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Working Paper

Optimal monetary policy under heterogeneity in currency trade

Bank of Finland Research Discussion Papers, No. 21/2007

Provided in Cooperation with:
Bank of Finland, Helsinki

Suggested Citation: Bask, Mikael (2007) : Optimal monetary policy under heterogeneity in currency trade, Bank of Finland Research Discussion Papers, No. 21/2007, ISBN 978-952-462-391-9, Bank of Finland, Helsinki,
https://nbn-resolving.de/urn:nbn:fi:bof-20140807508

This Version is available at:
https://hdl.handle.net/10419/212079

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Bank of Finland Research
Discussion Papers
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This paper has benefited from presentations at various conferences and seminars, and the author acknowledges all comments given on these occasions. The usual disclaimer applies.

All MATLAB routines that have been used in the preparation of this paper are available on request from the author. The impulse-response functions were computed using MATLAB routines developed by Michael Woodford. These are available at http://www.columbia.edu/~mw2230/Tools/.

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ISBN 978-952-462-390-2
ISSN 0785-3572
(print)

ISBN 978-952-462-391-9
ISSN 1456-6184
(online)

Helsinki 2007

Optimal monetary policy under heterogeneity in currency trade

Bank of Finland Research
Discussion Papers 21/2007

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Abstract

We embed an expectations-based optimal policy rule into a DSGE model for a small open economy that is augmented with trend extrapolation or chartism, which is a form of technical trading, in currency trade to examine the prerequisites for monetary policy. We find that a unique REE that is least-squares learnable is often the outcome when there is a limited amount of trend extrapolation, but that a less flexible inflation rate targeting may cause a multiplicity of REE. We also compute impulse-response functions for key macroeconomic variables to study how the economy returns to steady state after being hit by a shock.

Keywords: determinacy, DSGE model, least-squares learning, targeting rule, technical trading, monetary policy

JEL classification numbers: C62, E52, F31, F41

Heterogeeniset valuuttakurssiodotukset, optimaalinen rahapolitiikka ja talouden tasapainot

Suomen Pankin keskustelualoitteita 21/2007

Mikael Bask
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan rahapolitiikan toimintaedellytyksiä pienessä avoimessa taloudessa. Työssä käytettyä pienen avotalouden dynaamista stokastista yleisen tasapainon mallia täydennetään rahapolitiikan korkosäännöllä, jonka mukaan keskuspankki muuttaa ohjauskorkoaan, kun se ennakoi välittömien inflaatiopaineiden tai kustannuspaineiden voimistuvan tulevaisuudessa. Näin mallinnetun rahapolitiikan lisäksi työn teoreettinen malli poikkeaa kauttaaltaan rationaalisten odotusten mallista yhtäältä olettamalla, että osa valuuttamarkkinatoimijoista perustaa hinnoittelunsa ja kaupankäyntinsä tekniseen analyysiin. Toisaalta muiden taloudenpitäjien odotustenmuodostus tai oikeastaan ennusteiden laadinta perustuu talouden havaittuun liikelakiin, jota kuvataan tilastollisella mallilla. Mallin kerroinestimaatit päivitetään jokaisena periodina estimoimalla ne tuoreesta tilastoaineistosta. Näistä oletuksista seuraa erityisesti, että valuuttamarkkinoiden odotustenmuodostus on heterogeenista ja että mallitalouden dynaaminen tasapaino poikkeaa rationaalisten odotusten dynaamisesta tasapainosta. Tässä tutkimuksessa tarkastellaan kuitenkin, onko rationaalisten odotusten tasapaino opittavissa eli yhtäläistyvätkö taloudenpitäjien tilastolliset ja rationaalisten odotusten ennusteet ja miten teknisen kaupankäynnin levinneisyys valuuttamarkkinoilla vaikuttaa opittavuuteen. Tulosten mukaan yksikäsitteinen rationaalisten odotusten tasapaino on opittavissa, kun tekninen kaupankäynti on vähäistä. Toisaalta keskuspankin inflaatiovastaisuuden voimistuminen mahdollistaa useita rationaalisten odotusten tasapainoja. Työssä tarkastellaan myös keskeisten makrotaloudellisten muuttujien dynaamista sopeutumista pitkän aikavälin tasapainoon taloutta kohdanneen häiriön jälkeen.

Avainsanat: määrittyneisyys, DSGE-malli, pienimmän neliösumman oppiminen, tavoitesääntö, tekninen kaupankäynti, rahapolitiikka

JEL-luokittelu: C62, E52, F31, F41

Contents

1 Introduction

Interest rate rule

In 1993, Taylor (1993) demonstrated that Federal Reserve's policy could be described by the following interest rate rule

$$r_t = 0.04 + 1.5\,(\pi_t - 0.02) + 0.5\,(y_t - \overline{y}) \tag{1.1}$$

where r_t is Federal Reserve's operating target for the funds rate, π_t is the inflation rate according to the GDP deflator, y_t is the logarithm of real GDP, and $\overline{y}$ is the logarithm of potential real GDP. In particular, the rule in (1.1) prescribes setting the funds rate in response to the inflation rate and the output gap, where the latter is the difference between the two measures of GDP. Taylor (1999) also argues that since this rule describes Federal Reserve's policy during a successful period, one should adopt a rule like this in policy-making.[1]

Instrument or targeting rule?

Svensson (2002)–(2003) argues vividly that an instrument rule such as the Taylor rule in (1.1) is inferior to a targeting rule in policy-making since the instrument rule is not consistent with an optimizing behavior on the part of the central bank. The targeting rule, on the other hand, is derived from the optimization of an objective function. Thus, in a typical DSGE model that often is utilized when embedding an interest rate rule into a theoretical framework, all agents in the economy behave optimally; households maximize utility, firms maximize profit, and the central bank maximizes welfare (see Woodford, 2003).

Let us use Svensson's (2003) own words to make the point clear:

> 'Monetary policy by the world's more advanced central banks these days is at least as optimizing and forward-looking as the behavior of the most rational private agents. I find it strange that a large part of the literature on monetary policy still prefers to represent central bank behavior with the help of mechanical instrument rules.' (p. 429).

For this reason, we focus on a targeting rule, or an optimal police rule, in this paper.

A unique REE that is learnable?

Typically, in the literature, conditions for uniqueness of the rational expectations equilibrium (REE) are examined since the policy-maker would like to avoid coordination problems in the economy. For instance, without imposing additional restrictions into a rational expectations model, it may not be known in advance which REE that agents will coordinate on, if there will be

[1] See Clarida et al (1999) for an early review of interest rate rules in dynamic stochastic general equilibrium (DSGE) models, and Zimmermann (2003) for a more introductory text on the same topic. Woodford's (2003) seminal work on rules in policy-making should also be part of the reading list.

any coordination at all. To give an example, the effects of changes in monetary policy may not be known beforehand: is it the case that agents will coordinate on a REE that has undesirable properties, like a very high inflation rate, or on a REE in which the price level is stable?

Another problem is the actual computations of the time-paths of economic variables when agents have rational expectations since one cannot expect that they have perfect knowledge of the economy's law of motion. For example, it is a well-known fact among economists that the transmission mechanism for monetary policy has a complicated structure, and this also means that there are disagreements about the exact nature of this mechanism. The following question arises, however: may agents eventually learn the REE, if they can make use of data generated by the economy itself to improve their knowledge of its law of motion?

The concept of learning that we make use of in this paper is least squares learning, and to have a REE that is least squares learnable, the parameter values in the perceived law of motion (PLM) of the economy have to converge to the economy's actual law of motion (ALM), and this happens when the REE is characterized by expectational stability (ie, E-stability). Thus, this tool is not only able to single out the more interesting REE in a model, it also acknowledges the fact that agents may not be equipped with a perfect knowledge of the economy's law of motion. See Evans and Honkapohja (2001) for an introduction to this literature, and Bullard (2006) for a review of interest rate rules in DSGE models from a learning perspective.

Heterogeneity in currency trade

Questionnaire surveys made at currency markets around the world reveal that currency trade to a large extent not only is determined by an economy's performance or expected performance. Indeed, a fraction is guided by technical trading, meaning that past exchange rates are assumed to provide information about future exchange rate movements. A simple example of technical trading is trend following.

See Oberlechner (2004) for an in-depth discussion of two large questionnaire surveys conducted at the European and the North American markets, Gehrig and Menkhoff (2006) for a survey on trading behavior that includes references to several other surveys made at currency markets (eg, Cheung and Chinn, 2001, Lui and Mole, 1998, Menkhoff, 1997, Oberlechner, 2001, and Taylor and Allen, 1992), and Neely (1997) for a layman's guide on technical trading. Notice that other terms for technical trading are chartism and technical analysis.

Aim of the paper

We embed an optimal policy rule into Galí and Monacelli's (2005) DSGE model for a small open economy that is augmented with trend following in currency trade to examine the prerequisites for monetary policy. Specifically, the conditions for a unique REE that is least squares learnable are in focus. We also compute impulse-response functions for key variables in the model to study how the economy returns to steady state after being hit by a shock.

Relation to the literature

Since an optimal policy rule in the form of an expectations-based rule is derived (that we discuss what it is in Section 3), the present paper relates to Evans and Honkapohja (2003a)–(2006) who, from a learning perspective, examine the desirability of expectations-based rules in a DSGE model for a closed economy. Two recent papers that also focus on the learnability of a unique REE, but for an open economy, are Bullard and Schaling (2006) and Llosa and Tuesta (2006). The former paper examines optimal policy in a two-country model, whereas the latter paper examines instrument rules in the same DSGE model as in this paper. However, there is no technical trading in Llosa and Tuesta (2006) nor in the other papers.[2]

Outline of the paper

The model we examine is outlined in the next section. Thereafter, in Section 3, we derive the optimal policy rule for the central bank, whereas the conditions for a determinate and E-stable REE are in focus in Section 4. Impulse-response functions for key variables are computed in Section 5, and Section 6 concludes the paper with a short discussion.

2 DSGE model

A DSGE model with imperfect competition and nominal rigidities is presented in Galí and Monacelli (2005) for a small open economy, and their model consists of an IS curve

$$x_t = x_{t+1}^e - \alpha \left(r_t - \frac{1}{1-\delta} \cdot \left(\pi_{t+1}^e - \delta \left(\Delta e_{t+1}^{e,m} + \pi_{t+1}^{e,*} \right) \right) - \overline{rr}_t \right) \tag{2.1}$$

an AS curve

$$\pi_t = \beta \pi_{t+1}^e + \gamma \left(1-\delta\right) x_t + \delta \left(\Delta e_t - \beta \Delta e_{t+1}^{e,m} + \pi_t^* - \beta \pi_{t+1}^{e,*} \right) \tag{2.2}$$

and a condition for uncovered interest rate parity (UIP)

$$r_t - r_t^* = \Delta e_{t+1}^{e,m} \tag{2.3}$$

where x_t is the output gap, r_t is the nominal interest rate, π_t is the CPI inflation rate, e_t is the nominal exchange rate, and $\overline{rr}_t$ is the natural rate of interest. The superscripts e and e,m denote expectations in general and market expectations in currency trade, respectively, and an asterisk in the superscript denotes a foreign quantity. See the Appendix for the derivation of (2.1)–(2.2) using equations in Galí and Monacelli (2005).

$\beta \in [0,1]$ is the discount factor that is used when the representative household in the home country maximizes a discounted sum of instantaneous

[2] See Hommes (2006) for a survey of the literature on heterogeneous agent models in economics and finance, and De Grauwe and Grimaldi (2006) for an introduction to exchange rate determination in a behavioral finance framework.

utilities derived from consumption and leisure, $\delta \in [0,1]$ is the share of consumption in the home country allocated to imported goods, meaning that δ is an index of openness of the economy, and α and γ are functions of parameters in the Galí and Monacelli (2005) model.[3]

There are two types of behavior in currency trade: (i) trend following; and (ii) trading that is based on fundamental analysis. When trend following is used, it is believed that the exchange rate will increase between time periods t and $t+1$, if it increased between time periods $t-1$ and t.[4,5] When fundamental analysis is used, agents have rational expectations regarding the next time period's exchange rate change, meaning that market expectations can be summarized as

$$\Delta e_{t+1}^{e,m} = \omega \Delta e_{t+1}^{e,c} + (1-\omega)\,\Delta e_{t+1}^{e,f} = \omega \Delta e_t + (1-\omega)\,\Delta e_{t+1}^{e} \tag{2.4}$$

where $\omega \in [0,1]$ is the degree of trend following in currency trade, and e,c and e,f denote expectations according to chartism and fundamental analysis, respectively. Even though the superscript e denotes rational expectations, we also think of it as non-rational expectations when focusing on learning in Section 4.

3 Optimal monetary policy

The model is now closed by deriving an interest rate rule for the central bank that minimizes the value of the following objective function

$$L = \zeta x_t^2 + \pi_t^2 \tag{3.1}$$

[3] α depends on the openness index, δ, the intertemporal elasticity of substitution in consumption, the elasticity of substitution between domestic and foreign goods in consumption, and the elasticity of substitution between foreign goods in consumption; and γ depends on α, the discount factor, β, the intertemporal elasticity of substitution in labor supply, and the share of firms that set new prices in each time period (see Calvo, 1983).

[4] To minimize the number of structural parameters in the model, these two consecutive increases in the exchange rate are of the same size.

[5] More sophisticated trading rules, like the moving averages (MA) technique, could also be desirable to examine in the Galí and Monacelli (2005) model. However, this would complicate the analysis considerably, and it is not certain that the dynamics is affected that much compared to when simple trend following is used in currency trade. This conclusion comes from the asset pricing model in Bask (2006), where it was found that the exchange rate in time periods $t-t_0$, $t_0 \geq 2$, had a second-order effect on the current exchange rate, whereas the exchange rate in the previous time period had a first-order effect. However, see Bask (2007a) for the MA technique in a Dornbusch-style model.

given the economy's law of motion in (2.1)–(2.4). Thus, we assume that there is no commitment mechanism available for the central bank, meaning that the optimal policy rule is derived under discretion in policy-making. Then, the optimization problem for the central bank is[6]

$$\begin{aligned} \min_{x_t,\pi_t} L &= \min_{x_t,\pi_t} \zeta x_t^2 + \pi_t^2 \\ s.t. \qquad \pi_t &= -\left(\frac{(1-\beta\omega)\,\delta}{\alpha\omega} - \gamma\right)(1-\delta)\,x_t \end{aligned} \tag{3.2}$$

which has the first-order condition

$$x_t = \left(\frac{(1-\beta\omega)\,\delta}{\alpha\omega} - \gamma\right) \cdot \frac{1-\delta}{\zeta} \cdot \pi_t = A\pi_t \tag{3.3}$$

Be aware that the central bank controls the exchange rate change when optimizing the objective function in (3.1), even though this variable does not appear explicitly in the optimization problem in (3.2). The objective function in (3.1) is often referred to as flexible inflation rate targeting, where $\zeta = 0$ is strict targeting, and the condition in (3.3) is often referred to as a specific targeting rule.

As already stated in the literature, there is no unique way in which a condition for optimal policy can be implemented by the central bank as an interest rate rule. It is, however, shown in Evans and Honkapohja (2003b) that a fundamentals-based rule in a DSGE model for a closed economy is not associated with stability under learning. On the other hand, an expectations-based rule gives rise to this property of the economy since it is designed for this task.

For the sake of the argument, assume that the economy is in the neighborhood of a REE. Since a fundamentals-based rule is derived under the assumption that the economy is *in* the REE, there is no mechanism that corrects agents' private expectations. On the other hand, an expectations-based rule does not assume rational expectations. Instead, the interest rate set by the central bank is directly influenced by agents' private expectations, meaning that there is now a mechanism that may correct their expectations in such a way that the economy converges to the REE.

Therefore, after combing the economy's law of motion in (2.1)–(2.3) with the condition in (3.3), and not assuming that agents have rational expectations, we have the following optimal interest rate rule[7]

[6] Firstly, replace $\Delta e_{t+1}^{e,m}$ in (2.1)–(2.3) with the expression in (2.4). Secondly, neglect from constants and variables dated at time $t+1$ in the equations. Thirdly, replace r_t in (2.1) with the expression in (2.3). Finally, solve (2.1) for Δe_t, substitute this expression into (2.2), and the constraint is derived.

[7] Firstly, replace $\Delta e_{t+1}^{e,m}$ in (2.1)–(2.2) with the expression in (2.3). Secondly, replace x_t in (2.1)–(2.2) with the optimality condition in (3.3). Thirdly, solve (2.2) for π_t, and substitute this expression into (2.1). Finally, solve (2.1) for r_t, and the rule is derived.

$$r_t = const. + \kappa_x x^e_{t+1} + \kappa_\pi \pi^e_{t+1} + \kappa_{\Delta e} \Delta e_t \tag{3.4}$$

where[8]

$$\begin{cases} \kappa_x = \frac{A\gamma(1-\delta)^2 - 1 + \delta}{A(1-\delta)(\alpha\gamma+\beta\delta)-\alpha}, \\ \kappa_\pi = \frac{A(1-\delta)(\alpha\gamma+\beta)-\alpha}{A(1-\delta)(\alpha\gamma+\beta\delta)-\alpha}, \\ \kappa_{\Delta e} = \frac{A(1-\delta)\delta}{A(1-\delta)(\alpha\gamma+\beta\delta)-\alpha}. \end{cases} \tag{3.5}$$

Notice that the interest rate set by the central bank depends on the current exchange rate change, and not the expected exchange rate change.

4 A determinate and E-stable REE?

Let us now examine under what conditions the economy is characterized by a unique REE that is least squares learnable. To determine if the complete model has a unique REE, the number of predetermined variables is compared with the number of eigenvalues of a certain matrix that are outside the unit circle (see Blanchard and Kahn, 1980). If there is a unique REE, we also know that it is E-stable since the dating of expectations is time-t in our model (see McCallum, 2007). Therefore, since E-stability is closely related to least squares learning, all determinacy regions found below are also regions for a unique REE that is least squares learnable.

The complete model in matrix form is

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot \mathbf{y}^e_{t+1} + \mathbf{\Lambda} + \mathbf{\Xi} \cdot \overline{rr}_t \tag{4.1}$$

where[9]

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} & \alpha \\ -\gamma(1-\delta) & 1 & -\delta(1-\beta\omega) & 0 \\ 0 & 0 & -\omega & 1 \\ 0 & 0 & -\kappa_{\Delta e} & 1 \end{bmatrix} \tag{4.2}$$

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & -\frac{\alpha\delta(1-\omega)}{1-\delta} & 0 \\ 0 & \beta & -\beta\delta(1-\omega) & 0 \\ 0 & 0 & 1-\omega & 0 \\ \kappa_x & \kappa_\pi & 0 & 0 \end{bmatrix} \tag{4.3}$$

and

$$\mathbf{y}_t = [x_t, \pi_t, \Delta e_t, r_t]' \tag{4.4}$$

[8] $const. = \kappa_{\Delta e}\pi^*_t - \delta\kappa_\pi \pi^{e,*}_{t+1} + \delta\kappa_\pi r^*_t + \alpha\kappa_x \overline{rr}_t$.

[9] The first row is (2.4) substituted into (2.1), the second row is (2.4) substituted into (2.2), the third row is (2.4) substituted into (2.3), and the fourth row is (3.4).

The exact form of the matrices $\mathbf{\Lambda}$ and $\mathbf{\Xi}$ does not matter when examining whether the model has a unique and learnable REE. Then, since there is one variable that is predetermined, r_t, exactly one eigenvalue of the matrix $\mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta}$ must be outside the unit circle to have a determinate REE.[10] However, if more than one eigenvalue are outside the unit circle, we have an indeterminate REE, and if all eigenvalues are inside the unit circle, there is no stable REE.

However, deriving analytical conditions for determinacy is not meaningful for practical reasons since these expressions would be too large and cumbersome to interpret. Consequently, we adopt the same strategy as in other papers within this area and illustrate our findings for determinacy using calibrated values of the structural parameters. Therefore, the following parameter values are used in the analysis that are based on quarterly data:

$$\left\{ \ \alpha = \tfrac{1}{0.157}, \qquad \beta = 0.99, \qquad \gamma = 0.024, \qquad \delta = 0.2,\ 0.4 \right. \tag{4.5}$$

See Woodford (1999) for the closed economy parameters α, β and γ. Moreover, when the index of openness of the economy is $\delta = 0.2$, the index is slightly larger than the import/GDP ratio in the US, and when the index of openness is $\delta = 0.4$, which is the parameter setting in Galí and Monacelli (2005), the index corresponds roughly to the import/GDP ratio in Canada. We also use an alternative calibration in the analysis, where the closed economy parameters in Clarida et al (2000) are used[11]

$$\left\{ \ \alpha = 1, \qquad \beta = 0.99, \qquad \gamma = 0.3, \qquad \delta = 0.2,\ 0.4 \right. \tag{4.6}$$

Thus, the differences in α and γ in the two settings are quite large, and our focus below is on the findings when the parameters in (4.5) are used since this setting is more common in the literature.

See Figure 1 for regions in the (ω, ζ)-space that are associated with a determinate and E-stable REE, an indeterminate REE and no stable REE when the openness of the economy is $\delta = 0.4$, and the Woodford (1999) parameters are used.

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[10] It is not always self-evident which variables in a model that are predetermined. However, by looking at the entries in the relevant matrix, $\mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} = \begin{bmatrix} - & - & - & 0 \\ - & - & - & 0 \\ - & - & - & 0 \\ - & - & - & 0 \end{bmatrix}$, we can conclude that r_t is predetermined, and this is because there are predetermined *relationships* between current and expected values of x_t, π_t and Δe_t (see the first three rows in the matrix), and that r_t *only* depends on the expected values of the same variables and not the expected value of itself (see the fourth row in the matrix).

[11] Clarida et al (2000) estimate different interest rate rules to evaluate Federal Reserve's policy during 1960–1996 using a DSGE model, and they found that the policy during the Volcker-Greenspan period was more successful to stabilize the economy than the policy during the pre-Volcker period. Even though their evaluation of Federal Reserve's policy is somewhat simplistic, it is very intriguing.

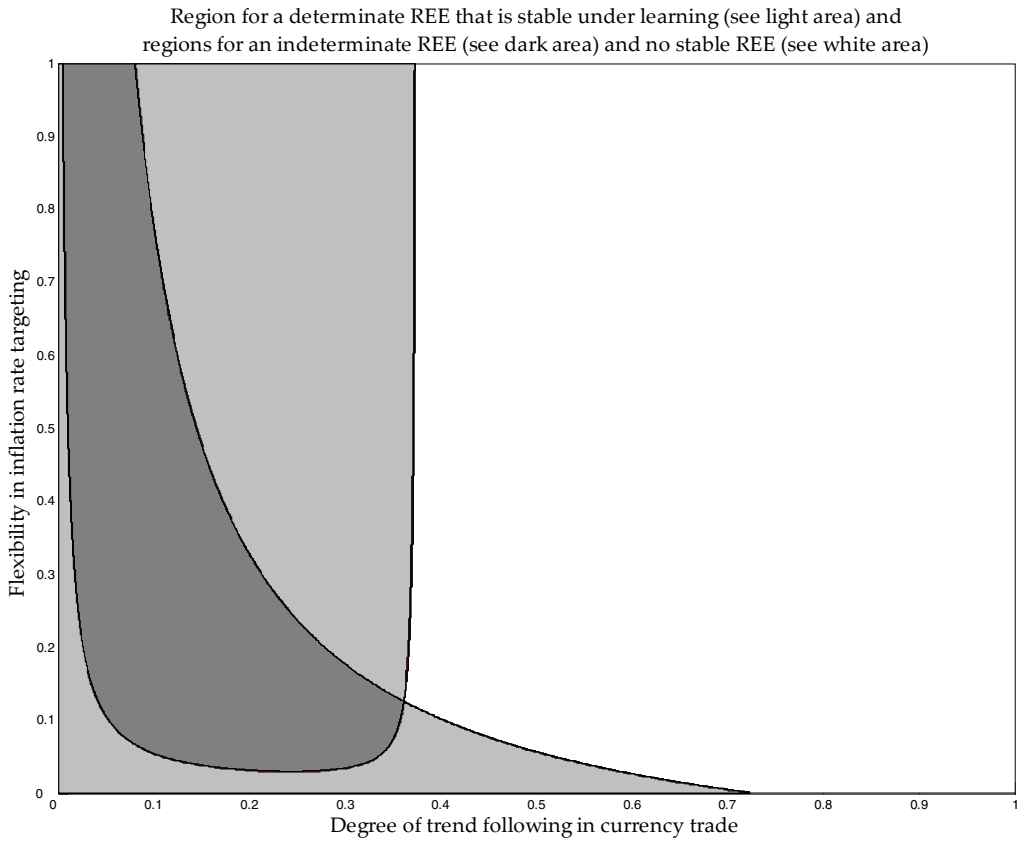

Figure 1. **The openness of the economy is 40 per cent (Woodford)**

Not surprisingly, when the degree of trend following in currency trade is large, there is no stable REE in the economy, and this is because there is no mechanism in technical trading that forces the economy to equilibrium. Moreover, when there is no trend following in currency trade, there is a unique REE that is least squares learnable, and this result is not either surprising since Evans and Honkapohja (2003b) show that a DSGE model for a closed economy has these properties when the central bank is using an expectations-based rule in optimal policy-making. Recall that the optimal policy problem in a small open economy is isomorphic to the problem in a closed economy (see Clarida et al, 2001).

When the amount of trend following in currency trade is more limited, there is either a unique or a multiplicity of REE in the economy. However, when inflation rate targeting is strict, or almost strict, there is always a determinate and E-stable REE. This is interesting since a qualified guess is that almost strict inflation rate targeting is optimal, even though we do not derive the optimal degree of targeting in this paper (since we have not established the exact relationship between the representative household's utility function and the objective function that the central bank is optimizing).[12]

In Figure 2, we show the same regions in the (ω, ζ)-space as in Figure 1, but the openness of the economy is now $\delta = 0.2$.

[12] Woodford (2003) has looked into this matter in a DSGE model and find that almost strict inflation rate targeting is optimal ($\zeta = 0.048$).

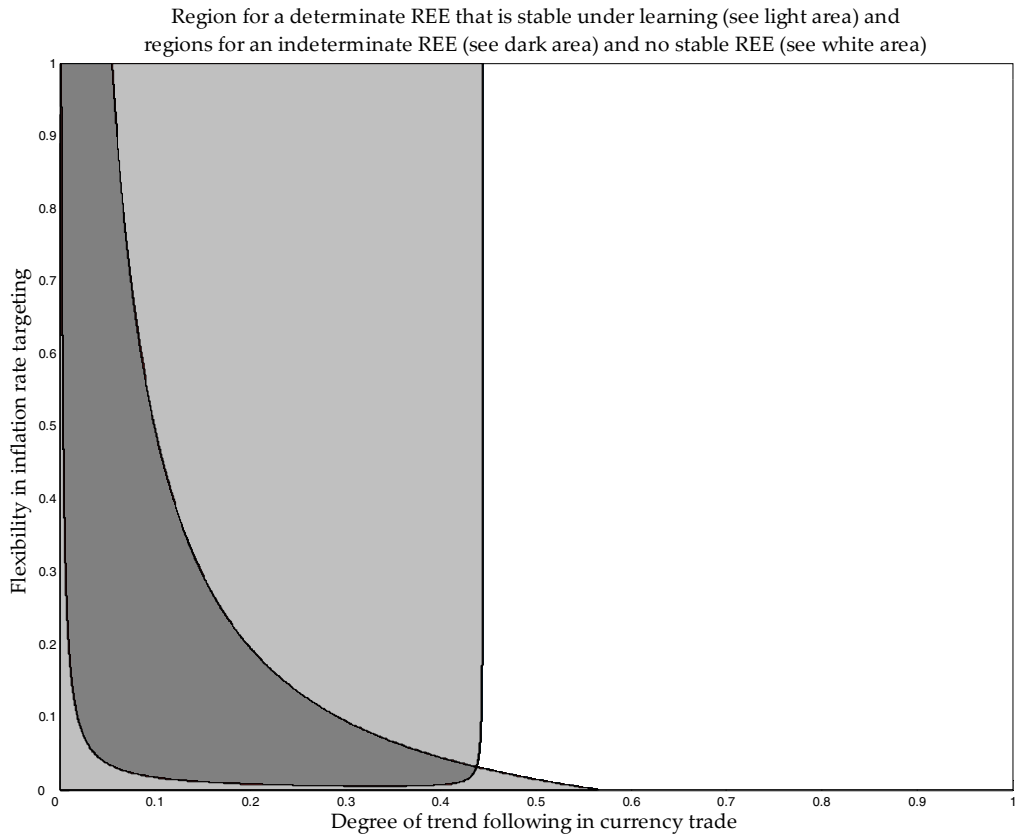

Figure 2. **The openness of the economy is 20 per cent (Woodford)**

We have the same findings as when the economy is more open, but there are, of course, quantitative differences. First, the maximum amount of trend following in currency trade to have a stable economy is somewhat larger in this case. The reason is that exchange rate movements, which may have a destabilizing effect on the economy due to technical trading, have a more limited effect when the economy is less open. Also, the region in the (ω, ζ)-space for a unique REE that is least squares learnable is larger, even though it is more likely to have an indeterminate REE when inflation rate targeting is more strict (eg, $\zeta < 0.1$). Unfortunately, it is not easy to give an economic intuition to the latter finding.

In Figures 3–4, we show the same regions in the (ω, ζ)-space as in Figures 1–2, but the Clarida et al (2000) parameters are now used.

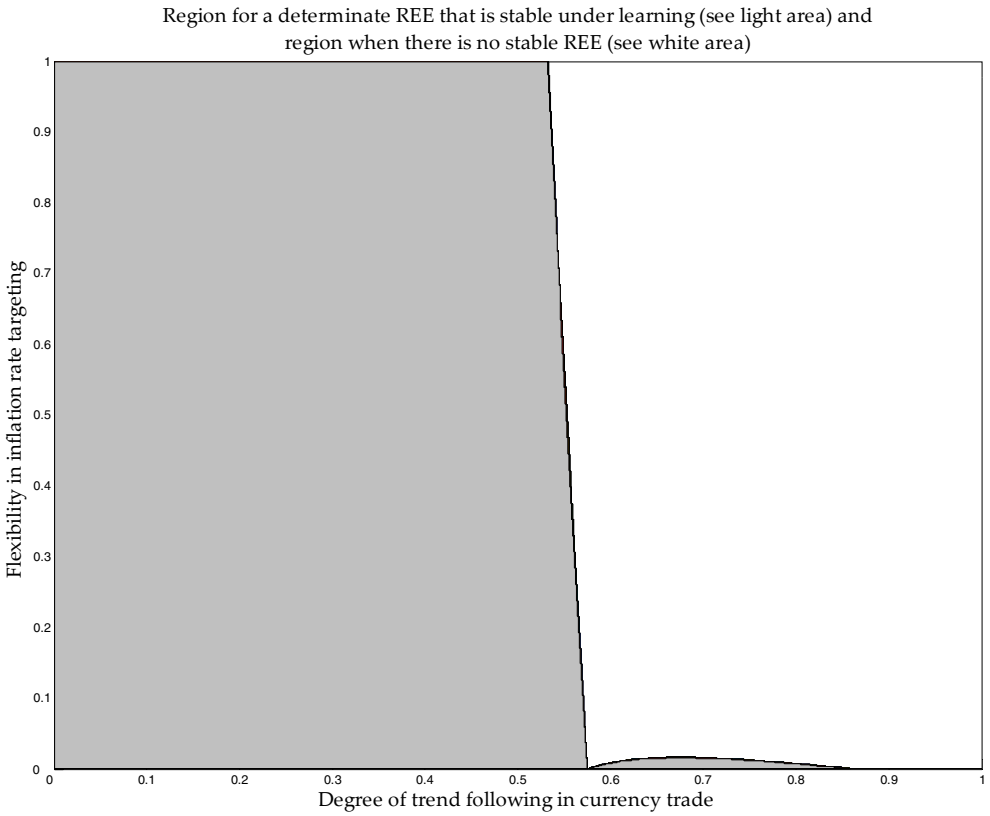

Figure 3. **The openness of the economy is 40 per cent (Clarida et al)**

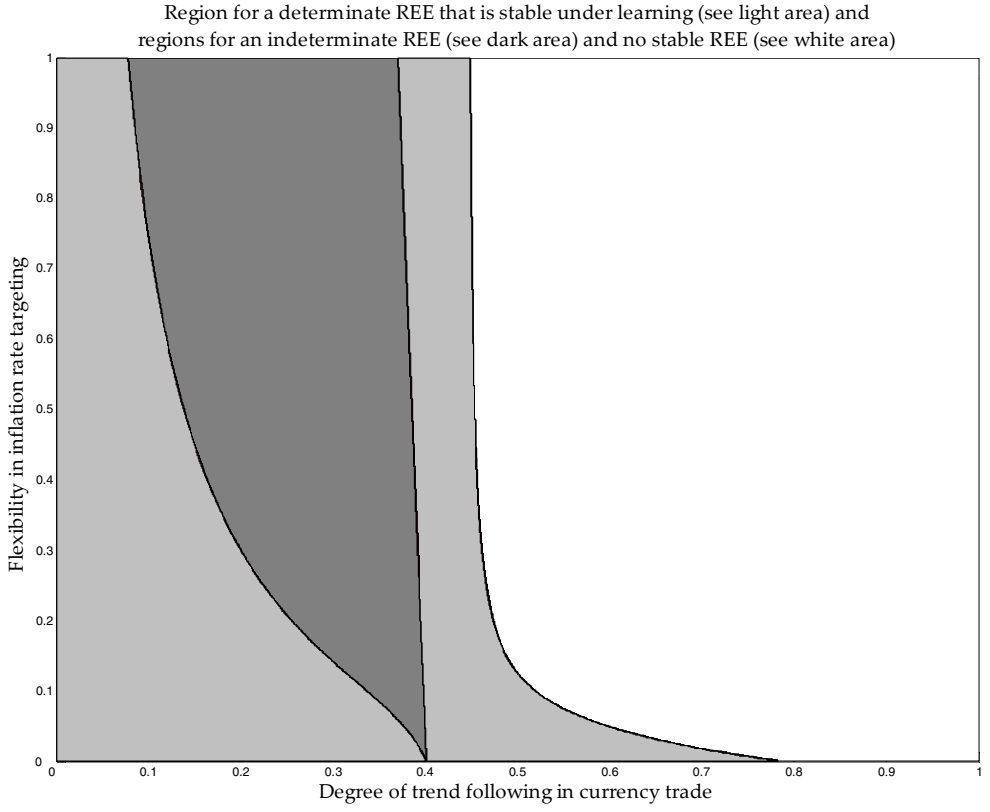

Figure 4. **The openness of the economy is 20 per cent (Clarida et al)**

Again, when the degree of trend following in currency trade is large, there is no stable REE in the economy, but the difference is now that this region in the (ω, ζ)-space is somewhat smaller. A qualitative difference between the two parameter settings, however, which applies when the openness of the economy is $\delta = 0.4$, is that there is always a unique REE in the economy when it is stable. Thus, there is no indeterminacy problem in the economy.

5 Impulse-response functions

Having established under what conditions the economy is characterized by a unique REE that is least squares learnable, we now shift focus to the computation of impulse-response functions for key variables. For this reason, we have to specify how the shock (impulse) hits the economy, and we assume that it is channeled through the natural rate of interest. Specifically, in time period $t = 1$, a shock is hitting the economy such that $\Delta\overline{rr}_1 = 1$.[13] Thereafter, when there is a unique and stable REE, the economy returns to steady state, and depending on the parameter setting in the economy, the convergence is fast or slow as well as oscillating or non-oscillating.

In Figures 5–7, impulse-response (IR) functions for the output gap, the CPI inflation rate, the nominal exchange rate change and the nominal interest rate are shown when the openness of the economy is $\delta = 0.4$, and the Woodford (1999) parameters are used.

[13] We have set $\pi_t^* = \pi_{t+1}^{e,*} = r_t^* = 0$ in all computations in this section. Also, $\overline{rr}_0 = 0$.

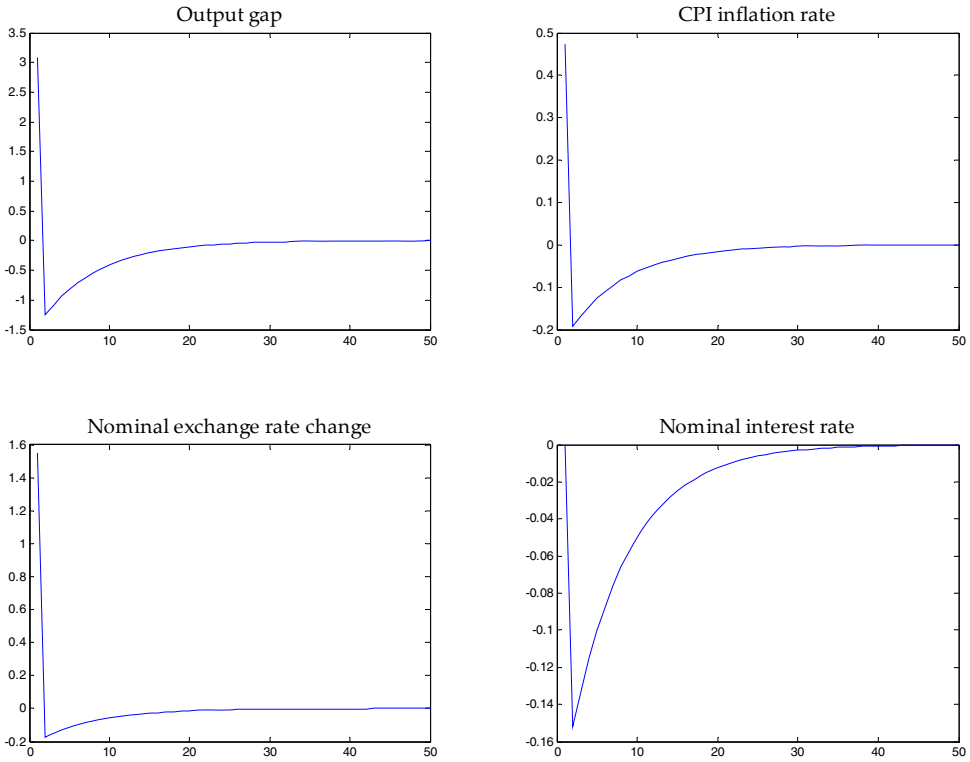

Figure 5. **IR functions when 10 per cent chartism and strict targeting (Woodford)**

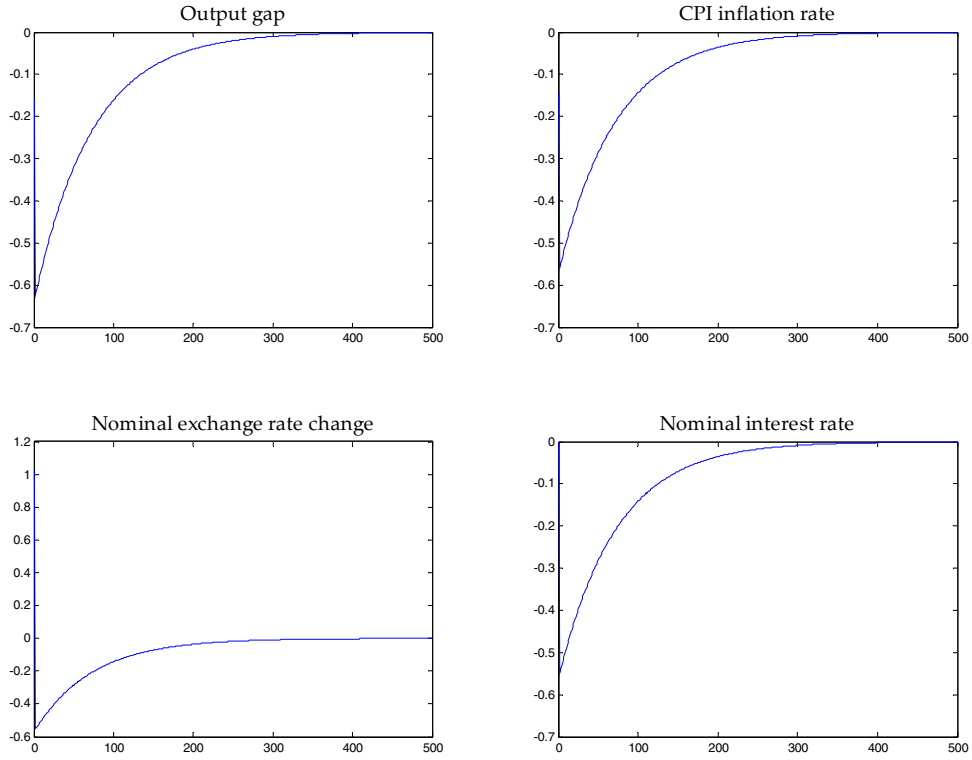

Figure 6. **IR functions when 35 per cent chartism and strict targeting (Woodford)**

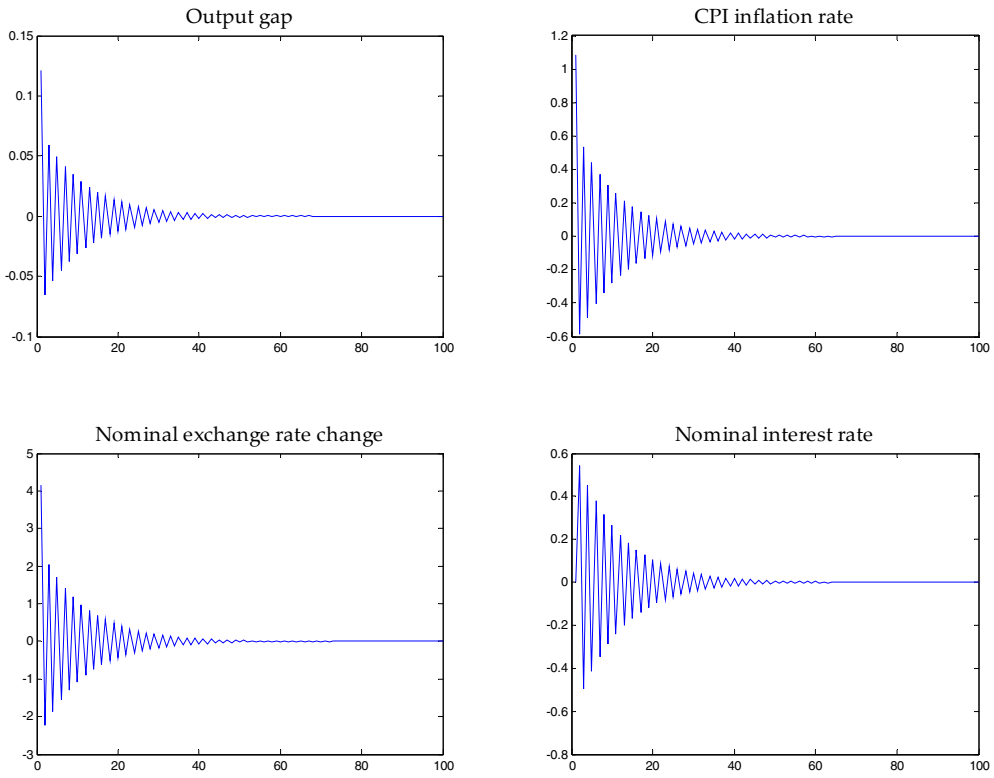

Figure 7. **IR functions when 35 per cent chartism and flexible targeting (Woodford)**

In Figure 5, there is a small amount of trend following in currency trade ($\omega = 0.1$) and inflation rate targeting is almost strict ($\zeta = 0.05$) with the effect that the economy's adjustment path to steady state is fast and non-oscillating. In Figure 6, when a larger amount of trend following is used in currency trade ($\omega = 0.35$), the adjustment path is still non-oscillating, but, on the other hand, the return of the economy to steady state is very slow. In Figure 7, when inflation rate targeting is more flexible ($\zeta = 0.5$) and the same amount of trend following is used in currency trade ($\omega = 0.35$), the adjustment path to steady state is again faster but now oscillating.

Obviously, the shock's effect on the output gap is smaller when inflation rate targeting is more flexible, and this also means that the effect on the inflation rate is allowed to be larger by the central bank. That the return of the economy to steady state is very slow in one case is not easy to give an economic intuition to, especially since the parametrizations of the optimal policy rule are not very

different in the three cases: $r_t = const. + 0.129x^e_{t+1} + 0.452\pi^e_{t+1} - 0.369\Delta e_t$, $r_t = const. + 0.098x^e_{t+1} + 0.934\pi^e_{t+1} - 0.045\Delta e_t$ and $r_t = const. + 0.095x^e_{t+1} + 0.994\pi^e_{t+1} - 0.004\Delta e_t$, respectively.[14,15] Notice that the Taylor principle is not satisfied in any of the rules (ie, $\kappa_\pi \not> 1$), even though the economy returns to steady state in each case, and that it is optimal for the central bank to 'lean with the wind' to exchange rate changes.

Impulse-response functions for the same variables are shown in Figures 8–10 when the openness of the economy is $\delta = 0.4$, but now are the Clarida et al (2000) parameters used.

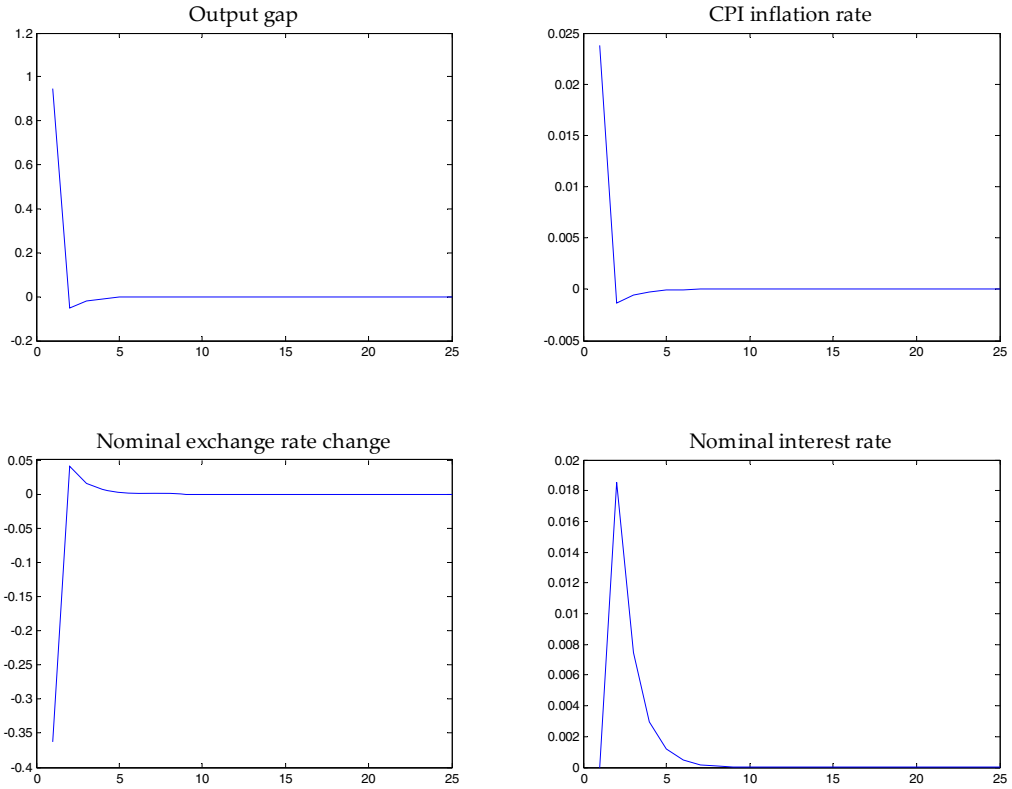

Figure 8. **IR functions when 10 per cent chartism and strict targeting (Clarida et al)**

[14] We also observe that the very slow return of the economy to steady state is even slower when the amount of trend following in currency trade approaches the limit to have a unique and stable REE. (This is not shown graphically in the paper.)

[15] That the economy oscillates back to steady state in one case is due to the fact that there are imaginary roots to the characteristic equation that describes the economy's law of motion. Thus, it is not possible to conclude that the economy must be oscillating by just looking at the parametrization of the optimal policy rule.

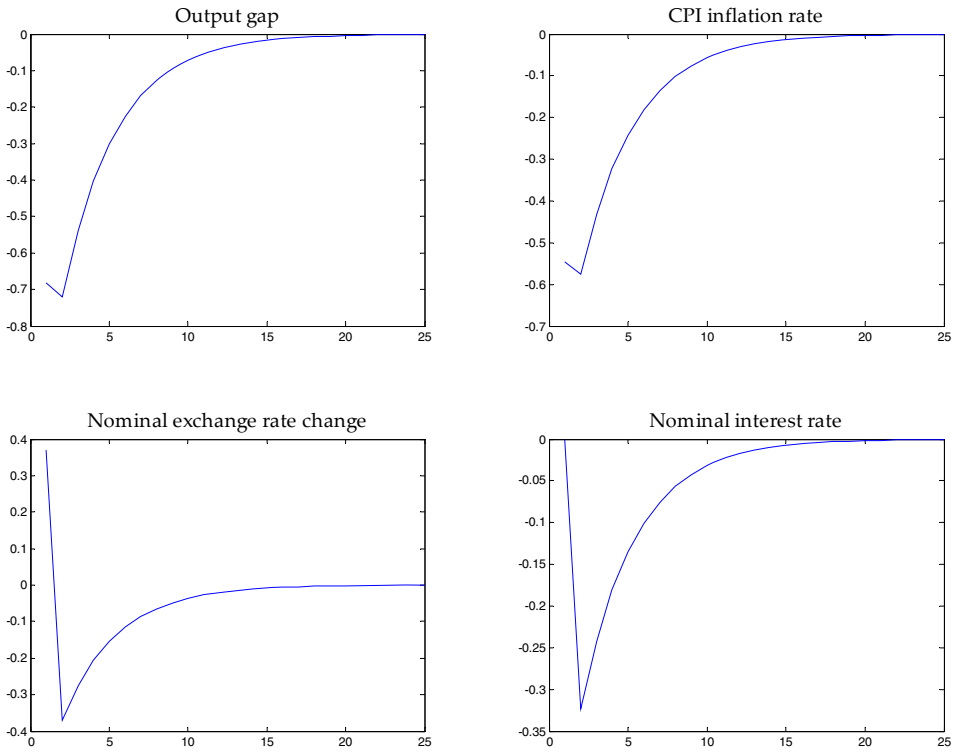

Figure 9. **IR functions when 50 per cent chartism and strict targeting (Clarida et al)**

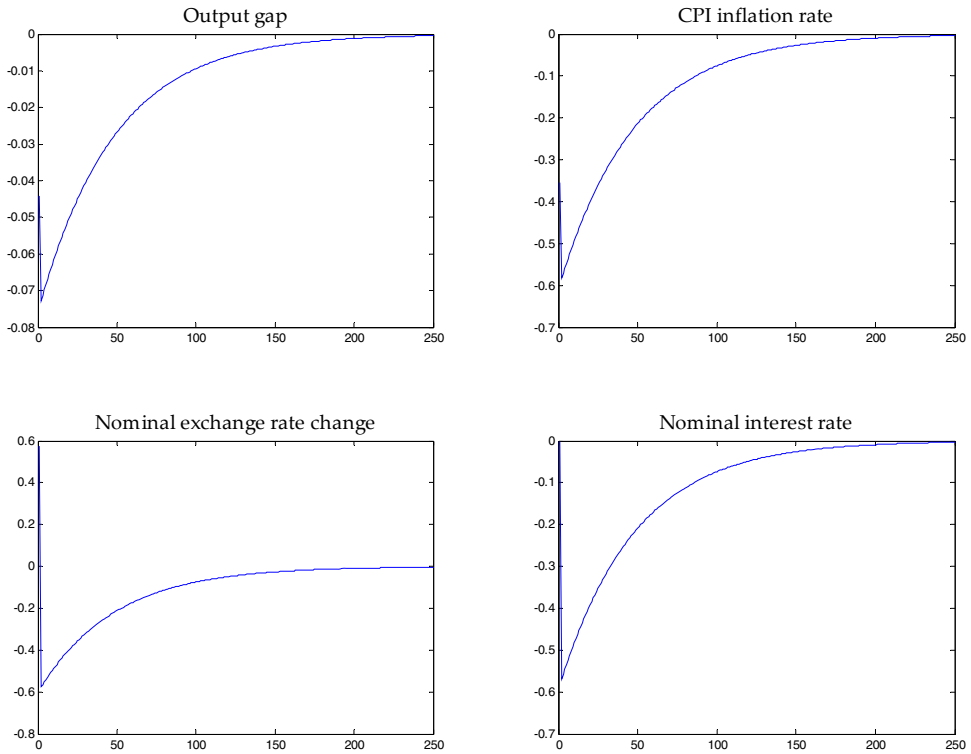

Figure 10. **IR functions when 50 per cent chartism and flexible targeting (Clarida et al)**

Roughly speaking, the assumptions behind Figures 5 and 8, Figures 6 and 9, and Figures 7 and 10 are the same, even though a somewhat larger amount of trend following in currency trade is allowed for in Figures 9–10 ($\omega = 0.5$). Again, the return of the economy to steady state is very slow in one case, but it is never oscillating in the cases examined. Also, the shock's effect on the output gap is smaller and its effect on the inflation rate is larger when inflation rate targeting is more flexible.

Turning to the parametrizations of the optimal policy rule, they are now $r_t = const. + 0.237x^e_{t+1} + 1.908\pi^e_{t+1} + 0.612\Delta e_t$, $r_t = const. + 0.972x^e_{t+1} + 0.071\pi^e_{t+1} - 0.626\Delta e_t$ and $r_t = const. + 0.619x^e_{t+1} + 0.953\pi^e_{t+1} - 0.032\Delta e_t$, respectively. Notice that the Taylor principle is satisfied in the first rule (ie, $\kappa_\pi > 1$), which corresponds to the case in which the economy's adjustment path to steady state is the fastest (of all six cases examined). Moreover, with the exception of the first rule, it is again optimal for the central bank to 'lean with the wind' to exchange rate changes. It is also worth noting that the parametrizations of the optimal policy rule are not that similar when the Clarida et al (2000) parameters are used as when the Woodford (1999) parameters were in focus.

6 Discussion

We have embedded an expectations-based optimal policy rule into Galí and Monacelli's (2005) DSGE model for a small open economy that has been augmented with trend following in currency trade to examine the prerequisites for monetary policy. We found that a unique REE that is least squares learnable often is the outcome when there is a limited amount of trend following, but that a less flexible inflation rate targeting may cause a multiplicity of REE. We also computed impulse-response functions for key variables to study how the economy returns to steady state after being hit by a shock.

Making the fractions of the two types of behavior in currency trade endogenous would, of course, be an interesting complement to this paper, and a setup similar to the one in Brock and Hommes (1997) could be used for this aim: (i) fundamental analysis is costly to use; and (ii) most traders use the trading strategy that has been more successful to predict exchange rate movements. In Bask (2007b), this setup is also used in a DSGE model in which the central bank is using a Taylor rule in policy-making with the result that chaotic dynamics and long swings may be present in the exchange rate. The implementation of an optimal policy rule in a similar setting is, therefore, part of future research.

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Appendix

Derivation of (2.1)–(2.2)

The Galí and Monacelli (2005) model can after extensive derivations be reduced to an IS curve and an AS curve

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \pi^e_{H,t+1} - \overline{rr}_t\right) \\ \pi_{H,t} = \beta\pi^e_{H,t+1} + \gamma x_t \end{cases} \tag{A.1}$$

where $\pi_{H,t}$ is the domestic inflation rate. However, (A.1) is not in an appropriate form since there are no expected exchange rate terms in the equations, which is necessary when incorporating different behaviors in currency trade. Fortunately, it is possible to use the following equations, which are derived in Galí and Monacelli (2005), to rewrite (A.1) into a suitable form

$$\begin{cases} \pi_t = \pi_{H,t} + \delta\Delta s_t \\ s_t = e_t + p^*_t - p_{H,t} \end{cases} \tag{A.2}$$

where s_t is the terms of trade, p^*_t is the index of foreign goods prices, and $p_{H,t}$ is the index of domestic goods prices. Firstly, shift the first equation in (A.2) one time period forward in time and rearrange terms

$$\pi^e_{H,t+1} = \pi^e_{t+1} - \delta\Delta s^e_{t+1} \tag{A.3}$$

Secondly, shift the second equation in (A.2) one time period forward in time and take differences

$$\Delta s^e_{t+1} = \Delta e^{e,m}_{t+1} + \Delta p^{e,*}_{t+1} - \Delta p^e_{H,t+1} = \Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1} - \pi^e_{H,t+1} \tag{A.4}$$

Thirdly, substitute (A.4) into (A.3), and solve for $\pi^e_{H,t+1}$

$$\pi^e_{H,t+1} = \frac{1}{1-\delta} \cdot \left(\pi^e_{t+1} - \delta\left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right) \tag{A.5}$$

Fourthly, shift (A.5) one time period backward in time

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi_t - \delta\left(\Delta e_t + \pi^*_t\right)\right) \tag{A.6}$$

Fifthly, substitute (A.5) into the first equation in (A.1), and (2.1) is derived. Finally, substitute (A.5)–(A.6) into the second equation in (A.1), solve for π_t, and (2.2) is derived.

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