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**Working Paper**

# On the importance of borrowing constraints for house price dynamics

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Essi Eerola – Niku Määttänen

# On the importance of borrowing constraints for house price dynamics

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# On the importance of borrowing constraints for house price dynamics

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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# On the importance of borrowing constraints for house price dynamics

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## Abstract

We study how a household borrowing constraint the the form of a down payment requirement affects house price dynamics in an OLG model with standard preferences. We find that in certain situations the borrowing constraint shapes house price dynamics substantially. The importance of the constraint depends very much on whether house price changes are driven by interest rate or aggregate income shocks. Moreover, because of the borrowing constraint, house price dynamics display substantial asymmetries between large positive and large negative income shocks. These results are related to the fact that the share of borrowing-constrained households is different following different shocks.

Keywords: house prices, dynamics, borrowing constraints, down payment constraint

JEL classification numbers: E21, R21

# Luottorajoitteiden merkitys asuntojen hintadynamiikan kannalta

## Suomen Pankin keskustelualoitteita 8/2008

Essi Eerola – Niku Määttänen
Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tässä tutkimuksessa tarkastellaan kotitalouksien asuntolainoihin liittyvien luottorajoitusten vaikutusta asuntojen hintadynamiikkaan numeerisen limittäisten sukupolvien mallin avulla. Tietyissä tilanteissa luottorajoitteet vaikuttavat asuntojen hintadynamiikkaan merkittävästi. Luottorajoitteen vaikutus riippuu oleellisesti siitä, onko asuntojen hintojen muutosten taustalla tulo- vai korkosokki. Luottorajoite vaikuttaa myös hyvin eri tavalla suurten negatiivisten ja suurten positiivisten tulosokkien yhteydessä. Nämä tulokset liittyvät siihen, että mallissa luottorajoitteisten kotitalouksien määrä vaihtelee.

Avainsanat: asuntojen hintadynamiikka, luottorajoitteet, omarahoitusosuus

JEL-luokittelu: E21, R21

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# 1 Introduction

This paper investigates the importance of borrowing constraints for house price dynamics. We consider an OLG model with owner housing. In the model, young households need to borrow in order to finance their housing and differences in household size create large differences in household leverage also among households of same age. We solve for the house price dynamics following different aggregate shocks contrasting two cases: one where household borrowing is unlimited and another where households face a constraint stating that they can borrow only up to a certain fraction of the value of their house. We are particularly interested in situations where a substantial fall in house prices reduces the net worth of leveraged households dramatically. In such situations, the borrowing constraint may become binding for many households.

In order to focus on the effect of the borrowing constraint, we make two key simplifying modelling assumptions. The first is that we assume perfect foresight. The second is that we abstract from transaction costs and other non-convexities in the household problem. These assumptions make it relatively easy to solve for the fully non-linear dynamics very accurately. Importantly, they also allow us to derive analytical results which are very helpful in developing intuition for our numerical results.

We calibrate the model to Finnish household data and compare the model dynamics to the recent experience in the Finnish housing market. The Finnish housing market is a particularly interesting example since it has recently been hit by two major shocks, a credit market liberalization in the late 1980s, which resulted into a drastic relaxation of household borrowing constraints, and a severe depression in the early 1990s. Both episodes were associated with very large house price changes. Computing the house price dynamics following similar shocks in the model and comparing them to actual house price movements helps in understanding the quantitative relevance of the model.

To briefly summarize our results, we find, first of all, that the model can explain a large part of the increase in house prices that coincided with the credit market liberalization as an equilibrium response to an empirically plausible relaxation of the borrowing constraint. This suggests that the model captures much of the actual relevance of borrowing constraints to aggregate housing demand. We also find that the remaining borrowing constraint can substantially shape house price dynamics especially following big adverse aggregate income shocks. In particular, after the impact effect of a negative income shock, the borrowing constraint tends to speed up the convergence towards the new steady state price. Related to this, the borrowing constraint creates substantial asymmetries in the house price dynamics that follow large positive and large negative income shocks. The borrowing constraint is much less important for house price dynamics that are driven by interest rate shocks. These results are related to the fact that the share of borrowing constrained households is different following different shocks.

In the next subsection, we discuss how our paper relates to the previous literature. In section 2, we describe the model and analyze the role of the borrowing constraint analytically. In section 3, we discuss the calibration and

the initial steady state. In section 4, we analyze the dynamics of the model. We conclude in section 5.

## 1.1 Related literature and our contribution

Stein (1995) was the first to stress the importance of borrowing constraints for house price dynamics. To see the intuition behind his results, consider a household that has a house worth 100 000 euros and a mortgage loan of 70 000 euros. It has no other assets or debts, so its net worth is 30 000 euros. The household wants to move to a bigger house. Banks require a 20% down payment. Hence, the household could buy a house worth 150 000 euros, which is 50 % bigger (in a quality adjusted sense) than its current one. Assume now that for some reason house prices fall by 10%. This reduces the net worth of the household to 20 000 euros. As a result, it can buy a house worth only 100 000 euros. Given that house prices have fallen, 100 000 euros will buy a bigger house than its current one, but only 10% bigger. Hence, because of the borrowing constraint, a house price fall may induce the household to buy a smaller house compared to the one it would have bought had house prices remained constant. In Stein's model, this link between house prices and buyer liquidity can give rise to a multiplier mechanism and even multiple equilibria.

Stein's model is essentially static, as he assumes that all trade takes place in one period. Ortalo-Magné and Rady (1999, 2006) are able to characterize how the interplay between aggregate income shocks, homeowners' capital gains or losses and borrowing constraints affects house price dynamics and the transaction volume in a fully dynamic model where houses are available in two sizes, or 'property ladders'. Like Stein's analysis, their analyses are qualitative rather than quantitative in nature. For instance, in order to keep the model tractable, Ortalo-Magné and Rady assume preferences that rule out consumption smoothing: in their model, all consumption of the composite consumption good takes place in the last period of households' lives.

Some recent papers incorporate housing with a down payment constraint into quantitative business cycle models with standard preferences. Iacoviello (2005) and Iacoviello and Neri (2007) are good examples. In these models, which are designed to analyze monetary policy, there are two types of households: patient and impatient. In the steady state, the impatient households are borrowing constrained while the patient households are not. Dynamics are analyzed around such a steady state. Restricting the analysis to the neighborhood of a steady state is computationally convenient because one can then use a linearized version of the model. However, by construction, the share of borrowing constrained households then remains constant over time.[1]

To put it very briefly, we contribute to this literature by analyzing the importance of borrowing constraints for house price dynamics following big aggregate shocks. Two features of our model are particularly important in

[1] The multiplier mechanism discussed in Stein (1995) is also close to the 'credit cycles' -mechanism in Kiyotaki and Moore (1997). Cordoba and Ripoll (2005) have analyzed the quantitative importance of that mechanism with a linearized model.

this respect. First, we use standard preferences with a consumption smoothing motive. We show analytically that the consumption smoothing motive is linked with the multiplier mechanism in question. Second, we solve for the fully non-linear dynamics. This means here that the fraction of households that are borrowing constrained may change over time. We show that this feature is quantitatively very important in the context of big shocks that create large capital losses or gains to highly leveraged households. We also believe that our analytical results are helpful in understanding more generally how the liquidity effect described by Stein (1995) works in a fully dynamic set-up.

However, it should perhaps be stressed that in several important ways, our model is simpler than the models of Ortalo-Magné and Rady's (1999, 2006). In particular, we do not model the timing of the first home purchase, which plays an important role in their analyses. Instead, in our model, all households are assumed to buy some housing in the first period of their economically independent lives. Also, our model does not provide predictions about the transaction volume since households can costlessly adjust their housing stock every period.[2]

There are many empirical papers analyzing house price dynamics. Following Case and Shiller (1989), many of them study whether and to what extent future house price changes are predictable. Some empirical studies also try to link borrowing constraints and house price dynamics. Lamont and Stein (1999) relate US city-level house price data to the data on household finances. Benito (2006) uses British Household Panel Survey. Both studies estimate the effect of income shocks on house price dynamics. They show that compared to other regions, house prices tend to overshoot or undershoot following aggregate income shocks more in regions where households are highly leveraged. These results are consistent with the multiplier mechanism à la Stein (1995), but do not testify to the importance of borrowing constraints for house price dynamics: households' asset positions may affect house price dynamics even in the absence of borrowing constraints. One purpose of this paper is to isolate the importance of borrowing constraints for house price dynamics in a theoretical set-up.

# 2 The model

We consider a model economy with overlapping generations of households. During the first $J$ periods of their lives, households derive utility from consumption, $c$, and from the stock of owner housing they have, $h$. We follow Gervais (2002), Davis and Heathcote (2005), and others in assuming that housing services are proportional to housing capital. In period $J+1$, they derive a terminal utility that depends only on their remaining net worth. Each generation is of the same size so that population remains constant over time.

[2] Ríos-Rull and Sanchez-Marcos (2006) have developed a quantitative model with a similar property ladders -structure as in the models of Ortalo-Magné and Rady. Their model features also idiosyncratic income uncertainty. However, they present results only for the stationary equilibrium, where the house price is constant.

In each generation, there are $I$ different household types, indexed by $i = 1, 2, .., I$. The intragenerational heterogeneity stems from households getting children at different ages. Children affect household savings behavior by changing the household size over the life cycle. As we will see, differences in the age at which households get children result in large differences in household leverage. The mass of households of type $i$ is denoted by $m_i$. We normalize the size of each generation to one. That is $\sum_{i=1}^{I} m_i = 1$. The periodic earnings are independent of household type and are denoted by $y_t^j$.

Housing involves some direct costs such as maintenance costs and property taxes. We assume that part of these costs are proportional to the size of the house and part of them (taxes in particular) are proportional to the value of the house.[3] We denote these two costs by $\eta$ and $\kappa$. There is also a financial asset, $a$. The interest rate that the financial asset earns is given by $R - 1$.

Households face a borrowing constraint which means that they can borrow only against their housing and that they have to finance part of their housing with own equity. Parameter $\theta \leq 1$ denotes the fraction of the value of the house that the household has to finance itself. This kind of borrowing constraint is often referred to as a down payment requirement. More generally, however, it can also be used to partly capture maturity constraints. In particular, if households can only take mortgages with a very short maturity, they have to pay a relatively large fraction of their housing during the first period.

The periodic utility function for $j = 1, ..., J$ is denoted by $u(c, h; s)$, where $s$ is the household size. The terminal utility is determined by function $v$. The subjective discount factor is $\beta$. We use superscripts to denote age of the household and subscripts to denote type and time period. The problem of a household of age $j = 1$ and type $i$ at time $t$ is then given by

$$\max \sum_{j=1}^{J} \beta^{j-1} u(c_{i,t+j-1}^j, h_{i,t+j-1}^j; s_i^j) + \beta^J v(b_{i,t+J-1}^J; s_i^{J+1}) \tag{2.1}$$

subject to

$$\begin{aligned} c_{i,t+j-1}^j + g_{t+j-1} h_{i,t+j-1}^j + a_{i,t+j-1}^j &= y_{t+j-1}^j + R_{t+j-1} a_{i,t+j-2}^{j-1} \\ &\quad + p_{t+j-1} h_{i,t+j-2}^{j-1} \qquad (2.2) \\ a_{i,t+j-1}^j &\geq -(1-\theta) p_{t+j-1} h_{i,t+j-1}^j \qquad (2.3) \\ h_{i,t}^0 &= a_{i,t}^0 = 0 \qquad (2.4) \end{aligned}$$

where $g_t = p_t + \kappa p_t + \eta$. In the utility function, for instance, $c_{i,t}^j$, denotes consumption of a household of age $j$ and type $i$ in period $t$, and net worth in age $J + 1$ is given by

$$b_{i,t+J-1}^J = R_{t+J} a_{i,t+J-1}^J + p_{t+J} h_{i,t+J-1}^J \tag{2.5}$$

[3] We introduce these two types of costs because they have different implications for equilibrium house prices. For instance, if there are large maintenance costs that are proportional to the size of the house alone, a large part of the user cost is unrelated to the house price. As a result, a relatively large change in the steady state house price is needed in order to create a given percentage change in the total user cost of housing. In this case, a relatively small change in aggregate household income, for instance, implies a relatively large change in the equilibrium house price.

The first constraint is the periodic budget constraint where $p$ denotes the price of housing. The second constraint is the periodic down payment constraint.

We consider a small open economy in the sense that the interest rate and the wage level are exogenously given. The only aggregate consistency condition is the market clearing condition for the housing market. We assume that the supply of housing is fixed at $\overline{H}$.[4] The market clearing condition reads as

$$\sum_{i=1}^{I}\sum_{j=1}^{J} m_i h_{i,t}^j = \overline{H} \tag{2.6}$$

The Lagrangian for the household's maximization problem is (we drop here the type index)

$$\begin{aligned} L \;=\; & \sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^j, h_{t+j-1}^j; s^j) + \beta^J v(b_{i,t+J-1}^J; s^{J+1}) \\ & + \sum_{j=1}^{J} \lambda_{t+j-1}^j [y_{t+j-1}^j + R_{t+j-1} a_{t+j-2}^{j-1} + p_{t+j-1} h_{t+j-2}^{j-1} - c_{t+j-1}^j \\ & - g_{t+j-1} h_{t+j-1}^j - a_{t+j-1}^j] \\ & + \sum_{j=1}^{J} \gamma_{t+j-1}^j (a_{t+j-1}^j + (1-\theta)\, p_{t+j-1} h_{t+j-1}^j) \end{aligned} \tag{2.7}$$

where $\lambda_t^j$ and $\gamma_t^j$ are the Lagrange multipliers for the budget constraint and the borrowing constraint for a household of age $j$ at time $t$.

We now discuss the importance of some simplifying assumptions we have made. There is no aggregate uncertainty in the model which means that we can consider only perfect foresight dynamics following completely unanticipated shocks. Clearly, this limits the way we can compare house price dynamics in the model to the data. However, with aggregate uncertainty, the model would become very difficult to solve since we would then have to use recursive methods with the distribution of households over their asset positions (or at least some moments describing it) as a state variable. Perfect foresight dynamics are the easiest way of illustrating how the borrowing constraint affects house price dynamics.

We also assume that there are no transaction costs. Again, transaction costs would make it much more difficult to solve the model since the household problem would then become non-convex and since we would then need a model with a continuum of households in different situations (in order to get a smooth aggregate demand function).[5] The absence of transaction costs means that households generally adjust their housing position every period, which is not realistic if the model period is relatively short. It also means that we cannot consider the dynamics of the transaction volume. However, as we show

[4] Without loss of generality, the aggregate supply of housing can be normalized to any strictly positive level. We choose the aggregate supply of housing so that the house price is equal to 1 in the steady states of the benchmark calibrations.

[5] Technically, we could handle convex transaction costs but not non-convex transaction costs, which are realistic in the housing market.

below, since the demand for housing in our model is affected by changes in household size (or children), the model nevertheless has the realistic feature that households undertake major adjustments to their housing only a few times in their life.

We take the supply of housing as fixed. This reflects the fact that our focus is entirely on the demand side. In any case, we believe that the supply side is not the key to understand the drastic house price movements that we have recently observed in Finland (which we describe below). While construction volume varies a lot over the business cycle, the level of investment is so small compared to the aggregate stock of housing that the aggregate stock changes very slowly. Related to this, we abstract from growth. An extended version of the model with income growth would have a steady state with constant house prices assuming that the supply of housing increases at the same rate.

## 2.1 Solving the model

We solve for the transitional dynamics of the economy following different completly unexpected shocks. We assume that it takes up to $T$ periods for the economy to converge to a new steady state after a shock. Using the household first-order conditions, the budget constraints and the borrowing constraint together with the housing market equilibrium condition for each period, we get the following system of equations (with $i = 1, 2, .., I$ and $t = 1, 2, ..., T$).

$$\beta^{j-1} u_{h_{i,t}^j} + p_{t+1} \lambda_{i,t+1}^{j+1} = g_t \lambda_{i,t}^j - \gamma_{i,t}^j (1-\theta) p_t \text{ for } j < J \tag{2.8}$$

$$\beta^{J-1} u_{h_{i,t}^J} + \beta_i^J p_{t+1} v_{b_{i,t}^J} = g_t \lambda_{i,t}^J - \gamma_{i,t}^J (1-\theta) p_t \tag{2.9}$$

$$\beta_i^{j-1} u_{c_{i,t}^j} = \lambda_{i,t}^j \tag{2.10}$$

$$-\lambda_{i,t}^j + R_{t+1} \lambda_{i,t+1}^{j+1} + \gamma_{i,t}^j = 0 \text{ for } j < J \tag{2.11}$$

$$-\lambda_{i,t}^J + \beta^J R_{t+1} v_{b_{i,t}^J} + \gamma_{i,t}^J = 0 \tag{2.12}$$

$$\gamma_{i,t}^j \left( a_{i,t}^j + (1-\theta) p_t h_{i,t}^j \right) = 0 \tag{2.13}$$

$$\gamma_{i,t}^j \geq 0, \; a_{i,t}^j + (1-\theta) p_t h_{i,t}^j \geq 0 \tag{2.14}$$

$$c_{i,t}^j + g_t h_{i,t}^j + a_{i,t}^j = y_{i,t}^j + R_t a_{i,t-1}^{j-1} + p_t h_{i,t-1}^{j-1} \tag{2.15}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} m_i h_{i,t}^j = \overline{H} \tag{2.16}$$

This set of equations fully characterizes the dynamics of the economy. With a multi-period life cycle, this is a relatively large system of non-linear equations. In our calibrated model, it consists of about 2000 equations. We solve this system using the broydn's algorithm. When solving the system, we impose a very strict error tolerance ($10^{-5}$). Hence, we solve for the dynamics very accurately. (We also have to check that the solution is not affected by our guess for $T$.)

## 2.2 The borrowing constraint and housing demand

As we discussed in subsection 1.1, the multiplier mechanism in Stein (1995) is essentially a link between house prices and buyer liquidity. Therefore, before turning to the numerical analysis, we derive analytically the effect of a marginal change in current house price on current and future housing demand comparing the behavior of an unconstrained household to that of a borrowing constrained household. This helps us to understand how the liquidity effect in Stein works in a fully dynamic set-up. The results will also be useful when developing intuition for our numerical results.

Let us consider a household of age $1 < j < J$. For notational convenience, we drop here time, age, and type indices. We denote the housing that the household owns in the beginning of the current period by $h^{-1}$ and its housing in the beginning of the next period by $h$. Similarly, $a^{-1}$ and $a$ denote the holdings of the financial asset in the beginning of the current period and in the beginning of the next period. We further denote current house price by $p$, and the next period house price by $p'$. We assume that the interest rate is constant over the two periods.

We can write the remaining lifetime utility of the household from next period onwards as a function of its next period net worth alone. We write it as $V(b)$, where

$$b = Ra + p'h \tag{2.17}$$

denotes household's net worth in the beginning of next period. Function $V$ is of the same functional form as the utility function. In particular, as long as the household has a consumption smoothing motive, we have $V_{bb} < 0$. As for the utility function, we assume here, for simplicity, that it is separable between consumption and housing so that the cross derivative $u_{ch} = 0$.

The problem of a household can now be formulated as follows

$$\max_{c,h,b} \{u(c,h) + \beta V(b)\} \tag{2.18}$$

subject to

$$y + ph^{-1} + Ra^{-1} = c + \left(p + \kappa p + \eta - \frac{p'}{R}\right) h + \frac{b}{R} \tag{2.19}$$

$$a \geq -(1-\theta)\, ph \tag{2.20}$$

We first ask how the current housing demand, $h$, of the household depends on the current house price, given its past housing and savings decisions, $a^{-1}$ and $h^{-1}$. The Appendix shows that in the unconstrained case, the effect of a marginal change in the current house price on current housing demand is given by

$$\frac{\partial h}{\partial p} = \frac{1}{D}\left[\underbrace{\frac{(1+\kappa)\, u_c u_{cc}}{R}}_{\text{negative}} + \beta R\left(\underbrace{V_{bb}(1+\kappa)\, u_c}_{\text{negative}} + \underbrace{PV_{bb}u_{cc}\left(h^{-1} - (1+\kappa)\, h\right)}_{\text{negative if } (1+\kappa)h > h^{-1}}\right)\right]$$

where $P = p + \kappa p + \eta - \frac{p'}{R}$ and $D > 0$. We assume here that $P > 0$.

The overall effect consists of three terms. The first term is independent of $V$. Hence, it is related to the intratemporal resource allocation alone. It is always negative: An increase in the current house price makes current housing more expensive relative to current consumption. The other two terms depend on $V$ and hence they are related to the intertemporal resource allocation. The second term is also always negative: An increase in the current house price makes current housing more expensive relative to future consumption and housing. The third term depends on whether $(1+\kappa)\,h$ is smaller or larger than $h^{-1}$. Intuitively, this term is related to an endowment effect: An increase in the current house price makes the household 'wealthier' if $h^{-1} > (1+\kappa)\,h$, that is, if it is a net seller of housing in current period. In that case, the third term works to increase housing demand when the house price increases. Note that last two terms would both go to zero if $V_{bb}$ goes to zero. Without a consumption smoothing motive, current housing demand would depend only on the relative price of current housing and current consumption.

When the household faces a binding borrowing constraint, the effect of a price change on current housing is given by the following expression (see the Appendix)

$$\begin{aligned}\frac{\partial h}{\partial p} &= \frac{1}{D^c}\left[\overbrace{-u_c\,(1+\kappa)}^{\text{negative}}+\overbrace{u_{cc}T\left((1+\kappa)\,h-h^{-1}\right)}^{\text{negative if }(1+\kappa)h>h^{-1}}\right] \\ &\quad+\frac{1}{D^c}\left[\underbrace{u_c\,(1-\theta)}_{\text{positive}}\underbrace{-u_{cc}T\,(1-\theta)\,h}_{\text{positive}}\underbrace{-S\beta V_{bb}R\,(1-\theta)}_{\text{positive}}\underbrace{-R\,(1-\theta)\,\beta V_b}_{\text{negative}}\right]\end{aligned} \tag{2.21}$$

where $T = p + \kappa p + \eta - (1-\theta)\,p$ and $S = p' - R\,(1-\theta)\,p$ and $D^c > 0$.

We assume here that $T > 0$ and $S > 0$. The overall effect now consists of six terms. The first term is related to the intratemporal resource allocation and is negative. The second term is related to the endowment effect, which is also negative as long as $(1+\kappa)\,h > h^{-1}$. Hence, these two terms have the same interpretation as the first and the third term in the unconstrained case. The second term of the unconstrained case is missing here: Because of the binding borrowing constraint, a marginal change in the relative price of current and future consumption (or housing) does not have a direct effect on current demand.

Compared to the above non-constrained case, there are four additional terms which depend on the borrowing constraint parameter, $\theta$. Assuming that $\theta < 1$, first three terms are positive. The first two terms come directly from the borrowing constraint: as the current house price goes up, the household can borrow more which increases the demand for housing. The third term is positive: When the household has a consumption smoothing motive (that is when $V_{bb} < 0$) the borrowing constraint prevents it from transferring resources from future to present. When the current house price increases, this constraint is relaxed, which also increases current housing demand. These positive terms means that in principle at least, the borrowing constraint can here give rise to

a similar multiplier effect as the one discussed in Stein (1995). We will refer to these terms together as the *liquidity effect*.

The fourth new term in (2.21) is negative and is related to the cost of transferring resources to the future. Intuitively, if a borrowing constrained household wants to save more for future, it buys more housing. When the current house price increases, this becomes more expensive. This term mitigates the liquidity effect.

In equilibrium, the current house price depends on both current and future housing demand. In order to analyze the effect of a change in the current house price on future housing demand, we note first that future housing demand must depend positively on household's next period net worth. Hence, we now consider how next period's net worth is affected by current house price changes.

The Appendix shows that in the unconstrained case we have

$$\frac{\partial b}{\partial p} = \frac{1}{D}\left[\underbrace{-u_{cc}P\left(1+\kappa\right)u_c}_{\text{positive}} + \underbrace{u_{cc}u_{hh}\left(h^{-1}-\left(1+\kappa\right)h\right)}_{\text{negative if } (1+\kappa)h>h^{-1}}\right] \tag{2.22}$$

where again $D > 0$. In the constrained case, this effect comes directly through the borrowing constraint so that we have

$$\frac{\partial b}{\partial p} = S\frac{\partial h}{\partial p} - R\left(1-\theta\right)h, \tag{2.23}$$

where $\frac{\partial h}{\partial p}$ is determined by (2.21).

In our numerical simulations, we have that $\frac{\partial h}{\partial p}$ is smaller for the constrained households than for the unconstrained households but still strictly positive. In that case, (2.23) shows that a decrease in the current house price must induce a borrowing constrained household to have a higher next period net worth. Through this effect, a decrease in the current house price works to increase next period housing demand. For the unconstrained household, the effect of a change in the current house price on next period net worth is ambigous as it depends on the endowment effect.

# 3 Calibration and the steady state

In this section, we describe the household data we use in the calibration, the calibration procedure, and the steady state of the model economy.

## 3.1 Household leverage in the data

We base our calibration on 2004 Wealth Survey conducted by Statistics Finland, which includes portfolio information from about 2500 Finnish households. We consider only homeowners. In the survey, they were asked an estimate of the current market value of their house.

The importance of borrowing constraints should crucially depend on household leverage. We characterize household leverage with the net worth-to-house value ratio (NWHV). Net worth is defined as the sum of the market value of household's residential property and its financial assets less all debt. Hence, the lower the NWHV of a household is, the more highly leveraged it is in the sense that it has more debt or less assets relative to the value of its house. Negative ratios mean that household's debts are larger than the value of its house. The distribution of the NWHV ratios in the data is shown in Figure 1.

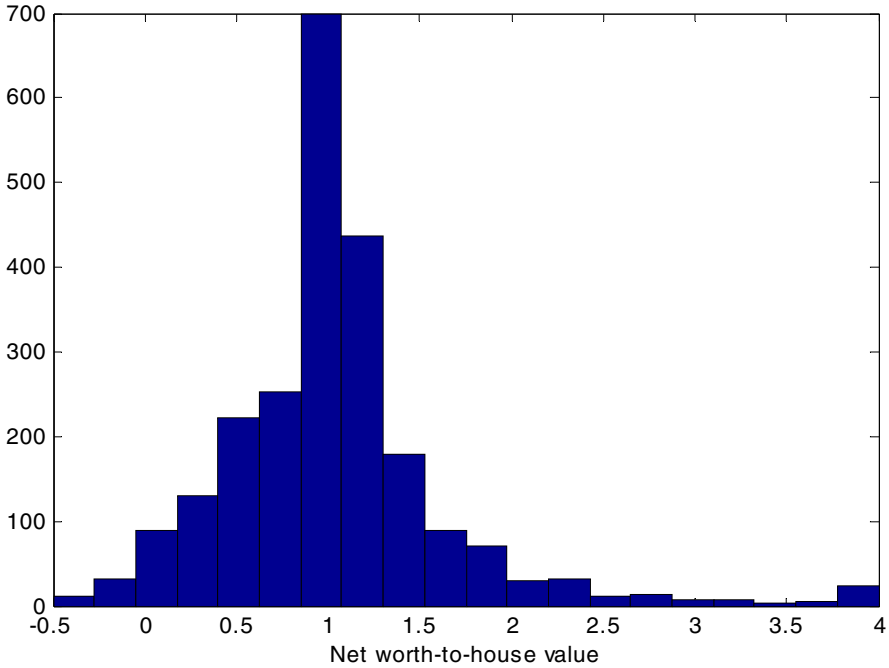


Figure 1. **The distribution of net worth-to-house value ratios in the data**

Figure 2 in turn shows the median NWHV ratio in different age groups. As should be expected, the young households are typically much more leveraged than older households. The median NWHV ratio increases from about 0.25 among households of age 25–29 to about 1.1 among households of age 70–74.

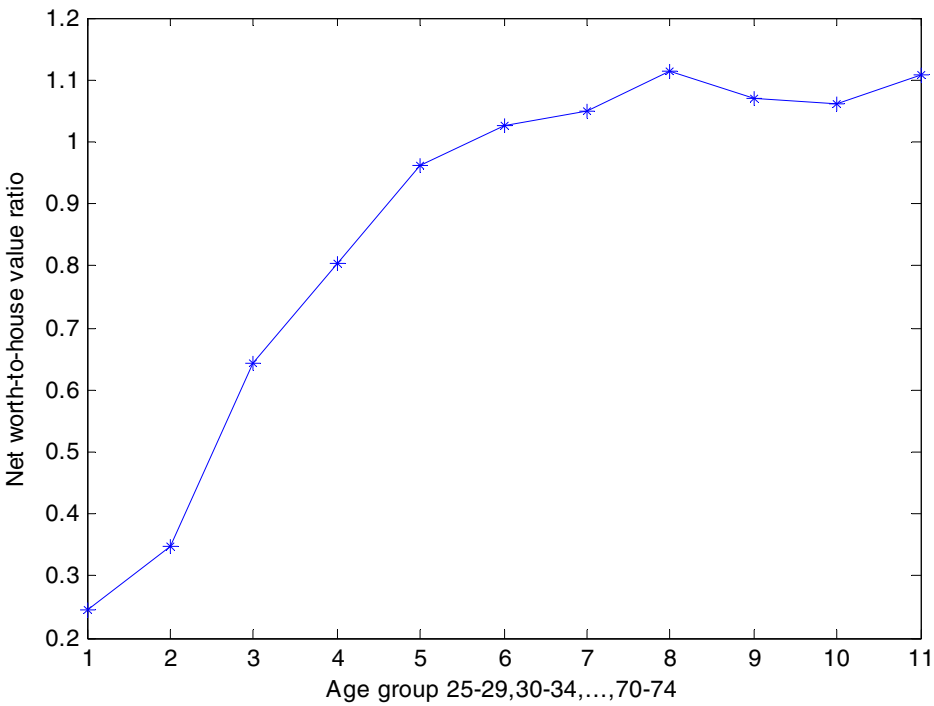


Figure 2. **The median net worth-to-house value ratio in different age groups in the data**

## 3.2 Calibration

The first thing to specify in the model is the number of periods in a life cycle. We take one model period to correspond to five years and assume that households' economically independent life lasts for 10 periods, that is $J = 10$. We interpret model age 1 as real ages 25–29 and model age 10 as real ages 70–74.

These choices are somewhat arbitrary, of course. However, there are a number of reasons why we do not want the model period to be much shorter than five years. First, the larger is the number of periods in households' life cycle, the harder it is to solve the model. With 10 periods, the set of non-linear equations to be solved here is already quite large. Second, a relatively long model period also seems more natural given that our model does not feature transaction costs related to moving. Third, as discussed above, a relatively long model period also allows us to partly capture maturity constraints with the borrowing constraint.

We assume three different household types, so that $I = 3$. As explained above, they differ only in the age at which they get children. We assume that households consist of two adults who get two children in model age 1, 2 or 3 and that children live within the household for four model periods (or 20 years). We compute the corresponding household sizes using the OECD scale for household consumption units. For instance, for households of type 2 (that get children in model age 2), this means that $s_2^1 = 1.7$, for $s_2^j = 2.7$, for

$j = 2, 3, 4, 5$, and $s_2^j = 1.7$ for $j \geq 6$. We assume $m_1 = m_2 = m_3$. That is, all household types are equally common in the population.

We take the income profile directly from the data. We first compute the average annual non-capital income in age groups 25–29, 30–34,..., 55–59. We then normalize the income profile so that the average income is one. Finally, we assume that average income after age 60 is 60% of the normalized average income. This is close to the current (after tax) replacement rate provided by the Finnish mandatory pension system. The resulting income profile is $\{y^j\}_{j=1}^J = \{0.87, 0.95, 1.13, 1.11, 1.03, 1.00, 0.90, 0.60, 0.60, 0.60\}$. We assume that both the periodic utility and the terminal utility are determined by a CRRA utility function

$$\begin{aligned} u(c,h;s) &= \begin{cases} s\frac{[(c/s)^{\alpha_c}(h/s)^{1-\alpha_c}]^{1-\sigma}}{1-\sigma}, & \text{for } \sigma > 1 \\ s[\alpha_c \log(c/s) + (1-\alpha_c)\log(h/s)], & \text{for } \sigma = 1 \end{cases} \\ v(b;s) &= \begin{cases} s\alpha_b \frac{(b/s)^{1-\sigma}}{1-\sigma}, & \text{for } \sigma > 1 \\ s\alpha_b \log(b/s), & \text{for } \sigma = 1 \end{cases} \end{aligned} \tag{3.1}$$

where $b$ denotes net worth in age $J + 1$.

We set the interest rate term at $R = 1.1$. This corresponds closely to the average yearly real after tax interest rate on mortgage loans during the period 2000-04, which was 1.95%.

We set the cost parameters at $\kappa = \eta = 0.1$. Hence, the annual maintenance and other direct costs of housing are about 4% of the house value and half of these costs are related to the house price level. We are then left with the borrowing constraint parameter, $\theta$, and the preference parameters, $\sigma$, $\beta$, $\alpha_c$, and $\alpha_b$. In the benchmark case, we assume that $\theta = 0.25$. This means that a household is required to make a down payment of 25% of the value of the house. We think of this a realistic borrowing constraint *after* the credit market liberalization. When considering the effects of credit market liberalization, we will start from a situation where $\theta$ is much higher. For comparison, we also consider the case where there is no borrowing constraint, which is equivalent to the case where $\theta$ is negative and large in absolute value. Since our results are likely to depend on the intertemporal elasticity of substitution, we consider two different but reasonable values, namely $\sigma = 1$ and $\sigma = 3$.

We then have four possible combinations of the borrowing constraint and the intertemporal elasticity of substitution. We choose the preference parameters $\beta$, $\alpha_c$, and $\alpha_b$ for each case separately so that

i) Average NWHV ratio in the model equals the median ratio for households of age 25–74 in the data.

ii) Average net worth-to-income ratio in the model equals the median ratio for households of age 25–74 in the data.

iii) Average net worth-to-income in age $J$ equals the median ratio for households of age 70–74 in the data.

In the data, the median NWHV is 0.9. The median net worth-to-annual income ratio is 3.0 for all households and 5.0 for households of age 70–74. Since the model period is five years, these ratios translate into net worth-to-periodic income ratios equal to 0.6 for all households and 1.0 for households of age $J$.

The resulting parameter combinations are shown in Table 1. Note that in order to get the same NWHV without the borrowing constraint as with it, we have to choose a higher discount factor. This reflects the fact that the borrowing constraint limits household borrowing.[6]

| | $\beta$ | $\alpha_c$ | $\alpha_b$ |
|---|---|---|---|
| $\sigma = 1$, $\theta = 0.25$ | 0.904 | 0.809 | 0.868 |
| $\sigma = 3$, $\theta = 0.25$ | 0.896 | 0.809 | 1.072 |
| $\sigma = 1$, no borr. constr. | 0.922 | 0.808 | 0.812 |
| $\sigma = 3$, no borr. constr. | 0.947 | 0.808 | 0.887 |

Table 1: Parameter combinations.

## 3.3 The steady state

Figure 3 displays the steady state housing profiles, $h^j$, for the three different household types in the case with logarithmic preferences (these profiles are not scaled by household size, $s$). Consumption profiles (not shown) are similar. Consider first households of type 3. These households get children at model age 3. They are never borrowing constrained and hence their housing follows closely household size. They move to a bigger house when they get children at age 3, and move to a smaller house at age 7. In contrast, households of type 1, who get children at age 1, are borrowing constrained until model age 4. This distorts their housing (and consumption) profiles over the life cycle. Households of type 2 are an intermediate case: they are borrowing constrained at ages 2 and 5.

Because of the absence of transaction costs, housing does not remain exactly constant between any two periods. These profiles are realistic, however, in the sense that in any time period, a small fraction of households wish to make *major* adjustments in their housing.

[6] When comparing the dynamics with and without the borrowing constraint, we could also keep all other parameter values fixed. Then, however, the initial distributions of household leverage in the model would be very different in the two cases.

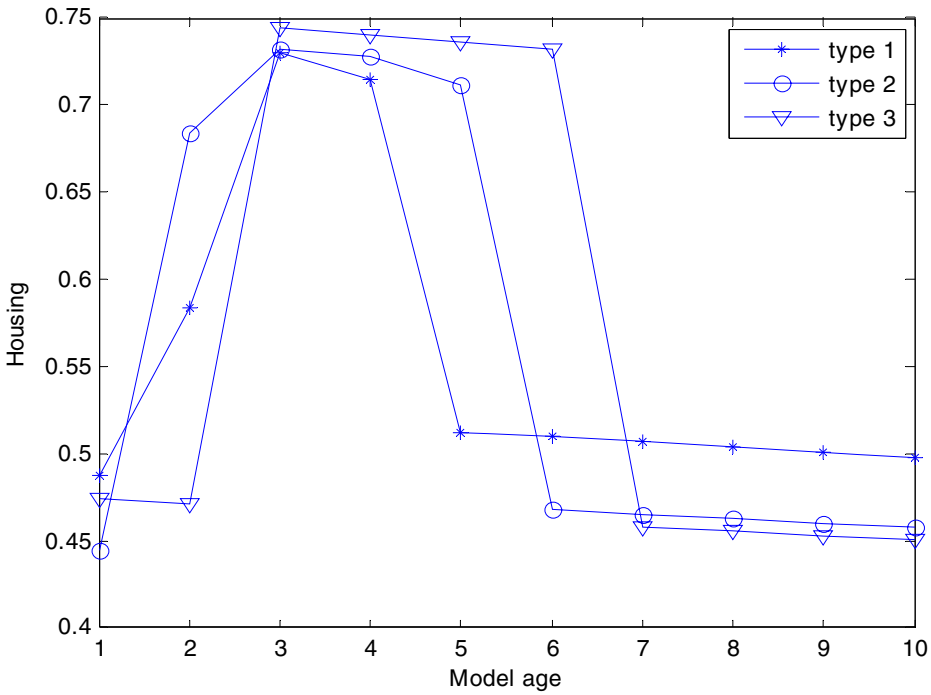


Figure 3. **Housing profiles over the life cycle in steady state**

Figure 4 displays the corresponding profiles for NWHV ratios as well as the average NWHV in different age groups. As in the data, it is the young households that are highly leveraged. The lowest NWHV allowed by the down payment constraint is 0.25. For all household types, the NWHV ratio is smaller than one during the first five periods.

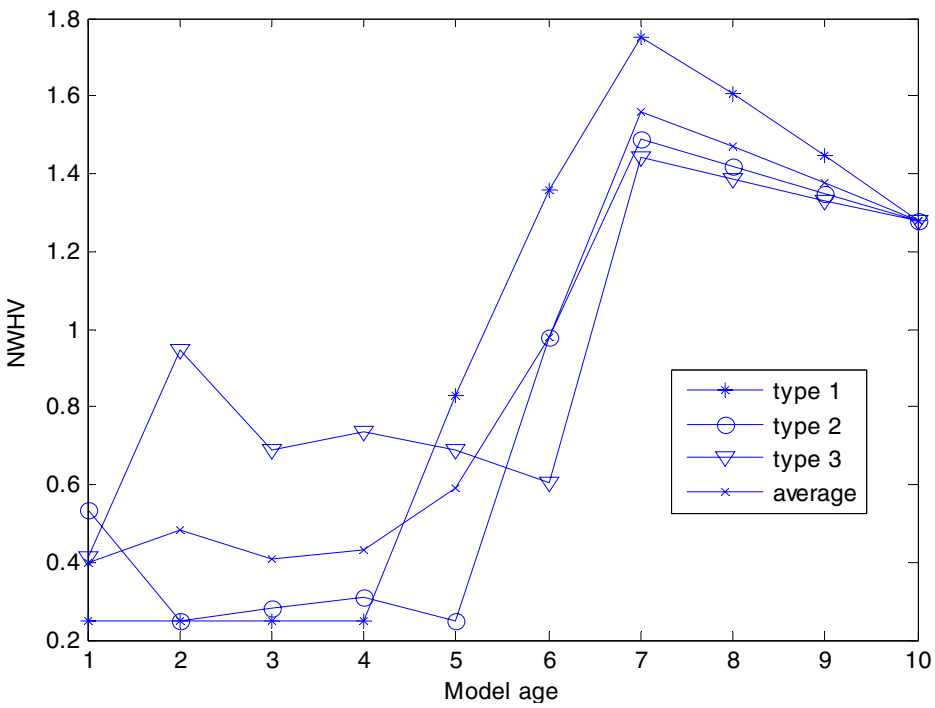


Figure 4. **Net worth-to-house value ratios over the life cycle**

Table 2 compares the distribution of household leverage in the data to the model when $\theta = 0.25$. For the table, we have divided the households into four groups according to their NWHV ratio and calculated the share of households in each group (in the model, these shares are exactly the same for $\sigma = 1$ and $\sigma = 3$). As the table shows, this distribution is more dispersed in the data than in the model. In the data, some households report to have NWHV less than 0.25, which is the lowest NWHV we allow for in the model. On the other hand, the model economy also features too few households with NWHV larger than one.

| | Net worth-to-house value ratio | | | |
|---|---|---|---|---|
| | $< 0.25$ | $0.25 - 0.5$ | $0.5 - 1.0$ | $> 1.0$ |
| Data | 6.7% | 8.6% | 28% | 57% |
| Model ($\theta = 0.25$) | 0% | 30% | 27% | 43% |

Table 2: Share of households with different net worth-to-house value ratios.

# 4 Dynamics

In this section, we analyze numerically the dynamics of the model following different shocks. We first describe, in subsection 4.1, recent house price dynamics in Finland as well as the evolution of per capita income and real after-tax interest rates. In subsection 4.2, we consider a sudden relaxation of the borrowing constraint that is similar to the one that was associated with the Finnish credit market liberalization in the late 1980s, and compare the price dynamics in the model to the data. Our aim is to see to what extent the model can explain the housing boom of the late 80s as a response to an empirically plausible relaxation of the household borrowing constraints.

In subsection 4.3, we consider how the remaining borrowing constraint shapes house price dynamics following big aggregate income and interest rate shocks. In subsection 4.4, we look at how a marginal change in the current house price affects housing demand at individual level. There, our aim is to provide further intuition for the results in subsection 4.3. In addition, this exercise allows us to discuss the possibility of multiple equilibria. In the last subsection, we consider a sequence of shocks where credit market liberalization is followed by a big adverse income shock. Our aim is to see how the fact that the credit market liberalization took place just before the depression, should have affected the house price dynamics following the Finnish depression.

## 4.1 The Finnish boom-bust-boom cycle

Figure 5 displays real house prices over the period of 1980–2006. It shows that real house prices first increased by about 50% from 1986 to 1989 and then fell by about 50% from 1989 to 1993. The boom was associated with a credit market liberalization that took place in the late 1980s. Before that, the Finnish banking system was highly regulated with tightly controlled and low lending rates which resulted in credit rationing. The abolition of interest rate controls in 1986 induced a huge growth of credit (see Koskela et al, 1992, and Laakso, 2000).

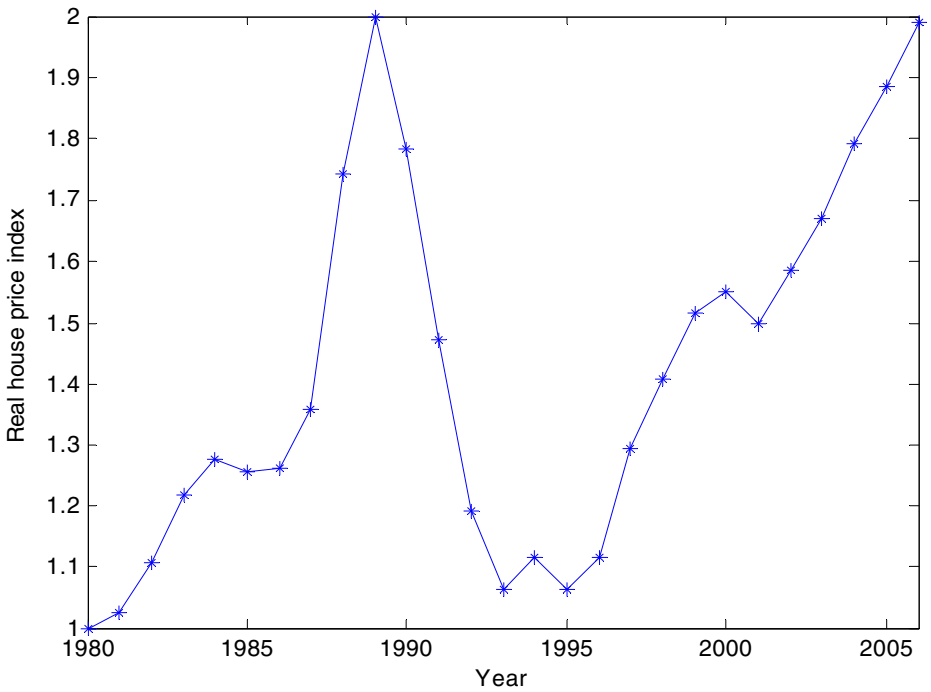


Figure 5. **Real house prices in Finland**[7]

The following bust coincided with a severe depression. Figure 6 shows the growth of real GDP per capita over the period of 1980–2006 as well as real after-tax mortgage interest rates. Real GDP decreased by over 10% from 1990 to 1993. Of course, such an income shock must have large effects on house prices.

After tax real interest rates have also varied a lot. To a large extent this has been due to relatively short-run changes in nominal interest rates and inflation. However, there has been at least one major shock which was of a more permanent nature. Until 1992, interest payments were deductible in general income taxation with average marginal rates close to 50%. In 1993, Finland moved to a dual tax system, where labor and capital income are taxed separately. In the new system, a tax payer could deduct 25% (the tax rate on capital income) of interest expenditures related to housing loans from taxes. The system has remained basically the same since 1993.[8]

[7] The source for the house price index series is Bank of Finland.

[8] However, the capital income tax has been increased to 28%.

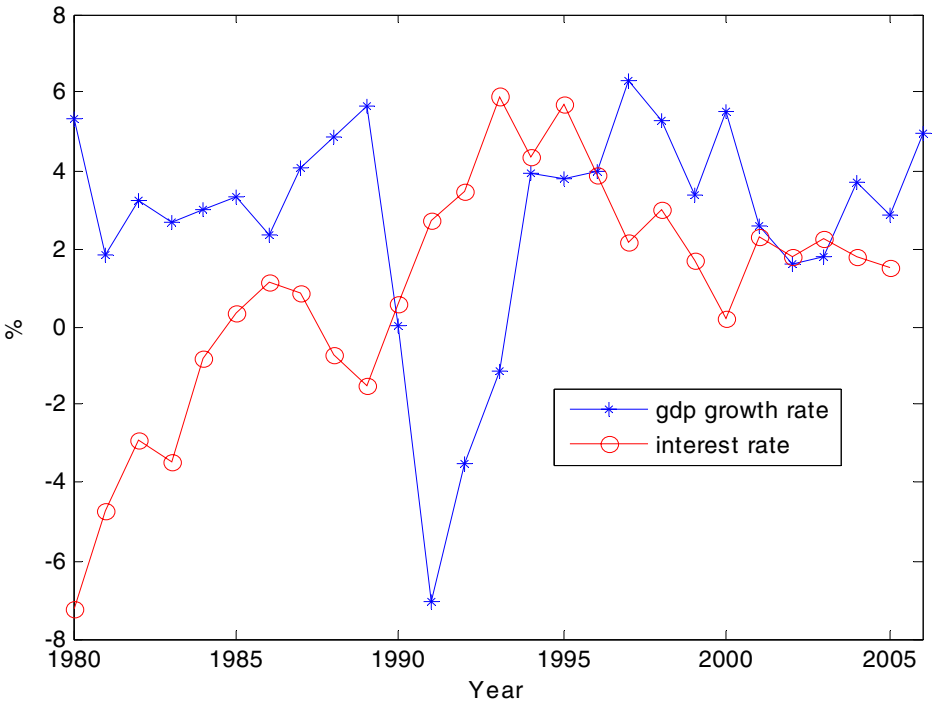


Figure 6. **The growth rate of real GDP per capita and real after tax mortgage interest rate**[9]

In what follows, we will first consider a similar relaxation of the borrowing constraint as the one that was associated with the credit market liberalization in the late 80s. We will then consider aggregate income and interest rate shocks that are of the same order of magnitude as those associated with the depression and the 1993 tax reform.

Specifically, when studying the effects of a relaxation of the borrowing constraint, we first solve for the steady state with a very tight borrowing constraint. In Finland, before the credit market liberalization, there was effectively credit rationing, and households were constrained by very short mortgage maturities. Based on discussions with market experts, our understanding is that for most households, it was impossible to get a mortgage with a maturity above 8 years. In addition, households typically needed to pay a down payment of around 30% of the house value. (See also Koskela et al, 1992). While we don't have a maturity constraint formally in the model, we can partly capture it by increasing the borrowing constraint parameter $\theta$. For a typical mortgage contract, a mortgage maturity of 8 years together with a down payment constraint of 30% means that a household needs to pay about 75% of the value of its new house investment during the first five years. This translates into $\theta = 0.75$.

As for the income shock, we note first that between 1900–1989, real GDP per capita has increased, on average, 11.8% every five years. In the five year

[9]The source for the GDP series is Statistics Finland and the source for the interest rate series is Oikarinen (2007).

period 1990–1994, however, the real GDP per capita decreased by 9.8%.[10] If we assume that just before the depression, households expected their income to keep growing at the average rate of 11.8%, the depression meant that by the end of the following five year period, household income was approximately 20% lower than expected.[11] Hence, when considering an adverse income shock that mimics the Finnish depression, we assume that income suddenly falls by 20%.

Of course, an important question is then how the depression changed expectations about future income. If we simply estimate an AR(1) process for the five year real GDP per capita growth, $x_t$, we get the following

$$x_t = 0.11 + 0.13x_{t-1} \tag{4.1}$$

This suggests that the effect of the depression on the level of aggregate income should be more or less permanent. On the other hand, the Finnish economy has been growing quite fast after the depression which means that the level of real GDP has partly recovered towards its previous trend. It is possible that households were able to anticipate that. We consider both permanent and temporary shocks.

As for the interest rate shock associated with the 1993 tax reform, it should first be noted that the effect of the lower deduction rate on the real after tax interest rate depends on nominal interest rates: the higher the nominal interest rate, the larger is the increase on the real after tax rate. However, even assuming a relatively low nominal interest rate (5%), the effect is at least a 1%-point increase in the real annual after tax interest rate. This corresponds to an increase in $R$ from 1.1 to roughly 1.15.

## 4.2 Relaxation of the borrowing constraint

Here we consider the transitionary dynamics following a sudden reduction of $\theta$ from 0.75 to 0.25. Initially, the economy is in a steady state. We consider the preference parameter combinations given in the first two lines of table 1 with $\sigma = 1$ and $\sigma = 3$. The initial house price is different for the two calibrations. By construction, the new steady state house price equals 1 in both cases.

Figure 7 displays house price dynamics following the shock. Period 1 corresponds to the initial steady state with a tight borrowing constraint ($\theta = 0.75$). The shock takes place in period 2.

Following the relaxation of the borrowing constraint, house prices first increase and then decline steadily towards a new steady state level. The reason why the house price increases on impact is simple: initially, the only thing that changes is that young, borrowing constrained households can buy more housing. Hence, the house price must go up. After the impact effect, the

[10] Disposable household income decreased by roughly the same amount.

[11] That is, if aggregate income is 1 in period $t$, households expected it to be 1.118 in period $t+1$. Given the income shock, the realized aggregate income in period $t+1$ is 0.902 instead of 1.118. The difference is 19.3%.

house price starts to decrease but remains higher than in the initial steady state. The reason to this gradual reduction in house price is the following: After the credit market liberalization, the young households borrow more than previously. Hence, they will be less wealthy at old age than the previous generations. Therefore, future generations demand less housing when old than the current old. Therefore, house prices must decline after the impact effect.

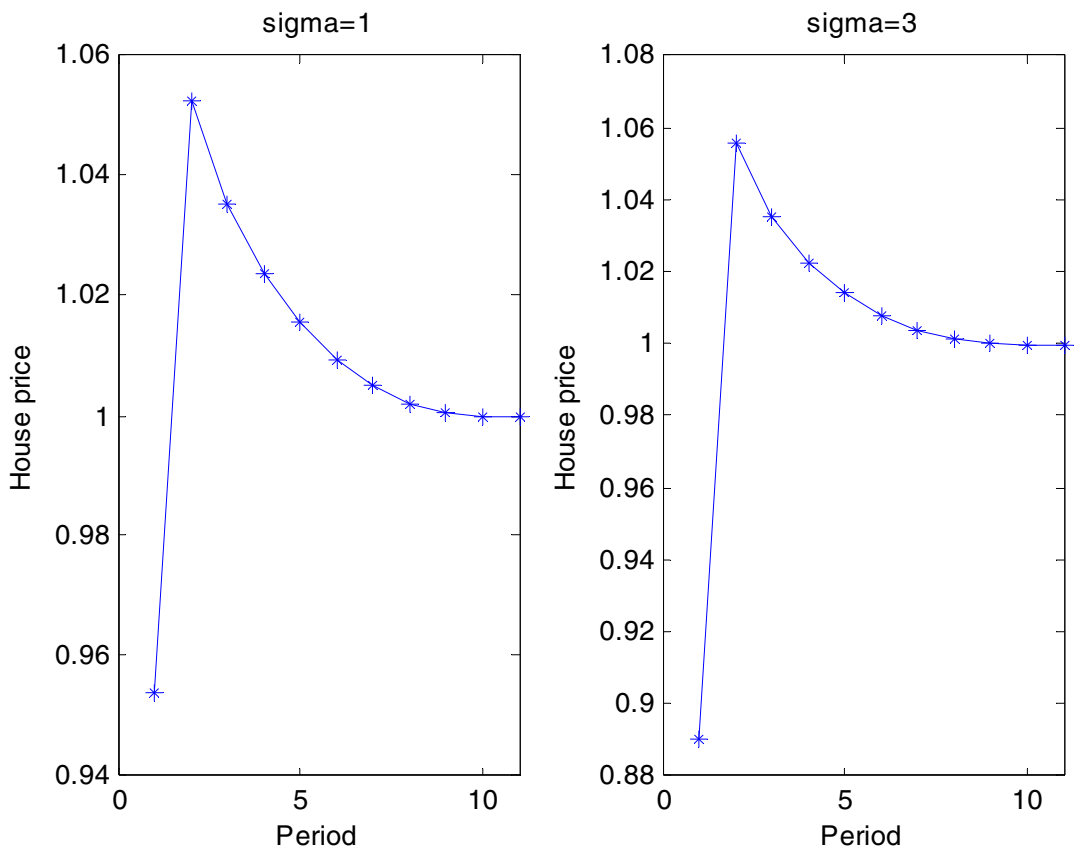


Figure 7. **House price dynamics following a relaxation of the borrowing constraint**

The impact effect is a house price increase of 11% with $\sigma = 1$ and 20% with $\sigma = 3$. In the latter case, the model explains almost half of the about 50% increase in house prices that followed the credit market liberalization in Finland. This suggests that the model captures a large part of the empirical relevance of borrowing constraints for house price dynamics.

## 4.3 Income and interest rate shocks

We now consider income and interest rate shocks. As we explained above, we consider income shocks that are of a similar magnitude as the fall in GDP during the Finnish depression. We consider both permanent and temporary income shocks. And in order to highlight the non-linearities of the model, we consider also positive income shocks.

We assume that the income shocks affect all households equiproportionally. Specifically, in the case of a permanent shock, we simply multiply $y_t^j$ by 0.8 or 1.2 from period 1 onwards. In the case of a temporary shock, we assume that aggregate income converges back to its initial level in four periods and multiply $y_t^j$ by either 0.80, 0.86, and 0.93 or 1.2, 1.13, and 1.06 in periods 1, 2, and 3, respectively. We assume that the economy is initially in a steady state (with $\theta = 0.25$). Households make their period 1 decisions after learning about the shock.

Figures 8 and 9 display the house price dynamics following the different income shocks. We have four different shocks and for all four shocks, we consider 4 cases: with and without the borrowing constraint and with $\sigma = 1$ and $\sigma = 3$. By construction, the initial house price in period 0 is equal to 1 in all cases.

The first thing to note from the figures is that the borrowing constraint shapes the price dynamics substantially only following negative shocks: following positive shocks, the price dynamics are remarkably similar with and without the borrowing constraint. Inspection of the dynamics after negative shocks (top part in Figures 8 and 9) suggests that the most important effect of the borrowing constraint is that it makes the house price increase more rapidly from period 1 to period 2. This means that there are relatively large anticipated capital gains to housing. Depending on whether we have a permanent or a temporary shock and on the intertemporal elasticity of substitution, these capital gains are about 2%–5% of the value of the house from period 1 to period 2.

The intuition behind this result is the following. The house price fall that follows a negative shock effectively tightens the borrowing constraint for those households for which it is binding. This reduces period 1 housing demand through the liquidity effect that we discussed in subsection 2.2. For the housing market to clear in period 1, the liquidity effect must be offset by sufficiently large capital gains to housing from period 1 to period 2, which induce households, especially those that are not borrowing constrained, to demand more housing already in period 1.

One might think that having these large anticipated capital gains means that the period 1 house price is much lower with a borrowing constraint than without it. This seems to be true in the case of a temporary negative income shock. With $\sigma = 3$, the period 1 house price is about 40% lower with the borrowing constraint than without it. However, in the case of a permanent shock, the impact effect is almost the same with and without the borrowing constraint. As we showed in subsection 2.2, a fall in current house price may decrease the future housing demand of unconstrained households but increase the future housing demand of constrained households. This is what happens here. In the case of a permanent and negative income shock, the housing demand just after period 1 is substantially higher with the borrowing constraint than without it. Hence, the anticipated capital gain that is needed to offset the liquiduity effect is created by a relatively high house price in period 2, rather than by a very low house price in period 1.

The fact that the borrowing constraint matters with negative income shocks but not with positive income shocks implies that the price dynamics are

quite asymmetric between positive and negative income shocks. The intuition behind this result is the following. In principle, the same mechanism that creates large anticipated capital gains following the impact effect of a negative income shock should create large anticipated capital losses in the case of a positive income shock. However, following a large positive income shock, the share of borrowing constrained households is very small: following the permanent positive shock considered above, the share of borrowing constrained households decreases from 6/30 in the initial steady state to 1/30 in period 1.[12] Therefore, a marginal increase in period 1 house price from the equilibrium price has a very small impact on the aggregate housing demand through the borrowing constraint. Consequently, following a large positive income shock, there is no need for large anticipated capital losses in balancing the supply and demand for housing in period 1. In contrast, following a large negative income shock, the share of borrowing constrained households increases to 8/30 making the liquidity effect all the more important.

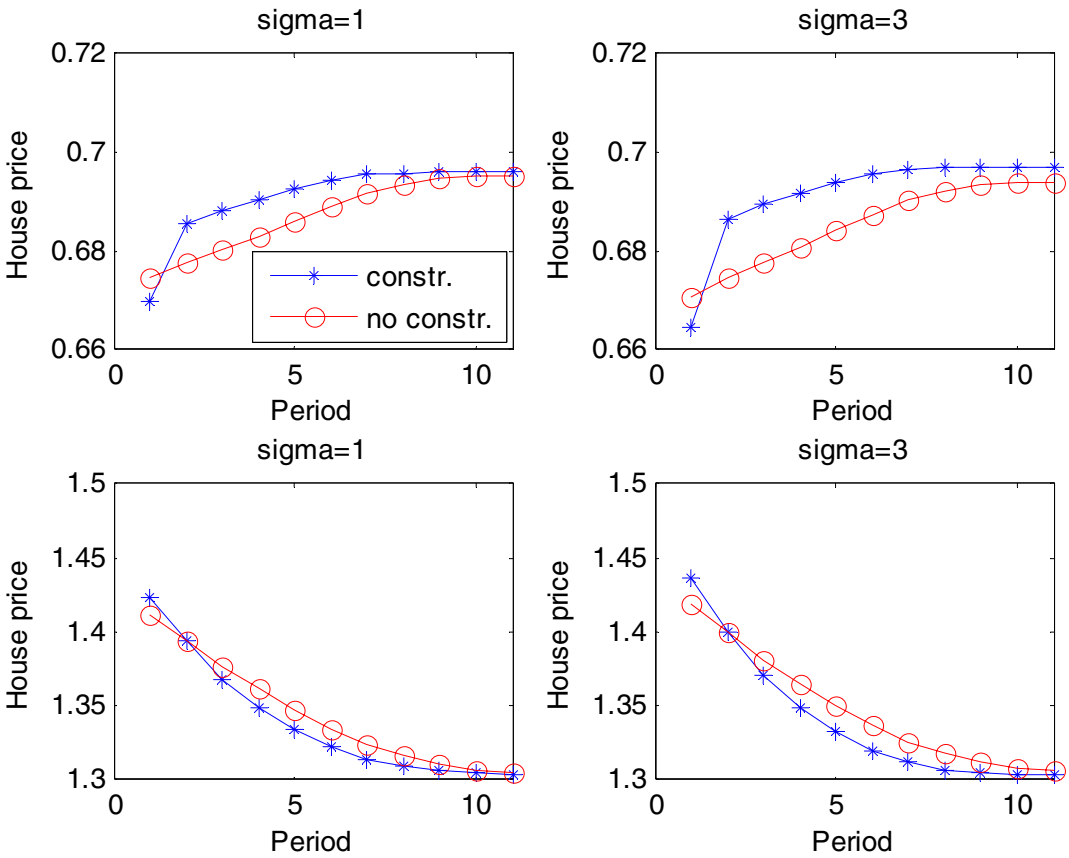


Figure 8. **House price dynamics following negative (top) and positive (bottom) permanent income shocks**

[12] At any point in time, there are 30 households of different age and type.

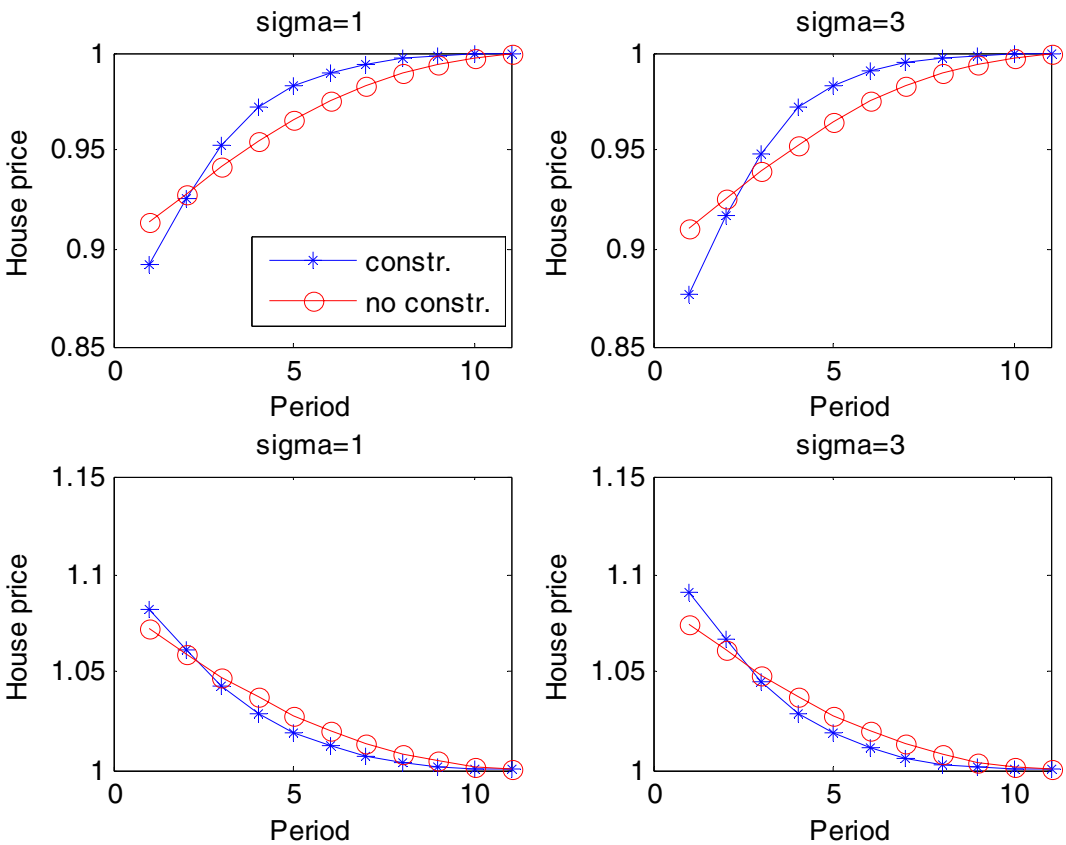


Figure 9. **House price dynamics following negative (top) and positive (bottom) temporary income shocks**

Figures 10 and 11 display the house price dynamics following the four different interest rate shocks. In the initial steady state $R = 1.1$ and the house price is equal to 1 in all cases. We consider a permanent increase to $R = 1.15$ and a permanent decrease to $R = 1.05$. The temporary shock lasts for three periods: In the case of an increase in the interest rate, we have $R_1 = 1.15$, $R_2 = 1.13$, $R_3 = 1.12$. In the case of a decrease in the interest rate, we have $R_1 = 1.05$, $R_2 = 1.07$, $R_3 = 1.09$.

Consider first the permanent shocks. There are now substantial differences in steady state effects with and without the borrowing constraint. The steady state effects are larger without the borrowing constraint, except for an interest rate increase in the logarithmic case. Apart from that, the borrowing constraint seems to shape price dynamics much less than in the case of (negative) income shocks. In the case of temporary shocks, the house price dynamics are almost identical with and without the borrowing constraint.

The reason why the borrowing constrained does not substantially affect house price dynamics following these interest rate shocks is related to the fact that now the share of borrowing constrained households does not change much on impact. An increase in the interest rate, for instance, induces a fall in the house price which tends to tighten the borrowing constraint. However, this effect is balanced by households' willingness to save more. A similar argument holds for the decrease in the interest rate: a lower interest rate reduces the cost of housing, thereby increasing demand and pushing up the house price. This tends to relax the borrowing constraint for all households. At the same

time, however, a lower interest rate makes borrowing more attractive, thereby making the borrowing constraint more binding.

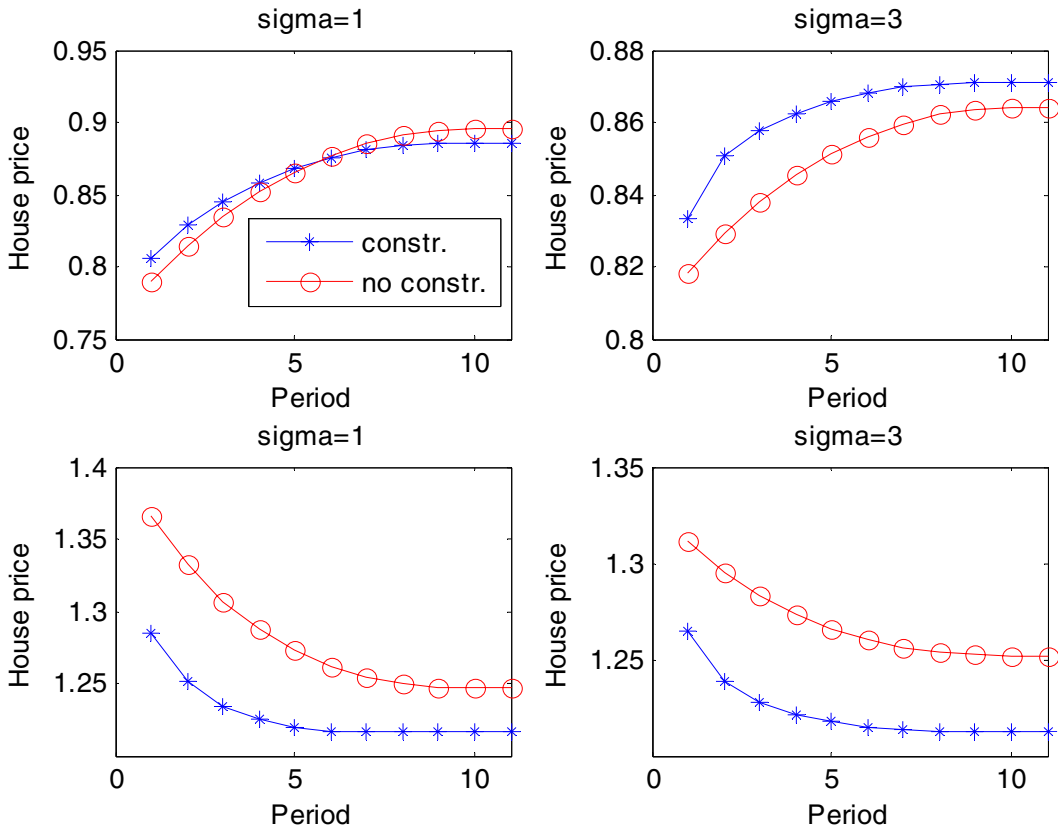


Figure 10. **House price dynamics following a permanent increase (top) and decrease (bottom) in the interest rate**

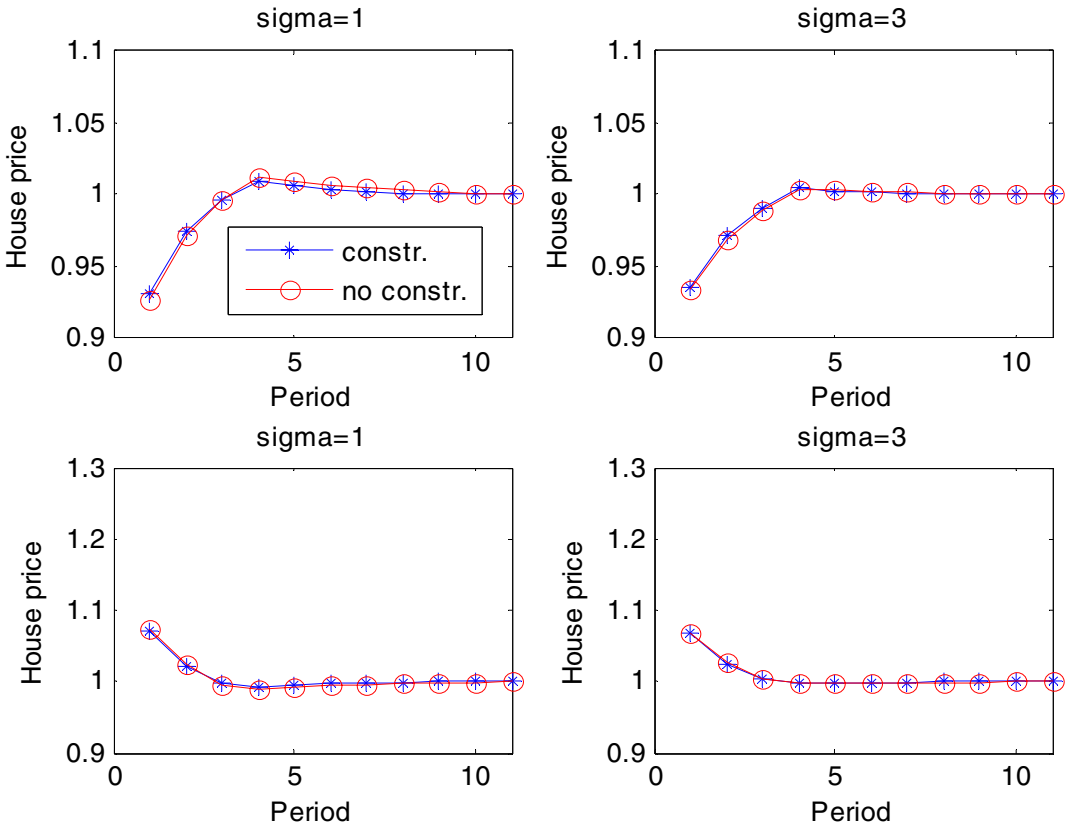


Figure 11. **House price dynamics following a temporary increase (top) and decrease (bottom) in the interest rate**

## 4.4 The effects of a marginal house price change

As we discussed in section 2, a change in the house price affects the demand for housing through various channels. The multiplier effect stressed in Stein (1995) relates to the fact that a fall in the house price may reduce buyer liquidity through the down payment requirement. This leads borrowing constrained households to demand less housing when the price falls. If this effect dominates in the aggregate demand response, there could even be multiple equilibria.

This issue can be investigated in our setting in the following way. First, we compute the equilibrium house price sequence following a permanent income shock in period 1 (the corresponding house price dynamics are shown in the top part of Figure 8). We then decrease the period 1 house price by 1% leaving other prices unchanged, and solve again the problem of all households.[13] Finally, we compute the change (from the level related to the equilibrium price dynamics) in housing demand for different household types and cohorts for periods 1 and 2. This gives us a measure of the elasticity of housing demand around the equilibrium path. As discussed above, for there to be potential for multiple equilibria, for some households at least, this price decrease should depress housing demand in period 1.

[13] This is a partial equilibrium exercise in the sense that with this new house price sequence, the demand for housing will no longer equal supply in every period.

Figure 12 shows the results for type 2 households of different ages with $\sigma = 1$ in periods 1 and 2. Following the negative income shock, households of age 2–5 are borrowing constrained.[14] The line with stars shows period 1 demand of different cohorts and the line with circles shows period 2 demand. To understand the figure, consider a household that is of age 2 in period 1 and, hence, of age 3 in period 2. The figure tells us that the (additional) 1% reduction in period 1 house price increases housing demand of this household in period 1 by about 1.6% and period 2 by 0.7%. For the household of age 1 in period 2, there is no demand change, as this household did not experience the 'disturbance' in period 1.

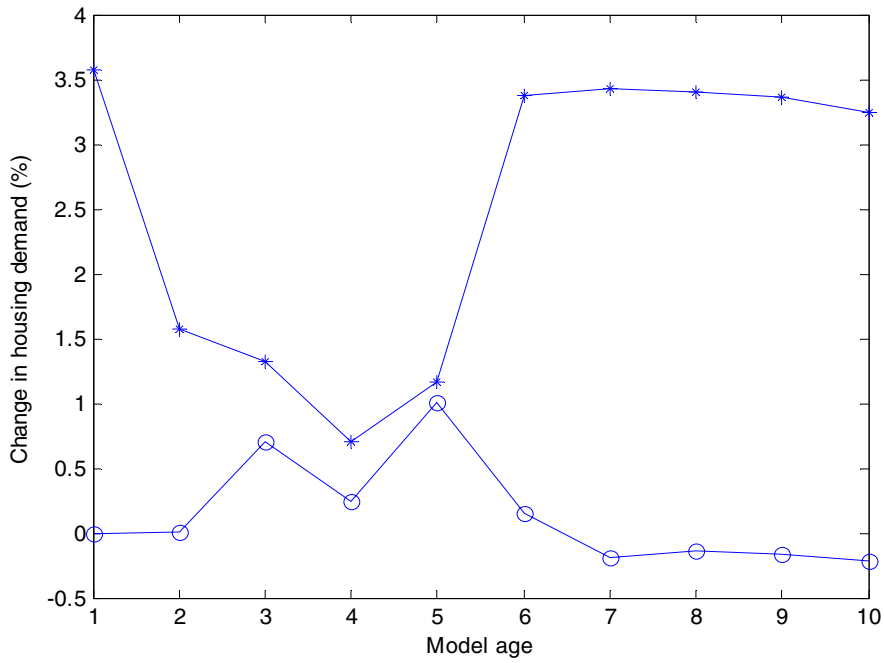


Figure 12. **Changes in housing demands following a 1% decrease in $p_1$**

The figure shows that a decrease in the period 1 house price increases the housing demand of all households of type 2 in period 1. Hence, in this sense, the liquidity effect does not dominate and there seems to be no scope for multiple equilibria. On the other hand, it is also the case that the variation in the demand response is large. In particular, the demand increase of the borrowing constrained households (of age 2–5) is much more modest than that of other cohorts. Hence, the borrowing constraint does substantially reduce the elasticity of housing demand to the current house price changes.

Finally, the figure shows, consistently with our analytical results in 2.2, that a fall in the current house price induces the borrowing constrained households to demand more housing in the following period. Intuitively, by effectively tightening the borrowing constraint, a fall in the current house prices forces

[14] Type 2 households get children at age 2 and the children live with the parents for four model periods.

households to save more. This induces them to demand more housing in the future. Those households who are not borrowing constrained in contrast demand less housing in the following period. For them, the only effect is that housing becomes cheaper in the current period. This induces them to buy more housing in the current period and less housing (and consumption goods) in the future.

## 4.5 Mimicking the Finnish boom-bust-boom cycle

We now consider a sequence of shocks, where first the borrowing constraint is relaxed and then, in the following period, the economy is hit by the depression which consists both of an income and interest rate shock as we think was the case in the Finnish depression. Our main interest is in seeing how the fact that the credit market liberalization took place just before the depression should have affected the house price dynamics following the depression.

Figure 13 shows the results. The initial steady state, with a tight borrowing constraint ($\theta = 0.75$), is now period 0. In period 1, the borrowing constraint is relaxed, and in period 2, the depression hits the economy. For comparison, we also display, in the same figure, the price dynamics that would follow the depression if the economy was in a steady state (with $\theta = 0.25$) in period 1. The interpretation of this case is that the credit market liberalization has taken place well before the depression. In that case, the price stays constant from period 0 to period 1. The permanent and temporary income and interest rate shocks analyzed here are the same as the negative shocks considered in the previous subsection. We consider here only the case with $\sigma = 3$.

The figure shows that according to the model, the credit market liberalization should have dampened the house price dynamics following the depression. In the case where the depression was preceded by a relaxation of the borrowing constraint, house prices stay at a higher level during the depression and increase more gradually after the depression. There are two reasons for this. First, as shown in Figure 7, the relaxation of the borrowing constraint alone should sustain higher, but gradually falling house prices for decades. Second, given that the borrowing constraint has just been relaxed, few households are borrowing constrained when the depression hits the economy. Both of these effects smooth away part of the undershooting in house prices that would have followed the depression had it not been preceded by the credit market liberalization.

Comparing these price dynamics with the empirical data in Figure 5 shows that when the depression is modelled with permanent income and interest rate shocks, the impact effect of the depression is similar to that in the data. That is, house price fall by about 50%. However, in the model, prices then stay almost constant following the depression, whereas in reality house prices increased quite rapidly after the depression. When the depression is modelled with temporary shocks, house prices do increase substantially following the depression. However, with temporary shock, the impact effect is then far too modest.

A plausible explanation for why house prices first fell so much and then recovered is that in the depression, households expected the income and interest rate shocks to be permanent and were positively surprised by the strong income growth and lower interest rate after the depression. We do not attempt to model these surprises, because that would not yield new insights into how the borrowing constraint affects the dynamics. Moreover, to fully address these issues, we would clearly need a stochastic model.

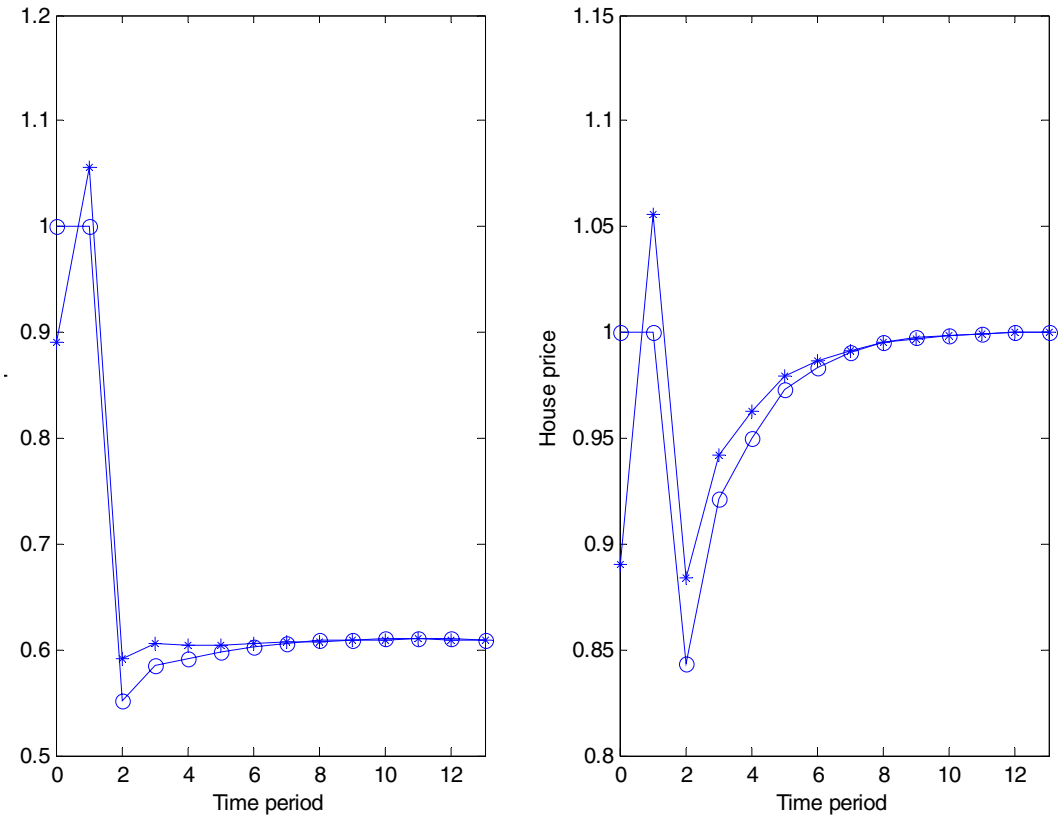


Figure 13. **Mimicking the Finnish boom-bust-boom cycle: permanent (left) and temporary (right) shocks**

# 5 Conclusions

We have analyzed how a household borrowing constraint affects house price dynamics in an OLG model with standard preferences. We found that in certain situations, the borrowing constraint can shape house price dynamics substantially. However, the effect of the borrowing constraint is very different following different shocks. Because of the borrowing constraint, the house price dynamics following large positive and negative shocks can also be quite asymmetric. These results are related to the fact that share of borrowing constrained households changes over time.

We believe that our results have some implications for empirical work. The literature (Lamont and Stein, 1999 and Benito, 2006) has tried to identify

the effect of borrowing constraints on house price dynamics by comparing the house price dynamics following income shocks between regions where household leverage differs; borrowing constraints should matter more in regions where many households are highly leveraged. However, this literature has implicitly assumed that the price dynamics are symmetric for positive and negative shocks. Our results suggest that we could learn more about the importance of borrowing constraints by considering separately positive and negative income shocks, and perhaps also relatively small and big shocks.

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# Appendix

## Housing demand and the effect of price changes

In this appendix, we derive the results of subsection 2.2. illustrating the effect of a change in $p_t$ for the housing demand and savings decision of household generation $j$ in period $t$.

### No borrowing constraint

Assuming that the borrowing constraint is not binding, the first-order conditions related to the household problem (2.18)–(2.20) are

$$\begin{aligned} u_c - \lambda &= 0 \\ u_h - \lambda\left(p + \kappa p + \eta - \frac{p'}{R}\right) &= 0 \\ \beta V_b - \lambda\frac{1}{R} &= 0 \end{aligned}$$

Combining the first-order conditions and using the budget constraint gives a system of three equations and three unknowns

$$\begin{aligned} u_c - \beta R V_b &= 0 \\ u_h - P u_c &= 0 \\ y + ph^{-1} + Ra^{-1} - c - Ph - \frac{b}{R} &= 0 \end{aligned}$$

where $P = p + \kappa p + \eta - \frac{p'}{R}$. Differentiating this system with respect to $p$ gives

$$\begin{pmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ u_{hh} & -P u_{cc} & 0 \\ -P & -1 & -\frac{1}{R} \end{pmatrix} \begin{pmatrix} \frac{\partial h}{\partial p} \\ \frac{\partial c}{\partial p} \\ \frac{\partial b}{\partial p} \end{pmatrix} = - \begin{pmatrix} 0 \\ -(1+\kappa)\, u_c \\ h^{-1} - (1+\kappa)\, h \end{pmatrix}$$

Note first that

$$D = \begin{vmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ u_{hh} & -P u_{cc} & 0 \\ -P & -1 & -\frac{1}{R} \end{vmatrix} = \beta R V_{bb}\left[u_{hh} + P^2 u_{cc}\right] + \frac{1}{R} u_{hh} u_{cc} > 0$$

Then we have that

$$\begin{aligned} \frac{\partial h}{\partial p} &= \frac{\begin{vmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ (1+\kappa)\, u_c & -P u_{cc} & 0 \\ -\left(h^{-1} - (1+\kappa)\, h\right) & -1 & -\frac{1}{R} \end{vmatrix}}{D} \\ &= \frac{1}{D}\left[\beta R\left(V_{bb}(1+\kappa)\, u_c + P V_{bb} u_{cc}\left(h^{-1} - (1+\kappa)\, h\right)\right) + \frac{(1+\kappa)\, u_c u_{cc}}{R}\right] \end{aligned}$$

And

$$\begin{aligned}\frac{\partial b}{\partial p} &= \frac{\begin{vmatrix} 0 & u_{cc} & 0 \\ u_{hh} & -Pu_{cc} & (1+\kappa)\,u_c \\ -P & -1 & -\left(h^{-1}-(1+\kappa)\,h\right) \end{vmatrix}}{D} \\ &= \frac{1}{D}\left[-u_{cc}P\,(1+\kappa)\,u_c + u_{cc}u_{hh}\left(h^{-1}-(1+\kappa)\,h\right)\right]\end{aligned}$$

Borrowing constraint

Assume now that the household faces a binding borrowing constraint. Then (2.18)–(2.20) can be written as

$$\max_{c,h}\left\{u\left(c,h\right)+\beta V\left(b\right)\right\}$$

subject to

$$y+ph^{-1}+Ra^{-1}=c+\left(p+\kappa p+\eta-\left(1-\theta\right)p\right)h$$

where

$$b=\left(p'-R\left(1-\theta\right)p\right)h$$

The first-order conditions then become

$$\begin{aligned} u_c-\lambda &= 0 \\ u_h+\beta SV_b-\lambda T &= 0 \end{aligned}$$

where $T=p+\kappa p+\eta-\left(1-\theta\right)p$ and $S=p'-R\left(1-\theta\right)p$. Combining the two first-order conditions and using the budget constraint gives two equations with two unknowns

$$\begin{aligned} u_h+\beta SV_b-u_cT &= 0 \\ y+ph^{-1}+Ra^{-1}-c-Th &= 0 \end{aligned}$$

Differentiating with respect to $p$ gives

$$\begin{pmatrix} u_{hh}+\beta V_{bb}S^2 & -u_{cc}T \\ -T & -1 \end{pmatrix}\begin{pmatrix} \frac{\partial h}{\partial p} \\ \frac{\partial c}{\partial p} \end{pmatrix}=\begin{pmatrix} R\left(1-\theta\right)\beta V_b+\beta SV_{bb}R\left(1-\theta\right)+u_c\left(\kappa+\theta\right) \\ -h^{-1}+\left(1+\kappa-\left(1-\theta\right)\right)h \end{pmatrix}$$

Note first that

$$D^c=\begin{vmatrix} u_{hh}+\beta V_{bb}S^2 & -u_{cc}T \\ -T & -1 \end{vmatrix}=-u_{hh}-\beta V_{bb}S^2-u_{cc}T^2>0$$

and therefore, we have

$$\begin{aligned} &= \frac{1}{D^c}[-R\left(1-\theta\right)\beta V_b-S\beta V_{bb}R\left(1-\theta\right) \\ &\quad -u_c\left(\kappa+\theta\right)+u_{cc}T\left(\left(1+\kappa-\left(1-\theta\right)\right)h-h^{-1}\right)] \end{aligned}$$

$$
\begin{aligned}
\frac{\partial h}{\partial p} &= \frac{\begin{vmatrix} R(1-\theta)\beta V_b + S\beta V_{bb} R(1-\theta) + u_c(1+\kappa-(1-\theta)) & -u_{cc}T \\ (1+\kappa-(1-\theta))h - h^{-1} & -1 \end{vmatrix}}{D^c} \\
&= \frac{1}{D^c}[-R(1-\theta)\beta V_b - S\beta V_{bb} R(1-\theta) \\
&\quad -u_c(\kappa+\theta) + u_{cc}T\left((1+\kappa-(1-\theta))h - h^{-1}\right)]
\end{aligned}
$$

Hence, we can write

$$
\begin{aligned}
\frac{\partial h}{\partial p} &= \frac{-u_c(1+\kappa) + u_{cc}T((1+\kappa)h - h^{-1})}{D^c} \\
&\quad + \frac{u_c(1-\theta) - u_{cc}T(1-\theta)h}{D^c} - \frac{S\beta V_{bb}R(1-\theta)}{D^c} - \frac{R(1-\theta)\beta V_b}{D^c}
\end{aligned}
$$

In addition, because $b = (p' - R(1-\theta)p)h$, we know that

$$
\frac{\partial b}{\partial p} = S\frac{\partial h}{\partial p} - R(1-\theta)h
$$

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