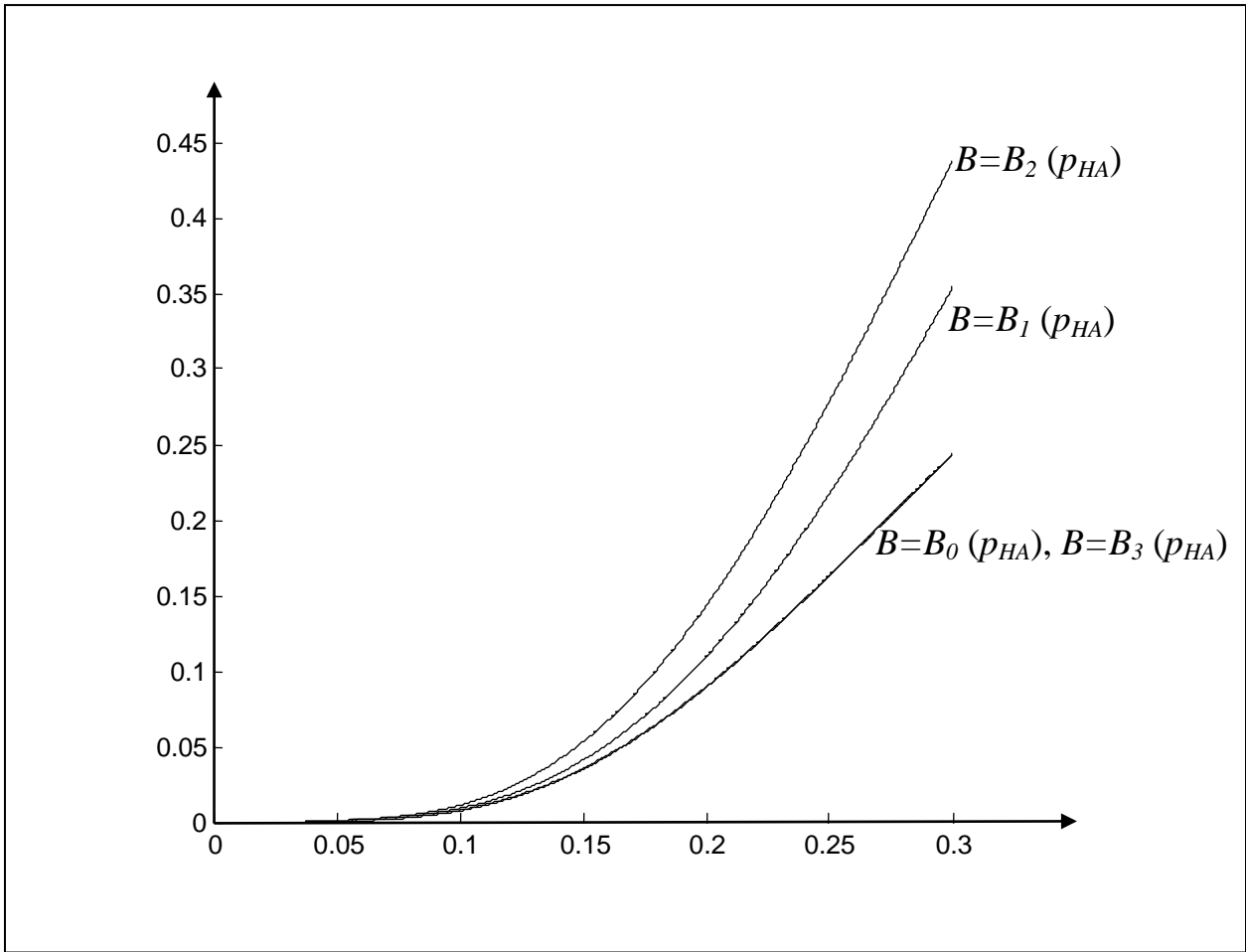
**Figure 6.** Expected number of bank failures as a function of the leverage ratio requirement



**Figure 7.** The regulation-dependent component of the welfare function as a function of  $k_{lev}$ .



**Figure 8.** Expected number of bankruptcies as a function of the actual low-risk loan default rate  $p_{La}$  in case of model error.



**Figure 9.** Expected number of bankruptcies as a function of the actual high-risk loan default rate  $p_{Ha}$  in case of model error.

**BANK OF FINLAND RESEARCH  
DISCUSSION PAPERS**

ISSN 0785-3572, print; ISSN 1456-6184, online

- 1/2011 Hanna Freystätter **Financial factors in the boom-bust episode in Finland in the late 1980s and early 1990s.** 2011. 39 p. ISBN 978-952-462-653-8, print; ISBN 978-952-462-654-5, online.
- 2/2011 Marko Melolinna **What explains risk premia in crude oil futures?** 2011. 37 p. ISBN 978-952-462-658-3, print; ISBN 978-952-462-659-0, online.
- 3/2011 Harry Leinonen **Debit card interchange fees generally lead to cash-promoting cross-subsidisation.** 2011. 37 p. ISBN 978-952-462-660-6, print; ISBN 978-952-462-661-3, online.
- 4/2011 Yiwei Fang – Bill Francis – Iftekhar Hasan – Haizhi Wang **Product market relationships and cost of bank loans: evidence from strategic alliances.** 2011. 39 p. ISBN 978-952-462-662-0, print; ISBN 978-952-462-663-7, online.
- 5/2011 Yiwei Fang – Iftekhar Hasan – Katherin Marton **Bank efficiency in transition economies: recent evidence from South-Eastern Europe.** 2011. 40 p. ISBN 978-952-462-664-4, print; ISBN 978-952-462-665-1, online.
- 6/2011 Arturo Bris – Yrjö Koskinen – Mattias Nilsson **The euro and corporate financing.** 2011. 63 p. ISBN 978-952-462-666-8, print; ISBN 978-952-462-667-5, online.
- 7/2011 Yiwei Fang – Iftekhar Hasan – Katherin Marton **Market reforms, legal changes and bank risk-taking – evidence from transition economies.** 2011. 41 p. ISBN 978-952-462-668-2, print; ISBN 978-952-462-669-9, online.
- 8/2011 George W Evans – Seppo Honkapohja **Learning as a rational foundation for macroeconomics and finance.** 2011. 60 p. ISBN 978-952-462-670-5, print; ISBN 978-952-462-671-2, online.
- 9/2011 Seppo Honkapohja – Arja H Turunen-Red – Alan D Woodland **Growth, expectations and tariffs.** 2011. 49 p. ISBN 978-952-462-672-9, print; ISBN 978-952-462-673-6, online.
- 10/2011 Ilkka Kiema – Esa Jokivuolle **Leverage ratio requirement, credit allocation and bank stability.** 2011. 50 p. ISBN 978-952-462-757-3, print; ISBN 978-952-462-758-0, online.

Suomen Pankki  
Bank of Finland  
P.O.Box 160  
**FI-00101** HELSINKI  
Finland



\*.2343\*