

Orjasniemi, Seppo

Working Paper

The effect of openness in a small open monetary union

Bank of Finland Research Discussion Papers, No. 18/2010

Provided in Cooperation with:

Bank of Finland, Helsinki

Suggested Citation: Orjasniemi, Seppo (2010) : The effect of openness in a small open monetary union, Bank of Finland Research Discussion Papers, No. 18/2010, ISBN 978-952-462-621-7, Bank of Finland, Helsinki,
<https://nbn-resolving.de/urn:nbn:fi:bof-20140807246>

This Version is available at:

<https://hdl.handle.net/10419/212175>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



Seppo Orjasniemi

The effect of openness in a small open monetary union



EUROJÄRJESTELMÄ
EUROSYSTEMET

Bank of Finland Research
Discussion Papers
18 • 2010

**Suomen Pankki
Bank of Finland
PO Box 160
FI-00101 HELSINKI
Finland
☎ +358 10 8311**

**<http://www.bof.fi>
E-mail: Research@bof.fi**



Seppo Orjasniemi*

The effect of openness in a small open monetary union

The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

* Email: seppo.orjasniemi@bof.fi

I thank Alfred Greiner, Kjetil Storesletten and Mikko Puhakka for comments. I have also benefited from comments by seminar participants at the Bielefeld University, HECER and in the Nordic Macro Symposium 2010.

<http://www.bof.fi>

ISBN 978-952-462-620-0
ISSN 0785-3572
(print)

ISBN 978-952-462-621-7
ISSN 1456-6184
(online)

Helsinki 2010

The effect of openness in a small open monetary union

Bank of Finland Research
Discussion Papers 18/2010

Seppo Orjasniemi
Monetary Policy and Research Department

Abstract

In this paper we build a dynamic stochastic general equilibrium model of a small open monetary union with optimal monetary and fiscal policy, to study the transmission of country specific shocks and associated exchange rate fluctuations. We show that movements of the monetary union's exchange rate stabilize the output fluctuations inside the monetary union, reducing the need for fiscal stabilization. We also show that, under the optimal policy, fluctuations in the exchange rate and the union-wide aggregates are affected by the differences in the degree of nominal rigidities among the monetary union member countries.

Keywords: monetary union, monetary policy, fiscal policy, exchange rate

JEL classification numbers: E52, E62, F41

Kansainvälisen kaupan merkitys pienessä rahaliitossa

Suomen Pankin keskustelualoitteita 18/2010

Seppo Orjasniemi
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä työssä tutkitaan dynaamisen yleisen tasapainon mallin avulla kansainvälisen kaupan vaikutusta pienen avoimen rahaliiton optimaaliseen raha- ja finanssipolitiikkaan. Tulosten mukaan rahaliiton avoimuus kansainväliselle kaupalle pienentää rahaliiton sisäisiä suhdannevaihteluita, mikä vähentää finanssipoliittisen vakautuksen tarvetta. Tulokset osoittavat lisäksi, että lyhyen aikavälin optimaalinen rahapolitiikka on riippuvainen nimellisten jäykkyyksien eroista rahaliiton maiden välillä. Heterogeenisessä rahaliitossa optimaalinen raha- ja finanssipolitiikka ei pysty kiinnittämään tuotantoa ja inflaatiota optimaaliselle tasolle lyhyellä aikavälillä.

Avainsanat: rahaliitto, rahapolitiikka, finanssipolitiikka, vaihtokurssi

JEL-luokittelu: E52, E62, F41

Contents

Abstract.....	3
Tiivistelmä (abstract in Finnish)	4
1 Introduction.....	7
2 A model of a small open monetary union.....	8
2.1 Households of the monetary union	8
2.2 Terms of trade and inflation	11
2.3 International risk sharing	11
2.4 Public sector	12
2.5 Firms	13
3 Equilibrium dynamics	14
3.1 Solving for equilibrium dynamics	14
3.2 The flexible price allocation	16
3.3 Dynamics with rigid price setting	17
4 Optimal monetary and fiscal policy under commitment.....	18
4.1 Optimization	18
4.2 Symmetric price rigidities	20
5 Numerial experiments.....	21
5.1 Parametrization	21
5.2 Symmetric nominal rigidities	21
5.3 Effect of openness	22
5.4 Asymmetric price rigidities	24
6 Conclusions	25
References.....	26
Appendix 1.....	27
Appendix 2.....	27
Appendix 3.....	28

1 Introduction

In the past few years an extensive amount of work has been done on optimal fiscal and monetary policies in micro-founded models of monetary unions (*MU*) with price rigidities. Recent papers include eg Beetsma and Jensen (2005), Galí and Monacelli (2008) and Ferrero (2009). Main argument in this literature is that fiscal policy is needed to stabilize inflation differentials inside the monetary union. Most of these studies are, however, conducted in the context of a closed monetary union. Only few studies, including Kirsanova, Satchi, and Vines (2004), Clausen and Wohltmann (2005) and Campa and González-Mínguez (2006), pay attention on the interaction of monetary union and the rest of the world. However, the analysis of exchange rate fluctuations and importance of the expenditure switching effects between goods produced in the monetary union and the rest of the world is still incomplete.

To study the effects of openness to international trade on the economic fluctuations inside the monetary union, we construct a model where monetary union consists of two open economies trading with a continuum of small open economies. With this setup we are able to examine the effects of the terms of trade movements and exchange rate fluctuations, associated with monetary union's openness to international trade, on the optimal monetary and fiscal policy. In particular, we are interested in the economic fluctuations caused by country specific productivity shocks. To extend the analysis of optimal monetary and fiscal policy by Galí and Monacelli (2008) we follow Benigno (2004) and Beetsma and Jensen (2005) and relax the assumption of symmetric price rigidity among the monetary union members. In our analysis fiscal policy is fully coordinated and policy planner is committed to the optimal policy.

As a benchmark, we analyze the optimal monetary and fiscal policy under commitment when the *MU* countries share the same degree of price rigidity. The optimal policy is to keep union wide levels of inflation, nominal interest rate, output and government spending at the levels associated with flexible prices. The result is that fiscal policies of both monetary union countries are used to stabilize the terms of trade fluctuations inside the *MU*. The resulting exchange rate fluctuations affect all countries inside the *MU*, reducing the need for fiscal stabilization. We study also the optimal policy mix when the degree of price rigidity varies across the *MU* countries. In this case optimal policy cannot keep the union wide variables on their flexible price levels and the resulting exchange rate fluctuations depend on the origin of the technological shock.

The rest of the paper is organized as follows. In section 2 we lay out the model and the equilibrium dynamics are derived in the section 3. Section 4 analyzes the optimal monetary and fiscal policy mix. The numerical experiments are presented in section 5. Section 6 concludes.

2 A model of a small open monetary union

2.1 Households of the monetary union

We construct a DSGE model with nominal rigidities and monopolistic competition. We mainly follow Galí and Monacelli (2005) and Galí and Monacelli (2008). In our model the world consists of continuum of small open economies indexed in the unit interval. The monetary union is formed by two of these small open economies indexed by H and U . The countries outside the monetary union are referred to as the rest of the world. Since monetary union is of measure zero its economic fluctuations do not affect the economies in the rest of the world. The economies are subject to idiosyncratic shocks on productivity and share the same preferences, technology and market structure. While the representative households share the same preferences, the countries of the monetary union are more open to the international trade than the countries of the rest of the world.

Utility of a representative household in country $i \in [0, 1]$ depends positively on consumption C^i and public spending G^i and negatively on labor output L^i . As in Corsetti and Pesenti (2001) and Galí and Monacelli (2008), private and public consumption are not perfect substitutes. In period t a representative household maximizes

$$E_t \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \log C_{t+s}^i + \kappa \log G_{t+s}^i - \frac{(L_{t+s}^i)^{1+\phi}}{1+\phi} \right\} \quad (2.1)$$

where $\phi > 0$, $\beta \in (0, 1)$ is a discount factor and E_0 denotes the mathematical expectations operator conditional to information available at period 0. The private consumption of the representative household in a MU country i is a composite index defined by

$$C_t^i = \frac{(C_{MU,t}^i)^{(1-\alpha)} (C_{F,t}^i)^\alpha}{(1-\alpha)^{(1-\alpha)} \alpha^\alpha} \quad (2.2)$$

where $C_{F,t}^i$ is an index of country i 's consumption of goods produced in the rest of the world. Variable $C_{MU,t}^i$ is an index of MU country i 's consumption of goods produced in the monetary union and is given by

$$C_{MU,t}^i = 2 (C_{i,t}^i)^{\frac{1}{2}} (C_{j,t}^i)^{\frac{1}{2}}, \quad i, j \in \{H, U\}, i \neq j \quad (2.3)$$

where $C_{i,t}^i$ is an index of country i 's consumption of domestic goods and $C_{j,t}^i$ is an index of country i 's consumption of goods produced in the another country of the monetary union. The consumption of goods produced in the rest of the world is given by

$$C_{F,t}^i = \int_0^1 C_{f,t}^i df$$

In the rest of the world the representative household in country i consumes domestically produced goods and imported goods. The consumption index of

such household is given by

$$C_t^i = \frac{(C_{i,t}^i)^{(1-\alpha)} (C_{F,t}^i)^\alpha}{(1-\alpha)^{(1-\alpha)} \alpha^\alpha} \quad (2.4)$$

where $C_{F,t}^i$ now includes also the monetary union. In the equations above the parameter $\alpha \in [0, 1]$ can be interpreted as a measure of openness. Using the equations (2.2) and (2.3) we see that the weight of domestic goods in the utility from private consumption of the households in MU is $(1-\alpha)/2$, while the equation (2.4) shows that for the households in the rest of the world the weight is $1-\alpha$, ie the MU countries are more open to trade than the countries in the rest of the world.

In each economy goods are produced by a continuum of monopolistically competitive firms producing differentiated goods. The consumption of goods originating from country i is given by

$$C_{i,t} = \left(\int_0^1 C_{i,t}(k)^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ denotes the elasticity of substitution between the differentiated goods produced in one country and k denotes the types of goods.

The aggregate consumer price index in a MU country i is given by

$$P_{c,t}^i = (P_{MU,t}^i)^{1-\alpha} (P_{F,t}^i)^\alpha$$

where the price index for goods originating from MU countries is

$$P_{MU,t}^i = (P_{i,t}^i)^{\frac{1}{2}} (P_{j,t}^i)^{\frac{1}{2}}, \quad i, j \in \{H, U\}, j \neq i$$

and the subindex $P_{F,t}^i$, expressed in MU 's currency, is given by

$$P_{F,t}^i = \int_0^1 P_{j,t}^i dj, \quad j \in [0, 1] \quad (2.5)$$

and is equal for both MU countries. The price index for goods produced in country j , denominated in country i 's currency, is given by

$$P_{j,t}^i = \left(\int_0^1 (P_{j,t}^i(k))^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}}, \quad i, j \in [0, 1]$$

The optimal allocation of expenditures of the representative household in MU country i implies the demand functions as follows

$$\begin{aligned} P_{i,t}^i C_{i,t}^i &= \frac{(1-\alpha)}{2} P_{c,t}^i C_t^i, & P_{j,t}^i C_{j,t}^i &= \frac{(1-\alpha)}{2} P_{c,t}^i C_t^i \\ P_{F,t} C_{F,t}^i &= \alpha P_{c,t}^i C_t^i \end{aligned} \quad (2.6)$$

where $j \in \{H, U\}$ and $j \neq i$. The demand for good k produced in country j by household in country i is given by

$$C_{j,t}^i(k) = \left(\frac{P_{j,t}^j(k)}{P_{j,t}^j} \right)^{-\epsilon} C_{j,t}^i$$

Consumption indices with associated price indices for the countries in the rest of the world can be derived analogously.

The exchange rate between monetary union's currency and the currency of country i in the rest of the world is denoted by $\mathcal{E}_{i,t}$ and the exchange rate between the currencies of countries i and j is denoted by $\mathcal{E}_{j,t}^i$. We assume the law of one price to hold for individual goods, ie $P_{j,t}^i(k) = \mathcal{E}_{j,t}^i P_{j,t}^j(k)$. Using the equation (2.5) we may write the consumer price index of goods originating from the rest of the world as

$$P_{F,t} = \int_0^1 (\mathcal{E}_{j,t} P_{j,t}^j) dj, \quad j \in [0, 1]$$

The effective exchange rate of the MU 's currency is defined as

$$\mathcal{E}_t = \exp \left(\int_0^1 \log \mathcal{E}_{i,t} di \right)$$

Using equations (2.6) we can write the consumption index of a household in MU country i as

$$P_{c,t}^i C_t^i = \frac{(1-\alpha)}{2} P_{i,t} C_{i,t}^i + \frac{(1-\alpha)}{2} P_{j,t} C_{j,t}^i + \alpha P_{F,t} C_{F,t}^i$$

where $i, j \in \{H, U\}, i \neq j$.¹

In each period and in each country, households earn labor income W . The households also have access to a complete set of state contingent claims traded internationally. The nominal payoff of the portfolio, including also the shares of firms, held at the end of period t is denoted by B_{t+1}^i . The payoff is paid in period $t+1$ and is discounted by gross nominal interest rate R_t^i . Representative household in country i maximizes its utility (2.1) subject to periodic budget constraint given by

$$P_{c,t}^i C_t^i + \frac{E_t \{B_{t+1}^i\}}{R_t^i} \leq B_t^i + W_t^i L_t^i - T_t^i$$

where T_t^i denotes lump-sum taxes. The optimality conditions of the utility maximization problem are given by

$$(L_t^i)^\phi = \frac{W_t^i}{C_t^i P_{c,t}^i} \tag{2.7}$$

$$E_t \{C_{t+1}^i P_{c,t+1}^i\} = \beta R_t^i C_t^i P_{c,t}^i \tag{2.8}$$

Equation (2.7) is the intratemporal optimality condition, and equation (2.8) is a conventional Euler equation. These optimality conditions hold for each $i \in [0, 1]$.

¹For the household i in the rest of the world we can write $P_{c,t}^i C_t^i = (1-\alpha) P_{i,t}^i C_{i,t}^i + \alpha P_{F,t}^i C_{F,t}^i, i \in [0, 1]$.

2.2 Terms of trade and inflation

The bilateral terms of trade between countries i and j is defined as $S_{j,t}^i = \frac{P_{j,t}^i}{P_{i,t}^i}$, ie the price of goods produced in country j in terms of price of goods produced in country i . The effective bilateral terms of trade of the country i in the rest of the world is defined by $S_t^i = \frac{P_{F,t}^i}{P_{i,t}^i}$ and the effective bilateral terms of trade between the MU country i and the rest of the world is defined by $S_t^i = \frac{P_{F,t}^i}{P_t^i}$. Denoting the logarithms of variables by lower case letters, we may write the consumer price index in a MU country i as

$$p_{c,t}^i = p_{i,t} + \frac{(1-\alpha)}{2} s_{j,t}^i + \alpha s_t^i, \quad i, j \in \{H, U\}, i \neq j$$

and the consumer price index in country i in the rest of the world as

$$p_{c,t}^i = p_{i,t}^i + \alpha s_t^i, \quad i \in [0, 1] \quad (2.9)$$

The domestic producer price inflation in country i is defined as rate of change in the price index of domestically produced goods, $\pi_t^i = \Delta p_{i,t} = p_{i,t} - p_{i,t-1}$. Using equation (2.9), the consumer price inflation in country i in the rest of the world can be written as $\pi_{c,t}^i = \pi_t^i + \alpha \Delta s_t^i$, and the consumer price inflation in MU country i can be written as

$$\pi_{c,t}^i = \pi_t^i + \frac{(1-\alpha)}{2} \Delta s_{j,t}^i + \alpha \Delta s_t^i, \quad i, j \in \{H, U\}, i \neq j \quad (2.10)$$

The consumer price inflation in the monetary union is defined as the average of country specific inflation levels, ie $\pi_{c,t}^{MU} = \frac{1}{2} (\pi_{c,t}^H + \pi_{c,t}^U)$. Noting that $s_{H,t}^U = -s_{U,t}^H$ and $p_{F,t}^H = p_{F,t}^U = p_{F,t}$, we may write the union wide consumer price inflation as

$$\pi_{c,t}^{MU} = \frac{(1-\alpha)}{2} (\pi_t^H + \pi_t^U) + \alpha \Delta \log \mathcal{E}_t + \alpha \pi_t^*$$

where $\pi_t^* = \int_0^1 \pi_t^i di$.

2.3 International risk sharing

We assume complete markets for state-contingent securities across world economy. The uncovered interest rate parity (UIP) condition between country i 's and rest of the world's interest rates is

$$R_t^i = \frac{E_t \{ \mathcal{E}_{t+1}^i \}}{\mathcal{E}_t^i} R_t^*$$

where R_t^* is the world interest rate and \mathcal{E}_t^i is the effective exchange rate of the country i , and the uncovered interest rate parity between the MU and a country i in the rest of the world can be written as

$$R_t = \frac{E_t \{ \mathcal{E}_{i,t+1} \}}{\mathcal{E}_{i,t}} R_t^i \quad (2.11)$$

where R_t is the gross nominal interest rate of the MU . Combining equations (2.8) and (2.11) we may write the following condition between the MU country i and a country j in the rest of the world

$$\frac{C_t^i P_{c,t}^i}{C_t^j P_{c,t}^j \mathcal{E}_{j,t}} = E_t \left\{ \frac{C_{t+1}^i P_{c,t+1}^i}{C_{t+1}^j P_{c,t+1}^j \mathcal{E}_{j,t+1}} \right\}, \quad i \in \{H, U\}, j \in [0, 1] \quad (2.12)$$

Iterating the above equation we can write the relationship as

$$C_t^i = \vartheta_j^i C_t^j \frac{\mathcal{E}_{j,t} P_{c,t}^j}{P_{c,t}^i}, \quad \forall t, \quad i \in \{H, U\}, j \in [0, 1] \quad (2.13)$$

where ϑ_j^i is a constant depending on steady state. As shown in the appendix 6, the assumption of symmetric initial conditions yields $\vartheta_j^i = 1$ for all i and j . Integrating equation (2.13) over j , we have an optimal risk-sharing condition as follows

$$C_t^i = C_t^* (S_t^i)^{1-\alpha} (S_{j,t}^i)^{-\frac{(1-\alpha)}{2}}, \quad i, j \in \{H, U\}, i \neq j \quad (2.14)$$

where $C_t^* = \int_0^1 C_t^i di$ is the worldwide aggregate consumption index. For the MU countries, logarithmic version of the above equation can be written as

$$c_t^i = c_t^* + (1 - \alpha) s_t^i - \frac{(1 - \alpha)}{2} s_{j,t}^i, \quad i, j \in \{H, U\}, i \neq j \quad (2.15)$$

and for the country i in the rest of the world

$$c_t^i = c_t^* + (1 - \alpha) s_t^i$$

Notice that using condition (2.14) and purchasing power parity (PPP) inside the MU the consumption levels of the MU countries are equal. Since all the countries have a home bias in their consumptions, the PPP holds only in the long run, ie in symmetric equilibrium.

2.4 Public sector

In each country the public spending yields utility to local households. Government of a country i buys only locally produced goods in a CES bundle

$$G_t^i = \left(\int_0^1 G_i(k)^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

With cost minimization the demand for locally produced good k by government is given by

$$G_t^i(k) = \left(\frac{P_{i,t}^i(k)}{P_{i,t}^i} \right)^{-\epsilon} G_t^i \quad (2.16)$$

As in Galí and Monacelli (2005) we assume that the governments issues an employment subsidy, τL_t^i , to local firms. The purpose of this subsidy is to neutralize the distortions associated with the market power of each firm.

The public spending and employment subsidy are financed by lump sum taxes T_t^i . The nominal government budget constraint can be written in form

$$T_t^i = \tau L_t^i + P_{i,t}^i G_t^i$$

The public consumption has two roles in this model. The first role is to provide utility to local households while the second role is to help the stabilization of the MU 's aggregate economy after country specific shocks, ie using the fiscal policy of both MU countries' the central planner can mimic the missing exchange rate fluctuations.

2.5 Firms

In each country there is a continuum of firms indexed in interval $[0, 1]$. The firms are owned by the local households. The demand for good k produced in a country i by households of country j is given by

$$C_{i,t}^j(k) = \left(\frac{P_{i,t}^i(k)}{P_{i,t}^i} \right)^{-\epsilon} C_{i,t}^j$$

and the demand by local government is given above in equation (2.16).

Each firm in country i produces a differentiated good with linear technology

$$Y_t^i(k) = A_t^i L_t^i(k), \quad i, k \in [0, 1]$$

where the country specific productivity level A_t^i follows the AR(1) process

$$\log(A_t^i) = \theta \log(A_{t-1}^i) + e_t^i \quad (2.17)$$

with $\theta \in [0, 1]$. Since production technology is linear and productivity level is common to all producers within a country, the real marginal cost, in terms of locally produced goods, in country i is given by

$$MC_t^i = \frac{(1 - \tau)W_t^i}{P_{i,t}^i A_t^i} \quad (2.18)$$

where τ is the employment subsidy. Labor is supplied by local households. With linear technology the aggregate labor demand in country i is given simply by

$$L_t^i = \int_0^1 L_t^i(k) dk = \frac{Y_t^i Z_t^i}{A_t^i} \quad (2.19)$$

where $Z_t^i = \int_0^1 \frac{Y_t^i(k)}{Y_t^i} dk$ and the aggregate product of country i is defined as $Y_t^i = \left[\int_0^1 Y_t^i(k)^{\frac{\epsilon-1}{\epsilon}} dk \right]^{\frac{\epsilon}{\epsilon-1}}$. In logarithmic terms, the first order approximation of aggregate labor demand in country i is thus given as $l_t^i = y_t^i - a_t^i$.

Price setting follows the rule by Calvo (1983). In each period in country i a fraction $0 < 1 - \xi^i < 1$ of firms are randomly and independently chosen

and permitted to choose their prices, while the prices of other firms remain unchanged. When permitted, a firm k chooses its price to maximize the present value of profits over the period when the chosen price is in effect. The optimal price for the monopolistic producer in period t is given by

$$\check{P}_t^i(k) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \left\{ \sum_{j=0}^{\infty} (\beta \xi^i)^j Y_{t+j}^i(k) P_{t+j}^i m c_{t+j}^i(k) \right\}}{E_t \left\{ \sum_{j=0}^{\infty} (\beta \xi^i)^j Y_{t+j}^i(k) \right\}} \quad (2.20)$$

In each period t the fraction $1 - \xi^i$ of firms in country i choose the same optimal price \check{P}_t^i . Now the consumer price index of goods produced in country i in period t is given by

$$P_t^i = \left(\xi^i (P_{t-1}^i)^{1-\epsilon} + (1 - \xi^i) (\check{P}_t^i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \quad (2.21)$$

Using log-linear approximations of equations (2.21) and (2.20) we can solve for the log-linear price index of traded goods produced in country i

$$p_t^i = \frac{\beta}{1 + \beta} E_t \{ p_{t+1}^i \} + \frac{1}{1 + \beta} p_{t-1}^i + \frac{(1 - \xi^i)(1 - \beta \xi^i)}{(1 + \beta) \xi^i} \widehat{m c}_t^i$$

where variables denoted with hat are the percentage deviation from initial steady state value, ie $\hat{x}_t = x_t - x$. From above equation we can solve for domestic producer price inflation

$$\pi_t^i = \beta E_t \{ \pi_{t+1}^i \} + \frac{(1 - \xi^i)(1 - \beta \xi^i)}{\xi^i} \widehat{m c}_t^i \quad (2.22)$$

3 Equilibrium dynamics

3.1 Solving for equilibrium dynamics

In the equilibrium national labor markets and international and national goods markets clear. The market clearing condition for goods originating from a MU country i yields

$$\begin{aligned} Y_t^i &= C_{i,t}^i + C_{i,t}^j + \int_0^1 C_{i,t}^f df + G_t^i \\ &= \left(\frac{P_{c,t}^i}{P_{i,t}^i} \right) C_t^i + G_t^i \\ &= C_t^i (S_{j,t}^i)^{\frac{(1-\alpha)}{2}} (S_t^i)^\alpha + G_t^i, \quad i, j \in \{H, U\}, f \in \{0, 1\} \end{aligned} \quad (3.1)$$

where the second equation is derived using the equations (2.6) and (2.13). Market clearing condition for country i in the rest of the world is

$$Y_t^i = C_t^i (S_t^i)^\alpha + G_t^i$$

and for the whole world economy

$$Y_t^* = C_t^* + G_t^*$$

The amount of goods produced by firm k at a country i is given by

$$Y_t^i(k) = \left(\frac{P_t^i(k)}{P_{i,t}} \right)^{-\epsilon} Y_t^i$$

where Y_t^i is the aggregate output of a country i defined as

$$Y_t^i = \left(\int_0^1 (Y_t^i(k))^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$$

By denoting the share of public spending in a steady state GDP by γ , we may approximate equation (3.1) around a symmetric steady state as

$$\hat{y}_t^i = (1 - \gamma) \left[\hat{c}_t^i + \frac{(1 - \alpha)}{2} s_{j,t}^i + \alpha s_t^i \right] + \gamma \hat{g}_t^i, \quad i, j \in \{H, U\} \quad (3.2)$$

and using equation (2.15) we can write the above equation as

$$\hat{y}_t^i = (1 - \gamma) (\hat{c}_t^* + s_t^i) + \gamma \hat{g}_t^i, \quad i, j \in \{H, U\} \quad (3.3)$$

The Euler equation (2.8) of a household in country i can be written in logarithmic terms as

$$c_t^i = E_t \{c_{t+1}^i\} - (r_t^i - E_t \{\pi_{c,t+1}^i\} - \rho) \quad (3.4)$$

where $\rho = -\log \beta$ and r_t^i is a logarithm of the nominal gross interest rate of currency used in country i .

Using equations (2.10), (3.4) and (3.2), we can write the dynamic IS equation for a MU country i as

$$\hat{y}_t^i = E_t \{\hat{y}_{t+1}^i\} - (1 - \gamma) (r_t - E_t \{\pi_{t+1}^i\} - \rho) - \gamma E_t \{\Delta \hat{g}_{t+1}^i\} \quad (3.5)$$

The same condition holds also for all countries in the rest of the world. We may write the dynamic IS equation for the monetary union as

$$E_t \{\Delta \hat{g}_{t+1}^{MU}\} = (1 - \gamma) \left(r_t - \frac{E_t \{\pi_{t+1}^H + \pi_{t+1}^U\}}{2} - \rho \right) + \frac{\gamma}{2} E_t \{\Delta \hat{g}_{t+1}^H + \Delta \hat{g}_{t+1}^U\}$$

Log-linearized version of equation (2.18) for real marginal costs of producers in a MU country i , can be written as

$$\begin{aligned} mc_t^i &= w_t^i - p_{i,t} - a_t^i + \log(1 - \tau) \\ &= c_t^i + \phi l_t^i + p_{c,t} - p_{i,t} - a_t^i + \log(1 - \tau) - \log(1 - \kappa) \\ &= c_t^i + \alpha s_t^i + \frac{(1 - \alpha)}{2} s_{j,t}^i + \phi l_t^i - a_t^i + \log(1 - \tau) - \log(1 - \kappa) \end{aligned} \quad (3.6)$$

where $i, j \in \{H, U\}$, $i \neq j$ and the second equality is derived using the equation (2.7). Using equation (3.2) we can write equation (3.6) as

$$\widehat{mc}_t^i = \left(\frac{1}{1-\gamma} + \phi \right) \hat{y}_t^i - \frac{\gamma}{1-\gamma} \hat{g}_t^i - (1+\phi) a_t^i \quad (3.7)$$

Substituting the equation (3.7) to the equation for domestic inflation, equation (2.22) we can write the New Keynesian Phillips curve in country i as

$$\pi_t^i = \beta E_t \{ \pi_{t+1}^i \} + \lambda^i (1+\phi) (\hat{y}_t^i - a_t^i) - \frac{\lambda^i \gamma}{1-\gamma} (\hat{g}_t^i - \hat{y}_t^i) \quad (3.8)$$

where $\lambda^i = \frac{(1-\xi^i)(1-\beta\xi^i)}{\xi^i}$.

3.2 The flexible price allocation

The optimal allocation from the viewpoint of the small open monetary union is a solution to a social planner's problem of maximizing the average utility of the representative households in the monetary union taking the rest of the world consumption as given. The social planner's problem is to maximize

$$V_t = \log C_t^H + \log C_t^U + \kappa \log G_t^H + \kappa \log G_t^U - \frac{(L_t^H)^{1+\phi}}{1+\phi} - \frac{(L_t^U)^{1+\phi}}{1+\phi} \quad (3.9)$$

subject to market clearing constraints (3.1) and technological constraints (2.19) and risk sharing conditions (2.14). Using the equations (3.1) and (2.14) we may write the consumption level in a MU country i as

$$C_t^i = (Y_t^H - G_t^H)^{\frac{(1-\alpha)}{2}} (Y_t^U - G_t^U)^{\frac{(1-\alpha)}{2}} (C_t^*)^\alpha, \quad i \in \{H, U\} \quad (3.10)$$

By plugging equation (3.10) to equation (3.9) we may write the first order conditions of the planner's problem as

$$\begin{aligned} \frac{dV_t}{dG_t^i} = 0 &\iff G_t^i = \frac{\kappa}{1-\alpha+\kappa} A_t^i L_t^i, \quad i \in \{H, U\} \\ \frac{dV_t}{dL_t^i} = 0 &\iff (L_t^i)^\phi = (1-\alpha) \frac{A_t^i}{Y_t^i - G_t^i}, \quad i \in \{H, U\} \end{aligned}$$

The solution to the social planner's problem is given by pair

$$(L_t^i, G_t^i) = \left((1-\alpha+\kappa)^{\frac{1}{1+\phi}}, \frac{\kappa}{1-\alpha+\kappa} Y_t^i \right) \quad (3.11)$$

ie the employment level is fixed and government consumption is a constant share of output. Notice that higher degree of MU 's openness to international trade decreases the natural level of employment and increases the share of government consumption in GDP. The optimal allocation for the countries in the rest of the world is equal to the allocation (3.11).

As shown in Galí and Monacelli (2005), above planner's solution can be supported as an equilibrium with flexible prices. With variables associated with the flexible prices denoted by upper bars, we may write the marginal costs in a MU country i as

$$\begin{aligned}
1 - \frac{1}{\epsilon} &= \overline{MC}_t^i \\
&= (1 - \tau) \frac{\overline{P}_{c,t}^i \overline{C}_t^i (\overline{L}_t^i)^\phi}{\overline{P}_{i,t}^i A_t^i} \\
&= (1 - \tau) \frac{(\overline{L}_t^i)^\phi}{A_t^i} \overline{C}_t^i (\overline{S}_{j,t}^i)^{\frac{(1-\alpha)}{2}} (\overline{S}_t^i)^\alpha \\
&= (1 - \tau) (\overline{L}_t^i)^{(1+\phi)} \left(1 - \frac{\overline{G}_t^i}{\overline{Y}_t^i} \right), \quad i, j \in \{H, U\}, i \neq j
\end{aligned}$$

To guarantee that the flexible price allocation yields the optimal outcome in both MU countries the governments must set $(1 - \tau)(1 - \alpha)$ to equal $\frac{\epsilon-1}{\epsilon}$ and government spending of each MU country must follow the rule

$$\overline{G}_t^i = \frac{\kappa}{1 - \alpha + \kappa} \overline{Y}_t^i = \kappa (1 - \alpha + \kappa)^{\frac{-\phi}{1+\phi}} A_t^i \quad (3.12)$$

Using equations (3.1) and (3.12) with assumption of full risk sharing we can write the bilateral terms of trade between the MU countries with flexible prices as

$$\overline{S}_{U,t}^H = \frac{A_t^H}{A_t^U}$$

and the difference between the producer price inflation rates as

$$\overline{\pi}_t^H - \overline{\pi}_t^U = -(\Delta a_t^H - \Delta a_t^U) \quad (3.13)$$

In logarithmic terms, the output and government spending associated with the flexible price equilibrium and optimal policy are $\overline{y}_t^i = \log \mu + a_t^i$ and $\overline{g}_t^i = \log \nu + a_t^i$, where $\mu = (1 - \alpha + \kappa)^{\frac{1}{1+\phi}}$ and $\nu = \kappa \mu^{-\phi}$. Below these variables are referred as natural levels of output and government spending. Notice that the logarithmic steady state values of output and government spending are $y^i = \log \mu$ and $g^i = \log \nu$, respectively.

3.3 Dynamics with rigid price setting

We define output and government spending gaps in a MU country i as logarithmic deviation of output and government spending from their natural levels, ie $\tilde{y}_t^i = y_t^i - \overline{y}_t^i$ and $\tilde{g}_t^i = g_t^i - \overline{g}_t^i$, respectively. The fiscal gap in country i is defined as: $\tilde{f}_t^i = \tilde{g}_t^i - \tilde{y}_t^i = (g_t^i - y_t^i) - \log \nu + \log \mu$. Now we are able to write the New Keynesian Phillips Curve, equation (3.8), of MU country i in terms of output and fiscal gaps

$$\pi_t^i = \beta E_t \{ \pi_{t+1}^i \} + \lambda^i (1 + \phi) \tilde{y}_t^i - \frac{\kappa \lambda^i}{1 - \alpha} \tilde{f}_t^i \quad (3.14)$$

Defining the union wide producer price inflation as $\pi_t^{MU} = \frac{1}{2}\pi_t^H + \frac{1}{2}\pi_t^U$, we can write the monetary union's NKPC as

$$\pi_t^{MU} = \beta E_t \{ \pi_{t+1}^{MU} \} + \lambda^{MU} (1 + \phi) \tilde{y}_t^{MU} - \frac{\kappa \lambda^{MU}}{1 - \alpha} \tilde{f}_t^{MU}$$

where $\lambda^{MU} = \frac{1}{2}(\lambda^H + \lambda^U)$. By using equation (3.5) we can write output gap in a MU country as

$$\tilde{y}_t^i = E_t \left\{ \tilde{y}_{t+1}^i - \left(\frac{\kappa}{1 - \alpha} \right) \tilde{f}_{t+1}^i \right\} - (r_t - E_t \{ \pi_{t+1}^i \} - \bar{r}_t^i) + \left(\frac{\kappa}{1 - \alpha} \right) \tilde{f}_t^i \quad (3.15)$$

where \bar{r}_t^i is the natural rate of interest in a MU country i given by

$$\begin{aligned} \bar{r}_t^i &= \rho + \left(\frac{1 - \alpha + \kappa}{1 - \alpha} \right) E_t \left\{ \Delta \bar{y}_{t+1}^i - \frac{\kappa}{1 - \alpha + \kappa} \Delta \bar{g}_{t+1}^i \right\} \\ &= \rho + E_t \{ \Delta \bar{y}_{t+1}^i \} = \rho + E_t \{ \Delta a_{t+1}^i \} \end{aligned}$$

The monetary union's output gap is given by

$$\tilde{y}_t^{MU} = E_t \left\{ \tilde{y}_{t+1}^{MU} - \left(\frac{\kappa}{1 - \alpha} \right) \tilde{f}_{t+1}^{MU} \right\} - (r_t - E_t \{ \pi_{t+1}^{MU} \} - \bar{r}_t^{MU}) + \left(\frac{\kappa}{1 - \alpha} \right) \tilde{f}_t^{MU} \quad (3.16)$$

where $\bar{r}_t^{MU} = \rho + \frac{E_t}{2} \{ \Delta a_{t+1}^H + \Delta a_{t+1}^U \}$. Finally, with equations (3.3) and (3.13) we can write the difference between changes of output gaps of the MU countries as

$$\Delta \tilde{y}_t^H - \Delta \tilde{y}_t^U = \frac{\kappa}{1 - \alpha} \left(\Delta \tilde{f}_t^H - \Delta \tilde{f}_t^U \right) - (\pi_t^H - \pi_t^U + \Delta a_t^H - \Delta a_t^U) \quad (3.17)$$

4 Optimal monetary and fiscal policy under commitment

4.1 Optimization

We restrict our analysis of the optimal fiscal and monetary policies to a case with full coordination, ie monetary and fiscal policy are practiced by a single policymaker to maximize the average welfare of households in the monetary union. The second order Taylor approximation of average welfare of households in the monetary union is derived in the appendix 6 and can be written as

$$\begin{aligned} V_t &= \Upsilon E_t \sum_{s=0}^{\infty} \sum_{i \in \{H, U\}} \left\{ \frac{\beta^{t+s}}{2} \left[\frac{\epsilon}{\lambda^i} (\pi_{t+s}^i)^2 + \frac{\kappa}{1 - \alpha} (\tilde{f}_{t+s}^i)^2 + (1 + \phi) (\tilde{y}_{t+s}^i)^2 \right] \right\} \\ &\quad + \text{t.i.p}, \end{aligned} \quad (4.1)$$

where $\Upsilon = -(1 - \alpha + \kappa)$ and the term t.i.p refers to terms that are independent to policy. Notice that utility parameter κ and the parameter

for openness of the monetary union α enter the utility function (4.1) in a weight for fiscal gap and they also affect the level of utility.

The optimal policy can be interpreted as rules for interest rate $\{r_t\}_{s=0}^\infty$ and national fiscal gaps $\{\tilde{f}_{t+s}^H, \tilde{f}_{t+s}^U\}_{s=0}^\infty$ that maximize (4.1) subject to the country specific New Keynesian Phillips Curves (3.14) and the difference between changes of output gaps of the *MU* countries (3.17). Following Galí and Monacelli (2008) we first solve for optimal processes for $\{\tilde{f}_{t+s}^i, \tilde{y}_{t+s}^i, \pi_{t+s}^i\}_{s=0}^\infty$, $i \in \{H, U\}$, and then solve for path of nominal interest rate that support the optimal process. The first order conditions of the maximization problem are

$$E_t \left\{ \frac{\epsilon}{\lambda^H} \pi_{t+s}^H - \Delta \psi_{\pi, t+s}^H - \psi_{y, t+s} \right\} = 0, \quad \forall s \geq 0 \quad (4.2)$$

$$E_t \left\{ \frac{\epsilon}{\lambda^U} \pi_{t+s}^U - \Delta \psi_{\pi, t+s}^U + \psi_{y, t+s} \right\} = 0, \quad \forall s \geq 0 \quad (4.3)$$

$$E_t \left\{ \tilde{f}_{t+s}^H - \lambda^H \psi_{\pi, t+s}^H + \psi_{y, t+s} - \beta \psi_{y, t+s+1} \right\} = 0, \quad \forall s \geq 0 \quad (4.4)$$

$$E_t \left\{ \tilde{f}_{t+s}^U - \lambda^U \psi_{\pi, t+s}^U - \psi_{y, t+s} + \beta \psi_{y, t+s+1} \right\} = 0, \quad \forall s \geq 0 \quad (4.5)$$

$$E_t \left\{ (1 + \phi) \tilde{y}_{t+s}^H + \lambda^H (1 + \phi) \psi_{\pi, t+s}^H - \psi_{y, t+s} + \beta \psi_{y, t+s+1} \right\} = 0, \quad \forall s \geq 0 \quad (4.6)$$

$$E_t \left\{ (1 + \phi) \tilde{y}_{t+s}^U + \lambda^U (1 + \phi) \psi_{\pi, t+s}^U + \psi_{y, t+s} - \beta \psi_{y, t+s+1} \right\} = 0, \quad \forall s \geq 0 \quad (4.7)$$

where $\psi_{\pi, t}^i$ are Lagrange multipliers associated with equations (3.14) and $\psi_{y, t}$ is associated with equation (3.17). Using equations (4.2) and (4.3) we can write the producer price inflation in the monetary union as

$$E_t \left\{ \pi_{t+s}^{MU} \right\} = \frac{E_t}{2} \left\{ \frac{1}{\epsilon} (\lambda^H \Delta \psi_{\pi, t+s}^H + \lambda^U \Delta \psi_{\pi, t+s}^U) + (\lambda^H - \lambda^U) \psi_{y, t+s} \right\} \quad (4.8)$$

With first order conditions (4.4) and (4.5) we can write the union wide fiscal gap as

$$E_t \left\{ \tilde{f}_{t+s}^{MU} \right\} = \frac{E_t}{2} \left\{ \lambda^H \psi_{\pi, t+s}^H + \lambda^U \psi_{\pi, t+s}^U \right\} \quad (4.9)$$

and using equations (4.6) and (4.7) we can write the union wide output gap as

$$E_t \left\{ \tilde{y}_{t+s}^{MU} \right\} = -\frac{E_t}{2} \left\{ \lambda^H \psi_{\pi, t+s}^H + \lambda^U \psi_{\pi, t+s}^U \right\} \quad (4.10)$$

Together the first order conditions (4.4) and (4.6), or equivalently (4.5) and (4.7), imply that

$$E_t \left\{ \tilde{f}_{t+s}^i + (1 + \phi) \tilde{y}_{t+s}^i \right\} = -\lambda^i \phi E_t \left\{ \psi_{\pi, t+s}^i \right\} \quad (4.11)$$

Using the conditions (4.9) and (4.10) above, we can see that under optimal policy

$$E_t \left\{ \tilde{f}_{t+s}^{MU} \right\} = -E_t \left\{ \tilde{y}_{t+s}^{MU} \right\}$$

By taking difference of equation (4.11) and plugging it to equation (4.8) we can write an optimality condition for the union wide producer price inflation as

$$E_t \{ \pi_{t+s}^{MU} \} = E_t \left\{ -\frac{1}{\epsilon} \Delta \tilde{y}_{t+s}^{MU} + \frac{(\lambda^H - \lambda^U)}{2} \psi_{y,t+s} \right\} \quad (4.12)$$

The rational expectations equilibrium for monetary union satisfies equations (3.14) and (3.17) and the first order conditions of the maximization problem for any stochastic processes $\{a_{t+s}^H, a_{t+s}^U\}_{s=0}^\infty$. We can write the rational expectations equilibrium as an allocation $\left\{ \pi_{t+s}^i, \tilde{f}_{t+s}^i, \tilde{y}_{t+s}^i, \psi_{\pi,t+s}^i, \psi_{y,t+s} \right\}_{s=0}^\infty$, with $i \in \{H, U\}$ and initial condition $\psi_{\pi,0}^i = 0$. Using equation (3.16) we can now write the equilibrium path of nominal interest rate of common currency as

$$r_t = \left(1 + \frac{\kappa}{1 - \alpha} \right) E_t \{ \Delta \tilde{y}_{t+1}^{MU} \} + E_t \{ \pi_{t+1}^{MU} \} + \bar{r}_t^{MU} \quad (4.13)$$

ie the nominal interest rate depends on expected change of union wide output gap, expected union wide producer price inflation and the natural level of interest rate.

Consider now the policy of setting $\pi_t^{MU} = \tilde{f}_t^{MU} = \tilde{y}_t^{MU} = 0$ for every period. Equation (4.12) implies that under this assumption we have $\psi_{y,t} = 0$ in every period. Thus suggested policy is not an equilibrium.

4.2 Symmetric price rigidities

In a special case with symmetric nominal rigidities, ie $\lambda^H = \lambda^U$, we have

$$\pi_t^{MU} = -\frac{1}{\epsilon} \Delta \tilde{y}_t^{MU} = \frac{1}{\epsilon} \Delta \tilde{f}_t^{MU}$$

In this case, setting $\pi_t^{MU} = \tilde{f}_t^{MU} = \tilde{y}_t^{MU} = 0$, for every period, maximizes the welfare of households in monetary union.² With staggered pricing only fraction of producer prices adjust, thus $\psi_{\pi,t}^i \neq 0$ when the economy is not in the steady state. By equation (4.11) we may conclude that setting $\tilde{f}_t^i = \tilde{y}_t^i = 0$ for every period is not an equilibrium. Note that under optimal policy we have $\tilde{f}_t^H = -\tilde{f}_t^U$ and $\tilde{y}_t^H = -\tilde{y}_t^U$ implying that also $\pi_t^H = -\pi_t^U$ in every period. In this special case optimal monetary policy is to keep the nominal interest rate of the common currency equal to its natural level.

Analogously one could solve for the optimal monetary and fiscal policies for a small country in the rest of the world. In that case, the optimal policy is to set $\tilde{f}_t^i = \tilde{y}_t^i = \pi_t^i = 0$ for every period. In this special case $r_t = \bar{r}_t^{MU}$ in every period, and the openness to trade with countries outside the MU does not an effect on the optimal policy.

²This result is common in literature, see for example Galí and Monacelli (2008).

5 Numerical experiments

In this section we present three numerical experiments. As a benchmark we first study the implications of an unit innovation in the productivity at country H under an assumption of symmetric price rigidities across the MU countries. In the second experiment we study the effect of openness to international trade on the fluctuations caused by country specific shock. Finally in third experiment we study the effects of the same shock under asymmetric price rigidities.

5.1 Parametrization

The time frequency of the model is interpreted to be one quarter. For the annual interest rate of four percent the discount factor β has a parameter value of .99, which is standard in quarterly business cycle models. As in Galí and Monacelli (2005) the steady state markup is set to be 20 percent, ie the value of elasticity of substitution between locally produced goods ϵ is 6. We also assume that elasticity of labor supply is $\frac{1}{3}$, ie ϕ has value 3. In the euro area average share of public spending in GDP is approximately 23 per cent and the openness to trade, defined as share of imports plus exports of GDP, in the whole euro area is 13 percent. To replicate these estimates we set the parameters α and κ , to have values .08 and .27, respectively. For the AR(1) process on labor productivity we use the estimates by Galí and Monacelli (2005) and set θ to have value .66. As discussed in the appendix 6, with this parameter setup the model is stable.

5.2 Symmetric nominal rigidities

First we study the dynamics effects of a productivity shock in MU country H under the optimal policy in the special case with symmetric nominal rigidities in both MU countries. As in Galí and Monacelli (2005) we assume that in each period 25 percent of firms set new prices, ie ξ^i has value 0.75. The impulse responses to a unit innovation in a_t^H are presented in Figure 1.

The one percent rise of productivity in country H increases the natural level of output by an identical fraction, while in country U the natural level of output stays unaltered. Rise of productivity in the monetary union also causes a decrease in natural rate of nominal interest rate. The optimal policy is to keep the nominal interest rate on its natural level, thus the nominal interest rate of common currency falls. By uncovered interest rate parity, the fall of the nominal interest rate causes instant depreciation of the common currency. As a result the effective terms of trade in both MU countries increase, inducing an expenditure switching effect from goods produced in the rest of the world to goods produced in MU .

With increased demand the local producers in country U are able to increase their prices, while producers in country H are able to lower their

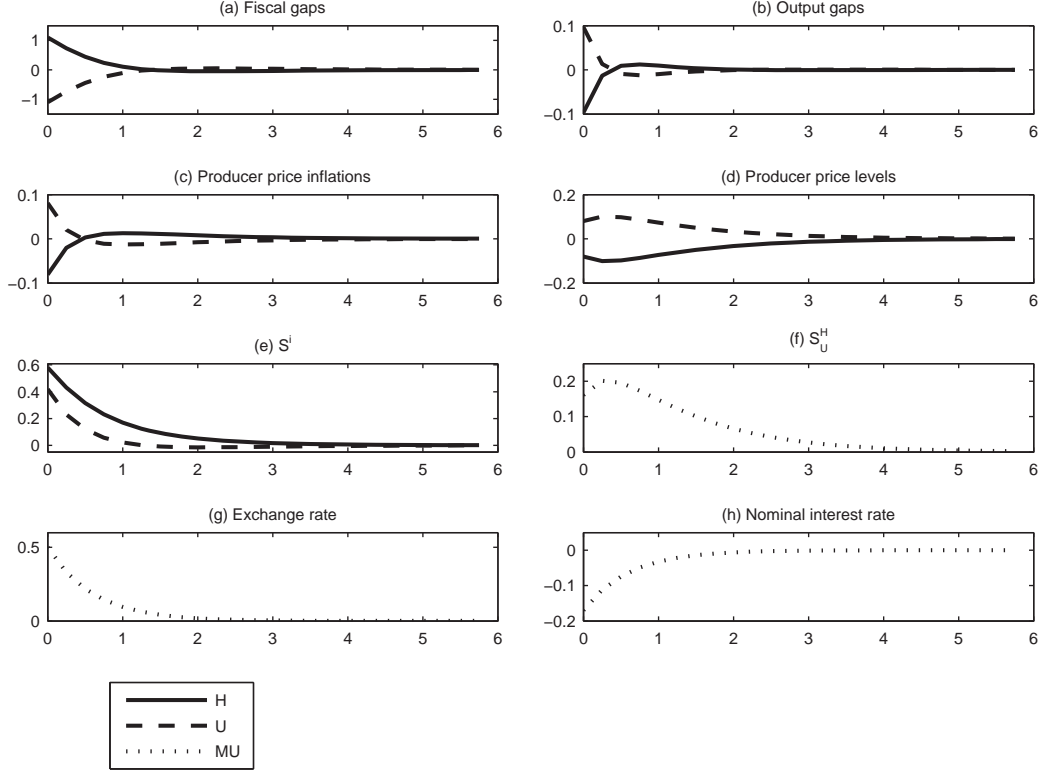


Figure 1: Effects of productivity shock in country H . The horizontal axes denote time in years and the vertical axes denote the variables' percentage deviations from the initial steady state.

prices with diminished marginal costs. Under the optimal policy, the fall of the producer prices in country H is balanced by an equal rise of producer prices in country U . As a result, output gap in country H is negative, ie output level does not reach its flexible price level, while in country U output gap is positive. In both countries these inefficiencies are smoothed by fiscal policy. As a result government spending exceeds its natural level in country H and the opposite in country U .

The lack of exchange rate fluctuations between the MU countries induces large expenditure switching effects between the goods produced in these two countries. Under the optimal policy, the central planner uses the public spending of both countries to reduce the resulting output gaps and producer price inflations. The public spending country with an increase in the productivity experiences a negative fiscal gap while in the country with lower productivity experiences a positive fiscal gap.

5.3 Effect of openness

We study the implications of openness to international trade under symmetric price rigidities. Keeping the values of other parameters constant we examine the differences between closed monetary union and open monetary union by

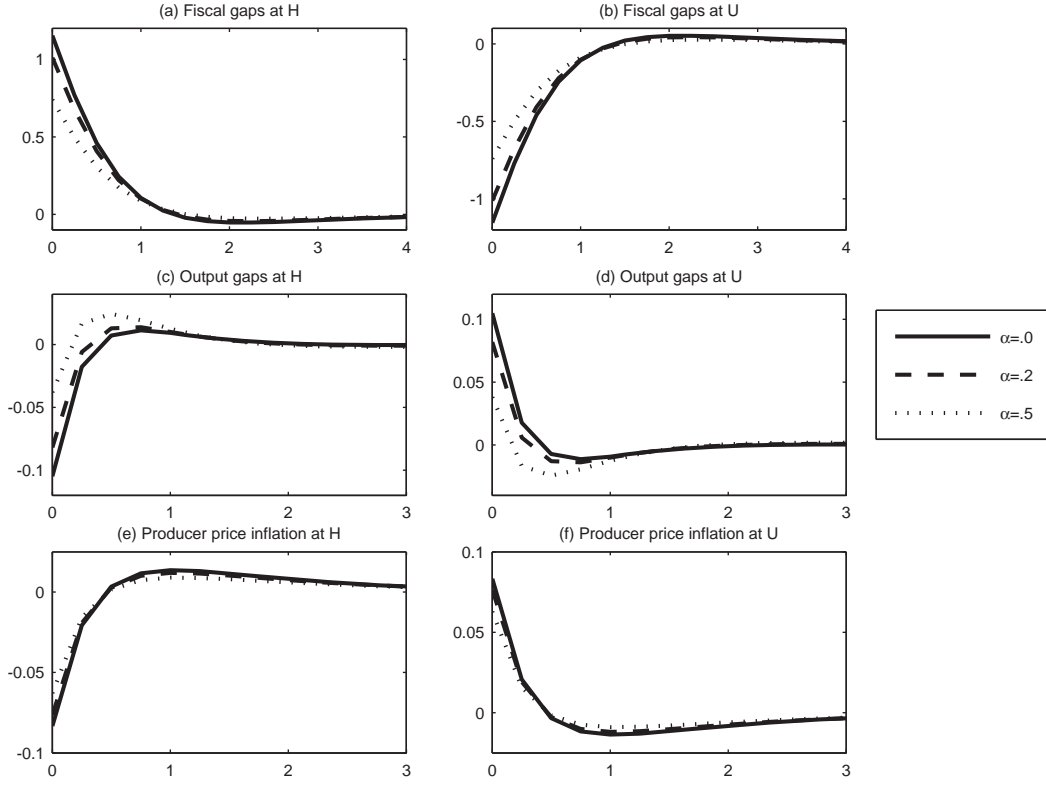


Figure 2: shock in country H . The horizontal axes denote time in years and the

setting degree of openness to have values .0, .2, and .5. Impulse responses for unit innovation on the productivity at country H are presented in Figure 2. As in the case above, under symmetric price rigidities and optimal policy, the union wide variables of output and fiscal gaps and producer price inflation remain at their natural level.

The depreciation of common currency increases the real demand of goods produced in the monetary union by households in both MU and the rest of the world, which decreases the expenditure switching between goods produced in MU countries H and U and stabilizes the monetary union's economy. As seen from Figure 2, fluctuations of both fiscal and output gaps and the producer price inflation are smaller when the monetary union is more open to the international trade.

With the unit elasticity of substitution between goods with different origins and logarithmic utility on consumption, the economic fluctuations in the rest of the world do not have an effect on the output and fiscal gaps and on the producer price inflations of the MU countries. We may conclude that the openness to trade with countries outside the MU stabilizes the economy and yields an utility increase to local households.

As mentioned above, under the assumption of symmetric price rigidities MU 's openness to trade does not have an effect on the optimal policy. Thus in this special case openness to trade yields only increases in the households utility and the outcomes of the model are close to ones of its closed counterpart.

5.4 Asymmetric price rigidities

In the third exercise we study the dynamic effects of a productivity shock in MU country H under the optimal policy when the price rigidity is different between the MU countries. We consider two cases. In the first case $\xi^H = .75$ and $\xi^U = .5$, ie prices are stickier in country H than in country U . In the second case $\xi^H = .5$ and $\xi^U = .75$, ie prices in country H are more flexible than in country U . The same parameter values are also used in the analysis by Beetsma and Jensen (2005). The impulse responses for both cases are presented in Figure 3. As noted above, setting $\pi_t^{MU} = \tilde{f}_t^{MU} = \tilde{y}_t^{MU} = 0$ for every period, when price rigidities are asymmetric, is not an equilibrium.

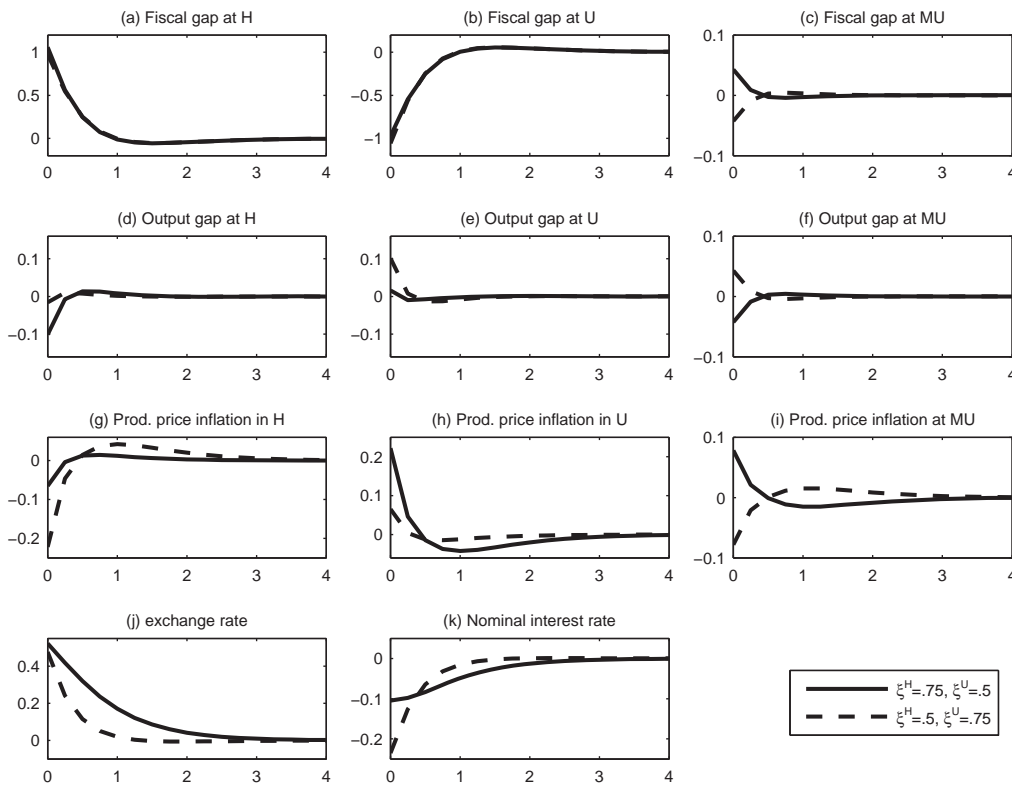


Figure 3: Effects of productivity shock in country H . The horizontal axes denote time in years and the vertical axes denote the variables' percentage deviations from the initial steady state.

Let us first look at the case with less nominal frictions in country U . As in our benchmark case, due to nominal rigidities prices react slowly to the fall of marginal costs in country H . As the prices are more flexible in country U , resulting producer price inflation is faster in country U than producer price deflation in country H , the resulting union wide inflation is positive. Due to difference in price rigidities, the output in country U is closer to its natural level than output in country H , inducing a union wide output gap. As argued above, the resulting union wide fiscal gap is negative of output gap.

In the second case, prices are more rigid in country U than in country H . Now the resulting producer price deflation in country H is faster than producer

price inflation in country U , resulting a union wide producer price deflation. With more flexible prices, output in country H is closer to its natural level than output in country U , thus the union wide output gap is now positive.

In both of the cases considered, increase of productivity in monetary union area causes a decrease in natural rate of nominal interest rate. As shown in equation (4.13) optimal path of nominal interest rate depends on expected union wide producer price inflation and expected change in union wide output gap. In the first case both of these are positive, thus the nominal interest rate is higher than the natural interest rate. In the second case the situation is exactly the opposite. Thus we may conclude that when the planner is maximizing the average utility of the households in MU fluctuations of the nominal interest rate and the exchange rate are driven by the economic fluctuations in the country with less rigid prices. Also note that the equation (4.13) shows that the optimal path of nominal interest rate is affected by the degree of openness. Since the openness to trade stabilizes the fluctuations of output gaps of MU countries it also reduces the fluctuations of the union wide variables.

6 Conclusions

This paper has explored the optimal monetary and fiscal policy mix in a micro founded model of a small open monetary union with price rigidities. The analysis of the optimal monetary and fiscal policy is restricted to full coordination and commitment. With this simple framework we have been able to study the exchange rate fluctuations of the common currency and the associated fluctuations of the terms of trade.

We have shown that the openness to international trade decreases the economic fluctuations inside the monetary union. When the countries of the monetary union are homogeneous, the degree of openness to trade does not affect the optimal path of the gross nominal interest rate. When the price rigidities are asymmetric across the countries, the optimal policy allows the union wide variables to deviate from their natural levels. Monetary union's openness to trade stabilizes these fluctuations.

The optimal coordinated fiscal policy is needed to reduce the fluctuations of the relative prices between the monetary union countries, ie its objective is to replicate the missing exchange rate fluctuations inside the monetary union. Our results show that the monetary union's openness to trade reduces the need for fiscal stabilization.

To derive the second order approximation to the average utility of the small monetary unions household we have restricted the parameter values of the model. Relaxing the assumptions on parameters, eg log utility, linear technology, unit elasticity of substitution between goods with different origin, would introduce more interesting dynamics to the model. Also relaxing the assumption of symmetric size and symmetric consumption allocation between the MU countries would bring the model closer to econometric applications. Also some comparison with a model with independent and discretionary fiscal policy could be fruitful.

References

- Beetsma, R M – Jensen, H (2005) **Monetary and fiscal policy interactions in a micro-founded model of a monetary union.** Journal of Internal Economics, 67(2), 320–352.
- Benigno, P (2004) **Optimal monetary policy in a currency area.** Journal of International Economics, 63(2), 293–320.
- Calvo, G (1983) **Staggered prices in a utility-maximizing framework.** Journal of Monetary Economics, 12, 383–398.
- Campa, J M – González-Mínguez, J M (2006) **Differences in exchange rate pass-through in the euro area.** European Economic Review, 50(1), 121–145.
- Clausen, V – Wohltmann, H-W (2005) **Monetary and fiscal policy dynamics in an asymmetric monetary union.** Journal of International Money and Finance, 24(1), 139–167.
- Corsetti, G – Pesenti, P (2001) **Welfare and Macroeconomic Interdependence.** Quarterly Journal of Economics, CXVI(2), 421–446.
- Ferrero, A (2009) **Fiscal and monetary rules for a currency union.** Journal of International Economics, 77(1), 1–10.
- Galí, J – Monacelli, T (2005) **Monetary Policy and Exchange Rate Volatility in a Small Open Economy.** Review of Economic Studies, 72(3), 707–734.
- Galí, G – Monacelli, T (2008) **Optimal monetary and fiscal policy in a currency union.** Journal of International Economics, 76(1), 116–132.
- Kirsanova, T – Satchi, M – Vines, D (2004) **Monetary Union: Fiscal Stabilization in the Face of Asymmetric Shocks.** CEPR Discussion Papers 4433, C.E.P.R. Discussion Papers.
- Uhlig, H (1995) **A toolkit for analyzing nonlinear dynamic stochastic models easily.** Discussion Paper 97, Tilburg University, Center for Economic Research.
- Woodford, M (2003) **Interest and Prices: Foundations of a Theory of Monetary Policy.** Princeton University Press, Princeto, NJ.

Appendix 1.

Stability of the model

Using the equations (2.17), (3.14), (3.17) and (4.2)–(4.7) our model can be written in form

$$\begin{aligned} 0 &= F_1 u_t + F_2 u_{t-1} + F_3 v_t + F_4 z_t \\ 0 &= E_t \{G_1 u_{t+1} + G_2 u_t + G_3 u_{t-1} + G_4 v_{t+1} + G_5 v_t + G_6 z_{t+1} + G_7 z_t\} \\ z_{t+1} &= H z_t + e_{t+1}; \quad E_t \{e_{t+1}\} = 0 \end{aligned}$$

where the vector $u_t = [\tilde{y}_t^H, \tilde{y}_t^U, \tilde{f}_t^H, \tilde{f}_t^U, \psi_{\pi,t}^H, \psi_{\pi,t}^U, a_t^H, a_t^U]$ is an endogenous state vector and $v_t = [\pi_t^H, \pi_t^U, \psi_{y,t}]$ is the vector of other endogenous variables. The vector $e_t = [e_t^H, e_t^U]$ represents the exogenous stochastic process. The matrices F_i , G_i and H collect the coefficients. As shown in Uhlig (1995) the model has a solution

$$\begin{aligned} u_t &= P u_{t-1} + Q e_t \\ v_t &= R v_{t-1} + S e_t \end{aligned}$$

which is stable if and only if all the eigenvalues of matrix P are inside the unit circle. With the parameter values used in this paper this condition holds, ie the model presented in this paper is stable and returns to initial steady state after exogenous shocks.

Appendix 2.

The perfect foresight steady state

In our model we assume that all the countries in the rest of the world are symmetric and the productivity level in each country is unity. We also assume that the government consumption in each country equals the optimal flexible price allocation (3.11). For simplicity we drop the time subindex for the variables associated with the perfect foresight steady state.

In the steady state productivity level of each country equals unity. Given the equal flexible price allocation in both MU countries, also the producer prices are equal in both countries and we can write the optimal risk-sharing conditions (2.14) as

$$C^i = C^* (S^i)^{1-\alpha}, \quad \forall i \in [0, 1] \quad (6.1)$$

In the appendix 6 we have shown that the model is stable. The iteration of the equation

$$\frac{C_t^i P_{c,t}^i}{C_t^j P_{c,t}^j \mathcal{E}_{j,t}} = E_t \left\{ \frac{C_{t+1}^i P_{c,t+1}^i}{C_{t+1}^j P_{c,t+1}^j \mathcal{E}_{j,t+1}} \right\}, \quad \forall t, i \in \{H, U\}, j \in [0, 1]$$

converges to the steady state and we may define the constant ϑ_j^i as

$$\vartheta_j^i = \frac{C^i P_c^i}{C^j P_c^j \mathcal{E}_j}, \quad i \in \{H, U\}, j \in [0, 1] \setminus \{H, U\}$$

Using the equation (6.1) and the definition of terms of trade, above equation can be written as

$$\begin{aligned} \vartheta_j^i &= \frac{C^i}{C^j} \left(\frac{P^i}{P^j} \right)^{1-\alpha} \\ &= \left(\frac{S^i P^i}{S^j P^j} \right)^{1-\alpha} \\ &= \left(\frac{\mathcal{E}}{\mathcal{E}^j} \right)^{1-\alpha}, \quad i \in \{H, U\}, j \in [0, 1] \end{aligned}$$

In the steady state nominal interest rate of each currency equals the world interest rate and also the exchange rates are equal. Thus we may conclude that also ϑ_j^i equals unity.

Appendix 3.

Welfare loss function

Using equation (3.10) we can write the sum of utilities of representative households in the MU as

$$\begin{aligned} V_t = E_0 \sum_{t=0}^{\infty} \beta_t \left\{ \right. & (1 - \alpha) [\log(Y_t^H - G_t^H) + \log(Y_t^U - G_t^U)] + 2\alpha c_t^* \\ & \left. + \kappa (g_t^H + g_t^U) - \frac{(L_t^H)^{1+\phi} + (L_t^U)^{1+\phi}}{1 + \phi} \right\} \end{aligned}$$

As shown in the appendix of Galí and Monacelli (2008), the second order Taylor approximation of term $\log(Y_t^i - G_t^i)$ can be written in terms of output and government spending gaps as

$$\begin{aligned} \log(Y_t^i - G_t^i) &= \frac{1 - \alpha + \kappa}{1 - \alpha} \tilde{y}_t^i - \frac{\kappa}{1 - \alpha} \tilde{g}_t^i - \frac{(1 - \alpha + \kappa) \kappa}{2(1 - \alpha)^2} (\tilde{g}_t^i - \tilde{y}_t^i)^2 \\ &\quad + \text{t.i.p} + o(\|a\|^3) \end{aligned}$$

where t.i.p denotes the terms that are independent of policy, eg aggregate consumption of the rest of the world, and $o(\|a\|^3)$ represents terms of the Taylor approximation that are of order higher than second, in the bound $\|a\|$ on the amplitude of productivity shock. In our simulations we consider only one percent shock to the productivity level.

Following Galí and Monacelli (2005) we can write the second order Taylor approximation of term for disutility from labor output about its flexible price

level as

$$\begin{aligned}\frac{(L_t^i)^{1+\phi}}{1+\phi} &= \frac{(\bar{L}_t^i)^{1+\phi}}{1+\phi} + (\bar{L}_t^i)^{1+\phi} \left[\tilde{y}_t^i + z_t^i + \frac{1}{2} (1+\phi) (\tilde{y}_t^i)^2 \right] + o(\|a\|^3) \\ &= (1-\alpha+\kappa) \left[\tilde{y}_t^i + z_t^i + \frac{1}{2} (1+\phi) (\tilde{y}_t^i)^2 \right] + \text{t.i.p} + o(\|a\|^3)\end{aligned}$$

where $z_t^i = \log \int_0^1 \frac{Y_t^i(k)}{Y_t^i} dk = \int_0^1 \left(\frac{P_t^i(k)}{P_{i,t}} \right)^{-\epsilon} dk$.

Collecting the terms above, we can write the second order Taylor approximation of MU's utility function as

$$\begin{aligned}V_t &= -(1-\alpha+\kappa) \sum_{i \in \{H,U\}} \sum_{t=0}^{\infty} \beta^t \left[z_t^i + \frac{\kappa}{2(1-\alpha)} (\tilde{f}_t^i)^2 + \frac{1}{2} (1+\phi) (\tilde{y}_t^i)^2 \right] \\ &\quad + \text{t.i.p} + o(\|a\|^3)\end{aligned}$$

Lemma 1. $z_t^i = \frac{\epsilon}{2} \text{var}_k \{p_t^i(k)\} + o(\|a\|^3)$.

Proof. Galí and Monacelli (2005).

Lemma 2. $\sum_{t=0}^{\infty} \beta^t \text{var}_k \{p_t^i(k)\} = \frac{1}{\lambda^i} \sum_{t=0}^{\infty} \beta^t (\pi_t^i)^2$, where $\lambda^i = \frac{(1-\xi^i)(1-\beta\xi^i)}{\xi^i}$.

Proof. Woodford (2003).

Now we can write the second order Taylor approximation of sum of utilities of representative households in the MU as

$$\begin{aligned}V_t &= -\frac{(1-\alpha+\kappa)}{2} \sum_{i \in \{H,U\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\epsilon}{\lambda^i} (\pi_t^i)^2 + \frac{\kappa}{1-\alpha} (\tilde{f}_t^i)^2 + (1+\phi) (\tilde{y}_t^i)^2 \right] \\ &\quad + \text{t.i.p} + o(\|a\|^3)\end{aligned}$$

**BANK OF FINLAND RESEARCH
DISCUSSION PAPERS**

ISSN 0785-3572, print; ISSN 1456-6184, online

- 1/2010 Timo Korkeamäki – Elina Rainio – Tuomas Takalo **Law and stock markets: evidence from an emerging market.** 2010. 34 p. ISBN 978-952-462-568-5, print; ISBN 978-952-462-569-2, online.
- 2/2010 Iftekhar Hasan – Heiko Schmiedel – Liang Song **Growth strategies and value creation: what works best for stock exchanges?** 2010. 48 p. ISBN 978-952-462-570-8, print; ISBN 978-952-462-571-5, online.
- 3/2010 Iftekhar Hasan – Heiko Schmiedel – Liang Song **Return from retail banking and payments.** 2010. 53 p. ISBN 978-952-462-572-2, print; ISBN 978-952-462-573-9, online.
- 4/2010 Sungho Choi – Bill B Francis – Iftekhar Hasan **Cross-border bank M&As and risk: evidence from the bond market.** 2010. 47 p. ISBN 978-952-462-574-6, print; ISBN 978-952-462-575-3, online.
- 5/2010 Hanna Freystätter **Financial market disturbances as sources of business cycle fluctuations in Finland.** 2010. 46 p. ISBN 978-952-462-576-0, print; ISBN 978-952-462-577-7, online.
- 6/2010 Patrick M Crowley **Long cycles in growth: explorations using new frequency domain techniques with US data.** 2010. 49 p. ISBN 978-952-462-586-9, print; ISBN 978-952-462-587-6, online.
- 7/2010 Paavo Miettinen **Information acquisition during a Dutch auction.** 2010. 30 p. ISBN 978-952-462-588-3, print; ISBN 978-952-462-589-0, online.
- 8/2010 Syed Mujahid Hussain **Simultaneous monetary policy announcements and international stock markets response: an intraday analysis.** 2010. 32 p. ISBN 978-952-462-590-6, print; ISBN 978-952-462-591-3, online.
- 9/2010 Antti J Tanskanen – Petri Niininen – Kari Vatanen **Risk-based classification of financial instruments in the Finnish statutory pension scheme TyEL.** 2010. 35 p. ISBN 978-952-462-592-0, print; ISBN 978-952-462-593-7, online.
- 10/2010 Hanna Putkuri **Housing loan rate margins in Finland.** 2010. 39 p. ISBN 978-952-462-594-4, print; ISBN 978-952-462-595-1, online.
- 11/2010 Efrem Castelnuovo – Salvatore Nisticò **Stock market conditions and monetary policy in a DSGE model for the US.** 2010. 62 p. ISBN 978-952-462-596-8, print; ISBN 978-952-462-597-5, online.
- 12/2010 Juha Kilponen – Torsten Santavirta **New evidence on implicit contracts from linked employer-employee data.** 2010. 25 p. ISBN 978-952-462-600-2, print; ISBN 978-952-462-601-9, online.
- 13/2010 George W Evans – Seppo Honkapohja – Kaushik Mitra **Does Ricardian Equivalence hold when expectations are not rational?** 2010. 26 p. ISBN 978-952-462-602-6, print; ISBN 978-952-462-603-3, online.

- 14/2010 Karlo Kauko **The feasibility of through-the-cycle ratings.** 2010. 32 p. ISBN 978-952-462-604-0, print; ISBN 978-952-462-605-7, online.
- 15/2010 Risto Herrala **Credit constraints and durable consumption. A new empirical approach.** 2010. 19 p. ISBN 978-952-462-614-9, print; ISBN 978-952-462-615-6, online.
- 16/2010 Patrick M Crowley – Aaron Schultz **A new approach to analyzing convergence and synchronicity in growth and business cycles: cross recurrence plots and quantification analysis.** 2010. 53 p. ISBN 978-952-462-616-3, print; ISBN 978-952-462-617-0, online.
- 17/2010 Esa Jokivuolle – Ilkka Kiema – Timo Vesala **Credit allocation, capital requirements and output.** 2010. 35 p. ISBN 978-952-462-618-7, print; ISBN 978-952-462-619-4, online.
- 18/2010 Seppo Orjasniemi **The effect of openness in a small open monetary union.** 2010. 31 p. ISBN 978-952-462-620-0, print; ISBN 978-952-462-621-7, online.

Suomen Pankki
Bank of Finland
P.O.Box 160
FI-00101 HELSINKI
Finland



.2343