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## What determines commercial banks' demand for reserves in the interbank market?

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Michal Kempa

# What determines commercial banks' demand for reserves in the interbank market?



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Michal Kempa\*

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# What determines commercial banks' demand for reserves in the interbank market?

Bank of Finland Research  
Discussion Papers 30/2007

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## Abstract

In this paper I analyse the determinants of commercial banks' demand for reserves in the interbank market. I first document the pattern in the Eurosystem, where banks deviate from the required reserves balance at the start of the maintenance period only to meet the requirements closer to the settlement day. Using my model I show that this behaviour can be explained by certain trade-related frictions and costs. Examples include potential extra expenses tied to large transactions or the asymmetry between the cost of borrowing and profits from lending. I also find that borrowing decisions can be largely unaffected by current liquidity, which has important implications for the implementation of central bank monetary policy: in order to influence the level of interest rates, the central bank must focus on controlling market expectations.

Keywords: money markets, EONIA, liquidity effect

JEL classification numbers: E52, E58, E43

# Liikepankkien reservien kysynnän määräytyminen pankkien välisillä markkinoilla

Suomen Pankin keskustelualoitteita 30/2007

Michal Kempa  
Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tutkimuksessa analysoidaan reservien kysyntää määrääviä tekijöitä pankkien välisillä markkinoilla. Tarkastelujen aluksi tutkimuksessa dokumentoidaan eurojärjestelmän liikepankkien tapa poiketa reservivaatimuksista reservien ylläpito-periodin alussa ja täyttää nämä vaatimukset vasta lähempänä tilityspäivää. Tämän jälkeen työssä osoitetaan, että eurojärjestelmän liikepankkien käyttäytyminen voidaan selittää pankkien välistä kaupankäyntiä vaikeuttavilla kitkatekijöillä ja kustannuksilla. Tällaiset kitka- ja kustannustekijät voivat liittyä esimerkiksi suurten kaupankäyntimäärien aiheuttamiin kustannuksiin tai lainanoton kustannusten ja lainanannon tuottojen epäsymmetrisyyteen. Tutkimuksen teoreettinen malli on muutoin suhteellisen tavanomainen ja yleisesti käytetty aihepiirin muissa tutkimuksissa. Tulosten mukaan pankkien lainanottopäätökset ovat lisäksi lähes täysin riippumattomia niiden likviditeettiasemasta. Tämä tulos on tärkeä rahapolitiikan toteutuksen kannalta, koska vaikuttaakseen markkinoiden korkotasoon, keskuspankin tulisi keskittyä markkinoiden odotusten hallintaan.

Avainsanat: rahamarkkinat, eonia, likviditeettivaikutus

JEL-luokittelu: E52, E58, E43

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# 1 Introduction

Commercial banks use the interbank market to trade funds that are used in daily payments and to satisfy the reserve requirement. Banks choose their demand (or supply) for funds, based on their individual liquidity and expectations of the future interest rate. This market is very important for the central bank, since the average transaction rate is used as the benchmark for the interest rate in the economy and therefore regarded a perfect tool to implement monetary policy.

This paper models the behavior of the interbank interest rate. The focus is on the issues indicated in Kempa (2006), where I found that under standard (in the literature) assumptions of risk neutrality, fixed expectations and certain, market specific conditions commercial banks' demand for funds cannot be uniquely determined undermining the effectiveness of central bank policy. Here, I find that market related frictions and trading costs might be used to explain the evolution of reserves demand along the maintenance period and link market liquidity and interest rate.

The interbank market has been analyzed quite extensively before. Perhaps the most influential contribution was Poole (1968) who linked the interest rate and market liquidity in a form of demand equation: the interest rate is equal to the expected cost of using standing facilities. More recently Hamilton (1996) was the first to state, the so called, *martingale hypothesis*: since the reserve requirement can be satisfied on any day of the maintenance period the funds should be perfect substitutes. But then to avoid arbitrage opportunities, the current interest rate must equal the expected rate in the future ( $i_t = E(i_{t+1})$ ) rather than depend on market liquidity. This would have a profound impact on the central bank operating policy as it would cut the link between market operations (targeting liquidity) and target interest rate. This hypothesis was rejected by Hamilton (1996) himself for the US market and he attributed the deviation to the transaction costs and trade limits. Later on Pérez-Quirós and Rodríguez-Mendizábal (2006) and Välimäki (2003) analyzed systems with reserve requirement averaging provision and using Eurosystem framework showed that the martingale hypothesis can be rejected even in the absence of market frictions. Banks are willing to pay extra to avoid, so called, a lock-in state, where the reserve requirement is satisfied and any positive balance is left to be deposited at the central bank at penalty rate. This results in increasing level of the interest rate toward the end of maintenance period which was indeed observed before introduction of Euro. Gaspar et al (2004) document the increasing volatility of interest rate toward the end of the maintenance period and explain it using a modified standard model with heterogeneity and trading groups. Similar models of interbank market for the US were constructed by Bartolini et al (2001), Bartolini et al (2002), Bartolini and Prati (2003) and Clouse and Dow Jr (2002) where they included US-specific market features (such as carry-over provision).

The structure of the aforementioned papers is similar: banks determine the demand for reserves by balancing the cost of market funds with the expected cost of using central bank's standing facilities as in the original Poole (1968) paper. Market specific features (such as averaging provision) are then used

to explain the deviation from martingale hypothesis that was indeed observed on the interbank markets in the late 90s. However in the Eurosystem, this deviation does not hold any more, as documented in empirical papers of Würtz (2003) and Moschitz (2004) where they fail to find any systematic pattern in the average interest rate on different days of the maintenance period. This leads to a discrepancy between empirical and theoretical results.

Before discarding the standard model note that Pérez-Quirós and Rodríguez-Mendizábal (2006) as well as other researchers assume that the uncertainty faced by the commercial banks is substantial and the volatility of so called liquidity shock – explained a bit later in detail – is equal to the average current account holding. With overdrafts forbidden, the buffer offered by reserve requirement is highly valued and its price is included in the market rate.

In my own paper Kempa (2006) I looked into the actual size of the liquidity shock. Using the standard model and estimates of the parameters for the Eurosystem I showed that the liquidity shock volatility is actually relatively small compared with the average current account value (around 10%). For commercial banks that means that the probability of using the standing facilities is very low and current interest rate is mainly driven by the expected level in the future. The no-overdraft condition is hardly binding and hence the benefit of reserve requirement buffer stressed by previous researchers is barely considered when deciding about trade volume. If the current interest rate is equal to the expected one, provided the bank's reserves are sufficient its borrowing decisions have no impact on the bank's profit as the revenues from lending one euro today is equal exactly to the expected cost of borrowing one euro tomorrow.

The fact that the borrowing value cannot be uniquely determined within the framework of that model is a shortcoming from the perspective of central bankers that need to decide on the allotment size in the open market operations. If the banks are indifferent between keeping different reserves at the same interest rate, the market becomes partially immune to the liquidity conditions and the interest rate becomes driven mainly by the expectations. Central banks can control liquidity (by open market operations) but controlling expectations is much more tricky especially in the periods of unrest. The issue of the demand determinants is then of crucial importance for the effectiveness of operating policy.

This paper focuses on other than simple profit maximization determinants of the reserves demand of the commercial banks answering several additional questions. Is there any specific pattern the banks follow during maintenance period? How important for banks are the standing facilities? How can be the demand modeled with the current interest rates equal to expected level (as the empirical data suggest)? And finally, what is the impact of the central bank allotment on the behavior of the market?

The paper is structured in the following way. Section 2 presents the data for the sample of Eurosystem banks and their actual reserve demand for a period of 2,5 years. I compare the actual liquidity holding to the required reserves, and it seems there is an interesting pattern: the banks tend to deviate from

the required level at the start, and compensate for it in the latter part of the maintenance period.

Section 3 contains the standard in the literature model of the interbank interest rate based on Poole (1968) and later modifications. I also illustrate the martingale hypothesis and its implications for the market behavior.

Section 4 contains the modifications to the original model, together with a Monte-Carlo simulation study. When looking at the reserve demand, the basic benchmark is of course the required reserve requirement but apart from that, I analyse other potential candidates that can play a significant role:

- existence of trading cost
- incentives to avoid excess trading
- increasing marginal borrowing cost
- asymmetry between borrowing costs and lending profits

The trading cost (suggested initially by Hamilton, 1996) might be the result of obligatory collateral (in case of secured transactions) or matching problems (cost of finding parties willing to trade). Avoiding volatility seems natural assumption that holds on most financial market and interbank is not exception here. As for increasing the marginal cost, excess borrowing on the interbank market might send a negative signal to other participants suggesting the bank faces liquidity problems and influence bank's reputation. Also increased borrowing cost is not likely to be matched by increased profits from lending creating certain asymmetry. All those issues are discussed in much more detail below.

The results of the simulation roughly follow the patterns documented in section 2. I find that including market related frictions and trading costs might indeed play significant role in the reserves demand and yields result that resemble the actual behavior of the Eurosystem.

## 2 Reserve demand of Eurosystem banks

This section contains a brief overview of the interbank market. The market is unique in many aspects and often overlooked or confused with regular bonds or stocks market. Later on I also present the data on the bank behavior obtained from the ECB.

### 2.1 Interbank market

The interbank market is used by commercial banks to trade overnight reserves that are then stored on the account in the central bank.<sup>1</sup> During the day, those reserves facilitate liquidity flows and transfers between the banks resulting from transactions between their customers (see below). It is however only the final end of the day balance that can be used to satisfy obligatory reserve requirement.

The reserve requirement is typically linked to the balance sheet of the bank and is known at the start of maintenance period. In many countries (including the Eurosystem), an averaging provision is used meaning the average (during maintenance period) rather than fixed reserves must be stored on the account in the central bank. Also required reserves are remunerated. The banking sector obtains the required liquidity from the central bank during open market operations that are either long term (1 month and more maturity) or short term (1 week maturity). The rate at which the required reserves are remunerated is closely linked with the rate of open market operations, hence central bank makes zero profit on that operations. Correspondingly, commercial banks bear no cost of the reserve requirement.

Each day commercial banks are processing large number of transactions. Since their number is so significant however, there are likely to be statistically significant patterns that can be anticipated in advance. For example mortgage bank can very well model loans repayments and modify their actions on the interbank market accordingly. From the bank perspective then, those liquidity movements should pose little problems in terms of meeting end of the day balance target. Even though certain flows drive the majority of the trade on the interbank market, in this paper I am interested in the random, unexpected transactions that require the bank to take extra measures. The reason is, those are more likely to have an impact on the behavior of the interest rate as will be explained later on.

In general, those random distortions are idiosyncratic, and I have decided to divide them into:

- those happening before the end of the trading
- those happening after the trading day is over

The first type result mainly from large, unexpected payments orders made by the customers that under some assumptions can be easily offset on the

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<sup>1</sup>Excellent description of various aspects of the interbank market and monetary policy implementation can be found in Bindseil (2004).

interbank market. Take for example two banks that start at the liquidity levels they find optimal for the purpose of satisfying reserve requirement. When an unexpected payment order occurs between those two, it will force one of them above and the other below optimal liquidity level which will create incentives for trade. Assuming no trade restrictions, transaction just reverting that payment order (returning banks to the optimal liquidity level) is the most profitable but also means early liquidity changes have no impact on the end of the day balance. The case where the restrictions are present is the main focus of this paper and is discussed in detail in section 4.

The customers orders have no impact on the aggregate market liquidity as they only result in liquidity flows between banks themselves. There are however also other operations, such as cash withdrawal or transfers to government accounts that have an impact on the aggregate market liquidity. Often referred to as ‘changes to autonomous liquidity factors’ they remain closely monitored by the central banks. The balance of those transactions is the benchmark against which the allotment (in open market operations) is decided. In case of the Eurosystem, the ECB has employed the policy of exact offsetting that changes, maintaining the market liquidity at stable level, equal to the aggregate reserve requirement. Historically the ECB has achieved high accuracy in predictions of market liquidity hence I decided not to deal with them in this paper and just assume the aggregate liquidity remains on constant level. An exception is scenario where I analyse the impact of the liquidity shortage in the market behavior discussed later on.

Finally there are changes to bank balances that cannot be offset on the interbank market, mainly because they happen after the trading day is over. Those are regarded far more interesting as they determine banks final day reserve holdings and ultimately reserve demand. When deciding its market participation value, the bank must take into account that it might be hit by a negative shock that exceeds its current account balance and will force the use of central bank standing facilities at penalty rates.

It is important to distinguish between commercial bank behavior on the interbank market and its regular activities (customer loans, assets management etc.) and those two aspects should be regarded separately as they refer to different time scale. The interbank market does not provide the liquidity for new customer loans that have typically longer maturities. Increasing customers deposits will eventually result in higher reserve requirement in the next maintenance period but the liquidity needed for that can be however easily obtained in open market operations at nearly no additional cost (since the required reserves are remunerated at the same rate as liquidity supply operations). This essentially means there is no direct link between interbank and other activities performed by commercial bank.

## 2.2 Eurosystem banks

The interbank market is a closed market and in the Eurosystem the aggregate reserves value is close to aggregate reserve requirement. What that means is one bank decision to front-load (maintain higher balance of current account

than required reserves) must be reflected in other bank's decision to back-load. When looking at market data, those two banks behavior will simply net each other, hence to observe those patterns one needs to look at individual data.

The sample obtained from the ECB includes information about 71 commercial banks, their current account balances and corresponding reserve requirement for the period 24 January 2003 – 31 May 2005.<sup>2</sup> That means, only end of the day positions are known but not the daily operations throughout the day or trade volume. The sample has been selected from all Eurosystem countries by choosing a fixed number of large, medium and small banks.<sup>3</sup> Unfortunately the sample choice is biased toward large banks. For example Germany was represented by 15 banks, 10 of which were large, 3 medium and 2 small. The benefit of that procedure is that data cover much larger part of market trade than raw number of banks suggest. Some basic statistics of the sample banks are presented in the table 2.1.

**Table 2.1** The sample statistics

Number of banks	71
Average reserves (1)	745,5 Eur mln
Total sample reserves (2)	52.925 Eur mln
Total market reserves (3)	135.907 Eur mln
Average difference between current account and reserve requirement (4)	0,4 Eur mln
Standard deviation of (4)	17,2 Eur mln

(1) Average reserves – average current account of sample bank,

(2) Total sample reserves – average sum of all current account balances in the sample,

(3) Total market reserves – average total Eurosystem liquidity in the corresponding period,

(4) (Current account) – (Reserve Requirement)

Source: ECB

First of all, due to the fact that largest banks dominate the sample, the reserves of analyzed banks (53 Eur bl) constitute almost 40% of total average market liquidity during that time (135 Eur bl). That also means, the patterns observed on the Eurosystem will be well reflected in my data. For example, the aggregate deviation of the current account from reserve requirement during the maintenance period is very close to zero which is a natural implication of the ECB liquidity supply policy (that is, the market is supplied with just sufficient liquidity to satisfy the reserve requirement). The average volatility of that deviation is also very small (in the sample its around 2% of the average current account) which is due to fairly frequent operations that offset changes to the autonomous liquidity factors which in turn stabilizes the market liquidity during whole period.

Another illustration of that is figure 2.1, that presents the average (across banks and maintenance periods) deviation of current account from reserve requirement on different days of maintenance period. I have also

<sup>2</sup>I am very grateful to ECB and Nuno Cassola from liquidity management division for the permission to collect and use the data obtained during the internship that took place in the period April – June 2006.

<sup>3</sup>Details on the sample selection can be also found in Kempa (2006).

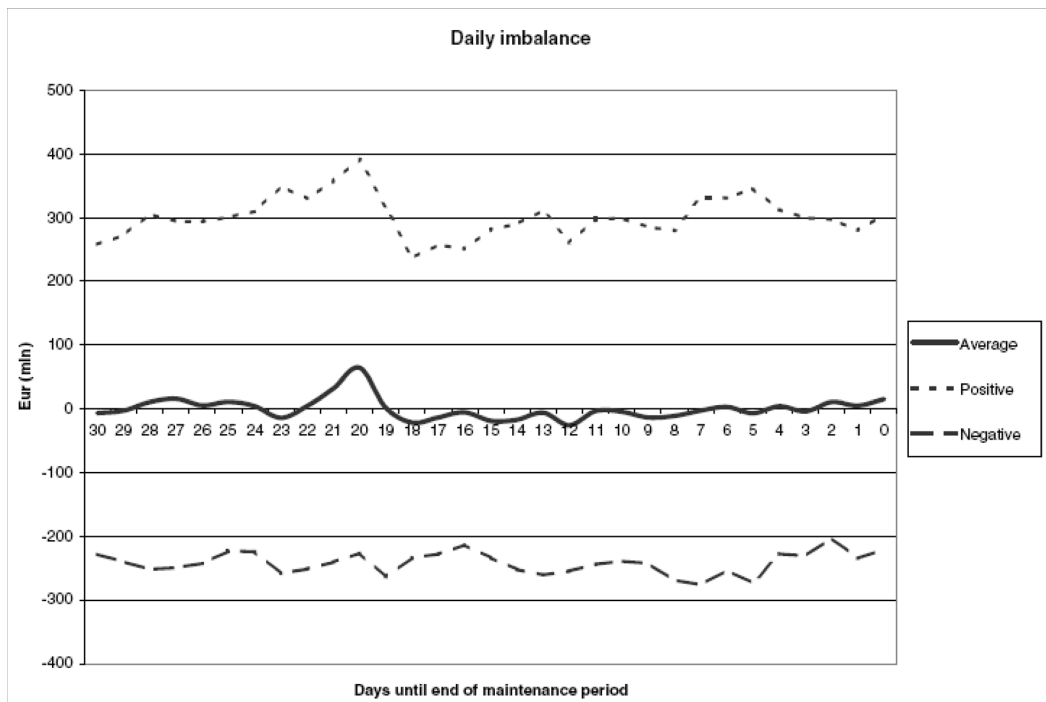
included average positive (current account higher than reserve requirement) and negative deviation value but they do not seem to follow any specific pattern apart from the decline in average positive imbalance 20–18 days before the end of the maintenance period. This decline can be attributed to the aggregate change in liquidity that was the result of ECB policy during analyzed period, rather than individual banks.

A more interesting picture emerged once I started to look into the behavior of accumulated imbalance for average front- and back-loading bank in my sample, presented on the figure 2.2. To plot that figure I have simply added all the deviations of current account balance from reserve requirement that happened since the beginning of maintenance period. The dashed lines indicate 95% confidence intervals. The front-loading bank will exhibit positive accumulated deviation in the beginning to drop toward the the end of maintenance period. An opposite effect will be observed for the back-loading bank.

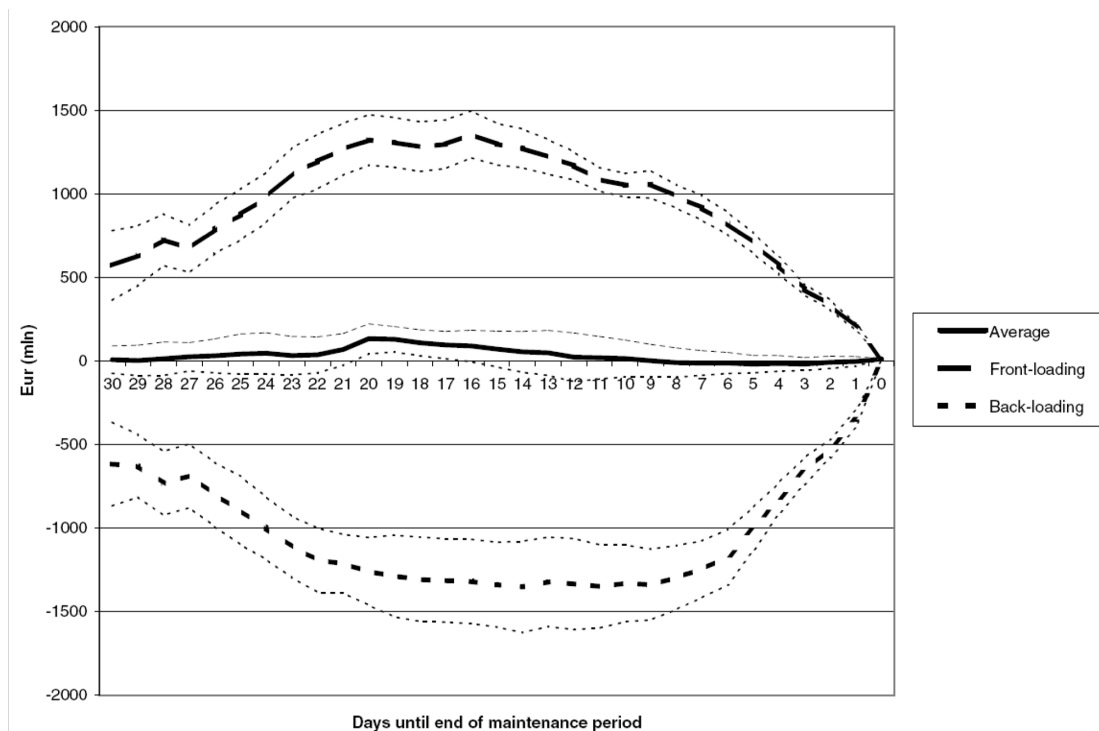
On an aggregate basis, those two series net each other resulting in very low accumulated aggregate imbalance (that result can be also obtained analyzing market data) thus creating an impression that most banks are aiming for current account value exactly equal to reserve requirement. When looking at the data however, both positive and negative deviations are increasing until approximately half of the maintenance period. This might indicate that the banks are not motivated to ensure their balances are close to the required reserves. As the end of the maintenance period is drawing closer, the banks realize the risks of failing to satisfy the requirement (or satisfying early) and reverse their previous behavior (so for example front-loading bank current account is falling below required reserves) so couple of days before the end of maintenance period they end up with accumulated deviation very close to zero.

An important question needs to be asked in this place, namely is that deviation significant? The maximum value of aggregate deviation from neutral liquidity is roughly 1.4 Eur bln, with the average reserves covered by the sample were equal to 745 Eur mln. To illustrate the importance, the bank with that sort of shortage would have to borrow twice its value of average current account value in order to immediately bring his reserves to required level. I have no information about average trade volumes of individual banks, but still it seems the scale of that deviation can be regarded substantial.





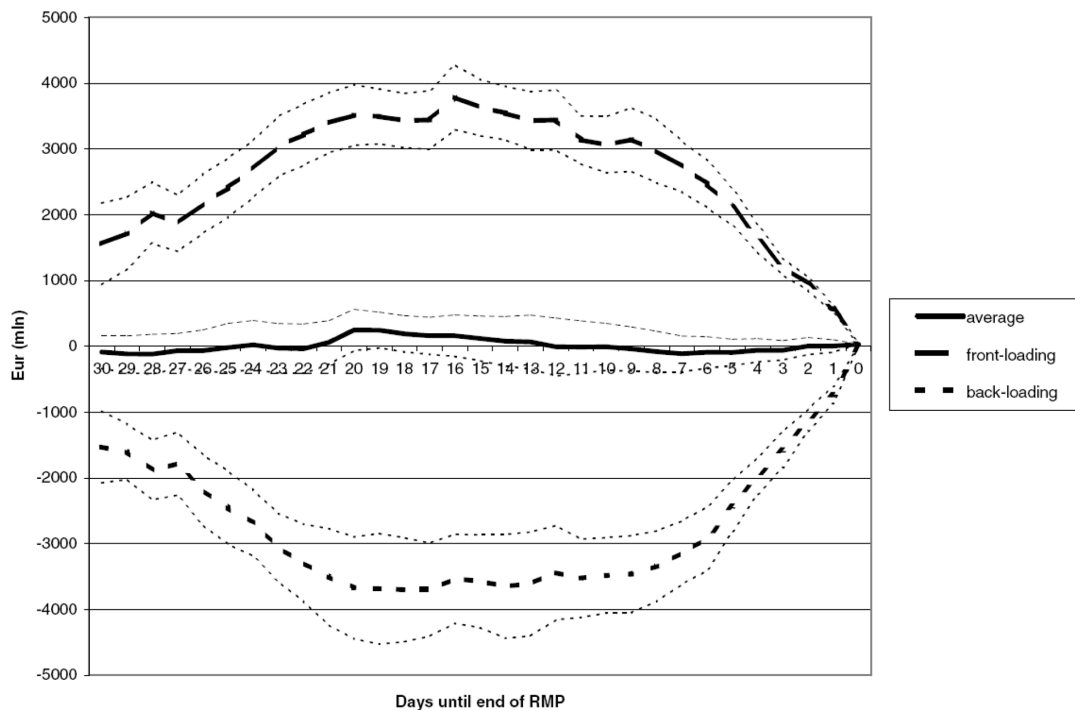
**Figure 2.1** Daily difference between current account and reserve requirement



**Figure 2.2** Accumulated imbalance in the sample

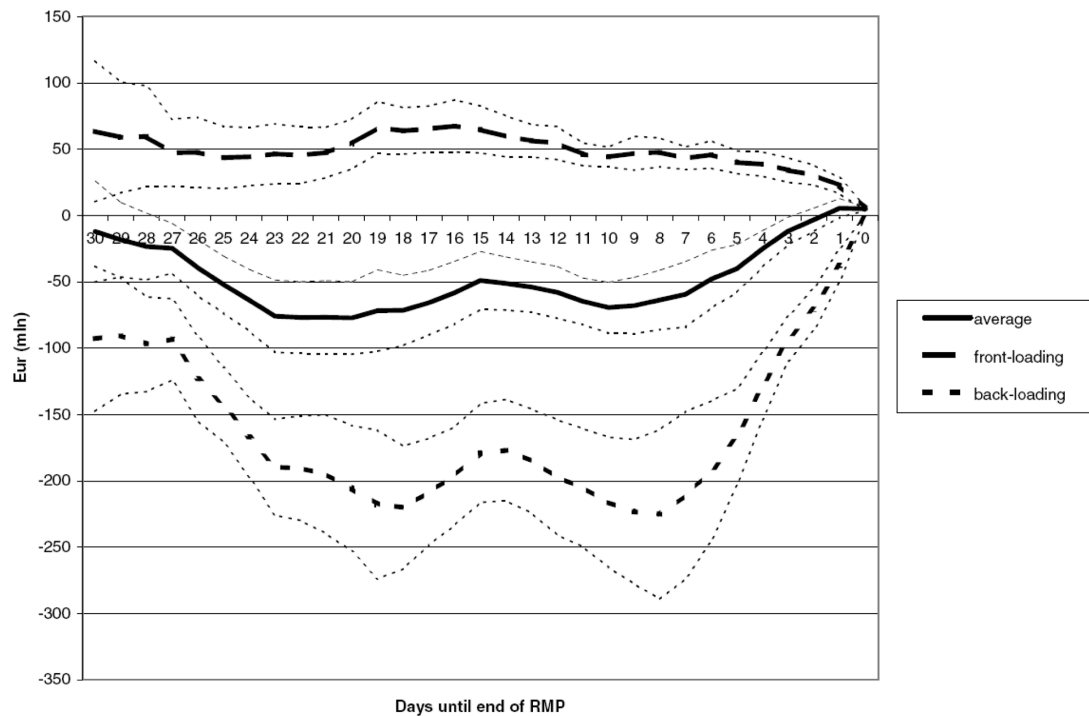
Another issue that might be interesting is the distribution of the deviation and in particular the impact of the size of the bank. To analyse that, I have picked 20 largest and 20 smallest banks from my sample and looked closer at

their behaviour. Figure 2.3 shows the largest 20 banks and not surprisingly the results are very similar to the whole market. This is due to the fact that the sample choice is heavily biased toward large banks. Perhaps more interesting is figure 2.4 that presents the accumulated deviation from neutral liquidity for smallest 20 banks in the sample. The results are not so clear here (note how wide are confidence regions<sup>4</sup>) but it seems the banks on average experience some aggregate back-loading behavior. The sample size does not allow for making any generalisations but this behavior is consistent with the theory and results presented in Kempa (2006). In this paper I showed that small banks might be more inclined toward the back-loading it has however no significant impact on the whole market.

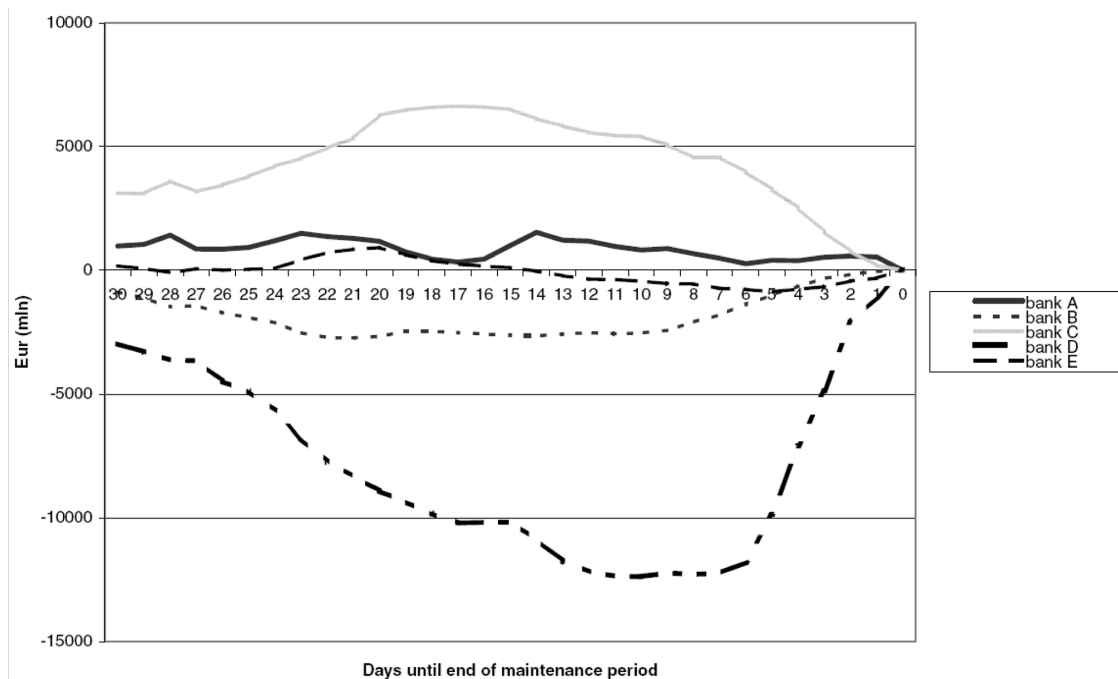


**Figure 2.3** Accumulated imbalance for large banks

<sup>4</sup>The confidence regions were computed using number of banks front- or back-loading and standard deviation of the aggregate positive (negative) deviation across the banks.



**Figure 2.4** Accumulated imbalance for small banks



**Figure 2.5** Accumulated imbalance for selected banks

The last question I would like to address is: do individual banks show some particular inclination toward back- and front-loading? Analysing the data reveals that there might indeed be a pattern in individual bank behavior. Some banks tend to be the front-loaders and some other are on average back-loaders

on a fairly regular basis. There is also a sizable group of banks that follow no specific pattern and sometimes are front- and sometimes back-loading. The illustration to that is figure 2.5 which pictures average value of deviation for 5 selected banks. This picture is however quite hard to interpret and conclusions must be drawn with care. For example, Bank C and D are both large banks from the same country but one of them exhibits on average front and other back-loading behavior. I do not have access to the names of the banks nor any other data, but this might indicate some additional factors that are bank specific might have an impact on its behavior. Those might relate to the either particular distribution of the daily liquidity flows (for example negative on average rather than zero) or bank's characteristics (for example market power). With no data available, I cannot address those issues in this paper.

The general picture from the analysis in this section seems to indicate that the banks typically choose to pick the reserves value that is only loosely related to the required reserves. Due to the fact that interbank market is closed, some banks front-loading must cause its counter-parties to back-load, which is reversed toward the end of the maintenance period, but the individual deviations from neutral liquidity are quite substantial. Banks seem to be heterogeneous in a sense that some exhibit preference toward back- and other front- loading but it does not seem to be related to their size or country of origin but rather some unobserved characteristics.

### 3 Standard model

In this section I introduce the standard model of the interbank market based on Poole (1968) and later extensions to the version with averaging provision by Välimäki (2003) and Pérez-Quirós and Rodríguez-Mendizábal (2006). In the next section, 4. I am going to introduce and motivate several modifications but the basic structure will remain very similar.

The starting point of the model is a commercial bank that minimises the cost of funding and needs to satisfy reserve requirement. It can obtain the funds either from open market operations, interbank market or standing facilities. Open market operations take form of tenders (at least in the Eurosystem) allotting liquidity to highest bidders. Since the funds obtained during liquidity tenders can be traded away on the interbank market, to prevent arbitrage opportunities the allotment and market rates must be the close to each other.<sup>5</sup> Hence, from the commercial bank perspective it should make no difference if the funds are obtained from the central bank or the market (in practice the collateral requirements differs a bit). Therefore in the model I focus on two sources of funding: interbank market and standing facilities.

The timing of the model is the following. Each bank starts the day with a current account balance  $m_t$  and deficiency (remaining reserve requirement)  $d_t$ .

$$\overbrace{m_t, d_t \quad \varphi_t \quad b_t \quad \varepsilon_t \quad m_{t+1}, d_{t+1}}$$

---

<sup>5</sup>A model of the relationship between the open market operations and interbank interest rate has been constructed by Välimäki (2006).

During the trading day, the bank faces both expected and unexpected liquidity changes based on its customers payment decisions. Those transactions were discussed in detail in the section 2.1 above and here I am interested in the part of them that has not been anticipated before. They are denoted  $\varphi_t$  and the bank is still able to offset them during the interbank trade by choosing the value of  $b_t$  (positive  $b_t$  means the bank is borrowing). After the trading day is over, the bank might face late liquidity shock  $\varepsilon_t$  that will be the final contribution to the end of the day balance.

In the remaining part of the paper I assume both early and late shocks are identically and independently distributed with mean zero and constant standard deviation. In the simulation study in section 4.2 I have normalised the shocks to follow normal distribution. Let me spend some time discussing this assumption. First of all, some banks are perhaps more likely to be hit by positive and other by negative shocks. There might be also connection between early and later shock realisation, but this is likely to be determined by factors such as individual bank characteristics, that are beyond the scope of this paper. Finally, the variance of the early shock might in reality be higher comparing to late one; the scale of the unexpected customer-driven transactions is much larger than random errors in processing. Differentiating the variance of the shock does not however introduce any new mechanism in place while it greatly complicates the computations due to the curse of dimensionality.

The final end of the day balance on the current account is then  $m_t + \varphi_t + b_t + \varepsilon_t$ . If that expression turns negative, the bank needs to borrow from the standing facilities. If positive, its either used to satisfy the reserve requirement or deposited at the central bank (if all the reserve requirement has been already satisfied). The equation of motion for deficiency is

$$d_1 = R \tag{3.1}$$

$$d_t = \begin{cases} d_{t-1} & \text{if } m_t + \varphi_t + b_t + \varepsilon_t < 0 \\ d_{t-1} - m_t - \varphi_t - b_t - \varepsilon_t & \text{if } 0 < m_t + \varphi_t + b_t + \varepsilon_t < d_{t-1} \\ 0 & \text{if } m_t + \varphi_t + b_t + \varepsilon_t > d_{t-1} \end{cases} \tag{3.2}$$

where  $R$  is the starting value of reserve requirement.

In the standard model a single period cost function  $K$  for commercial bank takes the following form

$$K_t = i_t b_t + E(c_t) \tag{3.3}$$

At the interest rate  $i_t$  the cost of borrowing from the interbank market is  $i_t b_t$ .  $E(c_t)$  is the expected cost of standing facilities  $E(c_t)$  that are used when the bank balance either falls below zero or exceeds the remaining part of the reserve requirement.

Since the bank can choose when to satisfy the reserve requirement, the problem has a dynamic nature captured by the following Bellman's equation where  $V$  is the value function

$$\min_{b_t} V_t = \{i_t b_t + E(c_t) + E(V_{t+1})\} \tag{3.4}$$

There is no continuation value beyond the end of the maintenance period (we assume no carryover provision similar to Eurosystem) so the problem on the last day is slightly different and takes the single period form

$$\min_{b_T} V_T = \{i_T b_T + E(c_T)\} \quad (3.5)$$

First order conditions<sup>6</sup> for equations (3.4) and (3.5) link the optimal borrowing and the interest rate

$$i_T = i^l F(-m_T - b_T - \varphi_T + d_T) + i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) \quad (3.6)$$

for last day of the maintenance period and

$$i_t = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_1 + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_2 - \underbrace{\int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t}_3 \quad (3.7)$$

for days before.  $m_t$  denotes assets holdings,  $b_t$  interbank borrowing (negative means lending)  $d_t$  is the remaining part of reserve requirement.  $F(\bullet)$  is the late shock ( $\varepsilon_t$ ) distribution function and can be interpreted as the probability of shock realisation falling below the expression in brackets.

Those conditions have the following interpretation: at the market rate  $i_t$  the bank would choose the borrowing value  $b_t$  so the expected cost of interbank borrowing ( $i_t$ ) is equal to expected cost of using standing facilities and dynamic cost factor that captures impact of extra unit of deficiency on the future cost of funding.

The demand curves based on above equations are presented on figure 3.6 In Kempa (2006) I have used them to draw several interesting observations. The most important one is perhaps that for the parameter range that resembles the values used in the Eurosystem, there exists a flat part in the demand schedule for days before the end of maintenance period. Its interpretation is following: given that the interest rate is equal to the expected level, the bank will be indifferent between several borrowing values. This is due to the fact that for large reserve requirement, the probabilities of using the standing facilities remain very small, hence the parts (1) and (2) from equation (3.4) vanish. It can be shown (Kempa, 2006, reproduced in appendix A2.2) that the last part, the dynamic cost factor value is very close to the expected interest rate, reducing eq. (3.4) to

$$i_t \approx E(i_{t+1}) \quad (3.8)$$

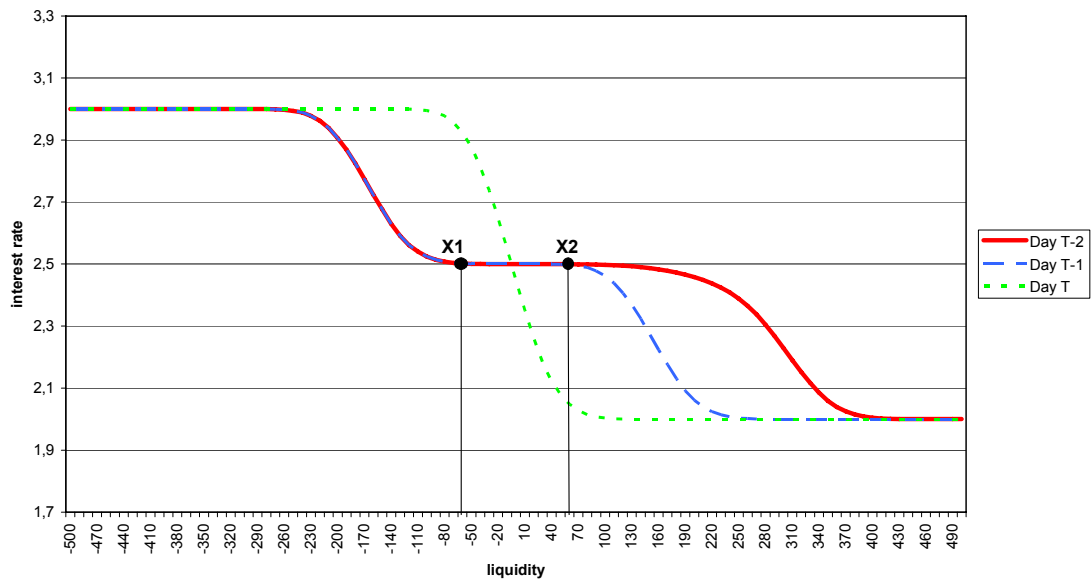
Intuitively, if the reserves are large enough comparing to the market volatility, small borrowing changes do not affect the probability of using standing facilities in the following days as well leaving the expected interest rates

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<sup>6</sup>Proofs of those equations can be found in Kempa (2006) and is reproduced in appendix A2.1.

as an alternative cost. This formulation of martingale hypothesis (from Hamilton, 1996) means however that there is range of bank decisions where exact borrowing value cannot be uniquely determined and the interest rate would stay close to the expected level regardless of the market liquidity.

In reality, that view might be somehow questionable. After all, it does seem the banks are following some specific policy. It might be front-, back-loading and the data from the sample (from section 2) indicate even that some banks might constantly change their policies. This is however corrected well before end of the maintenance period, and cannot be explained using standard model. In order to address that shortcoming, I will modify the original profit equation (3.3) to include some other additional motives.



**Figure 3.1** Standard model demand curves

## 4 Market frictions

The standard profit function assumes that the banks are risk neutral hence they care only for their absolute cost value. That assumption was key to the martingale hypothesis, that stated the funds on different days of maintenance period are perfect substitutes. In reality however banks might have other additional motives, when deciding on the borrowing value. In this section, I will explore the implications of banks trying to avoid excess volatility of their costs as well as certain asymmetry in the financing cost, referred to as market frictions in the remaining of the paper.

In the standard setup with risk neutrality, a scheme where one covers cost of several small borrowing operation with one-time revenues from lending is entirely possible. Consider now a following scenario where an individual bank lends almost all available resources (back-load) postponing satisfaction of the reserve requirement until the the last day of the maintenance period. If the rate

of borrowing and lending transactions is the same bank makes zero loss. The data from the sample I examined offer no indication such a behavior actually happens. Even though there is a number of banks front- and back- loading, they already reverse their behavior at latest around the middle of the corridor system, indicating they are willing to avoid large transactions on last days of maintenance period.

There are several possible explanations of this behavior and here I focus on two of them.

First, I address the possibility that banks want to avoid excess variation in the cost of financing from the market. That means, smaller scale transactions are preferred over large ones, even though they might sometimes results in lending one day just to borrow the day after.

Second, the market is subject to several risks including the probability of insolvency. Since bad loans are signaled by outflow of liquidity from the bank, bid for excess amounts of liquidity might be interpreted by others as indication of potential problems. This transfers into higher risk premiums charge in the unsecured market and in extreme case – refusal to trade.

In secured market the transactions between participants are backed by collateral which offers extra security but gives rise to another risk: the valuation of the underlying assets might change. This risk is asymmetric as it effects more borrowing than lending transactions. It is due to the fact that transition price captures both current market conditions (aggregate liquidity, expectations etc) and the quality of collateral used, and the last factor might be in some cases much more important.

In case of repo transaction, extra large borrowing of individual bank might expend its quality collateral and its trading parties will demand higher premiums for lower quality securities. Similar effect occurs when the valuation of the collateral drops, for example due to the crash on stocks markets. That means, excess borrowing might likely force the bank to trade at very unfavorable prices.

On the other hand, even though the market is closed, there are no dominant players that could corner it.<sup>7</sup> Thus single bank surplus of liquidity does not necessarily mean *significant* shortages at its corresponding parties that will continue to use their regular quality collateral. Hence the transaction price is very likely to stabilise around average market price.

This asymmetry might have implications for the behavior of the commercial banks. Depending how large is the pool of available collateral, the banks might exhibit larger or smaller propensity to maintain surplus amounts of reserves, especially at times of financial distress.

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<sup>7</sup>To see that, first recall there is no dominant player on that market: the average total liquidity in the sample period was 150 Eur bln, while average bank (and sample was biased toward large banks) reserves were 750 Eur mln. The HHI ratio calculated over current accounts (which can be used to estimate market power) was very low and only 0.04.



## 4.1 Model

In this section I construct the cost function that satisfies the properties described above. I assumed that the commercial bank has only two choices how to obtain funds. First, it can use the central bank standing facilities and I denote cost of using them by  $c_t$ . Second, it can use the interbank market which is a function of the interest rate  $i_t$  and borrowing volume  $b_t$ . I denote that function by  $\kappa(i_t, b_t)$ , so total cost function is

$$K_t = \kappa(i_t, b_t) + E(c_t) \quad (4.1)$$

Total cost function  $K_t$  captures the cost of financing from both those sources. Financing from the market is a function of interbank market rate and borrowing value denoted  $\kappa(i_t, b_t)$  and expected cost of using standing facilities is  $E(c_t)$  with exact expression given by (A2.1). Those two terms have quite different properties and I will discuss them separately.

### *Financing from the market*

In order for the capture the increased cost of borrowing the function must satisfy

$$\frac{\partial \kappa(i_t, b_t)}{\partial b_t} < \frac{\partial \kappa(i_t, b_t + \epsilon)}{\partial (b_t + \epsilon)} \quad (4.2)$$

for all  $b_t$  and positive  $\epsilon$ .

In order to capture asymmetric trading cost (borrowing more costly than lending), the cost function must satisfy

$$\kappa(i_t, b_t) > -\kappa(i_t, -b_t) \quad (4.3)$$

for any  $b_t > 0$ .

It is easy to see that both properties are satisfied for any non-decreasing, convex cost function such that:

1.

$$\frac{\partial \kappa(i_t, b_t)}{\partial b_t} > 0 \quad (4.4)$$

2.

$$\frac{\partial^2 \kappa(i_t, b_t)}{\partial b_t^2} > 0 \quad (4.5)$$

3.

$$\kappa(i_t, 0) = 0 \quad (4.6)$$

Those are not particularly restrictive conditions and in fact they can be captured by any quadratic function. The same results and easier computational burden can be also obtained by the following formulation

$$\kappa(i_t, b_t) = \rho^{-1} \exp(\rho i_t b_t) \quad (4.7)$$

This functional form directly satisfies conditions (4.4) and (4.5). Adding constant term  $(-\rho^{-1})$  would bring it in line with condition (4.6) with no impact on the model results. There are several additional restrictions that are not necessary (for example the maximum lending cannot exceed available assets) which are discussed later on, in the simulation description. The parameter  $\rho$  captures how significant trading costs are and I discuss its role later on.

#### *Financing from the central bank*

Commercial bank can also choose to finance through standing facilities. In the case its expected cost for days before the end of maintenance period<sup>8</sup> is given by following expression

$$E(c_t) = \underbrace{i^l \left[ \int_{-\infty}^{-m_t - b_t - \varphi_t} (-m_t - b_t - \varphi_t - \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right]}_1 - \underbrace{i^d \left[ \int_{-m_T - b_T - \varphi_T + d_T}^{\infty} (m_T + b_T + \varphi_T - d_T + \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right]}_2$$

Whenever negative shock realisation exceeds current account (term 1.) the bank needs to use lending facility paying  $i^l$ . If the current account balance is higher than required reserves, the surplus is deposited at rate  $i^d$ . The expected cost  $E(c_T)$  function is decreasing, convex function of market borrowing value  $b_t$  with proofs in appendix A2.3.<sup>9</sup>

The total cost of finance for the commercial bank is given by a following expression

$$K = \rho^{-1} \exp(\rho i_t b_t) + E(c_t) \quad (4.8)$$

which consists of both terms described before. Note an interesting property of such a formulation; the cost of obtaining extra euro from the market depends on the amounts requested and at extreme values can reach very high level. On the other hand, obtaining the same euro from the central bank cost always

<sup>8</sup>It is slightly different for last day as presented in appendix A2.1.

<sup>9</sup>Intuitively, the more bank borrows from the market, the more likely the use of deposit facility which reduces cost of central bank finance.

To see convexity, note that first order condition is just inverted function (3.7). plotted on figure 3.1 and is clearly increasing. Also note that the derivative becomes effectively flat at extreme low or high values of borrowing when the probability of using standing facility converges to unity. At that point extra unit of funds lent (borrowed) in the market results in exactly same increase in the use of standing facilities offered at fixed rates.

the same ( $i^l$ ). This means that in certain situations banks might prefer to choose central bank finance rather than using the market. When that exactly happens, depends on the parameters such as relative cost of standing facilities (spread between lending and deposit one) or degree of market frictions  $\rho$ .

Since total cost is sum of two convex functions, to solve for the optimal borrowing for individual bank on the last day of RMP, I use the first order condition

$$i_T \exp(\rho i_T b_T) = i^l F(-m_T - b_T - \varphi_T + d_T) + i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) \quad (4.9)$$

For the days before end of RMP, the first order condition takes form

$$i_t \exp(\rho i_t b_t) = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_1 + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_2 - \underbrace{\int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t}_3 \quad (4.10)$$

The proofs are very similar to the standard model and are presented in the appendix A2.3. The interpretation of those equations is very similar to standard model equations (3.6) and (3.7) In fact the right-hand side remains unchanged, as the expected cost of using standing facilities is the same. The market rate on the left-hand side is however modified by additional term, which follows

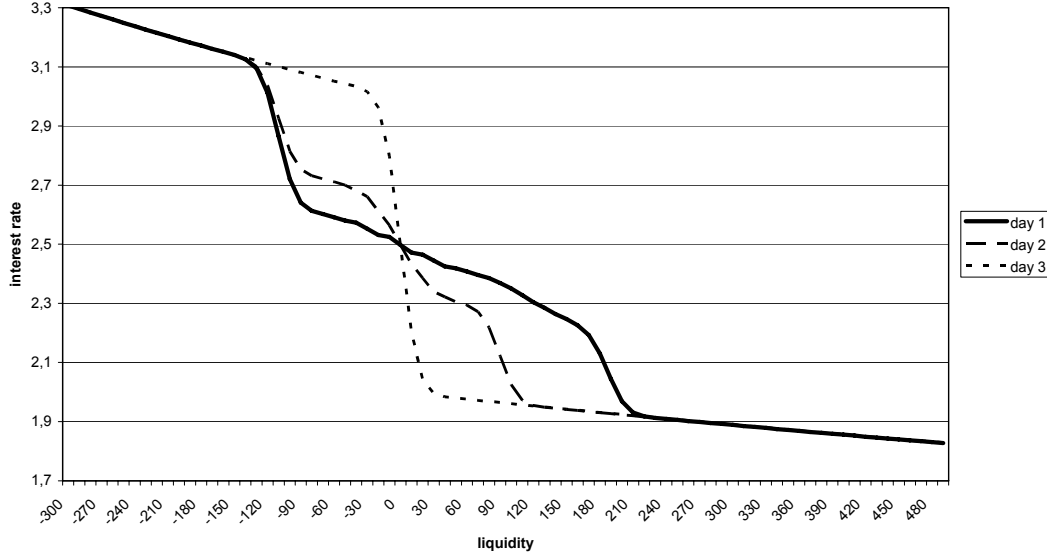
$$\exp(\rho i_t b_t) > 1 \text{ for } b_t > 0 \quad (4.11)$$

and

$$\exp(\rho i_t b_t) < 1 \text{ for } b_t < 0 \quad (4.12)$$

I come back to that equations in the next section, when I use them to explain the results obtained in the simulation.

Finally plotting the demand functions in manner similar to the ones above has been presented on figure 4.1. Those seem very similar, although note that all flat parts of the graph become steep. This offers early indication that the demand for funds is uniquely determined within this model.



**Figure 4.1** Demand curves with market frictions

## 4.2 Simulation

To verify the behavior of the whole market, I run a simulation of 10 day maintenance period with 10 homogeneous banks in the market. I assume both early  $\varphi_t$  and  $\varepsilon_t$  shocks are identically and independently distributed, approximated by normal distribution with mean zero and standard deviation 10% of the starting current account value. I assume any aggregate inflow and outflow of liquidity is adjusted in the open market operations, hence the sum of the shocks is normalised to be always zero. Given the distribution assumption (mean zero) this is not restrictive assumption. On the last day of the maintenance period additional fine tuning operations are performed, that correct for possible aggregate liquidity shortage (or surplus) from the use of standing facilities. For comparison I have also tried an experiment, and run the simulation with an aggregate shortage of funds for the whole maintenance period, that was corrected (by the central bank intervention) on the last day which is discussed in more details below.

I have verified 3 values of parameter  $\rho_1 = 10^{-2}$ ,  $\rho_2 = 10^{-3}$ ,  $\rho_3 = 10^{-5}$ . Those were chosen quite arbitrarily, based on the grid size and the relative size of assets. The value of the the parameter might seem small but recall the whole product  $(\rho i_t b_t)$  must be considered together as it enters first order conditions exponentially. Observe the right side of the equation (4.10) is a number between  $i^l$  and  $i^d$  which is typically linked very closely to market rate expectations. The left side is the market rate  $i_t$  multiplied by a factor: to keep those two close the factor must be close to 1, or the expression  $(\rho i_t b_t)$  close to zero. I return to that issue in more detail below.

The target rate of the central bank is 2.50%, which is in the middle of the standing facilities rate (2.0% and 3.0 % accordingly). Those rates were actually used by the ECB for majority of 2001–2003. Banks are homogeneous, with the average reserve requirement set to 100 units.

Finally, following empirical results of the interbank interest rate, I have assumed the martingale hypothesis holds, that is

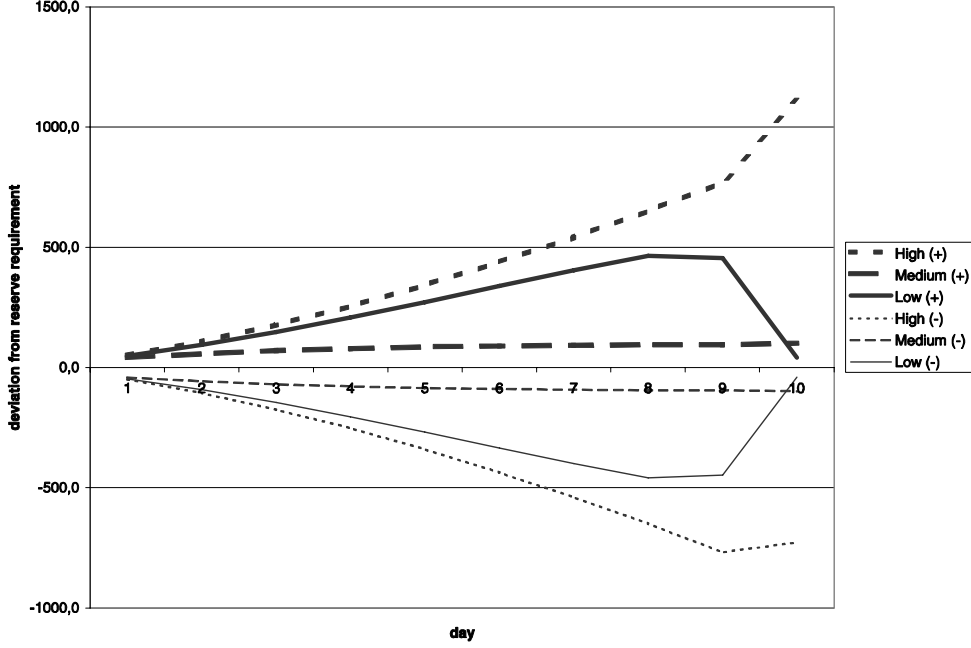
$$i_t = E(i_{t+1})$$

Given that the central banks are able to control the end of the day interest rate by adjusting liquidity I have used that assumption to set the expectations of the interest rate at the central bank target level. In some of the scenarios I present later I find that the actual average (across simulations) market rate slightly deviates from this level however the exact mechanism shaping banks expectations is beyond scope of this work. Maintaining this assumption in place allows to greatly simplify some of the calculations.

The results of the simulation are presented in table A1.1. Different columns present data for the scenarios with changing parameter  $\rho$  and liquidity shortage. Numbers 1–10 stand for days of the maintenance period. From the top of the table, I include the information about average interest rate, rate volatility (across simulations), total market accumulated negative and positive deviation from neutral liquidity (so the averages would be proportionally smaller). To obtain last data for the first day I have calculated the simple difference between the current account (after interbank trade) and average required reserve requirement (just the same as I did for my sample). For the following days, I have simply accumulated the differences from 1st day of the maintenance period and divided them into those with positive (+) and negative (-) deviations. Reported values are averages across simulations.

**Observation 1** *At the low level of market frictions banks do not keep the current account on the average required reserves level.*

This result was obtained for low levels of market frictions, for  $\rho = 10^{-5}$  and lower, which corresponds to third column in table A1.1 and scenario marked ‘low’ on figure 4.2.



**Figure 4.2** Deviation from reserve requirement

Inspection of the table and figure reveals that for the most of the period, the imbalances (deviation from neutral liquidity) increase meaning the banks do not fully adjust for the liquidity shock. The timing of these changes depends on the parameter  $\rho$  value and in my example it takes place on 8th day. At that point, the banks realise that in order to satisfy the required reserves they would have to either use the standing facilities or engage in substantial market trade, both of which are expensive. Since the penalty for borrowing (comparing to lending) is fairly small at this level of frictions, demand of one group matches supply of other and the interest rate remains at stable level.

This result can be also obtained by simple manipulations of the equation (4.10). copied here for convenience

$$i_t \exp(\rho i_t b_t) = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_{1.} + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_{2.} - \underbrace{\int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t}_{3.}$$

During the maintenance period, when both the reserve buffer  $d_t - b_t - m_t$  and current account  $m_t + b_t$  are very high comparing to shock variance, probability of shock exceeding those values is very low hence terms (1) and (2) from the right hand side of the equation are close to zero, essentially leaving

$$i_t \exp(\rho i_t b_t) \approx - \int_{b_t - m_t}^{b_t + d_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t \quad (4.13)$$

The remaining expression on the right hand side reflects the future value of satisfying the reserve requirement early. At the start of the maintenance

period, even relatively larger deviations from the average reserve requirement, do not change the probabilities of using standing facilities in the future hence the term on RHS is close to the expected market rate. Formal proofs are included in appendix A2.2. Intuitively, if the bank does not have to worry about standing facilities or trade restrictions, it can lend extra euro at the market interest rate. Similar to the martingale hypothesis from above, this leads to:

$$i_t \exp(\rho i_t b_t) \approx E(i_{t+1}) \quad (4.14)$$

If the arbitrage hypothesis is valid, the market rate  $i_t$  follows closely the expected level, at the optimal borrowing

$$\exp(\rho i_t b_t) \approx 1 \quad (4.15)$$

or

$$(\rho i_t b_t) \approx 0 \quad (4.16)$$

which means  $\rho b_t$ , should be close to zero. Large value of  $\rho$  implies restrictions on the trade volume must be small, as discussed below. In case of small  $\rho$  however, market frictions do not restrict the trade volume in significant way. That means the banks can avoid even relatively small trading costs knowing they will be able to compensate on last days of maintenance period.

Toward the end of the maintenance period, front-loading banks will find the probability of depleting all reserve requirement and consequent use of deposit facility higher. That will drive the right hand side of the equation (4.13) toward deposit rate, below the expected market rate. In order for the martingale hypothesis to hold,

$$i_t \exp(\rho i_t b_t) > E(i_{t+1}) \quad (4.17)$$

or

$$\exp(\rho i_t b_t) < 1 \quad (4.18)$$

and

$$b_t < 0 \quad (4.19)$$

In other words, the banks start to lend more in the market thus reverting slowly to neutral liquidity status, which is indeed observed in the market. Similar reasoning holds of course for back-loading bank and can be used to explain the reverse in trend in accumulated positive and negative deviation.

The interpretation of this scenario is following. Costs related to market frictions and trade encourage banks to avoid ‘noise’ trade. Sometimes temporary liquidity shortage can turn into surplus after several shock realisations, and early transactions would have to be reversed by the end of the period, which generates cost. On the other hand, even low  $\rho$  value still means large transactions are relatively more expensive and hence correcting the deviation from neutral liquidity on a single day of the maintenance period is not viable. Hence the adjustment process takes actually couple of days and starts a bit earlier.

**Observation 2** *At the high level of market frictions, banks substantially reduce trade volume*

This result holds for the high values of parameter  $\rho$ , such as 0.01 used in this example. At this point, the trading in the market becomes less favorable comparing to borrowing (or lending) from the central bank. Intuitively, the cost of using standing facilities is capped, while the cost of market finance scales with trade volume. That means, there must exist some maximum borrowing/lending values depending on the relative cost of market frictions. Those can be found using first order condition (4.10) copied here once again

$$i_t \exp(\rho i_t b_t) = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_1 + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)] - \int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t}_3$$

The right side of the equation is a probability weighted average of three terms in total resulting in a number between  $i^l$  and  $i^d$ . It is now possible to find the values of  $b_t$  at which the left side of the equation is equal to those boundaries. For maximum lending value, the profits from market must be higher than central bank deposit facility so

$$i_t \exp(\rho i_t b_t^d) \geq i^d \quad (4.20)$$

or

$$b_t^d \geq (\rho i_t)^{-1} \ln(i^d / i_t) \quad (4.21)$$

For the values used in the simulation:  $\rho = 0.01$ ,  $i^d = 2$  and the interest rate in the middle of corridor system  $i_t = 2.5$  the maximum trade volume the bank can lend  $b_t^d \geq -8.9$  and corresponding maximum borrowing is  $b_t^l \leq 7.2$ . Those values are below the standard deviation of the liquidity shock ( $\sigma = 10$ ) which means very likely they will become binding. That however means the imbalance is going to continue all the way including last day of the maintenance period. In addition, without balancing trade some banks will find themselves spending their reserve requirement buffer early forcing them to use deposit facility. Since those restrictions will also hold on the last day of the maintenance period, the imbalance will not be corrected until the end of the maintenance period which will induce substantial use of standing facilities.

Finally high value of  $\rho$  implies that imbalance between borrowing and lending will be substantial. Using the example from paragraph above, maximum borrowing is 20% lower than maximum lending which has the impact on the behavior of interest rate. Intuitively, costs of market borrowing are higher than profits from corresponding market lending and the potential standing facilities much more attractive. Hence, to induce market trade and attract those banks, the interest rate in the market must be lower than in a risk neutral case, which is what is in fact observed in column 2 of table A1.1.



**Observation 3** *At the medium level of market frictions banks follow the average required reserves level*

Using intermediate parameter value  $\rho = 0.001$  produced a very interesting outcome that has been marked as medium on figure 4.2. What happens is banks are actually reverting their daily fluctuations of liquidity (from shocks) maintaining stable account value which is very close to the average required reserves. This can be explained in a following way. Contrary to the scenario with low trade frictions, adjusting on the last day of the maintenance period requiring single massive transaction is very costly and hence banks try to avoid it. On the other hand, smaller values of trade are not penalized to the extent presented above, in the scenario with high  $\rho$ , and financing from the central bank is still more costly than market finance. Hence, banks find that the most profitable scenario is to remain in safe range, where the costs of daily trade during whole period are lower than cost of one large market transaction on last day or using standing facilities.

**Observation 4** *No liquidity effect in the market*

Recall that one of the most important issues for the central bankers is the strength of the liquidity effects, as it offers a link between market reserves (controlled in open market operations) and market rate. In the empirical works of Moschitz (2004) and Würtz (2003) the liquidity effects was found only on last days of the maintenance period. In my model I find confirmation and explanation for that result.

To analyse the impact of liquidity in the market I have run the simulation imposing an aggregate shortage of liquidity in the market (total value of it was 150 units comparing to market liquidity 1000 units). The shortage is corrected by the central bank on the last day of the maintenance period so the expected interest rate remained on constant level. I used the parameter  $\rho = 10^{-6}$  but results do not change for higher  $\rho$  values. This value is used as results resemble the actual behavior of the Eurosystem as discussed below. The results are reported in the last column of table A1.1. and perhaps surprisingly do not differ from the standard case.<sup>10</sup>

To understand that findings, first observe that in my model the demand for funds is uniquely determined. That means, the banks always choose specific borrowing value rather than remain indifferent within specific range (as was the case in standard model). This result should not be surprising. In both risk neutral and my model the right side of equation (3.7) is not changing for a range of borrowing values. In my construction however the trade volume also enters left side of (3.7) and even small change in borrowing changes it substantially. That means the value of borrowing that would match both sides is unique.

To see how the market liquidity affects the interest rate in the model let me rewrite the above equation

$$i_t \exp(\rho i_t b_t) \approx - \int_{b_t - m_t}^{b_t + d_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t$$

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<sup>10</sup>Apart from the obvious imbalance between front- and back-loading banks.

The right side captures the dynamic cost factor or the probability of using standing facilities in the future. With relatively high reserve requirement (as is the case in the Eurosystem) those probabilities are very low and that term remains very close to the expected interest rate level at wide range of not only borrowing ( $b_t$ ) but also the assets ( $m_t$ ) and deficiency ( $d_t$ ) values (see Kempa, 2006, for details). At the same time, the last two terms do not enter the left side of the equation. That means the borrowing decision will remain very similar for range of assets and deficiency realisations at least at the start of the maintenance period. If the borrowing decisions stay same, so does the interest rate which leads to the following conclusion: the market liquidity has little effect on the interest rate. This means the expectations of the future interest rate remain key determinant of the current rate, which is also the result obtained for standard model.

Intuitively, this comes from the fact that banks are balancing the cost of financing from the market and central bank. Large reserve requirement or long maintenance period means that current liquidity has little impact on the future use of standing facilities; there is plenty of time to revert positive or negative position. Provided the expected rate does not change much (as it was the case in the Eurosystem for much of the time between 2003–2006) there was no need for excessive and potentially costly borrowing actions even when faced with substantial liquidity shortages or surpluses.

This is potentially very important result for the central bankers, as it shows the implementation of the monetary policy should be more focused on controlling expectations rather than market liquidity.

The results presented above represent the key contribution of the paper. There is however several other issues that should be addressed as well, related to the key parameter  $\rho$  that measures the impact of trading frictions on the behavior of the commercial bank.

First of all, different values of parameter seem to induce different bank behavior. For high values of  $\rho$  increasing cost becomes quite important and banks are more reluctant to engage into to any trade turning to standing facilities instead. The exact numbers might be misleading, as they depend on the values of parameters used (such as reserves volume), but for comparison I find that for parameter value  $\rho = 0,01$  the banks refrain from any trade almost until the last day of the maintenance period (scenario marked ‘high’ on the figure 4.2). As the value of  $\rho$  decreases, so does the cost of daily trade and offsetting the shocks on a daily basis becomes viable option. Until  $\rho$  reaches very low level, adjusting to the required reserves in single transaction still is still very expensive and banks find it more profitable to plan in advance to avoid it. This becomes less an issue at parameter values around  $\rho = 10^{-5}$  where the convexity of the cost function almost vanishes.

Second is the issue how closely the market rate follows the central bank target which is implemented as expected future rate. Inspection of table A1.1 reveals that central bank goal is easily met for low and intermediate parameter values but the market deviates quite a bit at high values of  $\rho$ . At this point, the asymmetry between cost of borrowing and profits from lending become quite substantial and market clearing rate needs to adjust downward to compensate

for that. Intuitively, the banks with liquidity shortage are not as eager to borrow as the banks with surplus want to lend.

Finally one should compare the results of the simulations with the actual behavior of the Eurosystem. The analysis presented above is subject to several assumptions (such as bank homogeneity etc) yet it seems that out of three scenarios presented, the one with low level of market frictions seems to be the closest. First, it successfully replicates the pattern where the initial deviation from the target rate increases to drop in the later part of the maintenance period. Second, the interest rate remains closely tied to the central bank target or expected level, which was the actual behavior of Eonia in analysed period.

### 4.3 Section summary

The model presented in this section is a standard model of the interbank trade modified to include the asymmetry between borrowing and lending and potential market frictions related costs. I have then run a Monte-Carlo simulation for different parameters  $\rho$  value and I found I can duplicate some of the patterns observed on the Eurosystem market for relatively low level of the market frictions:

- The banks that initially deviate from the neutral liquidity, return to the required level later on during the maintenance period
- There is no liquidity effect in the market

Those can be interpreted in a following way. Early on, banks reduce their daily trade volumes in order to avoid market related costs. The adjustment to the required level takes place in the second part of the maintenance period, rather than on single day day, suggesting increasing cost of large transactions.

Comparing those results to the Eurosystem can lead to some interesting insights. It seems the market is subject to some trade related costs that are sufficiently high to encourage banks to reduce their transactions volumes early in the maintenance period. However the level of borrowing where the banks would have to incur extra expenses is realistically never reached, either due to market depth or the fact that banks in reality start to adjust for liquidity shortage/surplus a bit earlier than in the simulation. This is safe scenario from the perspective of the ECB, as the market rate tends to stabilise around expected level.

The second result have potential interesting implications for the central bank policy. Given the risk of using the standing facilities is small during the maintenance period, banks trading decisions are more affected by the expected interest rate level rather than their current liquidity. This is perhaps not a new finding and was indeed stated already by Välimäki (2003) and Kempa (2006) that analyse risk neutral case. This paper extends that results into more robust case where banks are facing certain limitations and market frictions.

Central banks could use that findings when deciding about frequency and scale of the liquidity supply operations. It seems that even if the central bank

supplies liquidity over or below benchmark level, the market rate will remain unaffected. Alternatively, more frequent (and thus exact) operations would not affect the interest rate level as well. This gives central bank some room for potential errors, but on the other hand complicates its job if the interest rate expectations run out of control.

## 5 Conclusions

The paper deals with the interbank market and bank's demand for reserves. I start with a brief introduction to the market mechanism and illustrate some of them using 71 banks sample from the Eurosystem. Contrary to popular belief, the commercial banks do not strictly follow the required reserves level on each day of the maintenance period. Some of them also exhibit propensity to front-and back-load (satisfy requirement early or late). This does not seem to be dependent on their size or country of origin.

Using modification of the standard model of the interbank market derived by Poole (1968) and Pérez-Quirós and Rodríguez-Mendizábal (2006) I then show how market related frictions can be used to explain that pattern. It seems that in the beginning of the maintenance period, banks are slow with reacting to liquidity changes. With high reserve requirement buffer, banks prefer to postpone adjustments reducing the cost involved with interbank trade. As the end of the maintenance period gets closer, the threat of being forced to use the standing facilities becomes more credible and banks are more willing to trade and revert to the neutral liquidity level. Instead of postponing that process until the very last day of the period banks choose to spread it avoiding large transactions that are likely to incur extra expenses. That suggests, both daily operational expenses and increasing marginal transactions costs are important in banks' decision making.

At the Eurosystem level of frictions, banks' individual patterns in the reserves demand seem to have little impact on the level of the interest rate. Indeed I find that the interest rate remains on a level very close to the expected with little volatility and the aggregate liquidity does not seem to affect it substantially. That is also somehow related to previous paragraph: if the banks trade volumes are only loosely connected to their individual liquidity (at least at the start of maintenance period banks do not fully respond to shocks), even aggregate shortage or surplus will have little impact. That is however assuming banks believe it will be corrected by the end of the maintenance period.

Those findings have specific policy applications. The central bankers control over the interest rate is exerted mainly through expectations channel rather than market liquidity. This is due to the fact that given the required reserves are high comparing to the volatility banks face, the probability of actually using standing facilities for commercial bank is very low.

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# Appendix 1

**Table A1.1** Simulation results

Average rate on day	$\rho = 10^{-2}$	$\rho = 10^{-3}$	$\rho = 10^{-5}$	liquidity shortage
1	2,50	2,50	2,50	2,50
2	2,49	2,50	2,50	2,50
3	2,47	2,50	2,50	2,50
4	2,47	2,50	2,50	2,50
5	2,46	2,50	2,50	2,50
6	2,45	2,50	2,50	2,50
7	2,45	2,50	2,50	2,50
8	2,44	2,49	2,50	2,50
9	2,45	2,49	2,50	2,50
10	2,38	2,46	2,50	2,50
Rate volatility on day				
1	0,031	0,000	0,000	–
2	0,030	0,003	0,000	–
3	0,030	0,003	0,000	–
4	0,030	0,004	0,000	–
5	0,038	0,006	0,000	–
6	0,044	0,009	0,000	–
7	0,052	0,011	0,000	–
8	0,058	0,017	0,001	–
9	0,074	0,027	0,002	–
10	0,093	0,076	0,000	–
Accumulated positive deviation				
1	51,5	41,6	49,4	0,00
2	108,0	56,8	94,9	0,05
3	177,2	70,6	147,8	0,10
4	254,3	78,8	207,6	0,42
5	342,5	85,5	271,6	0,77
6	438,9	89,4	338,7	1,47
7	541,3	92,2	404,4	2,00
8	650,7	94,9	463,9	0,15
9	770,0	94,5	455,4	0,00
10	1109,4	101,4	40,5	41,2

continued...

... continues

Accumulated negative deviation	$\rho = 10^{-2}$	$\rho = 10^{-3}$	$\rho = 10^{-5}$	liquidity shortage
1	-49,4	-41,6	-46,9	-157,0
2	-105,9	-56,8	-92,3	-307,4
3	-175,1	-70,6	-145,2	-457,6
4	-252,2	-78,8	-204,9	-606,7
5	-340,4	-85,5	-268,6	-753,0
6	-436,9	-89,3	-334,8	-898,4
7	-539,5	-92,3	-399,2	-1045,4
8	-649,1	-94,9	-458,1	-1192,1
9	-768,7	-94,5	-447,8	-1340,9
10	-727,4	-98,9	-40,3	-41,2

## Appendix 2

### Proofs

#### A2.1 Proof of results from section 3

Those proofs of the standard model follow almost exactly the ones presented in Kempa (2006).

The expected cost of using standing facilities for the last day of maintenance period is given by a following expression

$$\begin{aligned}
 E(c_T) &= i^l \left[ \int_{-\infty}^{d_T - m_T - b_T - \varphi_T} (d_T - m_T - b_T - \varphi_T - \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right] \\
 &- i^d \left[ \int_{-m_T - b_T - \varphi_T + d_T}^{\infty} (m_T + b_T + \varphi_T - d_T + \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right] = \\
 &= -i^l (m_T + b_T + \varphi_T - d_T) F(-m_T - b_T - \varphi_T + d_T) \\
 &- i^d (m_T + b_T + \varphi_T - d_T) (1 - F(-m_T - b_T - \varphi_T + d_T)) \\
 &\quad - i^l \int_{-\infty}^{-m_T - b_T - \varphi_T + d_T} \varepsilon_T f(\varepsilon_T) d\varepsilon_T - i^d \int_{-m_T - b_T - \varphi_T + d_T}^{\infty} \varepsilon_T f(\varepsilon_T) d\varepsilon_T
 \end{aligned} \tag{A2.1}$$

Substituting that equation into cost function (3.5)

$$V_T = i_T b_T + E(c_T)$$

and solving the first order conditions with respect to  $b_T$  yields

$$\begin{aligned}
 -i_T &= -i^l F(-m_T - b_T - \varphi_T + d_T) \\
 &+ i^l (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \\
 &- i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) \\
 &- i^d (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \\
 &- i^l (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \\
 &+ i^d (m_T + b_T - d_T) f(-m_T - b_T - \varphi_T + d_T)
 \end{aligned} \tag{A2.2}$$

which can be simplified to

$$i_T = i^l F(-m_T - b_T - \varphi_T + d_T) + i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) \tag{A2.3}$$

■

For days before the last one, the cost function can be rewritten using expected cost of using standing facilities:

$$\begin{aligned}
 E_t(V_t) &= i_t b_t - i^l (m_t + b_t + \varphi_t) F(-m_t - b_t - \varphi_t) \\
 &- i^d (m_t + b_t + \varphi_t - d_t) (1 - F(-m_t - b_t - \varphi_t + d_t)) \\
 &- i^l \left[ \int_{-\infty}^{-m_t - b_t - \varphi_t} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] - i^d \left[ \int_{-m_t - b_t - \varphi_t + d_t}^{\infty} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] \\
 &+ EV_{t+1}
 \end{aligned} \tag{A2.4}$$



The first order conditions

$$i_t = i^l F(-m_t - b_t - \varphi_t) + i^d (1 - F(-m_t - b_t - \varphi_t + d_t)) + E \frac{\partial V_{t+1}}{\partial b_t} \quad (\text{A2.5})$$

To calculate the last element of the the F.O.C. first note that the borrowing maturity is one period, hence it has no direct impact on the asset value and borrowing in the next period. It has however impact on the deficiency value, which must be analysed in 3 separate cases:

1. The shock forces the bank to use lending facility meaning the deficiency remains unchanged.
2. The shock forces the bank to use deposit facility meaning all deficiency gets satisfied.
3. The intermediate case, where the shock value only lowers the deficiency without forcing the bank to use any of the facilities

In first two cases, the deficiency either remains unchanged (comparing to previous period) or drops to zero, meaning the borrowing decision has no impact on its value.

$$\int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = 0 \quad (\text{A2.6})$$

and similar

$$\int_{-m_t - b_t - \varphi_t + d_t}^{\infty} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = 0$$

In the last case, whenever the deficiency is carried over one period, one more unit of borrowed funds decreases required reserves by the same one unit hence

$$\begin{aligned} \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon &= \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \\ &= \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon \end{aligned} \quad (\text{A2.7})$$

Leading to profit maximising conditions:

$$\begin{aligned} i_t = i^l F(-b_t - m_t - \varphi_t) + i^d [1 - F(d_t - b_t - m_t - \varphi_t)] \\ - \int_{-m_t - b_t - \varphi_t}^{d_t - b_t - \varphi_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon \end{aligned} \quad (\text{A2.8})$$

■

## A2.2 Proofs of the martingale property from section 3.

To prove the martingale property used in section 3 start with the equilibrium interest rate

$$i_t = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_{(1)} + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_{(2)} - \underbrace{\int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t}_{(3)} \quad (\text{A2.9})$$

As discussed in the paper, terms (1) and (2) converge to zero when the reserve requirement is sufficiently high. Hence

$$i_t \approx \int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t \quad (\text{A2.10})$$

The derivative of the value function with respect to deficiency can be solved using an envelope theorem ( $\frac{\partial V_t}{\partial b_t} = 0$ ) so the problem reduces to

$$\frac{\partial V_t}{\partial d_t} = \frac{\partial}{\partial d_t} (E(c_t) + EV_{t+1}) \quad (\text{A2.11})$$

Since the probability of using lending facility is not affected by the deficiency value, I can further simplify it by only taking the expected cost of using deposit facility so

$$\begin{aligned} \frac{\partial V_t}{\partial d_t} = & \frac{\partial}{\partial d_t} \left\{ -i^d (m_t + b_t + \varphi_t - d_t) (1 - F(-m_t - b_t - \varphi_t + d_t)) \right. \\ & \left. - i^d \left[ \int_{-m_t - b_t - \varphi_t + d_t}^{\infty} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] + EV_{t+1} \right\} \end{aligned} \quad (\text{A2.12})$$

Solving partial derivative

$$\begin{aligned} \frac{\partial V_t}{\partial d_t} = & i^d \{ (1 - F(-m_t - b_t - \varphi_t + d_t)) \\ & + (m_t + b_t + \varphi_t - d_t) f(-m_t - b_t - \varphi_t + d_t) \\ & + (-m_t - b_t - \varphi_t + d_t) f(-m_t - b_t - \varphi_t - d_t) \} + E \frac{\partial V_{t+1}}{\partial d_t} = \\ & i^d [1 - F(-m_t - b_t - \varphi_t + d_t)] + E \frac{\partial V_{t+1}}{\partial d_t} \end{aligned} \quad (\text{A2.13})$$

In a manner similar to above we analyse 3 cases:

1. The shock forces the bank to use lending facility meaning the deficiency remains unchanged.

$$\frac{\partial d_{t+1}}{\partial d_t} = 1 \rightarrow \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon \quad (\text{A2.14})$$

2. The shock forces the bank to use deposit facility meaning all deficiency gets satisfied so

$$\frac{\partial d_{t+1}}{\partial d_t} = 0 \rightarrow \int_{d_t - m_t - b_t - \varphi_t}^{\infty} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = 0 \quad (\text{A2.15})$$

The intermediate case, where the shock value only lowers the deficiency without forcing the bank to use any of the facilities is similar to case 1.

$$\frac{\partial d_{t+1}}{\partial d_t} = 1 \rightarrow \int_{-m_t - b_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int_{-m_t - b_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon \quad (\text{A2.16})$$

which results in

$$\frac{\partial V_t}{\partial d_t} = i^d [1 - F(d_t - b_t - m_t - \varphi_t)] + \int_{-\infty}^{d_t - b_t - m_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon \quad (\text{A2.17})$$

If the current account balance is high comparing to the size of the liquidity shock, the first part of that expression also converges to zero, leaving

$$\frac{\partial V_t}{\partial d_t} \approx E \frac{\partial V_{t+1}}{\partial d_{t+1}} \quad (\text{A2.18})$$

This can be solved recursively starting from last period.

$$\begin{aligned} \frac{\partial V_T}{\partial d_T} &= \frac{\partial c_T}{\partial d_T} = \\ & \frac{\partial}{\partial d_T} \left\{ -i^d (m_T + b_T + \varphi_T - d_T) (1 - F(-m_T - b_T - \varphi_T + d_T)) \right. \\ & \quad - i^l (m_T + b_T + \varphi_T - d_T) F(-m_T - b_T - \varphi_T + d_T) \\ & \quad \left. - i^d \int_{-m_T - b_T - \varphi_T + d_T}^{\infty} \varepsilon_T f(\varepsilon_T) d\varepsilon_T - i^l \int_{-\infty}^{-m_T - b_T - \varphi_T + d_T} \varepsilon_T f(\varepsilon_T) d\varepsilon_T \right\} \end{aligned} \quad (\text{A2.19})$$

which can be solved for

$$\begin{aligned} \frac{\partial V_T}{\partial d_T} &= \\ & i^d [(1 - F(-m_T - b_T - \varphi_T + d_T)) + (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T)] \\ & + i^l [F(-m_T - b_T - \varphi_T + d_T) - (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T)] \\ & - i^d (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \\ & \quad + i^l (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \end{aligned} \quad (\text{A2.20})$$

which can be simplified to

$$\frac{\partial V_T}{\partial d_T} = i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) + i^l F(-m_T - b_T - \varphi_T + d_T) = i_T \quad (\text{A2.21})$$

The result  $\frac{\partial V_T}{\partial d_T} = i_T$  can be then used recursively into eq. (A2.18) to obtain

$$\frac{\partial V_t}{\partial d_t} \approx E(i_{t+1}) \quad (\text{A2.22})$$

so the dynamic cost factor is equal to the expectations of the future interest rate.

### A2.3 Proofs of results from section 4.1.

To prove the convexity of the cost function, I need to show that

$$\frac{\partial^2 E(c_t)}{\partial b_t^2} > 0$$

For example for last day of the maintenance period  $T$  I have calculated above in (A2.2) that taking second order conditions yields

$$\begin{aligned} \frac{\partial}{\partial b_t} \left( \frac{\partial E(c_t)}{\partial b_t} \right) &= \\ &= \frac{\partial}{\partial b_t} [i^d + (i^l - i^d)F(-m_T - b_T - \varphi_T + d_T)] \\ &= (i^l - i^d)f(-m_T - b_T - \varphi_T + d_T) > 0 \end{aligned}$$

To prove equations (4.9) and (4.10) note that expression for expected cost of using standing facilities  $E(c_t)$  remains unchanged. Hence the most complicated part of the proofs presented in the section above remains the same.

The only expression that changes is cost of funding from the interbank market  $\kappa(i_t, b_t)$  which was given by  $i_t b_t$  in the standard model and  $\rho^{-1} \exp(\rho i_t b_t)$  in the presented model.

The derivative of  $i_t \kappa(i_t, b_t)$  with respect to  $b_t$  is simply

$$\frac{\partial \kappa(i_t, b_t)}{\partial b_t} = i_t \exp(\rho i_t b_t)$$

To prove the first order conditions (4.9) and (4.10) one needs to exactly repeat the steps from above section changing the marginal cost of finance from the market from  $i_t$  into  $i_t \exp(\rho i_t b_t)$ .

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